Assessing Understanding Through Reading and Writing in Mathematics

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ABSTRACT

The mathematics education community recognizes the integrality of reading and writing in learning and communicating mathematics knowledge. Unfortunately, many students have yet to significantly experience this integrality in their mathematics classrooms despite the power these tools offer teachers for assessing student knowledge. This paper explores the integrality of reading and writing in mathematics and outlines techniques that can be utilized in mathematics assessment to create experiences that promote reading and writing as tools for articulating mathematics understanding.
**Introduction**

The National Council of the Teachers of Mathematics (NCTM) argues that rather than being perceived as helpful add-ons to mathematics instruction, reading and writing should be recognized as an integral part of mathematics learning (NCTM, 1989, 2000). NCTM reflects a belief that, since reading and writing necessitates the use of verbal expressions, numbers, symbolic expressions, and graphical representations, it is a crucial component in the development of reasoning, communication and connections in mathematics. In the *Principles and Standards for School Mathematics*, NCTM further assert that, “Students who have opportunities, encouragement, and support for speaking, writing, reading, and listening in mathematics classes reap dual benefits: they communicate to learn mathematics, and they learn to communicate mathematically.” (NCTM, 2000 p. 60). Researchers in the field of mathematics education have, over the years, expressed similar sentiments.

Biancarosa and Snow (2006) contend that reading is a central skill in life-long learning. In mathematics, no less so, reading is viewed as a vehicle through which mathematical text and context are negotiated to construct mathematical knowledge (Borasi & Brown, 1985; Borasi, Siegel, Fonzi, & Smith, 1998; Siegel & Borasi, 1992; Siegel, Borasi, & Fonzi, 1998). Writing is similarly considered as a means to not only communicate but also to develop mathematical understanding (Emig, 1977). Researchers claim that writing sustains students’ development of reasoning, communication and connections and consequently deepens mathematical knowledge and extends thinking (Brandau, 1990; Doherty, 1996; Drake & Amspaugh, 1994; Gopen & Smith, 1990; Grossman, Smith, & Miller, 1993; Miller, 1992; Nahrgang & Peterson, 1986; Pugalee, 1997; Porter & Massingila, 2000; Rose, 1989; Shepard, 1993; Stehney, 1990). Reading
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and writing is thus recognized as integral tools in developing and assessing mathematical understanding (Bishop, 1988). Unfortunately, the integrality of reading and writing in mathematics have yet to be significantly experienced in mathematics assessment; a crucial component in the teaching and learning of mathematics.

Assessment

Assessment plays an integral role in mathematics teaching and learning. Some of the roles identified by Crooks (1988) include, but is not restricted to: reactivating or consolidating prerequisite skills and knowledge; focusing attention on important aspects of knowledge; giving students opportunities to practice and consolidate learning; providing knowledge of results and corrective feedback; influencing students choice of learning strategies and study patterns; communicating and reinforcing the broad goals of instruction and desired standards of performance to students; and guiding the choice of further instructional or learning activities. Assessment is thus a key component to be addressed in order to ensure purposeful experiences of reading and writing to learn mathematics.

NCTM (2000) views assessment as a component of instruction that best informs and guides teachers as they make instructional decisions and students as they make judgment of what is important to learn and their approaches to personal study. The Assessment Standards for School Mathematics (NCTM, 1995) recommends that classroom assessment should reflect the mathematics that students should know and be able to do. NCTM (2000) posits that assessment tasks can convey to students what kinds of mathematical knowledge and performance are valued. Thus, for reading and writing to be recognized as essential tools in learning and articulating
understanding in mathematics, an assessment regime where students and teachers have a shared expectation that learning and articulating understanding through reading and writing is a worthwhile goal. Unfortunately, assessment of mathematical understanding is bereft of purposefully directed reading and writing experiences (Marks & Mousley, 1990); lacking in assessments based on reading and writing.

**Current Practices of Reading and Writing in Mathematics**

Although it may be argued that students are continually assessed in mathematics classrooms through reading and writing, much of the current occurrences of reading and writing are not purposefully directed and as a result serve limited roles (Marks & Mousley, 1990). For example, in a typical math class, students may be asked to take notes, interpret directions for a worksheet, and read a math text or solve a word problem. In most of these classrooms, reading and writing serves as a means for extracting or receiving mathematical information (from text or teacher) rather than as a vehicle for articulating mathematical understanding (Borasi and Siegal, 1990).

