Different Modes of Communicating Geometric Shapes, through a Game, in Kindergarten

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Abstract

Kindergarten children's (5-6 years old) ability to communicate geometric shapes, to their classmates, in different modes—verbally, gesturally, schematically—through a game, was investigated. The game motivated the children to describe the shapes in the different modes, by emerging children's thinking process, communication capacity as well as their ability to transmit the surface-visual images of shapes to oral language, to gesticulations and to schematisations. Gestures, as well as written schematisations did not serve as a compliment of speech but on their own and contributed to the identification of the shapes. Both defining and non-defining features of the shapes were expressed in all three modes.

Introduction

Through (mathematical) communication young children develop their conceptual understanding, thinking, problem-solving skills and reasoning in mathematics (Poland, Van Oers & Terwel, 2009). For this reason, it is suggested that teachers encourage the communication of children's mathematical understanding in the classroom (Greenes, Ginsburg & Balfanz, 2004; Varol & Farran, 2006), within a critical and creative instructional design (Radford, Schubring & Seeger, 2011) offering them the necessary tools to create new and familiar ways of thinking and communicating. As mathematical cognition is mediated by actions, gestures, speech, written symbols, schemes, and other types of signs (Kim, Roth & Thom, 2011; Radford, 2009), specific types of communication, in the process of teaching and learning mathematics, can emerge.

The way students at different ages, communicate in the mathematics classroom, to explain their thinking, differs. Children usually express their thinking by touching, holding and gesturing (Roth, 2001). They also describe mathematical situations by using everyday language or their own words, actions, materials, examples, and informal recordings. Young children's explanations, usually have a practical and not mathematical basis, meaning that are not based on mathematical definitions and/or properties which are not necessarily formal but may serve as a bridge to formal explanations (Levenson, 2013). The ability of both teacher and children to decode verbal, written and bodily expressions and actions, in order to comprehend the mathematical content of the communication and give meaning to it, ensures the success in teaching and learning mathematics (Hümmer, 2011).

The importance of the different mathematical communication modes is also recognised by researchers who focus on the way, children of primary and secondary school, describe their thinking (Chen & Herbst, 2013). They investigate how language and gestures work together and influence each other (Alibali & Nathan, 2012; Arzarello, Paola, Robutti & Sabena, 2009), as well as, how children note their thinking on the paper (Carruthers & Worthington, 2006; Dijk, Van Oers & Terwel, 2004). They focus on the types of communication that occur in the mathematics classroom and mainly deal with teacher-student communication (Cobb & Whitenack, 1996).

Although theory and research defend the contribution of the diverse modes of communication in the process of mathematical teaching and learning, this is often not considered in practice. For example, in kindergartens, it is usually taken for granted that, children at this age encounter difficulties in expressing and communicating their mathematical thinking/understanding in any mode. Thus, in young children's learning, emphasis is given to teacher's verbal language and to children's observation and imitation rather than discourse (Flewitt, 2006). Due to the fact that kindergartners do not have the same degree of mathematical fluency, either verbally or in writing, as children in primary school (Johansson, Lange, Meaney, Riesbeck & Wernberg, 2014) and that they are not usually understood by others when communicating verbally, (Robotti, 2012) the communication in kindergarten is limited to one-word answers and the description of child's thinking to his/her peers is rarely encouraged. For this reason, there is a need to investigate young children's communication capacity, in order to redefine it.

The mathematical concept through which the communication skills of kindergartners investigated was geometric shapes because it is a mathematical concept that needs to be communicated in different modes, in order to be understood by students (Clements & Sarama, 2007). Geometric shapes are mathematical concepts that need to be communicated to and from the children in various ways, because shapes' iconic representations need 'observation', 'reading' and parallel 'description of this reading' by the children, in order to be understood both by themselves and others (Skoumpourdi, 2013). The 'observation' factor has to do with shape's visual perception. The 'reading' factor is focused on shape's figural properties. The 'description of this reading' factor requires the description/communication of the above. Due to the fact that children recognize shapes more easily than they describe them, the above three factors can help them face their difficulty to describe shapes in their own way, so as to develop gradually the shape's concept. When children are taught geometric shapes' names only, and these names are not accompanied by various descriptions (verbal, gestural or schematised), although they may remember these words, they make no sense to them (Kim, Roth & Thom, 2011).

