“Research Based” Mathematics Education Policy:  
The Case of California 1995-1998

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Abstract. In December 1998, the California State Board of Education adopted a new Mathematics Framework. The state had adopted Mathematics Standards the previous year and the legislature appropriated one billion dollars for new standards-based instructional materials. The foreword of the new Framework claims it provides “research-based information about how children learn”. The research described in the Framework supports an extreme (and expensive) shift in state policies on instructional materials, classroom instruction, professional development and assessment. In this article the authors examine the history and content of the Framework’s research base. Since the State Board adopted the Framework with such enthusiasm it is important for the research community to understand why. In part this was due to a public perception of a need for more “basics”. But a major reason is that prominent mathematicians authored key Framework sections and then endorsed the instructional strategies outlined by the psychologists who assembled the research. The Framework was supposed to present a balanced program including basic skills, conceptual understanding and problem solving. This paper takes a close look at the mathematician and psychologist authors views of each. The analysis reveals that although many of their ideas about mathematics teaching and learning are incompatible, overall their positions were combined in a way to mutually reinforce each other.
Recent education reform efforts in the United States frequently call for high standards and accountability. In fact, no political leader would risk a policy statement that could be interpreted as counter to these popular buzzwords. But in California, and in much of the nation as well, high standards and accountability are not enough. Educational reform efforts must meet the further criteria of being research based. The call for research based reform has been incorporated into California’s Education Code through legislation passed in 1995 that addresses the state adoption of instructional materials for grades K-8. It requires that materials:

Are factually accurate and incorporate principles of instruction reflective of current and confirmed research. (60200.c.3)

This paper examines how this one line of state law has been used by the California State Board of Education to direct policy in mathematics education based upon a narrow ideological view of mathematics learning, teaching, and research. Specifically, we explain how research was used in the development of the new Mathematics Framework. This is a story with important implications for education researchers, policy makers, as well as classroom teachers. The first section provides some of the background describing the state structures for setting policy, previous Mathematics Frameworks, and the 1997 adoption of instructional materials. The second section describes how the “research basis” for the new Mathematics Framework was developed, presented, and “authenticated” by the State Board of Education. It includes summaries of a presentation by E. D. Hirsch, writings by David C. Geary who became a major author of the Framework, a report funded by the SBE reviewing mathematics education research and reactions to the report by several groups. The third section compares the writings of the psychologists and mathematicians in the Mathematics Framework, looking closely at their views of basic skills, problem solving, and conceptual understanding. The final section describes a few recent events that indicate the directions California and the nation may be headed.

Background

In California, the Governor appoints members of the State Board of Education (SBE). The SBE is responsible for setting policy that is implemented by the California Department of Education (CDE) under the leadership of an elected Superintendent of Public Instruction. The Curriculum Development and Supplemental Materials Commission is an advisory body to the SBE and has the tasks of developing the initial drafts of curriculum frameworks and making recommendations for adoption of K-8 instructional materials.

Previous Mathematics Frameworks

There have been Mathematics Frameworks in California since 1963, although they were not well known or used prior to the 1985 Framework. Frameworks, which have been the major
state documents related to teaching and learning, were intended to be guides, not mandates. However, Frameworks have directly impacted schools. Criteria for adoption of K-8 instructional materials have been included as a portion of the Frameworks (sometimes in an appendix), and the Frameworks have been the basis for the development of state assessments in mathematics.

Both the 1985 and the 1992 Mathematics Frameworks were highly acclaimed, outside as well as in California. Like earlier Frameworks, the 1985 and 1992 Frameworks were not limited to a listing of mathematical content that was important for students at different grade levels to learn. They addressed instructional strategies and other critical issues, such as working with diverse students.

“Research” as a basis for instruction is not mentioned in the 1985 Framework (CDE, 1985). In fact, the compact 45 page document (including an Appendix) contains no references of any kind—no mention, nor a single citation, of the research literature. This is not to say its authors were unaware of education research, the (unwritten) policy at the time was not to include research citations.

The 1992 Mathematics Framework (CDE, 1992) concludes with a list of Selected References, and 10 of the 31 items might be considered as research articles. The document also includes informal mentions of research. For example in a section, Learning Mathematics (p. 32), one finds:

Over the past decade cognitive scientists and psychologists have looked closely at how children learn mathematics. . . . This research affirms the concepts Piaget, Vygotsky, and others put forward decades ago.

The 1992 Framework also includes a number of quotations and sidebars from research articles and mathematics education literature, such as Everybody Counts (1989). However, as in 1985, there was no attempt to describe a direct link between educational research and the instructional practices described in Chapters 1 and 2 (pp. 15-74). Also, the criteria for adoption of instructional materials do not refer to research.

Two Advisory Documents, 1995-96

After assuming office in 1995, California’s new Superintendent of Public Instruction formed a task force to “address the need to improve the mathematics achievement of California’s students” (CDE, 1995, p.1). The task force report, A Call to Action, made five recommendations titled: (1) Rigorous, Balanced Content and Performance Standards, (2) Assessment, (3) Quality Instruction for All Students, (4) Research to Improve Mathematics Education, and (5) Parents as Partners. The first recommendation included a phrase that has become the undisputed mantra for mathematics standards and programs in California: “a balance of basic skills, conceptual understanding, and problem solving.” The fourth recommendation states:

The State Superintendent of Public Instruction must act immediately to establish a management, research, and information system to answer basic questions about the operation and effectiveness of mathematics policies and programs, including the implementation and effectiveness of the state’s Mathematics Framework.
The elaboration of the recommendation states:

The Task force was astonished, therefore, to discover that the state has no means to determine the effectiveness of these state policies. The state has collected data on student achievement but not on the processes of teaching and learning that lead to student achievement.

In October 1996, the State Board adopted a Mathematics Program Advisory\(^5\) (CDE, 1996a). Its purpose was to provide guidance to schools and districts for reviewing and improving their mathematics programs. It includes brief descriptions of each of the components of a balanced program—basic skills, conceptual understanding, and problem solving. Although the Program Advisory never used the phrase “research based,” the Board integrated the Advisory’s calls for balanced programs with the research based requirement of Education Code 60200 in the Framework revision.

**Adoption of K-8 Mathematics Instructional Materials**

In California, the Education Code calls for Mathematics Frameworks to be adopted on a six or seven-year cycle with K-8 basic instructional materials being adopted at least two times within that cycle. The Curriculum Commission has the responsibility for making recommendations to the State Board on materials. The Commission appoints a panel to thoroughly review all submitted materials against the Instructional Materials Criteria (included in the Framework) using an evaluation instrument based on the criteria and approved by the State Board.

The last major adoption for mathematics was held in 1994, and all materials recommended by the Curriculum Commission were adopted.\(^6\) Several of the adopted programs were created with support from the National Science Foundation. These and other adopted programs generally encouraged the teacher to probe students’ understanding as a means to facilitate learning.

A supplemental adoption was held in 1997, after the section referencing “current and confirmed research” was added to the Education Code. At its September 9, 1997 meeting, the State Board rejected two Curriculum Commission recommended mathematics programs receiving high ratings. For the first time, research as criteria had played a key role in determining which instructional materials the state would purchase for its students.\(^7\) The explanation for their rejections was provided by State Board member Janet Nicholas (Nicholas, 1997), who in her memoranda stated:

> The Commission's report does not identify or mention the method it used to evaluate or determine that the submissions recommended for approval incorporate principles of instruction reflective of current and confirmed research (60200.c.3).

Legislation passed in 1998 (AB 2519) allocated funding for a special adoption of instructional materials (during spring of 1999) that are aligned with the state’s academic content standards.\(^8\) It included the requirement specified in Education Code 60200.c.3 that materials “incorporate principles of instruction reflective of current and confirmed research.” However, the criteria written and adopted by the SBE require that “current and confirmed experimental research should be reflected.” Presumably the word experimental was added as a result of the Board being advised to limit research to the experimental paradigm\(^9\). (See the next section.)
However, during the AB 2519 adoption process (February through June 1999) there were no discussions of research.

**State Mathematics Content Standards**

Like many other states, in 1997 California developed and adopted statewide standards in mathematics. Legislation required that a commission appointed by the Governor and the State Superintendent of Public Instruction develop and submit them to the SBE for adoption. The Commission first met in September 1996 and presented their version of Mathematics Content Standards to the SBE in October 1997. The development process included public meetings, fairly frequent public hearings throughout the state, and several drafts. The Commission listened to all points of view, and tried to develop standards that were rigorous and respected different beliefs about mathematics and different teaching approaches. However, after the standards were submitted for adoption, the State Board significantly revised the K-7 mathematics standards and completely rewrote the grades 8-12 standards. This revision was completed over a two-month period by four Stanford University mathematics professors, working outside of the public process. The SBE adopted the revised version in December.

Research from the Third International Mathematics and Science Study (TIMSS) influenced California’s development of standards in a key way. Based on presentations by William Schmidt and Jim Stigler, major researchers associated with TIMSS, the Commission early on decided that, to compete with top ranking countries, all students should begin to study algebra and geometry in eighth grade. The final document specifies traditional 9th grade algebra as the 8th grade content, currently a major challenge for California schools.

Both the Commission and the State Board had welcomed input from Schmidt in support of rigorous standards for all students. However, when the Board revised the standards, his input was ignored. In a letter to the Board on December 5, 1997, after giving numerous specific comments and suggestions, Schmidt concluded with the following. “Across the country I have said that those Standards sent to you by the Commission are world class and exemplary for other states in the nation. Sadly I can no longer say that.”

