Coming to ‘know’ Mathematics through being scaffolded to ‘talk and do’ Mathematics

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This paper adds to a discussion initiated by Askew (2007) about two contrasting views of scaffolding; as a ‘tool for results’ and a ‘tool-and-result’. The wider study the article is drawn from took place in four primary classrooms with Pasifika students within a low socioeconomic setting. Two classroom episodes drawn from one of the teacher’s classroom illustrate the two different perspectives of scaffolding. These are presented and the learning which evolves from each episode about how discussion and interaction affords mathematical learning are discussed. The paper illustrates that when scaffolding was used as a ‘tool for results’ the learning afforded was restricted. However, when the students were scaffolded within a tool-and-result perspective the mathematical knowledge and ways of doing and talking mathematics were generative.

Introduction

The use of the term ‘scaffolding’ has become commonplace when describing teaching and learning interactions in current mathematics classrooms. Most often users of the term draw on the literary work of Vygotsky (1978, 1986) and Bruner (1990, 1996) to use the term to describe learning which occurs as a result of social interactions between more and less knowledgeable individuals. In mathematics classrooms most often teachers are positioned as the more competent knower and students as the less competent knower. The students are therefore cast in the position of what Bruner described metaphorically as ‘scaffolding’ within a zone of proximal development (1996). Within this view of scaffolding, teachers as the more knowledgeable ‘other’ builds progressively higher platforms of mathematical knowledge and skills which scaffold the students to access more advanced mathematics. However, Askew (2007) in a recent PME paper challenged the appropriateness of this description of scaffolding. He suggested that in previous research (Bliss, Askew & Macrae, 1996) in English mathematics classrooms Askew and his colleagues found a paucity of scaffolded learning, and what was described as scaffolding more closely resembled teachers explaining or showing the students mathematical skills. Askew attributed this finding to the way differing views were held of “the nature of what was to be learnt” (p. 2-33) in the mathematical activity. In this article, this theme initiated by Askew is used to explore in two separate episodes what happens in classroom interactions, according to how the teacher views the nature of what is to be learnt. Specific attention will be given to the way in which the teacher scaffolds the students to engage in the interactions and discourse in the classroom.

Askew (2008) outlined how theorists like Lave and Wenger (1991) and Rogoff (1990) used scaffolded learning to describe learning as making available to the learner clear concrete outcomes of the learning (for example, a tailor knows the outcome of learning will be to sew garments). He suggested that scaffolded learning in this form can be described as a ‘tool for results’. But learning of mathematics contrasts sharply with the mediated
learning described by Lave and Wenger and Rogoff because the outcomes for the most part are not knowable in advance until after learning has occurred (for example, understanding about multiplication or fractions comes after the learning of them). This positions the learning process to be as much a part of the learning as is the outcomes of the learning. So, rather than describing the mediated learning in mathematics as a ‘tool for result’ Askew draws on the work of Newman and Holtzman (1997) to contend that a better description for the scaffolding in a mathematical context might be a ‘tool and result’. The descriptor ‘tool and result’ acknowledges the difficulties encountered in separating the outcomes from the learning process in mathematics. There are many articles on scaffolding used as a mediating tool set within the view of it as a ‘tool for results’ in the wider literature including that of mathematics. However, there appear to be few articles which have explored the view of scaffolding with a view of it as ‘tool-and-result’ within the dimension of mathematics education.

**Conceptual Frame**

According to Vygotskian thinking, conceptual reasoning developed in mathematics classrooms is a result of interaction between everyday spontaneous concepts and scientific concepts. Scientific concepts involve higher order thinking, which are used as students engage in more proficient forms of ‘doing and talking’ mathematics. Vygotsky (1986) maintained that “the process of acquiring scientific concepts reaches far beyond the immediate experience of the child” (p. 161). Although his work was not within the schooling system he suggested that school was the cultural medium, with dialogue the tool which mediated transformation of everyday spontaneous concepts to scientific concepts. Vygotsky’s suggestion was not, however, that scientific concepts are separate from spontaneous concepts, nor the act or practice of their development separate from their result. Rather, Vygotsky argued that they were an integral part of both the process and the outcomes. Askew (2007) illustrates Vygotskian theorising in his professional development work with teachers and students. In this study he persuasively illustrated that the performance and the creation of mathematical objectives is as much a priority for learning, as is the knowledge learnt. Through the construction of a learning environment in which students were both encouraged and required to talk mathematically, Askew illustrated how the immediate importance of the lesson learning outcome gave way to the bigger priority—that the students learnt to talk and act as mathematicians. Furthermore, through the specific scaffolding they received they learnt that they had the choice to continue to talk and act as mathematicians.

