Fostering Students' Preparation and Achievement in Upper Level Mathematics Courses

Mehmet Celik
Texas A&M University, Commerce
Mehmet.celik@tamuc.edu

Ali Shaqlaih
University of North Texas at Dallas
Ali.shaqlaih@untdallas.edu

This study describes an intervention to address both motivation, student engagement and preparation in upper-level mathematics courses. The effect of the intervention regarding students' achievements is investigated via students' opinions and data analysis from students' assessments. The results of this study show the featured intervention increases motivation for class preparation and fosters students' 'in and out-of-class interactions'. Moreover, the quantitative analysis performed on students' assessment scores reveals a significant increase on students' grades.

Keywords: Intervention in undergraduate mathematics teaching; mathematics education; fostering students' engagement.

In upper level mathematics classes such as Differential Equations, Fundamentals of Mathematics, and Abstract Algebra, students see the concepts for the first time during a class meeting. Instructors, in a traditional lecturing format, usually presume that students are able to grasp the new definitions and theorems, so instructors try to build on the newly introduced notions during the lecture. Refreshing the previously learned elementary concepts is essential for students to comprehend and make sense of the new ideas introduced in class. We believe that if students lightly recall previous material and lightly touch the new material before a class, they will feel more confident to join class discussions. Questioning mathematical ideas and concepts increases students' comprehension and retention (Park, 2005). Students should be encouraged to read, study, and practice the basic notations and definitions before class. The challenge for the instructor is how to initiate students' motivation, their potential to manage actions that are part of the system that controls emotion (Wæge, 2009; Hannula, 2004, p. 3).

Literature Review

Marks (2000) defined engagement as the attention, interest, investment, and effort students expended in the learning process. Student engagement is an essential factor in the learning process. Actively engaging students in their learning enhances their intellectual and social skills (Prince & Felder, 2007; Duch, Groh, & Allen, 2001; Jordan & Metais, 1997). Students learn more when they are engaged in and out of class. Research demonstrates a positive correlation between student engagement and academic achievement (Marks, 2000; Carini, Kuh & Klein, 2006; Coates, 2005; Furlong & Christenson, 2008; Park, 2005). Park (2005) found that student engagement has a positive correlation with their improvement in mathematics. Student engagement relies on the instructors' creativity and initiative (Protheroe, 2007).

In mathematics as well as in other fields, the achievement scores reflect the level of involvement (Finn, 1993; Axelson & Flick, 2011; Kuh, Kinzie, Buckley, Bridges, & Hayek, 2007). Newmann, Wehlage, & Lamborn (1992) argued that student motivation to work harder increases by engagement. Another major variable in students' engagement is the class size. Research suggests a positive relationship exists between smaller class-size and academic success (Finn, Gerber, Achilles, & Boyd-Zaharias, 2001; Nye, Hedges, & Konstantopoulos, 2001).

Students' views about the way they are being taught and the concepts they are learning have a strong influence on their commitment at school (The National Research Council, 2003, p.37). Unenthusiastic, insufficient self-regulatory manners and other negative forms of effect serve as initiators for students' declining confidence in their mathematics education (Malmivuori, 2001, 2007; McLeod, 1992; Schoenfeld, 1992). What students think about mathematics has a strong influence on how they will engage in learning and whether they will be successful in solving mathematics problems. Students' degree of learning is frequently decided by their views about the nature of mathematical understanding and capabilities, mathematical problem-solving, and their own mathematical proficiency (Hassi & Laursen, 2009). Hassi and Laursen found that students' gains in mathematical thinking depend on their views about the importance of rigorous reasoning and problem solving.

In a traditional lecturing environment, students need to capture the important parts of a lecture, relate those parts with each other and with the previous material if they want the lecture to be beneficial for their learning (Armbruster, 2000). During a lecture, students should write down these important points and notions so that after the lecture students should be able to recreate the lecture (Lew, Fukawa-Connelly, Mejia-Ramos, and Weber, 2016). Thus, reading the material before a class meeting becomes important. The way a student studies a concept is important (Fernald, 2004; Lewis & Hanc, 2012). However, research shows that students do not read the reading assignments (Burchfield and Sappington, 2000; Clump, Bauer, and Bradley, 2004). Clump, Bauer, and Bradley's research (2004) pointed out that more than 70% of students do not complete assigned readings before coming to class. Lei, Bartlett, Gorney, and Herschbach (2010) found out that low self-confidence, lack of interest in the subject matter, underestimation of importance of the subject, absence of regular homework assignments and in-class activities are some of the reasons behind this phenomenon. Lei, Bartlett, Gorney, and Herschbach (2010) also provided some insight for helping motivate students to read before coming to class, such as quizzes, participation points, and extra credit. The frequency of in-class assessments directly effects students' responses to out of class assignments. Some research reveals that during a semester administering periodic in-class quizzes significantly increases frequency of the reading process in advance (Connor-Greene, 2000; Tomasek, 2009; Ruscio, 2001; Agnew, 2000; Fernald, 2004; Sappington, Kinsey, and Munsayac, 2002; Marchant, 2002; Leeming, 2002).