**1997-98 Mathematics Framework Development**

Early in Spring 1996, the State Board of Education and the State Superintendent of Public Instruction called for the development of a new Mathematics Framework (about a year earlier than scheduled). It was supposed to reflect the balance of basic skills, conceptual understanding, and problem solving that was called for in the earlier Program Advisory. They also expected the Framework to include standards for every grade level.

Following its appointment, the Framework Committee met monthly between January and August 1997. At its first meeting the committee was presented its charge in a page titled *Focus of the Math Framework Revision from the State Board of Education and the Curriculum Commission*. It stated that the committee was to revise the 1992 Framework and “All aspects of the framework should address a balanced mathematics program for all students.” Two points from the second part of the committee’s charge are significant from the research perspective:
Additionally the Framework revision will address the following areas from the Math Program Advisory:

- A variety of research based instructional strategies designed to help students succeed and excel in mathematics;
- The need for professional development initiatives which address both mathematics content and effective research based instructional strategies appropriate to individual grade levels;

During its first two meetings the Framework Committee heard presentations on the TIMSS research by Drs. William Schmidt and Alfred Manaster. Each member also received a packet of “research” articles from Assemblyman Steve Baldwin. The packet contained an article critical of the Interactive Mathematics Program\(^\text{12}\) (Wu, 1995) together with most of the articles from an issue of Effective School Practices\(^\text{13}\) (Vol. 13, No. 2, 1994). Four articles reviewed the textbook series, Connecting Math Concepts.\(^\text{14}\) The Framework Committee chair announced that E. D. Hirsch would speak (see section below) to the State Board and urged Committee members to attend since he would supply the research for the Framework. However, during the eight months of committee meetings, the chair refused to allow discussion of research articles as agenda items, so at best, research was mentioned in incidental remarks during committee work.\(^\text{15}\)

At the July meeting, when the Framework Committee was assembling its final draft, State Board member Janet Nicholas announced that Professor Douglas Carnine from the University of Oregon would write the research basis for the document.\(^\text{16}\) The committee draft of the Framework, like the 1985 and 1992 Frameworks, contained no citations or discussion of research, although a list of 85 references was appended to the draft. These references consisted of random articles passed on by committee members to CDE staff, and many of their conclusions contradicted the approaches advocated in the final document.

**California's Research Basis for the Framework and for Mathematics Education**

In developing its Reading Program Advisory (CDE, 1996b) the SBE concluded that research could determine the best way to teach\(^\text{17}\), and attempted to set up a parallel course for mathematics. The reading advisory states, “There is sufficient guidance now available from research about how children best learn to read and about how successful reading programs work to ensure that virtually every child will learn to read well, at least by the end of third grade.” In February 1996, the State Board heard a special presentation from Douglas Carnine, Director for the National Center to Improve the Tools of Educators at the University of Oregon. He stated that, unlike early reading instruction, there is no research basis for mathematics. Carnine also said that the only scientific study done on the effectiveness of mathematics programs was Project Follow Through, and in that study, the direct instruction model proved superior.\(^\text{18}\)

In 1997 the SBE heard several other presentations related to mathematics education. Speakers included William Schmidt, James Stigler, and Alfred Manaster following the release of results of the Third International Mathematics and Science Study (TIMSS) and E. D. Hirsch. While the TIMSS presentations initially were received enthusiastically by Board members, they were not much utilized by the Board in the development of state policy. The Board also commissioned a review of research in mathematics education\(^\text{19}\) from Douglas Carnine in 1997 and had Prof. David Geary rewrite a significant portion of the Framework. Since they played
a pivotal role in setting final policy, the Hirsch presentation, Geary’s views of mathematics education, and the Dixon et al. report are summarized next.

**E. D. Hirsch's Presentation**

In April 1997, the State Board of Education invited E. D. Hirsch, Jr. to speak about research and education policy. Hirsch, a recognized advocate of back-to-basics’s curriculum and a professor at the University of Virginia, is the author of several books including *Cultural Literacy* and *The Schools We Need and Why We Don't Have Them*, and is the founder of the Core Knowledge program.

Beginning his talk, Hirsch acknowledged that Californians, both liberals and conservative had a sense of determination to “get education right this time around, to rise above politics, partisanship, and ideology and achieve a first-class improvement in the schools.” He also mentioned that “California law requires education policy to be research-based. That's the theme I shall focus on.”

Among Hirsch’s main points was that one has to be extremely careful before trusting educational research, and that psychology is the key field for educational research. He criticized educational research because it “. . . has been required to conform to a so-called constructivist ideology that does not represent the consensus in mainstream psychology, and is almost certainly incorrect.” He described the bias he finds in “mainstream educational research” as well as the peer review system, “used, for example, by journals such as Educational Researcher”

This is a situation that is reminiscent of what happened to biology in the Soviet Union under the domination of Lysenkoism, which is a theory that bears similarities to constructivism. . . . that nurture can transform nature. (p. 3)

Hirsch was so outraged by the control of educational research by the constructivists he stated:

Research cannot flourish under intellectual conformism. . . . If scientific information had been allowed to flow more freely during the past two decades, the school scene would almost certainly have a different face than it does now. California math and reading scores would almost certainly be higher.

Hirsch then went on to make it clear that nobody, except of course those he names, can be trusted to provide research-based advice. He suggested that the National Academy of Sciences could provide guidance on who are reputable researchers. Hirsch then spoke about research in the areas of assessment, mathematics education, and early childhood education.

He began his remarks about mathematics education by discussing the math wars as characterized in an article in Education Week in which the two sides are labeled “reform” and “anti-reform”:

That kind of ideological bias in reporting is characteristic of the education world, and well illustrates the need for constant vigilance by policy makers.

He then proceeded to characterize the position of the “reform” group:
The NCTM group stresses conceptual understanding over mindless drill and practice, while the dissident group stresses the need for drill and practice leading to mastery.

Hirsch suggested that there are three researchers who should be listened to: John Anderson, David Geary, and Robert Siegler. He asserted that there would be strong agreement among the three on the following point:

They would tell you that only through intelligently directed and repeated practice, leading to fast, automatic recall of math facts, and facility in computation and algebraic manipulation can one do well at real-world problem solving.

He closed his discussion of mathematics education by stating that Anderson, Geary, and Siegler would support their positions with highly reliable data from large scale classroom results as well as isolated lab experiments. And, Hirsch asserted:

If these top scientists agreed on all these points, that is the consensus you should trust, no matter how many pronouncements to the contrary might be made by national educational bodies.

Hirsch received a standing ovation from the State Board. And apparently, his views were influential since David Geary, one of his recommended researchers in mathematics education, was later brought in to rewrite the draft of the Mathematics Framework.

**David C. Geary’s Research**

David Geary’s research forms part of California's research base because he was asked by the SBE to rewrite major sections the Framework in August 1997, following the adjournment of the Framework Development Committee. He made no formal presentations on his research to the State Board. Geary is a cognitive psychologist at the University of Missouri-Columbia, who has written a number of articles and a book on mathematics learning, (Geary, 1994). (For a review of Geary's book by Robert Davis, see (Davis, 1996).) Geary has had little association with researchers within the mainstream mathematics education community.

Geary believes that there are two kinds of cognitive abilities, biologically primary and biologically secondary. Biologically primary cognitive abilities are found in all cultures and develop automatically in children, and therefore, don’t have to be taught. Examples include learning to talk and recognizing the quantity of a small number of items (3-4) without counting. Biologically secondary cognitive abilities will vary from culture to culture, and “their acquisition is generally, slow, effortful, and occurs only with sustained formal or informal instruction.” (Geary, 1995, p. 27) Examples of biologically secondary cognitive abilities include reading, writing, and arithmetic. Schools are needed for children to learn these kinds of skills, and the learning of these skills require deliberate practice which is not inherently enjoyable.

Geary states that a main weakness of constructivists’ view is their failing to distinguish between biologically primary and secondary learning (Geary, 1994, p. 265):

One of the implicit assumptions of the constructivist approach is that mathematics is a biologically primary domain. That is, given an appropriate social context and materials, children will be motivated and able to construct mathematical knowledge for themselves (Cobb et. all, 1992). In fact, this is probably not an unreasonable assumption for certain mathematical areas-such as number, counting, and some
features of arithmetic. . . . However, many other mathematical skills, such as solving algebraic word problems, are biologically secondary.

Geary criticizes mathematics education reformers because he believes that reformers believe that “. . . drill and practice and the development of basic cognitive skills, such as fact retrieval, are unnecessary and unwanted. . .” (p. 265). Since constructivists focus on conceptual knowledge at the expense of mechanical skills, “constructivism is not likely to lead to substantial long-term improvements in the mathematical skills of American children.” (p. 266)

Geary identifies two types of competencies, procedural and conceptual, and states that these different competencies require different forms of instruction:

Procedural learning requires extensive practice on a wide variety of problems on which the procedure might eventually be used. . . . Basically, the procedure should be practiced until the child can automatically execute the procedure with the different types of problems that the procedure is normally used to solve. (pp. 269-270)

Geary asserts that the development of conceptual understanding “requires a lot of experience but does not appear to require drill and practice per se.” (p. 270) Ways to develop conceptual understanding include: asking students to come up with as many ways as possible to solve a problem, presenting problems in familiar contexts, and teaching teachers about how children understand and solve mathematical problems. (pp. 270-271)

Geary devotes Chapter 3 of his book to problem solving and states (p. 95) “Learning to problem solve is an essential feature of children’s mathematical development, but unfortunately it is a skill that most children in the United States apparently do not master.” He reviews the literature and considers two types of problems. First he discusses arithmetic word problems, all of which involve single digit addition or subtraction and four classes are identified (p. 99), and second he considers algebra word problems that translate into linear equations. Both are limited to (traditional) one or two sentence written questions, where students are expected create a single symbolic expression that they then solve by a previously mastered method. As such, Geary ignores what much current curricula and research consider as problem solving. For example, asking first grade students to “find all addition expressions that equal 7” and then subsequently ask them to look for patterns in their collections, would not be covered by Geary’s discussion.

