EXPERIENCES OF STUDENT MATHEMATICS-TEACHERS IN COMPUTER-BASED MATHEMATICS LEARNING ENVIRONMENT

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Abstract

Computer technology in mathematics education enabled the students to find many opportunities for investigating mathematical relationships, hypothesizing, and making generalizations. These opportunities were provided to pre-service teachers through a faculty course. At the end of the course, the teachers were assigned project tasks involving investigation and discovery activities. This paper presents the mathematical thinking processes and pre-service teachers’ experiences in the course of investigation and discovery activities. The pre-service teachers claimed that they learned new mathematical concepts, as well as how to apply their mathematical knowledge. Finally, some useful examples of the mathematical processes experienced by the students during their project work are presented.

Key Words: mathematics education, computer-based mathematics learning, teacher education
Introduction

The advent of computers in the realm of mathematics education has brought with it optimistic expectations of adding new dimensions to mathematics education. With this new technology, the students are expected to abandon their traditional paper-pencil practices and discover mathematics on their own. However, most of these expectations did not come true. But important opportunities did emerge on how computers can be used in enabling students to learn mathematics better (Baki, 2001; Güven and Karataş, 2003). Given an opportunity to investigate as mathematicians, the technology would promote the development of students' thinking skills (Baki, 2001). Thus, when a computer is employed as a tool, it helps them to hypothesize, test their hypotheses, and derive generalizations. In so doing, the students may reflect on mathematical results, apart from taking the same steps as those of a mathematician in reaching mathematical results and developing unique thinking styles. In short, computer technology may promote the development of students' higher-level cognitive skills. The rapid advancement of computer technology in mathematics classes led to the development of Dynamic Geometry Software (DGS) and Computer Algebra Systems (CAS) which hold promise for improving mathematics education in terms of the objectives mentioned here.

The Role of Technology in Teaching and Learning of Mathematics

Technology is an essential tool for teaching and learning mathematics effectively. Computer technology in mathematics education is effective in that it focuses on higher-order cognitive skills, such as investigating, reasoning, hypothesizing, and making generalizations (Wiest, 2000). Wertheimer (1990) contends that technology motivates students to become more interested in exploring, investigating, conjecturing, creating, and discovering principles and
making generalizations; it helps students in becoming mathematical problem-solvers, and in enhancing their conceptual understanding of geometry. He further observes that the use of technology cannot replace conceptual understanding, computational fluency, or problem-solving skills. Yousef (1997) emphasizes that computer technology contributes to students’ cognitive development by allowing them to investigate, discover, hypothesize, and think creatively, besides solving problems based on mathematical models. The strategic use of technology enhances mathematics teaching and learning. Teachers must be prepared to serve as knowledgeable decision-makers in determining when and how their students can use these tools most effectively (NCTM, 2000). Therefore, the teachers may develop this skill when they learn how to use technology during preservice education. The main purpose of using technology in teacher education is to enhance teacher’s effectiveness and improve student’s learning. Thus, appropriate training of teachers is very important. Trainee teachers not only learn how to use technology, but also how to incorporate technology into their method of teaching. Programs in teacher education and professional development must continually update practitioners’ knowledge of technology and its classroom applications. Such programs should include lessons on the development of mathematics that take advantage of technology-rich environments and technology-integrated day-to-day instructions, besides instilling an appreciation for the power of technological tools and their potential impact on students’ learning and use of mathematics.

*Dynamic Geometry Software (DGS) and Computer Algebra System (CAS) in Mathematics Education*

Research shows that DGS provides more opportunities for students to concentrate on abstract structures when compared with traditional paper-pencil approach (Hazan and Goldenberg,
By providing appropriate dynamic software for geometry instruction, the instructors may be enabled to support students’ experiences and teach them geometry via investigation (Battista and Clements, 1995; Arcavi and Hadas, 1996). With this new approach, students may easily enter the research setting, investigate, hypothesize, test, reject, formulate, or explain these hypotheses. With the advent of computers and DGS in classroom settings, the nature of mathematical proving in classes has changed. With this new technology, students may discover mathematical relationships with induction, and learn how to easily draw simple or complex figures, analyze them, and express their own hypotheses as a theorem.

