KNOWLEDGE MAPS – TOOLS FOR BUILDING STRUCTURE IN MATHEMATICS

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The experience of mathematical structure may be supported by representing mathematical knowledge graphically in the form of networks. In this paper, two special graphical representations of mathematical networks, mind maps and concept maps, are presented. Both knowledge maps are means to show ideas and concepts connected with a topic, in a well-structured form. Their special fitting as a pedagogical tool for mathematics education is pointed out, especially with regard to build structure. Possible uses of the presented knowledge maps together with their advantages and limits are discussed and classroom experiences collected from 9 teachers of different schools are presented. It turns out that knowledge maps, like mind maps and concept maps, may be efficient tools for building structure in mathematics.

1. Introduction

Mathematical knowledge has the character of a network, as mathematical objects, i.e. for example concepts, definitions, theorems, proofs, algorithms, rules, theories, are manifold interrelated but also connected with components of the external world. Accordingly, there is a widespread consensus in the actual didactical discussion that mathematics should be experienced by students in its interrelatedness rather than a collection of isolated rules and facts (see e.g. NCTM Yearbook 1995, Preface, or NCTM Principles and Standards for School Mathematics 2000, p.64). The importance of this notion also becomes apparent in the recent PISA–Study, where interconnections and common ideas are central elements (OECD, 1999, p.48).

The network character of mathematics may be experienced but also learned by visualizing graphically structure in mathematics. Two especially suited means for this purpose are mind mapping and concept mapping, both representing a mathematical network around a topic in a well-structured graphical display. These two techniques are presented below. Their special fitting as a pedagogical tool for mathematics education, especially with regard to build structure, is pointed out and possible uses of mind maps and concept maps together with their advantages and limits are discussed, and where possible supported with classroom experiences collected from 9 teachers of different schools.

2 Theoretical Background

2.1 Mind mapping

Mind mapping was firstly developed by Tony Buzan, a mathematician, psychologist and brain researcher, as a special technique for taking notes as briefly as possible and also as interesting to the eye as possible. Since then, it turned out to be usable in many different ways other than just simple note taking. Mind maps have, among other things, been used in education, but rarely yet in mathematics.

The method of mind mapping basically takes into account that the two halves of the human brain are performing different tasks. While the left side is mainly responsible for logic, words, arithmetic, linearity, sequences, analysis, lists, the right side of the brain mainly performs tasks like multidimensionality, imagination, emotion, color, rhythm, shapes, geometry, synthesis. Mind mapping uses both sides of the brain (Buzan, 1976), lets them work together and thus increases productivity and memory retention. This is accomplished by representing logical structures using an artistic spatial image that the individual creates. Thus mind mapping connects imagination with structure and pictures with logic (Svantesson, 1992, p. 44).

2.2 Concept mapping

Concept maps were first introduced by Novak as a research tool, showing in a special graphical way the concepts related to a given topic together with their interrelations. The method of concept mapping "has been developed specifically to tap into a learner's cognitive structure and to externalise … what the learner already knows" (Novak and Govin, 1984, p. 40), according to Ausubel's statement: "The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Ausubel et al., 1980).

Although the primer intention was to use concept mapping in research, it was found this to be also a useful tool in helping students to "learn how to learn" (Novak and Govin, 1984; Novak, 1990, 1996). Consequently, concept mapping has been used also as an educational tool, above all in science, whereas experiences in mathematics education are rather seldom and not well-documented (Malone and Dekkers, 1984, p. 225; Hasemann and Mansfield, 1995, p. 47).

