1 Introduction

Is there really a dilemma between research and teaching (or the concern for the relationship between them)? In the dictionary we find that a dilemma is 'an argument based on two opposing suppositions leading to the same conclusion'. From this definition it follows that research and teaching (at any level) can be understood as distinct (and sometimes opposed) elements or ingredients of a process leading to a common, shared conclusion: improving the quality of life through improving teaching and learning.

However, the above is really at best nothing more than an optimistic interpretation of the definition. Certainly, there should exist a relationship between research and teaching, but all too often they tend to diverge. The reason for this lies, at least partially, with the members of each camp: sometimes, we researchers lose touch with current problems in teaching and with the teachers themselves, whilst on the other hand, teachers fail to give due credit to the research produced by the former. In the university sphere, where researcher and teacher are frequently the same person, one could even talk of a certain schizophrenia, in the sense that not only are the links between teaching and research issues broken on occasion, but that both areas come to be contradictory.

Research into Mathematics Teaching needs to maintain close links with teaching, given that its object of study is precisely such aspects as are related to the teaching and learning of mathematics. This is not to say that all research activity should display a direct, tangible relation with teaching and learning, nor that it should be of immediate use to teaching.

Teaching, the teacher, likewise needs these ties with research, as this offers a means of systematising approaches to professional problems, without which it would be difficult to make progress.

2 The research – teaching dilemma as opposed to the theory – practice dilemma

It is common to associate research with theory, and teaching with practice. Although such an association is not entirely correct, given that both research and teaching have their respective theory and practice, it is not a groundless supposition. Within the area of teacher-training, it is assumed that research should provide results applicable to teaching. However, the concept of applicability differs between researchers and teachers; in fact, it is precisely at this point that a distance between the two stances begins to emerge: whilst researchers complain of undue demands for immediacy from teachers, the latter are equally dissatisfied with the ambiguity in which proposals from the former seem to be couched, rendering them inadequate for solving their day-to-day problems. Research has its theory, in particular its fundamental framework, and its practice, primarily its methods and development which, in their turn, integrate theory.

Teaching has its theory, in particular general theories (of teaching and learning) and those related to the teaching and learning of the subject in question (relevant to the teaching of Mathematics), and its practice, made up by the day-to-day life in the classroom and the school, which throws up, it must be noted, innumerable occasions where decisions have to be taken.
To a certain extent, teachers view the work of the researcher as situated at the theoretical level, whilst theirs is on the practical level. This might be due to the fact that, on the one hand, researchers obtain results which in many cases refer to teaching which they do not experience, and on the other, teachers very often act without being aware of harbouring any kind of theory, but rather impelled by practice. This might well be a consequence of the lack of relevance of initial training programmes whereby primary and secondary school teachers forget decontextualised theories and set to work with good will, subject knowledge and a survival instinct – such is practice and theory stays put in the university!

It is important to make clear that researchers also tend to be teachers. However, as researchers usually teach at university where the theory of teaching overlaps considerably with subject content, there being no tradition of an applied element, the research – teaching dilemma tends to exclude university teaching. This article does likewise, assuming thus that it responds to the reader’s interest, although I will take the liberty of expressing some ideas with respect to the relation between university teaching and research in the final section.

In summary, the problem centres on the relation between what researchers do and what teachers (at non-university levels) do, or between both groups, which, as stated above, are apt to view things to their own benefit.

This, of course, is not the only educational dilemma facing teachers. Others include the relation Primary and Secondary education, the relation between the ‘Bachillerato’ (post 16) and university, and the relation between administrative proposals and the school reality. Nevertheless, the dilemma which concerns this paper has features which distinguish if from these others:

a) the vocation of educational research to bring solutions which in one way or another improve the professional life of teachers;

b) the role which the administration grants the teacher in managing the curriculum (though with ever-decreasing autonomy by virtue of the current Spanish Secondary Education reform); and

c) the benefit, put forward by the area of educational research (in particular in Mathematics education), of teachers interpreting their work as professional development, in which research exerts a significant influence over their practice.

This dilemma, then, is relevant to the current state of affairs, and it is up to us to make an effort to draft solutions or at least consider the means by which these two elements alluded to in the definition might be brought closer together so as to achieve a common objective.

3 An integrative framework: collaborative research

It is no easy matter to integrate the interests of research and learning, of researchers and teachers, as testified by the dissatisfaction mentioned above, which is often expressed by both groups.

