Error Analysis for Arithmetic Word Problems – A Case Study of Primary Three Students in One Singapore School

Lu Pien Cheng
National Institute of Education, Singapore
lupien.cheng@nie.edu.sg
1 Nanyang Walk, Singapore 637616
lupien.cheng@nie.edu.sg
Tel: (65) 67903954

In this study, ways in which 9-year old students from one Singapore school solved 1-step and 2-step word problems based on the three semantic structures were examined. The students’ work and diagrams provided insights into the range of errors in word problem solving for 1-step and 2-step word problems. In particular, the errors provided some suggestions to the design of an intervention programme to improve students’ abilities to solve word problems.

Brief author-biography:
Cheng Lu Pien, received her PhD in Mathematics Education from the University of Georgia (U.S.) in 2006. She is an assistant professor with the Mathematics and Mathematics Education Academic Group at the National Institute of Education (NIE), Nanyang Technological University, Singapore. She specializes in mathematics education courses for primary school teachers. Her research interests include the professional development of primary school mathematics teachers, tools and processes in mathematics education programmes for pre-service teachers. Her research interests also include mathematical thinking of children in the mathematics classrooms.

Introduction

One of the most common mathematics difficulties is difficulties with word problems (WPs) and with multi-step problems (Bryant, Bryant & Hammill, 2000). Grade 4 students from Singapore were reported to underachieve in mathematics in TIMMS 2007 “compared to their peers from Hong Kong and Chinese Taipei when comparing their mean scores up to the 5th, 10th, 15th and 20th percentiles” (Kuar, Koay, Foong & Sudarshan, 2012, p. 1). Such findings prompted studies on low attainers in primary schools in Singapore. One such study reported solving WPs as an area of poor performance for Primary 4 students (Koay, Chang & Ghani, 2012). Hence, deeper insight into students’ errors in solving WP is needed to shed greater light on a course of action to address those errors. Furthermore, the solving of mathematical WPs is a major component both within the instructional program as well as during formal assessments in Singapore primary mathematics curriculum. In particular, WPs involving addition and subtraction in the lower primary in Singapore are fundamental to mathematical WPs in the upper primary.

The purpose of this study is to examine students’ errors in solving 1-step and 2-step addition and subtraction WPs in the Singapore context. The workings and diagrams constructed by the students while solving the WPs were examined.

Word Problems in the Singapore Context

In Singapore’s primary mathematics curriculum, the model method or model drawing (diagram) is one of the heuristics to solve arithmetic and algebraic WPs. The model approach
is documented by the curriculum planning and development division, Ministry of Education, Singapore, as a resource book in *The Singapore Model Method* (MOE, 2009). Students in Singapore are taught the model method as early as primary 1. These models capture all the information in a WP and help the students to visualize the WP. According to Ng and Lee (2009, p. 285),

it was believed that if children were provided with the means to visualize a word problem, the structure underlying the problem would be made overt. Once children understood the structure of the problem, they were more likely to be able to solve it (Kho, 1987).

The structure of the model diagrams consists of a series of rectangles. In arithmetic WPs, the rectangles represent specific numbers but they can also represent unknown values. The structure of the model is used to help students “construct appropriate sets of step-by-step arithmetic procedures to solve given problems” (Ng & Lee, p. 285). The part-whole and comparison models are the two types of models used in Singapore in solving basic word problems. As shown in Figure 1, the part-whole model depicts quantitative relationship among three quantities: the whole and two parts while the comparison model is used to “compare two quantities to show how much one quantity is greater (or smaller) than the other” (MOE, 2009, p. 18).

The part-whole model for addition and subtraction (combine model) can be used to solve the following arithmetic WP, “A fruit basket has 5 apples and 2 oranges. How many fruit are there in the basket?” (Chan, 2014, p. 21). The comparison model can be used to solve the following arithmetic WP, “A fruit basket has 5 apples and 2 oranges. How many more apples than oranges are there?” (Chan, p. 21). The part-whole model and comparison model problems and their associated model diagrams are commonly found in Primary 3 textbooks (for example, see Collars, Koay, Lee, & Tan, 2015, Primary 3A, p. 29).

**Word Problems**

WP solving is considered a key component in learning mathematics (NCTM, 2000; OECD, 1999). WPs not only have the potential of motivating students, developing new mathematical concepts and skills meaningfully but also developing the skills to apply mathematics effectively in our day-to-day activities (Verschaffel, Greer, & De Corte, 2000; Boaler, 1993; Hiebert et al., 1996).
WP is defined as “textual descriptions of situations within which mathematical questions can be contextualized (Verschaffel et al., p. ix)”. A WP is a situation where “a mathematical model can be applied to represent the quantities and relations present in the text and to find a solution to the given question” (Nortvedt, 2011, p.256). Most of the WPs can be solved by a simple and straightforward application of one or a combination of the four basic arithmetic operations on the given numbers (Davis-Dorsey, Ross, & Morrison, 1991; Gravemeijer, 1997). A one-step arithmetic WP can be solved using one basic operation. A multi-step arithmetic WP can be solved using a combination of basic operations (Reed, 1999).

Standard WPs refer to these straightforward problems that can be found in the school curriculum. Realistic WPs refer to problems where the problem is embedded in a real-life context and the problem solver has to consider the realities of the context of the problem situation. As such, realistic WPs may not be solvable by the direct application of arithmetic operation(s).