Whilst the exact nature of how external articulation becomes thought has been extensively debated (Sawyer, 2006) sociocultural theorists are united in their belief that collaboration and conversation, is crucial to the transformation of external communication to internal thought. They suggest that this occurs as students and teachers interact in co-constructed zones of proximal development. The zone of proximal development has been widely interpreted as a region of achievement between what can be realised by individuals acting alone and what can be realised in partnership with others (Goos, Galbraith, & Renshaw, 1999). Traditional applications of zones of proximal development were used primarily to consider and explain how novices are scaffolded by experts in mathematical activity. Taking the view Askew (2007) proposes—the tool-and-results perspective—widens the frame and supports ways to consider the learning which occurs when levels of competence are more evenly distributed across the members of the zone of proximal
Mathematical learning in this form occurs during mutual engagement in collective reasoning discourse and activity (Mercer, 2000). Lerman (2001) describes collective participation in mathematical discourse and reasoning practices as pulling all participants forward into their zones of proximal development which he terms a symbolic space—“an ever-emergent phenomenon triggered, where it happens, by the participants catching each other’s activity” (p. 103).

Defining the zone of proximal development as a symbolic space provides a useful means to explain how participants in classrooms mutually appropriate each others’ actions and goals. In doing so, they are required to mutually engage and inquire into the perspectives taken by other participants. In such scaffolded learning environments teachers, too, are pulled into the zone of proximal development and are required to understand from the perspective of their students, their reasoning and attitudes (Goos, 2004). Mercer (2000) termed this process of inquiry into each other’s reasoning “interthinking” (p. 141). During interthinking Mercer outlined how the variable contributions of participants create a need for continual renegotiation, and reconstitution of the zone of proximal development. In extended discourse the contributions are critiqued, refined, extended, challenged, synthesised and integrated within a collective view.

The construct of interthinking—pulling participants into a shared communicative space—extends the view of scaffolding and the zone of proximal development. It supports consideration of the learning potential for pairs or groups of students working together with others of similar levels of expertise in egalitarian relationships (Goos, 2004; Goos et al., 1999). The partial knowledge and skills that group members contribute, support collective understanding. Opportunities are also provided for the group to encounter mathematical situations which involve erroneous thinking, doubt, confusion and uncertainty. Importantly, constructing a collective view is not always premised immediately on consensus. Dissension can also be a catalyst for progress either during, or after, a collaborative session (Mercer, 2000) and to reach consensus, negotiation requires participants to engage in exploration and speculation of mathematical reasoning—an activity which approximates the actual practices of mathematicians. Such scaffolded activity inducts students into more disciplined reasoning practices. The “lived culture of the classroom becomes in itself, a challenge for students to move beyond their established competencies” (Goos et al., p. 97) to become more autonomous participants in mathematical activity and talk.

The New Zealand context

Mathematical reform has been a common feature in many Western countries in recent times where the trend is to institute reform measures with a focus on improving numeracy achievement (Young-Loveridge, 2009). New Zealand has followed the international trend and at the start of the 21st century instituted a New Zealand Numeracy Development Project (Ministry of Education, 2004a). The New Zealand Numeracy Development Project (Ministry of Education) had as a central aim to develop teacher knowledge of student knowledge and strategy levels. Although the mathematical reform was designed primarily as professional development for teachers it contained specified and detailed learning outcomes and activity for teachers as they taught the mathematics in the project. Similar to international models the material had predetermined learning outcomes and a detailed script for teachers to use to develop student knowledge of a range of different strategies. Implicitly suggested within the model was the notion of teachers using scaffolding as a
mediating tool which would give results—student acquisition of mathematical knowledge and strategies through teacher led instruction.

There are approximately 760,000 students in New Zealand schools of New Zealand European (59%), Maori (22%) and Pacific Nations (9%) heritage. Many of the Maori and Pasifika students are English as an additional language, speakers and in their homes speak their own language. Mathematics achievement varies within each group but the New Zealand European students generally achieve above their Maori and Pacific Nations counterparts. The majority of the Maori and Pasifika students also attend high poverty schools. Despite the recent large-scale implementation of New Zealand Numeracy Development Project (see Higgins & Parsons, 2009), we continue to record significant levels of underachievement for these students who are from marginalized backgrounds. This underachieving group include a large percentage of Pasifika and Maori students (Young-Loveridge, 2009). For our part, in looking to redress the inequitable opportunities afforded Pasifika students in high poverty schools (Ferguson, Gorinski, Wendar-Samu, & Mara, 2008), we collaborated with a group of teachers of Maori, Pasifika and European backgrounds to explore how to enact inquiry teaching and learning practices that support students’ development of mathematical proficiency.

