Traditional vs Non-traditional Teaching and Learning Strategies – the case of E-learning!

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Traditional teaching approaches are generally teacher-directed, where students are taught in a manner that is conducive to sitting and listening. It is often argued that these approaches may not provide students with valuable learning skills, and rather that non-traditional approaches to teaching and learning might better endow students with such skills. The teaching of mathematics that is usually referred to as "non-traditional" uses constructivist philosophies as its bases; by implicating strategies in which the individual develops, constructs or rediscovers knowledge in an attempt to make sense of his or her universe, or by employing social constructivist ideals which suggest that group work, language and discourse are vital for learning. Also, it is often argued that non-traditional teaching is done using a problem solving approach, where the learner is the problem solver. Non-traditional teaching and learning (NTTL) in mathematics and engineering needs to be well understood so that appropriate and meaningful comparisons with traditional techniques can be made. The computer based teaching technology known as "e-learning" is now often used in tertiary level mathematics and engineering teaching. The e-learning methodology is considered to be more in-line with the non-traditional than the traditional teaching approaches. This paper critically reviews the literature that makes comparisons of the traditional and non-traditional teaching approaches in mathematics and engineering education, and specifically examines the advantages and disadvantages of these approaches along with the manner in which they influence the academic performance of students in mathematics and engineering courses.

Keywords: Traditional, Non-traditional, E-learning, Tertiary Mathematics Education, Science and Engineering

Introduction

This paper concerns the author's reflection upon more than twenty-five years of in class teaching of students in high school and tertiary levels across several countries. It is also a critical examination of much of research done by the author and colleagues concerning the teaching of mathematics and STEM (Science, Technology, Engineering and Mathematics) subjects more generally (Albano et al., 2013; Bryan, 2015; Caiazzo et al., 2013; Tularam 2013, Tularam and Amri, 2011; Tularam and Hulsman, 2013). In the past thirty years, there has been a concerted effort to develop, in STEM students, a strong conceptual understanding of mathematical principles, often at the expense of practice, memory, and negatively labelled instrumental learning. The case of demonizing any teaching related to rote learning, or instrumental or procedural understanding, has been espoused in numerous research publications (Abdulwahed et. al, 2012; Tularam and Hulsman, 2015). Throughout the same time, much change has occurred in the learning and teaching of mathematics, much of which the author has overseen in New Zealand and Australia. In particular, newly adopted methodologies tend to be derived from the following constructivist philosophies: Piaget's cognition, Vygotksy's socio-cultural and von-Glaserfield (Piaget among others) constructivism. It has been said that traditional teaching methods are not consistent with these philosophies, and hence a number of new methods have been devised (Abdulwahed et al., 2012). Also, new technology has enabled such philosophies to be adopted and persist in modern classrooms throughout the world. But as is the case with any new approach to mathematics teaching, there are a number of advantages and disadvantages. These will be highlighted in this paper, with the main aim being an examination of whether the shift towards new methods has meant that mathematics teaching has moved too far from the centre ("throwing out the baby with the bathwater", so to say). Given such low levels of performance currently observed in mathematics in STEM students in Australia, have we learned from past mistakes? Clearly, there is a need to critique any popular or newly adopted theory (or approach) in mathematics teaching (even that of constructivism), and particularly in mathematics taught in the physical sciences.

Background

The processes and methods used in teaching mathematics and engineering have long been a subject of study (Carlson and Bloom, 2015; Polya, 1970; Schoenfeld, 1992; Tularam, 1997a). Mathematics is often taught in a "practice makes perfect" manner, and many will agree that an ability to correctly employ even the simplest of mathematical principles (the "times table", for example) has resulted in the attachment of meaning to numerous scenarios encountered throughout life (Tularam, 2015). The Oxford university director of the Greenes tutorial college addresses this issue as: "Regular practice of applying mathematical rules and methods creates foundations for procedural mastery of many mathematical operations and develops mathematical skills. Procedural mastery develops the ability to use various mathematical tools and methods to solve problems in daily life, business and science. Mastery of mathematical procedures also develops confidence and comfort with numerical problems and can then lead to the understanding of mathematical concepts. Hence: "Practice makes perfect" (p. 1. https://www.greenes.org.uk/2012/03/learning-maths-how-practice-makes-perfect/) has been a saying that has been constantly used over the years and was indeed the foundation for early learning and teaching theorists.

Along with education researchers and practitioners, pure mathematicians have also put much effort into understanding how to successfully teach students a dauntingly large body of mathematical content over some given period of time (Herrera Owens, 2001). Some pure mathematicians believed that reducing mathematics to its elementary structural components, and then teaching the discipline from those components, would allow students to more easily access and comprehend the vast content of knowledge in mathematics in a structural manner; that is, by relearning the development of mathematical concepts in a structural manner, students would be able to grasp the concepts with some ease (Ellis and Barry, 2005; Hayden, 1981). But this movement was an "after the fact" analysis; that is, mathematics did not grow in a strictly structural manner even though its components can be ordered structurally (Wilder, 1953; 1960; 1981; 1998; 2013). In fact, in some places throughout the world mathematics developed as a consequence of cultural systems – an example being Vedic India (Wilder, 1953; 1960; 1981; 1998; 2013). This led to two schools of thought: formalists and intuitionists. What resulted was a change in the teaching of mathematics, with most education institutions adopting the "set theory approach". This shift in the education of mathematics later came to be well known as the "new mathematics movement" of the 1960's and 70's (Ellis and Barry, 2005).

Research shows that numerous positions within educational philosophy have changed over time (Abdulwahed et al., 2012; Tularam, 1997a). New ideas have influenced the teaching of mathematics, with particular examples being the behaviourist philosophy of

Thorndike (Thorndike, 1913), Skinner's condition-response method (Skinner and Holland, 1960), the cognitive learning approach of Piaget, Gagne's reductionism, and the multiple intelligence approach of Gardiner (Gardner, 1983; Savitz and Savitz, 2010). A notable shift in the teaching of mathematics occurred in the 1980's with the widespread adoption of Polya's problem solving approach (characterised by four steps: understand, plan, do and check). This approach was later revolutionized by a number of authors, leading to positions such as neo-Piagetian, and theories based on constructivist (radical and sociocultural) ideals. Of all the developments and new ideas witnessed in mathematics teaching and learning in recent times, the methods inspired by Polya's problem solving approach seem to be the most persistent and widely used (Abdulwahed et al., 2012; Tularam, 1997a, 1998).

Next we mention a non-traditional teaching approach known as "asynchronous learning" (Murphy et al., 2011). The term "synchronous learning" is used to describe scenarios in which teaching occurs most often at a particular place and time. Many traditional approaches are implemented by means of such scenarios – the lecture and classroom approaches being immediate examples. Asynchronous learning, on the other hand, describes scenarios in which teaching of a subject is not bound to temporal and locational constraints as found in synchronous learning (Mayadas, 1997). Asynchronous learning draws inspiration from Piaget's constructivist theory, and advocates learning via self-study combined with asynchronous discourse (and interaction) with teachers, instructors, or fellow students or learners in the same course of study (Wu et al., 2008). While asynchronous learning can be facilitated by many means, in modern times it is most often facilitated by computer networking and online resources such as email, discussion boards, forums (and other forms of threaded conversation), blogs, and wikis.

Advocates of asynchronous learning claim that asynchronous has many advantages not found in synchronous learning; such as a lack of temporal and locational constraints, interactivity for geographically or temporally separated students, accessibility to a large and varied body of prospective students, ease of administrative management; and maintenance, persistent availability of all course materials (typically through the use of electronic document archiving, or similar) (Bourne, 1998; Mayadas, 1997; Murphy et al., 2011). But some problems have been found, namely the requirement for a student to have a computer with internet access, costly setup and ongoing maintenance of all underpinning infrastructure (such as computer networks, data storage and recovery systems, and audio and visual equipment), software installation with ongoing technical support, and training of staff in the usage of such software (Palmer et al., 2008). Interestingly however, three commonalities has been found amongst successful asynchronous learning scenarios, namely i) the existence of a facilitator whose purpose is to guide, nurture and encourage relevant discourse amongst the students, ii) said facilitator recognizes and supports the individualistic construction of knowledge in each student, and iii) the participants in the scenario allow an amount of social development or interaction amongst themselves as part of the learning process (such as play or leisure time between or during periods of learning) (*Waltonen-Moore* et al., 2006).