The role of the communication of geometric shapes in their understanding is also highlighted from semiotics in which understanding of the shapes develops through the transformation of children semiotic representations. According to Kim, Roth and Thom (2011: 185), "from a semiotic perspective neither the cognitive activity of the individual nor his social interaction is primary; both co-exist and co-act in a synergistic manner to support the evolving process of sign interpretation and meaning-making. Thought and communication appear to be parallel and interrelated at the same time".

A context that can promote different communication modes of geometric shapes, in kindergarten, is games. International research results highlight the contribution of games in mathematics learning and teaching process (Afari, Aldridge & Fraser, 2012; Bennett, Wood & Rogers, 1997; Bragg, 2012; Edo, Planas & Badillo, 2009; Perry & Dockett, 2007; Skoumpourdi, 2014; Vankúš, 2005). These studies demonstrate the educational value of games, reporting the motivation they offer for participation in mathematics, as well as the experiences that children gain through them that promote mathematical communication and understanding.

Without minimizing the importance of multimodal communication, in this paper the kindergarten children's ability to communicate geometric shapes to their classmates, in different modes—verbally, gesturally, schematically—through a game, is investigated. The research hypothesis was, that, if the kindergartners manage to describe the shapes in a way to be understood by their classmates, then they have the ability to communicate effectively. Three research questions were posed: 1. Do kindergarten children have the ability to communicate geometric shapes to others through the different modes—verbally, gesturally and schematically—that are 'imposed' from the game? 2. How children use each mode of communication— verbal, gestural and schematic—to advance the information to their peers for each shape? 3. Which communication mode—verbal, gestural or schematically—seems to be more accessible for children and more effective for each shape?

Identifying and describing shapes in kindergarten

Children progress through levels of thought in geometry (Van Hiele, 1986). At the visual level, which is the first level, children identify shapes according to appearance and cannot form mental images of them. At the descriptive/analytic level, which is the second Van Hiele's level, children use shapes' properties to recognize and characterize shapes. Clements and Battista (1992) suggested a level before the visual one, the pre-recognition level, as the level 0, in which children may attend only a subset of a shape's visual characteristics. In this level, children cannot distinguish the basic shapes from their non-examples because they are in progress to form visual schemes of the shapes.

Kindergarten children have considerable informal knowledge concerning geometric shapes from their everyday activity before they enter school. They can recognize and name two-dimensional shapes such as circle, square, triangle and rectangle (Clements, 2004; Levenson, Tirosh & Tsamir, 2011) when they are represented in a prototype way. The effectiveness of their recognitions are influenced by the orientation of the axes of the shape and seem to be more accurate when they use shape's attributes (e.g. number of sides) in their justifications (Yin, 2003). However, children in this age have some difficulties to identify squares without horizontal sides (Clements, 2004) and other shapes like rectangles and triangles when represented in atypical ways.

Depending on whether the geometric shapes' features can be defined (such as the closeness, the straight sides and their exact number) or not (such as the orientation and the position, the sides' length and the angles' size) (Satlow & Newcombe, 1998), they can be represented by few or many ways, and thus to be recognized by young children easily or not. Circles and squares have more defining features because of their symmetry whereas rectangles and triangles have more non-defining features. Thus, circles and squares, do not have many different representations and they are identified relatively easily by young children whereas triangles and rectangles have many different representations and their identification may confuse children.

Verbal description of shapes

Children's external speech is very important and it shows their capability to think while talking, according to Vygotsky. It helps them to form their ideas and it is also a way to understand what is in their minds. Verbal skills and the ability to produce linguistic codes in interactions, play a crucial role both in teaching and learning mathematics (Rudd, Lambert, Satterwhite & Zaier, 2008) because they express and generate thought representing mathematical understanding (Chen & Herbst, 2013). Natural language is a tool that children can use from an early age and it supports students' cognitive processes providing them with different kinds of help for problem solving (Robotti, 2012).

In early year's mathematics, informal and formal language of mathematics takes place, during learning/teaching process (Carruthers & Worthington, 2006). Mathematical thinking begins to develop through the informal use of mathematical language which is essential for the teaching and learning of mathematics (Rabel & Wooldridge, 2013) and continues developing through its formal use.

