What happens when teachers and students are introduced to Mathematical Investigations: An exploratory study

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A mathematical investigation (MI) encapsulates the reform movement in mathematics education by addressing content and process simultaneously and providing a novel opportunity for students to develop thinking skills and good mental habits. However, introducing MI to teachers and students who are used to routinised teaching approaches and pen-and-paper testing presents many challenges. This study introduced MI to two junior classes and the mathematics teachers of a regular public high school. After the MI orientation workshops, the students engaged in the processes of pattern searching, problem posing, conjecturing, verifying, and proving. Data was gathered through focus group discussions, interviews, observations, teachers' reflection notes, audiotapes, and analysis of students' outputs. The challenges of doing MI included the teachers' and students' lack of exposure to and/or competence in the investigative processes, the teacher's traditional views and teaching practices, and some sociosystemic factors in schools. Issues raised here could give insights into the areas where teachers and students needed reinforcement and how teachers could redesign mathematics instruction to focus on the development of processes and thinking skills needed for investigations.

Keywords · mathematical investigation · investigative processes · authentic assessment · mathematics teaching and learning

INTRODUCTION

Integrating mathematical investigation (MI) in the teaching of mathematics has long been argued to have many benefits. Aside from developing students' mathematical thinking processes and good mental habits (Bastow, Hughes, Kissane & Mortlock, 1984; Jaworski, 1994; Orton & Frobisher, 2005; Bailey, 2014), engaging in MI makes students realise that learning mathematics is not about memorising formulas and following procedures. It deepens their understanding of the content of mathematics and challenges them to "produce" their own mathematics within their universe of knowledge (Ronda, 2005). In fact, doing MI encapsulates the calls for reform in mathematics education "to shift the learning of mathematics towards investigating, formulating, representing, reasoning and applying a variety of strategies to the solution of problems – then reflecting on these uses of mathematics – and away from being shown or told, memorising and repeating" (NCTM, 1995). However, evidence shows that while doing investigations in the classroom is a powerful activity for students' learning, it poses many challenges to teachers (Ponte, Segurado & Oliviera, 2003) as it runs counter to the usual routinised, direct instruction approach of teaching mathematics where the teacher's task is to present the information as clearly and as neatly as possible and the students' task is to process and absorb the information. Moreover, with participants in local MI competitions mostly coming from top science-oriented schools, there is a prevailing notion in the Philippines that MI is only for the mathematically gifted. Thus, despite its many benefits, students in regular mathematics classes of ordinary schools, rarely, if at all, get exposed to MI.

The main goal of this study was to find out what happened when teachers and students from an ordinary public school who were used to routinised teaching were introduced to the complex, challenging, messy and divergent nature of MI for the first time.

Background

This study adopts the view of MI as open with respect to its goals, processes and answers. It is what Yeo and Yeap (2010) referred to as an open investigative activity, which includes both problem posing and problem solving as a process. Orton and Frobisher (2005) described MI as a process-oriented mathematical activity that does not have a specific and recognisable goal or problem. Bailey (2007) described it as “an open-ended problem or statement that lends itself to the possibility of multiple mathematical pathways being explored, leading to a variety of mathematical ideas and/or solutions” (p. 103). After exploring a mathematical situation on their own, students are expected to choose their own paths, pose their own problems, and search for solutions to these problems. The open-endedness of MI provides students with the opportunity to develop independent mathematical thinking and engage in mathematical processes such as organising and recording data, pattern searching, conjecturing, inferring, justifying, and explaining conjectures and generalisations. In turn, these thinking processes enable them to learn more mathematics, apply mathematics in other disciplines and in everyday situations, and to solve mathematical (and non-mathematical) problems (Ronda, 2010).

According to Polya (1945), instead of presenting mathematics as a finished product, beginning with definitions and statements to go to examples and exercises, teachers may emphasise its development processes, starting with questions and issues, and showing how it is at the same time “an experimental and deductive science” (Pólya, 1945, p. vii). This suggests that learning mathematics needs to include opportunities for students to get involved in genuine mathematical activity (Ponte, Segurado & Oliviera, 2003) and to experience the development process by transforming the classroom into a small mathematical community (Schoenfeld, 1992).