Also important is Geary’s view that procedural mastery of skills must precede the form of problem solving he describes. Discussing addition word problems his summary states (p. 130):

Basically, the skills described in chapter 2 represent an essential component and building block for the development of problem-solving skills, just as number and counting concepts are essential for the child's developing arithmetic skills. . . . Thus, skilled problem solving must rest on a solid foundation of basic skills: Attempts to improve the mathematical problem solving of children should not be done instead of teaching basic skills, but rather in addition to teaching these basic skills.

In this way, Geary firmly rejects the notion of mathematics as sense making, where problem solving can be used as a basis for instruction. This also shows that E. D. Hirsch’s characterization of Geary’s opinions were accurate.
The Dixon, et al. Report

In his June 1997 response to the State Board’s RFP for a synthesis of research on mathematics instruction, Douglas Carnine stated his researchers would focus on “locating, evaluating, and synthesizing high quality experimental and quasi-experimental research on mathematics.” He stated their search would include minor journals, and he criticized the Journal for Research in Mathematics Education (JRME) because they had rejected a study simply because it was not compatible with NCTM Standards.

On March 13, 1998, Douglas Carnine presented to the State Board of Education the report, Review of High Quality Experimental Mathematics Research. (The report is authored by Robert C. Dixon, Douglas W. Carnine, Dae-Sik Lee, Joshua Wallin from the National Center to Improve the Tools of Educators, University of Oregon, and David Chard from University of Texas, Austin). In developing the report, the authors stated that only 110 studies (out of 956 articles that satisfied the minimum identification criterion of being an experimental study) were of sufficient high quality in terms of research design.22

A brief summary of each of the 110 studies was included in the Dixon et al. report. The studies were organized into six categories.23 Below is a summary of what the Dixon report concludes in two of these categories—Didactic vs. Discovery Learning and Manipulatives. We chose them to examine more deeply because they will highlight some of the differences between the 1992 and 1999 Frameworks.24

- **Didactic vs. Discovery Learning.** Since instructional approaches have been a prominent part of earlier Frameworks, one might expect that the Dixon research report would look at this area. However, it was difficult to find a category that encompassed what was intended in the 1992 (and earlier) Framework. The closest match was in the category Instructional Design—Didactic vs. Discovery Learning, which seems to indicate that the authors equate “discovery” with “constructivism.” The Dixon report looked at only three studies which “directly address the question of whether initial instruction should be given via a discovery mode (generally favored by constructivists), a guided discovery mode, or a direct mode (generally favored by cognitive and behavioral-cognitive psychologists).” (p. 17) On page 18, the report states, “Discovery was not found to be effective for any content or any group of students.” The three studies were published in 1970, 1972, and 1973, and their "discovery" treatments attempted to eliminate most student-student and student-teacher interactions. For example, (Orlander & Robertson 1993) states “An effort was made to have the pupil discover answers without the help of other pupils or the teacher,” while in the expository instruction, “pupils were encouraged to share ideas about a problem, the more able ones being assigned to help the slower ones.” In (Lackner, 1972), students used instructional booklets to completely eliminate interaction with teachers, and Dixon authors seem to have misinterpreted the “inductive” approach described there as being the same as discovery. We did not find evidence in these papers for the Dixon report’s statement that “Discovery was not found to be effective.”

- **Manipulatives in Instruction.** The Dixon report cites four studies related to manipulatives in instruction. The report states on page 13, “Clearly, the jury is still out on the use of manipulatives.” The Executive Summary (which was published after the
report) states, “Three of those studies—all conducted in elementary schools—found no benefit. One study, conducted with middle school students studying fractions and ratios, did find positive benefits.” Only one of these four papers (Pasnak, et al. 1996) describes research conducted after 1979. It studied low performing kindergarten students and, in contrast to the assertion in the Dixon report, noted substantial benefits for the experimental group receiving manipulative based instruction.

In none of the above examples did we find instructional approaches that resemble those advocated in either the 1985 or 1992 Frameworks. Neither can we see how they constitute “current and confirmed research” as required by California law. Analysis of research findings in the remainder of the Dixon report is similar to the examples above. Most of the studies are decades old, and the report’s summaries are questionable. For discussion of 35 papers from the report, see (Jacob, 1998).

There was no overall summary in the full Dixon report. However, in a printed hand out presented to the Curriculum Commission on April 28 (Carnine, Dixon, Kameenui, Simmons, 1998) the conclusions of the Dixon report were summarized as follows:

The research review supported a few prevailing suggestions for teaching mathematics such as cooperative learning and the use of computers. The review did not support many other suggestions for teaching math: grouping children heterogeneously, child-directed learning in which the teachers poses a challenging situation and then acts as a facilitator by asking thought provoking questions, the extensive use of manipulatives in the primary grades, stressing calculators while de-emphasizing the teaching of computation, relying on intrinsic motivation instead of external rewards, or teaching according to the learning styles of children. The research review has not established that these practices are harmful, but has found that these practices are not supported by high quality research that has been conducted up to this time.

Although the research review restricted its analysis to experimental studies, this quotation refers to “high quality research.” The state board in subsequent policy would adopt the notion that high quality research essentially means “experimental.” How accurate is the summary statement? There is little evidence for its accuracy.

An Executive Summary of the Dixon report was distributed and presented to the State Board some time after the full report. It included a section titled Effective Mathematics Lessons, which is purported to be a distillation of the report’s findings. This section criticizes what is referred to as the conventional two-phase lesson model and states the “lesson models for effective interventions most frequently followed a three-phase pattern.” See the diagram below for how the two- and three-phase models are characterized.25

<table>
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<th>Conventional Lesson Model (2-Phase)</th>
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<tr>
<td>Phase 1</td>
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<tr>
<td>Teacher demonstrates (often, teacher works one to four problems.)</td>
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<tr>
<td>Students observe passively</td>
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</tbody>
</table>
The three-phase model, which became the recommended instructional model in the 1999 Framework, was not based on any research findings in the Dixon report. There were no references to research in this section of the Executive Summary, and none of the research articles in the full report addressed these models for lessons. Neither of the two models are similar to the teaching approaches advocated in the 1992 (and earlier) Mathematics Frameworks. Rather, both are based on teacher presentations with students being able to independently reproduce, after sufficient practice, the content or procedures presented. The 1999 Framework recommended model calls for students to be “actively involved” rather than observing “passively” in phase 1. However, “active participation” doesn’t imply using manipulatives or discussing different approaches to solving a problem as in the 1992 Framework; it only requires that “the student attend to, think about, and respond to the information being presented or the topic being discussed.”

We believe the Dixon Report restricted its review to experimental design research in order to ensure that all articles were narrow in focus. The limitation of basing recommendations for instructional practices on research that is laboratory rather than classroom based is inappropriate. Real classrooms are not controlled like laboratories. In a dynamic classroom, it is difficult if not impossible to measure all of the variables that contribute to each student’s learning.

**How did other academics and the Curriculum Commission respond to the Dixon, et al. report?**

The AERA Special Interest Group for Research in Mathematics Education mailed a letter, signed by 73 people, to State Board President Yvonne Larsen on April 15, 1998. The letter describes several paradigms for “high-quality research” (other than experimental) and includes suggestions for the board to learn more about their implications:

> While we applaud your willingness to base policy decisions on research results, your decision to limit the research database to experimental research was a surprise to us, and one that we do not understand. . . . In the past two decades, we have come to recognize that many important questions regarding the teaching and learning of mathematics cannot be answered by experimental studies. While we continue to believe that the experimental method is appropriate for answering some questions, we know that it has severe limitations.

The Curriculum Commission minutes show that there was one discussion of research relating to the Framework (March 5, 1998). One member presented an article by Bonnie Grossen (Grossen, 1998) and explained that the article gave a definition of research and identified three levels, but that level 3 was the only kind of “real research.” State board member Janet Nicholas (in attendance) said that this was where SBE got their definition of research and
commented that math education had been “vastly harmed” by adults not posing the question “where’s the research?” The remarks by Mrs. Nicholas indicate she was familiar with the article by Grossen.

At the May 6 State Board Meeting, Curriculum Commission Chair Kirk Ankeny stated the Commission had heard from Professor Carnine on April 28, and felt that the experimental research was “was not broad enough.” He specifically asked that the Commission be allowed to include other (for example qualitative) methodologies in supporting instructional strategies, and received a mixed reaction from the Board (SBE Minutes, May 1998). No further discussion of research occurred in public sessions of the Curriculum Commission.