The progress in this area has led to a major movement in teaching that is driven by the recent inventions in computer technology, and can be labelled in a general sense as the "e-learning" movement. It is the latest in a succession of new learning approaches seen in recent times, and is believed to be based on the constructivist philosophy. E-learning as defined later has indeed made some remarkable changes to many learning contexts, largely due to its use of computers, videos, applications, and other electronic media. Yet as this paper will show, many findings emerging from research into e-learning approaches are not as clear or intuitive as one might expect or desire. Some researchers claim that academic gain (Seiler, 2011; Virtual labs), others show improvements in some social aspects of learning (Chang et al., 2011; Henderson and Broadbridge, 2007; Hirsh-Pasek et al., 2015; Wegerif et al., 2015), whilst some suggest little or no significant differences in learning concerning primary and secondary schools - over many years of comparative research between traditional and e-learning methods (Chang et al., 2011; Macgregor and Turner, 2009; Nusir et al., 2012; Tawil et al., 2010; 2011; Ward et al., 2012; Zang, 2005). While the debate continues, no clear consensus has been forthcoming from any prominent education sectors around the world. Essentially, the most modern of e-learning approaches (along with their related e-teaching approaches) have shown some notable gains, but when compared over an appropriate length of time, there seems to be no significant differences between the outcomes of non-traditional and traditional methods of teaching engineering and mathematics (Hirsh-Pasek et al., 2015 and Wegerif et al., 2015). One unpublished study even suggests that e-learning environments may in fact hinder, rather than promote, the development of problem solving skills within students but there is a long road ahead for educators to decipher the essential differences because of less focused studies thus far in this rather important area of research (personal communication; also see https://www.joomlalms.com/blog/guest-posts/elearning-advantages-disadvantages.html).

Problem solving is mathematical thinking - Mathematics is problem solving!

Tight et al. (2009) defined problem based learning (PBL) to be "a total education strategy based on the principle of using real-world problems as a starting point for the acquisition and integration of new knowledge." For its generality and comprehensiveness, we adopt this definition in our discussions of PBL. The focus of all learning that ensures is based upon a real life problem. However, it is noted that most work done in the eighties did focus learning and teaching on problem focus during teaching (Polya, 1970; Schoenfeld, 1992, 1994; Taplin, 2016) but this goal is yet to be achieved it seems. The notion of e-learning as used in this paper focuses more upon the nature of tools in terms of various modern technologies and computer based learning approaches that are used in tertiary teaching.

Problem solving has the various aspects such as exploring, conjecturing, testing, and reflecting or verifying as well as a number of other strategies that have been well revised over time (Anderson & Krathwohl, 2001; Polya, 1970; Bloom, 1956; Krathwohl, 2002; Krathwohl et al., 1964). Each is and probably all of these aspects are critical for the learning of concepts, skills and procedures, particularly in mathematics (but for all other courses as well) (Schoenfeld, 1994). What has not been understood well in the past has been the fact that it is just as critical for students to learn the "problem solving processes" themselves for them to become successful learners in mathematics; that is, to learn to reflect upon all learning and then conduct a critical review of their own learning – thus being critical during mathematical thinking itself. Expert and novice studies show that good mathematical thinkers demonstrate excellent problem solving styles and techniques (including metacognitive strategies) – the experts are to be good problem solvers and thus learn new mathematics easily by focusing on the deeper structures of the task at hand (Carpenter, 1989; Stacey & Groves, 1985; Thompson, 1985; Tularam, 1997a). Experts usually have a large knowledge base. A chess expert for example will know all the possible patterns on a chess board (Ross, 2006).

The experts' learning processes often relies on the use of higher order skills, which are demanding even for the best of us let alone high school and tertiary students (Carlos and Bloom, 2005: Leinhardt, 1989). So we as instructors need to know what, how, and when a student has gained sufficient problem solving skills and knowledge to then help students learn/acquire subsequent mathematical knowledge (sophisticated mathematical content) (Taplin, 2016); that is required to move on in the field. However, equally, and in turn with the more sophisticated mathematical knowledge now gained provides the student new ability and facility to increase the strategic problem solving processes and to apply this newly developed knowledge base to understand other higher mathematical knowledge once again. It is only when this circularity is well-understood that a student can become a successful learner of higher mathematical knowledge; that is, by being an active problem solver within the domain of mathematics and engineering courses (Ambrus, 2014; Carlos and Bloom, 2005).

Schoenfeld (1992) defined mathematical thinking as abstract thought, metacognition, and thinking that involves the understanding of the underlying structures in mathematics; as well as thinking that facilitates application to real life and to problem solving. It is therefore critical to include the mathematical thinking in the act of the problem solving itself (Wilson, Fernandez, & Hadaway, 1993; Tall, 2001, 2004, 2013). This however leads to many difficulties in the teaching process – this is a circular argument as noted; that is, "students with greater mathematical knowledge may be better mathematical problem-solvers and students who are stronger problem-solvers may then learn and solve problems in mathematics more efficiently" (Pajares & Kranzler, 1995). Clearly, there are many more variables that are in the involved in the process of learning and problem solving than simply knowledge variables, such as student self-efficacy, attribution, beliefs, motivation, persistence, attitude; and also the instructor variables. The research in each of these areas show that the affective variables are all related to successful performance in mathematical problem solving (Albano et 1l, 2013; Bidaki et al., 2013; Hiebert et al., 1996; Pajares & Kranzler, 1995; Pajares, 1996; Taplin, 2016).

So in the end problem solving is also a complex process and appears to be a course itself for students to do; that is to be learned and taught (Schoenfeld, 1992; Tall, 2004; 2013). It is the author's belief that this problem solving process and knowledge can be taught to students through the teaching of mathematics itself. Problem solving is more clearly and perhaps directly involved in the learning process of early mathematics as well in higher mathematics. This is reflected in the fact that mathematics courses are now more often believed to be critically important for all aspects of learning across disciplines (e.g., National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010).

Problem solving generally appears to be a prerequisite for the learning of mathematics itself as well as its applications (Ambrus and Barczi-Veres, 2016; Schoenfeld, 2001; Tall, 2001; 2004; 2013). Then how can those with less than satisfactory problem solving ability successfully learn early mathematics let alone higher mathematical knowledge? Importantly, we must also consider the case where in tertiary courses a student is taught to re-construct the knowledge of others so that they may own knowledge or retain their knowledge. Then the question arises in that how can this occur if the student is not capable to do this even at the tertiary level because of the lack of problem solving knowledge for example. So in turn another question that essentially arises is that why is this the case given that so many students who arrive at the universities (in Australia for example) have not gained the necessary skills after so many years of learning from Grade 1 onwards;

where such inquiry based learning and re-constructing processes has been learned and taught in classrooms (Bhakta et al., 2007).

Education researchers have continued to focus on whether a student has been learning through problem based learning or inquiry based learning and the like; that is to what level (compared to previously); and also whether the learning should be continued with the process of constructing of knowledge (Ambrus, 2014; Ambrus and Barczi-Veres, 2016; Pedaste et al., 2015; Tall, 2004; Tularam, 1997a). It is argued that perhaps it is indeed an important goal during the earlier years of learning but whether this is to be continued in each and every course in universities is still an open question; given the students have had much such inquiry and constructivist based learning backgrounds already from earlier years, and thus would be deemed to have had much expertise in problem solving – problem solving knowledge should have been acquired over so many years of learning. Even when such questions have not been addressed adequately but we have now moved to the computer learning environments. The current view is that computer technology can help in this regard (inquiry/problem based learning etc.) and traditional learning may have been the cause of the lack of success in mathematics learning and problem solving (even when it could be argued that this learning has been serving well gone for over many decades - at least in the teaching of teachers and teaching related higher degrees etc. given the high level of development that has gone over time till now for example). It would be reasonable to assume that novelty would increase student motivation to work on computers and the novel aspects of much computing software would be enough for students to engage much independent inquiry based learning; that is, by using videos, mobile phones, and in a more general way any of the latest available computer technology in teaching (Tularam & Hulsman, 2015). For the purpose of discussions of approach or type of learning environment in this paper such learning is more commonly referred to as the e-learning environments - here instructors use computer based methodologies to provide learning and teaching materials usually online and available at all hours; even though for the learning of content related to mathematics, science and engineering; it would be useful for our discussion to investigate asynchronous, open 24/7 and others here most of the literature work is based on software and platforms yet in some cases the open and anytime learning are also included. The other approaches including the abovementioned will be considered in another paper being presently developed.

