A framework for understanding teachers' promotion of students' metacognition

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Abstract

This is an ethnographic study of promotion of metacognition, focusing on the teaching practices in secondary mathematics classrooms of three teachers in the UK. With all three teachers, observations of their teaching and interviews regarding their teaching were conducted. The main aim was analysing and substantiating the parallels and differences among the teaching practices, providing an account of the patterns in the teachers' promotion of metacognition and the underpinning factors. An important finding of the study was the differences in the teachers' emphasis on metacognition throughout the stages of the lessons and the activities they used, and during their interactions with the students of different achievement levels and progress with the activities. Ultimately, I developed the Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM), comprising of four factors underpinning teachers' practices: their understanding of metacognition, their perceptions of students' features and needs, the distribution of mathematical authority and the external pressures teachers perceived. The role of a teaching approach and promotion of metacognition and possible uses of the developed framework for teacher reflection are discussed as implications of the study.

Keywords

Metacognition, thinking skills, mathematics teaching,

Introduction

The last decades of the 20th century witnessed important milestones in the UK regarding the discussions on the promotion of thinking skills within mathematics education. One such milestone was the influential Cockcroft report (1982). The Cockcroft committee implicitly indicated the importance of developing students' awareness, reflection on, and evaluation of their mathematical work and thinking processes through discussion. Over the following decade, researchers drew on the Cockcroft report to point to the importance of increased student autonomy, initiative and regulation through mathematics education (Ernest 1991; Jaworski 1994).

Another landmark in British mathematics education was the introduction of a National Curriculum (NC) in 1989. Together with the curriculum, various national strategies were also implemented. One of such strategies was the adoption of a three-part lesson approach. Three-part lessons, consisting of a short starter, the main teaching activity and a final plenary, were recommended to secondary schools by the government (Department for Education and Employment 2001). This was supposed to ensure different levels of engagement and use of a variety of skills that would be useful in different parts of the lesson.

In the post-NC era, students' progress through the curriculum has been assessed by national curriculum assessments (widely referred to as Standard Attainment Tests or SATs). These tests have been criticised by teachers, educators and educational researchers as influencing both the focus and implementation of the curriculum in a way that inhibited the development of students' higher-level skills, such as problem solving and critical examination of information (Broadfoot 1995; Gill 2004).

A new NC was introduced in the UK in September 2008. One of the major changes of this new curriculum is expressed in the curriculum documents as a focus on skills (Qualifications and Curriculum Authority [QCA] 2008a), which have been highlighted as key processes that need to be developed through the teaching of mathematics. Among the emphasised skills were:

- planning skills;
- monitoring skills such as identifying patterns, spotting mistakes;
- evaluation skills;
- and reflection on findings (QCA 2008b).

Prior to and accompanying the current changes in the curriculum, there have been numerous publications highlighting the need for further development of students' thinking skills and thinking about their thinking in particular (e.g. Kerry and Wilding 2004; Kounine et al. 2008; Ofsted 2008). Such publications addressed general educational processes as well as mathematics education.

Thinking skills are accepted to embody five dimensions as explained by Marzano et al. (1988, p. 4): "metacognition, creative and critical thinking, thinking processes, core thinking skills and relationship of content area knowledge to thinking". These are not discrete dimensions forming a taxonomy. Instead they intersect with and relate to each other. Metacognition, as will be explained for the context of the study in the next section, overlaps with the other four dimensions and constitutes a particular dimension of thinking skills having cognition as its object.

The context I have briefly summarised for the UK, pointed to the need for the development of a range of thinking skills. I argue that such needs are also valid for metacognition, considering that it is a key dimension within thinking skills, namely thinking skills targeting thinking. Therefore I claim that the context of mathematics education in the UK provided a unique opportunity in researching the development of metacognition through mathematics teaching.

A research report (McGuinness 1999) which studied various approaches for teaching thinking in educational settings for the case of UK, pointed to the lack of emphasis on thinking skills in teaching and the need to establish the teaching of thinking skills. This study strives to shed light onto the promotion of elements of thinking skills, and aims to enable a deeper understanding of teachers' practices, which would be central to any future changes. I studied the teaching practices of three middle school mathematics teachers in the UK, with a focus on their promotion of students' metacognition. The aim here is to investigate the patterns in which these teachers addressed and encouraged their students' metacognitive functioning, and to develop a lens for examination of the factors and issues which underpinned their considerations for encouraging metacognition.

Conceptual framework

Before going into the details of the conceptual approach, it is important to acknowledge that the term "framework" in this study is used as "a general pool of constructs for understanding a domain, not tightly enough organized to constitute a predictive theory" (Anderson 1983, p. 12). I adopted this term for both using metacognition throughout the analysis of this study and making sense of the findings.

Metacognition

A review of the literature indicated that there has been a lack of consensus among researchers' conceptualisation of metacognition (Veenman et al. 2006). Researchers' use of the term "metacognition" has varied from knowledge and awareness of cognitive processes (Wellman 1983 as cited in Brown 1987, pp. 106-107) to self-regulation for learning, which encompass monitoring, control and evaluation of cognition (Berardi-Coletta et al. 1995), or a joint use of the two (Flavell 1979; Garofalo and Lester 1985). Brown (1987, p. 106) posited that confusion as to the nature of metacognition, was due to its use as an all-encompassing, blanket term for various distinct concepts.

In his pioneering studies, Flavell (1979, p. 206) introduced metacognition by briefly defining it as “knowledge and cognitions about cognitive phenomena” and basically consisting of metacognitive knowledge and metacognitive experiences. Similarly, Garofalo and Lester (1985, p. 163) addressed the two aspects of the phenomena as knowledge of cognition and regulation of cognition. Indeed, most researchers accept metacognition as comprising of these two broad categories; as suggested both by studies during the early years of development of the concept (Campione 1987; Paris and Winograd 1990) and in recent reviews of work carried out on metacognition (Dinsmore et al. 2008; Georghiades 2004).

In this study, I used the terminology that is generally used in conjunction with Flavell’s definitions and focused on two main constructs throughout the fieldwork and the accompanying analysis: *metacognitive knowledge* and *metacognitive skills*. This first component of metacognition consists of an individual’s total knowledge base and beliefs relating to cognitive processes, tools and objects that the processes are directed towards. Metacognitive skills for regulating cognition comprises of cognitive actions in three major categories: planning by selection of strategies and courses of action; monitoring, also described by some researchers as awareness (Schraw 1998); and evaluation of outcomes and execution of the strategies and plans (Brown 1987; Garofalo and Lester 1985). I argue that the term regulation of cognition is used for the process one engages in while using metacognitive knowledge and skills. Hence the term regulation will be used in a context of using either one or both of knowledge and skills.

