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Teaching with Procedural Variation: A Chinese Way of Promoting Deep Understanding of Mathematics

Mun Yee Lai

School of Teacher Education, Charles Sturt University

Panorama Avenue, Bathurst, NSW 2795, Australia

61-2-6338 4337

mlai@csu.edu.au

Sara Murray

School of Teacher Education, Charles Sturt University

Panorama Avenue, Bathurst, NSW 2795, Australia

61-2-6338 4820

saramurray@csu.edu.au

Abstract

In mathematics education, there has been tension between deep learning and repetitive learning. Western educators often emphasize the need for students to construct a conceptual understanding of mathematical symbols and rules before they practise the rules (Li, 2006). On the other hand, Chinese learners tend to be oriented towards rote learning and memorization (Marton, Watkins & Tang, 1997). One aspect of the criticism is that rote learning is known to lead to poor learning outcomes (Watkins & Biggs, 2001). However, Chinese students consistently outperform their Western counterparts in many international comparative studies on mathematics achievement such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1997; Mullis, Martin, & Foy, 2008) and PISA (OECD, 2004; OECD, 2010). This paper aims to contribute to an understanding of the “paradox of the Chinese learners” (Marton, Dall’Alba & Lai, 1993) by exploring the procedural variation and its place in the development of mathematical understanding.

Keywords: procedural variation, theory of variation, deep learning, repetitive learning, the paradox of Chinese learners

Introduction

In Western culture repetitive learning is often positioned as the opposite of deep learning and understanding (Marton & Saljo, 1976). Western educators emphasise the need for students to construct a conceptual understanding of mathematical symbols and rules before they practise the rules (Li, 2006). Similarly, many Western educators hold the view that students should be encouraged to understand rather than to memorise what they are learning (Purdie, Hattie & Douglas, 1996) as they believe that understanding is more likely to lead to high quality outcomes than memorizing (Dahlin & Watkins, 2000). On the other hand, Chinese learners tend to be oriented towards rote learning and memorization (Marton, Watkins & Tang, 1997). Chinese educators have been criticized as not providing a learning environment which is conducive to “good learning” and using a teaching method which is merely “passive transmission” and “rote drilling” (Gu, Huang & Marton, 2004). One aspect of the criticism is that rote learning leads to poor learning outcomes. The evidence suggests, however that Chinese students consistently outperform their Western counterparts in many international comparative studies of mathematics achievement such as TIMSS (Beaton, Mullis, Martin, Gonzalez, Kelly & Smith, 1997; Mullis, Martin, & Foy, 2008) and PISA (OECD, 2004; OECD, 2010). This contradictory phenomenon is referred to by Marton, Dall’Alba and Lai (1993) as the “paradox of the Chinese learner”. Dahlin and Watkins (2000) argue that this so-called paradox raises two questions:

Firstly, how can Chinese students do so well in international comparative tests – especially in fields like mathematics and science – if they are ‘only’ rote learners? Secondly why do they report in both quantitative and qualitative investigations that they are trying to understand what they are learning while their Western teachers consider them as mere learners by rote? (p.67)

Watkins and Biggs (2001) suggest that the “paradox of the Chinese learner” might be a misconception by Western scholars arising from limited understanding of the philosophies and theories of learning and teaching in the Chinese context. Throughout his work, Marton (Bowden & Marton, 1998; Marton & Booth, 1997; Gu, Huang & Marton, 2004) has argued that the theory of variation may be central to solving the paradox of Chinese learners. This paper will explore the theory of variation and its place in the development of mathematical understanding, and in particular the Chinese conceptualisation of teaching with “procedural variation”. Procedural variation will be illustrated through three detailed examples. These examples serve two related purposes: firstly, they highlight the value of procedural variation (a particular type of repetitive learning) for children’s mathematical understanding in both Western and Chinese contexts, and secondly, in so doing, they contribute to an understanding of the paradox of the Chinese learner.