E-Learning and teaching traditional teaching – review of recent studies

There are number of definitions of e-learning (also called elearning or eLearning) but Stockley's (2003) definition appears appropriate for mathematics education. "E-learning involves the delivery of a learning, training or education program by electronic means. E-learning involves the use of a computer or electronic device (e.g. a mobile phone) in some way to provide training, educational or learning material." (http://www.derekstockley.com.au/elearning-definition.html). Clearly, the actual procedures used in E-learning environments can be varied such as online training or education, Internet or an Intranet, CD's and DVD's and so on. It seems that distance education was one of the first areas that used e-learning in its delivery. Interestingly, the learning through e-learning is considered "on demand" and as such overcomes many of the issues that plague the modern tertiary student; such as attending lectures on time, avoiding parking at universities and other difficulties faced in travelling to places of learning etc. A combination of various methods, including traditional based online 24/7 lectures and tutorials can also be included e-learning but this type of combined effort is usually referred to as blended learning.

Donkers et al. (2010) argued that "E-learning in the context of problem-based learning (PBL) refers to the use of software to: support the learning of specific knowledge and skills, support communication and group work, and support assessment and reflection." (p. 1); while Vestegen et al. (2016) stated that "E-learning includes a range of technological tools and facilities employed to support or improve the learning process of students, for example to support the learning of specific knowledge and skills, to support communication and group work, and to support assessment and reflection (Donkers, Verstegen, de Leng, & de Jong, 2010). E-learning is widely used to support distance learning and face-to-face learning" (p. 1). Similarly, Vestegen et al. (2016) defined "Problem-based learning (PBL) is a powerful student-centred educational approach, where learning is based on authentic problems (e.g., Barrows, 2002; Barrows & Tamblyn, 1980; Moust, Bouhuijs, & Schmidt, 2014). It seems that e-learning can be defined as "learning facilitated and supported through the use of information and communications technology" (Vestegen et al., 2016, p.1; JISC, 2014).

In brief, a computer that allows students to interact online and in real time may be referred to as an e-learning environment. A two or three dimension figure can be rather useful in demonstrating geometrical work and graphing in mathematics and engineering. The algebraic manipulations may also be presented in a more colourful and perhaps meaning ways using computer software such as Matlab and Mathematica etc. While not being concrete objects (of real life), it is in fact possible to represent pictorial and 3D graphs that help bring mathematics to real life all be it in two dimensions mostly (Hollebrands & Lee, 2012). There are many examples of such e-learning environments in mathematical (e.g., The Geometer's Sketchpad, GeoGebra, Cabri); and statistical environments (e.g., Fathom, TinkerPlots). In such programs students may be able to manipulate virtual objects etc. although not physical real life objects (Hollebrands & Lee, 2012). The software programs are designed to help students understand relationships, develop deductive and logical arguments using many examples that aid in practice. It is argued that the programs may help reinforce mathematical interconnections with exercises that provide almost instantaneous feedback. The pictures and objects tend to concretize mathematical concepts so that the learner can explore mathematical relations between variables (Baccaglini-Frank & Mariotti, 2010). Some research shows that students do form deeper and a more interconnected mathematical understanding in e-learning environments (Dick & Hollebrands 2011; Duval, 2006).

To critically examine the success of such e-learning environments however, there are mainly early grade school studies and a small amount of high school studies; with rather few tertiary based studies. In such studies the students are often asked to complete problem solving tasks while they are observed for their behaviour; both physically and cognitively through interviews. Their success is often measured by their ability to successfully perform some tasks while they talk aloud. The author has done much work in this area and examined problem solving in algebraic and mathematical problem solving studies over a period of more than 20 years (Tularam, 1997, 1998, 2013, 2014; 2015). In fact, there is a related study that is presently in press, which has investigated trajectories of students during their solution process when asked to complete tasks. Each student trajectories of problem solving were examined and the author noted that student access to technology in general may not assist in their mathematical problem solving and may at times hinder it even further (unpublished paper).

The e-learning environment is considered a new concept of teaching and learning but has probably its origins in distant learning. The aim is to increase the effectiveness of teaching and thus learning. However, Colace et al. (2014, p. 9) argued that the aim is to increase the quality and effectiveness but of the traditional teaching; which has slightly different implications and, as will be noted later, may indeed be in line with the present thinking in this important area. The purpose of this paper also falls into the general aim of their work in that Colace et al. (2014) wanted to know whether an e-learning classroom environment is an effective learning environment for children when compared with a traditional learning environment. The change in thinking has occurred mainly due to the advancements in technology and indeed the most current related e-learning type processes (smart mobile phones). Realizing which environment creates and engages students with a frame of positive learning motivation, attitude and behaviour; and that has a sound and meaningful academic basis will then aid the planning and implemental decision making in schools and universities.

There are a number of the variables that may confound the results of the studies when one analyses some of their findings more carefully. Some of these are: technological awareness of students, general mathematical ability, teacher competence - lack of training of teachers for modern/non-traditional teaching, student social mobility, motivations and perceptions of usefulness of tasks, and indeed the usefulness of e-learning itself just to name a few. Therefore, the studies that are included in this review are those that have controls on many of the variables mentioned and thus making them appropriate findings to discuss in such a systematic analysis.

A survey of Chemistry lectures from years 1-4 was undertaken in 2004 - 2005 published in 2007 (Shallcross and Harrison, 2007). This was to examine the differences between three delivery methods - category 1 used only electronic media to deliver courses, category 2 used a mixture of electronic and non-electronic and category 3 used non-electronic only. The interviews with both students and lecturers showed that non-electronic methods were preferred in general, but there were no significant differences noted among the methods. However, there were some difficulties noted: in the main the electronic presentations went through too much material thus covering a lot of work (learning time spans). It was also noted that students felt that they needed the hard copies of the notes and they were not provided. Also, the presentation contained particularly complicated diagrams or seemingly irrelevant images while the lectures were presented too quickly. Many of these aspects can now be improved and many of the difficulties noted could now be successfully managed. A current study by the author noted that a few students disliked colours on slides. Although a small number the comments are interesting about electronic media. This poses questions such as are we going to see a different set of learning styles considered for the online teaching methods.

In another study Zang (2005) examined the effectiveness of computer-assisted instruction (CAI) versus traditional lecture-type instruction on triangles. The students in the control groups were taught the concepts of triangles in their traditional classes, while the students in experimental groups were instructed in a computer lab. The groups and teachers including content variables were controlled. The analysis revealed that there was no statistically significant difference between the students' achievement in the control and experimental groups. The findings suggest that teachers use computer assisted instruction software that happens to be a novel item for students making their initial motivations high; but over time, the process of learning becomes more mundane so it seems then that computer learning may be used more as a supplemental tool for teaching and learning in the longer run.

In terms of constructive methodology, and perhaps inquiry learning, Tucky (2006) studied two teaching methodologies in a middle school algebra classroom, maintaining control on as many variables as possible. The study involved a pretest-posttest design. "The control group received the standard instructional practice, known as the "direct method", while the treatment group received the same "direct method" and additional "constructive" methodology in the form of manipulatives and group reading of the subject matter. No statistical difference was found between the means of the two classes on the posttest, therefore, it was concluded that the additional teaching methodology did not affect the student outcome" (p. iii, Tucky, 2006). This set of findings appropriately introduces the next section that reviews work on this important issue. A number of pure and applied mathematics as well as mathematics education professors worldwide are included in the analysis.

A qualitative analysis: analysis of mathematics professor comments (2014)

The inquiry based method (IBM or PBL) of learning is often pushed in schools as means of learning that allows knowledge to be retained in the longer term. While this may be true yet others may say that memorizing also allows us to retain much of what we want to retain. A lively discussion has occurred recently among a number of professors and case of this is presented in the following. The professors have many years of research experience including overseeing many different teaching programs over decades.

Dr. Mantyka provides remedial help at Memorial University's Math Learning Centre using discovery-based learning techniques; for example, she teaches methods for division and multiplication problems in a meaningful manner (http://www.universityaffairs.ca/features/feature-article/how-to-teach-math/). She has argued that while the inquiry or discovery learning method makes the process more transparent, but equally IBM/PBL makes the learning more complicated (Kang, 2016). In fact, Dr. Mantyka stated that the inquiry based approach appeared to be putting too many students on the math casualty list, in that they are unable to automatically recall math facts or use quick and efficient algorithms, in order the learn the processes that underpin the more complex mathematical concepts and skills. She has noted that around 19 percent annually failed a mandatory math test required for entry into any Memorial math course. The students unsuccessfully solved division questions even using the work that was learned through IBM - dots or successive subtraction. Importantly, the results are not encouraging when students had learned through IBM - the discovery-based techniques. While these students are not math majors, they need at least one math course for their chosen program in this college.

