Promoting Quantitative Literacy in an Online College Algebra Course

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Abstract
College algebra (a university freshman level algebra course) fulfills the quantitative literacy requirement of many college’s general education programs and is a terminal course for most who take it. An online problem-based learning environment provides a unique means of engaging students in quantitative discussions and research. This article reports on a US-based study that examined a traditional, lecture-based college algebra section versus an online, quantitative literacy focused section; the latter included weekly news discussions as well as problem-based learning projects requiring data analysis. Students in each section responded to survey questions at the beginning and end of the semester. Qualitative analysis of pre- and post-survey responses revealed differences in students’ mathematical disposition, attitude, and outlook on what utility mathematics has to offer, with the online section having the favorable outcome of each. Albeit modest, the results suggest that project-based learning in an online environment is a promising strategy for fostering the affective component of quantitative literacy in college algebra. More research is needed to capture the mechanisms through which such growth occurs.

Key words: quantitative literacy, college algebra, online courses, general education
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Introduction
Conveying the importance of mathematics in the lives of college students who have traditionally eschewed mathematics is a nontrivial but important endeavor. As quantitative literacy impacts nearly every aspect of students’ academic, professional, and personal lives, universities must rise to the challenge of finding strategies to bolster students’ quantitative literacy. Since college algebra (a university freshman level algebra course) is a course (terminal or pass-through) which many students experience, it is imperative that this course assists in student development of quantitative literacy.

Online courses have grown tremendously over the past two decades (Allen & Seaman, 2014). From a pedagogical standpoint, they facilitate student discussion, allow for choice in assignments, and foster students’ research and writing skills (Benson, 2003; Chinnappan, 2006; Larreamendy-Joerns & Leinhardt, 2006; Meyer, 2004; Perera-Diltz & Moe, 2014; Wegerif, 1998). This cadre of benefits aligns with those of a problem-based learning environment (Powers & Dallas, 2006), thus motivating a study on the effect of both in tandem.

This paper presents a qualitative analysis on the efficacy of problem-based learning (PBL) in an online environment designed to promote quantitative literacy for college algebra students. This US-based study compares the affective and communicative growth in quantitative literacy among students in a traditional college algebra class to those in an online college algebra class that employs additional PBL while covering the same content in college algebra. Two intertwined variables (face-to-face versus online instruction and conventional college algebra content versus similar content with the addition of some PBL) are investigated; no attempt is made to disaggregate the findings among these variables and the online course with the addition of some PBL is considered in the whole. This study presents promising results and provides suggestions for future directions in research.

Background and Framework for Quantitative Literacy
In 2004, the MAA formed a special interest group on quantitative literacy (SIGMAA) and has since released significant publications echoing the call for increased attention to quantitative literacy at both the secondary and postsecondary levels. The group states the following in its charter:

Quantitative literacy can be described as the ability to adequately use elementary mathematical tools to interpret and manipulate quantitative data and ideas that arise in an individual’s private, civic, and work life. Like reading and writing literacy, quantitative literacy is a habit of mind that is best formed by exposure in many contexts.

While communication and mathematical affect are not explicit in the SIGMAA definition, they are equally important in relation to mathematical ability. Wilkins’ (2000, 2010) multifaceted quantitative literacy construct encompasses ability, self-efficacy, and one’s beliefs about the utility of mathematics. Moreover, Lutsky (2008), Hallet (2003), and Wiggins (2003) argue for the importance of writing and argumentation in defining their view of quantitative literacy. Channeling these elements into one, this study views quantitative literacy as that described in the charter along with the elements of communication and mathematical disposition;
as discussed later, these latter two components comprise the lens through which the qualitative data is coded. With this working definition in hand, it is instructive to delineate its emergence over the past decades.

Whilst the underpinnings of the quantitative literacy movement developed in the late twentieth century, the genesis of its urgent promotion is found in the seminal text *Mathematics and Democracy* (2001), published by the National Council on Education and the Disciplines under the leadership of former MAA president Lynn Steen. Steen’s message was simple. Every day, students are inundated with numbers in relation to healthcare, crime, nutrition, the economy, politics, and many other issues; their ability to accurately interpret these quantitative arguments—and even produce their own—is not only important for their welfare, but also requisite for their ability to contribute to the nation’s democracy. Contemporaneously, the National Numeracy Network (NNN) formed with a mission of promoting quantitative literacy at all levels (Madison & Steen, 2008); it has held annual meetings since 2004, and biannually publishes a journal entitled *Numeracy*. As Earl, Watson, & Torrance (2002) and Liu, Aubrey, Pan, Godfrey, & Aunio (2008) make clear, the recognized importance of quantitative literacy transcends borders and is not solely an issue for the US issue.