Introducing innovations in the classroom designed to foster thinking is always easier said than done. As Deforges and Cockburn (1987, in Jaworski 2004) claim, “classrooms as presently conceived and resourced are simply not good places in which to expect the development of higher order skills currently desired from a mathematics curriculum” (p.139). In most classrooms in the Philippines and elsewhere, the teacher is the source of knowledge and mathematics is presented as a “finished product” and not as a development process that it is. Such practices do not support the goal of MI, which is to provide students with opportunities to produce their own mathematics and to develop thinking processes and mental habits. In MI, the teachers have to give up their “know-it-all” stance and become learners and explorers just like the students for they cannot predict nor control what problems the students will pursue. The challenge becomes even greater when the class is large, the students are not particularly inclined to mathematics, both the classroom and the curriculum are over-crowded, and the teacher’s primary concern is to finish the syllabus in time for the national achievement test, which is a standardised multiple-choice test.

The results of the study are particularly significant to mathematics education in the Philippines, where the Department of Education currently promotes performance-based assessment – assigning it 40% of a student's grade in mathematics, as reflected in D. O. No. 8, s. 2015 Policy Guidelines on Classroom Assessment for the K to 12 Basic Education Program (Department of Education, 2016).

Framework of the Study

MI is anchored on the theory of constructivism and authentic assessment. Constructivism views mathematical learning as a social construction of knowledge from shared meanings where a student is not considered as an object but rather as a subject who continuously acquires more importance within his relationships with others (Ernest, 1991). The quests and goals of a constructivist orientation are for students to take responsibility for their own learning, to be autonomous learners, to develop integrated understanding of concepts, and to pose - and seek to answer - important questions (Tobins & Tippins, 1993). From a constructivism perspective, assessments should focus on students acquiring knowledge, as well as the disposition to use skills and strategies and apply them appropriately (Steffy, 1995).

In doing MI, the following stages may occur, although not necessarily in the order listed (Bastow, Hughes, Kissane & Mortlock, 1984): preliminary skirmishing, gestating, exploring systematically, making conjectures, testing conjectures, explaining or justifying, reorganising, elaborating and summarising. The conduct of MI in this study followed these stages to a great extent. In the summarising stage, students were asked to submit a write-up of their investigation and present it before a panel of teachers and their classmates for feedback.

In line with its constructivist orientation, MI is a form of authentic assessment as it provides the students with an activity, realistic task, problem or simulation that relates to the standard being assessed (Steffy, 1995). The term authentic assessment is sometimes used synonymously with alternative assessment and performance-based assessment to mean "variants of performance assessments that require students to generate rather than choose a response" (Herman, Aschbacher, and Winters, 1992, p. 2, in Burke, 1999). Generally, authentic assessment assesses critical thinking and provides opportunities for synthesis of ideas. It offers a learning opportunity for students as it enables them to apply knowledge to solve problems, defend positions, and develop models. However, MI takes more time to score for it requires sophisticated scoring rubrics (Steffy, 1995). Clearly, MI is authentic assessment at its strictest sense. This study defines MI as an extended project where students pose and work on their own problems from a given unstructured point of view, which focuses both on content and processes, and involves both a written report and an oral presentation (Nivera, 2012). That definition contrasts MI in this study from the investigative mathematics teaching espoused by Jaworksi (1994).

The reform agenda of mathematics education envisions a systemic change in all aspects of mathematics - curriculum, instruction, and assessment. For assessment to be congruent to curriculum and instruction, it has to engage students in meaningful tasks that address both mathematical content and processes and use performance tasks and scoring rubrics. The integration of MI in the mathematics classroom reflects a change in all three aspects. However, this study only documented the experiences and views of the teachers and students as they went through two iterations of MI and did not include classroom observations of teachers' day-to-day mathematics teaching.