The importance and utility of mathematics and statistics is now agreed upon by most researchers. While in the past only those in science degrees did maths, Dr. Mantyka, confirmed that "now, even a degree like social work has a fourth-year course in research methods, which is essentially statistics". The low levels of mathematics skills have for many become a barrier to the higher learning. Support of this is noted worldwide; especially among advanced economies such as the US, Britain and Australia (Tularam and Amri, 2011; Tularam, 2014, Tularam and Hulsman, 2013; 2015).

The example of Canada is significant given that Canadian university mathematics departments have been complaining about this aspect for many years; but only in the more recent times the problem has been labelled a mathematics panic - it was noted. The Organisation for Economic Co-operation and Development's (OECD) Programme for International Student Assessment (PISA) showed that while Canada was still doing well, it had fallen 14 points, from 532 in 2003, to 518 in 2012; in fact Canada had fallen from the top 10. The OECD's assessment of adult skills – Programme for International Assessment of Adult Competencies indicated that Canada was below average in math skills with the new generation showing the "lowest" performance level.

In Canada, the discovery-based teaching approaches (PBL) were introduced from the kindergarten to Grade 12 schools mostly supported by the education researchers in order to counter the lowering of skills. The report also pointed out a lack of teacher education, but it was noted that the mathematicians complained about the discovery-based learning methods and related approaches that were being used to teach; in that the methods of teaching left our modern students with no firm grasp of fundamentals in mathematics. Clearly, there could be so many reasons such as those mentioned earlier that could be responsible for the low performance of students. However, the issue was important and to be addressed nevertheless after examining performance under controlled situations.

The Canadian traditionalists argued that students need to have automatic recall of math facts and standard algorithms otherwise during problem solving the cognitive load will be too much (Tall, 2004). They argued that with good memory skills, student can rely on their working memories to grasp the more complicated aspects of problems through reduction of cognitive load. In this manner they argued that students can perhaps learn to transfer their prior knowledge more easily and thus gain a "deeper" understanding of the higher mathematical content, skills and strategies. This would involve the use of the older direct and traditional teaching methods followed by practice – which was well enunciated in the Thorndikean and Skinner type theorists' methods of the old (Lutz and Huitt, 2003). While not advocating the older in full, many aspects of the older methods were to be involved and this they believed was well supported by the modern neural network connectionist theory of learning. The hope was that students would have a rather "solidified" knowledge over time – a highly developed knowledge that is inter-connected was deemed to be more deeply connected (Raghubar et al., 2010). The emergence of new knowledge would occur during problem solving from a well-established and highly connected database leading to creativity as well for example.

In contrast, and against memory based learning, Dr. Savard found that while some students could memorize multiplication tables they could not apply that knowledge successfully. Meanwhile, she had to give zeroes to students who showed sound reasoning in their problem-solving, but arrived at the wrong answer through calculation errors for example. According to Savard: "For students who don't have a good memory, they're done…". Savard argued that often "we observe that for students who do have a good memory…they can do it in school, but in their personal life, there's no [understanding] about it. It doesn't make any sense."

The above studies and discussion are leading to a Hegelian solution to this major problem in Canada. It seems that the discovery vs. traditional math-learning controversy should not become an either-or argument. There is a place for pure mathematician academic's view as well as an education researcher's view; importantly, both these academics now agree more than ever, that there is a place for discovery but equally there is also a place for mastery of mathematics skills. The discussion showed that the disagreement is in how much effort must be placed on discovery and equally how much must be on mastery of knowledge; it seems then the main question boils down to where should the emphasis be.

The education professor McDougall (University of Toronto) argued that discovery learning and traditional or direct instruction approaches are "both right,"…"We need to have skills instruction because there has to be some memorization. We need to know our math facts – multiplication, addition, subtraction – in order to continue on to do some discovery-based learning….and we can also use discovery-based learning to reinforce and to learn our math facts, to build a better understanding of the big ideas in math." (Ontario Institute for Studies in Education: Dr. McDougall).

In Canada, importantly the province of Quebec is well known for its mathematics innovation - the OECD student assessments ranked Quebec in the top 8 worldwide (PISA). The state uses a discovery-based curriculum (PBL) yet it emphasizes that students must memorize certain math facts and algorithms by specific grades – while other provinces in Canada require knowledge or understanding but not the memorization of the same. The public outcry for better math has led the Manitoba province to go back to explicit instruction of math facts and standard algorithms most recently. Alberta and Ontario have indicated similar changes will occur there. In early grades, the standard long division is sometimes left out of discovery learning for it is difficult to represent through conceptual models. Alternative methods such as estimation or successive subtraction are often used instead. However, it is argued that "*this neglects the fact that the standard algorithm is needed when students reach more advanced mathematics, such as dividing polynomials*" (Dr. Craigen: University of Manitoba - Western Initiative to Strengthen Education in Math (WISE Math). Dr Craigen argued that "*Long division is an important systematic process that is mirrored in a lot of higher-level skills*," and also that "*our entire society is built upon algorithms. People talk [about] 21st-century learning, and then one of the first things they do is they throw out algorithms*."

Nonetheless, Dr Dawson does accept that the discovery method may provide a better conceptual framework than "*simply learning a technique that someone else shows you*." But equally, he then states that for a tertiary student or high school student to learn "*a set of techniques that took some very bright people many hundreds of years to put together*" appears difficult and demanding for an average student perhaps to undertake. He added "*To expect everyone to discover this themselves is unrealistic*."

In the end, there is a need for a stronger foundation in mathematics, science and engineering concepts for the future generation of students to be successful (Tularam, 2016). This means that there has to be better trained teacherswho can develop or design activities for learning that is flexible enough to allow for individual differences, learning styles, age, maturity, motivation, attitude, beliefs and so on; in this manner the students may then develop their own conceptual connections, deductions, logical conclusions, and only through a deep exploration of real life problem solving tasks or thought experiments, can prior knowledge be applied to develop new and higher abstractions of lower level concepts (http://www.universityaffairs.ca/features/feature-article/how-to-teach-math/).

In response, Dr Mighton (from Toronto's Fields Institute for Research in Mathematical Sciences, JUMP) described a "third way"; one in which the standard mathematical rules and procedures are learned through repeated practice, while at the same time using concrete models etc. or physical activities that focus on and thus help develop students' conceptual, critical thinking and problem solving skills. It seems that that the "education biased" professors want to ensure understanding, but the mathematicians may more likely believe memorizing of rules and procedures are much more important; but however, what is more important to note is that the mathematician "knows" that memorizing does not necessarily exclude understanding. Mighton argues that "*There's miscommunication between the sides. I think you can have a hybrid or a very effective combination of discovery with guidance that doesn't draw these false dichotomies*."

It is indeed now the time to move on and accept the fact that both the traditional methods and the new inquiry based, discovery learning, e-learning, PBL, computer software driven learning are all equally important for education (Tularam, 2015; 2016). So then how does a lecturer or a teacher (both of whom a highly trained in their fields) develop methods of teaching in today's classrooms. In response, Dr. Mighton argued that

> "teachers must be freed to experiment with what works best, along with a rigorous tracking of student results and appropriate adjustment of approach in response. It is argued that students can perform much better than the best are performing presently so we need to lift student expectations at most colleges and universities for the sake of mathematics and sciences. To end this case study, it is said that "Even the advanced countries of the world who do better than [Canada] aren't even coming close to realizing the potential of children," (http://www.universityaffairs.ca/features/feature-article/how-to-teach-math/).

In light of the introductory section on research on e-learning, PBL and the above case study discussion, the proportionally small number of research studies and their findings related to e-learning, PBL, blended and standard teaching methods concerning mathematics teaching, and these are presented in the following section. In each case the studies are filtered out of many that were not able to control many confounding variables described earlier.

Further research studies concerning traditional teaching and e-learning?

It is true that the important research findings and work concerning traditional and e-learning relate to the work done in most recent times; mainly in the past 5 years because student ability to use technology and the nature of the technology itself have both changed dramatically. However, for the sake of completion some older studies on the issue of technology and the nature of their effectiveness in learning are included in this section. They also provide information on the nature of difficulties in the past, including how students view of technology may have changed over time, for example.

Harrington (1999) studied teaching methods and found that traditional face-to-face course did well overall, regardless of GPA, but the GPA was the major predictor. It was the intrusive academic advising or more personal contact with the instructor that was most important it seems particularly for those who were marginal students. So it seems then whether this was done via face-to-face, electronically, online chat, texting, or discussion boards etc. the continued success of students with marginal cumulative GPAs was based on the personal contact process regardless of course delivery mode chosen; but this aspect does require more research. Harrington found other main differences in that the online students were mainly female, older with more experience than those in the face-to-face instruction group. The online completions had greater college credit hours than face-to-face completions ($F=3.76$, $p<.01$). The face-to-face withdrawers had earned substantially fewer credit hours than online withdrawers. In both delivery types, the course completers who failed had significantly lower cumulative GPAs than either successful completers or withdrawers.