In the 1990s, the National Council of Teachers of Mathematics (NCTM) initiated a reform to increase awareness for mathematical communication and reasoning (NCTM, 2000). Recently, the topics of reasoning and proof have generated widespread interest among mathematics educators (Hemmi, 2010; NCTM, 2009; Varghese, 2011). No doubt that mathematical proofs are challenging for students; we believe that it will help students comprehend the material if students get familiar with the content before being introduced to proof techniques in a class meeting. The

United States Military Academy at West Point has been teaching its courses by what is known as the Thayer method of instruction (Shell, 2002; Rickey & Shell-Gellasch, July 2010): Students study the material prior to attending class. The learning is then reinforced in class through a combination of group learning and active learning exercises done primarily at the blackboard. This approach is similar to the flipped classroom approach (Shell, 2002; Rickey & Shell-Gellasch, July 2010; R.I.T., 2014).

The actual flipped classroom design is to require students to read course material before coming to class and then discuss the content during class (Frederickson, Reed & Clifford, 2005; Strayer, 2012). The flipped classroom approach heavily relies on technology to introduce students to course content, again with traditional lecture approach but outside of classroom via video-taped lectures and other web-based materials so that students can discuss the content further in the classroom (Baker 2000; Collins, de Boer, & van der Veen, 2001; Gannod, Burge, & Helmick, 2008; Lage, Platt, & Treglia, 2000; Strayer 2009; Strayer, 2012). With flipped classroom approach, there is evidence for D, F, and W grades rates decline (Koproske, 2012; R.I.T., 2014) and improvement on students' grades on course assignments (Day & Foley, 2006; R.I.T., 2014). However, in a flipped classroom approach students complain about the loss of face-to-face lectures and they show tendency to skip a class that focuses on discussions and practice related activities (EDUCAUSE, 2012, February). There are research studies presenting a relationship between high attendance rates and high grades (Brocato, 1989; Launius, 1997; Moore, 2003; Thomas & Higbee, 2000).

The intervention we introduce, in this paper, defines the instructor's role in students' engagement as well as describes an approach to foster motivation, student engagement and preparation in upper - level mathematics courses before a class. In our intervention, students are not expected or guided to study the entire material by themselves in advance, instead, in our intervention students are guided with a pre-assigned work-sheet to get lightly familiar with the material. Moreover, the course content is taught in class rather than asking students to study the material on their own as in a flipped classroom approach.

Purpose and Research Questions

The purpose of the study is to present an intervention which addresses both motivation, student engagement and preparation in upper- level mathematics courses. We investigate the following questions: (1) How can an instructor intervene to motivate students into getting familiar with the content before coming to class, in upper level mathematics courses? (2) Is there a correlation/causation between the used intervention and students' achievements? (3) What are students' opinions about the intervention?

Methodology

Two sets of data were collected one from a student survey and the second from the students' performance scores before and after the proposed intervention: (1) an anonymous survey administrated by the authors about students' perspectives on the intervention and (2) student assessment scores from different upper level mathematics classes taught in different semesters by different instructors. Qualitative and quantitative analyses are carried out, one on the survey results and another from the students' assessment scores. To evaluate the effectiveness of the intervention, final exam scores are considered. The variables measured are the final exam scores from two consecutive semesters. The experimental unit is the student; the population of interest is the final exam scores of all students who could possibly be taught using the intervention (Spring 2014) and the final exam scores of all students who are taught without using the intervention (Spring 2013). The final exam questions for both groups were the same and the duration of each semester was 15 weeks.