Stenhouse (1975) proposed fusing curricular research and curricular development, proposing for the teacher the role of researcher and manager of the curriculum. This idea of education based on research, presenting the teacher as apprentice, is amplified by Zeichner (1993) who, following the argument of Schön (1983, 1987) regarding reflection and action in professional practice, considers teachers as reflective professionals, and includes the concepts of action-research, teacher researcher and teacher training. But these proposals, despite clearly defining what is required of the teacher (and that counts for much), fail to explore the teacher’s role in relation to the researcher, whose assistance could be crucial to meeting the ever increasing demands of the profession. In fact,
these proposals could be placed within the framework of educational innovation, both necessary and integral to a concept of the teacher as someone committed to their professional development, but they do not establish the relation between the role of teacher and researcher.

There are various ways of understanding professional development. For some, professional development occurs wherever there is a development of the teacher’s conceptions from traditional tendencies to others more investigative or critical in nature. For others, the most characteristic feature of professional development is the acquisition of the habit of reflection. In this respect, following Climent (2001), I understand professional development

'as the process of ongoing learning as a professional both reflective and critical of their practice ... It would be taking account of said practice and of the learning of the students, and the analysis of this (analysis linked to action) taking into consideration more and more elements and adapting all this to the learning of the particular students. This assumes a continuous questioning of their conceptions and knowledge, and the enriching and enlarging of both. We therefore make a difference between the 'natural' development any teacher undergoes (as a result of experience, irrespective of self-reflection) and the phases or cycles which can be perceived in this process from what we have called professional development, the intentioned growth of the teacher towards a profile of reflection - action.' (p.1, author's translation)

Figure 1 intends, amongst other things, to clarify in graphic form, the role of reflection in the professional development of the teacher, understanding this as the culmination of the process and as a process in itself. Of significance here is the work of Irwin and Britt (1999), which sets out the case of teachers participating in a professional development project and which notes the ability to reflect on one's teaching as a measure of development.

Figure 1 also captures the idea of Elliot (1991) of teachers as researchers into their own practice, continuing the philosophy of Stenhouse and his concept of action research as a form of professional learning. It also reflects the role of reflective practice of Schön.

The complexity of practice, the questioning of concepts and knowledge, the tendency towards a profile of reflection-action, are parameters of professional development in Climent’s model, which is related to the concept of action-research of Jaworski (1998), who associates action-research with reflective and critical practice, highlights ongoing action-research from that subjected to established patterns, and highlights the role of external researchers, which is in its turn relates to collaborative research.

Reflection, on the right of the diagram, and research, on the left, are related to each other in aiming for professional development.
This is the point at which we return to the dilemma of the relation between research and teaching. Collaborative research offers a framework in which is located the above-mentioned relation. It is not a matter of considering how teachers might utilise the research results, nor how researchers might establish contact with practice so as to maximise the relevance of their research. In collaborative research, teachers and researchers work together in a team whilst maintaining their particular interests, but sharing assumptions about the role of research in the professional development of each.

Within the framework of collaborative research there is an attempt to minimise the above mentioned feeling on the part of the teachers of the scant usefulness of the research results. Ellerton and Clements (1998), making a distinction between theory-driven research and theory-generating research, claim that the area of Mathematics education is not subject to the laws which are independent of the culture and that, as a consequence, a theory can both be considered applicable to all contexts. These two researchers are in tune with the habitual feelings of teachers, who mistrust

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the applicability of the researchers' theories. Teachers need research within or close to the critical paradigm in educational research, in which the object of theory is the formation of the character in the habits of reflection, together with practice forming an indivisible whole; theory is not abstracted from reality, but rather forms part of the action: practice is theory in action.

However, working within the framework of collaborative research, that is to say, developing research projects within its theoretical framework, tends to be difficult if we take the definition of Feldman (1993):

'an equitable collaboration is one in which the following are equally applied for all the actors: work with parity; assume equal responsibility for problem identification, data collection, analysis, and report writing; share the same set of goals; and assure that needs are met.' (p. 343)

Ensuring that teachers and researchers come to a piece of research on equal terms is sometimes somewhat utopian. However, this is no reason why teachers and researchers cannot benefit from the collaboration, each defending their objectives. Climent and Carrillo (in press, a) emphasise the extent to which a research project is collaborative and how the roles of teacher and researcher are defined over the course of the process, without losing its collaborative nature, without ceasing to be a true piece of research, and maintaining the determined aim of the teachers to enhance their professional development.