The role of repetition and memorizing in mathematical understanding – the Chinese context

While repetitive learning, rote learning and memorisation have received criticism in some educational quarters, other researchers from both the West and East have challenged this critique. Marton, Wen and Wong (2005) make the obvious point that the likelihood of being able to recall something is higher if the learners hear or see something several times than if they do not. These researchers report that Chinese learners recognise the mechanism of repetition as an important part of the process of memorization and that understanding can be developed through memorisation. Likewise, Hess and Azume (1991) refer to repetition as a route to understanding. Dahlin and Watkins (2000) assert that the traditional Asian practice of repetition can create a deep impression on the mind and enhance memorisation, but they also argue that repetition can be used to deepen and develop understanding. Lee (1996)

concludes that memorising, repetition and understanding are interrelated and integrated. Finally, Chinese teachers regard memorizing and understanding are viewed as two intertwined components (Hess & Azuma, 1991; Marton, Dall'Alba & Tse, 1996; Dahlin & Watkins, 2000; Kember, 1996; Kember, 2000; Wang, 2006).

The theory of variation and its application to mathematics teaching

Based on these ideas, some authors (see for example, Marton & Booth, 1997; Marton, Dall'Alba & Tse, 1996; Gu, Huang & Marton, 2004) have argued that repetitive learning with certain "variations" could be meaningful. Lo and Marton (2012) point out that to see or experience a concept in a certain way requires the learner to be aware of certain features which are critical to the intended way of seeing this concept. With any given topic, there will be a number of critical features that need to be identified, if the concept is to be discerned. Likewise, Leung (2003) points out that discernment comes about when features of the concept are being focused and temporarily demarcated from the whole. Thus, the central idea of teaching with variation is to highlight the essential features of the concepts through varying the non-essential features (Gu, Huang & Marton, 2004). Variation theory is posited on the view that "when certain aspects of a phenomenon vary while its other aspects are kept constant, those aspects that vary are discerned" (Lo, Chik & Pang, 2006, p.3). In order for students to discern a concept or a set of concepts, they need to experience different forms of variation in which the critical features of the concept(s) will be "varied" and "not varied". Accordingly, certain form(s) of variation and invariance characterize certain ways of experiencing a phenomenon, and bring about particular ways of experiencing a particular phenomenon (Marton & Booth, 1997). In short, "variation enables learners to experience the features that are critical for a particular" (Leung, 2003, p.199). Variation theorists argue that this is how students' learning occurs (Marton & Morris, 2002; Marton & Tsui, 2004).

According to the Theory of Variation, a key feature of learning involves experiencing a phenomenon in a new light (Marton, 1999). In other words, “learning amounts to being able to discern certain aspects of the phenomenon that one previously did not focus on or which one took for granted, and simultaneously bring them into one’s focal awareness” (Lo, Chik & Pang, 2006, p.3). Thus, teaching with variation helps students to actively try things out, and then to construct mathematical concepts that meet specified constraints, with related components richly interconnected (Watson & Mason, 2005). Building on this idea, teaching with variation matches the central idea of constructivism, seeing learners as constructors of meaning (Watson & Mason, 2005).

Rote drilling or promoting deep understanding?

Gu and Marton’s identification of forms of variation offers a systematic way to look at mathematical exercises in terms of what is available for the learner to notice (Marton & Booth, 1997; Marton, Runesson & Tsui, 2003) by asking progressively and systematically “what changes and what stays the same” (Watson & Mason, 2006). Simon and Tzur (2004) provide a useful précis of the idea behind variation when they argue that a well-designed sequence of tasks invites learners to reflect on the effect of their actions so that they recognize key relationships. Likewise, Watson and Mason (2005) point out that mathematics is learned by becoming familiar with tasks that manifest mathematical ideas and by constructing generalizations from tasks. However, Watson and Mason (2006) caution that learning does not necessarily take place solely through learners observing some patterns in their work, even if they have generalized them explicitly. They argue this is so because “learners can do this by focusing on surface syntactic structures rather than deeper mathematical meaning - just following a process with different numbers rather than understanding how the sequence of actions produces an answer” (Watson & Mason, 2006,

p.93). Likewise, Burkhardt (1981) and Doerr (2000) recognise the limitation of a sequence of tasks if students' focus is on the calculations because the attention may be shifted from making generalizations to "rote drilling". Unless specific steps (and in this paper, we propose the use of the theory of variation together with appropriate teacher scaffolding) are taken to promote students' engagement and higher order thinking, advanced conceptual understanding will not be constructed.