Many teachers report that they assign biographical reports on famous mathematicians when asked about the types of purposefully directed assessments they provide to their students that involve reading and writing in mathematics. Bishop (1988) argued that this kind of purposefully directed assessment practice serve the limited role of promoting reading and writing as a mode of knowledge display rather than as a vehicle for learning and articulating mathematical understanding. Unfortunately, this is the most common form of purposefully directed reading and writing assessment activity observed in most mathematics classrooms. Few mathematics assessments are purposefully directed activities of reading and writing in mathematics, despite the power these tools offer teachers for assessing student mathematics
understanding.

**Need for Purposefully Directed Assessment Activities**

Reading mathematics is complex and non-linear. It often requires bouncing repeatedly from text to tables to diagrams to symbolic expressions to graphs and such (Freitag, 1997; Noonan, 1990). Additionally, mathematical texts are more conceptually dense than other genres of writing (Brennan & Dunlap, 1985; Culyer, 1988; Thomas, 1988) and are replete with linguistic and symbolic conventions which make navigating the text challenging (Adams, 2003). Thus, it cannot be assumed that students, who can fluently read the words in a math text, actually comprehend the text (Robb, 2003). Insisting that reading in mathematics is sufficiently different from reading in other fields, Fuentes (1998) and Reehm and Long (1996) recommend that purposefully directed classroom assessments are needed in order for students to read to learn mathematics.

Since writing mathematics requires different and additional skills to writing in most other subject areas, students also need to experience purposely directed classroom writing assignments in order to gain this skill. For students to effectively write mathematics, they must have a mastery of mathematical representations (numeric, symbolic, graphical, and verbal) and the connections among representations (Freitag, 1997). Additionally, in order for students to master the precision of the mathematical language, they must experience specific instruction regarding such and have ample opportunity at refining their writing. Moore (1993) and Shibli (1992) recommend that teachers both directly instruct students in the writing of mathematics and
develop assessment materials purposefully geared toward writing to learn mathematics.

Moreover, since standardized test questions are becoming increasingly open-ended, requiring students to read, understand the question, and then compose responses, the practice of assessment in mathematics classroom needs to improve in order to be compatible with, and to support and reinforce experiences where students read and writing to articulate understanding. That is, classroom assessment needs to change to purposefully afford students opportunity to communicate their understanding via reading and writing. Similarly, the content of assessment needs to improve to include items that require students’ to reflect on ideas, formulate definitions, read, and express ideas both orally and in writing and to communicate their thinking (Shield & Galbraith, 1998). Unfortunately, all too few resources and techniques have been provided to teachers to employ in their classes. In the subsequent sections, techniques are outlined for improving classroom assessment through reading and writing mathematics.

Techniques for Improving Classroom Assessment through Reading and Writing in Mathematics

The premise underlying this article is that in order for reading and writing to be purposeful in a student’s mathematical experience, a classroom culture where students read and write to articulate mathematics understanding may be needed. Herein, numerous assessment techniques are outlined that teachers can incorporate in assessment to simulate experiences that promote reading and writing as essential tools in learning and articulating mathematics understanding. These are separated into two types: Aptitude Assessment and Content Assessment.

The former assesses students’ involvement, interest, and engagement in mathematics.
The latter assesses student’s conceptual understanding and content knowledge. Most of these assessments have the purpose of delving more deeply into student understanding and revealing far more in respect to conceptual (mis)understanding while promoting reading and writing as integral tools in mathematics. Through reading, interpreting, and writing, each of these techniques forces students to investigate, reason, perform, and or report as they interact with mathematical concepts. Answers to these assessments are never single-step operations demonstrating one single skill at a time; rather, these assessments always necessitate the student summoning and revealing more of their understanding in the process of responding to the question/prompt.

**Aptitude Assessment Techniques**

Aptitude Assessments consider students’ involvement, interest, and engagement in mathematics without divorcing such from simultaneously investigating students’ conceptual understanding. Some of the ones identified in the literature include–journal writing (Borasi & Rose, 1989; Waywood, 1992; Clarke, Waywood, & Stephens, 1993), expository writing (Venne, 1989), proof writing, and rewriting lecture notes (Sipka, 1990). These can be regularly assigned in the classroom. Examples of these are provided below.