The most significant feature of DGS is its ability to drag individual elements of a figure or the entire figure (Hoyles, and Noss, 1994). While using this feature, the student may observe that some properties of the figure remain constant while some keep changing. This discovery may allow the student to propose a sound hypothesis. After that, the student may support or reject this hypothesis with enough examples. Cabri software enables mathematical thoughts by allowing modification of mathematical objects on the screen. Many relations, properties, and generalizations that cannot be handled in traditional settings can be easily addressed in this way (Güven, 2007).

Baki (2004) explored problem-solving experiences of student mathematics-teachers with the DGS Cabri. He reported that students had problem-solving experience dealing with experimental verification and generalization through DGS.

Güven, Çekmez, and Karataş (2010) show that DGS can be used in supporting deductive proof by showing empirical evidence as the source of insight for the proof. DGS can provide not only empirical data to confirm or reject a conjecture, but also ideas that lead to a proof. In the present study, the student combined inductive exploration with the deductive
structure of geometrical proof. This shows that DGS may be useful to students only in conjecturing but not in contributing to deductive proof.

By using DGS, Lavy and Shriki (2010) explored the changes in prospective mathematics teachers with regard to their mathematical knowledge in problem-posing activities in geometry. They found that the students improved their knowledge of geometrical concepts and shapes involved. And, in the process of creating the problem and working out its solution, they improved their understanding of the interconnections between the concepts and shapes involved.

CAS is the general term for software that can perform higher-order mathematical operations, quantitative and symbolic calculations, and 2D and 3D graphical representations. Derive, Mupad, Mathematica are some examples of CAS. There is evidence that CAS software has good potential in learning and teaching mathematics and more specifically algebra (Clements and Sarama, 1997; O’callaghan, 1998; Mayes, 2001; Jakucyn and Kerr, 2002). As a learning tool, CAS software allows the students to make simplification, test, and visualization. Moreover, these software environments enable the students to interpret and to use symbols and graphical and numerical representations along with symbolic ones (Çelik, 2007). The CAS also enables the student to switch easily between mathematical representations, such as graphs, tables, and formulae. This can lead to a more integrated and flexible use of these representations (Drijvers, 2000). Using a CAS allows the students to understand the concepts of proofs by induction and the principles of induction (McAndrew, 2010).

Serhan (2009) investigated undergraduate students' understanding of the derivative at a point by using graphing calculators. At the end of the study, students in the experimental group had a richer concept of the derivative image at a point when compared with that of the traditional students group. Students in the experimental group had the opportunity of viewing
the derivative concept by using different representations, and the visual image was enhanced by making use of the graphing calculator technology. This learning environment might affect the students’ understanding of the derivative concept.

Thus, these opportunities provided by technology to mathematics education should be offered to and be embraced by pre-service teachers. In the context of this study, the pre-service mathematics teachers were introduced to the software used in mathematics education and allowed to experience these opportunities.

*The purpose of the study*

The use of technology in mathematics teaching and learning is not for the purpose of teaching computers, but for the purpose of improving mathematics teaching and learning with computers. Furthermore, teachers who learn to use technology while exploring relevant mathematics topics are more likely to experience the potential benefits of technology and use them in their teaching (Garofalo, 2000). For this reason, pre-service teachers should experience the opportunities provided by technology to mathematics education. In this context, the pre-service mathematics teachers were presented with some examples of “Use of Computers in Mathematics Education (UCME)” course on using computer technology in mathematics education and investigating mathematical relationships. This paper attempts to present the mathematical thinking processes and preservice teachers’ experiences in the course of investigation and discovery processes.
Methods

Procedure

Pre-service mathematics teachers were educated about the properties and use of the dynamic geometry software, Cabri and Derive, in the context of the course. The course was taught to a class of 41 students for 11 weeks, at the rate of 3 h a week. The schedule of the course is shown in Table 1.

