Core Maths: A New Opportunity

Derek Robinson

Bishop Luffa School
Chichester, PO19 3LT

1. Background

One of the major risks in implementing Core Maths in the classroom is that we will not have a significant number of students willing to sign on for this course. Most of the targeted students will have struggled to get their Grade C or B and may be very reluctant to sign up for any more maths unless they can see its value and recognise that this will be a creative and interesting experience for them. So we need to ensure that whatever the examination boards design for their assessment, the delivery in schools is different to the approaches that have been taken in their GCSE courses.

We also need to understand why, for example, the free-standing mathematics courses and the Uses of Mathematics AS level course have not recruited significantly since their inception. Is this because they are not significantly different in their approaches, either to teaching or assessment, from traditional mathematics courses?

Current mathematics education reforms both here in the UK and abroad suggest “problem solving and investigative approaches are central to learning for all pupils” (Ofsted, 2011). It is certainly true in many of the countries the UK government wish to emulate.

For example, in Singapore, mathematical problem solving is central to mathematics learning (Ministry of Education, 2007). It involves the acquisition and application of mathematics concepts and skills in a wide range of situations, including non-routine, open-ended and real-world problems. The development of mathematical problem solving ability is dependent on five interrelated components, namely, *Concepts, Skills*, *Processes*, *Attitudes* and *Metacognition* (MOE, 2007).

And in Japan, the new Course of Study (COS) outlines the importance of developing new mathematical activities that inspire the students to “willingly engage in mathematics with purpose” by “trying to find new properties or to create new ways of thinking or to solve concrete problems.” (Japanese Ministry of Education, 2008). Isoda & Katagiri (2012) summarise this by reporting that the basic principal of the Problem Solving Approach is to nurture children’s learning of mathematics by/for themselves.

This is not a new idea as back in 1980 the National Council of Teachers of Mathematics (NCTM) suggested that "problem solving must be the focus of school mathematics" (NCTM, 1980, p.1). It concluded with the publication of *Everybody Counts* (National Research Council, 1989) and the *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989), both of which emphasize problem solving.

In the UK, Ofsted (2011) certainly seem to think that problem solving lies at the heart of outstanding teaching as can be seen from their current guidelines.

- Teaching is rooted in the development of all pupils' conceptual understanding of important concepts and progression within the lesson and over time.
- It enables pupils to make connections between topics and see the 'big picture'.
- Teaching nurtures mathematical independence, allows time for thinking and encourages discussion.
- Problem solving, discussion and investigation are seen as integral to learning mathematics.
- Constant assessment of each pupil's understanding through questioning, listening and observing enables fine-tuning of teaching.

This is echoed in their inspection guidelines for outstanding pupil achievement.

- They show exceptional independence and take the initiative in solving problems in a wide range of contexts, including the new or unusual.
- They think for themselves, and are prepared to persevere when faced with challenges, showing a confidence that they will succeed.

It is clear from these guidelines that Ofsted think that outstanding teaching includes teaching through problem solving.

2. Teaching through Problem Solving

One might conclude that there is general agreement that problem-solving should be at the heart of mathematics teaching with the aim that our students become competent problem solvers. (Schoenfled, 1992). However, as there is also international recognition that problem solving is a "fuzzy concept" (Pehkonen, 2008), this goal lacks clarity. For example, Schroeder and Lester (1989) identified three types of approach to problem solving:

- Teaching for problem solving. This generally means the teacher teaches a specific topic and then sets "problems" to practice the new skill. This was the American approach highlighted by Stigler and Hiebert in their report. It still is the approach adopted by most text books in the UK and apparently supported by Michael Gove (2010).
- Teaching about problem solving. This approach involves teaching students how to problem solve by identifying strategies that might be useful in approaching a task. For example, this could include draw a picture, look for a pattern, make an organised list. Identifying strategies provides a useful means for students to talk about their methods.
- Teaching through problem solving. This approach means that students learn mathematics through problem solving. It is almost the opposite of teaching for problem solving with the problem presented at the beginning of the lesson and the skill emerging through working on the problem during the lesson. This is the Japanese approach highlighted by Stigler and Hiebert (1999).

My argument is that for Core Maths to be sufficiently different to GCSE mathematics we need to move away from "Teaching for problem solving" to a course based on "Teaching through problem solving." For this to be successful, we also need to be clear in what we are trying to achieve. This means that we need an overarching aim that relates directly to the goals we have for our students' long-term development.