Method

This study was phenomenological in nature (Waters, 2016) as it documented the experiences, constraints, and changes the teachers and students underwent in the process of trying out MI as an extended performance task in their Geometry class. It was conducted in a large public high school in Metro Manila with less than adequate facilities and equipment, a high drop-out rate, and large class sizes (60 to 100 students per class) over two grading periods, or five months in SY 2010 -2011.

Participants

The study involved two junior classes, with students aged 14 to 15, and their Geometry teachers Jason, male, and Cathy, female, who had 12 and 15 years of teaching experience, respectively. During the initial interview, both teachers claimed to generally practice traditional, lecture-based approaches in teaching. Cathy believed that the best way to teach mathematics was by giving examples on the board and then giving seatwork to students. For Jason, students learned best through "*lectures, one step at a time… and recitation*".

Mark, male, and Ellen, female, served as external raters in the MI presentations. They both taught Geometry in the same school and participated in the MI orientation workshops, but they were not teachers of the classes involved at the time of the study. Ellen was also the mathematics department head. All names used in this article are not the participants' real names.

Procedure

As all participants had no prior exposure to MI, three separate half-day MI orientation workshops were conducted across three days, starting with the teachers and followed by the two junior classes. Each workshop allowed the participants to experience the various stages of MI – from exploring, problem posing and conjecturing up to verifying and proving. Each class was divided into groups of 5 or 6, composed of a mix of high and low performing students. All groups underwent two cycles of MI; however, only two groups from each class, or four groups in all, were selected as case studies. The interactions in these groups were audio- and videotaped and their outputs were subjected to careful analysis and assessment.

Each cycle covered a period of two months and began with a group exploration of the task and ended with an oral presentation of the outputs. Separate focus group discussions with the teachers and the select groups of students were conducted after each cycle. The activities will be discussed in greater detail below.

The study employed various data collection strategies such as interviews, FGDs, questionnaires, observations, field notes, analysis of video and audio transcripts, and content analysis of MI outputs. The comprehensive documentation served as the basis for the retrospective analysis of what happened in this study.

MI Orientation Workshops for Teachers and Students. During the orientation workshop for Jason and Cathy, the two teacher-participants of the study, five other mathematics teachers, including Mark and Ellen, joined for their own professional development. The teachers were asked to work in pairs in exploring the MI task "*Lines are drawn on the plane. Investigate.*" After an hour of skirmishing, they came up with one or two patterns. The researcher then guided them in posing problems and conjectures based on the patterns they observed and in showing that their conjectures were true. After the sharing of their initial work to the group, an open discussion of their difficulties and successes followed. We explored other possible problems and conjectures that could be generated from the given task. Sample proofs for a couple of conjectures were also discussed. After this initial experience with MI, the researcher then formally introduced the MI stages, processes and products. Then the teachers were asked to do a second task: *"Some numbers can be expressed as the sum of consecutive integers (e.g. 9 = 2 + 3 + 4, 11 = 5 + 6, and 18 = 3 + 4 + 5 + 6). Investigate."* Again, after an hour of exploring the situation, the teachers came up with one or two problems and conjectures. The teachers were asked to submit a write-up of their work the following week, but due to work commitments, the teachers only managed to submit partial results of their MI.

After the MI orientation workshop, Jason, Cathy, and I worked collaboratively to map out the scope and nature of the MI integration in their Geometry classes and to find the best way for conducting workshops with the students. As practitioners, the teachers provided realistic and workable ideas on the process of engaging students in MI such that its goals and constraints were drawn from the local context as well as the researcher's agenda.

A similar but separate orientation workshops were held for each of the two junior high school classes on two consecutive afternoons. With 70 students in a class, both workshops were held in the school's conference hall instead of their small classrooms to allow the students to sit around tables and work in groups conveniently. Since Jason and Cathy still lacked the confidence to facilitate the MI orientation workshops for their respective classes at this point, the researcher conducted the workshops for them, while they observed the process and assisted the students during the exploration and conjecturing parts. The students worked in groups of 4 or 5. They were encouraged to use tables, graphs, and sequential listing for more systematic explorations of the task. After about an hour's explorations, the students began to observe some patterns and to pose one or two conjectures. They were asked to share to the class the problems and conjectures that they had posed, and to try to show that their conjectures were true. Based on the observed weaknesses of the students in the initial conduct of MI, they were provided with additional activities in systematic exploration and proving of conjectures.

