Addition and Subtraction Word Problems in Greek Grade A and Grade B Mathematics Textbooks: Distribution and Children's Understanding

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Abstract

Mathematics textbooks are a predominant resource in primary school in Greece, as well as in many other countries. The present study reports on both a content analysis of Greek mathematics textbooks with regard to the types of word problems represented in them and a quantitative analysis of children's achievement in these problems. For the purpose of the study, mathematics textbooks of the first two primary school years were analysed using van de Walle's model (1998) of addition and subtraction and a total of 80 children from Grade A and Grade B were asked to solve addition and subtraction problems of the eleven types as determined by the model. The data from the analysis of school mathematics textbooks indicated that there was an emphasis on certain types of problems and a minimal representation of others. Children from both grades were found to perform better on problems that were overrepresented in the school textbooks than on problems with little or no representation. Our findings raise awareness of problems that need to be addressed in mathematics textbooks when dealing with addition and subtraction in order to build children's complete understanding of additive situations. Implications of the role that mathematics textbooks may play on children's mathematical understanding are considered.

Keywords: mathematics textbooks, word problems, additive problems

Introduction

In primary school classrooms where mathematics textbooks are used, this can have a substantial impact on mathematics teaching and classroom instruction. They identify the topics to be taught in accordance to the curriculum and provide the order by which students should explore them. They organize teachers' teaching and students' learning of mathematics. So far, a number of studies have examined the use of mathematics textbooks by teachers (e.g., Haggarty & Peppin, 2002; Johansson, 2006; Remillard, 2005) and students (e.g., Rezat, 2009). In these studies teachers and students seem to agree that mathematics textbooks are used as their primary resource: textbooks are used as an element in a lesson that is dominated by teacher discourse, as a vehicle for the transmission of knowledge by teachers to students. This finding not only provides us with important information about the great use of mathematics textbooks, but also indicates the strong relation between mathematics textbooks and children's mathematics learning. In other words, it makes us wonder about 'adequate' ways of presenting the mathematical topics in textbooks and the kind of learning that may be implied. Moreover, the fact that mathematical topics that are not included in textbooks are most likely not presented by the teachers (Reys, Reys, Lapan, Holliday & Wasman, 2003) -and thus not learned by the students- makes the need for a closer look into mathematics textbooks more than necessary. Although a teacher can use the textbook flexibly, adding to what is offered in the textbook or skipping specific ideas or sections, it is most likely that the textbook defines the boundaries of what students may perceive. Indisputably, mathematics textbooks have a great impact on what is taught and what students learn in primary school.

The value that problem solving has on mathematical education is highlighted by curriculum developers as well as in reports. Engagement in problem solving activities is connected to concept development and gives opportunities to students to apply their knowledge to a range of situations that lead to useful mathematical knowledge. Muir, Beswick and Williamson (2008) describe the skills needed in order for success in problem solving to be achieved: interpreting information, planning and working methodically, checking results and trying alternative strategies. Although these skills are considered very important, lack of problem solving focus in activities is identified in today's classrooms (Rogers, 2004) with great emphasis mainly being put on exercises that deal with conventions instead of problems that improve logic. Both exercises and problems can be found in mathematics textbooks. Ideally, learning mathematics with the mathematics textbook comprises activities as applying just procedures in one hand and solving problems that require both analysis and numerical solution in the other. However, according to Atkinson (1992), this can be problematic as she believes that these focus primarily on exercises. She views that meaningful mathematics cannot be accomplished when using textbooks as 'maths with reason is rooted in action – learning through doing' (p.13).

Word or story problems are often connected to problem solving in school mathematics curricula. However, this conception of problem solving is limited, as there are word problems that do not create problematic situations for the students to address. For example, when children are asked to solve an addition word problem and the algorithm of addition is given, they can add the numbers involved and obtain a correct answer by just knowing the rules to follow without understanding the concept of addition and the problem situation. Hence, such a word problem is considered as an exercise for children to perform, with no particular benefits or opportunities for different solution strategies to appear. Research recommends that students should be engaged in a variety of truly problematic tasks so that mathematical sense making is practiced (Marcus & Fey, 2003). In this sense mathematics textbooks may offer the opportunity to develop mathematical literacy by encouraging students to adequately understand and apply mathematical concepts in word problems containing more or less realistic contexts.