In my own school, our overarching aim is that

> "our *students will become independent thinkers (learners) who enjoy working together to produce creative solutions in unfamiliar situations"*

However, if this is to be successful, the "fuzziness" mentioned by Pehkonen needs to be removed by studying the approaches adopted by successful countries such as Japan.

3. Lessons from Japan: Open Approach

In *The Teaching Gap*, Stigler and Hiebert (1999) report that in all of the countries studied, problem solving is seen as an essential part of the mathematics curriculum. Their findings also indicate that in all high achieving countries, problem solving was used to actually drive learning not merely to test learning.

They also suggest that it is in the choice of problem and the thinking behind this choice that cultural differences begin to show. Problems are carefully chosen not only to help students develop their understanding of a particular concept but also to capture their interest by making the tasks thought provoking.

If we intend to "Teach through Problem Solving", we need to define a problem as follows:

> *"A problem occurs when students are confronted with a task, which is usually given by the teacher, and there is no prescribed way of solving the problem. It is generally not a problem if students can immediately solve it" (Nohda, 2000).*

Stigler and Hiebert emphasise that in Japan, the "teacher presents a problem to the students without first demonstrating how to solve the problem." In this way learning begins with a problem to be solved, and the problem is posed is such a way that students need to gain new knowledge before they can solve the problem. Rather than seeking a single correct answer, students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions. This process has been described as "structured problem solving" (Stigler & Hiebert, 1999).

Much of the current Japanese approach to problem solving can be traced back to the research carried out between 1971 and 1976 by Japanese researchers in a series of projects on methods of evaluating higher-order thinking in mathematics education which became known as the "Open Approach". (Shimada, 1997)

The aim of open approach teaching is to develop both the creative activities of the students and their mathematical thinking simultaneously through problem solving. It is based on the belief that students' perceptions of mathematics are formed by the work they are asked to do. If they are mainly asked to carry out pre-taught procedures in a set of exercises, they will think mathematics is about following a set of rules. If we want them to think that mathematics is about solving problems, then they need to spend most of their time solving problems (Hiebert et al., 1997).

When open problems are used in mathematics teaching, students have the opportunity to behave like creative mathematicians. In reality, to understand "openness" in Japanese problem solving means rethinking the way we teach mathematics.

Becker (2005) explains that problems can be open in three different ways:

- ***Process is open*** – where the learning comes from studying the different ways of solving a problem,
- ***End product is open*** – where the learning comes from studying the different answers,
- ***Ways to develop are open*** – where the learning comes from the students using the initial problem to generate new problems of their own.

Shimada (1997) also explains that we can classify open approach questions into three types:

Type 1: Finding relations

Students are asked to find some mathematical rules or relations. For example:

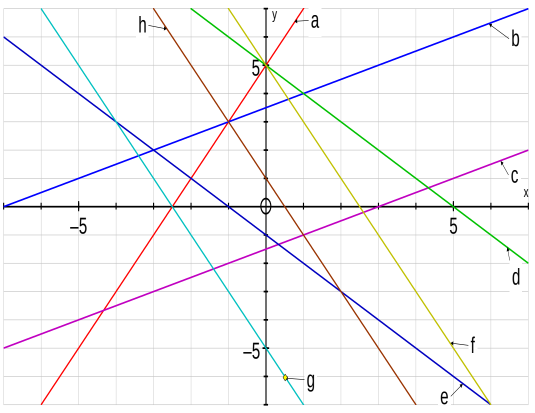
Find as many patterns as possible in the following table

1	2	3	4	5	6	7	…
6	9	12	15	18	21	24	…
27	36	45	54	63	72	81	…
108	135	162	189	216	243	270	…
…	…	…					

Type 2: Classifying.