4 An example

It is pertinent here to mention certain features of some research carried out as part of the project Professional development through collaborative research on problem solving [1] which involved two primary school teachers as well as the researchers. In particular, I shall emphasise the learning process of one of the teachers, underlining the work of the sessions of the project.

So anxious were the teachers to improve their practice that they were led to seek the help of the trainers, not to provide formal training, but rather to enable them to set up their own research project focusing on their practice so as to compare it with other methods of teaching. In particular, they aimed to redirect their methodology towards problem solving and discovery learning on the part of the student. For their part, the researchers aimed to go deeper into the characteristics of professional development, as well as enter into the practical shaping of a process of collaborative research.

The data presented below correspond to one of the teacher's class sessions in the fourth year of primary school, which encouraged subsequent reflection in the project's sessions. Clearly only an extract can be considered here (for further details, see Climent and Carrillo, in press, a). The proposed task demands an organisation of work and a consideration and taking of decisions on the part of the students sufficient to classify it as a problematic situation for their level.

After having worked on the definition of a polygon with the children, and having established that the triangle is the polygon with the minimum possible number of sides, the teacher moved on to study different types of triangles, with the aim of classifying them according to their sides and angles. In order to do this, the teacher gave the students a grid paper and the following instructions for them to work in groups:

"Draw all the triangles you can, but make them all different, and not just one a bit bigger and one a bit smaller, but all completely different, taking into account the elements of a triangle, which are the vertices, the angles and the sides."

The beginning and end of the transcription of the two sessions concerning the activity are given below (the limitations of space require certain things to be taken for granted).
The teacher draws on a grid on the blackboard:

Mateo:  *Do we have to draw normal triangles or ...*

Teacher:  *This is normal, why isn't it normal? What's wrong with it that makes it abnormal?*

Mateo:  *Because the base is ...* (another child says: *It's sideways, isn't it?) ... *it's not straight.*

Teacher:  *And so we've not found yet another group, number three (three equal sides), nobody actually did that one, but we've seen you could do it.*

I shall first examine various aspects of the pupils' learning with the aim of later considering the teacher's learning:

- The pupils introduce irrelevant variables into their classifications. They talk about sides which are long, medium and short, which leads them to distinguish a larger number of groups into their classification. In this way a triangle with two equal sides and one longer side would be put into a different group from another with two equal sides and a third shorter side.

- A restrictive influence is noticeable from the most typical images of geometric shapes, difficulty in recognising shapes when they aren't in their standard position, especially when it comes to finding the right angle in a right-angled triangle, and an over-reliance on the horizontal-vertical planes. The pupils understand the concepts, but are unable to identify them from the drawings.

It hardly seems chance that the first right-angled triangle that the pupils recognise in the files is placed in the most usual position (sides along the horizontal and vertical axes). There seems to be a discontinuity between the image of the concept and its definition.

What does the teacher learn and how?

Problem solving (or open situations) raises questions which are absent from more closed situations. When the teacher subsequently pondered extraneous variable in the project sessions, she realised the irrelevance of position, the restrictions caused by the grid paper, and other aspects of teaching the topic.

In effect, the teacher used the terminology long-medium-short. (*In other words, this is a triangle that has one longer side and two equal sides. So let's see if we can find on the sheet another triangle which is the same, which has two sides the same and one, the one at the bottom, different.*) She initially reinforced the consideration for the length of one of the sides, in comparison with the others, as a variable to take into account for classifying them, although she then says that they should simply look for one side which is different from the other two. She would have done better to allow other variables of importance to the children into the classification. This would have opened up the discussion within the project about the use of the pupil's knowledge in the construction of school knowledge, as well as the role of that knowledge in the construction of scientific knowledge.

The teacher was aware of the irrelevancy of the position of the sides and of the difficulty the association between position and type of triangle caused the pupils when it came to classifying.
Nevertheless, in this unit we can see how, perhaps betrayed by her own dominant conceptual images, she reinforces the restrictive concept consisting of one unequal side of an isosceles triangle is the one aligned along the horizontal plane.

The remaining comments correspond to aspects of teaching linked to content. Amongst these stand out the comments relating to grids as teaching material and their use by the teacher. They can be grouped according to:

a) the impossibility of drawing equilateral triangles,
b) the potential of grids as an aid to learning.

With respect to a), the teacher demonstrated her lack of knowledge. It is important to recognise the necessity of having an adequate understanding of teaching materials, and not seeing these as elements which are neutral with respect to the motivation of the pupil, which by their mere presence ensure spontaneous learning. As a consequence of working on this project, the teacher became aware of the need to explore and investigate the material before putting it to use in the class, and was also convinced that it is quite likely that unexpected results might appear when putting it into practice.