The rationale for combining two seemingly distinct constructs under a single term can be clarified through an analysis of the connections and interaction between the two. Metacognitive knowledge, as any knowledge base, is formed through individuals' cognitive experiences, owing to monitoring and evaluation of thinking, strategies used, tasks at hand etc. Moreover, metacognitive knowledge can play an important role in the regulation of cognition. Papaleontiou-Louca (2003) pointed to the fact that "metacognitive knowledge can lead somebody to select, evaluate, revise and abandon cognitive tasks, goals and strategies" (p. 15). Knowledge and beliefs can influence the depth and rigour of regulational processing, as well as the decisions to sustain effort, allocation of resources and other elements central to using metacognitive skills. This seems to be an indication of a mutual dependence and interrelationships between these two dimensions of metacognition.

Many educators believe that it is a worthwhile effort to develop students' metacognitive knowledge and skills. Metacognition has been emphasised especially in connection with the ability to learn how to learn (Georghiades 2004), mathematical problem solving (Gill 2004), and mathematical sense making (Henningsen and Stein 1997). There are a number of studies that report the positive influences of metacognition on problem solving and mathematical performance (e.g. Goos et al. 2002; Mevarech and Amrany 2008). Similarly National Council of Teachers of Mathematics (NCTM) in the US, pointed out the importance of metacognition for problem solving (NCTM 2000). Considering the evidence from the literature, I argue that the demands for a change in the focus of mathematics education have also manifested themselves as a call for developing learners' metacognition.

The efforts to improve students' mathematical performance through developing metacognition can be grouped under two main approaches: strategies targeting individual use and approaches drawing on teachers' impetus. In the first group, *thinking aloud* during problem solving (Berardi-Coletta et al. 1995) and students writing about their thinking (Pugalee 2004) were claimed to be effective tools for activating metacognition. A major implication of this for teaching is establishing the use of self-questioning and self-talk while students are individually engaged with mathematical tasks (Papaleontiou-Louca 2003). Approaches depending on teachers' impetus were basically teacher modeling the use of metacognition (Mayer 1998; Schoenfeld 1987), use of metacognitive questioning that was supported by the teacher (Goos et al. 2002; Schoenfeld 1987) and instruction based on the concept of *scaffolding* (Holton and Clarke 2006), which is an idea representing a temporary support system that is gradually reduced as the students gain competence (Wood et al. 1976).

Even though there is a range of methods and strategies proposed for developing students' metacognition, I have identified a lack of emphasis on the teachers' role and teaching practices within the efforts to incorporate metacognition into mathematics classrooms. As major actors in working towards the establishment of certain dispositions and methods in classroom settings, teachers' practices should constitute a central role in inquiries into metacognition in mathematics education (Goos et al. 2002, p. 220). However, it has also been claimed that teachers did not emphasise the explicit teaching of metacognitive skills (Desoete 2007). This current picture necessitated the investigation of the following research questions broadly expressed in two groups:

- To what extent do mathematics teachers promote (or not promote) students' metacognition in their classrooms? What practices do they use to encourage metacognition?
- What are the issues underpinning teachers' considerations for encouraging metacognition in mathematics education?

Methodology

I adopted an ethnographic methodology for this study. Use of such a methodology stemmed from a sensitivity to capture what happens in natural settings and making sense of the complex relationships between observable behaviours and participants' accounts and attitudes. In educational settings, various factors such as language, norms and practices forming the classroom culture, interaction patterns among students and with the teacher, influence the experiences of individuals. The major motive of my study was to explore the emphasis by mathematics teachers on metacognition in the classroom setting, by building up an extensive framework, addressing both the uniqueness and typicality of the studied settings, and capturing the complexity of teaching.

The context of the study and the teachers

Ethnographic studies are generally designed to encapsulate a small number of cases or situations, to provide detailed descriptions. Hence, my study had a small sample size, comprising of three teachers that I worked with in two schools.

The two middle schools where I conducted my research, which I call James King and Hillside Schools, had a tradition of enthusiastic cooperation with researchers. The provision of mathematics education was appraised as being very good in both schools by Ofsted, the responsible unit for inspecting and regulating education in the UK. Three teachers from these two schools, Alan and Tracey at Hillside and Keith at James King, volunteered to take part in my study. Two of the teachers, Keith and Tracey, were in their late twenties and new in teaching, with only one and two years of teaching experience respectively. On the other hand, Alan was the head of the mathematics department with 18 years of teaching experience. In my initial interviews with them, all teachers declared interest in development of metacognition through mathematics education and this was the adoption of a purposive sampling for the study.

Data collection

I collected data over a period stretching from December 2005 to January 2007. Due to the busy schedule of the teachers, I worked with Alan and Keith until the end of June 2006 and with Tracey from September 2006 until the end of my fieldwork. I observed lessons of teachers in one particular class each (with either 12 or 13 year olds). Due to the volume of visitors and trainees in both schools, with all three teachers, we decided that I could visit their classes for approximately one lesson every week. The number of visits to teachers' classes varied between 18 and 25. Each visit comprised of observations from an hour-long lesson. The classes were from different achievement levels, a high achieving group taught by Keith, a low achieving group taught by Alan and an average achieving group taught by Tracey.

Methods for data collection

I carried out my data collection by observations of the classrooms, audio recording of various lessons and interviews with the teachers. These were the primary sources of data, as widely seen in ethnographic studies (Eisenhart 1988). I also complemented these by collecting students' work and the materials used by the teachers, which constituted a secondary data source. Use of a variety of methods is in accordance with the emphasis in ethnography on capturing the complexity and uniqueness of the settings.

I spent almost all of my time in the classrooms, observing from one corner of the room and making notes. In each class, the teachers also used some activities we prepared jointly (two in Keith's class, three in Alan's and four in Tracey's) and the lessons in which the teachers used the activities were audio recorded. Together with the use of an ethnographic approach, there was an intervention to the teaching practices through the development of these activities. Ethnography can be accompanied by moderate levels of intervention in order to "gain further insights into the phenomena one is studying" (Gall et al. 2003, p. 493).