To minimize the possibility of random collections of items treated as individual tasks by students and to maximize the possibility of non-arbitrary relationship building, Watson and Mason (2006) suggest that teachers aim to constrain the number and nature of the differences they present to students and as a result, increase the likelihood that students' attention will be focused on the intended mathematical concepts. Likewise, Watson and Chick (2011) highlight the importance of teachers selecting mathematical tasks and examples with adequate variation to ensure that the critical features of the intended concept(s) are exemplified without unintentional irrelevant features (see also Goldenberg & Mason, 2008; Rowland, 2008). Thus, a crucial point in the use of variation is that it should be controlled and systematic in every case. What is varied and what remains invariant is intended to have direct impact on the pupils' discernment of concept(s). By systematically keeping some things invariant while others are varied and then changing what is varied and what remain invariant, students are able to "see" (to directly perceive or intuit) concept(s). Interestingly, despite repetition in exercises (Vincent & Stacey, 2008), some Western textbooks do not use variation in a systematic way such that it enhances students' capacity to generalize (Yeap, Ferrucci & Carter, 2006).

Thus, for teachers, it is critical to consider how to create proper forms of variation in mathematics classroom and form the necessary condition for students' learning in relation to

the intended learning goal(s) (Huang & Leung, 2004). Based on this rationale, in the context of the Chinese way of teaching, a sequence of similar tasks is not considered rote-drilling exercises but tasks that promote deep understanding.

Procedural variation

Gu (1981) identified two major types of variations in mathematics teaching: conceptual variation and procedural variation. Conceptual variation (see Lo & Marton, 2012; Leung, 2003) aims at providing students with multiple perspectives and experiences of mathematical concepts (Gu, Huang & Marton, 2004). Procedural variation aims to provide a process for formation of concepts stage by stage, in which students' experience in solving problems is manifested by the richness of varying problems and the variety of transferring strategies (Gu et al., 2004). Building on this idea, Watson and Mason (2005) argue that because some features of problems are invariant while others are changing, learners are able to see the general through the particular, to generalize, and to experience the particular. Huang and Leung (2004) point out that the function of procedural variation is to help learners acquire knowledge step-wise, develop progressively learners' experience in problem solving, and form well-structured knowledge. As the Chinese education system puts strong emphasis on drilling and procedural skills, and as this emphasis forms an important part of the paradox of the Chinese learner, this paper will focus on procedural variation.

According to Gu et al. (2004), procedural variation is derived from three forms of problem solving: (1) varying a problem: extending the original problem by varying the conditions, changing the results and generalization; (2) multiple methods of solving a problem by varying the different processes of solving a problem and associating different methods of solving a problem; (3) multiple applications of a method by applying the same method to a group of similar problems.

The first form of procedural variation: varying a problem

This form of variation aims to consolidate a concept by extending the original problem by varying the conditions, changing the results and making generalization. It provides students with an opportunity to experience a way of mathematical thinking, investigating the cases from special to general, from which students can see and construct mathematical concept(s) (Watson & Mason, 2005). The following detailed example, based on teaching division involving decimal numbers, will illustrate the idea of how students' conceptual understanding and higher order thinking skills can be enhanced by changing one condition of the original problem.

When teaching division involving decimal numbers a common misconception; that is, "division makes smaller", can interfere with student understanding. It is important to distinguish the two different meanings in division. Partitive problems refer to a group of items shared into equal groups where the number of groups is known but not the number in each group (e.g, You have 12 chocolates to share fairly with 4 friends. How many chocolates will each friend have?). Measurement problems refer to a group of items to be shared into equal groups (e.g, Each jar holds 3L of apple juice. If there are 9L of apple juice in the pitcher, how many jars will that be?). Many teachers find that the meaning of partitive problems is more easily understood and manipulated by students. This may be because the concept of sharing always goes in hand with answers which are smaller than the original number (i.e., dividend) in all kinds of division, including fraction and decimal. Students' misconceptions tend to occur in understanding the meaning of measurement problems. Firstly, the concept itself is quite confusing and difficult and secondly, the nature of results changes when the divisor changes from a whole number to a decimal and therefore, the generalization, "division makes smaller", is no longer applicable. A typical error can be illustrated by an example such as, "Each jar holds 0.5L of apple juice. If there are 10L of apple juice in the

pitcher, how many jars will that be?", students will give "0.5 is divided by 10" as the answer instead of "10 is divided by 0.5" as the former yields an answer smaller than 10. In order to help students construct and consolidate the concept of measurement model of division and therefore, address the misconception about "division makes smaller" progressively, procedural variation in the form of varying the conditions in the original problem can be adopted. For example:

Problem 1: There are 9L of apple juice and every 3L is put in a jar. How many jars are needed?