**Peer Evaluation.** Students can be asked to make written judgment about a peer’s performance on a given mathematical tasks or question. This technique assesses a student’s ability to understand mathematics produced by another and communicate mathematically regarding such. Thus, as collaboration is promoted in student work, so too can assessment be a vehicle for collaboration. Altogether, the teacher can use the results of this activity to assess the
understanding of both students involved and students gain practice with guidance at reading, questioning, and interpreting their own ideas and the ideas of their peers.

**Two Stage Test.** After students receive feedback from teachers on tests and assignments, students can provide written responses to teacher comments. This method of assessment provides information to both students and teachers (via writing) on how well a particular topic or content has been learned or understood and the area of difficulty and affords students the opportunity to relearn and rearticulate their understanding based on the feedback given.

**Portfolios.** Portfolios have long been promoted as an assessment tool to gain insight into students learning. Portfolios afford students the opportunity to longitudinally document their understanding and progress throughout the school year through purposefully selected artifacts of their work and written justifications of why the artifacts were selected to represent their understanding.

**Journal Writing.** Journaling can be utilized to prompt students to write about a specific concept or rule or to explain how to perform a mathematical procedure or to explain why a given mathematical outcome occurs. It can also be utilized as a means for students to record reflections on materials learned in class, reactions to readings or lectures or responses to open ended assignments. Journals can then be used to informally assess student understanding of the included mathematical concepts.

**Content Assessment Techniques**

Content Assessment techniques employ reading, interpreting and writing to look deeply into student understanding. Many of the types of examples outlined in here can be perceived as more difficult than traditional test questions. This occurs for a number of reasons. First, the style of
these questions is dissimilar from traditional test items. Second, all too infrequently students experience open-ended questions and even rarer are questions which seemingly ask the student to state all he can about the mathematical situation before him. Third, some of these question types necessitate heuristics which employ multiple strategies and the use of multiple simultaneous representations. Fourth, being able to “correctly complete” the problems is not as important as revealing all one knows about the problem. Fifth, many of these problems take multiple steps to solve and paragraphs to explain. All of these dimensions differentiate these problems from traditional test items and simultaneously make these items more revelatory of student understanding. While many of the examples herein are consistent with high school mathematics, most of these techniques can be modified to be grade appropriate. The style and concepts behind the technique are more important than the actual problems themselves.

Investigate and Explain
The Investigate and Explain technique interconnects many of the previously stated findings among reading, writing, and mathematics by focusing on different representations and leading to others. Reading and writing in mathematics is a non-linear process which integrates reading text, inspecting associated diagrams (graphs, tables, charts), considering symbolic expressions, and fluidly moving in and between each of these representations (Adams, 2003; Freitag, 1997; Noonan, 1990) and necessitates an understanding and use of a precise grammatical syntax. The Investigate and Explain technique simulates such reading and writing experiences for students. The following examples serve to illustrate ways in which the Investigate and explain technique
Example 1. Given that \(3x^2 + 5x + c = 0\), explain for which value(s) of \(c\) this will have; no real roots; one real root; two real roots.

Example 2. Explain mathematically what this diagram demonstrates.

Example 3. Explain the following. \[\overline{1.234234234234} + \overline{1.234523452345} = \frac{\overline{1.234523452345}}{2.468757686579}\]
Example 4. Tell all you can about the polynomial function which will produce
the accompanying graph of $f(x)$.

As a component of assessment, the *investigate and explain* technique furnishes teachers with
insight into students understanding while promoting reading and writing as essential tools in the
classroom (Armbruster, 1996). By affording students a chance to unpack these texts and
diagrams, this technique also allows students to articulate their understanding in many different
ways.

*Create Your Example*

This technique serves the purpose of allowing a student to both analyze a mathematical concept
and to express the concept in a creative manner which demonstrates her understanding of the
concept. The following examples serve to illustrate ways in which ideas from this technique can
be incorporated in assessment.
Example 1. Create a real world problem/scenario which would use or demonstrate the concepts in the following statement. For sets $A$, $B$, and $C$, and $n(S)$ denoting the cardinality of set $S$, then

\[
\begin{align*}
    n(A \cup B \cup C) &= n(A) + n(B) + n(C) \\
    &\quad - n(A \cap B) - n(A \cap C) - n(B \cap C) \\
    &\quad + n(A \cap B \cap C)
\end{align*}
\]

Example 2. Create a real world problem/scenario for which the use of the following theorem would be part of the solution. “The intersection of perpendicular bisectors of any two nonparallel chords of a circle is the center of the circle.”