Instruments

All MI tasks used in this study were taken from Bastow, Hughes, Kissane and Mortlock (1984). *Sums of consecutive integers* was chosen as the first MI task because of its openness to many possible patterns that were simple enough for students to find and prove. *Bounces*, the task for the second iteration of MI, required more analysis and applications of geometric concepts. An analytic MI rubric was used to evaluate the processes and products of MI. The discussions on how this MI rubric was developed and calibrated are discussed elsewhere (Nivera, 2012).

A set of FGD questions for the teachers and students were prepared to gain insights into their views about MI before, during, and after the two cycles. The FGDs also aimed to assess how doing MI influenced their views about mathematics, about working in groups and doing oral presentations, and about mathematics teaching and learning in general. The teachers were also asked to write down their thoughts about their MI experience and its workability in a regular mathematics class of an ordinary public high school in the Philippines.

Results and Discussion

The discussion of the results follows the steps used to introduce teachers and students to MI. It begins with the orientation workshops, followed by two iterations of MI and the reflections on the experience.

Teachers' and students' orientation workshops. The separate workshops for both teachers and students provided an overview of MI rationale, stages, processes, outputs, and assessment. Each workshop engaged the participants in an actual exploration and presentation of MI.

The MI orientation workshop with teachers revealed that the teachers themselves needed more training on problem posing, conjecturing and proving conjectures. In the FGD after the workshop, Jason and Cathy, the two teacher-participants, claimed that MI was interesting and challenging, but quite difficult even for them. Since their students had average to below average competence in mathematics, many of them doubted whether MI was suitable for them. Likewise, they worried about the time constraints, the demands of the curriculum and national examinations, and the additional workload that integrating MI into their course would entail. While they were not hopeful about its success, they were willing to try to do MI with their students. The role of the teacher's views about mathematics teaching and MI also came to the fore during the FGD. The teacher, who disliked group work and activities that "disrupted" the class, was also the one more resistant to doing MI.

The students' MI orientation workshop, on the other hand, showed that the students needed more guidance to get started and be systematic with their explorations and data presentations. Once they found an interesting pattern, the next challenge was how to pose the problem and state the pattern observed as a conjecture. It was clear that students needed further training in showing or proving that their conjectures were correct.

The observed inadequacies in posing problems and conjectures among the teachers and students reinforced previous research findings that "teachers often lack the necessary skills to develop students' writing skills in mathematics, and students are unable to communicate their ideas adequately" (Pimm & Wagner, 2003, p. 5, in Marshman, Clark & Carey, 2015). Despite these inadequacies, the study went ahead with the two cycles of MI to see what would happen when students were provided the proper guidance in doing MI.

First Mathematical Investigation (Cycle 1). The task for the students' first MI was the sums of consecutive integers.

Sums of Consecutive Integers
Some numbers can be expressed as the sum of consecutive integers e.g.
9 = 2 + 3 + 4
11 = 5 + 6
18 = 3 + 4 + 5 + 6
Investigate.

Figure 1. Sums of Consecutive Integers

At the outset, the researcher oriented the students per class on the expected MI outputs, format of the write-up, scoring rubrics, and assessment criteria to ensure that each group knew what was expected of them. The mathematics teachers guided their respective classes in their initial explorations of the task, and to some extent, on how problems and conjectures should be stated, and how conjectures could be shown to be true for all cases. The deadline for the write up was set 10 days later. During this period, the teachers were on hand to encourage and inspire their students to persist and persevere in their search for patterns and conjectures. After 10 days, the students did a mock presentation of their MI outputs in class. As expected, the results were crude and needed much improvement. At this point, it was difficult to rate the write-ups due to the inadequacies of the report. While some students were able to provide proofs to one or two simple conjectures, others could only provide verifications of the conjectures. To help out the students, the researcher had another workshop with them where she provided more examples and guidelines on how to phrase problems and conjectures clearly and concisely and how to prove some of their conjectures. The groups were given a week to revise and resubmit their first MI output.