In the light of problem solving activities that mathematics textbooks involve, great concerns may be raised about the actual implementation of word problems. Such concerns include the types and structures of word problems contained in the curriculum materials and the influence of the presence of such problems on students' ability to solve them. The present study focuses on the types of word problems that the mathematics textbooks involve and promote with regard to the topic of addition and subtraction. It also examines students' responses to varying problem types. Of particular importance to this study is the issue of the different types of addition and subtraction word problems proposed in the literature.

Riley, Greeno and Heller (1983) proposed that word problems requiring simple addition or subtraction operations can be grouped into different types according to: a) their semantic relationship describing the problem situation (combine, change and compare problems) and b) the position of the unknown quantity (first addend, second addend, total). Thus, Riley et al. (1983) suggested that there are a total of eighteen types of addition and subtraction word problems that cause great variation in difficulty for children. Given the critical role of semantic structure in problem solution, Carpenter and Moser (1984) postulated a similar classification that included change, combine, compare and equalize problems with the three first types being characteristic of most addition and subtraction word problems presented in elementary mathematics textbooks. Another classification of addition and subtraction word problems was also described by van de Walle (1998) who put emphasis on the role that word problems play in the development of number operations. According to him, change, combine and compare problems can further be divided into four types, namely join, separate, part-whole and compare problems, with each of these main types having two or three subtypes. Thus, eleven different meanings of addition and subtraction were proposed that show that addition and subtraction are connected.

A number of factors influence whether an addition or subtraction word problem is more or less difficult for a child to solve. These factors are related to issues such as whether the problem involves an action situation that the child can act out, whether a problem can be modeled in the order in which it is heard or whether the unknown quantity is placed at the end, middle or beginning of the problem (Carpenter, Fennema, Franke, Levi & Empson, 1999). Understanding these factors will help teachers decide on the word problems to be used.

Arithmetic operations are a topic central to the primary school mathematics curriculum. In case of Greece, basic work with the operations of addition and subtraction begins as early as Kindergarten and continues throughout the primary grades. Much of the work involving addition and subtraction occurs through the medium of word problems as they appear in mathematics textbooks. Particular types of word problems may affect children’s ability to understand the situations presented and the meaning of the operation (Greer, 1997). It might be the case that mathematics textbooks include only some problem types and thus children’s conceptual understanding of addition and subtraction is limited.

To investigate this perspective, Olkun and Toluk (2002) examined addition and subtraction word problems in primary school mathematics textbooks in Turkey using van de Walle’s model (1998). They found that addition and subtraction word problems were not used in textbooks in a systematic manner over grades: there were particular types of problems that were overrepresented, underrepresented or not represented at all. The fact that students were less successful on problem types that were underrepresented in textbooks made Olkun and Toluk argue that the emphasis on particular problem types in textbooks may hinder the development of student’s problem solving skills. Similar results were obtained by Parmjit and Teoh (2010) who analyzed the content of Malaysian Primary 1 and Primary 2 mathematics textbooks and measured top-class students’ success based on van de Walle’s model. Their findings revealed that the textbooks did not adequately distribute all types of addition and subtraction problems. They also found that most of the problems in the mathematics textbooks were those that children found easiest to solve suggesting a strong relationship between the distribution of problem types and children’s achievement. Both these studies suggest that excluding some types of addition word problems from textbooks could lead to children's incomplete understanding of the additive situations. The implication that is important to the present study is that children's difficulties may be due to limited exposure to certain types of problems in their school mathematics textbooks.