Problem 2: There are 9L of apple juice and every 1L is put in a jar. How many jars are needed?

Problem 3: There are 9L of apple juice and every 0.3L is put in a jar. How many jars are needed?

Problem 4: There are 9L of apple juice and every 0.1L is put in a jar. How many jars are needed?

Problem 5: There are 9L of apple juice and every 0.05L is put in a jar. How many jars are needed?

In this series of tasks, the total amount of apple juice was kept constant while the amount in a jar was varied from a whole litre to less than a litre. This exercise might be considered rote drilling if computing for a correct answer is the focus. However, an experienced mathematics teacher will organise this series of tasks hierarchically and provide scaffolding to illustrate and generalize the following mathematical ideas:

1. the smaller the divisor, the bigger the quotient;
2. dividing a whole number by a decimal less than one, will always result in a number more than the whole number (i.e., dividend);

3. measurement model of division can be described in the following way using an example of "10 is divided by 0.5": how many halves are in 10 or breaking up 10 into halves. Therefore, the result is not necessarily smaller than 10. In other words, division does not always mean smaller.

Some mathematics educators contend that it is important to start with simple decimals that students can visualize, as they find the ideas easier to understand if mathematical explanation is accompanied by pictures (Levenson, Pessia & Tirosh, 2010). However, Wu (1999) argues that we should not make students feel that the only problems they can do are those they can visualize. For example, what do the visual thinking skills have to say about "0.5 is divided by 0.133". He further states that if students only have enough understanding of simple decimals to do simple division, but nothing else, then this understanding is fragile and defective. In fact, the series of tasks above contains mathematical reasoning that would enhance students' higher order thinking skills and understanding of our decimal number system. In other words, this form of procedural variation by varying the conditions of the original problem provides a hierarchical system of experiencing processes through forming concepts (Gu et al., 2004). This teaching therefore can be considered meaningful "repetitive learning" but is definitely not "rote drilling".

Watson and Mason (2006) report that the results of generalizations created by students became tools for more sophisticated mathematics, and are a significant component of their mathematical progress.

Second form of variation: multiple methods of solving a problem

A second form of variation aims to foster students' problem solving skills by varying the methods of solving a problem. Gu et al. (2004) stated that students' experience in solving problems is manifested by the variety of transferring strategies. This idea will be illustrated by discussing the three different methods of solving simultaneous equations.

When teaching systems of equations in two variables, students often have difficulty in understanding that no single answer can be decided upon for an equation such as, $x + y = 30$. Clearly, from this equation, we can only say that if $x = 12$, then $y = 18$; if $x = 17$, then $y = 13$, and so on. With more information perhaps we can find a unique answer, for instance, $x = y + 6$. Therefore, a single answer can be sought by solving the system of equations:

$$x + y = 30$$

$$x = y + 6$$

In many secondary school mathematics syllabuses, three different methods (i) substitution, (ii) elimination, (iii) graphical method are presented. Teachers may introduce the three different methods separately and emphasis will be put on students understanding under what circumstances each method should be employed. For example, the substitution method is very effective and can be used to solve any kind of simultaneous equation. If you are to solve equations such as $x - 2y = 13$ and $5x + 2y = 5$, in which both equations have the same coefficient of y , elimination (i.e., addition method) will be more effective than substitution. Graphical methods are often introduced last. Students may be given opportunity to practise these three different methods until they are proficient in executing these computation skills.