Cathy, Jason, Mark, and Ellen were asked to rate the MI write-ups and oral presentations of the students using the MI rubric. Copies of the MI write-ups were given to the raters two days before the presentation to give them time to scrutinise the problems, conjectures and proofs. During the oral presentation of the MIs, other mathematics teachers and even the principal came to observe the process. Each group presented the explorations that led them to their problem and conjecture. They also provided some justifications on why the pattern was true for all possible cases. While there were some inaccuracies and weaknesses in the presentation, a number of the problems and conjectures posed reflected good mathematical thinking and interesting patterns that were, at least for them and for many of the teachers in the school, original. What surprised the teachers was how some of the students spoke with such confidence and pride in their discoveries. The students responded, sometimes with audacity, to the questions from the raters and other students. The interactions between and among the presenters, raters and audience transformed the room into what seemed like a community of mathematicians - certainly a novel experience in this school, according to the teachers. On this ground at least, the students exceeded everyone's expectations.

At the end of the first MI, Cathy, Jason, and Mark engaged in a focus group discussion about their experience. They claimed to be happy and proud to see their students communicating and reasoning mathematically and thinking and talking like mathematicians, which they did not normally do in class. They found the oral presentation to be an important part of the MI process as it allowed the students to explain their work and the depth of their reasoning more clearly than what they managed to do in their write-ups. Likewise, the oral presentations exposed the students' misuse of certain mathematical terms and symbols and incorrect way of reading some equations and notations. The teachers claimed that this experience with MI made them aware of some of their students' strengths and weaknesses in mathematics and highlighted the need to integrate investigative activities in their Geometry classes to develop the students' skills in looking for patterns and making conjectures. MI showed, in this instance, its power as an authentic assessment tool as it informed teachers of their students' strengths and weaknesses for the improvement of instruction.

Second Mathematical Investigation (Cycle 2). In the second iteration of MI, the students were presented with a more challenging task. *Bounces* involved the concepts of parallelograms, congruence, symmetry, angles and segments, among others.

BOUNCES

Imagine a rectangle on dot paper. Suppose it is a pool table.
Investigate the path of a ball which starts at one corner of the table, is pushed to an edge, bounces off that edge to another, and so on, as shown in the diagram. When the ball finally reaches a corner, it drops off a table.

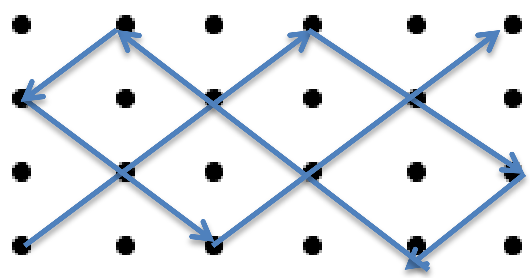

Figure 2. Bounces

This time Jason and Cathy presented the MI task on their own to their respective classes. Both teachers did well in facilitating the initial explorations and problem posing of the groups. The students were provided with 1 cm square grid papers, 1 cm square dot papers, rulers, pencils and crayons. The deadline for the write-ups was again set 10 days later.

As in Cycle 1, the students presented the outputs of their second MI in front of the school's mathematics teachers and their classmates. They were more relaxed and confident in presenting their work this time, although they still displayed some difficulty in highlighting their thinking and reasoning. Again, Cathy, Jason, Mark and Ellen rated the MI write-ups and presentations. As before, Mark, the external rater, actively engaged the students with his insightful questions. The raters took the time to discuss with the students the strengths and weaknesses of their investigations for improvement in their succeeding work. Both the teachers and students found this discussion with the students to be very significant and helpful.

As in Cycle 1, separate FGDs were conducted among the teachers and students after the oral presentations.