Based on previous findings, the present study represents an attempt to analyse the types of word problems in addition and subtraction in the first two years of primary school using van de Walle's (1998) model. Specifically, this study examines the following questions: a) What types of addition and subtraction word problems do mathematics textbooks of Grade A and Grade B involve? How these are distributed? b) Is there any connection between children's performance in problem types and the distribution of word problems in mathematics textbooks?

Method

A content analysis of the distribution of word problems related to addition and subtraction operations in Grade A and Grade B mathematics textbooks was conducted using the eleven categories of word problems proposed by van de Walle (1998). The methodological use of van de Walle's categories was decided because this framework was used by other studies (Olkun & Toluk, 2002; Parmjit & Teoh (2010) and would enable comparisons between results from different countries. In Greece, mathematics textbooks are approved by the Greek Education Ministry and are distributed free of charge to all students in state and private schools. Mathematics textbooks in Grade A consist of a total of two student's books and four workbooks, whereas in Grade B they consist of one student book and four workbooks. At first, all word problems that could be solved using addition and subtraction of natural numbers were included in the analysis. Problems that involved arithmetic expressions such as '6+2=?' which demand certain procedures and did not have any accompanying content were subsequently excluded from the analysis. Each of the authors independently read and categorized all the word problems from the textbooks into the given categories. The inter-rater reliability reached 92%. Those word problems that the two researchers were disagreed upon were dropped off.

Secondly, a quantitative analysis of Grade A and Grade B students' performance on addition problems was conducted. Eleven addition word problems were presented to the participants, all adapted from the eleven van de Walle's types of problems and devised by the researchers, with one representative problem for each type. As shown

Table 1

Addition and Subtraction Problems used in the study

Category	Type	Abbr.	Problems
Join Problems	Result Unknown	JRU	Maria had 8 pencils. George gave her 4 more. How many pencils does Maria have altogether?
	Change Unknown	JCU	Maria had 9 apples. George gave her some more. Now Maria has 15 apples. How many did George give her?
	Initial Unknown	JIU	Maria had some lollies. George gave her 5 more. Now Maria has 11 lollies. How many lollies did Maria have at first?
Separate Problems	Result Unknown	SRU	Maria had 12 flowers. She gave 5 flowers to George. How many flowers does Maria have now?
	Change Unknown	SCU	Maria has 15 fish. She gave some to George. Now Maria has 8 fish left. How many did she give to George?
	Initial Unknown	SIU	Maria had some biscuits. She gave 6 biscuits to George. Now Maria has 8 biscuits left. How many biscuits did Maria have at first?
Part-Whole Problems	Whole Unknown	PWWU	George has 4 blue pencils and 8 red pencils. How many pencils does he have altogether?
	Part Unknown	PWPU	Maria has 11 pencils. 4 of them are blue and the rest are red. How many red pencils does Maria have?
Compare Problems	Difference Unknown	CDU	George has 13 balloons and Maria has 2 balloons. How many more balloons does George have than Maria?
	Larger Unknown	CLU	Maria has 10 books. George has 4 books more than Maria. How many books does George have?
	Smaller Unknown	CSU	Maria has 15 sweets. George has 6 sweets fewer than Maria. How many sweets does George have?

Source: Adapted from van de Walle (1998) in Table 1, these problems differed in their semantic structure and were broadly classified into four distinct categories: joint, separate, part-whole and comparison problems. These broader categories referred to the types of relations involved in each problem. All problems were translated into Greek language and necessary changes regarding names and objects occurred with particular attention in the adaptation of these problems into young children's every day life and habits. Small numbers, whose sum did not exceed 15, were used in the problems in order to minimize children's difficulty with the arithmetical calculations.

Participants included a total of 80 students, 40 from Grade A (mean age 6 years and 4 months) and 40 from Grade B (mean age 7 years and 6 months) who were randomly selected from state schools in the city of Thessaloniki, in Greece. They were from predominantly middle socio-economic home environments and represented a range of school performance levels.

Participants were interviewed individually (Cohen, Manion & Morrison, 2007) by one researcher in a quiet area within their school. Paper and pencil were provided in case they wanted to use them and there was no time limitation for solving the problems. The research was carried out in the middle of the school year. No additional teaching focused on the different types of addition and subtraction word problems took place.