Research in arithmetic WPs

A corpus of research focusing on the assessment and understanding of young children's problem solving with respect to addition and subtraction on WPs can be found. According to Corte & Verschaffel (1987, p. 363), the research can be categorised into (a) the difficulty level of different types of WP; (b) “the strategies children use to solve those problems, and the nature of their errors”; and (c) “building explicit models of the knowledge structures and solution processes underlying children's performances on those problems”.

Various variables affect WP difficulty. They include context familiarity, number of words, sentence length, readability, vocabulary and verbal clues, magnitudes of numbers and sequence of operations (Caldwell & Goldin, 1979). The difficulty level of WP also depends on mechanics, (e.g. problem length, readability, order of data, etc.), format (e.g. written or oral format) and context (Hembree and Marsh, 1993).

Major areas of difficulties in solving WPs

Several factors affect students’ ability to solve word problems. Reading and computation are widely recognised as one of the factors (Balow, 1964; Cohen & Stover, 1981; Glennon & Callahan, 1968; West, 1977). Other factors include computational proficiency (Koay, Chang & Ghani, 2012), failure to recognize relationships between text elements and not fully understanding the WPs (Nortvedt, 2011). Interpretation of the problem situation, selection of the appropriate operation or operations can also affect WP solving ability (Kantowski, 1980). The misrepresentation of a single piece of information can result in an incorrect solution (Ng & Lee, 2009).

The study conducted in Singapore by Koay, Chang & Ghani (2012) shows that “Pupils’ responses showed a lack of understanding when either more than one-step or more than one operation is required to solve the problem.” (p. 47). Ballew and Cunningham (1982, p. 203) classified four different abilities involved in solving problems and suggested that each of these abilities can be investigated to explain what areas of solving WPs “present greatest difficulty to particular types of students”. The four abilities are (p. 203):

(a) the ability to read the problem;
(b) the ability to set up the problem so that the necessary computation is ready to be performed;
(c) the ability to perform the necessary computation, and;
(d) the ability to integrate reading, interpreting the problem, and computation into the total solution of a WP.

Causes of wrong solutions to WPs
One of the causes of erroneous solutions to WPs is number grabbing (e.g., Brekke, 1995; Hegarty, Mayer, & Monk, 1995; Reed, 1999). Number grabbing is explained by Hegarty et al as directly translating “the key propositions in the problem statement [key relational terms (such as "more" and "less")] to a set of computations that will produce the answer and does not construct a qualitative representation of the situation described in the problem” (p. 19).

Another category of wrong responses that is widely researched is solutions that do not make sense based on the context of the problem. That is, students did not apply common sense knowledge when solving WPs. The lack of ability to integrate commonsense knowledge with school-learned mathematics was reported in several studies (Greer, 1993; Reusser & Stebler, 1997; Verschaffel, De Corte, & Lasure, 1994; Yoshida, Verschaffel, & DeCorte, 1997). Some researchers reported that this difficulty in applying commonsense knowledge when solving WPs is a result of the instructional practices (Greer, 1993; Verschaffel, et al 1994). For example, school instruction may not require students to provide final solutions that are reasonable and based on the context of the problem. Instructional practices and experiences may also result in the belief that routine WPs can be solved using straightforward calculations without making reference to the context of the problems (Greer, 1993; Verschaffel & DeCorte, 1997). Perhaps, students will be able to apply common sense knowledge in solving WPs when the mathematics they learn is sensible to them. That is, they are able to “view that mathematics is a connected, coherent system in which there are reasons for such things as rules, procedures, and formulas, whether or not the individual yet knows or understands the reasons.” (Grady, 2013, p.5).

Quantitative operations, quantitative reasoning, numerical operations and number sense in WP solving
Quantitative operations, quantitative reasoning and numerical operations are important mathematical abilities required in solving WPs. When the problem solver is engaged in quantitative operation, the problem solver performs the mental operation of combining two or more quantities to produce a new quantity (Thompson, 1993). To reason quantitatively, the problem solver needs the ability to see “the problem situation as a network of quantities and their internal relationships and to reason with them by forming a series of quantitative operations in the establishment of the solution” (Lioe, Liu & Fang, 2008, p.2). Hence, mental operations and mental representations are important in WP solving.

The construction of an appropriate mental representation during WP solving can be facilitated by making the semantic relations in the problem more explicit. When the problem solver evaluates the new quantity, the problem solver is involved in numerical operation.

Number sense is a “key ingredient in the ability to solve basic arithmetic computations” (Gersten & Chard, 1999, p.20). Number sense can broadly be defined as “a child’s fluidity and flexibility with numbers, the sense of what numbers mean, and an ability to perform mental mathematics and to look at the world and make comparisons” (Gersten & Chard, p.20). Developing good number sense is important in solving WPs. Berch (2005) stated that possessing number sense ostensibly permits one to achieve everything from understanding the meaning of numbers to developing strategies for solving complex math problems; from making simple magnitude comparisons to inventing procedures for conducting
numerical operations; and from recognizing gross numerical errors to using quantitative methods for communicating, processing, and interpreting information (p. 334)

Developing number sense requires awareness of quantities, giving meaning to quantities and on relating the different meanings to each other (Van den Heuvel-Panhuizen, 2001). This insight into numerical relations can be achieved through splitting or decomposing and composing of quantities (Hunting, 2003; Steffe, Cobb, & Von Glasersfeld, 1988).