At this point, some students may question why they have to learn three different methods if the substitution method is effective enough to solve all kinds of simultaneous equations. In order to help students to make conceptual connections between the three different methods, an experienced teacher will adapt procedural variation by solving a set of simultaneous

equations with each of the three methods. In fact, this strategy can be just a form of rote learning if the sole focus is on the fluency of computation skills. However, using this form of procedural variation, students can be helped to understand how to interpret the graphical representation algebraically. The following example illustrates these ideas:

1. an linear equation with two variables can be represented by a straight line in a rectangular cartesian coordinate system and the ordered pairs on the straight line are the solution sets of the given equation;
2. therefore, graphically, the intersection of the two straight lines of a given set of simultaneous equations is the solution set as this ordered pair is a solution set for both straight lines;
3. the solution set obtained algebraically (i.e., substitution and elimination methods) when interpreted graphically is the intersection of the two straight lines;
4. the pitfall of graphical method is the estimation (i.e., rounding off) of the solution set when the values of x and/or y are/is not whole number(s);
5. the power of algebraic method lies in reducing two variables to one variable. When a single answer for that variable is obtained, the other variable can be sought out as well;
6. consider graphically, when two straight lines are parallel to each other, there is no intersection point and therefore, no one solution set can be obtained both graphically and algebraically. If two straight lines coincide, there will be infinite solutions;
7. two variables can be solved by a system of equations involving two equations, following this idea, three variables can be solved by a system of equations involving three equations and so on.

Gu et al. (2004) stated that by creating this form of procedural variation, students are able to comprehend different components of a concept and hence upgrade their structure of knowledge, in the meantime, a non-arbitrary relationship between different components of

the concept can be built. In other words, this form of procedural variation offers a structured and structural approach to exposing underlying mathematical forms and therefore, can enhance students' conceptual understanding of a series of related concepts. Likewise, through extending the relationships between and within different mathematical tasks and through developing familiarity with these tasks, learners can gain fluency and facility in associated techniques (Watson & Mason, 2005).

Third form of variation: multiple application of a method

This form of variation aims to manifest a series of concepts by applying the same method (i.e., invariant) to a group of similar problems (i.e., to be varied). It is not surprising that students and teachers will consider it a rote learning if students are asked to repeat a similar group of problems by using similar methods or strategies. However, Gu et al. (2004) suggested that during the process of solving problems, if separate but interrelated learning tasks are reorganised into an integration, it can provide a platform for learners to make connection between some interrelated concepts. Likewise, Zawijewski and Silver (1998) pointed out that variation is a tool to scaffold the construction of different tasks that are conceptually related. Thus, the structure of the tasks as a whole, not the individual items can promote common mathematical sense making (Watson & Mason, 2006). The following examples illustrate this idea.

Problem 1: In a room with 4 people, everyone shakes hands with everybody else exactly once. How many handshakes are there?

We can employ a visual representation strategy by drawing a diagram (Posamentier & Stepelman, 2002) as in Figure 1.

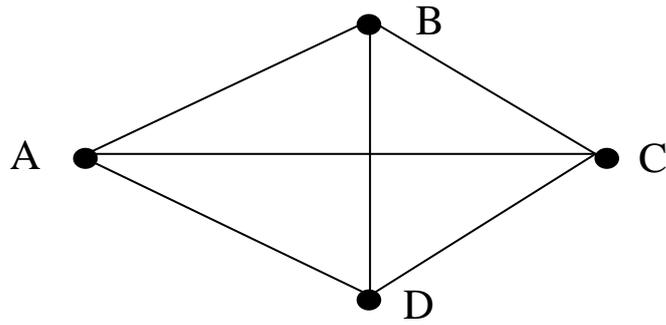


Figure 1

The 4 points represent the 4 people. Begin with the person represented by point A. We join A to each of the other 3 points, indicating the first 3 handshakes that take place. Now, from B there are 2 additional handshakes (i.e., C and D). Similarly, from C there will be 1 line drawn to the other point (i.e., D). Thus, the sum of the handshakes is $3 + 2 + 1 = 6$. We can use this idea to solve some extended problems for examples, how many handshakes if there are 5, 6, 7 and 8 people. The visual representations are as shown in Figure 2:

Number of people	Number of handshakes	Visual representation
5	10	
6	15	

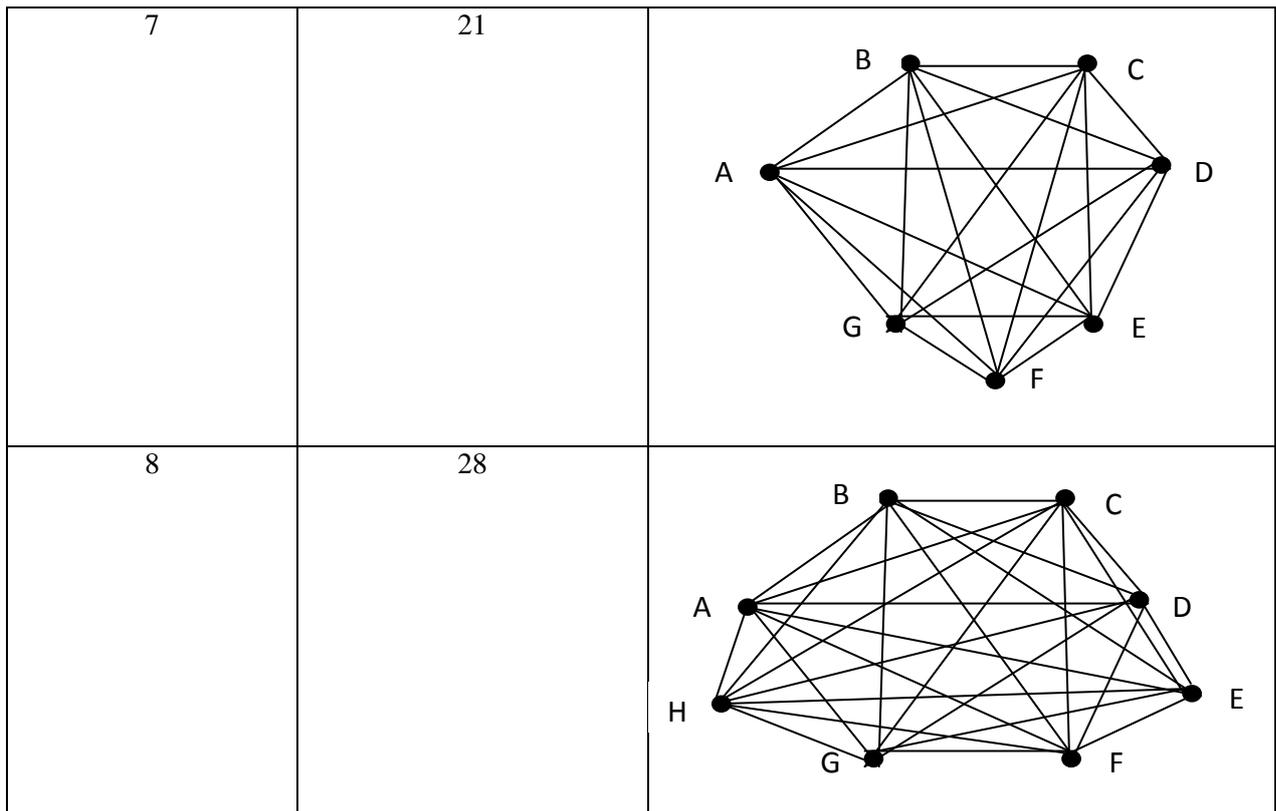


Figure 2

To many students and teachers, the series of tasks shown above, may appear to be rote drilling or just a repetition *if the focus is merely on the application of the formula*, $n + (n-1) + (n-2) + (n-3) + \dots + 2 + 1$, to this specific type of problems. However, when we examine carefully the visual representations of the 4, 5, 6, 7 and 8 people handshakes and look for a pattern, the problem has now become a geometry problem, where the answer is the sum of the number of sides and diagonals of a n -sided polygon (Posamentier & Stepelman, 2002). So, we can make use of this idea to solve the following problem.

Problem 2: How many diagonals in dodecagon?

Since the sum of the number of sides and diagonals of a dodecagon = $11 + 10 + 9 + 8 + \dots + 2 + 1 = 66$, therefore, the number of diagonals in a dodecagon = $66 - 12 = 54$. In fact, we can approach the solution for the handshakes problem by considering the combinations formula

of N things taken 2 at a time (i.e., ${}_N C_2$). This idea can now be applied to solve the following problem.