Results

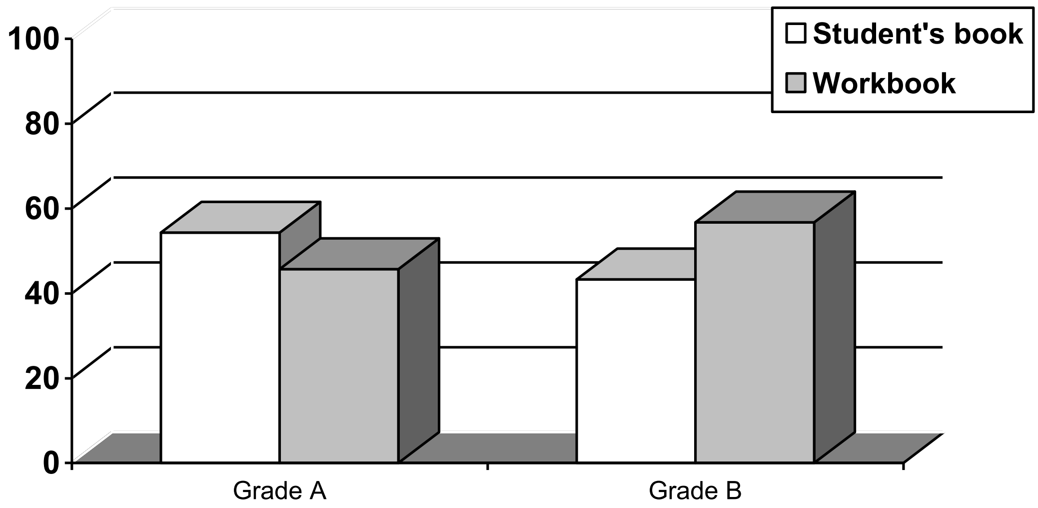
a. Grade A and Grade B Mathematics Textbooks

The analysis conducted in mathematics textbooks showed an overall increase in the number of addition and subtraction word problems from Grade A to Grade B textbooks: a total of 83 word problems were found in Grade A textbooks and a total of 111 word problems in Grade B textbooks. As shown in Figure 1, Grade A student's books consist of more word problems than workbooks, whereas the opposite is true for Grade B mathematics textbooks.

All the addition and subtraction word problems found in Grade A and Grade B textbooks were further classified into eleven categories based on van de Walle's model. The distribution of these problems in the categories reveals that there isn't an even spread of the problems between the two grades. In grade A, Join problems were the most frequently found problems (58% of all problems) with the JRU category being the most highly represented problems (47%). However, the rest of the categories are not evenly distributed. Separate problems - all of the SRU type - have

Figure 1

Distribution of addition and subtraction word problems in Grade A and Grade B mathematics student books and workbooks



high representation (24%), leaving the rest of the categories mostly unrepresented or not adequately represented gathering a total of about 20%. Surprisingly, there was no representation of JIU, SCU, SIU and CSU categories and a very low representation of CLU (one problem in workbook) and PPU categories (one problem in student's book). In Grade B mathematics textbooks the category with the highest representation is JRU (34%), followed by SRU (15%), CDU (12%), and SCU (9%). Categories JIU and CLU did not have any representation. Figures 2 and 3 indicate the percentage of the problems in Grade A and Grade B textbooks respectively.

b. Children's overall performance

Statistical significant differences were observed in children's overall performance between the two age groups ($F(1,79)=15,315$, $p<.001$), with the second graders being more successful (65%) than first graders (45%). This difference was found in all types of problems, as seen in Figure 4. Both first and second graders' correct responses were above 85% on problems of the JRU, PWWU and CLU categories. Low level of success in similar percentages that did not exceed 50% in both grades was observed in problems of the SCU, SIU, PWPU and CDU categories, showing that these were the most difficult problems for all participants. Interestingly, the operation of addition was involved in each of the categories for which the participants were more successful, whereas the operation of subtraction was needed in the categories with the