**Research design**

This paper reports on episodes drawn from a larger classroom-based design research study (Hunter, 2007a). The larger study was conducted at a New Zealand urban primary school and involved four teachers and 120 Year 4-8 students (8-11 Year olds). The students were from low socio-economic backgrounds and were pre-dominantly of Pasifika or New Zealand Maori ethnic origin. At the start of the larger study this group of students were predominantly at lower levels of achievement. The teachers were all expert teachers with at least five years of teaching experience. They had completed the professional development programme in the New Zealand Numeracy Development Project (Ministry of Education, 2004a). The teachers reported at the start of the study that their students had poor mathematical achievement levels. They also considered that asking their Maori and Pasifika students to explain their reasoning, or challenge the reasoning of others’ had considerable difficulties both socially and culturally for this grouping of students. However, at the conclusion of the study they were all united in their belief that all students can talk and interact in mathematically appropriate ways.

A year-long partnership between the researcher and the teachers using a design research approach supported the design and use of a ‘Participation and Communication Framework’ and a ‘Framework of Questions and Prompts’. The ‘Participation and Communication Framework’ was designed as an organising tool to assist the teachers to scaffold students’ use of proficient mathematical practices within reasoned inquiry and argumentation. The ‘Framework of Questions and Prompts’ was a tool co-constructed during the study to deepen student questioning and inquiry. More detail of the Frameworks and how these scaffolded teacher change and student change can be found in recent publications (Hunter, 2007a, 2007b, 2008). Student development of mathematical practices is not the focus of this paper but how they were inducted into the discourse of inquiry and argumentation influenced how they were scaffolded into, and interacted in zones of proximal development. For this article, two separate mathematics lessons were selected to analyse and discuss in relation to how scaffolding used by one of the four teachers influenced how
the students engaged in ‘talking and doing’ mathematics. The first excerpt was selected as representative of those which occurred at the beginning of the study and the second excerpt occurred towards the completion of the study when the use of exploratory talk had become a consistent feature of the mathematical talk the students used.

Within the wider study, data were collected from multiple sources. These included field notes, classroom artifacts, teacher interviews, and video and digital photo records of mathematics lessons (with focus on teachers establishing small group and large group mathematical activity). The analysis presented here draws primarily on the audio and video recorded classroom episodes of the teacher and students interacting during problem solving group work. Data analysis drew on qualitative methods. In the first phase open coding (Glaser & Strauss, 1967) was used on the observation transcriptions to look for emerging themes in relation to the area of inquiry informed by our conceptual framework. The following table provides an example of the initial coding.

**Table 1: Examples of the codes**

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
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<tbody>
<tr>
<td>Teacher tells students to ask for an explanation which is different.</td>
<td></td>
</tr>
<tr>
<td>Teacher asks group members to support each other in explanation.</td>
<td></td>
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<tr>
<td>Teacher asks for collective support when student states lack of understanding.</td>
<td></td>
</tr>
<tr>
<td>Teacher asks students to predict questions related to strategy solution.</td>
<td></td>
</tr>
<tr>
<td>Student expects support when stating lack of understanding or asking for help.</td>
<td></td>
</tr>
<tr>
<td>Student adds ideas or questions that advance collective thinking.</td>
<td></td>
</tr>
<tr>
<td>Student tells group members that group work involves team work.</td>
<td></td>
</tr>
<tr>
<td>Student makes sure that everyone else understands strategy solution.</td>
<td></td>
</tr>
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This analysis was conducted concurrently with analysis of the field notes of classroom episodes. A concurrent analysis of both sources of data provided the opportunity to compare and refine emerging patterns and themes in the video and audio records against shifts in the social and sociomathematical norms of the classroom. Connecting the themes supported interpretation and the building of narrative of teacher and student responses to learning and doing mathematics as it was enacted in the classroom over the year.

**Results**

In the initial stages of the study the four teachers in the larger research study closely adhered to the structured lessons provided in the New Zealand Numeracy Development Project material (Ministry of Education, 2004a). The scripted lessons were often followed word for word from the curriculum material provided by the developers. In this section I illustrate what happens when the scripted lessons from the New Zealand Numeracy Development Project scaffold what the teacher says and does. Clearly illustrated is how the lesson outline as enacted by the teacher provides the students with scaffolding, and in which its goal leads it to become a ‘tool for results’.

