Abstract

Although mathematics is visual language of symbols and numbers it is also expressed and explained through written and spoken words. For students to excel in mathematics, they must recognize, comprehend and apply the requisite vocabulary. Thus, vocabulary instruction is as critical in content areas as it is in language arts. It is especially critical for students who live in poverty and may enter school with limited background knowledge. This paper presents aspects of vocabulary and its impact on mathematical comprehension and performance based on representative vocabulary from standardized examinations. Direct and indirect instructional methods for math vocabulary are discussed. Instructional strategies for fostering vocabulary development are also provided.
Introduction

Mathematics is a visual language of symbols and numbers. However, mathematics is also expressed and explained through written and spoken words. Although students may excel in computation, their ability to apply their math skills will be hindered if they do not understand the vocabulary utilized in instructions and word problems. Research (Biemiller, 2001) has indicated that vocabulary knowledge is strongly related to overall academic achievement in school. The relationship between vocabulary mastery and scholastic performance has been clearly established in the research; this is particularly true in the area of mathematics. It has been shown that students must understand math vocabulary if they are to master content and be able to apply it in future situations (Thompson & Rubenstein, 2000). Thus, teaching vocabulary in the mathematic content area is a critical element of effective instruction.

Importance of Vocabulary Instruction

Vocabulary instruction is one of the essential elements of a student’s academic development (Snow, 2002). Although commonly associated with language arts, vocabulary proficiency is a significant contributing factor in learning and mastering mathematical concepts. Assisted by classroom instruction, vocabulary grows rapidly. Anglin (2000) states that, during grades one through five, the recognition vocabulary growth rate averages twenty words per day.

Enhancing vocabulary development is a critical instructional element, particularly when working with students who live in poverty. Children in poverty often enter school with a limited range of vocabulary. While there are several potential factors for this statistic, one key indicator of future vocabulary proficiency is the amount of time spent reading to the
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child. Neuman’s (2001) research indicated that “Children in poverty are read to an average of 25 hours before entering kindergarten; while those in middle class homes are read to an average of 1,000 hours before entering kindergarten”. This limited exposure to vocabulary through books and associated concepts can contribute to limited vocabulary capabilities when the student enters school (Sénéchal & Cornell, 1993). Boyer, in the Carnegie Foundation for the Advancement of Teaching Report (1991), indicated that approximately “35% of U.S. children enter kindergarten unprepared to learn, with most lacking the vocabulary and sentence structure crucial to school success.”

Furthermore, children in poverty who have had limited language opportunities typically have less problem solving abilities (Guerra and Schutz, 2001), a critical skill in mathematics. As they progress in school, students lacking a firm foundation of vocabulary knowledge have more difficulty obtaining meaning while reading. This difficulty leads to reading less and, as a result, less exposure to new vocabulary words. This pattern of reading less leads to a persistent, widening gap of vocabulary proficiency compared to students with strong vocabularies who read more frequently, thereby expanding their understanding through new vocabulary and continually improve their skills. Walberg and Tsai (1983) termed this the “Matthew Effect”: the students with rich vocabulary skills continually enhance their vocabulary knowledge while the vocabulary of students with poor vocabulary skills ebbs. The effect of poor vocabulary skills are compounded in the mathematics domain if students do not possess the requisite vocabulary to understand word problems, verbally express mathematical concepts and formulate solution approaches to problems. Math vocabulary instruction is needed to help these students build the critical background knowledge that students who have been exposed to a wide range of vocabulary through reading have attained.

Relevance to School Curriculum
Although mathematics is described visually through symbols and numbers, the symbols have associated words and meaning. More importantly, mathematical problems are often expressed in words. Thus, vocabulary is essential to achieve math proficiency.

As part of planning a math curriculum, vocabulary words should be identified based on student needs, previous curriculum exposure, activities such as field experiences, films or labs, and vocabulary used during testing. Marzano (2004) found that teaching academic vocabulary could positively influence standardized test scores by as much as 33%. Recent research by Gifford and Gore (2008) showed that underperforming math students who received vocabulary instruction showed standardized test gains as high as 93%, supporting Marzano’s findings.

Most states publish a list of words by subject that students may expect to see on achievement tests. For example, the Colorado Department of Education (2007) and the Montana Office of Public Instruction (2006), like many state department of education offices, offer a list of essential mathematics vocabulary test terms. Additionally, some governments publish math vocabulary lists such as the Department of Children, Schools and Families in the United Kingdom (2000). Classroom instruction should include the words in these essential vocabulary lists when they appear in textbook passages or in curriculum activities.