**Motivation for Quantitative Literacy**

As seen in the immediately following paragraphs, despite historic attempts to promote numeracy, significant reasons yet remain connoting its importance, including the link between quantitative literacy ability and one’s overall well-being, the dubious status of first-year postsecondary math courses, and a growing movement for accountability in general education outcomes. These components are briefly addressed here.

Socioeconomic status is arguably a major contributor to human well-being, and it has been found that mathematical achievement is connected to both wage increases and likelihood of fulltime employment (Eide & Grogger, 1995; Levy, Murnane, & Willett, 1995; Rivera-Batiz, 1992). Quantitative literacy is linked with better decision-making (Jasper, Bhattacharya, Levin, Jones, & Bossard, 2013; Dickert et al., 2006), nutrition label understanding (Rothman et al., 2006), and even risk comprehension in healthcare (Fagerlin, Ubel, Smith, & Zikmund-Fisher, 2007; Lipkus & Peters, 2009). Altogether, quantitative literacy is linked with positive life outcomes.

Due to the impact of quantitative literacy on one’s long-term well-being, Steen (2001) argues that educators should strive to develop it amongst their students. However, Madison (2003) argues that the geometry, algebra, trigonometry, and calculus (GATC) sequence most commonly followed in US secondary education does not foster a quantitatively literate citizenry, as it sifts “through millions of students to produce thousands of mathematicians, scientists, and engineers,” producing a small proportion of students who actually pursue such careers (p. 154). It is important to note that mathematical and quantitative literacy are not equivalent. For instance, Steen (2001) articulates that, while mathematical literacy provides a firm foundation for quantitative literacy, it does not directly effect it, and it would be misleading to claim there is a simple relationship between the two. As an example, one may be able to solve a quadratic equation, but have no idea of how to interpret a percentage reported in a news article; such a student exhibits mathematical literacy but not quantitative literacy. Conversely, another individual may be able to interpret the reported percentage and discuss the author’s argument, but not solve the quadratic equation.
Hallett (2003) suggests that it is incumbent upon secondary and college faculty to follow through on foundations established in early secondary school and foster quantitative literacy as students grow in their development in communication and critical thinking. Accordingly, universities must carefully examine the courses they designate to fulfill general education requirements for quantitative literacy. It can no longer be assumed that a student having experienced the GATC sequence will be quantitatively literate. If a field of study does not require that a student take a course past college algebra (e.g., music, anthropology), and college algebra fulfills their general education requirement, the college algebra course should demonstrate some quantitative literacy benefit aside from preparing a student for future math courses (Kennedy, 2001). Kennedy (2001) notes that university mathematics departments must: recognize that content alone does not necessarily produce quantitative literacy; accept at least a partial responsibility for producing both mathematicians and numerate individuals in all disciplines; and be held accountable for the quantitative literacy (or lack thereof) of all students who earn a degree. A reason for calling attention to this responsibility is that college algebra—a frequently taken terminal course—has gained notoriety for its complicity in failing to foster quantitative literacy (Steen, 2004). In addition to high withdraw and failure rates, Small (2006) notes that students often leave the course with negative attitudes towards the discipline of mathematics. Steen (2004) describes the course’s lack of relevancy to students:

Focused entirely on a wide range of relatively specialized algebraic techniques that students rarely remember beyond the final exam, college algebra neither prepares students well for courses in other quantitative disciplines nor their civic employment, or personal needs. (p. 38)

To increase quantitative literacy in college algebra, Small (2006) suggests that courses can be significantly improved by: replacing lectures with a considerable number of small-group activities and projects; focusing on real-world, ill-defined modeling rather than traditional word problems; containing a strong focus on communication; and employing more alternative assessments and fewer traditional assessment. Unfortunately, survey data suggests that college algebra at most US universities does not fulfill his recommendations.

The importance of investigating college algebra is due to the very large number of students enrolled in a great number of sections at universities both nationally and internationally. The American Mathematical Society (AMS) reports that 29% of all non-remedial intro-level collegiate math enrollments are in college algebra (Blair, Kirkman, & Maxwell, 2010). As its enrollments exceed that of precalculus, statistics, and other liberal arts math courses, it is vital that the course have worth for students. The AMS survey reveals that, of the undergraduate programs surveyed, only 16% of college algebra sections required writing assignments, and 65% used a “traditional” lecture-based approach assessed through tests and quizzes, demonstrating no significant change in content and delivery methods since 1990. Additionally, math for the liberal arts courses—designed as an alternative to precalculus or college algebra and meant to fulfill quantitative literacy requirements—are not known for developing quantitative literacy among students (Ganter, 2012). Ganter (2012) notes that most often these courses cover a broad survey of math topics (or simply those topics that faculty desire to teach) and are rarely designed to develop student ability in quantitative reasoning. Richardson and McCallum (2003) use an apt analogy, suggesting that such courses teach one to appreciate a work of art (mathematics), rather
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than produce the art for oneself. In sum, it is clear that many required college math courses, including college algebra, do not fulfill their quantitative literacy promise.