**The Writings of Psychologists and Mathematicians in the Mathematics Framework**

During spring and summer 1998, three mathematicians were assigned by the SBE to write sample problems aligned with the Standards, and during summer 1998, the SBE asked Dr. David Geary to rewrite much of the Framework. Geary added nearly all of the research citations that appear in the final document (CDE, 1999), most of which were based on his work as a cognitive-psychologist. Finally, in October and November mathematics professors H-H Wu and James Milgram were given the responsibility to perform a “technical review of the framework”, and they revised and added some discussion. We consider it likely that other mathematicians and State Board members were involved in the development of the final draft, but the public record lacks details. The Board accepted the Dixon, Geary, and the mathematicians work for the Framework without question, and incorporated it as part of their new instructional materials and professional development initiatives. When the Framework was adopted on December 10, 1998, SBE President Larsen proclaimed she was proud to have a “mathematically correct” document.

**Influence and Discussion of the Dixon Report**

Of the 62 listings in the Framework’s “Works Cited”, none are from the 110 "high quality" studies identified and included in the Dixon report, although 17 do appear in a separate collection of “Additional References”. Because of this we infer that the Framework’s authors paid little attention to what they actually contained. None the less, the Dixon report is given a prominent place near the beginning of Chapter 4 in the 1999 Framework, under the section titled “Instructional Models: Classroom Studies” (pp.180-183), where its three-phase instructional model is advocated. The discussion concludes in a way that implies the model is research-based:

> While current and confirmed research such as that reported in the Dixon study provides a solid basis on which to begin to design instruction, research from cognitive psychology provides insights into when and how children develop mathematical thinking. (p. 183)

Unlike the Executive Summary of the Dixon report, there is a reference to research in the Framework. The Framework (p. 181) describes the second phase of the 3-phase model as the “‘help phase’—where the students gradually transition from ‘teacher-regulation’ to ‘self regulation’” citing (Belmont, 1989). Belmont’s article is not mentioned in the Dixon report, nor is it about lesson models. Rather, it is a discussion of research related to strategy instruction and how Vygotsky’s concept of the zone of proximal development.
Geary’s Work on the Framework

Geary’s rewrite of the Framework draft was presented to the Curriculum Commission in August 1998. He rewrote much of the document and added his own citations to the draft, none of which came from the Dixon Report. Chapter 3, Grade Level Considerations; Chapter 4, Instructional Strategies; and Chapter 5, Assessment; are, for the most part, his work.31

Geary added a section on learning, Instructional Models: View from Cognitive Psychology, to Chapter 4. It is based on his own theory of natural and academic learning discussed previously. Chapter 4 also has a section called General Suggestions for Teaching Mathematics. This discusses what research shows helps foster procedural and conceptual competencies, and cites twelve articles.32 These citations are not limited to experimental research and include theoretical papers and syntheses of research as well. The Framework lists three guidelines for teaching procedures and three for teaching concepts (p. 186) which summarize Geary’s research discussion:

**Teaching Procedures**
1. Provide practice until the procedure is automatic, which is until the student can use it without having to think about it. Automaticity will often require practice over many school years.
2. Provide practice in small doses (e.g. 20 minutes per day) over an extended period of time; practice on a variety of problem types mixed together.
3. Once automaticity is reached, include some additional practice of the procedure as part of review segments for more complex material. This facilitates the long-term retention of the procedure.

**Teaching Concepts**
1. When possible, present the material (e.g. word problems) in context that is meaningful to the student.
2. Solve some problems in more than one way. This would typically be done after students have developed competence in dealing with the problem type.
3. Discuss problem solving errors; use errors to diagnose and correct conceptual misunderstandings.

The practice guideline (see #2 above under Teaching Procedures) is attributed to (Cooper, 1989). Cooper’s paper is a synthesis of research on homework that in fact makes no such recommendation. Instead the paper offers a generally positive appraisal of homework and discusses how practices should differ according to grade level. In further discussion under Fostering procedural competencies, the Framework states:

> . . . studies of human memory and learning indicate that most of the benefit, in terms of learning, occurs during the early phases of a particular practice session (e.g., Delaney, et al., 1998). (p. 187)

The Delaney et. al. paper studies fractal power laws relating practice time to solution time on rote arithmetic tasks. While Geary’s statement might be interpreted as a consequence of the power law (provided you accept the notion that learning is the same as implementing rote procedure), the main conclusion of the paper is that “improvement of solution time is better explained by practice on a strategy, than by practice on a task.” (p. 1) So we are not sure why this article is cited.

In his discussion of Fostering conceptual competencies, Geary states, “A number of teaching techniques can be used to foster student’s conceptual understanding of problems (Cooper & Sweller, 1987, Sweller, Mayer, and Ward 1983).” (p. 189)
The Framework lists the importance of using context, multiple approaches, and analysis of errors. These emphases indicate that teachers should consider fewer problems in greater depth, and is supported by (Cooper & Sweller, 1987), where one finds in the paper’s conclusion:

In an educational context, the results suggest that learning to solve algebra transformation problems can best be done not by solving large numbers of conventional problems, but rather by placing a stronger emphasis on worked examples. This contrasts with current teaching techniques. (p. 359)

In this case the citation is appropriate. However, key points of other articles cited conflict with other suggestions in the Framework, and they are missing from the discussion.

**Comparison and Discussion of the Influence of Mathematicians and Geary on the Framework**

As noted above, several university mathematicians wrote major portions of the final version of the 1999 Framework. Because of their content expertise, their views of mathematics education were accepted, blended, and used to bolster the positions described in the previous section. Since the Framework’s overarching theme was supposed to be balance\textsuperscript{34}, we will examine their views of basic skills, conceptual understanding, and problems solving. Put together, their beliefs lead naturally to a formalistic presentation of curriculum and the reductionist view of learning evident in California’s research basis. One also finds an accompanying distrust of school mathematics teaching, which in turn adds to the appeal of a prescriptive approach.

Their views can be traced through the written record of the Framework development. We begin with a summary, with elaboration to follow. We present each topic in two forms; the first based upon statements by the experimental psychologists and the other based upon statements by the mathematicians. Each reflects different perspectives, but they are closely related and it is important to note how they reinforce each other. Of course, these views are not representative of all experimental psychologists or mathematicians—the summary is our interpretation of the views of those named by the SBE whose writing formed the Framework and its research report.
### Skills

<table>
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<tr>
<th>Psychologists</th>
<th>Mathematicians</th>
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<tr>
<td>Skills are procedures; they must be learned to “automaticity,” and may be divorced from meaning. Components of procedures must be mastered before the procedure itself. Understanding and problem solving comes later and is the result of practicing procedures.</td>
<td>Mathematical skills should be learned according to their logical structure. The deductive systems of formal mathematics may provide explicit organization (as in Euclidean geometry), or define organization implicitly (say in lower grades.) Conceptual understanding is derived from using this logical structure.</td>
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### Problem Solving

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<th>Psychologists</th>
<th>Mathematicians</th>
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<tr>
<td>Problem solving is executing practiced procedures. It may include (ii) translation of a question into (ii) a known mathematical representation where (iii) a previously learned procedure can be applied to find an answer.</td>
<td>Problem solving involves knowing when to apply a general mathematical result (such as a theorem or an established algorithm) to a specific situation to obtain desired information.</td>
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### Conceptual Understanding

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<tr>
<th>Psychologists</th>
<th>Mathematicians</th>
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<td>Conceptual understanding is measured by students knowing when to apply a previously learned procedure. Speed and accuracy in procedural performance is an essential component and measure of conceptual understanding.</td>
<td>Conceptual understanding is measured by students being able to use correct mathematical language and reasoning to support their answers. The deductive structure of formal mathematics, including a clear formulation of hypotheses before drawing conclusions, should provide the basis for this reasoning.</td>
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**Computational and Procedural Skills**

**Psychologists’ Influence.** The primacy of skills has permeated the California discussion. E. D. Hirsch stressed automaticity in skills, and in order to ensure that students won’t have to think about what to do, instruction must present component skills in the right order. For example, in Appendix A, Sample Instructional Profile, the Framework states “Component skills need to be thoroughly taught before they appear in complex applications.” (p. 238) While in a general sense, few (if any) educators would dispute the importance of certain concepts and skills as requisite to learning others, the distinctions made by Geary (and in the California discussion more generally) are far more detailed than this. We found most examples are tied to highly structured procedural approaches, with sufficient rules and regulations for their use so that students need not understand what is going on (also see the quote from VanLehn below). For example, in a discussion of subtraction of fractions in a draft Appendix, Geary wrote:

> Easier problem types are to be taught before more difficult types . . . . For example, problems that require borrowing (2 1/2 – 1 3/4) are much more difficult than problem that do not require borrowing (2 3/4 – 1 1/2).

We ask, is the subtraction (10/4 - 7/4) significantly harder than (11/4 - 6/4)? And what if students are learning about fraction operations using geometric representations—is the first more difficult in this setting? This illustrates how the procedural hierarchy determines difficulty, which in this case is Geary’s expectation that “borrowing” must be used. This rigidity in skill sequencing has had significant impact on California’s materials adoptions, the Standards, and on the AB 2519 Adoption Criteria.35
Mathematicians’ Influence. The Framework’s mathematician authors elaborate on the belief that skills must be divided up and mastered in rigid sequence. The September 5, 1997, draft Framework’s Instructional Materials Criteria was initially authored by university mathematicians and these same points remained in the final version. For example, the criteria include these statements (p. 231):

- Concepts should be developed in logical order, . . .
- Prerequisite skills and ideas should be addressed or presented before the more complex topics that depend upon them.
- Coverage starts with the easy cases, and proceeds step by step, too increasingly more complex problems within the topic area.