Students are asked to classify according to different characteristics which may lead them to formulate some mathematical concepts. For example:

Classify these graphs into two groups, one group that have some mathematical feature in common with graph ***h****, and the other that does not. Find as many different methods to classify as you can, and explain the basis for your classification.*



a: $2x - y + 5 = 0$,

b: $x - 2y + 7 = 0$,

c: $x - 2y - 3 = 0$,

d: $x + y - 5 = 0$,

e: $x + y + 1 = 0$,

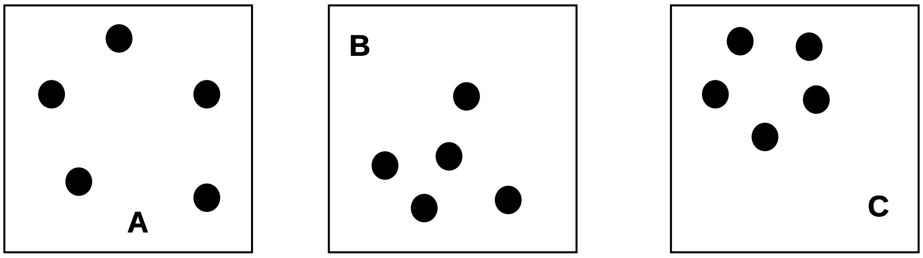
f: $2x + y - 5 = 0$,

g: $2x + y + 5 = 0$,

h: $2x + y - 1 = 0$

Type 3. Measuring

Students are asked to assign a numerical measure to certain phenomena. For example:



Three students, Alisha, Ben and Catherine, each threw five marbles, which came to rest as shown. In this game, the winner is the student with the smallest scattering of marbles. The degree of scattering seems to decrease in the order A, B, C. Devise as many ways as you can to express numerically the degree of scattering.

4. Choosing the Problem

Sawada (1997) suggests that the success of the open-ended approach depends very much on the choice of problem. He argues that the open-ended approach begins by choosing a suitable problem that all students will be able to access.

He describes how Japanese teachers firstly determine if the problem is appropriate by asking three questions:

- Is the problem rich in mathematical content and valuable mathematically?
- Is the mathematical level of the problem appropriate for the students?
- Does the problem include some mathematical features that lead to further mathematical development?

They then develop their lesson plan by:

- Listing the students' expected responses to the problem
- Making the purpose of using the problem clear
- Devising a method of posing the problem so that students can easily understand the meaning of the problem or what is expected of them
- Making the problem as attractive as possible
- Allowing enough time to explore the problem fully

Sawada also suggests that as the approach places 'special emphasis on the mathematical thinking of individual students', it is important that the teacher remains neutral. Shimada warns that 'the openness is lost if the teacher proceeds as though only one method is presupposed as the correct one.'

He emphasises that a suitable problem for learning mathematics should have the following features:

The problem must begin where the students are. The design or selection of the task should take into consideration the current understanding of the students. They should have appropriate ideas to engage and solve the problem and yet still find it challenging and interesting. In other words it should be within their zone of proximal development.

The problematic or engaging aspect of the problem must be due to the mathematics that the students are to learn. In solving the problem or doing the activity, students should be concerned with primarily making sense of the mathematics involved and therefore developing their understanding of those ideas. Although it is acceptable and even desirable to have contexts or external conditions for problems that make them interesting, these aspects should not overshadow the mathematics to be learned.

The problem must require justifications and explanations for answers and methods. Students should understand that the responsibility for determining if answers are correct and why rests with them. Students should also expect to explain their solution methods as a natural part of solving problems.

It is important to remember that mathematics is to be taught through problem solving. That is, problem based tasks and activities are the vehicles through which your curriculum can be developed. Student learning is an outcome of the problem solving process.

Teaching with problem based tasks is student centred rather than teacher centred. It begins with and builds on the ideas our students have available – their workings and understanding. It is a process that requires faith in children, a belief that all children can create meaningful ideas about mathematics.

In Japan, all mathematics teachers are familiar with Shimada's work and plan many of their lessons around his open approach. As this approach is common in all Japanese primary and secondary schools, students are familiar with both the wording and style of this type of question.

Lessons generally follow a four-step process (Natsusaka, 2011):

- Presenting and identifying the problem of the day
- Developing a solution
- Progression through discussion
- Highlighting and summarising the main point

Natsusaka adds that this model is based on the earlier work of Polya (1945), Dewey and Wallas who all studied the problem-solving process.

Polya's four phases were:

(1) Understand the problem,
(2) devising a plan,
(3) carrying out the plan, and
(4) looking back.

Dewey's five phases were:

(1) Experience a difficulty,
(2) define the difficulty,
(3) generate a possible solution,

(4) test the solution by reasoning, and
(5) verify the solution.

Wallas's four phases:

(1) Preparation,
(2) incubation,
(3) illumination, and
(4) verification

These strategies for problem solving are all very similar but the key difficulty we face when delivering a problem solving approach for Core Maths is that in the UK , neither the students nor their teachers are familiar with this approach.