Fostering Quantitative Literacy in a Problem-based, Online Learning Environment

As college algebra will undeniably remain prominent in undergraduate math programs, it is essential to make every effort to redesign it in a form that will fulfill its quantitative literacy designation. Since an online environment possesses many qualities conducive to the task (discussed later), a promising approach may be to situate college algebra online with a problem-based learning (PBL) structure. Savery (2006) broadly characterizes PBL as “an instructional (and curricular) learner-centered approach that empowers learners to conduct research, integrate theory and practice, and apply knowledge and skills to develop a viable solution to a defined problem” (p. 9). The PBL method of teaching and learning is commensurate with Small’s quantitative literacy recommendations and those from others. Indeed, Ganter (2012) states, “Teaching methods for quantitative literacy courses are not lecture and listen, but they may involve group work, projects, writing, and many of the approaches advocated by those in the calculus reform movement” (p. 8). Additionally, Packer (2003) notes:

> The short answer is that typical tests are mere proxies for real performance. They amount to sideline drills as opposed to playing the game on the field. Assessment of quantitative literacy requires challenges that are essentially not well structured or even well defined; problems that are, well, problematic. (p. 127)

Through a meta-analysis of face-to-face PBL research, Strobel and Van Barnevald (2009) found that PBL exceeded the traditional lecture approach in respect to long-term content retention, skill development, and satisfaction of students and teachers.

While putting college algebra online may seem somewhat strange, a digital learning environment actually possesses many qualities conducive to the task. In light of the fact that 33.5% of students in higher-education institutions enrolled in at least one online course in 2012 (up from 18% in 2005), the availability of online courses is becoming an expectation among college students (Allen & Seaman, 2014); common factors for making courses available online include students’ demand for flexible schedules, making more courses available, and increasing student enrollment (Jaggars 2012; Parsad & Lewis, 2008). Research has also found that PBL works well in online environments (Powers & Dallas, 2006), as evidenced by significant increases in measures of students’ critical-thinking skills (Cheaney & Ingebritsen, 2005; Sendag & Odabasi, 2009). Cheaney and Ingebritsen (2005) divided an online biotechnology course into two sections for a five-week unit, one of which was centered around PBL and the other around video lectures and traditional exams; through pre- and post-unit exams, researchers found that PBL students scored slightly lower on the factual portion of the exam, but evinced higher-order thinking ability. Sendag and Odabasi (2009) performed a similar study in a semester long undergraduate computer science course split into PBL and traditional sections; using similar measures of factual knowledge and critical thinking skills, they found no significant differences in content acquisition, but significant increases in critical thinking skills among the PBL students. Moving beyond PBL to the features of online learning, Blackmon (2012) reports that online discussion forums—when used to support big ideas in a class and not for the sake of busywork—are powerful tools to increase student interaction and achievement. Additionally, online frameworks permit student collaboration (whether through discussion forums or wikis),
reflection time during communication, as well as increased student writing (Benson, 2003; Chinnappan, 2006; Larreamendy-Joerns & Leinhardt, 2006; Meyer, 2004; Perera-Diltz & Moe, 2014; Wegerif, 1998). Summarily, well designed PBL and well designed online instruction share many commonalities.

Interestingly, while some meta-analyses have found that online learning is no different from, or superior to, face-to-face instruction (Bernard et al., 2004; Caldwell, 2006; Means, Toyoma, Murphy, Bakia, & Jones, 2009; Russell, 2001; Zhao, Lei, Yan, Lai, & Tan, 2005), others have found the opposite (Figlio, Rush, & Lu, 2010; Rovai, 2002; Tanyel & Griffin, 2014). These studies point to the notion that high-quality instruction is what leads to student learning, regardless of course delivery mode (Oncu & Cakir, 2011; Xu & Jaggars, 2011). Accordingly, those designing online courses should focus on developing instructional materials that utilize all the benefits of the online environment (Rovai, 2000). Relating these findings to the present study, one sees that an online PBL environment has the potential to foster critical thinking among students, and—assuming it is designed well—will likely do no harm with respect to student achievement in college algebra. Moreover, in line with the study’s framework for quantitative literacy, such has the capacity to increase students’ mathematical communication abilities.