During the 1997 Framework development committee conversation, frequently heard was the assertion, that even though one might not teach children a formal system, the teaching of (symbolic) skills should be organized according to the sequence and structure dictated by the axiomatic formulation. This point is made in Chapter 3’s “Preface to Grades 8-12” where the Framework states, “Problem solving and symbolic computations are nothing more than different manifestations of mathematical proofs” (p. 154). The mathematician author of this section illustrates this point with a discussion of solving $x – 1/4(3x – 1) = 2x – 5$, by showing the solution to this problem as the result of a sixteen step two-column formal proof of the following: “Theorem: A number $x$ satisfies $x – 1/4(3x – 1) = 2x – 5$ when and only when $x = 3$.” After presenting the proof, the mathematician adds:

In practice, it would be impractical to demand such detail each time a linear equation is solved. Nevertheless, without the realization that such a mathematical proof is lurking behind the well-known formalism of solving linear equations, an author of an algebra text or a teacher in an algebra classroom would most likely emphasize the wrong points in the presentation of beginning algebra. (p. 155)

This illustrates the belief that conceptual understanding is derived from using this logical structure, even if not explicitly. To remind the reader that this crosses grade levels, the author adds (p. 156) “every technique taught in mathematics is nothing but proofs in disguise.”

Problem Solving

Psychologists’ Influence. Geary authored the K-6 grade level discussions for Chapter 3 of the Framework (including Key Topics and Elaborations), and although his work at grades 5 and 6 was revised by other authors, his K-4 discussions are intact and have a number of references to “problems.” Consistently, the references to problem solving in these sections refer to rote symbolic procedures. For example, the grade 3 elaborations contain nothing other than symbolic manipulation of number. Among other tasks called “problems” we find:

Gr. 1: committing basic addition and subtraction problems to memory (p. 117)
Gr. 2: Initially, problems should be limited to those requiring borrowing or carrying across one column (p. 123)
Gr. 3: memorizing answers to simple multiplication problems (p. 128)
Gr. 4: multiplication and division problems using the standard algorithm (p. 134)

We emphasize that, while all these tasks could in fact be formulated as new situation for students to resolve (and thus be a “problem” in a reform sense), the discussions focus on prior procedural mastery of component skills and the direct instruction needed for students execute rote procedures. There are few hints that they could be presented in any other way.
The view that problem solving is synonymous with executing practiced procedures is also implicit in the discussion of pedagogy in Chapter 4 under *Fostering conceptual competencies*:

Extensive studies of problem-solving errors indicate that most are not trivial but are systematic (VanLehn, 1990). Generally, errors result from confusing the problem at hand with related problems, as in stating $3 + 4 = 12$ . . . (p. 190)

We find the citation by Geary of VanLehn’s book interesting, for VanLehn is quite careful to distinguish the procedural skills he studies from problem solving. In the introductory chapter of his book, VanLehn describes his research goals:

The main advantage of arithmetic procedures, from a methodological point of view, is that they are virtually meaningless to most students. They seem as isolated from commonsense intuitions as the nonsense syllables used in the Ebbinghaus paradigm for studying verbal learning. In the case of the subtraction algorithm, for example, most elementary school students have only a dim conception of its underlying semantics, . . . (p. 12)

Unfortunately, this quote of VanLehn was not included in the Framework—but indeed it could not be, as it is incompatible with the Framework’s views on problems. We do remark that there is a brief discussion of three problem solving steps, “translation,” “representation,” and “solution” in the Framework’s first chapter (p. 10). The discussion is based upon the 1996 *Mathematics Program Advisory* except for the addition of two references (Geary, 1994) and (Mayer, 1985). But the Framework offers no further explanation or examples involving these three steps.

**Mathematicians’ Influence.** For the mathematicians writing the 1999 Framework, problems should be solved applying a known mathematical theorem or algorithm, and the process is linked to their proofs of correctness. We contend their views greatly blur the distinction between problem solving and skills learned by rote. This is illustrated by the sample problems the mathematicians created for the document, where one might expect a few purely procedural examples, but where instead, such examples dominate. For example, at grade 5 the complete list of problems for Algebra and Functions (prepared by a mathematician) is:

A. Joe’s sister Mary is twice as old as he is. Mary is 16. How old is Joe?
B. $3x + 2 = 14$. What is $x$?
C. Plot the points $(1,2), (-4,-3), (12,-1), (0,4), (-4,0)$. (p. 54)

Problem C is executing a graphing skill, while A and B involve solving a linear equation (a skill specified in the standards.)

Presumably, because procedure can be implemented by rote, the Framework mathematician authors believe some very challenging tasks can be solved at an early age. For example, the following problem for the Algebra and Function strand is included for third grade:

When temperature is measured in both Celsius (C) and Fahrenheit (F) it is known that they are related by the following formula: $9x \times C = (F - 32) \times 5$

What is 50 degrees Fahrenheit in Celsius? (Note the explicit use of parentheses.) (p. 38)

We wonder how students who are just beginning to learn about multiplication are supposed to access its symbolism in meaningful way.
Conceptual Understanding

Psychologists’ Influence. Most discussions in the Framework and its cited research describe conceptual understanding through the discussion of problem solving situations, and therefore the view of “what is a problem” must be kept in mind when interpreting views of conceptual understanding. In the section Fostering Conceptual Competencies (p. 189) Geary states, “Without conceptual understanding, students often use procedures incorrectly.”

On the surface, the statement appears true enough. But if this statement is misconstrued to mean, “if students do not have conceptual understanding, then they will use procedures incorrectly,” then one has the logically equivalent contrapositive “if students use procedures correctly, then they have conceptual understanding.” We argue that this latter (fallacious) form is how it will be used in California given the Frameworks lack of attention to students’ conceptual development. The Assessment chapter of the Framework sets out this principle quite clearly. It states,

But certain methods, like timed tests, play a more basic role in mathematics assessment than they do in other areas of the curriculum in measuring understanding and skills and in checking whether students have an adequate knowledge base—whether they understand the material to the depth required for future success. (p. 197)

Geary does mention the importance of having students solve problems set in contexts and having them learn multiple approaches to a problem. But the focus of his discussion of conceptual competencies is on examples of incorrect procedural use:

Typically errors will be the result of confusing related topics, such as addition and multiplication, a common memory retrieval error even among adults (Geary 1994). For other problems, the error will reflect a conceptual misunderstanding, such as confusing the rules for solving one type of problem with the rules for solving a related type. (p. 190)

In order to foster appropriate conceptual competencies, the corresponding advice is:

Students can practice using the associated procedures as part of their homework, or practice can occur in school, with a focus on practicing and problem solving occurring on alternating days. (p. 190)

Mathematicians’ Influence. For the mathematicians, conceptual understanding is inseparable from their professional notions of rigor and proof. At the conclusion of the Preface to Grades 8-12 section, the Framework states, “The preceding discussion explains that mathematical proofs are the underpinning of all mathematics” (p. 157). In this discussion the author points out that although a mathematical question may have multiple methods of solution, this should never lead to “the view of mathematics as an imprecise discipline where a problem may have more than one correct answer.” The tone is both fearful and abrasive—it reads as if the use of intuitive or informal reasoning in mathematics, along with ill-posed problems, has destroyed mathematics education. Moreover, this view extends to the early grades. In Chapter 3, in its Preface to Grade K–7, the Framework offers the following discussion about kindergarten:

The students are given a picture which shows in succession a rectangle, triangle, square, rectangle, triangle, square, blank, triangle, square. The students are asked to fill in the blank.

While this may seem to be a reasonable problem (and an example of problems that all too commonly appear in the mathematics curricula of the lower grades) it cannot be considered a problem in
mathematics. From a mathematical point of view there is no correct answer to this problem unless more data are supplied to the students. Mathematics is about drawing logical conclusions from explicitly stated hypothesis. Because there is no statement about the nature of the pattern in this case (e.g., does the pattern repeat itself every three terms? every seven terms? or is it every nine terms?), students can only guess what should be in the blank spot. (p. 110)

The use of a problem where students might use implicit assumptions, or they have to think and decide upon additional assumptions is rejected as inappropriate for developing mathematical reasoning. Referring to the above kindergarten problem the Framework adds:

But if students were to start thinking that every mathematical situation always contains a hidden agenda for them to guess correctly before they can proceed, then both the teaching and learning of mathematics would be tremendously compromised. (p. 110)

The issue of using patterns in mathematics classrooms was also troublesome for the four mathematicians who revised the Standards. In their revision, almost all references to patterns in elementary school were moved out of the Algebra and Functions strand and into the Data Analysis strand. They did not value patterns in developing important number or (pre)-algebra concepts. Discussions in the Framework Committee may reveal why. Attempts by a few members to include patterns in the Number Stand were vigorously resisted because they were not part of the logical structure necessary to derive results in arithmetic, and therefore should not be considered as mathematical concepts (except perhaps in statistics). They explained further that the deductive structure of formal mathematics provided the basis for developing concepts, and so using patterns in algebra was also inappropriate. In the final version of the Framework, all discussions of patterns were removed from the Strands (the section describing branches of mathematics).

When the Framework was presented in December 1998, its supporters would argue that content experts had validated its research-based pedagogy. The document would proclaim to balance skills, concepts, and problem solving. But its authors’ formalistic beliefs about school mathematics resulted in such restricted views of teaching and learning that we consider its claims of being research-based and balanced completely without merit.

The Impact of the Framework

Although it is too soon to evaluate the Framework's impact on student achievement, several recent events do give an indication of the direction the state and the nation are headed. Further discussion can be found in Becker & Jacob (2000).