Figure 2

Distribution of addition and subtraction word problems in Grade A mathematics textbooks

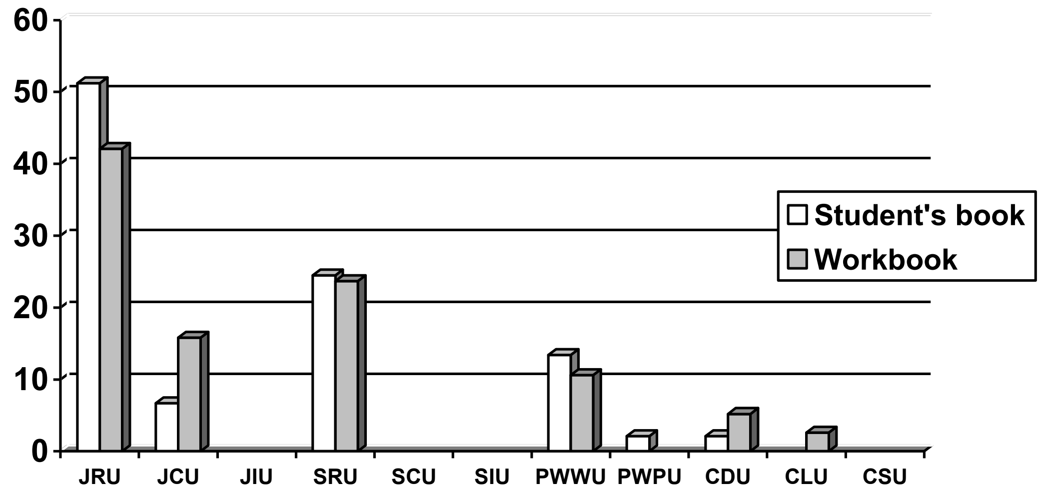
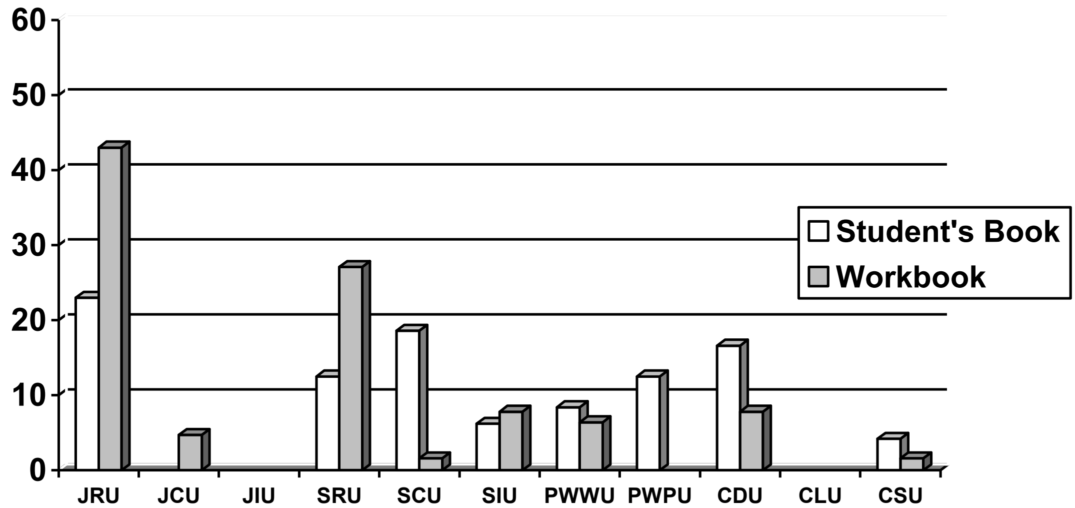