Methodology

Participants
Study participants included a convenience sample of 57 students enrolled in one of two different college algebra sections at a midsized southeastern university in the US. Twenty-eight of these students took the online section of the course, while the rest took the course in a traditional face-to-face lecture format (i.e., instruction via lecture; students complete homework outside of class; in-class exams and quizzes; little or no group-work or projects). While students independently enrolled in a particular course delivery format, the demographics of the students in the two classes were very similar in a number of ways including: distributions of race and age; mathematical backgrounds; incoming mathematical abilities and attitudes; and the percentages of students who were taking the class as a terminal course and those taking following mathematics courses. Notwithstanding the similar demographics among the two groups of students, the researchers concede that it is unknown whether there were specific factors that affected students’ choice.

The two sections used the same textbook and covered almost identically the same chapters from within the text. Therefore, from the context of mathematical content, these two courses were very similar. The face-to-face course was traditional and lecture-based, tests comprised the majority of students’ grades, and no specific additional focus was placed on quantitative literacy – understanding that the course as designed was intended to meet university quantitative literacy requirements. While the face-to-face course covered the same mathematical material as the online course, the latter necessarily placed somewhat less emphasis on a few mathematical topics in order to permit time for student discussions on real-world topics and readings. The mathematical topics sacrificed included some coverage of quadratic equations and the conceptual notion of a derivative. As a whole, students in the online course spent slightly more time on writing and discussing applications of the math content, and slightly less on computational homework problems.

Online Course Configuration
The structure of the online course included traditional college algebra content quizzes, short Geogebra assignments, weekly discussion forums, and four major projects. There were no tests and the discussions and data-driven projects comprised the majority of students’ grades. Many of the projects required that the students examine news articles or data sets involving: analyzing the population growth of several countries through the lens of concavity; modeling data from the college through linear regression; and analyzing news articles about the Ebola outbreak through exponential growth. For the final assignment, students created a website to model data sets of their choice from Gapminder.org. All of the assignments were contextualized in some way and required students to communicate arguments or opinions using mathematics. Three of the news articles that students analyzed were taken from *Case Studies for Quantitative Reasoning: A Casebook of Media Articles*. Two quantitative literacy experts, Madison and Dingman (2010), spent several years crafting the text for a course in quantitative reasoning. Students analyzed these articles during weeks when the article’s content was pertinent to the course. As a whole, the course was designed so that, in addition to learning the content of college algebra, students would develop skills in mathematical communication, modeling, online research, and data analysis in Microsoft Excel. Altogether, the online course had two characteristics distinguishing it from the lecture-based section: the course was offered online rather than face-to-face and the content instruction included the experience of students performing four project-based learning activities.

**Task and Procedure**

Remaining consistent with the study’s framework regarding quantitative literacy, students would be deemed successful in quantitative literacy growth if they improved their communication skills in general (not singularly associated with communication within the mathematical content), mathematical disposition, and ability to solve contextualized problems using elementary mathematics. This study focused on the first two of these dimensions through an open-ended survey question posed at both the beginning and end of the semester. (The third dimension is discussed in (Tunstall & Bossé, 2015).) Face-to-face students answered these questions in a computer lab with approximately twenty minutes to complete them; this is comparable to the maximum amount of time students in the online course likely would have spent on them. The prompt was the following: *It has been said that “The world is awash with numbers.” Do you use math in your daily life, or do you avoid doing calculations?* Answers to this question had the potential to address these first two dimensions. Also used as data were comments from the online course’s discussion forum. Because the online forum was clearly not a part of the face-to-face course, comparisons between the online and face-to-face courses could not be made and were used only as supplementary evidence.

**Analysis**

Analysis of the open-ended survey question proceeded in four stages. After compiling the data, two researchers analyzed the pre- and post-answers of both classes. The researchers agreed beforehand to look for themes within in the post-course answers for the coding. Each researcher then developed a set of themed codes perceived to have a significant presence in the data (Bogdan & Biklen, 2007; Creswell, 2009; Strauss & Corbin, 1998). Following this initial analysis, the researchers compared the set of codes to develop a unifying collection. With this in hand, one or more codes were assigned to each student’s response. The final stage consisted of meeting to ensure there was agreement on coding assignment. In addition, comments from the
online discussion forum were used when such provided additional insight into either a theme from the codes or student responses.

**Research Question**
This study investigated whether the students in the PBL online course with the alternative assessment measures would have greater gains in the two elements of quantitative literacy: mathematical communication and disposition. Here the former element is one’s ability to use quantities in communication or arguments, and the latter is one’s inclination to do so. Notably, this study investigates students’ quantitative literacy and not their understanding of the mathematical content associated with college algebra.