Table 1: The schedule of the course activities

<table>
<thead>
<tr>
<th>Week</th>
<th>Course content</th>
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</thead>
<tbody>
<tr>
<td>1st week</td>
<td>Learning about Cabri 2D</td>
</tr>
<tr>
<td></td>
<td>Forming basic geometric objects (line, segment, ray, vector, triangle, circle, etc.)</td>
</tr>
<tr>
<td>2nd week</td>
<td>Activities about measuring angles, perimeters, areas of geometric figures.</td>
</tr>
<tr>
<td>3rd week</td>
<td>Drawing perpendicular line, parallel line, midpoint of segment, angle bisector, perpendicular bisector</td>
</tr>
<tr>
<td>4th week</td>
<td>Reflection, symmetry, translation, rotation, dilation</td>
</tr>
<tr>
<td>5th week</td>
<td>Activities about measurement transfer, geometric locus problems</td>
</tr>
<tr>
<td>6th week</td>
<td>Learning about Derive</td>
</tr>
<tr>
<td></td>
<td>Calculating derivative, limit, and integral of functions</td>
</tr>
<tr>
<td>7th week</td>
<td>Factoring algebraic expression, solving equation, and drawing graphics</td>
</tr>
<tr>
<td>8th week</td>
<td>Matrix, vector, sum and product</td>
</tr>
<tr>
<td>9th week</td>
<td>Exploring geometric and algebraic theorems</td>
</tr>
<tr>
<td>10th week</td>
<td>Presentation of projects</td>
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<tr>
<td>11th week</td>
<td>Presentation of projects</td>
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</tbody>
</table>
Throughout the course, the potential of using technology in mathematics education was highlighted with project and research questions. Example projects of algebra and geometry theorems were assigned and the students helped in experiencing the discovery processes with the help of technology. The pre-service teachers were grouped into pairs and assigned project tasks involving investigation and discovery activities of algebra and geometry. Students investigated these projects in groups using Cabri and Derive. During the last weeks of the course, the students explained the results of the projects and presented the mathematical procedures they learned during the course. The Researcher observed the performance of the students during their project presentations. The processes the students went through were recorded with observation notes. Additionally, at the end of the term, the students were asked to narrate their experiences and mathematical gains during the course and also to present their views about the course. The experiences of students and their views about the course are presented in this paper.

Sample

The subjects of the study were 41 student mathematics-teachers pursuing a 5-year undergraduate program at the Faculty of Education in Karadeniz Technical University. In the final year of their undergraduate program, they got enrolled in the required instructional technology-based mathematics course. The teachers enrolled in the course constituted the data source.

Data Collection and Analysis

The data of this study include observation notes of the researchers and pre-service teachers’ experiences in the course of project process and their views about the course. In this paper, the views and experiences of students were descriptively analyzed and exemplified.
Results

In this section are presented the experiences of students during project completion process and their views about the course.

The students were assigned to investigate the following project: “A bridge will have to be constructed between the parallel banks of a river to join villages A and B. Where should this bridge be constructed to minimize the distance between the two villages? The location of the bridge is expected to change as the locations of the villages A and B change.” This project is an example of the problems of maximum and minimum values as an application of derivative in calculating the distance between two points. The students were expected to calculate the minimum value by calculating the distance using variables and taking the derivative in terms of a single variable in determining the position of the bridge connecting the villages A and B.

First, the students realized that for changing the location of the bridge as the locations of the villages A and B change, one must use Cabri software. They drew a river between randomly located villages A and B by using parallel lines. They tried to find the shortest distance between the villages by trial and error. First, they drew a perpendicular line from village A to the river banks and the points where this line intersected them (C and E) marked the bridge. After that, by joining points E and B, they completed joining the villages A and B. Repeating the procedure by drawing a perpendicular line from village B, they obtained the second path. Furthermore, they drew the line joining the villages A and B by picking a random point O on the river. Thereafter, the lengths on the drawn path were calculated and the shortest path was sought (see Fig. 1).
Making use of the dynamic property of Cabri, they determined ‘O’ as the point joining the two villages. When they moved point O, they observed that the distance between the two villages became minimal in a certain interval. Thus, they felt the need for using mathematical knowledge to find the location of point O on the river. As they could not calculate the distance using Cabri, they expressed the equation of the 3rd path joining the two cities as shown in Fig. 2.