To see the effect of the intervention on students' achievements, the first part of the course in Spring 2014 was taught without intervening and the second half of the semester Spring 2014 was taught with the intervention. Data was collected from both halves of the semester. Three upper level mathematics classes participated in this study: Fundamentals of Mathematics (class size: 9 students), Differential Equations (class size: 7 students), and Abstract Algebra (class size: 7 students). Each class met twice per week for 80 minutes per class. The same three courses were also taught in a prior semester by the same professors without intervening. The final exams covered the same material and the questions assessed the same level of knowledge in both semesters. This gave the authors the opportunity to compare students' final exam scores from both semesters and to see better the effect of the intervention on student achievements.

The Proposed Intervention

To foster students' engagement, preparation and motivation for class we exercised the following intervention: at the end of each class a set of questions from the content that is going to be covered during the next class meeting is handed out to students, (see Appendix A). Every student has a textbook from which answers to those questions can be obtained. Students are informed that at the beginning of the next class they are going to be assessed on one random question from the set, each student gets a different question. To answer all the questions in the set it requires students to study the definitions, examples, notations, and theorems in that section. This practice aims to foster students' conceptual understanding. Moreover, it helps students know what is expected from them in class (McKeachie, 2002). The number of questions in the set is about seven. Questions are focused on the definitions, notations, and statements of the theorems, as well as some straight applications of corollaries. It may include some procedural questions in case the concept is computational, such as using reduction of order to find a second solution for a homogenous linear second order differential equation. Students are expected to spend about an hour answering the set of questions before the class meeting. Each quiz takes 5 to 10 minutes depending on the questions in the set. At the beginning of the class, each student randomly picks one question from the set and answers it in writing.

Analysis and Discussion

1. Students' Perception about the Proposed Intervention

The sample of this study included 23 undergraduate mathematics students enrolled in Abstract Algebra, Differential Equations, and Fundamentals of Mathematics. Abstract Algebra discussed groups, rings, integral domains, polynomial rings and fields; Differential Equations discussed first-order equations, existence-uniqueness theorem, linear equations, separation of variables, higher-order linear equations, systems of linear equations, series solutions and numerical solutions; and Fundamentals of Mathematics tackled mathematical logic, mathematical induction, relations and functions, combinatorics, counting techniques, graphs and trees, and finite automata theory. All the students in those three courses were exposed to the intervention practiced during the second half of the semester. At the end of the semester, students were invited to participate in an online survey; the data was reviewed by the two authors to ensure accuracy. 18 students (out of 23) participated in the survey. A qualitative analysis along with descriptive statistics was carried out. 50% of the participants were males. 89% were Hispanic, 11% were from other ethnicities. The survey was designed to have nine objective questions and one open ended question. The table 1 below summarises the output for the objective part of the survey. Students' responses are grouped as Agree, Disagree, and Undecided in percentages.

Table 1:

Survey Results (figures are in percentage)

Questions	Agree	Disagree	Undecided
1) The intervention has motivated me to study the subject in advance before it is worked in class.	100	0	0
2) The intervention helped me to follow the discussions in class easily by making me familiar with the notation, notions, and concepts in the new subject.	100	0	0
3) The intervention helped me to interact with my professor and class mates in class easily by making me familiar with the notation, notions, and concept in the new subject.	88.9	0	11.1
4) I can say that this intervention make me more efficient in learning a new subject in mathematics.	88.9	0	11.1
5) The intervention increased my interaction on the subject with my classmates outside of class.	94.4	0	5.6

6) Since we started the intervention my quiz and exam grades went up.	72.2	0	27.8
7) The questions in the intervention were homogeneously distributed and were covering the whole aimed lecture.	100	0	0
8) The time amount, provided in class for this new type quizzes, was appropriate.	100	0	0
9) I would love to have the intervention in my future classes.	100	0	0

From table 1 data, from the students' responses to question (1) it appears that 100% of students feel that the intervention motivated them to prepare for the course material before it is introduced in class. From the responses to questions (2), (3) and (5) it can be concluded that at least 89% of the participants think that the intervention engaged them in and outside of class. In fact, it has been observed by both instructors that the intervention not only has students interact by asking questions in class, but also facilitates the involvement of students in mathematical discussions outside of class. The responses to question (4) stand as evidence that the intervention increases the confidence level and the self-esteem of students in learning mathematics. 89% of the students noted that they are more effective in learning new mathematical concepts, demonstrating that they believe in their ability to learn mathematics by themselves. This is a recognizable achievement for the intervention as increasing the level of self-learning for students is one of the most important goals of education. With the little amount of time that are given for quizzes, the answers to question (8) point out that all of the students feel that the provided time was appropriate. This is a compelling indicator for how well-engaged and motivated the students are.