Views of the SBE and Framework Authors

California’s most recent appointment to the SBE, Nancy Ichinaga, spoke with the Los Angeles School Board about the adoption of math materials on May 2, 2000. She said:

I am here to ask for your support in requiring all the K-12 schools in Los Angeles to adopt a rigorous and systematic mathematics program which meets the state content standards. Gertrude Stein wrote, a rose by any other name is still a rose. A rose is a rose is a rose. Integrated math, reform math, CPM or college math, no matter what you call it, it is still watered down math, fuzzy and substandard math, and does not meet the new state math requirements. … Integrated math instruction is akin to the now discredited whole language instruction in reading. Our kids need systematic instruction in
both reading and math. ... The district must support its teachers by giving them a systematic skill-based (math) program which tells them what to teach and how to teach it.

Her statement indicates the SBE will push for the programs they believe are based upon practice of computational procedures. At the meeting the LAUSD board was considering a report outlining a district plan to align curriculum with the Standards and Framework. The Executive Summary of the Dixon report was included in the LAUSD report, but was not discussed in the meeting.

On September 22, 1999 the California Curriculum Commission sponsored a public meeting for publishers to learn about the new Framework, in preparation for submission of materials for adoption in August 2000. Prof. James Milgram discussed the Framework’s K-6 Grade Level Considerations and Prof. H-H Wu the 7-12 Grade Level Considerations. This meeting provides insight into the beliefs of these two key Framework authors and how they expect it to influence curriculum in the coming decade.

Prof. Milgram began by emphasizing two points. He explained that Marion Joseph and Janet Nicholas asked him to correct the problems with the Standards and they were told they could not throw anything away. He didn’t like this because he felt some were much more important than others were. He then stated that Chapter 3 of the Framework gives the emphasis topics in each grade level. He said publishers are “obliged by law to emphasize the emphasis topics” but it’s more than the law because “these are the topics that matter.” In this way the Standards are not a laundry list any more. Milgram also emphasized that publishers should study the sample lesson plans that appear in the appendices. About the Compound Interest plan (which he authored) he said, “it’s been a long time since compound interest has been taught”. Milgram said this plan was “our price for working on the Standards”, adding that teachers don’t teach it because they don’t understand it.

Milgram also prepared answers to previously submitted written questions. Responding to a question about calculators he said:

Why do we recommend that calculators not be used prior to grade six? The issue here is fairly simple. It has been noted consistently that when calculators are used, students do not learn basic skills – it’s that simple. When calculators are used, students do not learn basic skills – they learn basic button pressing skills. But they do not learn how to add or how to subtract.

When asked about research to back his assertion Milgram had the following conversation with a member of the audience:

Milgram: What research would you like? Okay, how about the research of Dr. David Geary? Would that be acceptable to you?

Audience: Well it’s one individual’s opinion.

Milgram: Okay, but I mean it’s the research of a professional developmental psychologist. And the research involved is his famous work on a - well let me tell you about the work, let me tell you what it says. This is the paper in which he compares the arithmetic competence of the men and women in China and in Russia and in one or two of the eastern European countries and in the US. So here is the picture. ... (a graph is displayed) ... So this would be the age group of 23 to 27, this would be to about to about age 35, this would be about age 49, and this would be about age 50 and above. This is picture of basic math skills in the US that he found and here is the
picture of basic math skills that he found in China. And more or less in Japan and more or less in some of the Eastern European Countries and in Russia, it looked like this. This was done a number of years ago and so you backtrack from this point 50 years old and you find out this is exactly coinciding with the introduction of the new math in the US. This is the research of David Geary and there are numerous other studies that tend to show the same thing. Typical results show that students coming into high school that from have come in from programs the same groups of people that have come in from programs that involved extensive calculator use show a two to three a year lag in skills and in basic skills. So this is the situation which we tend to deal with. It is only for that reason that we wrote that calculators are not recommended, only for that reason.

The same member of the audience pointed out that students in the new math didn’t use calculators. Milgram agreed but said:

What it means is that the new math is the first time a for which the first for the primary part of the math education was no longer basic arithmetic.

Later in the same presentation, Professor Wu offered some specific comments for publishers about grade 7 mathematics:

The main thing is how to multiply rational numbers. Let me give you the context of this. Now this is grade 7 material. Grade seven number sense. It says ‘add subtract multiply and divide rational numbers integers, fractions and terminating decimals’. And take rational numbers to all number powers. Or whatever it is, the main thing is if I want to multiply rational numbers, so this is one of the standards.

He then added the following details:

I’m going to amplify this and treat specifically the situation of fractions, so you imagine that the (problem is) minus two over five times seven over four. Obviously you want to say this is equal to the negative of two over five times seven over four. That’s what you want to say. And now I’m going to try to explain it. I’m offering you one way. There must be other ways, that’s Okay.

Wu then repeated (essentially) the following derivation three times:

\[
(-2/5) \times (7/4) + (2/5) \times (7/4) = \frac{-2 + 2}{5} \times \frac{7}{4} = 0 \times \frac{7}{4} = 0.
\]

Therefore, since \((-2/5) \times (7/4) \text{ and } (2/5) \times (7/4)\) add to 0 , we have \((-2/5) \times (7/4) = - ((2/5) \times (7/4)).\)

In response to repeated questions from a confused audience he said:

Whatever you have to do to help students to understand just do it, but ultimately they have to see this. They cannot replace this by manipulatives.

The intent of Wu’s discussion is very clear. As noted in the last section, teaching approaches at this level must be guided formal mathematics.

New Instructional Materials for California

A standards aligned mathematics adoption was completed during summer 1999 which has a $250 million allocation for AY 1999-2000. Mathematics materials aligned with the 1999 Framework are scheduled for SBE adoption in January 2001. The SBE now controls the adoption process more tightly than in prior years. Adopted programs must pass a Content Review Panel (CRP), all of whose members must have a Ph.D. in mathematics (a
doctorate in education is not sufficient). Although programs are also reviewed by an Instructional Materials Advisory Panel (IMAP) which does include teachers, during summer 1999 it was the CRP opinions that counted in the end. After the IMAPs were dissolved in April, CRP members negotiated with publishers prior to the final SBE votes in June and July. In late June the SBE asked Prof. David Klein to write an addendum to the CRP on Everyday Mathematics\textsuperscript{39} in which he recommended rejection citing “Missing or drastically abridged presentations of the standard algorithms of arithmetic” and “Promotion of calculator use contrary to the California Mathematics Standards” among other objections\textsuperscript{40}. Summarizing, Klein stated:

The Everyday Mathematics curriculum makes clear its hostility to proficiency in arithmetic through the standard algorithms, its opposition to drill and practice, and its support for reliance on calculators in arithmetic.

Like Prof. Klein, most mathematicians selected for the CRP’s have been visibly allied with groups actively opposed to mathematics reform efforts on a national level.\textsuperscript{41}

Members of the organization Mathematically Correct (including Professor Klein) spoke at the May 2 LAUSD meeting, advocating LAUSD immediately adopt one program district wide for K-8, suggesting either Saxon K-8 or Sadlier K-8\textsuperscript{42}. Both programs were adopted by the SBE in summer 1999. Across the state pressure is being applied for districts to choose new instructional materials.

**Some Events Beyond California**

Following the US Department of Education’s release of a list of ten “exemplary and promising mathematics programs” (USDE, 1999), a full page advertisement appeared in the Washington Post (Klein, et. al, 1999) containing an “open letter” asking Secretary of Education Riley to withdraw the recommendations. The approximately $67,000 cost of the advertisement was paid by the Packard Humanities Institute, a California foundation that funds implementation of SRA’s Open Court, a back-to-basics reading program. Four of the advertisements six primary authors served on the California’s 1999 CRP. The letter criticized a 1994 statement of Steven Leinwand questioning extensive use of pencil-and-paper computational algorithms in elementary grades. It then cited a 1998 American Mathematical Society report stating:

*We would like to emphasize that the standard algorithms of arithmetic are more than just ‘ways to get the answer’—that is, they have theoretical as well as practical significance.*

So the mathematicians’ beliefs outlined in the last section are driving the national discussion, namely that teaching approaches in elementary school should be set according to the theoretical underpinnings of the subject. The letter was signed exclusively by University professors (mostly in mathematics), and the exclusion of K-12 educators was deliberate. Secretary Riley noted this in his reply, for although he agreed about the importance of basic computational skills he went on to say:

*However, we do not agree with your assertion that both the panel and the criteria it used were outside of the existing mathematics education mainstream. It is important to note that the Panel concluded that each of the ten programs had demonstrated a measurable difference in student learning.*
The Washington Post letter was followed by a Congressional hearing on the subject on February 2, 2000. In his testimony Prof. James Milgram criticized the exemplary and promising programs using the California Standards as a benchmark (Milgram, 2000):

The high school programs, Core-Plus and IMP, both place heavy emphasis on topics such as discrete mathematics at the expense of basic algebra, and do not come near the level indicated in e.g., the new California Standards for most of the topics there.

During fall 1999 and winter 2000, a draft of the Massachusetts Framework was revised in a manner paralleling the California revisions during late 1997. Again mathematicians played prominent roles in the revision, and again they stressed the importance of direct teaching of standard algorithms, and cited “vagueness” of draft standards and their lack of “measurability”. Massachusetts Education Commissioner Sandra Stotsky, in a memo dated June 10, 1999 stated:

I also find the language expressing standards to be more pedagogical than standards-like. For example, the verbs “explore,” “investigate,” and “develop” are not words for standards; they are words for a curriculum document in the schools. Those are not measurable verbs. … When it uses "constructs" or "creates," it's even worse. I don't know what we are asking children to know or do when a standard expects them to “construct, explain, and use algorithms for addition and subtraction of multi-digit whole numbers.”