**Limitations and Delimitations**
As referenced above, a number of variables differentiated the two classes involved in this study. The face-to-face course was instructed and assessed through very traditional means. The online course introduced four additional project based learning (PBL) activities, writing critiques of a small number of articles, and being assessed through alternative assessments rather than through traditional exams. Altogether, these variables set the stage to confound each other. Disaggregating the data to speak to one variable at a time would be impossible. Thus, this study could not speak to any one dimension of the online, PBL, alternative assessment course; all components were considered simultaneously.

**Results**
As discussed above, the initial plan was to code the textual data using constructs that emerge from the texts. However, an initial glance at students’ responses revealed an unanticipated result that connotes a blatant distinction between the two groups of students: the difference in pre- and post-test word-count on student open-ended responses was significant. The average pre-response word count for the face-to-face group was roughly 19 words with a post-response average of 18 words. On the other hand, the pre-response average for the online group was 20 words with a post-response count of 71 words. Notably, this significant distinction between the word-count differences is not a function of the face-to-face versus online class structures, since both groups had the same instructions, both took the exam on a computer, and both had sufficient time to answer the writing prompts. It is more probable that this word-count distinction was due to students in the PBL online course had developed significantly more comfort discussing mathematics.

Analysis revealed a number of common conceptual structures in the data. These conceptual structures became the codes through which the data was further analyzed. The coding is roughly structured in ascending order from a negative disposition toward numbers and mathematics to a strongly positive disposition. These codes are listed below. Because all textual data was contextualized in student comments, multiple codes were applicable for any statement, depending on the text and ideas surrounding the coded text.

0. The student dislikes numbers and sees no use for them in day-to-day life (e.g., “I use very little math in my day-to-day life. I use a calculator on the rare occasion I need it.”).

1. Students say “yes, I use numbers” without articulating how or why (e.g., “I use math at least once a day.”).
2. The student notes they would use math more if they were better at it (e.g., “I enjoy using math when I can, but I wish I knew how to apply more complicated algebra to daily life.”).
3. The student states they do not see the need for any math beyond basic arithmetic in their life (e.g., “Yes I do use numbers every day, but not algebra or anything like that.”).
4. The student articulates their use of basic arithmetic (e.g., “In the summer I work as a waitress so I use math all the time in calculating how much money I will make, how much change to give customers, etc.”).
5. The student says we all use math every day, even if we don’t realize we’re doing so (e.g., “After this course...I believe we all do math in our heads even if we don’t realize it. Someone may say that they don’t use math or numbers everyday, but they probably do.”).
6. The student states that math surrounds us, even if we’re not the ones using it (e.g., “Now I recognize that I not only use math more than I used to, but also that other people are using it all the time.”).
7. Students have immediate retrieval of various new math applications they’ve learned over the semester (e.g., “I have seen in class projects how we can use math to predict population growth, and disease outbreak.”).
8. The student has become less fearful of math, seeing no bound to the level of math they might learn or use if they are interested or if it is required by their career (e.g., Now that I see math has more use than I thought, I imagine I will end up using it doing research in psychology or sociology.”).

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<th>3</th>
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<td>F2F Tally (n = 29 students)</td>
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<td>20</td>
<td>8</td>
<td>8</td>
<td>5</td>
<td>1</td>
<td>2</td>
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</table>

Table 1. Tally distribution of themes

Figure 1. Double bar graph of code distribution

Table 1 and Figure 1 depict the numeric count of the codes in the textual data compared between the face-to-face and the PBL online students. A clear divide exists between the tally
distributions of each class, with the lecture-based course having the majority of students answer in a fashion similar to “Yes I use numbers everyday.” Moreover, four of the students in the face-to-face course ended the semester with a fear or disgust for math, while only one student in the online course was coded as having the most negative disposition. Eight of the face-to-face students felt they would use math more if they knew how to use it in ways relevant to their lives and another eight felt that arithmetic was the only type of math they felt necessary in their lives and careers. Depictive quotes from some face-to-face students include:

- I use math and numbers in my daily life. If I knew how to use them correctly I would use them more.
- I avoid them because I just don't see it as necessary in my day to day life.
- I believe that simple math maybe enough.