Question (6)'s outcome shows that 72% of the participants agree that their grades significantly improved. Question (9)'s outcome indicates that students think the intervention motivated and engaged them to the extent that all of them would love to have the intervention implemented in their future classes. Students' self-motivation was also witnessed by the authors from the students' use of office hours. It is worth mentioning that there were not any 'disagree' responses on the survey, which supports the conclusion that students appreciated this new practice.

Students were also asked to answer an open-ended question about what they liked or disliked regarding the intervention. Their responses can be grouped into three trends: the intervention (1) encourages interaction; (2) boosts self-confidence; and (3) reveals the expectations from the class.

1.1 The Intervention Encourages Students' Interaction

The first trend is on *interaction inside and outside of class* and interaction with the professor and classmates. Some of the students' responses are as follow: *(1) I like this new approach because it leads me to be able to go to my professor for any questions or doubts. (2) The class interacted more than any other class I've been. (3) Whatever I didn't understand I could ask during lecture, which was more helpful to me. (4) What I liked the most about this approach was the fact that I was able to make connections between the lecture and the questions requested from the professor for the quiz in order to understand the material*. The vast majority of students indicated that this approach encourages collaboration. We believe that this is due to students' preparation before class. It is clearly observed that the students can follow the class discussions better and ask more specific and explanatory questions. Moreover, it is also noticed that students intensified their study-group participation before and after the class, especially for debating different approaches for solving the assigned problem.

1.2 The Intervention Boosts Students' Self-confidence

The second trend is *self-confidence*. Some of the students' responses are as follows: *(1) I like this new approach because it has reduced my test anxiety on quizzes as it makes me feel more confident to know that I am prepared for my quizzes. (2) I liked how it gave me a chance to see material and try to see what I can understand on my own before the lecture. (3) The part I like the most is that I am more motivated to cover the material on my own and try to prepare by reading the book and studying my notes*. It is clear from the students' comments that practicing the intervention increases the level of self-confidence of students to study mathematics on their own. Students become more willing to devote more effort to studying mathematics by themselves. It increases the students' perseverance.

1.3 The Intervention Reveals Class Expectations

The third trend is *students realising the expectations for class*. Some of the students' responses regarding this trend are: *(1) what I like is that I knew what to expect in the new lesson. (2) I like that I knew what to expect to learn in the new lesson. (3) What I liked the most about this new approach is that I knew what to expect to learn in the new lesson. (4) I like that I knew what I will be learning ahead of time. (5) I like that this approach prepares me for the lecture class in advance. (6) I liked how we were given a chance to work on problems in advance*. Many students indicated that coming prepared to class is a helpful practice in order to better understand the concepts in class. Indeed, McKeachie (2002) argued that knowing the course expectations helps students to prepare more effectively for the class. The difference that this preparation brings into students' responses, discussions and interactions is clearly observed by the authors; the number of students involved in the discussions on the material has noticeably increased.

1.4 Other Factors about the Intervention in Students' Opinions

Students were also asked to criticise the intervention. Some of these comments are as follows: *(1) the quiz questions do not prepare the student for the type of questions that should be expected on the tests given that the quiz questions are kind of reading comprehension type questions and not actual problems to be solved, which is still helpful nonetheless. (2) I disliked some of the questions that I felt needed more explanation by the professor than the book itself to complete.* As a response to these comments, it is important to mention that the questions in the implemented intervention are not meant to prepare students for exams, but to motivate them to prepare themselves for the class. This is also explained to students. The gains through this practice, such as, the interactions among students and professor and the boost on students' self-confidence to study mathematics, are intended to increase students' performance in the class as a whole. Furthermore, the idea behind this intervention is that if students don't understand a concept, they will go to the professor, a classmate, or ask questions about it during the class. In either case the intervention helps create opportunities for improving learning skills.

2. Statistical Analysis on Student Assessment Scores with and without the Intervention

The sample of this part of the study included 23 undergraduate mathematics students enrolled in Abstract Algebra, Differential Equations, and Fundamentals of Mathematics. A comparative analysis of the total score of all assessments (all in-class quizzes and exams) from the first half of the semester while the intervention is not implemented and the total score of all assessments (all in-class quizzes and exams) from the second half of the semester while the intervention is implemented.