Benard et al. (2004) noted that the background variables significantly contributed to the academic performance for online and face-to-face students. It was important for students to delve into active decision making, for the study noted that many even set themselves up for failure. The students were simply not prepared for the courses yet those who did the work did not perform poorly on online when compared to the face-to-face learning. Also, the withdrawal and failure rates were both not significant when compared.

Descamps et al. (2006) examined the role of e-learning technologies including the potential that on-line content can bring to education generally; they argued that this was not realized as much in mathematics. The materials presented are mainly formulae with some concrete and pictorial representations and interactivity is difficult on-line. The developments such as Skype and the various conferencing systems have helped and while the nature of the personal interaction between instructors and students has increased; there was too much demand placed on teachers or lecturers to develop new ways to present notes etc. or refresh to develop their ability to become facilitators or coaches online. It is true that the most effective on-line content is expensive to produce and much funding is required and clearly there is a lack of it particularly at the tertiary level. Descamps et al. (2006) argue that e-learning may become another fad that in the long run may in fact not improve teaching mathematic for it is critical that all equipment "to work perfectly" for students almost all the time. However, they do agree that student education without the information networks is not possible (Descamps et. al, 2006).

University of Helsinki and Florida State University examined how the WebALT Project can bring on-line learning to classrooms (Seppala et al., 2006). Their studies show on-line instruction is an available alternative to traditional instruction in most learning areas. Online was believed to be improving learning yet also reducing costs. While there were some positive results noted a number of disadvantages were also found: these related to lack of allowance for different learning styles, collaborative learning and discussion type instruction etc. The authors argued that each technology needs further investigation and research for there may be issues unique to each aspect of e-learning. It is possible that technological tools may be the answer to students' mathematical difficulties yet some of the results of their studies show that e-learning did not help as much for those with weak mathematical skills; and moreover, e-learning did not seem to help much in problem solving either. Further, those with weak technology knowledge or negativity towards technology may indeed become further frustrated and thus not acquire the necessary conceptual understanding as hoped.

Sepalla et al. (2006) and Caprotti et al. (2007) noted that the technical barriers hindering the development of on-line learning were not allowed for earlier. They stated that it was now practical to teach on-line using video conferencing for example. In their study, they noted that the students tested embraced any synchronous and asynchronous on-line learning at the university level. Their data indeed showed that the on-line instruction method was a viable alternative to traditional instruction. Also, the online can improve learning and at the same time lower the operating costs.

Smith and Ferguson (2005) of Stony Brook University (US) studied student attrition in mathematics e-learning. Their qualitative studies show that when dealing with mathematics e-learning does not work as well. Equally their quantitative study looked at problems of e-learning in mathematics. Student attrition was used as a measure of student satisfaction and course viability with attrition in e-learning and comparing attrition in face to face courses. In fact e-learning recorded higher attrition with significantly higher attrition for math courses than other non-mathematics courses. In the case of face to face courses, there were no differences in attrition rates for math versus non-mathematics. It was hypothesised that the online student populations were different from the face to face group. Also it would seem that the learning systems may not have been appropriate for mathematics learning but nonetheless, the attrition of online mathematics was significant, while the traditional case did not perform poorly when compared to other courses. Whether gender plays a role in online enrolment has also not been studied much at all. Some studies have noted that females are more likely to undertake online classes. In fact they tend to gain better higher grades than males and generally do better in the online environment (McCann, 2006; Friday, Friday-Stroud, Green, & Hill, 2006).

Smith et al. (2008) studied discipline specific e-learning that has been largely not studied when investigating the effectiveness of the e-learning course design. This study investigated how instructors of mathematics viewed e-learning when compared to other disciplines: whether e-learning met the challenges of their discipline. The mathematics instructors suggested different challenges and e-learning solutions than other courses. They found that significantly less mathematics instructors were likely to view the existing e-learning models and management systems of the time as appropriate for their discipline. This was probably a negative bias of the mathematics teachers probably held at that time towards technologically based learning approaches.

Wilson and Allen (2011) studied a historically African American based university (HBCU) and found a marked difference in the success of students taking online courses versus students taking face-to-face courses. The online students had a higher withdrawal rate, failure rate and had difficulty completing assignments on time. It seems students are more likely to withdraw from online courses than traditional lecture based courses where direct contacts are made (McClaren, 2004; Lawrence & Singhania, 2004). However, when such courses are redesigned often Temple (2013) noted fewer withdrawals. Therefore, there are divergent findings in this regard it seems.

Aral and Caraltepe's (2012) examined learning styles and e-learning fields in order to synthesize an answer to the question: "Does considering learning styles improve e-Learning performance?" It was clear that different teaching methods that allowed for different learning styles did help students' understanding of mathematics. There are only a few studies on either traditional learning or e-learning and their differences, hence it is difficult to enable reliable deductions to be made it seems. A small number of studies show positive outcomes but then much less work has been done in the tertiary sector. Clearly, adaptive teaching methods based on learning style would tend to improve mathematics performance as noted in the past literature on mathematics education (Tularam & Hulsman, 2013: 2015; Temple, 2013).

One tertiary level study was done by Tawil et al. (2013). They studied students' perception towards the importance and usefulness of modern technologies such as e-learning (WILEY PLUS) in comparison with the more traditional lecture classes. The sample included tertiary students who have taken mathematics and statistics. This study showed a significant difference between WILEY PLUS and lecturing. Overall, the traditional lecturing was favourable in the learning process for both courses. It was the explanations, notes and assignments provided by the lecturer that assisted their understanding of topics. In this case, teacher level preferences may be one of the main effects of the differences noted.

A positive outcome was noted by So and Ching (2011) who studied the two teaching methods. They commented that "lessons are more interesting when blended using technological resources" (p. 10). They found student academic achievements were slightly higher in the electronic learning environment when compared with the traditional method. The e-learning approach included differentiated lessons made suitable for students at different levels. There were differences in this study such as the parents were also involved in that they helped with homework; in this way, parents can continue with the lessons to help their academic progress at home thus changing the conditions slightly.

Albano et al. (2013) studied online learning concerning professional development of mathematics teachers. The survey results showed that e-learning needs to be well planned: the related tools and activities should be designed in full detail and the scope. The focus and use of activities ought to be well stated to the students, in order to avoid simply their participation. The training should include blended learning; learning should fully involve the use of relevant communication tools and collaborative activities; if there is to be constructive critical thinking, reflection and discussion during the development of teachers.

Academic partnerships review (2011) shows that while there is evidence of students performing as well online than within a traditional teaching setting, there are equally many studies that show little or no significant difference noted between them. Some studies show that the lecturer is the more important factor in that the instructor assures multimodal learning during teaching (Jackson, 2014; McCann, 2006; Walker et al., 2011); so there is much student and teacher interaction (Bidaki et al., 2013; Abdous and Yen, 2010) within the teaching and learning process. It may be that effective teaching may be the most important factor no matter which delivery or teaching and learning method may be involved.

In case of the distance learning studies, there is also the sense of isolation or low level of self-directedness as noted by the Hanover research (Hanover Research Council, 2009). Temple (2013) noted the blended type flipping classroom teaching and learning methodology together with quick student feedback on assessments led to much less lower grades than other methods; and similarly low withdrawal rates from courses. The students tended to take less attempts to pass and were satisfied with courses they took.

Most recently the online teaching of the master’s level courses has increased greatly with the business and education programs being most popular (Academic Partnerships, 2011). But as noted earlier, some studies have showed no significant differences in student performance with respect to the methods. Rather the GPA had a significant effect on exam performance (Trawick et al., 2010) instead. This appears to suggest that students need to be much more prepared before choosing a particular course for themselves.

Tunstall and Bosse (2015; 2016) compared a traditional, lecture-based college algebra teaching with an online literacy learning method; the e-learning was based on weekly news discussion as well as problem-based learning projects that involved data analysis. The survey showed differences in students’ mathematical disposition, attitude, and outlook with respect to the use of mathematics; with the online group showing favourable outcome in each. This suggests that project-based e-learning environment is a promising strategy for fostering the affective component of literacy but there is some cautiousness in that novelty aspect of the work may have influenced the findings. Not surprisingly, the authors argued that much more research is needed to capture the mechanisms through which such growth occurs.

In a large study, Borokhovski et al. (2016) conducted a meta-analysis using 674 independent primary studies that compared higher degrees of technology use in the experimental condition with less technology in the control condition. The differences concerned the effects on student learning outcomes in tertiary education. The result was a low positive but significant effect on learning outcomes of technology. Further analyses revealed positive effect of educational technology in student-centered pedagogical frameworks.