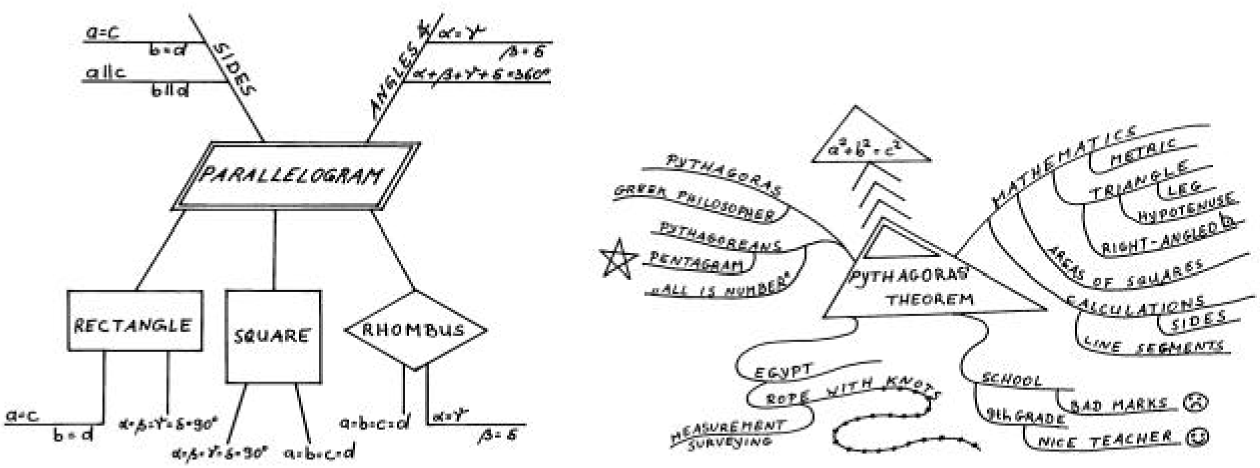
3 Rules for making knowledge maps

Both, mind maps and concept maps, are hierarchically structured. They are produced following the rules given below (for more details see e.g. Buzan and Buzan, 1993; Svantesson, 1992, pp. 55-56; Novak and Govin, 1984; Brinkmann, 2003a, 2003b).

3.1 Mind maps

Use a large sheet of paper. Place the topic of the mind map in the center. From the topic draw a main branch for each of the main ideas linked to the topic. Write keywords denoting the main ideas directly on the lines. Starting from the main branches you may draw further lines (sub branches) for secondary ideas (subtopics) and so on. The order follows the principle: from the abstract to the concrete, from the general to the special. Use colors when drawing a mind map; add images, sketches, symbols, such as little arrows, geometric figures, exclamation marks or question marks, as well as self-defined symbols.