The equation of the path was expressed as \( \sqrt{a^2 + x^2} + l + \sqrt{b^2 + (c-x)^2} \) in terms of variables a, x, b, and c. By using Derive, they found the solution \( \frac{ac}{a+b} \) by calculating the first derivative of this equation, and found point O in terms of other variables. By using Cabri, they determined that point O did exist and obtained the same according to the locations of A,
B, and the river, and by using Derive, they determined how to construct the shortest path between A and B.

The students were assigned another investigation project, which goes thus:

“The number of animals living in a national park is expressed by a function of time as

\[ s(z) = \frac{300}{1 + 7(0.69)^z} \]. Find and graphically analyze

a) the number of animals after ten years,

b) the number of animals after twenty years, and

c) the maximum number of animals the park can reach.

In investigating this project, the students were expected to calculate the maximum value that an algebraic expression may take in terms of variable z. In this calculation, the limit of the function was asked as the value of z approached \( \infty \) in the algebraic expression. Furthermore, it was expected that the student could draw the graph of an algebraic expression and perceptually observe the maximum number of animals that live in a park.

During the investigation, the students first attempted to decide by drawing the graph of the function. They explained that they could find the solutions to questions (a) and (b) by substituting 10 and 20, respectively, for ‘z’ in the function. But, they could not directly find the answer to question (c). So, they obtained the graphic of the function algebraically in terms of variable ‘z’ using Derive as shown in Fig. 3.
On examination, they found that the graph was progressing horizontally after a certain ‘z’ value. The dominant view was that the value was approximately 300 as the number of animals in the park tended to infinity. They confirmed this hypothesis using Derive by calculating the limit of the function as 300. Moreover, in the algebraic expression of the function, $(0.69)^z$ tends to 0 as $z$ tends to infinity. This is because the value 0.69 is between 0 and 1 and thus they decided the result would be 300.

Similarly, rational functions were given as projects related to Derive applications. The students were asked to draw the graph of an algebraic function and to determine the vertical and horizontal asymptotes. In finding the horizontal asymptote, the limits of $x$ for $-\infty$ and $+\infty$, and in finding the vertical asymptote, the values that make the algebraic expression undefined were asked.

The students in the group expressed their inability to investigate the given complex function, although they knew the concepts of vertical and horizontal asymptotes. For example, when examining the asymptotes of the function \( \frac{5x^3 - 1}{2x^3 + 5x^2 + 5x + 3} \), they told that they calculated the horizontal asymptote by calculating the ratio of coefficients; however, they could not calculate the vertical asymptote easily (see Fig. 4).

![Figure 4](image-url)
They realized that they drew the graph of the function in Derive and that a real point was intersecting the x-axis. They obtained the roots of the algebraic expression in the denominator by using “solve equation” in the software and then drew the graph of the function along with its asymptotes. In this project, the students could find an opportunity to gain mathematical experience and to practice the concepts relating to asymptotes.

The students were asked to investigate the relationship between any quadrilateral and the quadrilateral obtained by joining the midpoints of the sides of that quadrilateral. For this, they were given quadrilateral ABCD, and the quadrilateral KLMN obtained by connecting the midpoints of the sides of the quadrilateral ABCD. By realizing that KLMN was a parallelogram, the students were expected to discover the relationship between the areas of these quadrilaterals, and if the relationship is independent of quadrilateral ABCD being convex or concave.

First, the students drew Fig. 5 using Cabri. They intuitively developed the hypothesis that quadrilateral KLMN was a parallelogram and by using the menu of the software, they measured the side lengths and angles, and checked whether the sides were parallel.