Verstegen et al. (2016) studied the importance of blended learning in PBL contexts and noted that PBL in a blended learning format is appears to be an effective strategy. The principles of PBL can be unified in PBL when using blended learning, although the extent to which a principle can be implemented may differ. Vestegen et al. (2016) conducted a major review of the literature insight in how e-learning was used to support PBL. The contextual and collaborative learning was promoted by PBL problems with multimedia, “but also by enabling students to execute authentic learning tasks, taking on authentic roles so that they learn and apply new knowledge in an environment that is similar to their (potential) future workplace, sometimes also using real or simulated equipment” (p. 25). E-learning was used to supplement the PBL learning experiences including “problem cases and practice-related skills together in a virtual environment” (p. 25).

Some e-learning tools were used to improve collaborative learning and this led to some interaction. The authors noted that online discussions can help to enrich the learning experiences that may to involve real stakeholders such as domain experts in the PBL discussions. Vestegen et al. (2016) highlighted factors such as changes in one element can influence other elements; which in the end may have negative impact on student learning generally (Barrows, 2002); (a) “in some asynchronous discussion tools it is difficult to follow the thread of the conversation; (b) even if tools work well they may become a distraction, cause cognitive overload, or learning to use them may take valuable time and attention away from PBL discussions; (c) prescribing the same resources to all students for self-directed learning leads to poorer discussions; and prescribing a sequence of interaction forms in detail may lead to a teacher-led, directive processes rather than student-centred learning” (p. 26). Moreover, in studies the effect of e-learning led to PBL being influenced in that students “solving problems individually with a computer rather collaborative small-group learning” (p. 26).

Similarly, de Jong et al. (2017) found that: “PBL in a blended learning format is perceived to be an effective and feasible educational strategy. The four key learning principles of PBL (stimulating constructive, self-directed, collaborative, and contextual) can also be unified in PBL with a blended learning format, although the extent to which each principle can be implemented can differ depending on the structure of the educational program and the student population. Educators should carefully consider the role of distance learning in PBL education and consider several factors, including, “getting-to-know-activities at the start of education”, “lectures that are suitable for distance learning purposes”, “guidelines for behaviour in online sessions”, and “the importance of visibility and feedback during education” (p. 210).

In the end developments such as Skype and various novel conferencing systems have all helped the e-learning case. While the nature of the personal interaction between instructors and students has increased via email there is much less actual social interaction. There is a much increased demand on teachers or lecturers to develop new ways to present notes etc. and to become facilitators or coaches rather than lecturers. The attrition of online mathematics students was found to be significant but in the traditional case, students did not in the end perform significantly poorly. It is true that the most effective on-line content is expensive to produce and much funding is required but there is a great lack of funding particularly at the tertiary level. There are some such as Descamps et al. (2006) who argue that e-learning may become another fad and in the long run may not improve the teaching of mathematics much. This is supported by some studies. However, they do agree that student education without the information networks is not possible in 2016 onwards; but what is important is that nowadays students expect all equipment “to work perfectly” all the time (24/7). There will be a need for the redesigning of mathematics and statistics courses given the existing learning systems may not have been appropriate for learning anyway as noted by the not changing longer term levels of achievement over time in countries such as Australia and New Zealand for example; although ipad is now available but again the use and application will require designing for lecture type of teaching in almost all cases. There are indeed a number of hidden curricula that is taught at universities that will be left out in online teaching such as socializing, and meeting and interacting with fellow students, dealing with studying diversity, conducting discussing work in face to face interactions and networking etc. that have all been assumed in the past when student learn in traditional environments for example. Clearly, this is not so good for social adjustment of our students in that interactions occur in multicultural learning environments now more generally. Moreover, it can be argued that each aspect of online technology may now need further in depth investigation and research for there may be a number of issues that are unique to each aspect of e-learning itself, that need to be better understood if it is in indeed to be used an appropriate alternative in modern times.

Discussion

Notwithstanding the differences in the many variables involved in learning generally, the reviewed studies have clearly shown that the e-learning option is more popular when compared with traditional classroom learning; but there are many studies that show not so significant improvement findings or even no significant differences between the two methods overall. The little difference does suggest however that e-learning may be more desirable due to the existence of a number of positive factors such as flexibility in learning, time, and location for example. Nonetheless the studies do show a number of advantages and disadvantages in both. Some issues has been highlighted that include: discipline to discipline differences, lack of "real" interaction and collaborative learning opportunities, gender differences as females may favour online learning, differing personal learning styles causing difficulties in both cases, isolation factor particularly in distance learning, lack of student preparation for online study, or inappropriate choice of online courses, withdrawal rates from online courses are greater than traditional ones, fear of or inadequate knowledge of the tools of technology – thus frightening off many to use online courses are just a few important issues for consideration. For example, the students who require flexible schedules, independent work environments, and possess strong motivation levels, the e-learning method appeared to be desirable. However, when students prefer real life explanations, hardcopy notes, "real" discussion and conversations with others when learning or solving problems, the traditional method appears to be more appropriate. It seems that collaborating in "real" groups or discussing and/or interacting about work is considered to be more appropriately achieved in traditional teaching.

There are problems with disciplines such as mathematics in both the e-learning and traditional methods (Tularam, 2015). The online students tend to be older and perhaps working more hours than others who are full time, in which case online is most appropriate. Equally however, many of the online students may have had "long break" from studies and thus find mathematics learning problematic; since the content is cumulative and rather large gaps in knowledge may exist. Such difficulties are generally not able to be solved so easily online. In mathematics it is especially difficult to develop good quality online mathematics courses with appropriate assessments tools for such cases; there seems to be in the end no other way than one that involves a teacher actually presenting the material and providing a blended learning and teaching case for example.

The above difficulties do in fact exist and clearly if it did not then there would be no need for traditional schooling at all for all could be done online from home for example; and the teaching institutions would have all necessary education materials online, from child care, primary through to tertiary. Importantly, also, there would have been empirical observations of rather deep and significant improvements in learning already for example, in student performance, given that computer technology enhanced teaching methods have been around now for more than 20 years at least; methods have been extensively used in our institutions over this period of time and more. The findings more generally suggest that students must learn both mathematical content and technological knowledge, in both independent as well as a collaborative discussion type working environments. While it is not appropriate to assume the application of technological and online methods involving mathematical problem scenarios will automatically lead to students deeply understanding the required mathematics with appropriate related heuristics; because there is a lack of strong evidence in this regard in the literature, it is nevertheless important to provide much exposure and support with regard to e-learning technology to students and indeed celebrate their use in learning and problem solving.

It is noted that when technology is used in mathematics via e-learning or when it is used to support PBL, students still require general problem solving strategic knowledge for consolidating learning at the same time to be successful. It seems then that student weaknesses noted in e-learning is of the same order as in other more traditional learning methods. Therefore, there seems to be no one best method of learning that fits all and this is mostly because of the complexity of learning and teaching processes generally for individuals. It is believed that during problem solving there are multiple entry points and multiple heuristics but students do not necessary rely on these, but are often rather tunnel-visioned following a particular heuristic (and usually not in favour of choosing another even when the existing fails to provides further insights). But even when the students change and choose another method they often seem to return to the earlier methods probably due to familiarity with that method used. In the end it seems the "emotions" appear to be of major importance in that "only when students are simultaneously "strong" in both mathematical content and the use of technology do both aspects truly enhance the other" (Tularam, unpublished).

The studies show that both types of environments provide students the opportunity to demonstrate self-engagement, problem solving efforts, and opportunities to experience varying levels of success (Jackson, 2014). While, e-learning can provide students with differentiated individualized work that are appropriate to their needs; the traditional can provide the opportunity self-evaluate, collaborate, and discuss or question their understandings with friends or teachers while learning in a live interactive manner (Walker et al., 2011). Both environments offer positives and negatives so it seems that our modern lecturer's teaching mind frame may remain the same yet required some adjustments to account for the nature of technologies etc (Rausch and Crawford, 2012; Tawil et al., 2012). It seems that on balance we need to retain a natural balance in learning environments rather than totally leaning towards one or the other as history shows. It is critical for students to receive personalized and individual instruction but retaining the traditional classroom learning where collaboration and social understandings can be questioned or discussed within a large lecture design as well as smaller group tutorials; in the tertiary case, the now favoured small tutorial groups do not reflect the complex nature of all types of individual inputs; or indeed where a large number of views cannot be shared and discussed together as is the case in larger classes that may be an efficient way to contribute to cost management as well as passing big picture information that is also needed in the more abstract tertiary learning where students are encouraged to rise above their prior types and levels of engagements (Tularam, 2015).