Use of activities for data collection

The activities were tailored for tackling common student misconceptions about the topics being covered according to the teachers' term plans. There was an inherent assumption that activities based on such an approach provided opportunities for promoting the use of metacognition (for a detailed description and discussion of the activities see **Author –** blinded for anonymity). While I created the ideas for some of the activities, some already existed in published materials (Swan 1996; Swan and Wall 2005). I discussed these ideas for activities with the teachers. I consistently emphasised flexibility; teachers could choose the activities to be used, make alterations to the resources for the activities, and make decisions on patterns of interaction and time allocations within the activities. In this way, teachers' decisions and use of activities could provide insights into how they approached emphasising and promoting metacognition.

I needed to be cautious in making sense of teachers' use of the activities and their accounts of their teaching and avoid imposing my ideas regarding how the activity could be used on my interpretations of the teaching practices. An important manifestation of my sensitivity on this issue was encouraging the teachers to have flexibility in incorporating the activities into their teaching. I endeavoured to analyse their decisions on what sort of teaching practices they prioritised while using the activities, in order to unravel their teaching practices from a perspective of promoting metacognition.

An important point for me to consider during teachers' use of the activities was that it was not common practice for them to build activities on students' misconceptions. Therefore this constituted a deviation from the natural flow of their teaching. Taking part in a research study could also cause teacher reactivity as we had discussed previously what this research study focused on. Teachers could be influenced to show me what they thought I expected to see as a researcher. Moreover, most of the activities were used as the main activities of the lessons and students spent 15 to 40 minutes on each activity. This was another deviation from the teachers' common classroom practices. However, having observed them over long periods of time, I carefully analysed their teaching both in comparison with their regular teaching practice and by reflecting on their teaching with them during our interviews following the activity lessons.

Data analysis

Analysis of data for my research had a dynamic nature occurring in a number of ways. An immediate analysis was taking place informally while I was taking notes during my observations and being selective in paying attention to some incidents and issues, and through my decisions for bringing certain issues up for discussion in the interviews. The formal data analysis after the completion of my fieldwork, displayed the elements of initial theorising through perception; comparing, contrasting, aggregating and ordering; establishing linkages and relationships; and speculation (making inferences) (LeCompte and Preissle 2003).

Having transcribed the audio recorded data from the classrooms and interviews, I coded the transcript data and my field notes. The coding emerged as I studied my data, and it was basically used for representing teachers' actions and probes for eliciting various forms of student functioning. The coding framework comprised of teachers' encouragement of students' use of metacognitive knowledge and skills (i.e. knowledge, planning, monitoring/awareness, evaluation), pointing to the cognitive operations for the students to engage in (e.g. telling the specific operation, asking for recitation of formulas) and teachers carrying out the cognitive operations themselves (e.g. evaluating students' answers, explaining students' answers). I categorised the coded data by continuously comparing and contrasting and refining and making alterations to the categorizations whenever necessary. I also used the concept of teacher guidance of students' cognitive and metacognitive operations as "background ideas informing the research problem, used for seeing, organising and understanding experience" (Charmaz 2000, p. 515). This was beneficial for enabling coding and categorization of teachers' practice. In the section on findings, I provide the codes on the right hand column parallel to the extracts from data of the study.

While using guidance as a tool for analysing teachers' practice, I considered teacher guidance in two dimensions: *guidance of cognitive operations* and *guidance of metacognitive operations*. High teacher guidance of cognitive operations in classrooms can be generally characterised by a high density of closed-ended questions and tasks highly structured and directed by teachers, pointing to ways in which students can work. Regarding teachers' guidance of metacognitive functioning, a high level is indicated by encouraging students' activation of metacognition through suggesting strategies and by modelling the use of such strategies.

Findings

All three teachers generally adopted the 3-part lesson approach recommended to secondary schools by the government. They got the students to work in pairs or individually during the main teaching section of the 3-part lessons followed by an extensive teacher-led whole class discussion. My analysis focused firstly on my first research question regarding the function of teachers' practice as they relate to the promotion of students' metacognition.

Manifestations of promotion of student metacognition

During introduction and early stages of the lessons

A common pattern, particularly in Keith and Tracey's lessons, was their encouragement of students' use of metacognition during their introduction and explanations of the work in which the students were expected to engage. The teachers generally asked their students to explain their ideas, progress or answers, mainly by using written methods during the activities. Such an emphasis during the early phases was in line with their regular teaching practices.

Keith often emphasised various aspects of metacognition in a casual manner while students were starting to work on the exercises. He was making suggestions to his students about what sort of metacognitive skills they could adopt. He made the following comments in his classroom:

> "It's very interesting to see the choice of method: traditionals not using calculators, free thinkers using the tree method. You're free to use whichever method you like to factorise. Just pay attention to the method you're using."
>
> "You need to have resilience. Sometimes you can try doing things and see!"

There were different elements of metacognition in these extracts. Asking the students to be aware of the methods they used was a part of his teaching practice that Keith regularly emphasised in his class. He urged his students to notice that there were numerous methods they could employ and encouraged awareness of their cognitive strategies. He was suggesting that he expected the students to make conscious decisions on their mathematical thinking. He also emphasised the importance of reflecting on what they were doing and evaluating their progress. His suggestions above and other similar ones conveyed a sense of expectation regarding generic habits of working mathematically. Yet he did not resort to providing a set of questions, methods or a rubric as a means for establishing a classroom ethos for working metacognitively as widely suggested in the relevant literature (e.g. Holton and Clarke 2006; Schoenfeld 1987). A similar episode given below is from Tracey's introduction of an activity:

> "OK, I'm going to give you a problem. I'm not going to tell you anything about it to start with. All I want you to do is I want you to read through the problem and just check you understand it. So, check you understand what it's talking about. And then I want you to do... There's a question at the end. OK, what I'd like you to do is explain what you...explain what you think the answer is. OK, and jot it down in your book. Alright and if you

can, explain why you think that is the correct answer. And then I want you to think about how we might actually check and see if we're right."

Tracey's explanation contained indicators of teacher guidance, considering that she was clearly telling the students the thinking processes they were expected to carry out. Yet, she was encouraging them to monitor and be aware of their cognitive functions. Her directions were addressing students' metacognition during their involvement with the task. She was trying to set them up for the task, by making sure that they understood what the mathematical problem involved, to get them to monitor their thinking by jotting their ideas down and trying to explain them, and finally to evaluate their solutions by checking their plausibility. This short period of teacher guidance of the metacognitive domain, which can be interpreted as an act of scaffolding, can help her to initiate students' engagement with the tasks metacognitively. However, it is important to note that Tracey adopted a slightly different approach from her practice during this activity. She refrained from giving answers to students' questions asking for her approval of whether they were on the right track or not. Students reacted to her persistence for encouraging them to work without her guidance by asking her further questions. This was an indication of how social norms for expectations in a classroom setting can shape the interactions among the teacher and the students. Tracey had to invest considerable amounts of time to persuade the students to work in the ways in which she asked.