**Scaffolding as a ‘tool for results’**

The teacher began the lesson by stating a learning intention that signalled what he expected the outcome of the lesson on fractions should be for the students. He begins by reading:
Teacher: So what we are doing today is that we are learning to find fractions of a set.

He continues reading the script (See Ministry of Education, 2004b, p. 7).

Teacher: Here is a farm [as he reads from the text he draws a line across a rectangle representing two fields on a piece of paper]. The farmer uses an electric fence to make her farm into two paddocks. She has ten animals.

He stops and directs one student to use plastic animals to represent the animals in the problem:

Teacher: Hinemoa you count out ten of those.

Hinemoa counts out one by one the ten animals as the teacher continues to read the problem to the children:

Teacher: She wants to put one-half of the animals in one paddock and one-half in the other. How many animals do you think will be in each paddock?

A student interjects and is ignored by the teacher and all the other students:

Jo: I already know five because five and five are ten.

Without acknowledging Jo’s interjection the teacher directs the students to work together in pre-set groups of three with the animals:

Teacher: We all need to take ten animals and share them into the two paddocks in our groups. You need to turn to your partners because you are working together in your groups of three and talk about what you are doing.

The students turn and in their groups take the ten animals which they count out one by one in unison into the two groups as directed. The only talk they do is counting out the animals one by one, then counting the two sets and agreeing that there are five animals in each set. They each take a turn to do this. The teacher watches and when he observes that all the groups have completed the task he returns to reading from the script:

Teacher: Could we have worked out the number of animals in each paddock without sharing them out?

Jenny: Yes we could say five plus five is ten.

The teacher picks up a group of five animals in each hand and shows them to the students as he says:

Teacher: Yes you can use your doubles and say five and five.

He then continues the lesson posing similar problems and directing the students to use materials. The students always follow the set pattern established initially where they count out the animals one by one and then share them out in sets in order to find the answer to the problem. Each time the groups complete each problem he asks one student from a group to explain and represent with the animals what they did. Student response is always modelled on the first example the teacher enacted with Hinemoa:

Hone: We had fourteen bears so one, two, three, four, five, six, seven, eight, nine, ten, eleven, twelve, thirteen, fourteen so seven and seven are fourteen.

The lesson is quiet and orderly and the students respond almost as if they are performing, acting out a game of turn-taking, counting out the animals one by one, then sharing them into sets and saying the matching script related to double addition modelled
by their teacher. The teacher has followed the script closely. At the conclusion of this lesson the teacher reiterates the goal of the lesson as he tells the students that they now know how to find a fraction of a set. In a follow-up discussion with the researcher the teacher states that he is confident that this group of students understood the concept of adding using doubles and are now ready to move to the next lesson outlined in the NZ Numeracy Project curriculum material.

Considering this lesson it appears that both sets of individuals, the teacher and the students, have been scaffolded to play specific roles. The teacher uses the script to play out a role in which he can address a specific and narrow learning outcome which he has detailed at the start of the lesson. The role he takes is to show and tell as he takes what he perceives to be his teacher ‘role’. The students in turn adopt the role he casts them in, and play out their student ‘role’ to acquire the specific piece of mathematical knowledge. They are ‘talking and doing’ mathematics. However, a question which needs to be asked is what knowledge of themselves as mathematical users and doers are they developing?

In the following section I contrast this lesson excerpt which occurred in the first week of the study with a later mathematics lesson which occurred towards the end of the study. As outlined in the Methods Section extensive scaffolding for the students had been provided by the teacher through use of a Communication and Participation Framework and Framework of Questions and Prompts. The teacher had guided the students into the use of a range of proficient mathematical practices. These included using reasoned mathematical explanations, justification, representations, and generalisations within mathematical argumentation. The students had also been supported to develop a repertoire of questions and prompts to use to inquire into their own sense-making and the sense-making of others during classroom mathematical discussions. In relationship to the New Zealand Numeracy Development Project (Ministry of Education, 2004) the teacher continued to foreground the intent of the project; that the students would develop a range of informal strategies and he continued to draw on the curriculum material to provide guidance for his lessons. However, he no longer followed the script outlined in the materials for each lesson, instead he used the materials as a guide but he wrote contextual problems which better matched the interests of his students.