Theoretical Framework

Semantic representation in arithmetic WPs

Three main types of semantic schemas were distinguished in studies on WPs requiring simple addition or subtraction operations, that is, one-step addition and subtraction problems (Riley and Greeno, 1988): (a) change structure; Change problems are those in which an initial quantity is increased or decreased. The change quantity results in a final quantity e.g. Anna had 3 pencils, Bala gave her 5 pencils. How many pencils does Anna have now? (b) combine structure; Combine problems are those in which two quantities are combined into a whole quantity e.g. Anna has 3 pencils. Bala has 5 pencils. How many pencils do they have altogether? (c) compare structure; Compare problems are those in which a quantity is compared to another quantity with a quantitative difference the two quantities e.g. Anna has 5 pencils. Bala has 8 pencils. How many pencils does Bala have more than Anna?

“Specific features within each semantic structure, like the identity of the unknown quantity, must also be taken into account” (Riley, Greeno, & Heller, 1983, p. 165), when trying to understand students’ ability to solve WPs. Riley et al grouped the one-step addition and subtraction WPs into four major classes and their specific subtypes within each class. Change, combine and compare problems are the same as above. An additional class is the equalize problems (a variant of the change problems) with equalization indicated by the phrase, “to have as many as”. For example, “Rose has 7 marbles. Dora has 10 marbles. How many marbles must Rose get to have as many as Dora?” (Morales, Shute, & Pellegrino, 1985, p. 43). 16 problems from the specific subtypes can be found in (Riley et al, p. 160; Morales et al, p. 43). Some of the problems are as follows:

Examples of easiest problems:

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change problem CH1</td>
<td>Pete had 7 marbles. Then Sam gave him 5 more marbles. How many marbles does Pete have now? (p. 43)</td>
</tr>
<tr>
<td>(Result Unknown)</td>
<td></td>
</tr>
<tr>
<td>Combine problem CB1</td>
<td>Fred has 7 marbles. John has 5 marbles. How many marbles do they have altogether?</td>
</tr>
<tr>
<td>(Total Set Unknown)</td>
<td></td>
</tr>
</tbody>
</table>

Examples of the most difficult problems:

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Change problem CH5</td>
<td>Emily had some marbles. Then Ana gave her 8 more marbles. Now Emily has 14 marbles. How many marbles did Emily have in the beginning?</td>
</tr>
<tr>
<td>(Start Unknown)</td>
<td></td>
</tr>
<tr>
<td>Combine problem CB2</td>
<td>Eddie and Roy have 11 marbles altogether. Eddie has 4 marbles. How many marbles does Roy have?</td>
</tr>
<tr>
<td>(Subset Unknown)</td>
<td></td>
</tr>
<tr>
<td>Compare problem CP5</td>
<td>Jerry has 10 marbles. He has 4 more marbles than Bob. How many marbles does Bob have?</td>
</tr>
<tr>
<td>(Referent Unknown)</td>
<td></td>
</tr>
</tbody>
</table>

Examples of problems of intermediate levels of difficulty:

<table>
<thead>
<tr>
<th>Type</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td>Compare problem CP3</td>
<td>Bill has 9 marbles. James has 7 more marbles than Bill. How many marbles does James have?</td>
</tr>
</tbody>
</table>
The semantic structure of the WP plays an important role in the design of WPs for primary school students (De Corte & Verschaffel, 1991). The semantic content causes different levels of difficulty in arithmetic WPs (Riley & 1988). When the range of structurally similar problems are given to students, the unsuccessful problem solvers may find it difficult to transfer what they have learnt to those problems because they focus on the surface features of problems (Silver & Marshall, 1990). Successful problem solvers look for structural information, that is, problem schemata of the problem. However, the semantic schema may become less noticeable or incomprehensible for young children as a result of the linguistic features of a problem. (De Corte, Verschaffel, & De Win, 1985).

Semantic structures play a part in determining pupils' performance in the solving of mathematical word problems (Koay, Chang & Ghani, 2012; Carpenter, Moser, & Bebout, 1988; Yeap and Kuar, 2001). “Comparison structure is deemed more challenging and complex in comparison to joint and change structures” (Koay, Chang & Ghani, p. 42). “Problems involving more than one semantic relation (multi-related) yielded poorer performance than those of common semantic relation (single-relation)” (Koay, Chang & Ghani, p. 46). The same result was reported by Yeap & Kaur (2001).

Chan (2014, p.21) emphasizes the benefit of knowing the classification of problems “if we can identify the type children usually have problems in, we will then be able to zero-in on the specific areas of concern”. In the study with low attainers, Kaur, Koay, Foong & Chang (2012, p. 161) suggested that when teaching word problems, teachers should “expose pupils to WPs involving similar operations in varied semantic structure. Moreover, class discussion on the alternative solutions to a problem should be included”.