There were differences in the way Alan made introductions to the activities in his class, in comparison to Tracey and Keith. Alan used questions and prompts to guide both cognitive and metacognitive operations. He was pointing towards specific directions about the task while also highlighting elements of metacognition. An incident where Alan encouraged the use of metacognition during the early whole class discussion is below. He was engaged in a conversation with a student, in order to urge her to explain her thinking.

Alan:	Well, I want to know what you had as an understanding of the word biased. So I gave you two opportunities I've said "it's a dice with lots and lots of 3s on it" or "a dice that has got more chance of landing on a 3" and you said a dice that has more of a chance of landing on a 3. I'm wondering why you've said that.	
(silence)		
Alan:	Is there anything on that piece of paper that makes you think that will be the situation?	Teacher's encouragement of explanation of thinking
Diane:	Yeah.	
Alan:	What's that?	
Diane:	Frequencies.	
Alan:	The frequency table. So you looked at the frequency table to try and get some evidence of that.	Teacher's encouragement of awareness and monitoring of thinking

Alan first recapitulated what he expected Diane to do, but when she did not give an explanation, he shifted the focus of his questioning onto how Diane's thinking was influenced by the information in the activity. He aimed at promoting her reflection on her thinking, by asking her to connect her thinking with the materials she had been thinking on. He also endeavoured to create an environment where Diane could talk about her thinking more comfortably, since the activity materials served as a framework for expressing the progress of her ideas.

Emphasis on elements of metacognition during individual interactions

Data I collected from Keith and Alan's classes did not include a detailed account of their individual interactions with every student. The recording was done at a whole class level. Therefore, my analysis regarding this aspect of promotion of metacognition from my fieldwork with Keith and Alan was mainly confined to a number of incidents from the lessons and their comments on their own practices. But as the important nature of such interactions unfolded through my fieldwork in their classes, I focused more on interactions with the students at an individual level, in the later stages of my research, in Tracey's class.

An interesting incident took place while Keith interacted with two students, Christine and Tamara, during an activity on Pythagoras Theorem. In the activity, the students had various triangles (acute, right and obtuse) with only certain side lengths given and they were asked to find the length of the missing sides. They were expected to compare their calculations of the perimeter and their measurements afterwards. During their conversation, Tamara initially claimed that she thought the calculations would give her the correct answers as opposed to measurements, whereas Christine, Tamara's partner, insisted that the answers they got from their measurements were to be reliable for correctness. After asking them to articulate their approaches, Keith left them to discuss and think about their ideas as a pair. He encouraged both students to discuss their ideas and reflect on their approaches. In this way, he emphasised a metacognitive mode of operation.

He brought this incident up for further discussion at a whole class level at the end of the activity. He asked Tamara to explain what process they went through and the conclusion they reached. Even though he had not followed up Tamara and Christine's further discussion, he expressed in my conversation after the lesson that he anticipated that it was likely for Tamara to spot a pattern in the contradictions in their calculations and measurements. Why he opened Tamara's thinking for a whole class discussion was strategically related to the fact that he considered Tamara as one of the highest achieving students in his class and expected her to have made progress. However, Tamara commented on the results she reached without referring to the thought process she went through and Keith brought the whole class discussion to an end by highlighting her statement. Even though he invited his student to comment on her thought process, a metacognitive activity, when she did not do so, Keith did not persist for a metacognitive emphasis. He explained in my interview with him that he felt he was running out of time. This indicated the complex nature of teaching decisions and how various agendas (e.g. in this case an emphasis on metacognition, students' contributions to interactions and running out of time) influence such decisions.

Tracey encouraged her students to reflect on their solutions, including both the steps they had gone through and their outcome, by questions and prompts during her individual conversations with the students. Her approach drew on means of guiding and pointing to metacognitive operations, hence structuring and supporting students' reflection on and often evaluation of their thinking. In this way, she initiated a joint effort with the students where she facilitated the evaluation of their cognitive functions by monitoring their written work and explanations of their progress.

During the interviews all teachers commented on their general preferences to engage in one-to-one conversations in order to support students' reflection on their thoughts. They claimed that interactions at an individual level were more appropriate for encouraging metacognitive functioning, due to three main factors:

- the relatively informal nature of individual interactions when students' work was in progress;

- being away from the competitive peer pressure in whole class interactions;
- idiosyncrasies in students' thinking approaches making them personal thinking features.

While personalising the use of metacognitive elements through individual interactions still helps the promotion and development of students' metacognition, these efforts could be examined together with the emphasis on metacognition at the whole class level, as suggested by the studies on classroom culture (De Corte and Verschaffel 2007).

Lack of Emphasis on Metacognition during Whole Class Interactions

Unlike the teachers' general preferences to promote students' metacognitive processing during individual work, a common pattern in their teaching was the limited emphasis on metacognition during whole class interactions with the students. The interactions I address here took place during discussions about students' ongoing work, and discussions at the end of the activities and lessons.

Keith and Alan's teaching displayed high levels of teacher guidance of students' cognitive operations. They monitored, evaluated and planned further action regarding students' mathematical work. Unlike their introduction to the activities, their interactions at the whole class level manifested teacher regulation of students' cognitive operations during problem solving, as opposed to a sustained emphasis on students' metacognitive engagement with their work.

For example, Alan used questioning in a way that regulated students' progress, by prompting and pointing in ways for them to conduct various lower cognitive functions, during the discussion at the end of an activity on ordering decimals. Students were asked to order the given decimals using any strategy they wished. Alan engaged in a whole class discussion to bring the activity to an end. His prompts aimed at demonstrating a strategy for ordering decimals.

Alan: Amir's thinking 0.375 is bigger than 0.4. But you're telling me it's not. How do you know it's not, Josh?

Josh: Because 4 comes after 3.

Alan: Because 4 comes after 3. So when you're placing your numbers on the number line, I know that this number here is a little bigger than 0.3, because it has got some more things following it, but it's certainly less than 0.4. Because I look at the first number after the decimal point, don't I? So I always need to look at the first number after the decimal point. So which one of these numbers has got the smallest number after the decimal point? Let's work through and see. Chris, what number has this got after the decimal point?