The role of language in developing the concepts of shape is crucial. Children first use words such as *circle* and *square* to answer the question "what shape?" (Clements & Sarama, 2007). Then they match these words with specific shapes examples. After that, children combine the correct shapes' names with the prototypical examples of the shapes. When children are asked to explain their decision they are induced to describe shapes' features, using, initially, a subset of the shape's visual characteristics. Although children's primary descriptions may include various terms and attributes they usually base these descriptions in the comparison with the prototype shapes of each category. For example, they can say "this shape is very thin" for a rectangle in which the width is much smaller than the length.

Children's descriptions usually rely on physical features of a diagram to describe it (Herbst, 2004). In a study, 4-6 year old children recognized circles and squares relatively easily while they had difficulty in describing them. The children that made a description said for circle that is "round" and they described squares using their attributes, such as the quantity and the size of sides (Yin, 2003). In another research study, on kindergarten children's verbal and nonverbal descriptions of shapes during their play, it appeared that children in their questions usually referred to the quantity of shapes' angles (Skoumpourdi, 2013). Later, the children became more capable of distinguishing attributes and adding to their descriptions other visual-spatial elements like right angles.

Gestural description of shapes

Gestures, as a type of embodied communication, which according to Vygotsky (1997) is writing in the air, play an important role in many aspects of mathematics as components of understanding and are considered not only as communication tools but also as learning tools. When children negotiate new ideas or express their knowledge, gestures can organize their thinking in a visible and concrete way. Students, often express new knowledge by gesturing, before they express it in speech, when trying to communicate concepts they are learning (Alibali & Nathan, 2012).

There is a growing interest in recent research in how gestures, in relation with verbal language, influence the construction of mathematical meanings and concepts (Arzarello, Robutti & Thomas, 2015; Johansson, Lange, Meaney, Riesbeck & Wernberg, 2014). McNeill's view (1992) is that speech and gestures are elements of a single integrated process, like two sides of the same coin. Sfard (2009) supports the idea that language and gestures have two different functions and that only verbal language can describe abstract mathematical concepts. For her, gestures are used before verbal language. In Radford's view (2009), verbal language and gestures, together with objects, tools, devices and signs, are semiotic means for objectification, explaining that they can be used individually or together.

Various classifications of gestures are proposed. McNeill (1992) proposes four gestural dimensions: the pointing (deictic) gestures, the iconic gestures, the metaphoric gestures and the beat gestures. Gestures as described by Radford (2003) act iconically, indexically and symbolically. Arzarello, Paola, Robutti and Sabena (2009) mention that gestures, as personal signs, may consist of two categories: institutional, (mathematical) if established by an institution and personal, if it is an idiosyncratic production of the subject. Kim, Roth and Thom (2011) analyze the emergence of gesticulation with and without speech in four categories: a) gestures with no talking and no apparent communicative purpose, b) gestures with no talking but with apparent communicative purpose, c) gestures accompanying talk, oriented towards others, and d) gestures accompanying talk not directed towards others.

Although there are many studies investigating and emphasizing the importance of children's gestures in constructing geometric concepts, even before and without verbal expressions, their focus is on primary and secondary school. Research on kindergarten children is mainly focused on gestures for spatial reasoning, as supplements to verbal explanations (Elia, Gagatsis, Michael, Georgiou & Van den Heuvel-Panhuizen, 2011; Ehrlich, Levine & Goldin-Meadow, 2006). A research study on kindergarten children's verbal and nonverbal descriptions of shapes, during their play, revealed that children's gestures played a crucial role when shape's description got difficult (Skoumpourdi, 2013). The posed questions were of two types: a) independent ones and b) those dependent on their gestures. The latter happened when the remaining shapes were all of the same kind (i.e. triangles) and it was difficult for the child to describe verbally the shape's particularities. For example, to describe an atypical triangle a child tried to recreate triangle's image with gestures, requesting confirmation for its sides: "Is this big, this smaller and this like this?" (representing its sides by gesturing). Generally, their gestures referred to the sides, the image, the size, as well as to the shape's orientation.

Schematisations of shapes

Writing is another communication modality and it includes inscriptions, notations, jottings, drawings, schematisations etc. Research in this area is generally not focused on its contribution to young children's mathematical learning but is mostly focused on writing as a critical emerged literacy skill that lays the foundation only for children's later literacy skills and reading achievement (Dijk, Van Oers & Terwel, 2004).