Figure 3

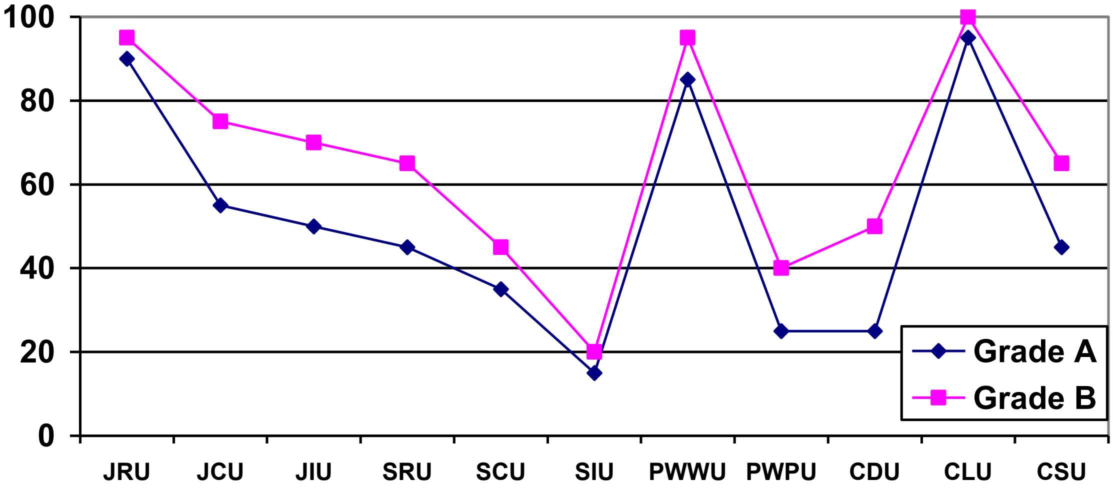
Distribution of addition and subtraction word problems in Grade B mathematics textbooks



average lowest. Generally, although scores obtained in Join and Separate problems were quite consistent, high variation in children's performance across the categories in Part-Whole and Comparison problems was found. This means that there wasn't a common difficulty in the categories of the same type: there was a category that was easy for children (e.g., PWWU) and another category of the same type (e.g., PWPU) that was very hard.

Figure 4

Percentage of correct responses in each category by grade



c. Addition word problems in Grade A and Grade B mathematics textbooks and children's performance

Correlation statistical analyses using Pearson R were carried out in order to examine the relation between the distribution of word problems in mathematics textbooks and children's performance in problem types. Interestingly, no statistically significant correlation existed neither between Grade A children's achievement and the distribution of the word problems across the categories in the Grade A mathematics textbooks (r=.503, p=.115) nor between Grade B children's scores and distribution of problem types in Grade B mathematics textbooks (r=.142, p=.676). This result may happen because there were certain categories (e.g. JRU, SRU) that were well represented or moderately represented in the textbooks and all children showed a very good performance in these problems (above 90% for JRU category). Similarly, children from both grades had a poor performance in SIU problems that were almost unrepresented in the textbooks. However, although the CLU category was under-represented (1,3%) in Grade A textbooks and unrepresented (0%) in Grade B textbooks, children reached ceiling performance of above 95% for both grades. Similar trends with low represented categories and high children's performance were observed with regards to JIU and CSU problems. Last, children's performance was quite low in CDU problems (25% and 50% for Grade A and Grade B children, respectively), although this category was adequately represented in the textbooks (3,65% and 12,2% for Grade A and Grade B textbooks, respectively).

A very strong statistically significant correlation was found between the distribution of the eleven categories in Grade A and Grade B mathematics textbooks (r=.890, p<.001). This correlation shows that there was a consistency in the appearance frequency of the categories between the two grades' mathematics textbooks: categories that appeared very often in Grade A textbook were also highly represented in Grade B textbook. Similarly, the underrepresented categories appeared in similar percentages in Grade A and Grade B textbooks.

Grade A student's performance was also highly correlated with Grade B student's performance with regards to the eleven categories (r=.965, p<.001). This correlation indicates that categories that were well answered by Grade A students tended to be well answered by Grade B students. Moreover, poor scores obtained by Grade A students in certain categories tended to be in accordance to Grade B students' low scores in the same categories.