From the Table 2 data, first, we compare the means of '*total score of all assessments prior to the intervention*' and '*total score of all the assessments with the intervention'* by Paired Samples T-test. The null hypothesis is *the total score in each group do not differ*. It is observed that the p-value is .003 (t(22)=-3.38) less than .05 which means that the *total score* in the groups do, in fact, differ. The null hypothesis is rejected. We conclude that the implemented intervention is effective. Students did better after applying the intervention than they did prior the intervention. Moreover, the standard deviation decreased by 5.5 which may indicate less individual differences and hence a better learning environment.

Table 2

Descriptive Statistics on Total Scores

	N	Minimum	Maximum	Mean	Std. Deviation
total score prior the intervention	23	24	90	60.8	19.6
total score with the intervention	23	39	95	69.3	14.1

Independent Samples T-test is also conducted. We have one dependent variable (testing variable) which includes Final Exam Grades for the total of 46 participants in the sample. We group them into two groups: "Final Exam Grades from a semester prior to the intervention", and "Final Exam Grades from the semester in which the intervention was employed". Sample size in the group of Final Exam Grades from the semester prior to the intervention is 23 and in the group of Final Exam Grades from the semester with the intervention implemented is 23. The descriptive statistics is summarised in the Table 3 below.

From the Table 3 data, the null hypothesis is that the Final Exam Scores in both groups do not differ. The means are compared by Independent Samples T-test. The p-value is .01 (t(44)=-2.66) less than .05 which means that the groups' means do, in fact, differ. Hence the null hypothesis is rejected and therefore there is a statistical significance between the two means. We conclude that the intervention is effective; the difference in "Final Exam Scores between the two groups" is statistically significant. Students do better in the Final Exam after employing the intervention for that semester than they did prior to employing the intervention.

Table 3

Descriptive Statistics on Final Exams

	N	Minimum	Maximum	Mean	Std. Deviation
Final Exam scores no intervention	23	10	100	49.5	25.6
Final exam scores with intervention	23	18	94	67.0	18.5

Conclusions and Recommendations

In this study, an intervention to address the effect on students' engagement and achievements is implemented. The results based on students' responses reveal that the intervention helps students gain confidence in asking questions in class. Coming to a class with familiar questions in mind, for the lecture, fosters students' engagement in class. Moreover, students feel that the intervention encourages them to interact more with classmates and the instructor before and after class. Students' acknowledgment about class expectations is the most important characteristic of the intervention. For students, knowing explicitly what is expected from them in class, motivates them to get lightly familiar with the material before class and then to participate in discussions during class. Moreover, the intervention generated a significant increase on the students' grades. The results confirm a positive correlation between students' preparation to class and students' achievement.

The findings of this study are also consistent with prior results in the literature. For example, in order to motivate students toward reading before coming to class, some researchers reported success if frequent in-class quizzes are incorporated (Connor-Greene, 2000; Tomasek, 2009; Ruscio, 2001; Agnew, 2000; Fernald, 2004; Sappington, Kinsey, and Munsayac, 2002; Marchant, 2002; Leeming, 2002). Leeming, (2002) reported that frequent in-class quizzes increase the perception of the material; demonstrate learning; motivate students to read; and create an environment for class discussions (Ruscio, 2001).

In this study we also observed that giving credit to students for participating in the intervention serves as an important motivator. This is consistent with the findings in (Daniel, 1988; Carkenord, 1994; Green & Rose, 1996; Roberts and Roberts, 2008). For example, Daniel (1988) reported that the number of students who usually read the material before class jumped from about 10% to about 90%.

One challenge the authors had as instructors while employing this intervention is the preparation of suitable questions from the new content to serve the purpose of the intervention. Coming up with appropriate questions turned out to be quite time and effort demanding, especially in the proof writing courses. Based on the results of this study the authors urge mathematics instructors to employ this intervention in their mathematics classes. However, the study has limitations such as the results are for upper level mathematics courses. Inventing a parallel intervention for the introductory mathematics courses is left for a future project.

References

Agnew, M. (2000). DRAW: A motivational reading comprehension strategy for disaffected readers. *Journal of Adolescent & Adult Literacy*, 43, 574-576.

Armbruster, B. B. (2000). Taking notes from lectures. In R. F. Flippo & D. C. Caverly (Eds.), *Handbook of college reading and study strategy research* (pp. 175-199). Mahwah, NJ: Lawrence Erlbaum Associates.