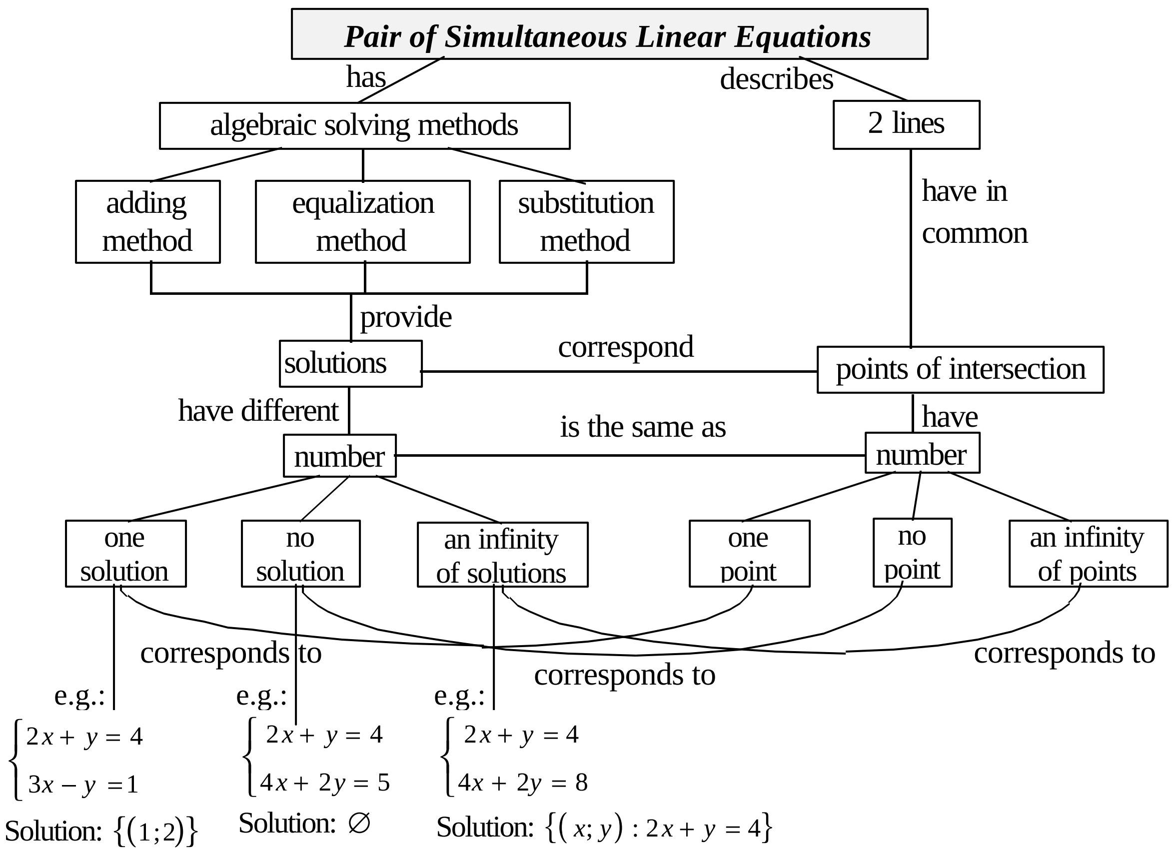
Figure 1: Mind maps



3.2 Concept maps

Use a large sheet of paper. Position the topic at the head of the map. Arrange the other concepts beneath it on several levels, the more inclusive, general, abstract concepts higher, the more specific, concrete concepts lower. If possible, arrange the concepts so that ideas go directly under ideas that they are related to. Note beneath the last row some examples to the concepts situated here. Draw lines from upper concepts to lower concepts that they're related to; do the same for any related concepts that are on the same level.

Figure 2: A concept map



On the connecting lines, write words or phrases that explain the relationship of the concepts. Beneath the last row, put examples to the concepts situated here and connect the examples with the concepts they belong with (as linking words write a phrase like “for example”). Draw circles around the concepts, do not draw circles around the examples.

4 The adequacy of knowledge maps as tools to build structure in mathematics

The hierarchical structure of mind maps and concert maps conforms to the general assumption that the cognitive representation of knowledge is hierarchically structured (Tergan, 1986). Mathematical knowledge may thus be organized in a mind map or a concept map according to this knowledge’s mental representation.

As for mathematics, this “is often depicted as a mighty tree with its roots, trunk, branches, and twigs labelled according to certain sub disciplines. It is a tree that grows in time" (Davis and Hersh, 1981, p. 18). Similarly, the structure of a mind map resembles a tree seen from the top: from the trunk in the middle, representing the topic of the mind map, the lines for the ideas linked to the topic branch off like tree branches. Also concept maps resemble on the whole a tree, only seen from another perspective. In addition, concept maps show also links between concepts of different branches, their structure is thus even more in accordance with the character of interrelatedness of mathematics. Relations between mathematical objects around a topic may thus be visualised by mind maps and concept maps in a structured way that corresponds to the structure in mathematics (Brinkmann 2001a, 2002, 2003a, 2003b).

5 Uses of knowledge maps in mathematics education and classroom experiences

Below there are listed and explained some of the most important uses of mind maps respectively concept maps, that might be profitable also in mathematics education especially for building structure in mathematics. Where possible, they are supported with reports about classroom experiences (written in italics), mainly collected from 9 teachers of different German schools (including the authors own experiences), but also from literature. 7 out of the 9 teachers introduced only mind mapping in their classes, 2 (the author included) introduced both mind mapping and concept mapping.

5.1 Mind maps

- Mind maps help to organize information.

The special structure of a mind map allows to organize hierarchically structured mathematical knowledge. A clear and concise overview of the connectedness of mathematical objects around a topic is enabled. Moreover, this is supported by the use of colors and pictures.

In addition, mind mapping supports the natural thinking process, which goes on randomly and in a nonlinear way. As mind maps have an open structure, one may just let one's thoughts flow; every produced idea may be integrated in the mind map by relating it to already recorded ideas, and this with virtually no mental effort.

Teachers reported that especially students who were not good in mathematics benefited from mind mapping as a help to organize their knowledge. These students often first realized connections between mathematical concepts while producing a mind map. Further on, they told their teachers that only after having drawn a mind map they could "see" the structure of the respective mathematical knowledge.

- Mind maps let cognitive structures of students become visible.

Mind maps drawn by students provide information about the students' knowledge. In broad outline, a learner's knowledge structure around a given topic gets visible by means of mind maps for both the teacher and the learner.

  - The student develops an awareness of his or her own knowledge organization. This process might be enhanced by having the students, in small groups, construct mind maps, as by it students have to discuss about the concepts to be used and the connections to be drawn.

    I could observe that students not only developed an awareness of the knowledge organization they had, but also an awareness of missing links between isolated concepts they knew were belonging to the map topic. For example, a student had difficulties to integrate the equation $f(x)=x^2$ in a map on quadratic parabolas.

  - Wrong connections in a students' knowledge become visible and can be corrected by the teacher. It is recommendable to first ask the student why the (wrong) connection was drawn; the explanation given by the student might bring more insight into the underlying cognitive structure than the simple and reduced representation in the map.