Teachers' Views on MI and their Students' Outputs. After two cycles of MI, Cathy described MI in her reflection notes as *"interesting and challenging*" and as a "*good activity that needs more time and good attitude towards it*". She believed that with the right planning, she would succeed in including MI in her classes. On whether she would promote the use of MI to other mathematics teachers, Cathy wrote: *"Yes, because this is a chance for the students and teacher to explore... They could experience lots of different problems to investigate which would make them realise the wonder of investigations."*

Jason was more pragmatic and pessimistic. In his reflection notes, he wrote: *"MI cannot be successfully implemented in our school because we have huge class sizes, we have a syllabus to finish, and we do not have enough time for mathematical investigations."* He maintained that he would be able to integrate MI in his classes only if the class size was small.

Mark also saw the benefits and challenges of MI. He claimed in the FGD that while MI was a great activity for students as it challenged them to think, it required much work on the part of the teacher. Cathy disagreed. To quote: "*Hindi naman burden sa teacher. Kung ikaw nag-che-check ng project mo, parehas lang din sa project*." [It is not a burden to the teacher. (Checking MI outputs and presentation) requires the same amount of work needed for checking projects.].

Both Jason and Cathy were satisfied with their students' performances. One of the groups in Cathy's class came up with nine problems and their corresponding conjectures and proofs, way beyond the two problems required. Their outputs certainly exceeded Cathy's expectations. To quote: "*Nagulat ako sa nangyari talaga."* [(Their performance) really surprised me.]

With respect to the oral presentations, the students still had much to improve, but the teachers were happy and satisfied with their efforts. To quote Cathy: *"Medyo di pa masyadong confident mag-deliver ng report. Pero OK na talaga ang ginawa nila for beginners."* [They were not yet very confident in delivering their report. But what they did was good enough for beginners.] Jason agreed. He said, *"Sa akin, yung enthusiasm nila 100%. Involved sila 100%. Yun lang OK na. Tsaka yung misconceptions or mistakes nila, we can talk about them with the students para maayos kasi it doesn't stop there eh. We have to continue.*" [For me, it is enough to see that they have 100% enthusiasm and 100% involvement. We can talk about their mistakes and misconceptions in class so that these can be corrected because it doesn't stop there. We have to continue.]

Both Jason and Cathy agreed that the oral presentation was a good experience for the students. *"Eto a ang time na matuto ang students na mag-defend ng ideas nila sa teacher, to contest their teacher's views and to argue."* [This is the time for the students to learn to defend their ideas before the teacher, to contest their teacher's views, and to argue.] Cathy added: *"Kailangan nga magganyan kami para masanay sila."* [We need to have activities like this so that the students will get used to it.]

On what they observed as the students' strengths and weaknesses in MI, Cathy replied: *"Magaling sila mag-explore ng patterns pero hirap sila minsan mag-interpret kung ano ang nakita nila. Di sila familiar sa mga notations at symbols at hirap sa proving."* [They were very good at exploring patterns but they had difficulty in interpreting the patterns observed. They were not familiar with the notations and symbols, and they had trouble proving (the conjectures).] Jason said: *"Strengths – nakita ko talagang critical thinkers sila. Talagang magaling mag-isip. Kaya lang hirap mag-deliver."* [I saw that they were critical thinkers. They were very sharp. But they had difficulty delivering (their findings orally)].

It is interesting to note that when MI was first introduced to the teachers, Jason resisted the idea of group work and explorations due to the huge class size and the need to finish the syllabus. Now, after seeing how the students enjoyed working on their investigations as a group and how interesting the outputs produced by such collaborations were, Jason became convinced that group exploration and discussion of the task could significantly improve students' chances of coming up with good problems and conjectures and staying on task.