WP solving process and diagrams as a strategy to solve WPs
Students go through several phases in order to solve a WP, from comprehending the text to modeling, executing, and evaluating the answer (Verschaffel, Greer, & De Corte, 2000). Before a student can solve a WP, he or she must first understand what the WP is about. That is, the student must read the WP, form a situation model that serves as a basis toward solving the WP (Thevenot, Devidal, Barrouillet, & Fayol, 2007). This situation mental model “is a temporary structure stored in working memory that contains, in addition to the mathematical information necessary to solve the problem, nonmathematical information that is related to the context in which the situation described by the problem takes place” (Thevenot et al., 2007, p. 45). After the problem solver access and model the mathematical situation embedded in the text, only then can the problem solver solve the WPs.

One of the strategies use in solving WPs is the use of diagrams. A diagram is “a visual representation that displays information in a spatial layout” (Diezmann & English, 2001, p. 77). Diagrams are powerful strategies in solving WPs because they can be used to unpack the structure of a problem, simplify a complex problem, make abstract concepts concrete (Diezmann & English; Novick, Hurley, & Francis, 1999). Drawing a diagram to solve WPs has been strongly advocated by many researchers (Diezmann, 2000; NCTM, 2000; Shigematsu & Sowder, 1994). Kuar, Koay, Foong & Chang (2012) also suggested that “organizing information in word problems such as with a diagram / chart would help to improve pupils’ performance” (p. 161). The use of visual and concrete representations in improving performance in solving WPs is also reported by Lewis (1989), Willis & Fuson, (1988), Davydov (1962), Davydov & Steffe (1991) and Freudenthal (1974). However, researchers warned that it is possible for this strategy to interfere with the solution process.
which may in turn results in poor problem-solving performance (Diezmann, 2000; Larkin & Simon, 1987). This is especially so when the problem solver has difficulty with the representation and has not grasped the representations fully (Hegarty & Kozhenvikov, 1999).

Method

This study is a case study of one Singapore Primary School that seeks to understand the students’ solutions in 1-step and 2-step WP.

Participants

The study was conducted with 251 Primary Three children at the beginning of the year after completing the whole numbers topic. Primary Three students in Singapore are 9-year-olds. The children were from one mainstream government school with students representing different socio-economic groups in Singapore. The students were already streamed into the various classes according to their academic performances in language and mathematics from their final end of the year examination in Primary Two. The low-attainers were from Class 1, 2 and 3 (L1, L2 & L3). The middle-achieving children were from Class 4, 5 and 6 (M1, M2 & M3). The high achieving students were from Class 7 and 8 (H7 & H8). 70 students from L1, L2 and L3, 103 students from M4, M5, M6, and 78 students from H7 and H8 participated in this study.

The children had experience solving 1-step and 2-step WPs and were familiar with the part-whole (combine), comparison situations (compare) and change problems. All the students were familiar with writing number sentences and the use of model diagrams to represent the problem situation.

Professional development on the design of 1-Step WPs

One of the main goals of the teachers in this study was to level up the mathematical competency skills and knowledge of their Primary Three cohort before the students were promoted to Primary Four. The teachers were concerned with Primary Three students who were unable to solve 1-step and 2 steps WPs. The teachers wanted to examine the errors in solving WPs before planning for intervention programmes to address those errors. A diagnostic test was conducted to identify the errors made by the students.

This study is from a larger study that examines school-based professional development based on the needs of the participating school in two different phases. Phase 1 of the professional development involved designing 1-step WPs using the three semantic structures and diagnosing student errors in solving the WPs. Phase 2 of the study involved designing an intervention programme to address those errors. This study reports on Phase 1 of the professional development program.

A total of eight teachers from the same school participated in the professional development (PD) and research. Three teachers had more than 5 years of teaching experience. The rest of the teachers had 4 months, 1 year, 2 or 3 years of teaching experience.

The teachers had been using the following three phases of problem solution WPs (Ng and Lee, p. 289) to guide their students in solving 1-step.

- Phase 1: Text Phase (T): Children read the information presented in text form.
- Phase 2: Structural Phase (S): Children represent the text information in the structure of the model. Children can alternate between text and the model to check that the model accurately depicts the textual information.
• Phase 3: Procedural-Symbolic Phase (P): Arithmetic equations are formed

In short, the teachers elicit the information from the text (Phase 1), organize the information using drawn models (Phase 2), and identify the operations to be used, form the arithmetic equation and solve the problem (Phase 3). The teachers in the school called the teaching method chunking. “Each chunk may refer to a specific numerical quantity or specific information related to an unknown quantity” (Ng & Lee, 2009, p.291). The teachers would read the text several times and “chunk” the text before representing the chunk with model strips. Either part-whole or comparison model will be selected for the model diagrams. The teachers are especially concerned with the performance of students in L1, L2 and L3 classes and they felt that the errors in solving WPs in the three classes are largely due to language difficulty, not fully understanding the word problem and not being able to recognise relationships between the text.

A total of three 1-step WPs, each from the three semantic structures, were designed for the diagnostic test. Problem 1, 2 and 3 contained WPs associated to the combine, change and compare structures respectively. The teachers wanted the 1-step WPs to be simple and reflective of the semantic structure. Three 2-step WPs were also designed. Problem 4 was designed according to the combine-combine structure, problem 5 the combine-compare structure while problem 6 the combine-change structure.