**Scaffolding as a ‘tool and results’**

In this lesson the teacher’s goal is for the students to explore the strategy of partitioning but he has selected numbers which support a contextual problem and emergence of multiple ways of reasoning towards a solution strategy. The lesson consists of two components; small group problem solving where the students work in groups of three or four students and then a class discussion of the different groups’ strategies. The following excerpt describes the first section of the lesson in which a group of three students have been given a problem and asked to discuss and develop a number of solution strategies. Each student has had individual time to think about and develop their own strategy.

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1 Bart Simpson had five different coloured marbles. He had 756 of each colour and Lisa wants to know how many he has altogether. Can you help him tell Lisa how many he has? Lisa might challenge him to prove he has more than her so can you work out some different strategies he could use?
solution. Then they have begun discussing the different ideas they have developed. Hine is recording across a large sheet of paper as the different individuals speak:

Saawan: What about five times 700 and then…

Hine: Five times fifty, and then five times six.

Sonny has been listening and watching closely what Hine is representing and as he analyses what he recorded in his individually developed strategy he notes:

Sonny: Hey mine’s the same but mine’s starting from the six, fifty, and then seven hundred. Hey all our ways are the same, well kind of, because you can start both ways.

Sonny has introduced the use of the distributive property to explain his reasoning. Saawan looks at both representations and starts to explore whether the concept can be generalised:

Saawan: Well let’s see if that right…so you say we can start both ways, yeah that’s cool it works.

The three students work together, interthinking about the problem and how solution strategies can be constructed using the distributive property. They continue to discuss and explore whether the order of how the factors are distributed affects the solution as they record have Hine record them on the sheet in the different ways. Then, as Sonny studies the recordings he introduces the group to an alternative idea. This strategy is one which draws on distributing the factor of five rather than the factor of 756:

Sonny: I have just thought and I know another way. Can you do seven hundred and fifty six times two and then plus it so the times two becomes…becomes times four…equals…

Sonny is playing with the idea of the generalisation the group have collectively constructed. He introduces it as he thinks out loud and Saawan’s answer indicates that although he has not yet made sense of what Sonny is saying he is open to the new contribution but needs to see it as a representation.

Saawan: What? Let’s write it down.

Sonny shows that his thinking is still being formed when Hine records it vertically as 756 + 756. As he watches he tells her:

Sonny: No times two is easier.

Saawan follows Sonny’s reasoning closely and his argument indicates that he is linking back to their previous reasoning. He then extends his reasoning and that of his peers when he argues that multiplication is repeated addition:

Saawan: Times two yeah but doing it that way is the same way really, you can say it as a plus because that’s the same as times like before when we went the other two ways not just one way.

Hine, listening to the exchange crosses out the recording, replacing it with 756 x 2. Then Sonny continues with the new thinking as Hine and Saawan track closely and examine and analyse the reasoning section by section:

Sonny: Seven hundred and fifty six times two equals one thousand six hundred and twelve…

Hine: Wait, one thousand… [Lapses into silence as she records 700 x 2 then writes 50 x 2 and 6 x 2].
All three students examine the recordings and check the total. Then Saawan takes the pen, from Hine and he records 1512 x 2 as he continues to explain:

Sawaan: And then we times, no we add them together then times it by two and add seven hundred and fifty six on to it [Records 3780].

Sonny and Hine are actively watching as Saawan records each step and then Hine says:

Hine: But hang on how did we get that?

Sonny responds using the recording as a representation to explain each step of the mathematical explanation:

Sonny: [Points at each piece of the recording as he explains] By timsing this by two, and this by two, and then adding.

Hine: [Nods her head] Yeah I get it now.

The teacher has been silently listening and observing the interaction. Now he notices Hine’s facial expression which indicates her continuing uncertainty and so he prompts her to question, emphasising that she needs to do so until she has complete understanding:

Teacher: You look like you are still a bit puzzled. Look at what he has explained and if you need to, ask more questions. Make sure you are convinced that it works. Think about a good question and ask it.

Hine studies the recording carefully and then she asks:

Hine: Why did you times one thousand five hundred and twelve by two?

Saawan: Because it’s like…because then when we times that by two [he points at the second two] it is like that will be like four and then we only have to add seven hundred and fifty six. It’s just doubling.

The teacher’s prompt for further questioning has left the mathematical agency with the students. After closely listening to the student provided explanation he presses them to further explore and extend the reasoning related to multiplication as repeated addition:

Teacher: By adding this [He points at + 756] what’s another way of saying that because I think maybe that…how could you say it differently instead of saying adding seven hundred and fifty six?