In the end, Cathy was more optimistic in her reflective journal. While she found MI to be interesting but difficult, she saw how the experience engaged the students in doing mathematics. In fact one of her students got so excited with his investigations, he kept at it all night and did not notice that it was already dawn. While the experience with MI seemed to have opened Jason's mind to the benefits of group work and investigations, he maintained that the use of MI in his class would be more realistic if the class sizes were reduced and if he did not have to finish the syllabus to prepare the students for the national achievement test (NAT). Clearly, as shown in Jason's case, many teachers are hindered from teaching mathematics the way it should be taught because of sociosystemic factors in schools such as the huge class size, overcrowded curriculum, and pressures from the national assessment. However, one can also argue that the difference in the views of the two teachers towards MI reinforces the idea that the success of any innovation lies, to a large part, on the commitment of the implementers. While one teacher was willing to work around the existing constraints, the other wanted to integrate MI in the teaching of mathematics only when the constraints had been addressed.

Students' Views on MI. Two groups from each class identified early on as case studies were involved in the FGDs. Thus, four groups with 5 to 6 students per group participated in the separate FGDs per class. Reports from the FGDs showed that students found MI to be "*fun, interesting, difficult and exciting*". At the start, they found the absence of a question to be answered very unusual to the point that some of them were at a loss on what to do. After experiencing two cycles of MI, they welcomed the idea of posing their own questions as it allowed them to be creative and gave them the freedom to explore different paths. It also forced them to think. The students claimed that to succeed in MI, they needed to be patient, focused, and hard working. They also needed more exposure, practice and training especially in framing conjectures and proving them. They claimed that working in groups helped in keeping them on task. Interestingly, the students were convinced that anyone could do MI, not just the smart ones. They took pride in their experience of presenting and defending their work in front of many students and teachers. MI not only made mathematics exciting for them, it also enhanced their social and communication skills. For some students, the experience made them realise their potentials. As one student said: *"Pwede pala akong maging mathematician. Wag lang akong susuko."* [I realised I could be a mathematician if I persevered and refused to quit.]

While the lack of short-term goals and immediate rewards in MI were frustrating and demotivating for the students at the start, they got used to it as the two cycles of MI unfolded. More importantly, the students engaged in mathematical processes such as exploring, searching for patterns, and communicating mathematical information in both written and oral forms, among others, which is a relevant goal in mathematics education. Judging by the enthusiasm the students displayed during the oral presentations and FGDs, it can be argued that MI seemed to have improved their view of mathematics and its processes.

Conclusions and Implications

This study explored what happened when MI was introduced as an extended project or performance task to a group of students and their teachers from a regular public high school in Manila. The results showed a number of real challenges and constraints, but also some very encouraging outputs and responses from the students, and some useful insights for the teachers towards the improvement of mathematics teaching. The students lacked skills in exploring and searching for patterns systematically, in writing problems and conjectures, in using correct and precise terms and phrases, in proving the conjectures, and in explaining the salient features of their work to show their thinking and reasoning. The students' difficulties in the conduct and presentation of their MI drew the teachers' attention to the processes and skills that needed to be emphasised in their teaching of mathematics, thus providing the base for teachers to make adjustments in their teaching to optimise learning. Indeed, being aware of the processes and skills necessary to undertake investigations enables teachers to plan and implement mathematically rich tasks that develop students' investigatory abilities (Diezmann, Watters & English, 2001).

The study also revealed the teachers' reluctance and difficulties of doing MI because of their own lack of exposure to its processes. Some reluctance also stemmed from sociosystemic factors such as large class size, over-crowded curriculum and classroom, and pressures from national assessment, which are realities in many Philippine schools. Other factors that came into play were the teachers' views about mathematics and how it should be taught, their anxieties about the cognitive demand of MI, and its open-endedness and messiness, which compromised the teacher's complete "control" over how the class proceeded. As Bailey (2014) claimed, the process of undertaking MI was not without struggle and negative emotion. Uncertainty and perseverance were an accepted part of the process for the teachers and students alike.

Based on the results of this study, this paper argues that with the right orientation to MI and with sufficient training in investigative processes, even ordinary teachers and students in less than ideal settings can do it. Doing MI can reveal students' strengths and weaknesses to inform instruction, which makes it a powerful assessment tool. It can also uncover some students' depth of mathematical reasoning and their ability to find unusual and interesting patterns .