Table 1
WP Types

<table>
<thead>
<tr>
<th>Problems designed by teachers</th>
<th>WP Types and Corresponding Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1-step WP</strong></td>
<td></td>
</tr>
<tr>
<td>(1) Jim has 132 red marbles and 45 blue marbles. How many marbles does he have altogether?</td>
<td>Combine &lt;br&gt;132 + 45 = 177</td>
</tr>
<tr>
<td>(2) John has 225 marbles. Paul gives John 30 marbles. How many marbles does John have now?</td>
<td>Change &lt;br&gt;225 + 30 = 255</td>
</tr>
<tr>
<td>(3) Ahmad has 120 marbles. Raju has 50 more marbles than Ahmad. How many marbles does Raju have?</td>
<td>Compare &lt;br&gt;120 + 50 = 170</td>
</tr>
</tbody>
</table>

| **Multiple-step WPs**         |                                     |
|-------------------------------|                                     |
| (4) Linda bought a jigsaw puzzle. She completed 68 pieces on the first day, 75 pieces on the second day and 57 pieces on the third day. How many pieces were completed? | Combine-combine <br>68 + 75 + 57 = 200 <br>Or <br>68 + 75 = 143 <br>143 + 57 = 200 |
| (5) Steven spends $240 on transport each month. Jane spends $80 on the MRT and $35 on buses each month. How much more does Steven spend on transport each month? | Combine-compare <br>240 - 80 - 35 = 125 <br>Or <br>80 + 35 = 115 <br>240 - 115 = 125 |
| (6) Samy and Devi caught 104 fish altogether. Samy caught 50 fish. Devi gave away 10 fish to her friend. How many fish has Devi left? | Combine-change <br>104 - 50 - 10 = 44 <br>Or <br>50 + 10 = 60 <br>104 - 60 = 44 |
Standard format that followed the standard textbook problems in Primary Two and Three in the students’ textbooks were used. The problems within the 1-step WP were nearly as identical as possible with respect to the following variables:
(a) concrete problems
(b) algorithm used to solve each problem is the same for all the three problems 3-digit add 2-digit without renaming.
(c) number of sentences is not more than 3.
(d) number of words in each problem is about 14.
(e) each problem used only vocabulary appropriate for Primary Three students, words are adapted from the Primary Three mathematics textbook used by the school.
(f) each of the problem from one semantic structure
(g) problem can be solved in the sequence in which the given values were given

The 2-step WPs involved two step-by-step arithmetic procedures involving either one or two semantic relations. Similar to the 1-step WPs, concrete problems were used. The mathematical skills involved addition and subtraction of whole number with regrouping within 200. The number of words in each problem is less than 30 and vocabulary familiar to the primary three students was used to construct the problem. This study investigated the following research questions:
(1) What errors were found in the Primary Three students’ solutions to the 1-step and 2-step WPs?
(2) What do the diagrams tell us more about the errors of the Primary Three students in solving 1-step and 2-step WPs?

Administration of the diagnostic tests
The six problems were administered by the teachers in this study as a piece of classwork during regular mathematics instructional hours. The students completed the problems individually during curriculum hours and submitted the class work once they completed the problems. Class L1, L2, L3, M5, M6 attempted the 1-step WP, problem 1-3. A total of 173 Primary Three students attempted the 1-step WPs. H7 and H8 did not attempt the 1-step WP because the teachers felt that the questions were too easy for the students. Class M4, M5, M6, H7, H8 attempted the 2-step WPs, problem 4-6. A total of 181 students from 5 classes attempted the 2-step WPs.

Problems 4-6 were thought to be too challenging by the teachers and they were not administered to L1, L2, L3. The students were asked to show all their workings in the space provided in the problem sheet. The WPs were not read to the students.

Data analysis for research question 1 & 2
All the interpretations were based on analysis of the students’ work. Students were not interviewed in this study. All the samples of students’ work were collected for analysis. During the first round of analysis, the correct and incorrect responses were identified. Correct responses were considered as responses with the final correct solutions with the correct operations and computations applied to the problem. Responses with correct final answer but no working were also considered correct responses. Responses were considered incorrect if wrong operations and computation errors were observed in the solution strategy.

During the second round of analysis, deeper analysis of the correct responses was identified followed by the incorrect responses.
• For the correct responses, the method used to solve each WPs was identified and categorized.
• For the incorrect responses, the type of errors made were identified and categorized. The classification of errors was described in detail below.

The errors were classified into (1) clerical/transfer error (2) computational errors (3) wrong operations used (4) no response, (5) incorrect response offering no clues (6) special case. The error classification was adapted from Knifon & Holtan (1976). They considered clerical errors “endemic to human activity and have little to do with understanding or WP ability” (p. 108). For an error to qualify as a clerical error,

(a) the correct answer was worked out and copied incorrectly or not at all into the answer space; (b) the correct operation was used but the wrong numeral was inserted because a numeral was miscopied from the problem to the computation, … or (c) the work was correct but the student became confused, skipped a problem, and copied the right answer into the wrong problem space (p. 108).

Computational errors are errors as a result of wrong calculations. In the category where the wrong operation was used, the error occurred when the problem was worked out using an incorrect operation or an incorrect/ incomplete series of operations. In the category where no response was given, these errors included the students skipping the problem and thus leaving an empty space in the students’ problem sheet. For the category where no clues were offered for the incorrect responses, errors without accompanying working belonged to this group. Special cases included cases where students’ work cannot be understood or incorrect unique solutions were given to the problem.