The attitudes that manifest in the online students’ responses are markedly different than those from the face-to-face course. Many of the online students were able to articulate how they used numbers in their daily lives; note this is different from simply stating they regularly use numbers. (This distinction is later explored.) Some students also felt that math was ubiquitous in their lives and that they became more self-conscious of its presence. Additionally, seven students discussed specific real-world topics investigated in the course as evidence for the importance of numbers and seven also elucidated how they became more confident with numbers over the course of the semester. Examples of student response include:

- In my initial post on this subject I said that I felt that our world is indeed awash with numbers; they surround us in every aspect of our lives. After taking this class I feel that to still be true, and maybe hold an even stronger belief in that. I have seen in class projects how we can use math to predict population growth, and disease outbreak…Math is pretty handy!
- I believe everyone uses numbers or math in their daily life. I know I use math commonly when leaving a tip or calculating sales percentages to see how much something costs. Numbers are everywhere, and I think we sometimes use them without even thinking about it.

Notably, since the demographics of the students in both the face-to-face and the online course are similar and the face-to-face course was offered in a traditional manner, it appears that the distinctions in the quantitative literacy results between these two groups are due to the nature and characteristics of the online course. While it cannot be precisely determined whether the positive results of the online groups is due to the course being offered online or that there were additional PBL experiences in the course, the differences in results between these two groups are notable. While the findings can be concisely simplified to the face-to-face class demonstrating lower quantitative literacy and the online class exhibiting higher quantitative literacy, the data speaks to more salient notions, which are examined in the following discussion section.

Discussions
The following discussions are separated into two sections: preliminary discussions and extended discussions. The preliminary discussions were born from dimensions in the student responses that were anticipated by the researchers as they planned the open-response survey questions. Extended discussions evolved from unanticipated dimensions discovered in student responses, comments made by faculty members and early readers of this paper that highlight some ideas tangential to these findings and discussions, and additional connections to the literature. Altogether, the preliminary and extended discussions provide much more extensive analysis of the data and results of this study.

**Preliminary Discussions**

It is interesting to note that 20 of the 29 participants in the face-to-face group responded in the post-test that they use numbers every day. Recognizing that a significant component within quantitative literacy resides in a person seeing that she lives in a world of numbers and mathematics, it may initially seem that this result speaks to a high level of quantitative literacy among this group. Indeed, this is far from the case. First, according to the coding hierarchy, claiming a daily use of numbers with no additional description of the nature of that mathematical use was recognized as among the lowest of the levels of quantitative literacy. Students who responded with particular types of mathematical applications associated with their daily use of numbers were coded as having higher quantitative literacy. An example of this distinction in the breadth of student answers is given below:

*Student in the online PBL course:* Surely, math is used in everyday life. Whether it is seeing how much longer we can sleep in before missing our bus, to calculating tips, to crunching numbers on performance evaluations, math cannot (and probably should not) be avoided. I think math can certainly be used more if used correctly. For starters, it is much easier to judge news articles as reliable when graphs and stats are understood. It is more understandable if a virus is really growing exponentially or if that word is just used incorrectly.

*Student in F2F course:* I use math when necessary.

Second, the frequency of this response may have an altogether different rationale. It is possible that students responded that they use numbers every day because they believe that to disagree with this statement is culturally inappropriate – tantamount to claiming to be illiterate. This would skew this response higher, since students would rarely admit the taboo of not daily using numbers. This is all the more probable since students were responding to the prompt in the context of being enrolled in a university mathematics course.

Third, although the writing prompt asked if students used “math or numbers” every day, comments from the two groups demonstrate differing interpretations of the these terms. Students in the face-to-face group interpret “math or numbers” to represent basic arithmetic up through addition, subtraction, multiplication, and, to a limited extent, division in contexts minimally including decimal arithmetic and outside of the realm of fraction and algebra; the comments students in this section made in reference to mathematics were actually about arithmetic. However, students in the online group recognized “math and numbers” to mean a broader notion of mathematics. (Examples of such are provided below.) While this is addressed in greater detail
in following discussions, differing interpretations of “math and numbers” could certainly account for such a high number of these responses among the face-to-face students.

Student in online PBL course: I do use math in daily life for things such as budgeting, financial planning, tipping, time management, cooking, and taking care of my car. I enjoy calculating things when I understand how to do the calculations and when the results are useful to the project at hand.

Student in F2F course: I do use them in my everyday life. There are a lot of things that require numbers and to live without arithmetic would be very difficult.

If it is indeed the case that students perceived it a taboo to state that they did not daily use math and numbers, this taboo also proved inadequate to keep four students from the face-to-face group from stating unequivocally that they disliked math and saw no use for them in day-to-day life. All the more so, this response, while in the context of being enrolled in a university math course focusing to some extent on quantitative literacy, connotes the extensiveness of this antipathy toward mathematics. Furthermore, recalling that the prompt asked about a student’s daily use of “math and numbers” and that this group consistently interpreted this as somewhat less than mathematics, the response of these four face-to-face students connotes the deepest level of hostility toward even the lowest level of arithmetic. This result was not found in any of the responses of the online students. Three of the aforementioned comments from the face-to-face group are listed below:

I avoid them at all cost and no, I hate math with a passion.