A prime stage of a schematising activity is drawing, a capability that children have from a very young age (before 4 years old). Children, at that age, usually give meaning through their drawings (Matthews, 1999) and this must be cultivated since the reflection between meanings and drawings seems to be a thinking activity. As children grow older they are required to use schemes, especially with respect to mathematical activities. Through schemes they can organize their knowledge and thoughts. Undoubtedly, this is a complex activity because it demands both knowledge of the kind of schema to be used for demonstration and ability to decide whether the schema used expresses its intended message.

Schematising activities are very important for kindergarten mathematics for three crucial reasons (Dijk, Van Oers & Terwel, 2004). The first is that schematisations form the bridge between the concrete practical thinking of young children and the logical-symbolic thinking in later development (Poland, Van Oers & Terwel, 2009). The second, concerns the communicational function of mathematics and specifically the need to understand the underlying meaning of the symbolized messages in the notations. Investigations have observed that young children are willing to be involved in schematising activities only when they have a clear communicative function. The third, concerns the enrichment it provides for the playful activities for young children.

Concerning shapes, very young children (even from 3 years old) can be engaged in the shape construction process and this process changes with development which includes analysis of a shapes parts and combination of those parts to form the shape (Clements & Sarama, 2007). For example, for cross construction, very young children construct a cross by using four separate line segments, older children construct it with a long vertical and two horizontal line segments, and adults construct it with two vertical line segments. In general, young children when constructing a shape take into consideration the shapes independent parts whereas older children relate parts across intersections. The same happens with the simple closed shapes in which the closeness and the contour help students to consider the shape as a whole. Thus, children's written schematisation for a circle is something closed and 'rounded', for a square something closed with almost equal sides and nearly right angles, for triangle something closed and 'pointy' and for a rectangle something closed with 'long' like-parallel sides (Clements & Sarama, 2007: 504). But in visual contexts children may not be able to construct an image of shapes, or a representation of the image because "before level 1, children lack the ability to construct and manipulate visual images of geometric figures" (pp. 505).

Methodology

The designed game

In order to investigate kindergarten children's ability to communicate geometric shapes to their classmates in different modes—verbally, gesturally, schematically—a game was designed. For the creation of the game, specific principles for designing educational materials enriched with the principles for designing games were taken into account. To name some: need assessment and formative evaluation (Romberg, 1992), as well as, mathematical, cognitive and pedagogical fidelity (Zbiek, Heid, Blume & Dick, 2007).

Specifically, for the creation of the game the need of this game was investigated and recorded (need assessment). Then, the game was constructed in such a way that the desired objectives can be achieved (formative evaluation). To be specific, the game playing reflects accurately the expected mathematical characteristics (mathematical fidelity) and expresses students' thoughts and strategies (cognitive fidelity). The game and its materials allow students to act mathematically in ways that reflect the nature of the mathematics learning and are consistent with the practice of teaching (pedagogical fidelity). Additionally, for the games design and construction, several board games features were taken into consideration. The features of board games negotiate a particular context in which underlies the construction of the rules, of the board, of the route, of the pawns, of the cards and of the means used to set the order of the players and the continuation of the game (Skoumpourdi, 2014).

The game "Seeking the shape", was built on these principles and is a board game for geometric shapes which can be played in teams. The game board consists of 18 squares in a path without numbers (Fig. 1). It includes four pawns in different colours, a dotted (1-3) dice, hourglass, spinner (in order to find which type of card to take) and three types of cards with shapes: lip cards, gesture cards and pencil cards. In the first card type, with the lips, the player must describe verbally to his/her team mates the figure depicted on the card, without saying the name of the shape. In the second card type, with the gesture, the player has to describe the shape depicted on the card, with his/her hands, without speaking. In the third card type, with the pencil, the player has to schematise the shape depicted on the card, on a piece of paper. After the player describes to his/her team the shape on the card, in verbal, gesture or written form, the members of his/her team try to identify the described shape before the time is over. If a team stops in a place that has two hourglasses, it has double time available to answer questions. The winning team is the one who reaches first the end of the path.