Chris: 7.

Alan: 7. What number has this got after the decimal point?

Students: (all at once) 0.

(They went one by one through all the numbers on the board)

Alan: So which number has got the smallest number?

Students: (all at once) 0

Alan: The second one. So this one I know, and we think and we are certain because we used our logic, is the smallest number.

(He continued checking one by one the four orderings already on the board)

In this extract from the final discussion of this activity, Alan's description of the strategy was followed by questions that took his students step by step through the procedure. Alan regulated the entire process, since he was planning and monitoring the cognitive operations to be carried out. He did not direct his questioning towards students' work on the number line or their reflections on their comparisons of their answers, which could require them to take responsibility for explaining their thought processes and hence operate metacognitively.

Variations according to students' progress and teacher perceived achievement levels

The teachers' emphasis on the use of metacognition depended vastly on students' mathematical achievement in general and their difficulties in dealing with the classroom tasks in particular. They tended to guide the cognitive operations while working with the struggling students. Without a teacher emphasis on metacognition, the students conducted the lower level cognitive operations set for them by their teachers. Such an interaction pattern was particularly observable in Tracey's individual interactions with her students and Alan's general teaching preferences for his lower achieving class.

In the following example, Tracey stopped by a group of students working together and delved into their work.

Tracey: How are you getting on?	Teacher's encouragement of
Emma: Trying it out.	monitoring through verbalising
Tracey: Is everything all right? What have you	thinking
started off with the price of?	Teacher's encouragement of
Emma: 2 pounds.	monitoring of the steps in the
Tracey: 2 pounds, OK. And what has happened	problem solution
to that price then?	Teacher's encouragement of
Emma: It would increase. (pause) I think it's	monitoring of the steps in the
like…	problem solution
Tracey: OK, so what happens to it first? What	
do you do? Do it in two stages.	Teacher directing the operations
Emma: What, there?	to be executed

Tracey: Yeah. What's the first thing, it's gone up by?	Teacher directing the operations to be executed
Emma: 20	
Tracey: 20%. So what happens if that price goes up by 20%?	Teacher pointing to the performance of a specific operation
Emma: 2.40.	
Tracey: OK, so it's 2.40. So it's costing 2 pound 40. And then you're going to get a discount of 20%. So what happens if you discount that by 20%?	Teacher directing the operations to be executed
Emma: (inaudible)	
Tracey: Is it? Work it out. What happens if you discount that price?	Teacher pointing to the performance of a specific operation
Elizabeth: Aaaa!	
Emma: It's not coming back. 24 p.	
Tracey: OK, that's 10%.	Teacher evaluation of students' cognitive operations
Emma: Yeah, 48 p off. Then less I think.	
Elizabeth: 1 pound 96.	
Emma: 1 pound 92, isn't it?	

Tracey initially asked questions about where they started their inquiry and in which direction they had moved. In her first utterance, she encouraged the students to take control of their problem solving process and to explain their efforts on the task by asking a general question about their progress, but in the second and third, she gradually started guiding the metacognitive operations and asked specific questions about the steps in their solution. These questions gave Tracey a short time to observe the students' progress and to encourage the students to explain their work. However, after her third utterance, (she claimed this was the moment she saw that the students were not making considerable progress) Tracey started guiding the students towards certain cognitive operations, starting with her suggestion to solve the problem in two stages. From that point on

Tracey was engaged in a joint problem solving task with the students, where she was leading them through the steps of the solution. She posed a series of questions to the students, the majority of which entailed the execution of simple mathematical operations, rather than working metacognitively.

Metacognition as retrospective evaluation

Metacognition is a multi-component construct, and individual teachers' priorities of the components encapsulated in metacognition can vary. While teachers highlighted various components within metacognition, a common practice was to encourage the evaluation of thinking retrospectively.

Considering the teachers' emphasis on metacognition, I posit that they often encouraged students' evaluation of their mathematical work as a means for checking their answer. This can be seen in Keith's questioning during the whole class discussions on students' evaluations of their completed work. Alan's use of the activity sheets as a tool for emphasising the need to check their answers and repeatedly urging the students to check their work also pointed to his priority of students' retrospective evaluation of cognitive functions. Similarly, Tracey's use of repeated admonishments about checking their work on the activity (e.g. in an interview, she explained students' lack of regulation of their work as "I think it all comes back to that they don't check their work") highlighted this issue.

Factors influencing teachers' promotion of students' use of metacognition

Producing an account of the three teachers' promotion of metacognition and categorisation of these patterns enabled me to further my analysis by discussing the issues underlying considerations for promoting metacognition. I developed the Framework for Analysing Mathematics Teaching for the

Advancement of Metacognition (FAMTAM), comprising of closely linked factors that inform and influence teachers' decisions regarding their practice (Figure 1). The four factors are:

- teachers' conceptualization of metacognition;
- teachers' perceptions of students' features and needs;
- distribution of mathematical authority in the classroom
- external pressures perceived by teachers.

In the following sections, I will provide an account of the development of these factors through my analysis.

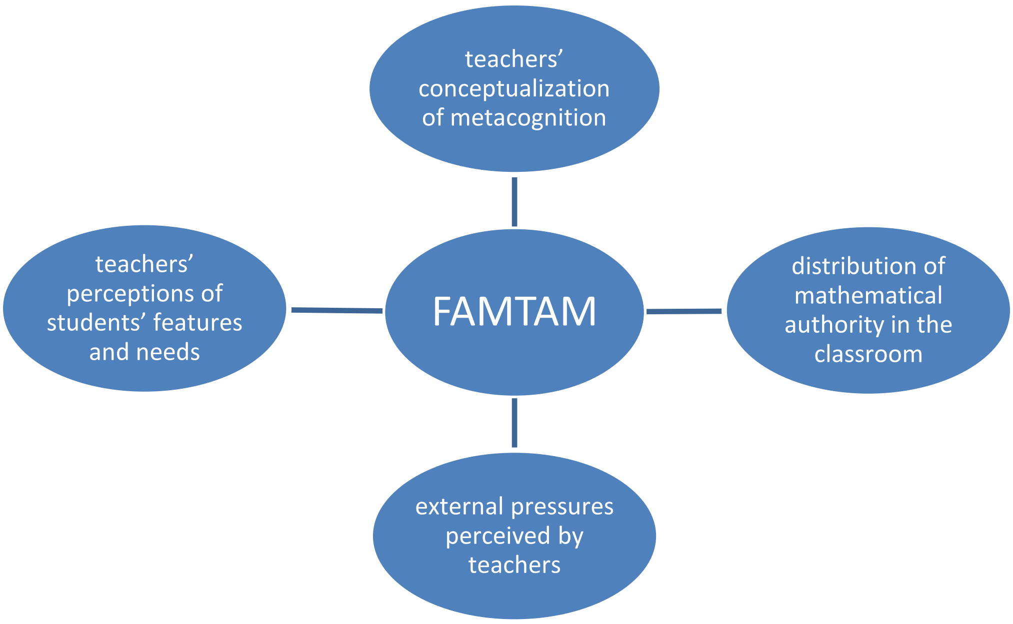

Figure 1: Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM)