I avoid them because I just don't see it as necessary in my day-to-day life.

I've never really had to use math in my daily life, and no I wouldn't use them more.

Extended Discussions
In addition to students in the face-to-face group scoring 40 of 48 scores in the four categories denoting the lowest levels of quantitative literacy, only three scored in the four categories denoting the highest quantitative literacy and none in the categories two denoting the highest level of quantitative literacy. Since the face-to-face students enrolled in this college algebra class could be considered a relatively random collection of this genre of students, it can be hypothesized that this group represents a broad spectrum of university students in college algebra throughout the nation. Among responses from this group, none provided recollection of any new mathematical applications they learned over the semester and none confirmed that they had become less fearful of math. This speaks strongly regarding the attitudes and quantitative literacy of a vast number of students who take college algebra and other courses designated to fulfill university quantitative literacy requirements. This result gives birth to many significant dimensions.

First, this indicates that traditional college algebra and university courses—even those designated to fulfill quantitative literacy requirements in students’ general education coursework—may have little effect regarding students’ development of quantitative literacy.
This aligns precisely with the opinions of Madison (2003) and Steen (2004) that effective, restructured, introductory-level qualitative literacy courses have yet to proliferate and highlights Small’s (2006) vision for such courses. These results speak for a continued push to significantly rethink these courses throughout the nation.

Second, these findings may inform university faculty that content alone does not necessarily address quantitative literacy – particularly if students do not clearly recognize the connections of the mathematics to their respective career and life interests (Eide & Grogger, 1995; Levy, Murnane, & Willett, 1995; Rivera-Batiz, 1992). Since quantitative literacy includes attitudes toward mathematics, developers of mathematical investigations and activities should ensure that these investigations address career and life concerns in ways that ensure that sufficient mathematical content is still addressed. As seen in the following quote from university faculty of the traditional college algebra course at a university in the Southeast US, the difficulty of this may be exacerbated by the fact that some faculty may believe that teaching traditional content in traditional ways is the faculty’s primary responsibility and that it is the students’ responsibilities to independently apply this foundation and content in their respective career pursuits.

*University faculty member:* I teach the algebra and use many of the applications provided in the book. It is then for students to learn how to adjust those applications to their own interests. I can’t do it all, and quantitative literacy means that the students can themselves make the applications to what interests them.

Again, students in the face-to-face group could not recall any applications of daily mathematics use in their own lives at the end of a traditionally offered college algebra course; some even went as far as to note that they would use math in their daily lives *if* they recognized how to do so. Consistent with the opinions of Madison (2003) and Steen (2001), examples from face-to-face students comments may indicate that general education requirements attached to traditional college algebra courses are less than adequate at meeting students’ needs and desires:

*I do use numbers daily, either with school or with my own activities. I would definitely use more if I knew how to use them.*

*Yes, I usually do use numbers with everyday activities. Yes I also think if I knew how to use them better I would be using them more often. Every day I find myself using numbers.*

In stark contrast to the results of the face-to-face student group, the online group scored 55 of 66 responses in the categories denoting the five highest levels of quantitative literacy. While it is difficult to fully assess whether this was the result of the course being offered online, the course’s emphasis in alternative and project-based assessments, or the course’s focus on career and life applications of mathematics, these results clearly demonstrate that quantitative literacy can be significantly affected in an online college algebra course. The following responses are from students in the online PBL course:
This online class made me feel way more comfortable when it comes to learning math I've never seen before. I don't love math, but I know I'm going to need it to succeed in the future. I can learn any math if I try hard enough.

I chose to take this course online because of my living situation and schedule. I learned much more math than I anticipated and I learned how to apply it to MY life and not simply solve word problems. It may have changed my view of both math and online courses.

You do use numbers every single day of your life, whether you're figuring out how much time you have before work or counting the days until you go somewhere. I don't avoid them. I try to embrace them because I need to accept that they are apart of my daily life. This online course made this more clear to me than any other course I have taken.

As we have seen, there were stark differences in both the quantitative and qualitative nature of students’ responses between the online and F2F sections of the course. With respect to the qualitative differences, in addition to seeing distinctions in students’ mathematical affect, more nuanced themes that emerged included: how students define and apply mathematics, exhibit (or not) math phobia, as well as demonstrate a willingness to learn applicable mathematics. These are discussed below.