Discussion

The purpose of this investigation was to focus on the connection between the types of word problems in addition and subtraction in Grade A and Grade B mathematics textbooks using van de Walle's (1998) model and children's achievement in these problems. This article was not meant to argue that mathematics textbooks are ineffective. On the contrary, our aim was to examine mathematical understanding as depicted in the textbooks and to explore how a significant content of different problem situations regarding addition and subtraction contributes to that understanding.

Results revealed an uneven spread of problem types according to van de Walle's classification in both Grade A and Grade B mathematics textbooks. This finding which supports the conclusions of Parmjit and Teoh (2010) and Olkun and Toluk (2003) is based primarily on differences in the representation of the eleven types of additive problems. More specifically, textbooks emphasize on problem types of JRU, SRU and PWWU categories. There is some representation in categories JCU, PWPU and CDU. However, there were no problems in category JIU in mathematics textbooks of both grades, whereas problems in categories SCU, SIU and CSU were totally unrepresented in Grade A. The overrepresentation of JRU problems were also found in Turkish mathematics textbooks (Olkun & Toluk, 2003), but not in Malaysian textbooks where SRU problems were mainly represented (Parmjit & Teoh, 2010). Irrespective of the small differences in the degree of problem types representation, the present study further supports previous findings about the implications that the over representation or the under representation of a particular problem type may have on children's understanding of additive situations. Children seem to be more often exposed to particular additive situations – mainly join problems – while others (e.g., separate and compare problems) are ignored. This may hinder them from mastering additive reasoning and from developing problem solving skills regarding to addition and subtraction.

The quantitative analysis of children's achievement revealed that children from both age groups obtained high scores on problems that were mainly represented in the school mathematics textbooks and low scores on those that were minimally represented. It is particularly interesting that the operation of addition was more easily identified and correctly applied compared to the operation of subtraction that posed difficulty to children. This finding may be explained by children's greater exposition to addition problems rather than subtraction problems, as noted earlier. The easiness of particular problem types that are more often presented to children is also suggested by Parmjit and Teoh (2010) and Olkun and Toluk (2003). Of course, we need to mention that we do not know what results Parmjit and Teoh (2010) would have obtained if they had offered the same items to mathematically less experienced students instead of students selected from the top classes. They may have found even worse results for problem types that were under represented in school textbooks. If this is the case, the need for bigger variation of problem types in the school mathematics textbooks is even greater.

However, children's level of performance varied over the study, sometimes considerably. Some children demonstrated high scores in particular problems of one type and poor scores in other problems of the same type. This variability might be an indication that some of the primary school children in our study are going through a period of developmental change in additive reasoning development. As Siegler (1997) suggests, young children often have multiple strategies available to them to solve a given problem. Over time, they seem to gather data that help them decide which of the available strategies works better in a particular problem situation. Developmental changes in young children's additive reasoning together with their great exposition to addition problems may explain for the unexpected finding concerning CLU problems: although CLU problems were underrepresented in Grade A textbooks and unrepresented in Grade B textbooks, children from both grades showed a very good performance of above 95%. Children's exciting performance may also be due to their limited exposure to subtraction problems that made them reveal –as found in the present study- better performance on addition problems compared to subtraction problems. Thus, more work on additive reasoning problems involving greater variation of problem situations should be presented to children.

Conclusions

Children's poor performance on some of the problem types appears to be due to the lack of experience with such situations. Participants in the present study tended to be less able to deal with situations that were less represented in their mathematical textbooks. The fact that children's good performance on problem types showed similar pattern as the distribution of the problem types in the textbooks further supports such a claim. Nevertheless, we cannot yet conclude that children's exposure to particular situations is the only thing that matters for their development of mathematical concepts of addition and subtraction. Although the analysis of mathematics textbooks revealed a variation in the representation of problem types and children's performance was in line with such a variation, this relationship could be affected by the sample size or the sample characteristics. Further larger studies are required to confirm these results. Additionally, the need to better understand what really happens in actual classroom teaching has to be acknowledged. As mathematics educators, we should be very interested in looking at what is in the textbooks with the focus on how the material is presented and used by children and teachers as well as the kind of learning that may be implied. It may be the case that textbooks shape the way we teach and learn mathematics in such a way that misconceptions are strengthened or even created (Kajander & Lovric, 2009).