It is true that the internet provides some opportunities for discussion and collaboration but Quigley (2011) has noted that "the internet limits opportunities for young students to develop social interaction skills that are critical to their overall emotional and social development" (p. 749). But clearly research shows that students tend to be more attentive in e-learning and PBL environments but this does not directly imply that learning is taking place even when some amount of gains are noted in academic testing results over the traditional methods. Students need to learn to be analytical, metacognitive - reflective and evaluative of their efforts as well as of others. These sorts of actions include identification of faults in arguments or finding errors in solutions presented in class or in texts. Such processes and teaching actions are more easily and perhaps more effectively done in traditional learning contexts of lectures for example. The concept of social engagement with peers help in the longer run to develop students' abilities to cope with the "pressures of real work environments"; where support and collaboration is important and required. Learners will need to understand that some criticism of their work may also be forthcoming in real work places and as such must experience the same in learning environments. Placing appropriate levels of work pressure such as attending on time to workshops, lectures, tutorials, and other class engagements are all important real life experiences that must be taught in learning environments. In this manner, being subjected to some criticism either by the lecturer or other students etc. may be helpful for learners in the longer run in any case.

In contrast, e-learning banks on the independence and self-motivation of learners themselves to be driven and directed in many cases (Tawil et al., 2010). Although some real life interactions do occur in teacher emails or on Skype discussions these are not considered "real" interactions or face to face interactions that students (humans) are often looking for; where real feelings and "emotions" also play a part in learning and problem solving contexts. In such recorded media, teachers are more cautious in their approach to solving problems in any case for the fear of what may be on record perhaps. In mathematics for example, emails do not provide an appropriate communication platform where typing of symbolic material often requires a lot of time on computer etc. presently (Seppala et al., 2006). A direct video link helps but once again a recording normally places undue pressure on the teacher to always be correct so real problem solving of blind alleys or incorrect approaches may not be shown to the learner for fear of recordings where the teacher not able to solve the problem for example directly at hand; when in fact this is the real approach to problem solving and one that is particularly useful for students to experience in real life problem solving especially when there are no existing solutions for example.

A US (Common Core Standards state from Engage NY, 2014) study did note that "students engage in rich and rigorous evidence based conversations about text" (p. 1) but this appeared to be particularly discipline specific. There are difficulties with learning styles, with low levels of attentiveness, and low self-directedness noted in e-learning and other PBL cases. Rather, students often have to take on self-responsibility for decisions about what and how they are to learn in online cases some. These aspects cannot be assessed as easily in online type learning whereas each tends to be better acquired/learned in traditional classes - mainly through real live student-lecturer and student-student interactions. In all cases in tertiary education, time management is vitally important throughout a degree program. The pressure of being on time for the lecturer for class actually makes the student plan their personal life, to wake up on time so that they can get to class on time etc., student beliefs and motivations may be more easily be exposed during their work in traditional classes, for example; it is indeed important for general human learning that we need to make the learner or the student responsible for their decisions in real life where they are exposed often to problem solving contexts. In this case it seems that assessing whether a student is demonstrating a positive action is better assessed through real interactions rather than online assessment techniques or methods etc.

This does not mean we should not change for the technology is often changing and requires constant learning and updating of knowledge. Often students will need to become familiar with the varied technology devices, such as Matlab, SAS, Mathematica, SPSS, skype, message banks, smart phones etc. in order to be an active participant but this does take some time and individual personalized instruction in most cases. In the end, learners will need to seek and learn to enjoy discovery, which is probably also done better in both e-leaning, PBL and traditional learning contexts where a real/virtual teacher may be present to assess, facilitate and guide the process (Bryan, 2015).

The studies critically examined above can be overviewed by the following note from Abdulwahed et al. (2012) whose comments were particularly appropriate regarding the literature and approaches taken in studies and experiments:

> "Thus, the actions of the teacher are seen to derive from the theoretical stance. The teaching approach only makes sense theoretically if the theoretical principles on which it is based are made explicit. This is lacking in many of the articles reviewed. We might ask whether this matters, or why it matters. The methodology of any research project needs to explain why what is done is done in the way that it is done (Burton, 2002). If research takes a constructivist perspective, then whether it is rooted in Piagetian or Vygotskian psychology, we assume it is looking at the constructions of the individual learner, that it is involved with seeing insights to individual cognition. This is useful if we wish to look closely at the ways in which the individual construes particular mathematical ideas or concepts (see for examples, the clinical interviews of Steffe, 1983, following Piagetian traditions). Such a perspective, with its focus on the individual, has no tools to address the wider social factors that impinge on the learning context and influence its outcomes. Taking a Vygotskian focus draws attention to the ways in which social factors impinge on the individual?[s] consciousness. However, to look at learning and teaching in the full sociocultural contexts in which they are located requires an alternative to constructivism." (p. 60).

What the literature shows is that learning should at least become flexible with the availability of a high level of technology, PBL and e-learning methods. Educators should now focus on how to improve student outcomes in mathematics, science and engineering with a balanced, appropriate and effective implementation of the implications of the research findings that make common sense but also has a sound philosophical basis (Jackson, 2014; Sepalla et al., 2006). But there are also questions about whether some students are simply not prepared for the choice they make thus setting themselves up for failure before starting. There are indeed questions we may pose such as; should students be restricted from taking online courses if they have not reached a certain GPA? Or should students who fail or withdraw from an online course be required to take to traditional courses? This is because online students seemed to have a higher withdrawal rate, failure rate and seemed to have trouble completing assignments by the deadline, if at all in some cases. Yet students' academic achievement rate, engagement, and positive behaviour were more likely to be displayed within the electronic and PBL learning environments. It is true that some social collaboration self-evaluations and persistency of effort as supported by real life support agents were more likely to be exhibited within the traditional learning environments. Moreover, students rated their behaviours more positively within the e-learning environment in comparison to the traditional classroom environment. Also, more on-task attention and engagement was noted within an electronic learning environment, whereas students showed persistent effort within the traditional environment. The effort level was related to their social networks, friendly collaborations, their feeling of some self-ownership of their work within a group etc. There was evidence of self-reflection of their understanding in group discussions; which helped to check their work based on others comments etc. There was some evidence that a real life instructor feedback had a desirable effect when compared to e-learning online feedback method. Some evidence suggested that feedback was inadequate within the e-learning environments. Others said that not all active PBL learning environments are successful. Problem-based learning may be appropriate for some circumstances but not others, for some students, and not others, and for some learning contexts but not others (Vestegen et al., 2016).

Summary, Conclusion and Implications

In Australia, and worldwide, all educators are serious about increasing college or university attendance and particularly in our general attempts in trying to improve STEM at tertiary levels. Thus there is a need to improve students' preparation of learning mathematics in tertiary colleges. Now online e-learning has been considered a viable option to our traditional university teaching but the question is what role technology may play in the learning and teaching process in terms of outcomes. The reflection and review have explained some aspects but many more issues have been put forward as well. A possible solution would be to develop student competence to a greater extent and level in the preparation of mathematics and science before they arrive at college. Helping students learn the rigors of college type of study and appropriate preparation in mathematics and science could be done while in high school with the technology availability - this may also be done through either traditional or online pre-university preparation courses.

But exactly "what learning and how much" can be acquired through online learning and teaching and what role should it play in the learning and teaching of mathematics for example still seem to be open questions. Evidently, there are some positive as well as negative effects of e-learning – in the case of online learning and teaching of mathematical problem solving successes. It is noted that the online and blended learning options are suitable for particular groups of students such as those who are highly motivated learners - particularly those who are simply lifelong learners, or motivated migrants and the minority communities, where the motivation for learning is often strongly motivated by gaining of opportunities to be employable; or simply fitting into a new community tasks such as learning English courses online for example. This means that online learning may be more appropriate for particular contexts or disciplines than in others perhaps.

Also, there are some clear advantages of e-learning systems over traditional schooling methods in that students can get extra help 24/7 together with personalized instruction. The online offers tutorials and lessons that explore a given concept in various ways – differentiated instruction exists allowing for different levels of exploration of concepts with feedback etc. The online material also provides extra material for the traditional teacher for developing in students in depth understanding of abstract/conceptual aspects taught. Many students prefer to use e-learning because it is "convenient and cost-effective". Also, students often need personal tuition and the e-learning methods can be a rather convenient way to deal with this aspect; one may learn at home saving travelling and parking costs etc. Yet there are a number of questions we should be asking in tertiary contexts such as are students setting themselves up for failure by taking online courses. Should some restrictions be placed on students from taking online courses such as a minimum level of achievement or a high motivation level. There are other issues such as should the students who fail or withdraw from an online course allowed to attend traditional classes or continue online etc.