As previously evidenced through student responses (with additional responses below), the responses of face-to-face students connoted their interpretation of daily applications of numbers to mean little more than addition, subtraction, and multiplication (with very limited consideration of division) in contexts including integers and simple decimals and mostly excluding even simple applications of fractions. Many of their comments referred to applying mathematics—or rather to avoidance of applying such—to their current lives. Based on their responses, they did not look beyond their current experiences and limited mathematical needs to the mathematical needs of future careers. Interestingly, the face-to-face group seemed to imply that their career goals were constrained by their perceived abilities and attitudes regarding mathematics rather than being willing to learn more mathematics in order to open additional career paths and possibilities. They seemed to have little belief that they could learn mathematics at a level even slightly above what they were currently investigating.

I use numbers to calculate time between classes, taking naps. I do a lot of adding and subtracting when it comes to calculating my expenses. If I took more math courses, I believe I would still use them about the same as I do now.

I do use basic arithmetic in everyday life, but nothing more advanced. I don't mind doing calculations, and I don't think I'd use more in daily activities if I knew more.

In contrast, the online group recognized their daily application of math numbers to mean any level of doable mathematics for which they recognized applicable to their current or future lives. They commonly stated that they would be willing to learn any level of mathematics that they recognized as valuable to their current and future lives and careers. They believed that
mathematics was a gatekeeper to numerous careers and that they could master the mathematics associated with any career interest they might have. Thus, they were commonly willing to look beyond their current uses of mathematics to future needs. As seen below, they were much more positive that they had the ability to learn any mathematics that they recognized as applicable.

Yes, I do believe I would use calculations more if I knew how to do them. I think that would greatly benefit and ease certain situations.

Since answering these questions at the beginning of the class, I believe that I have been paying more attention to how often I use numbers. I use them daily whether I'm in the grocery store trying to stay under a budget, calculating how many hours I have before class, or how many pieces of pie I want to eat at Thanksgiving... I am always using math, no matter how difficult or simple. I think I do understand a little more about numbers after this class. I also think I need to take a financing class because that is one thing I still think I would have difficulty doing on my own.

I use math in my daily life for other courses such as my Planning and Design of Leisure Facilities class, performing ratios for site and master plan designs. I also use quite a bit of math in my part-time job as a waitress. To me, math is a crucial part of my personal knowledge that I believe is important.

Conclusions and Future Directions
Altogether, the findings of this study are quite clear: university college algebra courses can be developed that significantly affect students’ quantitative literacy. The results further bolster the notion that teaching methods for quantitative literacy are not lecture in tandem with traditional assessment (Shavelson, 2008). This study—in tandem with its quantitative counterpart (Tunstall & Bossé, 2015)—demonstrates that PBL in an online environment has the ability to develop students’ quantitative reasoning, disposition toward mathematics, as well as communicative ability. Math instructors should convey both content and its utility; accomplishing the latter is not done through rote exams, but rather through student exploration and research. Having an instructor stand in front of a room and give examples of mathematics is perhaps worthwhile; however, should the same instructor also tell the students what the application is of such content, or should students search for themselves? If quantitative literacy is a goal for the course, this study suggests the latter approach, whether the class is online or in person. Additionally, because accountability for general education outcomes is of increasing importance—and quantitative literacy is a significant part of any university’s general education program—teachers have the responsibility to answer the calls for fostering quantitative literacy with nontraditional teaching methods. The 2010 statistic revealing that 65% of college algebra sections employed such a strategy is not acceptable in an age where quantitative literacy is of increasing importance.

The central goal of this exploratory study was to examine the efficacy of an online, problem-based learning environment in promoting quantitative literacy in a college algebra course. The results of the study provide strong preliminary evidence that such an approach is effective. Moreover, because growth in college algebra was not the focus of this analysis, the findings speak to promoting quantitative literacy in any PBL course. As discussed earlier, due to the logistics of the study, there are limitations to the findings. The study does not prove
causation, as it does not account for whether the online environment, PBL activities, or a mixture of both is responsible for gains in students’ quantitative literacy. The authors believe that one would find similar results in a PBL face-to-face course. Note that while one could have similar discussions in a face-to-face environment, the very nature of the projects required students to complete them on the computer. Hence, based on the results of this study, it is difficult to speculate what would occur if one taught a similar PBL course without computer projects. Accordingly, future studies should compare all of these variations in order to isolate key factors. Because the online environment is likely to remain prominent as a course delivery medium for the foreseeable future, mathematics instructors must understand the environment’s ability to foster quantitative literacy. Indeed, in a world “awash with numbers,” we are the lifeguards keeping students afloat.

References


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