Vocabulary Retention

Stahl and Fairbanks (1986) indicate that students must be exposed to words at least seven times over spaced intervals in order for retention to occur. Repetition is critical for the retention of new vocabulary. Learners will have increased retention of vocabulary words when they manipulate them in a variety of ways (Miller, 2007). An action research study that looked at the impact of teaching and testing math vocabulary terms to third grade students on a regular basis indicated that students retained math vocabulary longer when the words were manipulated and emphasized in the curriculum (Bradley, 2003). Incorporating vocabulary on
multiple occasions and utilizing varied instructional methods may increase the likelihood of student retention.

### Instructional Methods

To be effective, vocabulary instruction must provide more than simple definitions. Stahl and Fairbanks (1986) indicated that students need more than just a surface knowledge of the word and that teaching vocabulary words solely as definitions does not assist students in comprehending the word when found in the text. Students must be actively engaged in building background knowledge using key content-specific vocabulary. Marzano (2004) identified eight characteristics of effective vocabulary development that builds background knowledge. These vocabulary instruction characteristics should:

1. Not rely on definitions
2. Be demonstrated in both linguistic and non-linguistic ways
3. Involve the gradual shaping of words through multiple exposures to meaning (comparing & contrasting, analogies, classifying, interpreting using multiple methods including linguistic and non-linguistic)
4. Teach word parts to enhance understanding (triangle – tri = three, angles)
5. Include different types of instruction for different types of words
6. Allow students to play with words (i.e. parallel = the double ‘ll’s’ in the word run parallel to each other)
7. Include student discussion of words being learned
8. Focus on terms that have a high probability of increasing academic performance not just the words in bold print.

Vocabulary can be developed through both direct and indirect instructional methods. It is important to understand the differences in delivery and effectiveness between indirect and
direct instruction. Indirect methods focuses on student-centered inquiry and discovery based learning. Direct instruction takes a systematic, goal-oriented, teacher-directed approach to learning. The following sections look at both methods and discuss the pros and cons of each.

**Indirect Methods**

Indirect vocabulary instruction focuses on learning words primarily through exposure in conversations and through reading (Beck et al, 2002). Marzano (2004) proposed several principles for the indirect teaching of vocabulary. These include:

- **Activate background knowledge:** Background knowledge is organized, networked, and stored in linguistic and non-linguistic modes (bimodal) that become more generalized over time. It is essential that background knowledge be activated so that networks can be established between the new knowledge and previously learned material.

- **Types of memory and use:** There are three types of memory that can be enhanced. These include sensory, permanent, and working memory. Sensory memory is gathered by the senses and is stored as temporary memory. This is one of the reasons why incorporating manipulatives in teaching mathematical concepts is critical. Permanent memory is long term memory that students use as their knowledge base. This is an area that educators activate when they activate prior knowledge through K-W-L and other strategies. The last type of memory, working memory, processes information from both sensory and permanent memory. Successful processing leads to the formation of permanent memory. Working memory can be limited. For example, Miller’s Magic Number (1956) is the amount of information that can be remembered after one exposure. The typical limit is 5 to 7 items. This is why Social Security Numbers and telephone numbers are grouped and limited to foster
remembrance. To increase the effectiveness of working memory teachers should use repetition. For example, Nuthall, 1999, states that 4 exposures over not more than two days promotes deep processing (adding details), and elaboration (associating the new information with known information.

- Prior experience: Background knowledge is multidimensional and varies in each individual based on previous experiences; having knowledge in one subject area does not mean it will apply to other areas.
- Multi-level knowledge: Surface level knowledge is hierarchical. Some knowledge is quicker to recall (surface level) than other knowledge (bottom level). When teachers use appropriate questioning techniques in the classroom and incorporate both lower and higher order questions they activate both surface and bottom level knowledge. In order to be successful in a curriculum area, bottom level knowledge must be available that will act as background knowledge. Students must be familiar with topic’s terminology and definitions.

Lower socio-economic students often have less exposure to vocabulary. When a teacher teaches vocabulary they are creating background knowledge for the student; the vocabulary word becomes a label for a packet of information to be stored in permanent memory. One way to do this is through virtual experiences such as reading about the topic to create a ‘being there’ experience, providing language experiences including listening to others and talking about the topic, and, watching educational programs or videos on the topic.