The Core Maths initiative could and should challenge us as to how maths can be taught effectively to students who have already been through five years of secondary mathematics and who have possibly "passed" GCSE by learning set methods to standard questions. In order for Core Maths to be successful, teachers beliefs on teaching and learning need to be challenged and changed. If our teachers' beliefs remain unchanged then our students' learning opportunities will be restricted to a conventional approach that we all know has failed many of our students for decades.

References

Becker J.P. (2005) *Reflections on U.S.-Japan Collaborative Research in Mathematical Education.* Improving Mathematics Teaching and Learning through Lesson Study, May 20-21, 2005, Chicago.

Becker J.P. & Shimada S. (1997). *The Open-Ended Approach: A New Proposal for Teaching Mathematics.* NCTM

Department for Education (2010), *The Importance of Teaching: The Schools White Paper 2010*, London: DfE

Gove, M. (2010). In Vorderman, C., Porkess, R., Budd, C., Dunne, R., & Rahman-hart, P. (2011). A world-class mathematics education for all our young people. London.

Hiebert J., Carpenter, T.P., Fennema, E., Fuson, K.C., Wearne D., Murray, H., Olivier, A., Human, P. (1997) *Making Sense: teaching and learning with understanding.* Heinemann

Isoda, M., Stephens, M., Ohara, Y. & Mijakawa, T. (2007). *Japanese Lesson Study in Mathematics.* World Scientific Publishing.

Isoda, M., Katagiri, S. (2012) *Mathematical Thinking: How to develop it in the classroom*. World Scientific Publishing

Japanese Ministry of Education, 2008. Available on line at http://www.globaledresources.com/products/books/guide_arithmetic_7-9.html

Kishimoto, T. & Tsubota, K. (2007). What are the features of lesson study projects conducted in elementary school mathematics departments? In: Isoda, M., Ohara, Y., Miyakawa, T. and Stephens, M. (Eds.), J*apanese Lesson Study in Mathematics: Its Impact, Diversity and Potential for Educational Improvement*. (pp. 124-127) World Scientific Publishing Co. Pte.Ltd., New Jersey.

National Council of Teachers of Mathematics. (1980). *An agenda for action: Recommendations for school mathematics of the 1980s.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Natsusaka, S. (2007). *Discussion-orientated teaching methods and examples:* The problem-solving orientated teaching methods and examples. In: Isoda, M., Ohara, Y., Miyakawa, T. and Stephens, M. (eds.), J*apanese Lesson Study in Mathematics: Its Impact, Diversity and Potential for Educational Improvement*. (pp. 92-101) World Scientific Publishing Co. Pte.Ltd., New Jersey.

Nohda, N. 2000. Teaching by Open-Approach Method in Japanese Mathematics Classroom. In: *Proceedings of the PME-24 Conference* (eds. T. Nakahara & M. Koyama), Vol.1, 39–53. Hiroshima University (Japan).

Ofsted, (2011). Supplementary subject specific-guidance for mathematics. .[Online] Avalable at: http://www.ofsted.gov.uk/resources/generic-grade-descriptors-and-supplementary-subject-specific-guidance-for-inspectors-making-judgemen [Accessed 10th January 2013]

Pehkonen, E. (2008). Problem solving in mathematics education in Finland, p.3, www.unige.ch/math/EnsMath/Rome2008/WG2/Papers/PEHKON.pdf

Polya, G. (1945) *How to Solve It: A New Aspect of Mathematical Method.* Princeton, NJ: Princeton University Press.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In: D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334–370). New York: MacMillan

Schroeder, T.L. & Lester, F.K. 1989. Developing Understanding in Mathematics via Problem Solving. In: *NCTM Yearbook 1989* (ed. P.R. Trafton), 31-42. Reston (VA): Council.

Sawada, T. (1997). *Developing Lesson Plans*. In: Becker J.P. & Shimada S. (1997). *The Open-Ended Approach: A New Proposal for Teaching Mathematics.* NCTM

Shimada, S. (1997). *The Significance of an Open-Ended Approach*. In: Becker J.P. & Shimada S. (1997). *The Open-Ended Approach: A New Proposal for Teaching Mathematics.* NCTM

Singapore Ministry of Education. (2007) Available on line at http://www.moe.gov.sg/education/syllabuses/sciences/files/maths-primary-2007.pdf

Stigler, J.W. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York: Free Press.