An angry national email debate erupted in December after reports that Commissioner Stotsky had sharply criticized the draft as advocating “constructivist pedagogy” and explicitly pointed out the example “Construct triangles with straight edge and compass and by other means.” Stotsky insists the report is false, but others attending the meeting assert it is correct and the stalemate over this event greatly damaged the ability of two sides to come together to discuss the document. On February 9, 2000 the entire Massachusetts Framework authoring panel resigned stating in their letter:

Since our last input to the document, changes have been made behind closed doors, without the opportunity for review by our panel.

**Professional Development Initiatives in California**

California is now preparing to implement expensive mathematics professional development aligned with the Standards and Framework. In May 2000, $43 million in AB 1331 mathematics funds were allocated to counties and districts for AY 2000-2001. The SBE had to approve all AB 1331 providers, and providers had to submit professional development materials to two SBE appointed mathematicians who demanded revisions. Prof. David Klein rewrote the California County Superintendents program in grades 4/5. The number module includes the following definition of the counting numbers “A counting number is an attribute of a set”. Peano’s axioms for natural numbers are included, as are explicit step-by-step outlines of how to implement the standard algorithms for addition, subtraction, multiplication and division.

The Governor has proposed Mathematics Professional Development Institutes to involve 20,000 teachers during summer 2000. The RFP for the Institutes (UCOP, 2000) includes “a content specification for the Elementary Institute, by Hung-Hsi Wu of UC Berkeley”. The RFP states (p. 13):
Wu’s elementary statement is formal and axiomatic. It is a statement of the mathematics that a number-and-operations Institute could be about, beginning with the whole numbers and moving up to the rationals.

As noted in the last section, the mathematicians involved in the development of the Framework believed that mathematics teaching should be rooted in the deductive structure of formal mathematics. This view continues to prevail and the implications for curriculum and professional development are substantial. Interestingly, in spite of the initial fanfare, the research used in development of California’s Framework has played almost no role in subsequent discussions.

**Concluding Remarks**

This paper about how the selective use of research has influenced the content of the 1999 Mathematics Framework is only part of the story about how state policy has affected mathematics education in California. The development of state Standards, assessment, and new requirements for professional development of teachers has also been highly politicized and controversial. Research has played its role in these debates as well. But in our opinion, the Framework story provides a lens that can be useful in understanding how ideology in the guise of research can be used to shape policy that will directly affect schools, teachers, and students.

The core beliefs guiding California’s recent back-to-basics movement are aligned with the beliefs about mathematics and education examined in this paper. Most sad from our view is the continued devaluing of teachers’ expertise and the belief that the optimal learning occurs automatically with a proper sequencing of facts and procedures. The research selected for the Framework (and how it was interpreted) examined instruction of a limited selection of symbolic mathematics, largely designed to “teacher-proof” the curriculum and also to weed out the “procedural bugs” from student work.

The psychologists and mathematicians who crafted the 1999 California Mathematics Framework wove their views that the skills needed for problem solving must be taught following a procedural hierarchy that is (structurally) rooted in the formal proofs that mathematicians treasure. As such, they believe each stage is explicitly measurable and must be attained before the next incremental step is made. This rules out the indirect and informal processes which qualitative researchers and teachers have noted as essential to the development of students mathematical understandings, both cognitive and affective.

How should mathematics educators respond? There is a great need to move beyond the ivory tower and study how to be more effective in communicating with the public. All participants in the teaching for understanding movement (researchers, curriculum developers, teachers, and parents) must make public advocacy a serious component of their effort. Moreover, they must prepare in advance for the simplistic views of teaching and learning espoused by “experts” who capture support from a naïve public whose experience with mathematics has been limited to rote computation.
Especially important is to move beyond over simplified public statements that are easily dismissed as rhetoric or educators talk. A few examples: (1) what mathematics do adults need and how do they use it? It is no good to merely say “for the 21st Century.” But if we can be specific, it will be clear that thirteen years of precalculus symbol manipulation is not what students in K-12 need. (2) Continued discussion of equity is also critical. But beyond the data and efforts increasing awareness, we need to examine how to help policy makers and the public understand that a challenging, thinking curriculum, attracts more students and involves them in learning. (3) Linked to both (1) and (2) we need to help the public reexamine what mathematics is and what it means to understand mathematics. In order to understand understanding, one must really experience it, and for students progressing through school it is not the same as a mathematician’s final product or efficient algorithm. It is popular today to consider sense making as a critical component of mathematics, but parents and policy makers do not yet understand this idea. Researchers need to explore how to communicate it more widely across the public sector. A main lesson learned by the California Framework committee in 1997 was that “limiting policy to content” won’t reduce controversy, and we hope others will not fall for this trap.

Our hope, even though it is currently being disregarded by policy makers in California, is that what we have learned from the last thirty years research about how students learn mathematics will continue to guide more effective teaching for understanding. This research cannot be repudiated, although being recognized will not be enough to turn the situation around. If enough people speak out, and if enough people learn that the new ideological policies won’t adequately prepare students for the future, then we will return to a more supportive stance of schools and the implementation of sense-making curricula.

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**Endnotes**

1 In the case of teaching reading, Douglas Carnine and Hans Meeder (Carnine & Meeder, 1997) suggested that the U.S. Congress establish “an expert reading panel to authoritatively synthesize research-based knowledge about reading” that would determine approaches used in federal education programs (from Head Start through professional development). Two months later, the House of Representatives passed the Reading Excellence Act (H.R. 2614) whose preamble states it will improve the reading skills of students “through the use of findings from reliable, replicable research on reading, including phonics.”

2 Requirements for including research in inservice training for reading and mathematics were added to the Education Code in 1996 (reading, Sec. 53002,53006) and in 1998 (mathematics, Sec. 44721). The mathematics section calls for the inservice training for teachers of grades 4-12 to address “replicated” research on “how mathematical skills are acquired” and on “methods of teaching mathematics that produce measurable growth on a validated assessment of at least one grade level during a school year of instruction.”

3 The 1985 Framework, Chapter 3, *Delivery of Instruction in Mathematics*, had subsections stressing teaching for understanding, problem solving, use of concrete materials, and cooperative learning groups.

4 In the past there were some attempts to include learner verification, or proof that materials work with students, as part of the adoption process. However, since almost all materials submitted for adoption have
copyright dates that are quite close to the submission dates, this proved unworkable. Also, there was concern that publishers would only provide data that are supportive of their materials.

3 Earlier a program advisory for early reading instruction had been successfully developed by a committee that included researchers. The development of the Mathematics Program Advisory was not as smooth—there was considerable disagreement about the definition of terms, although all parties involved ultimately supported the final document. There were no researchers in mathematics education involved in the development process.

6 The criteria used in 1994 were from the 1992 Framework, and adopted materials reflected these criteria. The SBE added three programs not recommended because they felt that more traditional materials needed to be included, and that these programs also met the criteria.

7 See (Jacob, 1999) for more discussion, as the rejection involved other issues as well. For example, Janet Nicholas’ memos identified a problem involving a “pizza pirate” as violating the Patriotism and Morality section of the Education Code (Sec. 60200.5). She also asserted that rewriting 36 x 45 as 9 x 30 x 6 was a “factual error” because “the number 30 is not, however a factor of either 36 or 45”.

8 This special adoption in mathematics and language arts was established by AB 2519, and the SBE approved its Criteria in October 1998. They did not utilize the Frameworks and the legislature has committed $250 million per year for four years for approved programs.

9 Experimental research means a study with both treatment and control groups. Current mathematics education research also includes qualitative, exploratory, summary, theoretical, and other paradigms.

10 In Fall 1996 the Curriculum Commission recommended a list of members to be appointed to a committee to develop the Framework. The State Board removed ten of the fifteen people recommended and replaced them with fourteen others, whom they believed would be more favorable to traditional views. As with the preparation of the Mathematics Program Advisory, the SBE choices ensured that no mathematics education researchers were on the committee.

11 The development of the Framework overlapped the development of state Standards in mathematics, even though the Framework was to be aligned with the Standards. Although the Framework committee kept apprised of the work of the Standards Commission, a majority of their effort was devoted to writing and re-writing their own grade-by-grade level expectations, many of which were significantly different from either the Commission’s version or the final version adopted by the SBE.

12 The Interactive Mathematics Program, published by Key Curriculum Press, is an integrated high school mathematics program. It was developed with support from Eisenhower Funds and the National Science Foundation.

13 This journal is a publication of the Association for Direct Instruction, Eugene, Oregon, with most authors affiliated with the National Center to Improve the Tools of Educators, Eugene. This is the center commissioned by the SBE to write the report on a review of research related to mathematics education.

14 Connecting Math Concepts is authored by Siegfried Engelmann, Douglas Carnine, Owen Englemann, and Bernadette Kelly and is published by Science Research Associates (SRA). One of the research articles states “Copyright 1994, Science research Associates, Reprinted with Permission”, and is essentially a publishers advertisement authored by Bernadette Kelly (1994), who was a guest co-editor for this particular Effective School Practices issue.

In June 1999, the SBE approved the 1996 edition of Connecting Math Concepts in their AB 2519 adoption in spite of the fact that both the SBE appointed Content Review Panel and Instructional Materials Advisory Panel recommended its rejection for failing to align with the Standards. The program advertises and stresses its direct instruction pedagogy. For example, in its Level F (fifth grade) Teacher’s Guide one finds (p. 8) “Remind students about two important rules for doing well in this program: Always work problems the way they are shown. And: No shortcuts are permitted.”