Fig. 1 The game "Seeking the shape"

On the game cards, only the basic two-dimensional shapes (circle, square, triangle and rectangle) that are included in the Greek kindergarten curriculum are depicted. These representations are both typical (Fig. 2) and atypical (Fig. 3) so as to create rich and various experiences to children regarding shapes. The presentation of several shapes' images on the cards, intends to enable students to grasp their common internal structure and their common characteristics in order for the children to identify not only shapes' schema but also to understand the shape's concept, so as to describe it to the others.

Fig. 2 Typical shapes

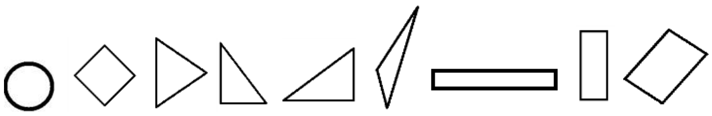

Fig. 3 Atypical shapes

Three dimensional shapes were not included for two reasons. The first reason was that even advanced kindergartners had difficulty in identifying drawings of the solids (Leeson, Stewart & Wright, 1997). The second reason was that most of the students have difficulty naming solids and they use plane figures names for solid shapes (Clements, 2004).

Sample and game process

The game was played by 16 children (5-6 years old), in a public kindergarten school, in two teams with 8 children each (4 girls and 4 boys in each team). The kindergarten was in an area with middle socioeconomic status. The children had been taught about shapes. Their teacher had provided them with opportunities to explore two and three dimensional shapes and to develop their knowledge of geometric shapes in various ways. She had taught them the shapes' names, their images and their main characteristics.

The researcher did not give any specific instructions to the teacher regarding her intervention during the game. The teacher just read and explained the rules of the game to the children. In each round of the game a different child was a describer and the rest children of his/her team were the listeners/identifiers. The describer picked up a card, from the pile indicated by the spinner (lips, hands or pencil). After he/she saw the shape on the card, he/she put the card under the pile and tried to describe the shape he/she saw to his/her team members. Due to the fact that the cards were in a random sequence, the shapes that children had to describe through the different modes of communication were not the same (quantity and kind). The game process was videotaped and related ethics were taken into consideration.

Method

The focus of the analysis was children's descriptions when they communicate the geometric shapes to their classmates. The correctness of mathematical expressions, the accuracy of gesticulations, as well as the precise schematisations of shapes' images were analyzed independently, but also with respect to whether they had been understood from the other children. The descriptions used by children and their relationship with the card's shape were coded in three categories: 1. Descriptions that match the shape on the card: Verbal expressions, gestures and written schematisations that give specific information about the shape, with many details. 2. Descriptions that partly match the shape on the card: Verbal expressions, gestures and written schematisations that give incomplete data about the shape. 3. Descriptions that mismatch the shape on the card: Verbal expressions, gestures and written schematisations that give confused or no details about the shape.

Results-Discussion

Children's verbal descriptions

There were ten cards that children picked randomly in order to describe verbally the shapes: three with squares, four with rectangles, and three with triangles.

Children's verbal descriptions for squares were all mathematically based. They described a square as the shape which "has four angles" or the shape that "has four sides". Although they used correct mathematical expressions, their descriptions were usually incomplete, mentioning a part of square's characteristics, which did not represent shape's particularities effectively, and partly matched the shape on the card. Team members, the listeners/identifiers, found the shape after two or three incorrect answers (see table 1). This may have happened because the expressions used were very general and provided the listeners with incomplete data for the kind of shape on the card. However, answers like "it is a triangle", that listeners gave cannot be justified rather can be characterized as random. No child used the combined shape's attributes giving a complete description for a square. For example, nobody said that square is the shape that has four angles and four equal sides. There was a child (1/16) who could not describe the square at all. She had spent about 90sec watching the card and finally she said "square". She named the shape, although it was forbidden in the game's rules.

Mathematically based verbal descriptions were also used for rectangles. The verbal descriptions of the rectangles were more descriptive than those of the squares, especially in the atypical example. In this case, in which the shape was not in the prototype form, a child gave more details in his description, which matched the shape on the card (Table 1). Although the describer recognized the atypical example of the rectangle and described it to his team members in a very detailed way, they did not manage to recognize it when they heard the description. Even when they found it they doubted the correctness of their answer when they saw the shape on the card (the names of the children are not their real ones and in the brackets there are the researcher's comments):

> George: It has four angles and four lines [simultaneously he made four vertical movements with his finger IIII, like counting using 1-1 correspondence] … [since nobody tried to find the shape, he continued his description in order to help his team]
>
> George: … and is thin.
>
> Team: Triangle.