![Figure 5](image)

The students observed that the opposite sides of quadrilateral KLMN were “parallel.” To determine whether quadrilateral KLMN was parallel in any case, they dragged the corners of quadrilateral ABCD and found that quadrilateral KLMN was a parallelogram. Furthermore, they noticed that even when quadrilateral ABCD was concave or convex, the properties of
quadrilateral ABCD remained the same. So, they concluded that quadrilateral KLMN was a parallelogram, regardless of quadrilateral ABCD being concave or convex.

Then, the students investigated whether any relations existed between the areas and circumferences of the quadrilaterals ABCD and KLMN. One student thought that because quadrilateral KLMN was created by joining the midpoints of the sides of quadrilateral ABCD, there should be a relationship between the areas and circumferences of these two quadrilaterals. Accordingly, the student calculated the areas and circumferences of those quadrilaterals and tried to find out the respective ratios. The student stated that as quadrilateral ABCD was changed by dragging, she discovered that the ratio between the areas remained constant, but the ratio between the circumferences was changing continuously. Finally, the students came to the conclusion that the ratio between the area of a quadrilateral and the area of the quadrilateral obtained by joining the midpoints of the sides of that quadrilateral was 2.

When the students were asked to prove the same mathematically, they drew the diagonals of quadrilateral ABCD, calculated the ratio of congruence, and found the relationship between the areas.

Further in the project, the students were asked to draw quadrilaterals laterally, one in another, and investigate the relations between their areas and circumferences. The students accordingly drew the figure (Fig. 6) and began exploring the relationships. They calculated the area and circumference of each quadrilateral.

![Figure 6](image-url)
The dominant view among them was that the ratio between the areas was always 2 and that there was no relationship between the circumferences. But, a student in the group had a view that, because KLMN was a parallelogram and the other quadrilaterals created by joining the midpoints of the sides of that quadrilateral were also parallelograms, there should be a relationship between their circumferences. The students began investigating this hypothesis and found that the ratio of the circumferences between 2nd and 3rd quadrilaterals was equal to that between 4th and 5th quadrilaterals, and the ratio of the circumferences between 3rd and 4th quadrilaterals was equal to that between 5th and 6th quadrilaterals. They also discovered that this relationship holds good regardless of quadrilateral ABCD was convex or concave.

In another project, the students were given the following geometry problem: “Draw equilateral triangles outwards on the sides of any triangle ABC. Construct a new triangle by joining the centers of gravity of these triangles. Investigate the properties of that triangle.” The students did not find any difficulty in accomplishing this task, because during the lectures they were taught how to draw an equilateral triangle on the side of any triangle with the help of a circle. Also, they knew that the center of gravity of a triangle is the intersection point of its medians, by using this knowledge; the students drew the required triangle KLM (see Fig. 7).

Figure 7.
Then, they intuitively guessed that triangle KLM might be an equilateral triangle. By using some properties of Cabri, they measured its angles and sides, and confirmed that it was an equilateral triangle. Then, the students were asked to find out the relation between the areas and circumferences of the triangles ABC and KLM. Some students reported that there was no relation between the two triangles. But, they reported having discovered that the triangle created by inward drawing of equilateral triangles on the sides of the triangle ABC and by joining their centers of gravity, was also an equilateral triangle (see Fig. 8).

![Figure 8](image.png)

The group members tried to first find if there was any relation between KLM and PRS equilateral triangles in terms of area and circumference, but they found none. Then, they looked at the ratios between the areas and circumferences of ABC, KLM, and PRS triangles to ascertain if there was any relation. Finally, they found that the sum of the areas of KLM and PRS equilateral triangles was equal to the area of the triangle ABC and there was no change in this result as the triangle ABC was moved. On the contrary, by using Cabri, they found that there was no relation between the circumferences of these two triangles.

In another project, the students were asked to calculate the area of the function $x^4 + 4x^3 - 9x^2 - 16x + 20$ confined within the boundaries of lines $x = -1$, $x = 4$, and $y = 0$. In this project, the students were expected to draw the graph of a polynomial function and to find
the area between the determined boundaries using integrals. It was quite a difficult exercise to draw the graph of a fourth-degree function. Also, determination of the area of a function confined between two lines was similarly difficult. The students drew the function and the graphs of the lines with the help of Derive. They showed the areas of the required regions by a drawing (see Fig. 9).