The create your example technique is similar in scope to that proposed by researchers and curriculum developers such as Alvermann & Moore (1991), Rothstein, Rothstein, & Lauber (2007). As a component of assessment, this technique may offer teachers insight into students understanding and also serve to promote reading and writing as essential tools in articulating mathematics understanding. In that when students create their own real-world examples and or applications of math concepts, teachers can immediately recognize the students’ depth of understanding. Also, when a student creates an inappropriate example for a given mathematical concept, much can be deduced regarding the nature of the conceptualizations developed by the
Definition & or Theorem Altering

This technique focuses on assessing students’ mastery of both vocabulary and concepts via their articulations on purposefully and precisely altered definitions and theorems. The following examples serve to illustrate ways in which ideas from this technique can be incorporated in assessment.

Example 1. Discuss the validity of this statement: A function is continuous on the closed interval \([a, b]\) if it is continuous at each point in the interval.

Example 2. Discuss the validity of this statement: If \(f\) and \(h\) are functions that are continuous at \(x = c\) then their quotient function is also continuous at \(c\).

The Definition & Theorem Altering technique melds suggestions regarding formal expository mathematical writing with creative mathematics writing assignments (Alvermann & Moore, 1991; Borasi & Rose, 1989; Clarke, Waywood, & Stephens, 1993; Sipka, 1990; Venne, 1989; Waywood, 1992). Since mathematical understanding is strongly correlated to fluency, through this technique, teachers can begin to assess much regarding student understanding of theorems they read. (Earp & Tanner, 1980; Helwig, Rozek-Tedesco, Tindal, Heath & Almond, 1999; Stahl & Fairbanks, 1986). Also students’ discussions lead to both more careful reading and an
appreciation for the precision by which mathematics is written.

*Connect Representations.*

This technique is similar to the sketch-to-stretch strategy proposed by Borasi et al (1998), where students develop nonlinguistic representations, and elaborate on such, of mathematical concepts discovered in mathematical texts. The following examples serve to illustrate ways in which ideas from this technique can be incorporated in assessment.

**Example 1.** Use text and pictures to explain the concept, “If $f(x)$ is a polynomial of degree $n$, with $n \geq 1$, then $f(x)$ can be expressed as a product of linear factors in the following way: $f(x) = a(x - c_1)(x - c_2)\cdots(x - c_n)$, where $c_1, c_2, \ldots, c_n$ are complex numbers and $a$ is the leading coefficient of $f(x)$. That is, every complex polynomial function of degree $n \geq 1$ has exactly $n$ (not necessarily distinct) zeros.”

**Example 2.** Draw a pictorial representation of the theorem: The points of a line can be placed in a correspondence with the real numbers such that; (1) to every point of the line there corresponds exactly one real number; (2) to every real number there corresponds exactly one point of the line; and (3) The distance between two distinct points is the absolute value of the difference of the corresponding real numbers. Or Every line can be made into an exact copy of the real number line using a 1-1 correspondence.
Example 3. Using both text and diagrams, explain the following theorem: On an interval, given any $n+1$ points in the form $(x_i,y_i)$ in which $i = 0, 1, 2, 3, \ldots, n$, and each $x_i$ is unique, there exists a polynomial function of degree no greater than $n$ that maps through the $n+1$ points within the interval.

Example 4. Write the theorem depicted by the diagram below

![Diagram](image-url)

As with other types of assessment techniques demonstrated in this paper, *Connect Representations* employs the process of reading, interpreting and writing mathematics. Another version of this techniques is provided by Readence, Bean, and Baldwin (2001), where students discuss verbal concepts through pictures and diagrams. It is said that when students *connect representations* in order to explain mathematical concepts, their work and explanations reveal much regarding their understanding of the representation at hand (NCTM, 2000). Since students are not told precisely how to make the connections, both the product and the process are revelatory of student understanding. Consequently as a tool for assessment, *Connecting*
Representations provide teachers with the opportunity to assess student understanding of each dimension of this process: during the reading of the representation, the interpretation of the representation, or the creation of other explanatory representations.

Conclusion

Teachers can deeply assess student mathematical understanding when assessment strategies employ reading and writing components. There are multiple assessment techniques to promote successful integration of reading and writing in mathematics and as mathematics teachers, we should explore these and share our successes. Herein, a few such assessment techniques are outlined based on their emergence in the literature. It is hoped that these examples will generate greater interest in assessing mathematics through reading and writing and will encourage others to develop additional assessment methods.
References


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