Unit 5 *Growth and Decay* Introduction

Learning objectives

In this unit we focus on practical problems that illustrate growth and decay, introducing the exponential function, which has widespread applications. After completing this unit you should

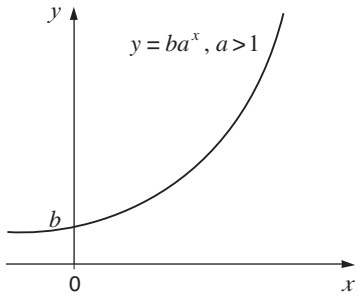
- appreciate the characteristics of exponential functions of the form $y = a^x$
- understand how to model population growth
- be able to use the methodology of Carbon Dating to date artifacts
- understand the concept and characteristics of the exponential function $y = e^x$
- appreciate that $\ln x$ is the inverse of e^x
- be able to solve exponential equations
- understand the properties of logarithms
- be able to undertake currency conversions, including commission rates.

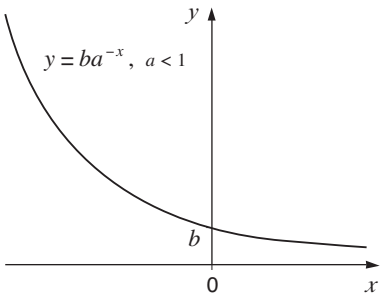
Introduction

In this unit we use and apply exponential (or power) functions of the $y = a^x$, where a is a positive constant. We also determine the value of a so that $\frac{d}{dx}\left(a^x\right) = a^x$; this is the number e, found on scientific calculators and used in many applications, including Carbon Dating.

Facts to remember

- The graphs of the exponential (power) function, $y = ba^x$, for constants a and b, is given by (for $b > 0$):





- If $y = e^x$, $\frac{dy}{dx} = e^x$
- If $e^x = a$, then $x = \ln a$
- $y = e^x$ and $y = \ln x$ are inverse functions

Unit 5 *Growth and Decay* Introduction

- Properties of logarithms:

$$\ln\left(a^x\right) = x\ln a$$

$$\ln(ab) = \ln a + \ln b$$

$$\ln\left(\frac{a}{b}\right) = \ln a - \ln b$$

Glossary of terms

- *Exponential (Power) Functions:* are of the form $y = a^x$ where a is called the base and x the exponent
- *The Exponential Function*: a special power function, $y = e^x$, where the number e is such that $\frac{dy}{dx} = e^x$, i.e. the gradient of the function is the function
- *Population Model*: the function of time t that represents the growth (or decay) of a population