In order to prevent multiple listings, each incorrectly solved problem was assigned to only one type of error. If two or more errors were made within each problem was observed, a subcategory was created to record such errors. The results of the analysis were tabulated (see Appendix A, Table 10 for a sample of how results were tabulated). Next, from the correct responses, the typical correct responses were identified. The sample of students’ work from each category of errors made was examined to identify the nature of these errors. For example, one of the errors stems from incorrect use of mathematical equations.

Data analysis for research question 3
The students’ work was re-sorted into 3 categories: (a) no diagrams; (b) diagrams with correct answer; and (c) diagrams with incorrect answer. Diagrams with correct answers were then checked for variation in model drawings and the different types of model diagrams used (e.g. part-whole model and comparison model). Diagrams that yielded incorrect answer were teased out for further analysis to gain deeper understanding about students’ errors in solving WPs.

Results
Table 2 shows the performance of students for 1-step WPs. Performance for 1-step combine problem (Problem 1) for the 6 classes were the best followed by compare problem (Problem 3) and change problem (Problem 2). The percentage of students who gave the correct solutions was mostly higher for M4-M6 than L1-L3. The change problem (Problem 2) posed the most challenge for 4 out of the 6 classes.
Table 2

**Percentage of Students Who Answered 1-step WPs Correctly**

<table>
<thead>
<tr>
<th>Problem</th>
<th>L1 (n=16)</th>
<th>L2 (n=20)</th>
<th>L3 (n=34)</th>
<th>M4 (n=35)</th>
<th>M5 (n=35)</th>
<th>M6 (n=33)</th>
<th>Total (n=173)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
</tr>
<tr>
<td>1</td>
<td>7(43.75)</td>
<td>15(75)</td>
<td>33(97.06)</td>
<td>34(97.14)</td>
<td>32(91.43)</td>
<td>31(93.94)</td>
<td>152(87.86)</td>
</tr>
<tr>
<td>2</td>
<td>6(37.5)</td>
<td>11(55)</td>
<td>17(50)</td>
<td>29(82.86)</td>
<td>29(82.86)</td>
<td>23(69.70)</td>
<td>115(66.47)</td>
</tr>
<tr>
<td>3</td>
<td>8(50)</td>
<td>16(80)</td>
<td>16(47.06)</td>
<td>32(91.43)</td>
<td>31(88.57)</td>
<td>23(69.70)</td>
<td>126(72.83)</td>
</tr>
</tbody>
</table>

Table 3 summarizes the students’ performance for the 2-step WPs. For the three 2-step WPs, the performance for the combine-combine addition WP was the best. The percentage of students who was able to solve the 2-step WPs correctly was mostly higher for H7 and H8. One exceptional case was problem 4 with the lowest percentage of students that solved the problem correctly from H7.

Table 3

**Percentage of Students Who Answered 2-step WPs Correctly**

<table>
<thead>
<tr>
<th>Problem</th>
<th>M4 (n=35)</th>
<th>M5 (n=35)</th>
<th>M6 (n=33)</th>
<th>H7 (n=39)</th>
<th>H8 (n=39)</th>
<th>Total (n=181)</th>
</tr>
</thead>
<tbody>
<tr>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
<td>n(%)</td>
</tr>
<tr>
<td>4</td>
<td>27(77.1)</td>
<td>30(85.7)</td>
<td>30(90.9)</td>
<td>30(76.9)</td>
<td>36(92.3)</td>
<td>153(84.5)</td>
</tr>
<tr>
<td>5</td>
<td>22(62.8)</td>
<td>21(60)</td>
<td>18(54.5)</td>
<td>30(76.9)</td>
<td>34(87.2)</td>
<td>125(69.1)</td>
</tr>
<tr>
<td>6</td>
<td>18(51.4)</td>
<td>26(74.3)</td>
<td>21(63.6)</td>
<td>34(87.2)</td>
<td>37(94.9)</td>
<td>136(75.1)</td>
</tr>
</tbody>
</table>

Comparing, Table 2 and 3, the performance for 1-step addition WP was generally better compared to 2-step addition and subtraction WPs for M4-M6. However, performance for 2-step combine-combine addition WPs was better than 1-step change and 1-step compare problems for M4-M6. Performance for 2-step WPs (problem 5, combine-compare; problem 6, combine-change) is poorer than 1-step addition and subtraction WPs. Table 4 shows that for 1-step combine problem, the errors were mainly due to computation and transfer errors. For 1-step compare and change problem, the errors were mainly due to incorrect operations.