It is hoped that the experiences and insights shared in this study will contribute to an understanding of how MI focuses attention to the developmental process of mathematics and the breadth and nature of the challenges and rewards that teachers and students experience in doing it.

References

Bailey, J. (2014). Mathematical investigations for supporting pre-service primary teachers repeating a mathematics education course. *Australian Journal of Teacher Education*, 39(2). Retrieved from http://dx.doi.org/10.14221/ajte.2014v39n2.8

Bastow, B., Hughes, J., Kissane, B. & Mortlock, R. (1984). *40 Mathematical Investigations.* The Mathematical Association of Western Australia: Nedlands, WA.

Burke, K. (1999). *The mindful school: How to assess authentic learning (3rd Ed)*. Illinois: SkyLight Professional Development.

Department of Education (2016). K to 12 mathematics curriculum (August 2016 version). Retrieved October 1, 2016 from http://www.deped.gov.ph/sites/default/files/order/2015/DO_s2015_08.pdf

Diezmann, C. M., Watters, J. J. and English, L. D (2001). Implementing mathematical investigations with young children. In *Proceedings 24th Annual Conference of the Mathematics Education Research Group of Australasia*, pages 170-177, Sydney. Downloaded from http://eprints.qut.edu.au/1505/1/1505.pdf.

Ernest, P. (1991). *The philosophy of mathematics education: Studies in mathematics education.* London: The Falmer Press.

Jaworski, B (1994). *Investigating Mathematics Teaching: A constructivist enquiry.* London: The Falmer Press.

Jaworski, B. (2004). Grappling with complexity: Cp-learning in inquiry communities in mathematics teaching development. *Proceedings of the 28th conference of the International Group for the Psychology of Mathematics Education, Vol. 1, pp. 17 – 36.*

Marshman, M., Clark, D. & Carey, M. (April 2015). The use of mathematical investigations in a queensland primary school and implications for professional development. *International Journal for Mathematics Teaching and Learning*: online journal available at http://www.cimt.plymouth.ac.uk/journal.

National Council of Teachers of Mathematics (1995). *Assessment standards for school mathematics.* Reston: VA.

Nivera, G. (2012). Mathematical investigations and its assessment: Implications to teaching and learning. *The Normal Lights Journal on Teacher Education,* 6 (1), pp. 56 -66.

Orton, A. & Frobisher,L. (2005). *Insights into teaching mathematics*. London: Continuum.

Pólya, G. (1945). *How to solve it: A new aspect of the mathematical method.* Princeton: Princeton University Press.

Ponte, J. P., Segurado, M.I. & Oliviera, H. (2003). A collaborative project using narratives: What happens when pupils work on mathematical investigations. In A. Peter-Koop, V. Santos-Wagner, C. Breen & A. Begg (Eds.), *Collaboration in teacher education: Examples from the context of mathematics education* (pp. 85 – 97). Dordrecht:Kluwer.

Ronda, E. (2005). *Mathematical investigations.* Hand-outs given during the Facilitators Training in Science and Mathematical Investigations. Diliman, Quezon City: UP NISMED,

Ronda, E. (2010). *What is mathematical investigations*? Retrieved on April 3, 2015 from http://math4teaching.com/2010/03/09/what-is-mathematical-investigation/.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: Macmillan.

Steffy, B. (1995). *Authentic assessment and curriculum alignment*. Massachusetts: Pro-Active Publications.

Tobin, K. & Tippins, D. (1993). Constructivism as a referent for teaching and learning. In K. Tobin (Ed.), *The practice of constructivism in science education* (or TPCSE, pp. 3-21). ??

Waters, J. (2016). Phenomenological research guidelines. Capillano University. Retrieved January 5, 2016 from https://www.capilanou.ca/psychology/student-resources/research-guidelines/Phenomenological-Research-Guidelines/

Yeo, J. B. W., & Yeap, B. H. (2010). Characterising the cognitive processes in mathematical investigation. *International Journal for Mathematics Teaching and Learning*: online journal available at http://www.cimt.plymouth.ac.uk/journal.

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