Teacher: Did you hear carefully what George said?

George: It has four lines four angles… [showing at the same time his four fingers] … and is thin [while joining the two markers in a line]

Team: Triangle.

Teacher: Has the triangle four lines?

Team: No.

George: And it is thin [while joining both palms].

Team: Square.

George: No.

Team: Rectangle.

George: Yes.

Teacher: [Show the card to the children]

Team: This is not a rectangle this is like a belt.

The children did not pay enough attention to their classmate's verbal description. In this episode, like in the square, the listeners initially mentioned triangle even though the describer said "four angles and four lines". The atypical rectangle, with the width much smaller than the length, seemed to confuse children. This confirms the research results demonstrating the difficulty of identification of atypical rectangles (Clements & Sarama, 2007).

Another describer (1/16) gave specific information for the shape saying that "it has four angles and two big lines", a description that matched the shape on the card and in that case the members of her team found it at once (Table 1). Other describers' expressions were the following: "the shape that has four angles" or "the shape that has four angles and four lines". These descriptions partly matched the shape on the card and maybe for this reason listeners/identifiers first reported the square (Table 1). Squares and rectangles were described in general, with the same defining features. Non-defining features, such as the orientation, the position, the sides' length etc. were not at all described except in the cases of atypical rectangles.

Although a triangle was the shape with more non-defining features (Satlow & Newcombe, 1998) that has many different representations and due to this it is hardly identifiable, for kindergarten children was the easiest recognizable shape. The describers said that this shape "has three angles" or that it "has three angles and three lines" or that it has "has three sides" and their teams found the shape at once (Table 1). The verbal communication of shapes was limited to defining features such as the number of sides and/or angles. This, in most cases (6/9) resulted in descriptions that partly matched the shape on the card but always in effective communication. The effective communication for the triangles verbal description can be explained by the fact that children did not know any other shape with three angles. Triangle in Greek language is 'trigono' which means three angles and therefore children did not confuse it with any other shape. The fact that the children provided the same verbal description for the various types of triangles can be considered as a positive finding for children's triangle concept understanding. It is an indication that they focused on the critical attributes of the triangles and not on their appearance.

Shape on the card	**Describers' verbal expressions**	**Type of matching**	**Listener's answers**
Square	"It has four angles"	Partly match	1. "Triangle" 2. "Triangle" 3. "Square"
Square	"It has four sides"	Partly match	1. "Rectangle" 2. "Square"
Triangle	"It has three angles"	Partly match	1. "Triangle"
Triangle	"It has three angles and three lines"	Match	1. "Triangle"
Triangle	"It has three sides"	Partly match	1. "Triangle"
Rectangle	"It has four angles and four lines … and is thin"	Match	1. "Triangle" 2. "Square" 3. "Rectangle" ?
Rectangle	"It has four angles and two big lines"	Match	1. "Rectangle"
Rectangle	"It has four angles and four lines"	Partly match	1. "Square" 2. "Rectangle"
Rectangle	"It has four angles"	Partly match	1. "Square" 2. "Rectangle"

Table 1: Describers' verbal expressions in relation with the shape on the card and with identifiers' answers

Gestures

There were six cards that children picked randomly, in order to describe by gestures: one with circle, three with triangles and two with rectangles.

The circle together with the equilateral triangle was the easiest to describe and identify shapes. The describers showed with their fingers the image, for example, the circle in a way (Fig. 4) that matched the shape on the card (Table 2) and the listeners immediately identified it and named it. When the triangles were atypical, their description and identification were more complicated. For example, a boy, as a describer, showing a right-angled triangle connected his fingers in a way that did not satisfy him and for this he reconstructed the shape with his gestures trying to be more accurate, till he represented it as it was on the card (he asked his teacher to show him the card again) (Fig. 5).