Teachers' conceptualization of metacognition

While dealing at a professional level with complex phenomena having various components, teachers' conceptualisations have implications for the practical domain. I analysed teachers' approach to encouraging students' use of metacognition, in comparison to their remarks and explanations manifesting their understanding of metacognition. This enabled me to study the links between them.

It is important to note that the interaction between the conceptualization of metacognition as a construct and practical considerations for teaching is a reciprocal one. While practical experiences have an impact on understanding of how metacognition can fit into classroom practice, understanding of metacognition, and its educational implications, informs to what extent promotion of metacognition exists in teaching practices.

In my interviews with the teachers, we discussed their understanding of metacognition. I have discussed earlier, the manifestations of promotion of metacognition in a retrospective manner through evaluation and there were similarities in teachers' ideas about metacognition. This extract from my interview with Tracey helps to reveal her understanding of the concept:

Researcher: . . . to what extent do you think metacognitive skills should have a role in education?

Tracey: Thinking about what you're (pause) sort of develop, get into the point where you regulate what you're doing, sort of have that feedback. I mean that's just, that's one of the key things we'd like kids to be able to do isn't it, when you leave school? Certainly if you go onto further study or you go onto a job, you want to be an independent thinker. And part of being an independent thinker is knowing (pause) looking at what you come up with and being able to say "right, is that correct, what do I need to do, have I done it?", you need to be able to do that loop yourself. ...And a lot of kids will get to the end and they won't see anywhere to go from there. That's the end. They've got their answer. Check their work and answer, they see it as a chore.

Her comments did not explicitly describe her conceptualization of the phenomenon. However, she used the term *have that feedback,* while referring to the use of metacognitive processing and indicated her perception of using it as an ultimate process in cognitive tasks. Using examples of self-questioning, she elaborated on how students could develop evaluation skills for their work. Rather than describing metacognition as a process that an individual could exploit during all the stages of mathematical problem solving (e.g. planning, monitoring and evaluating progress), she described it succinctly as assessing products.

Both Alan and Keith's practices also had influences from their conceptualisation of how metacognition can be activated in classrooms. Keith's preferences about asking students questions about their ideas and explanations of their answers and using writing as tool for reflecting on thinking were influenced by a project going on at James King School. Learning to Learn (L2L) project had been implemented at the school for over a year. L2L project was conducted in separate lessons, where students practiced strategies and methods for developing their awareness and retrospective reflection on their learning. The skills emphasised within this project influenced the priorities Keith gave to some metacognitive skills more than others. Similarly, in my interview with Alan, his contention about lower achieving students being able to carry out evaluation of their work and accompanying thinking, but finding it harder to engage in planning and monitoring pointed to the underpinnings of his relative emphasis on retrospective evaluation among metacognitive skills.

In addition, Tracey claimed that explicitly engaging in metacognitive processing was very difficult for students struggling with cognitive functioning and confessed that she did not believe she could promote their metacognitive work. Hence there was almost a separation in her efforts to facilitate cognitive and metacognitive functioning of her students. In her approach cognitive functions had precedence over the metacognitive ones, rather than the two sets of skills developing together.

An implication here is that it is important for teachers to develop awareness of their perceptions (Kerry and Wilding 2004), particularly relating to the nature of the metacognitive domain. This can enable them to engage in a quest for developing students' metacognitive knowledge and skills, for all components of metacognition and for all students..

Teachers' Perceptions of students' features and needs

The second factor influencing teachers' efforts to promote students' metacognition has its roots in what Jaworski (1994) has defined in her investigation of mathematics teaching as "sensitivity to students". Sensitivity to students was described as a domain of teaching, comprising of "teacher's knowledge of students and attention to their needs and the ways in which the teacher interacts with individuals and guides group interactions" (Potari and Jaworski 2002, p. 353). I extended the ideas underpinning "sensitivity to students" to promoting metacognition in mathematics teaching.

A good example of the influence of perceptions about the students' need on teachers' considerations for promoting metacognition was evident in Tracey's practice. Tracey's interactions with the students regarding their work in various steps of the activities displayed a general pattern. She started by inquiring into their thoughts and progress on the task, a request for general comments about how they were getting on with the task and for the students to explain their progress. However, Tracey's further comments and approach varied according to the responses of the students. She encouraged students to explain their thoughts if they had produced ideas and made progress towards a solution, but she guided the students who struggled with the mathematics through a step-by-step process towards a solution, using questions and prompts. While in various interactions of the former cases Tracey addressed metacognitive elements, she did not address metacognition in the latter.

In my interviews with Tracey, we discussed her activity lessons in detail. In these interviews, the level of her guidance and directing in the activity lessons was one of the recurring themes and her comments were in parallel with my claim that she was inclined to guide the cognitive operations, especially with students having problems in understanding the mathematical content of the activities.

> "I think it depends on...I usually start out with a fairly open, like "how are you getting on" or (pause) to see where they're going and if I think they're floundering, if I think they really don't understand, I usually take more control. So I usually see how they're feeling about it. It's hard to tell but you can tell looking at (pause) a kid when they genuinely don't really understand what's going on. And in that case you've got to help them, you've got to step in a little bit for them... So I think it depends on how much understanding you can see they've got of the task in hand."

The teaching practice Tracey described above revealed the underpinnings of her teaching practice, particularly the influence of her perceptions of the students' needs on her efforts to promote students' metacognition. Similarly, Alan's description of his practice as "frogmarching the lower achieving students to an endpoint" pointed to the important influence of his beliefs about what students need and his considerations about efforts to promote students' metacognition.