Axelson, R. D., & Flick, A. (2011). Defining student engagement. *Change, 43*(1), 38-43. http://dx.doi.org/10.1080/00091383.2011.533096

Baker, J. W. (2000, April). *The ''classroom flip'': Using web course management tools to become a guide by the side.* Paper presented at the 11th international conference on college teaching and learning, Jacksonville, FL. Bower, G. H. (1970).

Brocato, J. (1989) How much does coming to class matter? Some evidence of class attendance and grade performance. *Educational Research Quarterly, 13, 2-6.*

Burchfield, C., & Sappington, J. (2000). Compliance with required reading assignments. *Teaching of Psychology*, 27, 58-60.

Carini, R., Kuh, G. & Klein, S. (2006). Student engagement and student learning: Testing the linkages. *Research in Higher Education, 47*(1), 1-32.

Carkenord, D. (1994). Motivating students to read journal articles. *Teaching of Psychology,* 21, 162-164.

Clump, M., Bauer, H., & Bradley, C. (2004). The extent to which psychology students read textbooks: A multiple class analysis of reading across the psychology curriculum. *Journal of Instructional Psychology*, 31, 230.

Coates, H. (2005). The Value of Student Engagement for Higher Education Quality Assurance. *Quality in Higher Education, 11*(1), 25-36.

Collins, B., de Boer, W., & van der Veen, J. (2001). Building on learner contributions: A web-supported pedagogic strategy. Educational Media International, 38(4), 229–240.

Committee on increasing high school students' engagement and motivation to learn, National Research Council (2003). *Engaging schools: Fostering high school students' motivation to learn.*

Connor-Greene, P. (2000). Assessing and promoting student learning: Blurring the line between teaching and testing. *Teaching of Psychology*, 27, 84-88.

Daniel, J. (1988). Survival cards in math. *College Teaching*, 36, 110.

Day, J., & Foley, J. (2006). Evaluating web lectures: a case study from HCI. Paper presented at the Conference on Human Factors in Computing Systems, Montreal, Quebec, Canada.

Duch, B. J., Groh, S. E., & Allen, D. E. (2001). Why problem-based learning? In B. J. Duch, S. E. Groh, & D. E. Allen (Eds.), *The power of problem-based learning* (pp. 3-11). Sterling, VA: Stylus Publishing.

EDUCAUSE (2012, February). 7 thinks you should know about flipped classrooms. *Educause*. Retrieved from https://net.educause.edu/ir/library/pdf/eli7081.pdf

Fernald, P. S. (2004). The Monte Carlo quiz: Encouraging punctual completion and deep processing of assigned readings. *College Teaching*, 52(3): 95-99

Finn, J. D. (1993). *School engagement & students at risk*. Washington DC: National Center for Educational Statistics Research and Development Reports. (ERIC Document Reproduction Service No. ED 362322).

Finn, J.D., Gerber, S.B., Achilles, C.M & Boyd-Zaharias, J. (2001) 'The Enduring Effects of Small Classes.' *Teachers College Record*, 103(2): 145–183.

Frederickson, N., Reed, P., & Clifford, V. (2005). Evaluating web-supported learning versus lecture-based teaching: Quantitative and qualitative perspectives. *Higher Education, 50(4),* 645–664.

Furlong, M. J. & Christenson, S. L. (2008). Engaging students at school and with learning: A relevant construct for all students. *Psychology in the Schools, 45*, 365-368.

Gannod, G. C., Burge, J. E., & Helmick, M. T. (2008). *Using the inverted classroom to teach software engineering. Proceedings of the 30th international conference on software engineering,* Leipzig, Germany.

Green, A., & Rose, W. (1996). The professor's dream: Getting students to talk and read intelligently. PS: Political Science and Politics, 29, 687-690.

Hannula, M. S. (2004). Regulation motivation in mathematics. Paper presented at the 10th International Congress on Mathematical Education, http://www.icme10.dk/, TSG 24, Copenhagen, Denmark.

Hassi M.L., Laursen S. (2009), Studying undergraduate mathematics: exploring students' beliefs, experiences and gains. *Proceedings of the Thirty First Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*. 5, 113-121.

Hemmi, K. (2010). Three styles characterizing mathematicians' pedagogical perspectives on proof. *Educational Studies in Mathematics*, 75, 271-291.