Table 4

**Types of Errors for 1-Step WP**

<table>
<thead>
<tr>
<th>Incorrect solutions based on 173 participants</th>
<th>Problem 1 (Combine)</th>
<th>Problem 2 (Change)</th>
<th>Problem 3 (Compare)</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct operations → compute incorrectly</td>
<td>7</td>
<td>6</td>
<td>1</td>
<td>14</td>
</tr>
<tr>
<td>Correct operations → transfer errors → compute correctly</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>9</td>
</tr>
<tr>
<td>Correct operations → transfer errors → compute incorrectly</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Incorrect operations → compute correctly</td>
<td>3</td>
<td>39</td>
<td>34</td>
<td>76</td>
</tr>
<tr>
<td>Incorrect operations → compute incorrectly</td>
<td>2</td>
<td>8</td>
<td>9</td>
<td>19</td>
</tr>
<tr>
<td>Incorrect response with no clues</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Special Case</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Total</td>
<td>21</td>
<td>58</td>
<td>47</td>
<td>126</td>
</tr>
</tbody>
</table>
Table 5 shows that the types of errors for 2-step WPs were similar to 1-step WPs, mainly, computation errors, transfer errors, incorrect operations. Missing or additional arithmetic operations were errors unique to the 2-step WPs. The use of wrong operations accounted for 42.9% and 55.6% of the errors for the second and third 2-step WPs respectively. Next, examples of typical correct responses and the types of errors that surfaced from students’ work are discussed.

Table 5

*Types of Errors for 2-Step WP*

<table>
<thead>
<tr>
<th>Incorrect solutions based on 181 participants</th>
<th>Problem 4</th>
<th>Problem 5</th>
<th>Problem 6</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct operations → compute incorrectly</td>
<td>20</td>
<td>20</td>
<td>7</td>
<td>47</td>
</tr>
<tr>
<td>Correct operations → transfer errors →</td>
<td>6</td>
<td>7</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>compute correctly</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Correct operations → transfer errors →</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>compute incorrectly</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Incorrect operations → compute correctly</td>
<td>0</td>
<td>22</td>
<td>21</td>
<td>43</td>
</tr>
<tr>
<td>Incorrect operations → compute incorrectly</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>Incorrect response with no clues</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Special Case</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Miss an operation</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Additional operation</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>No response</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>28</strong></td>
<td><strong>56</strong></td>
<td><strong>45</strong></td>
<td><strong>129</strong></td>
</tr>
</tbody>
</table>

*Typical correct responses*

Figure 2 shows two typical students’ work for WPs. The solution consists of a mathematical working column, mathematical sentence(s), and mathematical statement(s). Diagrams (model diagrams) were used in some of the students’ work. Most of the students represent problems with mathematical symbols both for solution efficiency with larger numbers and as a general strategy for representing problem situations mathematically.

**Problem 1 M5**

**Problem 4 H7**

*Figure 2. Typical correct students’ work.*

Students who gave the correct response mostly used the standard number sentences with the unknown to the right of the equals sign \((a + b = ?)\). In most cases, for the 1-step WP, the
 Errors in Students’ Solutions to 1-Step and 2-Step WPs

From the above diagnostic results, one key problem appeared to be a lack of competency in ‘picking up’ solutions that do not make sense numerically e.g. numerical value of the answer does not make sense in the part-whole structure. This type of error in WPs is found across all the three groups of students in this study for all the six problems. These errors are categorized into the following themes:

(a) the whole is smaller than the parts (e.g. 132 + 45 = 8 in Problem 1 L1; 104 - 50 -10 =144 in Problem 6 M5)
(b) the whole is much larger than the parts (e.g. 68+75+57=1100 in Problem 4 M5; 55+30=550, Problem 1 L2; 75+68+57=280, Problem 4 H7)
(c) underestimate value of sum of the parts (e.g. 62+57+75=143 in Problem 4 M6)
(d) underestimate the value of part of a whole (e.g. 240 - 80 = 60 in Problem 5 H7; 240-35=105 in Problem 5 M6)
(e) overestimate the value of part of a whole (e.g. 104 – 50 = 95, Problem 6 H8)

The second error appeared to be misinterpretation of the problem. For example, 16 out of 181 (8.8%) across all the classes that attempted problem 6 interpreted that “Samy caught an additional 50 fish”, contributing to an increase in the total number of fish that Samy and Devi had caught. This interpretation of the problem translates to the arithmetic expression 104+50.

Another error is solutions that do not make sense when fitted back to the problem. For example, for Problem 3, one student in L3 gave 70 as the final solution. However, the problem statement was Ahmad has 120 marbles and Raju has 50 more marbles than Ahmad. From the problem statement, the student is expected to be able to imply that Raju must have more than 120 marbles.

A fourth error observed was the incorrect use of mathematical equations (104-50=54-10=44, Problem 6, L2; 104+50=154 -10 = 144, Problem 6, M6). Although only two students were observed to write incorrect mathematical equations, this error is a serious mathematical error and needs to be addressed.