Problem 3: In a village there are 20 streets. All the streets are straight. One lamp post is put up at each crossroads. What is the greatest number of lamp posts that could be needed?
(answer: 190)

But the visual representation of the handshakes problem may not be good enough to illustrate lamp post problem or may even confuse students. We can consider the followings as in Figure 3:

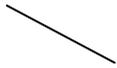
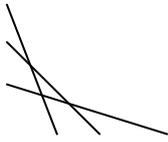
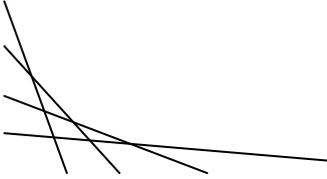
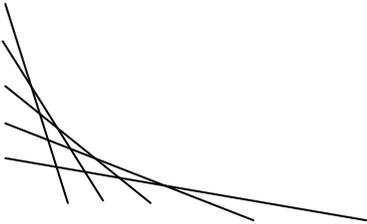
Number of street	Number of lamp post	Visual representation
1	0	
2	1	
3	3	
4	6	
5	10	

Figure 3

If we examine the visual representations of the lamp post problem carefully and use our imagination, it is not difficult to realize that they are pictures of line design! Figure 4 is an example of 10 lines (i.e., 10 streets in the lamp post problem).

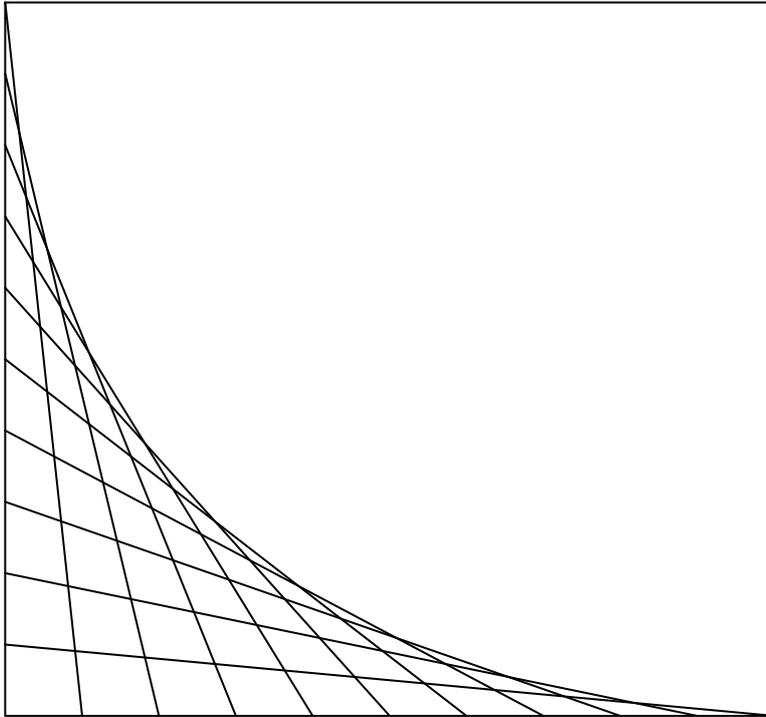


Figure 4

The preceding three problems illustrate how application of the same method to a group of similar, but varying problems can provide a scaffold for learners to make connections between interrelated mathematical ideas. As noted earlier, in this form of procedural variation, it is the structure of the tasks as a whole which encourages mathematical sense making (Watson & Mason, 2006). This form of procedural variation provides a final illustration of the way in which a sequence of similar tasks can promote deep understanding.

Unfolding the paradox of Chinese learners

The discussion and illustrations in this paper highlight the ways in which strategies of variation can foster meaningful student learning – regardless of cultural context. When non-arbitrary and substantive connections between knowledges are well structured, students can be helped to understand the essential features of different mathematical concepts. The three examples as outlined in the previous sections fully illustrate this point.

As noted earlier, studies (such as Marton, Dall’Alba & Tse, 1996; Biggs, 1996; Marton, Watkins & Tang, 1997; Dahlin & Watkins, 2000; Kennedy, 2002; Wang, 2006) consistently find that Chinese teachers do not see repetition and understanding as separate but rather as interlocking processes, complementary to each other (Watkins & Biggs, 2001). Thus, those Western educators who reject rote and repetitive learning may have failed to understand the learning strategy in the Chinese context. We argue that equating repetitive learning with “surface learning without understanding” oversimplifies and misinterprets the intrinsic meaning of the Chinese notion of learning. The value of procedural variation for enhancing student understanding goes some way to explain why Chinese students consistently outperform their Western counterparts in many international comparative studies, and thus to unfolding the paradox of the Chinese learner.

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