Distribution of mathematical authority in the classroom

Another factor, that informed teachers' emphasis on students' metacognition, is the distribution of the mathematical authority in their classrooms. One should be careful while describing the authority role in a classroom on a bipolar continuum, having teacher authority on one end and authority lying with the student on the other. The key issue here is how they exercise this authority. Boaler (2003) discussed the mathematically empowering influence of establishing a classroom practice where mathematics itself acts as an authority. In such an approach, students are encouraged to "consider the authority of the discipline - the norms and activities that constitute mathematical work" instead of turning to the teacher for evaluation and approval of their mathematical ideas (p. 8). Similarly, Schoenfeld (1994) claimed that individuals exercise the authority given to them by mathematics, i.e. mathematical ways of working accepted by mathematical communities. Deflecting the authority role from teachers to students requires encouraging students' use of particular ways of working, not just telling them to work on their own. Within this study, mathematical authority is viewed as where the responsibility for evaluating and commenting on the plausibility of the mathematical work lies in the classroom. My conceptualisation of authority of the discipline here also embodies metacognition as a norm for mathematical problem solving. Thus, I claim that a teacher, deflecting authority to mathematics in the classroom, could also encourage students' use of their knowledge, planning, monitoring and evaluation of their cognition. These would constitute the canonised ways of working metacognitively to exercise this authority.

In the teachers' introductions to the activity lessons, the mathematical authority in the classroom stood out as an important issue in terms of their emphasis on student metacognition, consistent with a classroom environment where mathematical thinking was presented as a source of authority. They discussed the thinking processes students were expected to go through. Similarly in the conversations with individual students early in the activity, particularly in Tracey's classroom, she was trying to encourage them to think and talk about their initial thoughts and she tried to avoid evaluating their thinking. These episodes were marked by an emphasis on promoting metacognitive skills in the classroom. Even though she was telling them how to work on the given tasks, i.e. working metacognitively, she was inviting her students to exercise the authority of mathematics. The point here is that teachers who are ready to deflect authority away from them towards students' use of authority of the discipline might create a working environment where metacognition is promoted.

Evidence of a link between Tracey's ideas about classroom authority and promoting metacognition was seen during the whole class discussion on an activity. Rose, one of the high achievers, explained her method for finding the answer. When Rose explained her ideas, I was thinking about what Rose meant, since I could not understand clearly what she was trying to explain. From Tracey's facial expressions I also had the impression that Tracey was not exactly sure what Rose's explanation meant. As her next move, Tracey chose to guide the student's cognitive operations, by picking out a part of Rose's explanation that Tracey thought would be useful for directing the discussion towards where she wanted it to go. In an interview, she commented on this event:

> "I think my initial thought at that point was: 'I am not really sure where she's going at'. And perhaps, maybe I haven't thought about this problem

enough, and I was just a little bit insecure about what I was doing. So at the whole class level, I wouldn't want to do that unless I was really secure in understanding why (pause) what she was saying and what she was getting on and where she was making that error... It's all a confidence thing though... I think what I was worried about in a whole class situation is saying to her (pause) asking her these questions and then not really following again sort of what she was saying."

Tracey's response clearly indicated that her desire to determine the direction of the discussion in the teaching settings prevailed over the idea of questioning Rose about her explanation, and possibly promoting metacognition in a classroom context. She did not want to confront the risk of not comprehending the students' explanations. This was an incident where Tracey acted as the mathematical authority in the classroom and the accompanying classroom exchanges did not emphasise use of metacognition. Alternatively, during such incidents, teachers aiming to deflect the authority to mathematics could take on the role of a participator in an inquiry of students' mathematical ideas. But the more Tracey adopted the authority role to direct mathematical work, the less metacognitive students' work needed to be.

Keith's comments on his role as a teacher and what his students expected from him, especially during whole class interactions (e.g. "the students think, whatever they say they all get some feedback as to whether or not it's right or wrong, and they get that experience from me", "Obviously for me, the massive important thing was they did, by the end of the lesson, need to know what the right answer was.") indicated that he deemed it necessary to take on the role of an evaluator of students' answers and ideas. In return his students expected him to act as the authority for evaluation during whole class discussions. While Keith regarded individual interactions as possibilities for encouraging students' reflection on their thinking, during whole class interactions, he judged and evaluated students' answers and prioritised providing them with the knowledge they needed to have. Therefore, his perceptions of teachers' roles in relation to the underlying approach to learning influenced the extent to which he emphasised the use of metacognition.

External pressures perceived by teachers

Having discussed the influence of factors concerning personal disposition, I combine the external issues that influence their practice in this fourth factor. By labelling them as external, I address issues arising from wider educational contexts beyond the dynamics of mathematics classrooms, such as issues stemming from policies of the educational system and the accompanying conditions within the educational institutions. Perceived as pressures on teaching practices, these issues became internal anxieties for the teachers and influenced their efforts to promote metacognition.

A major source of external pressure was the time constraints and the curriculum content that needed to be covered. A manifestation of how time pressure influenced Tracey's practice was the way in which she guided and pointed to specific cognitive operations during her interactions with the students. While trying to enable all the students to have a basic understanding, she tried to avoid spending long periods of time with individuals, in order to be able to supervise and cater for all the students in her classroom. However, this left her with a challenging task of promoting students' understanding within interactions lasting for short periods of time, in some cases less than a minute. She dealt with this issue, by often guiding and directing the cognitive operations in those situations.

Meanwhile, Alan, as the head of department at Hillside, commented widely on the importance of exam results and the time constraints. The extract below, from an interview, emphasises this issue:

> "Sometimes you have to frogmarch them to your endpoint because that's the structures that are put upon you. Two weeks before the exam understanding how you go about adding fractions probably isn't that important. What's much more important is, can you actually do an algorithm to do that."

His words in the given quote indicated an inherent tension between focusing on the processes building up mathematical work and emphasising the products at the expense of an understanding of these processes. Even though he started this particular activity he was referring to, by encouraging students' building up of their knowledge through reflecting on their thoughts, towards the end of the activity he was trying to transmit knowledge regarding use of a strategy. This ultimate goal diminished the opportunities for reflection and working metacognitively.