This prompts Sonny and Hine to link their understandings to the reasoning Saawan introduced that multiplication is repeated addition. This has now become a part of the common knowledge within their collective understandings:

Sonny: You could multiply it by one…

Hine extends it further and clearly illustrates that she understands that not only can seven hundred and fifty be partitioned and then distributed, but also five as a factor can be broken into smaller factors which also can be distributed:

Hine: Okay, I get it now so multiply by one yeah so when we times two, times two, times one because the whole thing is seven hundred and fifty six times five, so times five yeah, [she laughs then refers to the context of the problem] huh that’s a good one Lisa better understand from Bart.

In this second lesson scaffolding took a different form from that reported in the first lesson excerpt. Clearly illustrated in this lesson excerpt is that scaffolding had become a
tool which mediated the mutual engagement of all participants in the collective reasoning. The use of problem solving groups where mathematical expertise was more evenly distributed across the members changed their interactions. This resulted in each individual’s role emerging and changing minute by minute in the discussion, as they were pulled into a shared communicative space. The different contributions resulted in the group being extended beyond their own capabilities to develop collective reasoning grounded in the discipline of mathematics. Importantly, the mathematical understandings they were developing was of equal importance to what they were learning about acting as mathematicians and ‘talking and doing’ mathematics.

**Discussion and conclusion**

This paper sought to explore and examine scaffolding used in two different ways in two classroom episodes and the mathematical learning which emerged as a result. What was clearly illustrated in the paper was that when scaffolding is used as a tightly controlled tool within what Askew (2007) describes as a “technical-rationalist view of teaching and learning” (p. 2.39) the roles the teacher and the students hold and the mathematical talk they use are limited. In this context the teacher cast himself in the role of expert and his role became one of ‘shower’ and ‘teller’ rather than scaffolder. His actions matched those described by Lave and Wenger (1991) and Rogoff (1990) in which he had clear concrete outcomes and these controlled the direction of the lesson. As a result the students’ learning to ‘talk and do’ mathematics in ways mathematicians do were restricted. In contrast, the second lesson episode shows that when scaffolding was used within a widened dimension that affirmed both the importance of the construction of mathematical knowledge and the manner in which it was constructed the learning potential for all participants was enhanced. Although the teacher had a key focus on the students developing solution strategies using partitioning they were able to extend their understandings towards the distributive property in rich and meaningful ways.

This paper confirms the results in Askew’s (2007) PME paper but extends what Askew illustrated to show the learning potential available when teachers scaffold primary aged students to work together to construct a collective mathematical view within zones of proximal development. Clearly evident is the way in which students, in the act of discussing solution strategies used collaborative conversations to construct and reconstruct their reasoning. As other researchers (eg., Goos, 2004; Goos et al., 1999; Lerman, 2001; Mercer, 2000) have illustrated, the act of interthinking and developing a collective view was a key factor in how these students learnt to talk and do mathematics in a form in which they were agents in their own learning. In the first excerpt the zone of proximal development more closely resembled a traditional view in which the students were led by ‘a more expert other’ and opportunities to contribute to the conversation were reduced. The teacher led actions supported the concept of scaffolding as a tool for result (Askew, 2009). However, in the second lesson notions of the zone of proximal development are widened to encompass considering it as a tool and result. The more evenly distributed levels of expertise caused all participants to be drawn into what Lerman termed a symbolic space. Within this space the participants were able to appropriate each others’ mathematical thinking and reasoning, and develop and extend this further to the benefit of all. This resulted in the students developing the key mathematical understandings the teacher had set as a lesson goal but also they were afforded opportunities to renegotiate and reconstruct their reasoning in ways which parallel the work of mathematicians.
In the second lesson of importance also, was the careful teacher preparation which drew on the New Zealand Numeracy Project as a tool for classroom activities rather than a rigidly followed formula. The use of grouping and the careful selection of numbers allowed the lesson to unfold and provided the teacher with opportunities to press the students to extend their thinking. Through this the students were able to improvise and play with the numbers, in a form of mathematics which was generative.

Implications of this study suggest the need for mathematics educators to consider not only the importance of the development of mathematical knowledge but also how it is constructed through collaborative and discursive talk. In this form scaffolding needs to be metaphorically viewed as a ‘tool-and-result’ and teachers need to not only address how they teach but also how their students learn. National projects such as the New Zealand Numeracy Project (Ministry of Education, 2004) have an important place as a professional development tool but teachers need to develop their own script rather than use these rigidly.

References