Indirect instructional strategies for teaching vocabulary are varied and can include games, physical movement activities, mnemonics, and visualizations. The list below provides suggestions that may be incorporated in the classroom.

- **Categorize by Preference:** Categorize words by personal preferences: easy versus hard words to learn, words they know versus don’t know, or by likes and dislikes (for
example in the topic of sports, likes might be hockey, basketball and fencing/dislikes might be football, tennis and rock climbing).

- **Categorize by Type:** These types can be by similar genre, by number of syllables (square is a one syllable word, circle and oval are two syllables, triangle and trapezoid are three syllable words but all are shapes).

- **Create a Story:** Prior to reading the chapter, show students a list of essential vocabulary. Review definitions of all words. Have students write a story (fiction or non-fiction) using all or a specified number of the words.

- **Demonstrate the Word:** Moats (2001) study showed that disadvantaged kindergarten students were often unable to name pictures that demonstrated the meanings of words like ‘sewing’ and ‘parachute’. She calls this “word poverty”. Using visualization and/or demonstrations helps students connect word meaning to word recognition.

- **Hangman:** Using the vocabulary list, play the game of hangman with the students. After the students recognize the word and definition, have a student come up and direct another hangman game using a synonym for the word.

- **Student Generated Examples:** After students have been introduced to the word, have the students generate examples for the word. For example, if the term is ‘quadrilateral’ (a plane figure with four sides and four angles), have the students identify quadrilaterals in the environment (windows are generally rectangular and desks might be trapezoidal – both are quadrilaterals). If a student misidentifies the term, discuss why the chosen object is a non-example.

- **Total Physical Response (Asher, 2000):** Have students physically move to demonstrate the word. For example, if the word is “intersection” the students might hold their two arms in a crossed fashion. Movement helps the brain to imprint (Hardiman, 2003) the vocabulary words.
Use Mnemonics and Memory Techniques: For example, students often have difficulty distinguishing the numerator from the denominator. Having students remember that denominator begins with the letter “D” and so does the word ‘down’ helps them to remember that the denominator is the number on the bottom of a fraction.

Visualization Strategies: When introducing vocabulary words, have students close their eyes and create mental images of the word. For example, for the math term ‘slope’ the students might be instructed to visualize a ski slope or for ‘apex’ to think of the ape, King Kong, at the very top of the Empire State Building. Students can be asked to help develop these images; teachers may be surprised at what they visualize!

Incorporating the indirect method is just one of the ways to teach mathematics vocabulary. Another method, direct instruction, will be addressed next.

Direct Methods

In direct instruction, the teacher assists students to focus on specific words by studying word parts (i.e. bisect - bi equals two), stimulating word comparisons, and through games. Direct instruction has been proven to work well with students who have little prior knowledge (Vacca, et. al., 2008). Pressinger (n.d.) lists seven direct instruction steps to achieve vocabulary memorization:

1. Introduce the Words – Categorize, phonically sound out, and define the words.
2. Provide Group Practice – Teacher makes a ‘game’ of mispronouncing the words and having the students recognize the errors.
3. Recognition Memory Test – Students are given a list of vocabulary words. The teacher provides a definition and the students identify the correct word. The teacher then provides the correct response.
4. Practice #1 – Students complete word scrambles, crosswords, and other prepared ‘game’ sheets (a website that teachers can utilize to create these sheets is http://www.puzzlemaker.com).

5. Time Delay – Wait 1 to 2 hours between step 4 and step 6.

6. Practice #2 – Students complete a worksheet that requires them to match definitions, antonyms, or synonyms to words.

7. Sentence Writing – Students create sentences using the words. The sentences must be handwritten (for sensory based brain imprinting), contain a minimum of seven words, and not begin with the word “I”.

Collier (2007) recommends that vocabulary words be taught through context clues and word parts, word families (triangle = 3 angles, tripod = 3 legs, etc.), graphic organizers, word walls, and the use of cognates. This is especially useful for English Language Learners who may discover patterns within the English language as well as language similarities.

By utilizing direct instruction strategies such as those presented below, teachers can incorporate techniques that may interest students while enhancing vocabulary mastery.

- **Antonyms**: Powell (1986) recommends using antonyms (a word that means the opposite of another word) to teach vocabulary. The teacher creates a list of vocabulary words that begin with the same letter. A second list is created with words (beginning with any letter) that are antonyms to the first list. The students are asked to attempt to match the words from the two lists without any reference materials. After approximately five minutes the students are allowed to use references and work in pairs to accurately complete the list. A discussion of the words is led by the teacher after the students have matched the words.