Tracey also mentioned in our discussions that, at Hillside, they tried to create a balance between preparing the students for the sorts of questions they would be asked to answer in their national tests, which are closed-ended problems emphasising products of mathematical work (Kounine et al. 2008), and providing them with open-ended tasks that promote students' engagement in mathematical investigations. However, there are serious tensions arising during this balancing act. Tracey claimed that she was trying to encourage her students to answer questions about explaining their solutions and thoughts, and develop habits for asking themselves these questions. Yet she also felt obliged to impose certain mathematical operations and guide their cognitive functioning, as a manifestation of efforts to prepare them for SATs. Those two approaches were also evident in her use of the activities. While she encouraged students' thinking about their thoughts, particularly at the early stages of the activity lessons, she guided struggling students with her questions and prompts through the steps of the solutions. She felt the pressure regarding the short-term outcomes of students' mathematical work. Such a decision by Tracey to resort to ways in which she guided the cognitive operations of the students, once again implies how external factors influence her efforts to develop students' metacognition.

My interpretations of Keith's understanding of elements and possible uses of metacognition indicated that it was mainly built on evaluation of students' cognitive functions. Keith contended that metacognitive skills were represented by the L2L project which conceptualised reflection on a retrospective basis, and this was reflected in his selection of questions while talking about activating students' metacognition in a generic way (e.g., "how have you done that?", "how would you know if it wasn't working?"). He also asked his students to write down their explanations, a strategy promoted by the L2L project. Consequently I posit that this conceptualisation of metacognition, and the methods he used for activating metacognition were influenced by the L2L project, another manifestation of the influence of an external factor on his considerations for promoting metacognition.

Conclusions

This concluding section is for presenting a brief account of the implications of the findings in relation to the purposes of the study as well as discussing the suggestions for further action.

Implications for promoting metacognition

Teachers' practices, that did not encourage or support students' use of metacognition, were marked by their adoption of a didactic approach to teaching. Embodiments of this approach were apparent in their guidance of their students' cognitive operations, through conducting the required functions and presenting the students with segments of work and algorithms comprising of lower level cognitive functioning. Likewise, certain teacher roles during discussions of students' work on the activities, in which the teachers evaluated and judged students' ideas and answers, indicated a didactic approach and a lack of emphasis on metacognition. Here, I use the term *didactic* for an approach to teaching marked by efforts to transmit knowledge (Crawford 1999; Ruthven and Hennessy 2002).

The teachers' promotion of metacognition, however, drew on their support for the students' reflection on their thinking and evaluation of their existing knowledge structures. During the introduction and early stages of the lessons, the students were urged to reflect on their initial work and their knowledge regarding the mathematical concepts involved. These were classroom practices encouraging use of metacognition. Hence the links between a didactic approach to teaching and promotion of students' metacognition, and strategies through which teachers can promote metacognition needs to be further studied.

FAMTAM – A useful framework

An important goal of this study was to analyse common factors informing teachers' considerations for their practices. Hence I developed the Framework for Analysing Mathematics Teaching for the Advancement of Metacognition (FAMTAM), comprising of four factors influential on teachers' considerations and decisions regarding promotion of metacognition in their teaching. These factors individually, and even more so viewed together, provide opportunities to unravel the complexities surrounding the teachers' practices and to make sense of them. It is important to note that FAMTAM is not a framework developed to describe generic teaching practices. Rather, it is a tool for offering insights into promotion of metacognition within teaching.

The factors within FAMTAM are organically related to each other, and whether the factors can be clinically unravelled while exploring teaching practices is a moot point. I claim that addressing the interrelationships and overlapping among the factors enriches the analysis of practices, by encouraging consideration of multiple factors as the underpinning influences. A manifestation of such an interrelationship was apparent in the way L2L project at James King School informed Keith's considerations for promoting metacognition as an external factor, as well as potentially influencing his conceptualisation of classroom metacognition to be mainly consisting of evaluation of cognitive functions retrospectively.

The first two factors of FAMTAM are about teachers' beliefs and conceptions and how these can inform their practices. I did not initially intend to focus on investigating teacher beliefs and the links between the findings and teacher beliefs appeared as a result of the emergent nature of qualitative inquiries. Belief research has been marked with contradictions about possible links between beliefs and practice (Thompson 1992). A linear causal relationship has not always been the case. By using FAMTAM, I argue that this difficulty can be taken into consideration through attending to multiple factors influencing teachers' practice. Especially the fourth factor about external pressures can bring social contextual influences into the picture, and help researchers make sense of teachers' contradictory considerations informing their practice. Moreover, a methodological issue in studying beliefs is limiting research conclusions to teachers' professed views (Thompson 1992). I tried to tackle this issue by collecting data through multiple sources, from observations of instructional practice and teachers' comments on specific instructional incidents and contexts.

It is important to consider that the influences of different factors might have conflicting effects on teachers' considerations for promoting metacognition. While a teacher's considerations of various factors within the framework support the encouragement of student metacognition, another factor can urge the teacher to guide students' cognitive operations and not to emphasise students' reflection on their thinking. Tracey took on the mathematical authority role and evaluated students' explanations at the whole class level, even though she encouraged and supported students' reflection on their thinking at the early stages of the activities. My interpretations and discussions with Tracey revealed that, although she conceptualised students' use of metacognition as a useful means for facilitating understanding, her priorities about acting as the mathematical authority during whole class discussions and the influences of external factors such as preparing the students for the tests and in the least time consuming way were prioritised in informing her practices.

While FAMTAM enabled me to analyse the issues underpinning the promotion of metacognition, it can also serve as a useful tool for the teachers to analyse and understand their practice. This can be achieved through teachers' reflection on their teaching, using the factors in FAMTAM as strands for developing awareness of the perceptions, tensions and constraints on the promotion of metacognition. Such reflection can encourage a critical analysis of their practices. Classrooms, as natural settings of educational practice, are the best places to develop praxis supporting students' metacognition. Teachers should orchestrate classrooms for promoting students' metacognition by using tools like FAMTAM to critically reflect on their practice towards that purpose.

Suggestions for further research

There is still a pressing need for classroom-based research exploring the socio-cultural complexities of teaching practices by attending to the immediate interactions between the teacher and the students (Cobb 2005), in order to facilitate the establishment of metacognition in mathematics classrooms. Future research in this area would benefit from projects involving work with teachers over longer periods of time, and promoting teacher research (Hall et al. 2006). An important outcome of this study is the development of FAMTAM to understand teachers' promotion of metacognition. Yet, FAMTAM requires further elaboration, with different teachers and in various educational settings, in order to explore the ways in which it can be consolidated and improved to enable constructions of detailed accounts of fostering metacognition within teaching practices.

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