![Graph of a function](image)

Figure 9.

The students observed that one of the shaded regions in the graph was under x-axis. They decided that they must finally take the absolute value of the result to find the area of this region. They needed to know the x intercepts to calculate the areas. They obtained the roots of the function by using the equation-solving property of Derive. Therefore, they found the total area by calculating the partial integrals. With the completion of this project, the students gained the experience of identifying the regions confined between two curves and demarcating the borders of the confined regions by using Derive.

The following is another example of a project assigned to the students: “Draw a triangle ABC. Determine its orthogonality center. Locate the symmetries of the orthogonality center with respect to each side of the triangle. Determine the property of these three points.” In this project, the students were expected to determine the symmetrical properties of the orthogonal center with respect to each side of the triangle. The students located the orthogonal center and its symmetries with respect to each side of the triangle using Cabri Geometry Software. By using the dynamic property of the software, they investigated the movements of
these three points. The first impression of the students was that these three points followed a circular path. As each corner of the triangle was moved, they noticed that the circular motion of these three points passed through the corners (see Fig. 10).

![Figure 10](image)

Point H is the orthogonal center of the triangle, and the symmetrical points of H with respect to the sides of the triangle are points D, E, and F. From their observations, the students discovered that points D, E, and F were on the circumference of the triangle (Fig. 6). As a consequence, they decided that because ABC was a random triangle, this property could be generalized. Therefore, the project was found to contribute to students’ learning process.

One more project assigned to the students was as follows:

“Complete the operations below, step by step, and investigate the question:

a) Draw an irregular pentagon,

b) Find the diagonals of this pentagon by skipping every second vertex,

c) Hide the pentagon and label the vertices of the formed star,

d) Create a table presenting the sum of the angles,

e) Make your observations about the angles of the star with five vertices.”

In this project, the students were expected to discover the relationship between the sum of the interior angles of the polygons obtained by drawing the diagonals of any polygon
and then making a generalization. Using Cabri Geometry Software, the students began their investigation to find out the sum of the internal angles of the star they would create from the pentagon. They drew Fig. 11 and with the help of the table they observed that the sum of the internal angles of the star was 180°.

![Figure 11](image)

They found that the sum of the angles did not change when the vertices of the pentagon were moved. Although this demonstration seemed sufficient for the students, they also prepared an algebraic proof and found that the sum of the internal angles did not change; it was 180°. Then, the students investigated how the results would change when a hexagon, heptagon, or octagon was used. Then, they formed a hexagon and drew diagonals by skipping every second vertex. Realizing that the figure formed was composed of two triangles, they decided that the sum of the angles would be 360°. When the polygon was a heptagon, they determined that the sum of internal angles of the created figure would be 540°.
The Views of Students about UCME Course

At the end of the projects, the students were asked to express their views on the course and its projects, and share their ideas about their gains.

They expressed that the software may be helpful in teaching concepts by concretizing the abstract concepts. Particularly, one group of students expressed their views thus: “Previously we could not know more than what we memorized when drawing function graphs and determining how these graphs changed with the change in coefficients. However when we used Derive in completing the projects, we found the opportunity to reach conclusions we never reached before.” They expressed that they could easily reach some conclusions by observing how the graph of the function changed with respect to its coefficients.