- **Ask Me About**: The teacher selects several critical vocabulary words for the subject being discussed. After the student understands the definition, the teacher places a pin
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or sticker on the child that states “Ask me about……(vocabulary word)”. As the student moves through the day, have other professionals and staff in the school ask the student about the word. The student then defines the word in his or her own terms and gives an example. This reinforces the word and increases the likelihood that the word will be remembered. Teachers can select one child per day to be the ‘Word Wizard’.

- **Daily Vocabulary:** When students enter the room in the morning or the period, have three to five of the essential vocabulary words written in a contextual sentence (the vocabulary word is underlined). Have the students write the word and, through the use of the cloze procedure, define the word using synonyms. The class then discusses the vocabulary words and possible definitions, clarifying any misunderstandings.

- **Deep Processing Questions:** After developing the definitions of vocabulary words, have students move into deeper processing of the words. For example, for the vocabulary word ‘triangle’ a deep question might be “What characteristics must all triangles possess?”, “A square would become a triangle if…….”, and “Examples of triangles include…..”

- **Draw:** Represent the word through an interpretive drawing or as a drawing with labels (i.e. a square drawn by the student with the elements labeled corner, side, angle, etc.).

- **Graphic Organizers:** Place a vocabulary word in the middle of a word web. Branch the web with words that are synonymous. Another graphic organizer is to have students create a four square. Place the word in the middle, in the top left square place the dictionary definition of the word. In the top right square, have students rewrite the definition using their own words. In the bottom left have students list words that are synonyms of the vocabulary word. In the bottom right have students create a drawing that will help them to identify the word in the future. These may be
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kept in a vocabulary notebook and used for the hangman game to review words.

Templates are effective vocabulary graphic organizer. Kinsella (2005) has assembled a variety of vocabulary templates. A particularly interesting chart asks the students to identify the part of speech for the vocabulary word and then place the word in various other forms such as noun, adjective, verb, and adverb. Examples of this and other graphic organizers can be found at http://www.scoe.org/content.php?SubsiteId=10 and at http://www.eduplace.com/graphicorganizer/.

- **Match Vocabulary Words to Definitions:** Divide a piece of paper into three vertical columns. List essential terms in column one and definitions (not in the same order) in the second column. Have students correctly match the pairs and then rewrite, in the last column, the definition in their own words.

- **Preview the Text:** Have groups of students look at the chapter being discussed and create a list of words, including any bold text words, which are essential for understanding the chapter. The list may include words that are familiar or unfamiliar to the group. Have the students then group the words by category.

- **Provide Examples and Non-Examples of the Word in Context:** It is important for students to understand what a vocabulary word means as well as what it doesn’t mean. For example, when using the word ‘net’ in mathematics (a flat shape that can be folded into a three dimensional solid), it is important for students to have the understanding that ‘net’ as a mathematical term does not mean the same as the net used to catch butterflies. A graphic organizer that incorporates well into teaching examples and non-examples is the Frayer Model (1969). (An example of the Frayer Model can be found at: http://www.longwood.edu/staff/jonescd/projects/educ530/aboxley/pdffiles/2.pdf).
• **Reward the Word:** Every time the students in the class apply the word in context during the week the vocabulary word is taught, they get a mark or counter. For example, if a vocabulary word is “array” (a set of objects grouped in order such as in rows or columns) and a student entering the lunchroom states “Look, the lunch room tables are organized in an array,” the class would earn a mark or counter. After receiving a specified number of marks or counters, the class receives a reward. This practice encourages students to apply the words meaning to situations outside the classroom.

• **Shape the Word:** Prior to reading, have students perform a text walk through the chapter paying close attention to the bold text. Using a list of bold words as a guide, the teacher should create a word ‘shape’ (outline of the word) for the students. Students refer to the text to guess the word. If multiple words have the same shape, the teacher may indicate where vowels, prefixes, root words, or suffixes occur. After the students have discovered the word, have the students develop a definition of the word.

• **Show and Define:** Place essential vocabulary words on the board or overhead. Ask students if they know the meaning of the first word. Accept all responses, clarifying if necessary. Show a list of definitions and ask students to pick the definition that most closely aligns with the discussion. Have the students write the word and paraphrase the definition in their own words. Continue with the next word.