In the end some students may be unfamiliar with mathematical and engineering programs that involve many technological aspects, so it is important that instruction is provided in this regard, which then takes time away from students learning the more important content, skills, and processes learning during a semester (Jungic, 2012). This is often noted in Applied Mathematics and Statistics courses where finally the lecturers/tutors end up having to spend many hours correcting students' SAS attempts/programs in major assessment projects during the semester; just to help run the program successfully because students failing to spend enough time on preparation for projects etc. In this manner, students gain minimal SAS coding knowledge for example. So we must conclude at this point in time that a suitable and appropriate blending of these two types of environments is preferred than demoting one or the other in tertiary institutions if we are to retain excellence in the level of mathematics and statistics in engineering and science degree programs more generally. A large number of studies "also yielded examples of how e-learning tools and facilities can be used to stimulate cognitive elaboration and the (re)structuring of information, thus helping to counteract known bottlenecks in PBL such as superficial discussions in tutorial groups or "PBL fatigue" resulting in the skipping of vital steps" (Vestegen et al., 2016; p. 27). There are certain areas that require research and this concerns "learning analytics and the support of self-directed learning, but this would require a longitudinal approach supporting the development of students over a longer period of time" (Vestegen et al., 2016; p. 27).

It is important that all students learn via a self-directed multimedia training session on how to take online courses and this indeed could be a prerequisite for the first time users of online tools. It seems that the teachers will need to help engage students by teaching them how to learn from online learning or how learning can be maximized from technological e-learning or PBL environments; such as having face-to-face weekly sessions, particularly to emphasise the need to be on time for assignments, workshop sessions, exams etc. For these reasons, there is a need for many more studies on e-learning, asynchronous and PBL learning environments particularly in discipline and domain specific areas. Also, in each of the aspects, studies should also be conducted on attitudinal measures that help identify at risk students. Further work on the nature of relationships between the amount of non/personal contact (between students and the teachers via email, chat and discussion boards) and deeper level success in learning etc. needs to be done as well.

It is the goal of educators to educate students to take self-responsibility for the acquisition of *knowledge* necessary to respond creatively to problem tasks– that is, actively reflect and evaluate outcomes of work. This should be the basis for non-traditional teaching and learning strategies regardless of the academic course level. In this manner, less emphasis is placed on information transmission and greater emphasis placed on developing skills, attitudes and values in the domain areas. This will lead students to engage in higher order of thinking - *analysis, synthesis*, and *evaluation*. Educators need to decide if their courses are intended to require the regurgitation of facts and figures or assist students into developing critical thinking skills with regard to the subject at hand – this process may be done in separate courses. A lecture format may be entertaining and efficient delivery of information, but does it facilitate the best way to learn? Bosworth and Hamilton (1994) posed the following a while ago; do you want to be "*the sage on the stage or the guide by the side*". They argued that we must decide now if education is about "*your teaching*" or "*student learning*". However, is this really an appropriate question to pose in education at all? The complex nature of teaching and learning may not allow a simple either yes or no answer; yet it is clear that in the past education research has focused on either one or the other as being the most appropriate, instead of taking a rather more "holistic" and a systematic position. In a systematic approach, it is assumed that if a section or part of our existing evolved system is removed, there will be consequences to the system and its evolution and this is probably what has occurred in STEM learning over time. What is critical however is the ability of the teacher or lecturer to have the training which allows them a required level of autonomy in order to act appropriately in a given context of the learning situation - that change within a lesson, daily, weekly or semester by semester. This is what must be assumed given the teachers are indeed experts in their fields.

In concluding their work, Moeller and Reitzes (2011) argued that technology may be used for assessing students' needs and allowing for "flexible scheduling, pacing, advising, presenting content in alternative ways, project-based learning, and involving the community" (p. 45). They added that technology has been mostly integrated in curriculum-based and school-based approaches at all levels of primary and secondary schooling. But importantly, technology alone may not transform traditional learning environments into student-centred online learning in any of the areas including tertiary. But, the online technology will support the existing either student-centred or peer interactive, collaborative type of teaching, learning and problem solving with the ongoing advancements.

A Hegelian balancing the nature of the two environments may allow students to learn and progress better than in the past (Lewis and Harrison, 2012; Rausch and Crawford, 2012; Weber and Lenon, 2007). A US meta-analysis found that instruction combining online and face-to-face elements is better in terms of student outcomes than purely face-to-face instruction or purely online instruction (US Department of Education). The literature review in this paper also showed that the research considering learning styles appropriately enhances the learning experience but there are not many studies that examine the impact of learning styles in specific subjects (Mathematics/Statistics). There are also some questions about the methods used in comparative studies; for example, in learning style detection/determination methods are either questionnaires or automated systems that track user behaviour in an e-Learning environment. If students do not develop self-awareness skills or are not motivated in the case of questionnaires, then this and other studies may lack of reliability/validity or both due to the many confounding variables, and the complex nature of student learning and instructor teaching process itself - even when care is taken to appropriate chose studies that are controlled to some degree. Also, simply gearing work to suit the various learning styles seems pointless when in real life problem solving no such position is possible or available; such as when learning through non-routine and novel problem solving. It seems that enhancing all types of learning styles to a satisfactory level should then be the goal of education more generally. Vestegen et al. (2016) noted that "teachers and researchers have little insight in what students do and how this works (Bridges et al., 2012). It is a challenge for instructional designers, teachers, and researchers to start thinking about e-learning to support PBL at the curriculum level, to employ different forms of e-learning at different moments in time in order to provide extra scaffolds for PBL group meetings and self-study early on and to help students gradually develop self-directed learning skills up to a level where they can function in PBL groups with less tutor support." (p. 27).

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Appendix

Taplin, M. (http://www.mathgoodies.com/articles/problem_solving.html; retrieved 11th June 2016)

What Is A 'Problem-Solving Approach'?As the emphasis has shifted from teaching problem solving to teaching *via* problem solving (Lester, Masingila, Mau, Lambdin, dos Santon and Raymond, 1994), many writers have attempted to clarify what is meant by a problem-solving approach to teaching mathematics. The focus is on teaching mathematical topics through problem-solving contexts and enquiry-oriented environments which are characterised by the teacher 'helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying' (Lester et al., 1994, p.154). Specific characteristics of a problem-solving approach include:

- interactions between students/students and teacher/students (Van Zoest et al., 1994)
- mathematical dialogue and consensus between students (Van Zoest et al., 1994)
- teachers providing just enough information to establish background/intent of the problem, and students clarifing, interpreting, and attempting to construct one or more solution processes (Cobb et al., 1991)
- teachers accepting right/wrong answers in a non-evaluative way (Cobb et al., 1991)
- teachers guiding, coaching, asking insightful questions and sharing in the process of solving problems (Lester et al., 1994)
- teachers knowing when it is appropriate to intervene, and when to step back and let the pupils make their own way (Lester et al., 1994)
- A further characteristic is that a problem-solving approach can be used to encourage students to make generalisations about rules and concepts, a process which is central to mathematics (Evan and Lappin,

1994).

Schoenfeld (in Olkin and Schoenfeld, 1994, p.43) described the way in which the use of problem solving in his teaching has changed since the 1970s:

> My early problem-solving courses focused on problems amenable to solutions by Polya-type heuristics: draw a diagram, examine special cases or analogies, specialize, generalize, and so on. Over the years the courses evolved to the point where they focused less on heuristics per se and more on introducing students to fundamental ideas: the importance of mathematical reasoning and proof..., for example, and of sustained mathematical investigations (where my problems served as starting points for serious explorations, rather than tasks to be completed).

Schoenfeld also suggested that a good problem should be one which can be extended to lead to mathematical explorations and generalisations. He described three characteristics of mathematical thinking:

1. valuing the processes of mathematization and abstraction and having the predilection to apply them
2. developing competence with the tools of the trade and using those tools in the service of the goal of understanding structure - mathematical sense-making (Schoenfeld, 1994, p.60).
3. As Cobb et al. (1991) suggested, the purpose for engaging in problem solving is not just to solve specific problems, but to 'encourage the interiorization and reorganization of the involved schemes as a result of the activity' (p.187). Not only does this approach develop students' confidence in their own ability to think mathematically (Schifter and Fosnot, 1993), it is a vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others (NCTM, 1989). Because it has become so predominant a requirement of teaching, it is important to consider the processes themselves in more detail.