A group of students associated the students’ success with the use of technology and expressed their views thus: “I started believing the idea that the applications may promote students’ achievements in mathematics. Especially by using software such as Cabri and Derive, I think students will understand algebra and geometry concepts better and their involvement will increase. Indeed, as pre-service teachers, we also had the experience of learning some concepts in this way.” The students adopted the idea that using software such as Cabri and Derive in teaching algebra and geometry will contribute to their conceptual learning. Conceptual learning, in turn, was expected to increase students’ success in mathematics. Similarly, another group stated that with this course, they learned how to use technology in mathematics education. They conveyed their views thus: “I think I developed skills in using technology in mathematics education through this course and its projects. Therefore, I am confident that I can teach students the concepts of function, limit and continuity via the graphs of functions.”
Another group explained their view thus: “In Cabri we can calculate the area, circumference and length of the figure we draw; we hypothesize and check our hypotheses by moving the figure. We can generalize our hypotheses with the results obtained.” Thus, the students conveyed that they had the opportunity to test their hypotheses with their experience from using dynamic Cabri Geometry software. Having gained perspectives of conducting proofs with software, they further observed thus: “We’ve learnt new concepts in the projects and found opportunities to apply them in new situations. We also had opportunities to prove arguments intuitively and visually in Cabri, which we find difficult to prove analytically.” It can be deduced that technology makes it possible to make intuitive proofs before conducting analytical proofs.

Conclusions

Students were observed to discover mathematical relationships and concepts in computerized settings while investigating projects. With the help of DGS Cabri and Derive, students gained the experience of discovering a mathematical relationship and making generalizations from that relationship. Pre-service teachers’ experiences in the process of project completion were shared with classmates during the project presentations and thus an environment congenial for discussion was set. So, they had the opportunity to answer the questions of classmates and confirm mathematical results. The experiences of pre-service teachers in this study align with the claim of Wiest (2000) that technology provides opportunities to students in making hypotheses and assumptions and then in deriving generalizations. Similarly, Wertheimer (1990), Lavy and Shriki (2010), and Yousef (1997) consider that computer technology contributes to understanding the concepts in geometry-teaching, and provides opportunities to pre-service teachers in investigating geometry.
concepts and theorems. Besides, pre-service teachers used problem-solving procedures effectively for mathematical projects. This finding is in line with that of Baki (2004). In the process of pre-service teachers’ investigating mathematics projects, it was found that technologies such as Cabri and Derive gave perceptual ideas to students before conducting analytical proofs.

This case study conducted from observations on student mathematics teachers’ experiences in a technology-based environment has hopefully shed light on the fact that new mathematical relationships can be discovered by students. Previous studies have shown that Cabri can be an excellent tool for teaching and learning mathematics through problem-solving (Baki, 2004; Guven, 2007). It can be said that Cabri is an excellent tool for learning mathematics via explorations. In addition, computer technology such as Cabri and Geometry Sketchpad Software can be used for construction of proofs, conjecturing, and experimental verification in teaching and learning geometry.

The students inquired about the graphics and properties of functions using Derive. In this process, the students got the opportunity of understanding mathematical concepts algebraically and graphically. This finding matches with the results of Serhan’s work (2009) on the concept of a derivative. The finding also supports the claims of Çelik (2007) and McAndrew (2010) who consider that using CAS in teaching mathematics contributes to students’ conceptual understanding.

**Pedagogical implications**

The graphs of algebraic equations and the change in these graphs with respect to coefficients of the equations can be examined dynamically in computer-assisted environments and this
enables learning new concepts which is not possible in traditional settings. Furthermore, learning mathematics in computerized settings supports conceptual learning.

Computer-assisted environments contribute to concretizing abstract mathematical relationships. Therefore, appropriate environments enabling concretization of mathematical relationships should be provided in teacher-education institutions for pre-service teachers to live these experiences.

This was found to help in the discovery of new mathematical relationships intuitively. Therefore, sample cases about how to use technology in mathematics education should be provided to pre-service teachers.

Computer-assisted environments, well established with software, such as Cabri or Derive, may help in constructing strong bridges between mathematicians and students. Once these bridges are constructed, students may cease to perceive mathematics as esoteric; instead, they may indulge themselves in mathematical activities and engage in higher-level cognitive activities, such as hypothesizing, generalizing, testing, or rejecting. This will directly enhance the development of students’ problem-solving skills. It is also found that computers enable the students to behave as mathematicians and provide them with a functional learning experience. As a result, these experiences of pre-service teachers may be considered as models for their professional lives.

References