Jordan, D., & LeMetais, J. (1997). Social skilling through cooperative learning. *Educational Research*, 39, 3-21.

Koproske, C. (2012). *The promise and perils of innovation: Part* I. Education Advisory Board webinar aired August 15, 2012. Available on-demand at http://www.eab.com/Research-and-Insights/Academic-Affairs-Forum/Events/Webconferences/2012/The-Promise-and-Perils-of-Innovation-Part-I

Kuh, G.D., Kinzie, J., Buckley, J.A., Bridges, B.K., & Hayek, J.C. (2007). Piecing together the student success puzzle: Research, propositions and recommendations. ASHE Higher Education Report (Vol. 32, No. 5) San Francisco, CA: Jossey-Bass.

Lage, M. J., Platt, G. J., & Treglia, M. (2000). Inverting the classroom: A gateway to creating an inclusive learning environment. *The Journal of Economic Education*, 31(1), 30–43.

Leeming, F. (2002). The exam-a-day procedure improves performance in psychology classes. *Teaching of Psychology*, 29, 210-212.

Lei, S., Bartlett, K., Gorney, S., & Herschbach, T. (2010). Resistance to reading compliance among college students: Instructors' perspectives. *College Student Journ*al, 44, 219-229.

Lew, K., Fukawa-Connelly, T., Mejia-Ramos, J. P., Weber, K. (2016). Lectures in advanced mathematics: Why students might not understand what the mathematics professor is trying to convey, *Journal for Research in Mathematics Education,* Vol. 47, No. 2, pp. 162-198.

Lewis, M. and Hanc, J. (2012) Encouraging Students to be Readers: Survey Results of Successful Practices. *Teaching Journalism and Mass Communication*, Vol. 2, no. 1, pp. 12-20.

Launius, H. H. (1997) College student attendance: Attitudes and academic performance. *College Student Journal*, *31, 86-92.*

Malmivuori, M. L. (2001). The dynamics of affect, cognition, and social environment in the regulation of personal learning processes: The case of mathematics. *Research report* 172. Helsinki, Finland: Helsinki University Press.

Malmivuori, M.L. (2007). Understanding student affect in learning mathematics. In C.L. Petroselli (Ed.), *Science education: issues and developments* (pp. 125-149). NY: Nova Science Publishers.

Marchant, G. (2002). Student reading of assigned articles: Will this be on the test? *Teaching of Psychology*, 29, 49-51.

Marks, H. M. (2000). Student engagement in instructional activity: Patterns in the elementary, middle, and high school years. *American Educational Research Journal*, 37 (1), 153-84.

McKeachie, W. J. (2002). Teaching tips *(11th ed.).* Boston : Houghton - Mifflin.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 575-596). New York: Macmillan.

Moore, R. (2003) Helping students succeed in introductory biology classes: Does improving students' attendance also improve their grades? *Bioscience, 29, 17-25.*

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics.* Reston, VA: Author.

National Council of Teachers of Mathematics. (2009). *Focus in high school mathematics: Reasoning and sense making.* Reston, VA: National Council of Teachers of Mathematics.

Newmann, F. M., Wehlage, G. G., & Lamborn, S. D. (1992). The significance and sources of student engagement. In F. M. Newmann (Ed.), (1992). *Student engagement and achievement in American secondary schools.* New York: Teachers College Press.

Nye, B., Hedges, L.V. & Konstantopoulos, S. (2001) 'Are Effects of Small Classes Cumulative? Evidence from a Tennessee Experiment.' *The Journal of Educational Research*, 94(6): 336–345.

Park, S.Y. (2005) Student Engagement and Classroom Variables in Improving Mathematics Achievement, *Asia Pacific Education Review.* Vol. 6, No. 1, 87-97.

Protheroe, N. (2007). "What Does Good Math Instruction Look Like?" *Principal* 7(1), pp. 51 – 54.

Prince, M., and Felder, R. (2007). The many facets of inductive teaching and learning. *Journal of College Science Teaching*, 36(5), 14-20.