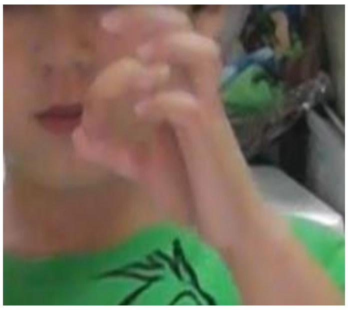

Fig. 4 “Circle”

Fig. 5 “Triangle”

For an obtuse triangle, a boy in his effort to represent it like it was on the card got confused. Initially, trying to show the biggest side of the shape, he made a shape like a circle. Then, trying to be more accurate, he closed his fingers and he made two triangles. Then he re-gestured, but did not manage to construct it. He finally made a classical isosceles triangle (Fig. 6-8). His first two gestures mismatched the shape on the card whereas the third one partly matched (Table 2).

Fig. 6-8 “Obtuse angular triangle” 1st, 2nd and 3rd try

The rectangles were presented by a boy and a girl. The boy constructed it with his two thumbs together with his forefingers (Fig. 9) whereas the girl depicted it by putting in parallel her two forefingers (Fig. 10). As a result of the fact, that her gestural description in the beginning was not very stable, the members of her team got confused and said that the shape was a triangle. Then, the accuracy of her gestural construction led children to identify and name the shape correctly.

Fig. 9 “Rectangle”

Fig. 10 “Rectangle”

Shape on the card	Describers’ gestures	Type of matching	Identifiers’ answers
Circle	Figure 4	Match	1. “Circle”
Triangle	Figure 5	Match	1. “Triangle”
Triangle	Figure 6-7	Mismatch	1. “Triangle”
Triangle	Figure 8	Partly match	1. “Triangle”
Rectangle	Figure 9	Match	1. “Rectangle”
Rectangle	Figure 10	Partly match	1. “Triangle” 2. “Rectangle”

Table 2: Describers’ gestures, in relation with the shape on the card and with identifiers’ answers

The gestures used during the game were gestures with no talking but with a communicative purpose as Kim, Roth and Thom (2011) describe. They were also iconic, according to McNeil’s (1992) and Radford’s (2003) categorization, mimicking the shape on the card, not always successfully. Although gestures were associated with the physical characteristics and the position of the depicted shape, they were idiosyncratic productions of each student and for this they can be also characterized as personal, in accordance to Arzarello, Paola, Robutti and Sabena’s (2009) categorization. It cannot be assured that all gestures showed a clear shape. But it appeared that in most of the cases, mathematical thinking existed, both through the movement of describers’ hands and the interpretation of gestures by identifiers.

Written Schematisations

There were eleven cards that children picked randomly, in order to describe the shapes in written form: two with circles, two with squares, four with rectangles and three with triangles.

Describers schematised circles (Fig. 11-12), squares (Fig. 13-14) and rectangles (Fig. 15-17), relatively easily and in general their schematisations were matched with the shapes on the cards. In a case (Fig. 16), a describer, in order to schematise an atypical rectangle, asked her teacher to show her the card again.

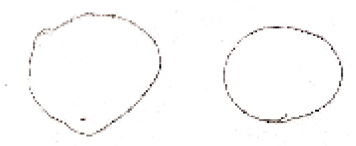

Fig. 11-12 “Circles”

Fig. 13-14 “Squares”

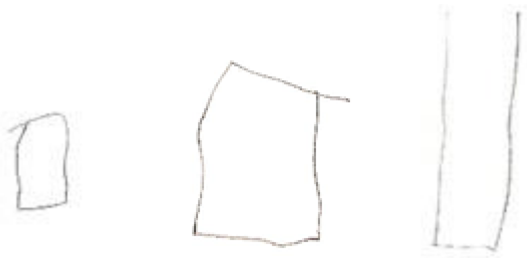

Fig. 15-17 “Rectangles”

In another case, the written schematisation of a rectangle caused difficulty to the identifiers as they confused it, initially, with the square. Then they mentioned a towel, meaning rectangle (Fig. 17). This was the only practically based example that children gave:

Kostas: [he schematises a rectangle and he shows it to his team]

Team: Square

Kostas: No

Team: Towel

Teacher: Tell us the name of the shape

Team: Rectangle

The triangles schematisations caused difficulty to the describers who asked their teacher to show them the card again in order to construct those triangles on the paper (Fig. 18-20).

Although the triangles were both typical and atypical, and the schematisations matched or partly matched the shapes on the cards (Table 3) due to the lack of geometric accuracy in the constructions, the identifiers were successful.

Fig. 18-20 “Triangles”

No one used a tool neither to make a straight line nor to make an accurate circle. This may mean that they have (or have not) explicit ideas of the concepts of shape in their mind but those were their capabilities in schematising shapes.