    One of my students connected the concept "congruent transformation" with the sub concept "Thales' theorem". When asked to explain this connection that seemed strange to me, he showed me that in the textbook "Thales' theorem" occurs as a sub heading under the heading "congruent transformation", so obviously these concepts had to be linked in some way.

  - The students' growth in the understanding of a topic can be checked when asking them to create both a pre- and a post-unit mind map (Hemmerich et al., 1994). The teacher might see e.g. if supplementary concepts are linked to the topic, in a meaningful way.

- Mind maps can be used as a memory aid.

Each mind map has a unique appearance and a strong visual appeal. Thus information may be memorized and recalled faster, the learning process is speeded up and structured information becomes long living.

- Mind maps can be of help to repetition and summary.

At the end of a teaching unit the subject matter of the treated topic can be repeated and structured by composing a mind map; this mind map then serves as a good memorizable summary.

Several teachers who introduced mind mapping in their mathematics lessons could observe that some of their students began on their own initiative to construct mind *maps at home, especially when preparing for an examination, in order to get a structured overview on the subject matter.*

One teacher told about a ten year-old girl that showed her proudly a mind map she had drawn as decoration for her exercise-book: The map represented the contents of this exercise-book in a structured way.

- A mind map may summarize the ideas of several students.

A mind map may grow as the common task of an entire class: The teacher might write the topic in the middle of the chalkboard and ask the students what main ideas they connect with it. For each idea the teacher draws a main branch of the mind map. Further on, students are asked to tell all other ideas they link to these main ones. Due to the open structure of a mind map, each single contribution can be integrated. The complete mind map should be redrawn by each student, in his or her personal style.

Classroom experiences show that this is a very fruitful way of action. Students discuss about the proposed ways of integration of the single concepts. Sometimes they make the valuable experience that a concept may be sub ordered under different super concepts, as a mathematical concept may be related in more than one way to others.

- Mind maps help meaningfully connect new information with given knowledge.

New information can be integrated into an existing mind map and related to previously learned concepts. Such an activity of the students has to be initialized by the teacher, who has the overview of already created mind maps and of how new concepts fit to old topics. Of course this can be done only for few new information because of space limitations.

- New concepts may be introduced by mind maps.

Entrekin (1992) reports that she used mind maps to introduce new concepts in mathematics classes. The new concept "is written on the chalkboard or transparency. As the concept evolves in later lessons, the teacher may add additional components and form an extended mind map. This visual representation serves to help students relate unknown concepts to known concepts."

- Mind maps may show connections between mathematics and the rest of the world.

As a mind map is open for any idea someone associates with the main topic, non-mathematical concepts may also be connected with a mathematical object (see figure 1, mind map on Pythagoras' theorem). Thus it becomes obvious that mathematics is not an isolated subject but is related to the most different areas of the "rest of the world" (Brinkmann, 2001b).

This option of mind maps is generally not obtained automatically, as students are used that they are expected to think and argue only within mathematics. Thus it is of need for teachers to give a respective hint when introducing mind mapping in a class. Students often express their surprise, that they are allowed to insert non-mathematical terms in their maps, but also their good feelings when doing so.

5.2 Concept maps

Concept maps have been found to be useful in a variety of applications, in the teaching of the different sciences but also of mathematics at all levels ranging from primary school to senior high school. Concept maps can be used for example in the following situations (Novak and Govin, 1984; Novak, 1990, 1996; Malone and Dekkers, 1984):

- Concept maps help to organize information on a topic.

In order to be useful, knowledge must be organized so as to facilitate understanding and problem-solving ability. A concept map organizes knowledge into categories and sub-categories so that it can be easily remembered and retrieved.

From classes where concept mapping has been used, I received the feedback that there is a widespread consensus among students that concept maps can help to organize information, especially if these concept maps are drawn by the students themselves. For the usefulness of concept maps in this point, the degree of complexity of a concept map (defined by the represented number of hierarchy levels, of concepts and of links) seems to play a crucial role: maps with a great degree of complexity seem to be rather confusing than helpful. As a problem it must be seen that a productive degree of complexity is dependent on the individual, or at least on the achievement level of a learning group.