Rickey, Frederick and Amy Shell-Gellasch. (July 2010) "Mathematics Education at West Point: The First Hundred Years.", Retrieved from http://www.maa.org/book/export/html/116858

R.I.T. (2014) Teaching Elements Flipped Classroom Version 1.0, 5/27/2014, Retrieved from: https://www.google.com/url?sa=t&rct=j&q=&esrc=s&source=web&cd=1&ved=0ahUKEwjMhZKiso7UAhVrsFQKHRJfA0cQFggkMAA&url=https%3A%2F%2Fwww.rit.edu%2Facademicaffairs%2Ftls%2Fsites%2Frit.edu.academicaffairs.tls%2Ffiles%2Fdocs%2FTeaching%2520Element-Flipped%2520Classroom.pdf&usg=AFQjCNEJ4FEuYbAJEch7uGTcoftvFn6bxA&sig2=1xQt44S0cAqpk01SvOsbUA&cad=rja

Roberts, J, & Roberts, K. (2008). Deep reading, cost/benefit, and the construction of meaning: Enhancing reading comprehension and deep learning in sociology courses. *Teaching Sociology*, 36, 124-140.

Ruscio, J. (2001). Administering quizzes at random to increase student's reading. *Teaching of Psychology* 28:204–06.

Sappington, J., Kinsey, K, and Munsayac, K. (2002). Two studies of reading compliance among college students. *Teaching of Psychology*, 29, 273-274.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), Handbook of research on mathematics teaching and learning (pp. 334-370). New York: Macmillan.

Shell, A. E. (2002). The Thayer Method of Instruction at the United States Military Academy: A Modest History and A Modern Personal Account, *PRIMUS, 12:1*, 27-38.

Strayer, J. F. (2009). *Inverting the classroom: A study of the learning environment when an intelligent tutoring system is used to help students learn.* Saarbru¨cken: VDM Verlag.

Strayer, J.F. (2012). How learning in an inverted classroom influences cooperation, innovation and task orientation. *Learning Environment Research, 15(2)*, 171–193. DOI 10.1007/s10984-012-9108-4.

Thomas, P. V. and Higbee, J. L. (2000) The relationship between involvement and success in developmental algebra. *Journal of College Reading and Learning, 30, 222-232.*

Tomasek, T. (2009). Critical reading: Using reading prompts to promote active engagement with text. *International Journal of Teaching and Learning in Higher Education*, 21, 127-132.

Varghese, T. (2011). Possible student justification of proofs. *School Science and Mathematics*, 111, 409-415.

Wæge, K. (2009). Motivation for Learning Mathematics in terms of Needs and Goals. *Proceedings of CERME* 6, January 28th-February 1st 2009, Lyon France © INRP 2010 www.inrp.fr/editions/cerme6

Appendix

Sample Problems

Abstract Algebra

1. Write the following cycle as a ***product of transpositions***.

2. Write the identity permutation as a ***product of transpositions.***

3. Consider the identity matrix ***Does interchanging any rows changes the sign of the determinant of the resulting matrix?*** Let be a permutation of the rows of then what would be ***the sign of determinant*** of the resulting matrix**?** Let be a permutation of the rows of then what would be ***the sign of determinant*** of the resulting matrix**?**

4. Define an ***even*** permutation and an ***odd*** permutation notion.

5. State the definition of the ***alternating group*** of ***n letters***. What is the order of?

6. Is the set of odd permutations a group? ***Why?***

7. Is the following permutation ***even*** or ***odd***?

Discrete Mathematics

1. **Prove or disprove:** For all integers if divides then divides.
2. **Prove or disprove:** The sum of any three consecutive integers is divisible by**.**
3. **Prove or disprove:** A necessary condition for an integer to be divisible by is that it to be divisible by
4. **Prove or disprove:** A sufficient condition for an integer to be divisible by is that it be divisible by 16.
5. **Prove or disprove:** The product of any two even integers is a multiple of .
6. **Prove or disprove:** The sum of any two odd integers is even.
7. **State and prove** the transitivity of divisibility theorem.
8. **State** the fundamental theorem of arithmetic.

Differential Equations

1. State the ***linear dependence*** notion of a set of functions on an interval**.**
2. Is the set of functions ***linearly dependent*** on . Present your reasoning.
3. Define ***Wronskian*** determinant.
4. What is the ***Criterion for Linearly Independent Solutions***
5. Define what ***fundamental set of solutions*** is.
6. The functions and are both solutions of the homogenous linear equation on the interval ***Inspect the solutions whether they are linearly independent on the x-axis (do the observation with Wronskian determinant). Can we call a fundamental set of solutions? What is the general solution of the equation on the interval?***
7. In order to solve a non-homogenous linear differential equation, we first solve ____ and then find ____. The general solution of the non-homogenous equation is then ____.