Shape on the card	**Describers’ written schematisations**	**Type of matching**	**Identifiers’ answers**
Circle	Figure 11-12	Match	1. “Circle”
Square	Figure 13-14	Match	1. “Square”
Rectangle	Figure 15-16	Match	1. “Rectangle”
Rectangle	Figure 17	Match	1. “Square” 2. “Towel”
Triangle	Figure 18-19	Match	1. “Triangle”
Triangle	Figure 20	Partly match	1. “Triangle”

Table 3: Describers’ written schematisations, in relation with the shape on the card and with identifiers’ answers

Like Clements’ and Sarama’s (2007) claim, the written schematisations that the describers made were for a circle a closed and ‘rounded’ shape, for a square a closed with almost equal sides and nearly right angles shape, for a rectangle a closed with ‘long’ like-parallel sides and nearly right angles shape and for a triangle a closed and ‘pointy’ shape. In addition, describers' schematisations showed shapes orientation and position, taking into consideration both defining and non-defining features of the shapes.

Conclusion

Children used the three different modes of communication to provide information to their peers but not always in an effective manner. It seemed that the kind of the shape presented influenced children's communication effectiveness. Triangles and circles were easily identified by children when verbally described due to the fact that they had only one significant feature. When the clue was something with three it could only be the triangle. In the same way the circle was also easy identifiable. From the children's descriptions it was indicated that they recognized the different types of shapes, some of the shapes basic characteristics, as well as some of their attributes, but this was not obvious in all different modes. For example, the square and the rectangle were not immediately identified by the children when verbally described; this was due to the fact that the describers used the same expressions for both of them. Both defining and non-defining features of the shapes were expressed in some descriptions. Specifically, for rectangles in verbal as well as in written form and for triangles, in written form.

The describers' verbal expressions were only mathematically based and were about the number of angles, and/or sides/lines, as well as the size of the shape's sides. More complex expressions related to shapes' specific characteristics like the equality of shapes' sides or the existence of right angles etc., as well as children's own words and examples were not used, except in a case in which children said towel instead of rectangle.

Gestures as well as written schematisations did not serve as a compliment of speech but were used individually, such as in Radford's view (2009), as semiotic means for objectification that encourage mathematical communication. Each mode of communication contributed on its own to the identification of the shapes. Although gesturing and schematising with no talking but with apparent communicative purpose are complex activities, since they demand the ability to decide how to represent the shape in order to be identifiable from those who cannot see it, children in general responded very well. However their gestures as well as their schematisations were not accurate in cases in which they had to describe an atypical shape. For example, instead of representing an obtuse triangle they depicted a 'classical' isosceles triangle.

The game "Seeking the shape" motivated children to describe the shapes in the different modes—verbally, gesturally and schematically—by emerging children's communicational capacity. From the game playing, it appeared that young children have an ability to communicate geometric shapes to their peers, transmitting effectively the surface-visual images of shapes in verbal language, in gestures and in written schematisations separately. These modes of communication became cognitively active and through them children expressed the shapes not only in an abstracted geometric way but in tactile and kinesthetic actions and representations. The attempt to put thought into a description helped students to structure and clarify their shapes' 'observation' and their shapes' 'reading'. The descriptions of the shapes emerged effortlessly from the children in order to end the game as winners and not because it was asked by the teacher and had to be done.

Taking for granted that in most kindergartens multimodal communication is underestimated, the present study can inform teachers for the role of the separate modes of communication in mathematics education, to understand better students' ways of reasoning as well as students' communicating capacities, in order to be used as a tool for their teaching practice. They can rely on children's diverse descriptions of shapes and use these personal creations of children to support the transition to formal ones. Understanding more in depth how these modes of communication become cognitively active for developing geometric shapes' concepts in classroom situations, they can cultivate each mode itself and in relation with the others, in order to be considered as an important aspect in developing young children's thinking, understanding and reasoning in mathematics.

Some limitations of this study are the small number of children that participated in it as well as the limited number of shapes described by students due to the game's nature. In future research the sample will grow, cards with shapes' counterexamples will be included to investigate the type of communication that will take place, and the metaphoric dimension of iconic/deictic gestures, as defined by Arzarello, Robutti and Thomas (2015) will be explored.

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