The statement that concept maps may facilitate problem-solving abilities is not well-founded. For this purpose, I have carried out a study (Brinkmann, in press) in 4 high school mathematics courses. The students had to work on several mathematical problems, that were unusual for them and that afforded a well-organized conceptual knowledge on the concerned topic. All students could use their textbooks, half of the students could additionally use a concept map representing the needed knowledge in a structured form and constructed by the author. The students had been distributed equally according to their achievement levels into the 2 groups. It turned out, that those students using the concept map were more successful in solving the given problems. Moreover the study showed, that the given concept map was more helpful for normally higher-achievers than lower-achievers. The lower-achievers expressed, that the given concept map was too complex for them, they would have preferred a map showing only an excerpt of it, or the division of the concept map into two maps. Perhaps it is of importance, that the students that participated at this study were not accustomed to the use of concept maps, this representation form was new for them.

Thus, there remain a lot of open questions: what degree of complexity is optimal for students with a certain achievement level, which differences can be observed between students that are accustomed to concept mapping and novices, is it more helpful for problem-solving to use a (comprehensive) map constructed by the teacher or the whole class or to use a self-constructed map, …

- Concept maps facilitate meaningful learning, they aid in organizing and understanding new subject matter.

- Concept maps are a powerful tool for identifying students' knowledge structures, especially also misconceptions or alternative conceptions.

This helps the teacher to plan effective lessons by taking into account what a learner already knows. A student himself gets awareness of his own knowledge organisation. Possibly wrong connections in a student's knowledge become visible to the teacher and can be corrected by him. (See also the remarks in 5.1 to this point.)
Concept maps are more likely to show students' knowledge structures than mind maps, as there are intended more linking lines and the description of the represented relations by linking words. But experience based it is very hard for students to find suitable linking words, also if they can describe correctly, in longer terms, the represented relation. Thus some additional discussions of the teacher with the single students might be helpful.

- Concept maps may serve as a memory aid.

As a concept map is a graph, a pictorial representation, it may be grasped at once, and due to its unique appearance committed well to one's memory and recalled faster.

- Concept maps may be used for revision of a topic.

At the end of a topic a concept map can be constructed, as repetition and in order to get a lasting and well organized overview of this topic.
In classes where concept mapping has been introduced, it could be observed, that some of the students constructed by their own initiative concept maps in order to prepare for written tests at the end of a topic. Higher achievers generally constructed more detailed and comprehensive maps than lower-achievers.

- Concept maps can be used as design of instructional materials.

Teachers found that concept maps were useful tools for organizing a lecture or an entire curriculum. Moreover, they were not only aided in planning instruction, but also their own understanding of the subject matter was increased (Novak, 1996).

6 Limitations

It has to be considered that the methods of mind mapping and of concept mapping can be used only if one has got familiar with them. Moreover, when using concept mapping it has to be calculated that it takes some time to construct a concept map (more than to construct a mind map).

In spite of their well-structured and ordered contents concept maps as well as mind maps may sometimes have a confusing effect.

Mind maps are very individual graphic representations. As different people have different associations with the same topic they also draw different mind maps. The right grasp of the relations represented in a mind map affords the right associations to the used key words. Hence, mind maps that someone wants to use should be self-done or done with one's own involvement.

In a mind map, each main branch builds up a complex whole with its sub branches. Connections between the single complexes are as a rule not drawn in order to increase the clarity of the map. Thus, the existing relations to the map topic are probably represented incomplete. In contrast to mind maps, the concepts of a concept map are linked by lines whenever they are related in some way, moreover, every single relationship is described by linking words written on the linking lines. Thus, a concept map provides much more information on a topic than a mind map, but it has not got that open structure allowing to add easily every new idea one might associate to the topic.

7 Final remarks

The methods of mind mapping and concept mapping were not invented as educational tools, but it turns out that these methods may be useful in a variety of applications in teaching and learning processes, especially also for building structure in mathematics. Yet, up to now, mind mapping and concept mapping have been rarely used in mathematics education. However, the feedback of teachers that took part in further education events which I offered on the topic of mind mapping and concept mapping in mathematics is full of enthusiasm throughout.

Of course, depending on the pursued goals, teachers respectively students have to decide which of the two methods they particularly want to use. When my students had the free choice between mind mapping and concept mapping in order to summarize and structure their knowledge on a topic, they usually constructed a mixture of both: they typically centered the topic (this is advantageous with regard to space problems), drew 3 to 5 main branches in the style of mind mapping, constructed the complex to each main branch rather in the style of concept maps and described the represented relations only partly by linking words. Advantageously, they used the possibility to give examples also in giving example problem solutions. Regarding these facts it might be of benefit to optimize the rules for making knowledge maps, especially with regard to build structure in mathematics.

As knowledge mappings can be expected to be efficient tools for building structure, an enhanced usage of these methods in mathematics education should result.

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