On the other hand, the angle template aid was very positively valued, as was the triangle cut out of paper, which, when rotated, allowed the independence of the type of triangle and its position to be visualised, both of which ideas were suggested by the teacher herself and tried out in subsequent lessons. This knowledge, as part of her subject knowledge, is considered desirable by members of the project. In this way, a training is brought about which is not so much a kind of craft exercise of imitating an 'expert' teacher, so much as the critical incorporating of the aspects of practical knowledge. Throughout this process the development of metacognitive features are promoted through a continuous appeal to reflection, critical thinking, and questioning, the authentic intelligent GPS of our task.

In short, the richness of the reflections by members of the group are confirmed, emphasising the appropriateness of including activities such as this, in which the development of a primary class task is analysed, within in-service training.

5 Conclusion

As mentioned in the second section, I shall now allow myself to briefly note several ideas with respect to the relation between teaching and university research. Within the topic which concerns us, that of teacher training, it is now important to speak of initial training. In too many cases, an initial training usefulness is identified as an administrative requirement rather than as a period in which concepts, procedures and attitudes and values are forged, these being of use in carrying out professional activity. Paradoxically, those entrusted with this training are usually researchers and, despite serious limitations resulting from a minimal treatment of mathematics teaching on university programmes and from financial and organisational questions, we have to assume some kind of responsibility.

Russell (1994) criticises the disparity between the advances in educational research and the antiquated models of teacher training, and adds

'[one should extend] our visions of what teacher education could and should be, to increase the pressure for deliberate and thoughtful change in programs of teacher education.' (p. 206)

One of the changes in the philosophy of teacher training would be the consideration of continuous training, with initial and on-going periods, entailing a relation between experienced, probationary,
and future teachers. Such a relation could be achieved by means of greater integration between theory and practice from initial training:

Processes that aim to integrate theory and practice ... contribute to forming a connection between initial and in-service training. Moreover, they offer a frame of reference for future teachers to conduct research upon their own practice, thus establishing the basis of a desirable role as future professionals in the teaching field. (Carrillo and Coriat 1000 p. 123)

But besides doing research on their own practice, the university researcher-teacher can bring into their classes cases such as the one referred to in section 4, which are amenable to analysis from various angles (the pupils' learning, subject knowledge of the teacher, methodology and so on), thus making a link between research and teaching 2. In this way, the professional development of various practising teachers is related to educational research, and both are related to the training of future teachers: research, training and teaching constitute, in this manner, a triad which strengthens the assumption of roles and commitment by the researchers and teachers, amongst which must be mentioned the commitment of experienced teachers in the training of future teachers and probationers. 3

With respect to the example in section 4, this aims to highlight a model or framework of work, based on section 3, which overcomes the polarity of the research-teaching dilemmas, offering a context (shared projects) in which the links between researchers and teachers is a fact.

In like fashion, collaborative research can be extended to other levels of teaching. The focusing on Primary in the example helps one understand those aspects which have been the object of analysis, but by no means circumscribes this type of research to primary teachers alone (see, for example, Jaworski (1998), mentioned above, for the case of Secondary teachers).

Whatever the case, it would be necessary to carefully analyse to what extent a piece of research might be considered collaborative. Such an analysis could not be carried out on the basis of the definition of the aims of the project, but rather on the basis of the study of the features of the process, the role that each member assumes, and the characteristics of each person's task. However, more important than determining the degree of collaboration of the research is that the teachers and researchers view this context as empowering their capabilities, and as a meeting point for their interests, becoming, by this good fortune, real powerful environments for the professional development of both.

2 A proposal along these lines is presented in Climent and Carrillo (in press, b).

3 Teacher training at Secondary level deserves special mention. Here, the above proposals are, at the present time, scarcely viable, owing to the fact that the spirit of this training, from its administrative structure (in Spain), answers to the almost exclusive importance of the specific contents of the subject matter. The changes indicated by the law have yet to be finalised, although there are universities which are experimenting with it.

Some suggestions can be found in Carrillo and Coriat (1999).
References


Climent, N. & Carrillo, J (in press, b). Ejemplificación de una propuesta formativa: el uso de situaciones de primaria en la formación inicial. In L.C. Contreras & L. Blanco (Eds.) *Aportaciones a la formación inicial de maestros en el área de Matemáticas: una mirada a la práctica docente*.