• **Spell the Word:** A strategy that incorporates movement is to have the students spell the vocabulary words through movement. If, when written, the letter touches the top line on a piece of handwriting paper (letters like ‘l’, ‘h’, ‘b’, etc.) Students reach their arms above their head. Letters that, when written, go to the mid-line (letters like ‘e’, ‘m’, ‘o’, etc), the students place their hands on their waist. For letters that go below
the bottom line when written (letters like ‘y’, ‘g’ and ‘j’) the students bend down and touch their toes. Anytime an ‘s’ appears in a word, the students make an \( \mathfrak{s} \) shape with their hips. For example, with the word square – the students would say ‘s’ while moving the hips, touch their toes when they say the ‘q’, and place their hands on their hips each time they say the letters ‘u’, ‘a’, ‘r’ and ‘e’. (This is primarily an elementary strategy.)

- **Teach Break-Apart Strategies**: Help students to look at a new word and assess if there are prefixes or suffixes. With words that contain a root word, guide the students to discovering the root word.

- **Use Think-Pair-Share**: Show a vocabulary word to the students. Students talk with their partner to define the word and give an example of the word’s meaning. After a brief ‘pair-share’, the teacher asks for possible definitions and examples.

- **Vocabulary Knowledge Rating Process** (Blachowicz, 1986): Students receive a list of the vocabulary and self-assess their knowledge of the words on a 1 – 3 scale (1 = no knowledge, 2 = recognition of word on a surface level, 3 = deep conceptual knowledge of the word). Based on these ratings, the teacher can assess the vocabulary strength of the class and focus instruction based on class need. After teaching the words, the students complete a reassessment.

- **Word Associations**: Create word associations by asking students if the word reminds them of another word or idea. For example – the vocabulary term ‘Acute Angle (an angle less than 90 degrees) might remind the student that the ‘A’ in the word Acute forms an acute angle.

This list of strategies may be incorporated singularly or in combination during mathematics instruction to assist students in mastering critical vocabulary and assist in increased understanding of concepts.
Direct versus Indirect Discussion

So what is the best method of teaching vocabulary: direct or indirect? While indirect instruction is focused on the non-linear construction of knowledge based on prior experience, direct instruction provides students sequential steps with the content broken down into smaller units. Haberman (1991) argues that it is neither direct nor indirect instruction that matters but rather the relevance of the activity to the student. This relevance is especially critical when working with high-poverty students who need to see a connection established between the vocabulary being mastered and the students’ real-world.

Contrary to Haberman, Marzano’s (2004) research supports the use of indirect instruction. He states that background knowledge manifests itself as vocabulary knowledge and that when a student is taught vocabulary they are creating essential background knowledge. Others, such as Shostak (2002) feel that vocabulary acquisition should not be left to chance and should include direct, guided instruction. A study by Drake and Meyer (2003) looked at the influence of direct and indirect instruction on testing outcomes for pre-med students. Their study defined indirect instruction as research topic assignments and direct teaching as lecture, laboratory and assigned reading. They discovered that test results were higher on the direct teaching than the indirect method. Przchodzin, Marchand-Martella Martella & Azim (2004) contend that only direct instruction teaches students the necessary vocabulary and strategies that are critical for communication in mathematics. Other research has indicated that while direct instruction may be beneficial to some students it may not be beneficial to all. A three year study (Ryder, Sekulski & Silberg, 2003) regarding poverty and direct instruction indicated that direct instruction demonstrated less sensitivity to the issues of poverty, race and culture and that students showed better academic gains with less structured approaches.
Research has also indicated that the use of both methods is critical when teaching vocabulary. The Partnership for Reading (2006) states that both direct and indirect instructional strategies have validity; indirect instruction provides words in context through real-world exercises such as conversations and reading while direct instruction allows students to learn words through specific vocabulary instruction and master word-learning strategies.

While the research supports the use direct and/or indirect instruction each classroom is unique. The decision rests on the teacher to choose which method or combination works best for their classroom. They must also consider the most appropriate time for instructional delivery (Bay-Williams & Livers, 2009). Having instructional strategies available that support both methods is critical for student success in vocabulary acquisition.

Conclusions

This paper presents reasons why vocabulary development is critical within the mathematics curriculum. It provides a description of techniques that may be implemented to teach vocabulary using direct and/or indirect instruction.

The vocabulary of mathematics must be taught with the same vigor as any other vocabulary term. By the utilization of a variety of methods, such as those presented here, teachers are better able to meet the needs of their students and increase vocabulary retention.
References


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