15 At the first meeting of the Framework Committee, following her election as chair, Deborah Haimo canceled an agenda item discussion of a research article (Hiebert, et. al, 1997) that previously had been sent by the Curriculum Commission to committee members.

16 The main focus of Carnine's research has been special education. He was a major author of DISTAR, a “direct instruction” instructional program in reading, arithmetic, and language first published in the 1970s by Science Research Associates (SRA). This program utilizes scripted lessons for teachers to present with choral responses by students and is similar to Connecting Math Concepts discussed in endnote 15, of which Carnine is also an author.

17 In May 1996, the California Assembly Education Committee heard testimony on what research proves is the best way to teach reading. For discussion, see (Taylor, 1998), where the accuracy of their statements is brought into question. A number of these same researchers in early reading, including
Marilyn Adams, Douglas Carnine, Linda Diamond, Adria Klein, Sheila Mandel, John Shefelbine, and G. Reid Lyon, were involved in the development of the *Reading Program Advisory*.  

Follow Through was a U.S. Office of Education program, designed in the 1960s as an extension of Head Start to improve the schooling of disadvantaged children in the early grades. The multi-million dollar program compared several models of innovative educational approaches. A major portion of the evaluation was done by Abt Associates and published in 1977. For a critique of the Follow Through evaluation, see (House, et. al., 1978) and for a response by Abt authors see (Anderson, et. al, 1978). In spite of many differences, both papers argue against 1977 press reports asserting the superiority of any one model of instruction over another. The response states (p. 165) "Let us repeat once more: the major finding of our report is that differences between sites within a model were larger than differences between models."

The SBE issued a Request for Proposals (RFP), as required by law, for this report. It was done so quietly that other prospective responders and staff of the CDE was unaware until the deadline for responding was near.

Hirsch provided an anecdote as evidence of bias where Educational Researcher turned down an articles (refuting claims of situated learning) by three of the "most distinguished cognitive scientists in the country," John Anderson, Lynn Reder, and Herb Simon (the latter a Nobel prize winner). He also stated that the educational division of the National Science Foundation has a deservedly poor reputation because it is dominated by the constructivist ideology.

Hirsch was critical of performance assessment, and stated that "almost all the nasty things said about multiple choice tests are incorrect." He suggested consulting with Professor Lee Cronbach, Samuel Messick, Robert Lind, and Eva Baker for research on assessment. Hirsch was critical of the National Association for Education of Young Children (NAEYC) and asserted that its position on "developmentally appropriate practices" is ideology rather than science. He mentions that leading researchers in this area are Rochel Gelman, James Stigler, Kevin Miller, and Sandra Scarr.

We remark that the References in the NCTM Standards 2000 draft (NCTM, 1998) contains 88 research entries that should have appeared in the Dixon database of 8,727 articles. Of these, four appear on the list of 954 experimental studies. All four were rejected at level 1 of the analysis, meaning that they did not meet the studies criteria for "minimal construct, internal, and external validity," or were not "pertinent to the analysis." This indicates the extent of the difference between the NCTM authors and the Dixon Report of what type of research should advise practice.

They are (parenthesis indicate the number of studies in the category): Cooperative and/or Peer Work (13); Instructional Design—Didactic vs. Discovery Learning (3), Strategy Instruction (13), Selection and Sequencing of Examples (4), Mastery Learning (2); Technology—Calculators (5), Logo (3), Computers (10); Grouping (5); Reinforcement and Motivation (6); Assessment (5); and Manipulatives (4).

Chapter 2 of 1992 Framework focused on classroom and program characteristics that would promote student learning of mathematics, emphasizing instructional approaches that would encourage students to take responsibility for their own learning, with teachers as facilitators rather than imparters of information. The 1992 Framework was based on the theory that, "Students construct their understanding of mathematics by learning to use mathematics to make sense of their own experience.” (p. 33) The messages of the 1999 Framework are quite different. The 1999 Framework promotes direct instruction and rote mastery of symbolic procedures and doesn't mention the use of manipulatives. Both issues of pedagogy and manipulatives were sources of tension in the 1997 Framework committee, and remained controversial during the Curriculum Commission discussions.

In the Framework’s version, the description includes a starting point “Central Focus,” a “Help Phase” where students are transitioned from “teacher regulation” to “self regulation,” and concludes with a “Closure” phase where students work independently and demonstrate their ability with the skill or concept.

At the April 28 Curriculum Commission meeting Dr. Dan Fendel (attending in audience) asked Dr. Carnine if the three-phase model was backed by research. Dr. Carnine replied that he did not know of any research supporting the model, and that the model was inferred from looking at research.

They were, Hung Hsi Wu of U.C. Berkeley (grades K-3), Ralph Cohen of Stanford (grades 4-7), and Scott Farrand of CSU Sacramento (grades 8-12).

Between October and November, the introductory material in Chapter 3, Grade Level Considerations, was revised and expanded, Chapter 10 was rewritten, and the final November work session on the Framework included Dr. Wu, Dr. Milgram, SBE member Janet Nicholas, and CDE staff. The Chapter 3 prefaces to Grades K-7 and 8-12 were added at the last moment and did not appear in the October
version, the last version disseminated for public comment. The K-4 discussion in Chapter 3 can be traced
to Geary's August draft, with grades 5-6 started by Geary but modified by others.

While discussing the sample problems at Curriculum Commission meetings, SBE member Janet
Nicholas did mention there was a "peer review" set up for the problems, but declined to mention where,
when, or who was involved. Had such a group of more than two people convened to provide advice on the
problems it would have been a violation of California’s public meeting act.

The Dixon report is providing the "research based instructional strategies" that are required by the SBE
approved providers of professional development in mathematics (AB 1331, funded for $43 million for
2000-01). Applicants to be SBE approved providers were expected to work off a January 1999 version of
the Dixon report.

What is striking is that all 17 come from the 18 papers listed in the Dixon report in the category,
Studies related to Computers and Technology. In the September 8, 1998 Framework draft, in Chapter 10
(The Use of Technology) one finds the generic references “Carnine report” or “Carnine study.” These
citations appear to have been subsequently added by CDE staff.

The draft did blend some prior writing into these sections and contains some California specific
information which was not Geary’s work. Also, his format was reorganized, and the Chapter 3 sections
Preface to Grades K-7, 8-12 were added in November by the mathematicians. When questioned, no
explanation was provided by the SBE describing this revision process. Had more than two persons
 collaborated in the revision, the work would have violated California's public meeting act.

In his book, Children’s Mathematical Development, he refers to these as “biologically primary and
biologically secondary cognitive skills.”

(Siegler & Crowley, 1994), (Fennema, Wolleat, Pedro, & Becker, 1981), (Siegler & Stern, in press),
(Sophian, 1997), (Cooper & Sweller, 1987), (Sweller, Mawer, & Ward, 1983), (Gear, 1994), (Cooper,
1989), (Delaney, Reder, Staszewski, & Ritter, 1998), (Bahrick & Hall, 1991), (Siegler, 1995), (Van Lehn,
1990)

In the Introduction of the Framework the description of balance is similar to what was included in the
Mathematics Program Advisory. It states “An important theme stressed throughout this Framework is the
need for balance in emphasis on computational and procedural skills, conceptual understanding, and
problem solving.”

Education Code 60200.4 (a) (added by AB 170 in 1995) requires that SBE adopted instructional
materials in “reading and mathematics are based on the fundamental skills required by these subjects,
including, but not limited to, systematic, explicit phonics, spelling, and basic computational skills.” The
AB 2519 Mathematics Criteria (p. 4) not only requires evaluators to determine if basic computational
skills are present, but additionally requires them to determine if materials “provide for basic skills
instruction that is systematic and explicit.” To our surprise, the descriptors “systematic and explicit” are
applied by the SBE to mathematical computation, not only phonics.

This was added after Geary’s August work, and so we are uncertain who the actual author is.

These are Instructional Profile: Subtracting Fractions with Unlike Denominator, Grades Four through
Seven, (pp. 237-244), Elementary School Sample Lesson: An East Asian Approach (pp. 245-253), Middle
School Sample Lesson Plan: Compound Interest (pp. 254-278).

The discussions are from a tape recording of the public session. Repeated words are edited out to
improve readability.

Everday Mathematics representatives had been involved in two months of discussions with members of
the Curriculum Commission and the original CRP, and made numerous revisions in order to remedy
the objections raised in April. The SBE delayed a final vote pending verification of changes made, but the
new issues raised in the Klein report superseded all previous work and the program was rejected.

As an example, Klein cites a use of a nonstandard long division algorithm in grade 4. Like the standard
algorithm, the procedure in Everyday Math is based on repeated subtraction, but students do not have to
determine the maximal single digit multiple prior to subtraction.

The entire 1999 California Mathematics CRP signed the November 18 Washington Post letter to
Secretary Riley and ten of the initial twelve CRP appointments for the summer 2000 adoption signed the
letter.

The Saxon program has been widely promoted by Mathematically Correct and is based on sequenced
and highly scripted drill and practice of standard algorithms.

Proposed legislation will increase the program’s $500 per teacher allocation to $1600 per teacher.
The current projection is for a maximum of 2500 teachers, mostly coordinated by the California Mathematics Projects. The California Subject Matter Projects, which includes the Math Projects, will have its $12.5 million annual budget augmented by $20 million.