Important educational implications can be drawn from these findings that provide strong evidence to support the effect that presentation of only some types of problems has on the quality of children's learning. By identifying and presenting word problems that increase learner's familiarization with several additive situations of different problem types, it is hoped that the quality of mathematical understanding will increase.

References

Atkinson, S. (1992). *Mathematics with reason. The emergent approach to primary maths.* London: Hodder and Stoughton.

Carpenter, T.P. & Moser, J.M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15,* 179-202.

Carpenter, T.P., Fennema, E., Franke, M.L., Levi L., & Empson, S.B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth: Heinemann.

Cohen, L., Manion L. & Morrison, K. (2007). *Research methods in education*. NY: Routledge.

Greer, B. (1997). Modelling reality in mathematics classrooms: the case of word problems. *Learning and Instruction, 7(4),* 293-307.

Haggarty, L. & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: who gets an opportunity to learn what? *British Educational Research Journal, 28(4),* 567-590.

Johansson, M. (2006). *Teaching mathematics with textbooks: A classroom and curricular perspective*. PhD thesis, Department of Mathematics, Lulea University of Technology, Sweden.

Kajander, A. & Lovric, M. (2009). Mathematics textbooks and their potential role in supporting misconceptions. *International Journal of Mathematical Education in Science and Technology, 40*(2), 173-181.

Marcus, R. & Fey, J.T. (2003). Selecting quality tasks for problem based teaching. In H.L.Shoen & R.I. Charles (Eds.), *Teaching mathematics through problem solving: Grades 6-12* (pp. 55-67). Reston, VA: NCTM.

Muir, T., Beswick, K. & Williamson, J. (2008). 'I'm not very good at solving problems'. An exploration of students' problem solving behaviours. *Journal of Mathematical Behavior, 27(3),* 228-241.

Olkun, S. & Toluk, Z. (2002). Textbooks, word problems and student success on addition and subtraction. *International Journal for Mathematics Teaching and Learning*. Retrieved on 20 June 2013. http://www.cimt.org.uk/journal

Parmjit, S. & Teoh, S.H. (2010). An analysis of addition and subtraction word problems in mathematics textbooks used in Malaysian primary school classrooms. *Brunei International Journal of Science and Mathematics Education, 2(1),* 68-85.

Reys, R., Reys, B., Lapan, R., Holliday, G. & Wasman, D. (2003). Assessing the impact of standards-based middle grades mathematics curriculum materials on student achievement. *Journal for Research in Mathematics Education, 34(1),* 74-95.

Remillard, J.T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Educational Research, 75(2),* 211-246.

Rezat, S. (2009). The utilization of mathematics textbooks as instruments for learning. *Proceedings of CERME 6, Working Group 7,* Lyon, France.

Riley, M.S., Greeno, J.G. & Heller, J.J. (1983). Development of children's problem solving ability in arithmetic. In H. P. Ginsburg (Ed.), *The development of mathematical thinking* (pp. 153-196). New York: Academic Press.

Rogers, J. (2004). Autonomy and mathematical problem-solving: The early years. *Education 3-13, 32(3),* 24-31.

Siegler, R.S. (1997). Beyond competence - Toward development. *Cognitive Development, 12*, 323-332.

van de Walle, J.A. (1998). *Elementary and middle school mathematics: Teaching developmentally*. New York: Longman.

Vergnaud, G. (1982). A classification of cognitive tasks and operations of thought involved in addition and subtraction problems. In T.P. Carpenter, J.M. Moser & T.A. Romberg (eds.), *Addition and subtraction: A cognitive perspective* (pp. 39-59). Hillsdale, NJ: Erlbaum.