Understanding What Students Can Do in WPs Through Their Diagrams

Table 6 shows the frequency of diagrams used for the 1-step WPs. Less than 50% of the students used pictures for each of the 1-step WPs. Table 7 shows the frequency of diagrams used for the 2-step WPs. About 50% of the students used pictures for each of the 2-step WPs. Compared to 1-step WPs, more students drew diagrams for the 2-step WPs. Those who used diagrams mostly arrived at the correct answer for the WPs.
Table 6
Number and Percentage of Students that Used Diagrams for 1-step WPs

<table>
<thead>
<tr>
<th>Problem</th>
<th>Use diagrams answer correct n (%)</th>
<th>answer incorrect n(%)</th>
<th>Total no. of students who drew diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 1</td>
<td>44(89.8)</td>
<td>5(10.2)</td>
<td>49</td>
</tr>
<tr>
<td>Problem 2</td>
<td>33(70.2)</td>
<td>14(29.8)</td>
<td>47</td>
</tr>
<tr>
<td>Problem 3</td>
<td>40(90.9)</td>
<td>4(9.1)</td>
<td>44</td>
</tr>
</tbody>
</table>

Table 7
Number and Percentage of Students that Used Diagrams for 2-step WPs

<table>
<thead>
<tr>
<th>Problem</th>
<th>Use pictures answer correct n (%)</th>
<th>answer incorrect n(%)</th>
<th>Total no. of students who drew diagrams</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 4</td>
<td>78(79.6)</td>
<td>20(20.4)</td>
<td>98</td>
</tr>
<tr>
<td>Problem 5</td>
<td>68(75.6)</td>
<td>22(24.4)</td>
<td>90</td>
</tr>
<tr>
<td>Problem 6</td>
<td>78(85.7)</td>
<td>13(14.3)</td>
<td>91</td>
</tr>
</tbody>
</table>

Types of correct diagrams students generated to solve 1-step additive WP

From the sample of students’ work that provided correct diagrams, the part-whole and comparison model drawings were used to represent the arithmetic situation for the 1-step WPs (see Table 8). The students’ model diagrams reflected students’ knowledge of the part–whole relationship of numbers. Although the 1-step WPs have the same mathematical structure, students who drew the model diagrams correctly were able to recognise that their underlying conceptual structures are different. Most of the students that used the correct model diagrams to solve Problem 3 demonstrated “an integrated and well-organized knowledge base of how to represent information such as … comparative relationships more than” (Ng and Lee, 2009, p. 308).

Table 8
Correct Diagrams Found in Students’ Drawing for 1-step WPs

<table>
<thead>
<tr>
<th>Problem Type</th>
<th>Part-whole model</th>
<th>Comparison model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Combine (problem 1)</td>
<td>M5 L3</td>
<td></td>
</tr>
<tr>
<td>Change (Problem 2)</td>
<td>M5 M5</td>
<td></td>
</tr>
</tbody>
</table>
Diagrams used in 2-step WPs
The part-whole and comparison models were found in correct students’ drawings for 2-step arithmetic addition and subtraction WPs. The students continued to use the structure of the combine and comparison model to help them construct appropriate sets of step-by-step arithmetic procedures to solve the 2-step WPs (Ng and Lee, 2009). Some of the diagrams for the 2-step WPs reflected a series of independent arithmetic expressions that matched, step-by-step, the structure of the model. In addition, combinations of part-whole and comparison models were also observed in the students correct diagrams. Samples of correct model diagrams for the 2-step problems are in Table 9 (see Appendix A).

Variation to diagrams
From the correct solutions, three variations of model diagrams were observed (see Figure 3). Variation 1 shows the rectangles start at different ends. Variation 2 shows more details given to the diagram, e.g. the use of an arrow to indicate the action of giving (Paul to John). This fine detail reflects even more clearly the students’ understanding of the problem situation. Variation 3 shows the text information translated into a set of arithmetic expressions (TP) before being used to construct a model (TPS). In this case, the arithmetic expression presented in the final phase provided evidence of students’ interpretation of the model (Ng and Lee, 2009). The third variation suggested that diagrams play multiple roles as a cognitive tool in problem solving.

Variation 1: Rectangles start at different ends
Problem 2 L2

Variation 2: Diagram indicates the change
Problem 2 M5
Variation 3: Translate text → Arithmetic expressions → Construct model

Problem 6 M5

Figure 3. Variations in model diagrams.

More Errors Revealed for 1-Step and 2-Step WPs Through Diagrams

The diagrams of incorrect solutions were teased out to gain deeper understanding of the students’ errors in solving WPs. The findings are discussed below according to the categories that emerged during data analysis.

Models do not make sense

In this category, the representation was problematic for the student. Either the diagram did not make sense or the diagram did not model the problem situation correctly (see Figure 4).

Problem 2 L2

Problem 2 M4

Figure 4. Diagram does not make sense.

Identify the wrong unknown to be evaluated

In this category, the diagrams were partially correct (e.g. see Figure 5). A question mark was placed in the wrong rectangle to identify the unknown to be evaluated.

Problem 2 L2

Problem 5 H7

Figure 5. Essential information misrepresented.
**Solve only one of the steps correctly in a 2-step WP**

The diagram in Figure 6 shows errors due to solving only part of the problem.

![Problem 4 H7](image)

*Figure 6* Solve only part of the problem in a 2-step WP.

**Inaccurate diagrams**

Figure 7 shows that rectangles were used to represent specific numerical values (240 and 115). However, the non-proportional sized rectangles (115 longer than 240) makes it impossible to evaluate the unknown value. This presents confusion to the problem solver.

![Problem 5 H7](image)

*Figure 7* Non-proportionately sized rectangles.

**More about the Diagrams in Correct Solutions**

**Labelling of diagrams**

Some model diagrams were poorly labelled in WPs in the following areas; (i) Question marks to identify the unknown value to be evaluated were in the wrong place (see Figure 8i); (ii) Did not indicate the unknown values to be evaluated with a question mark. For 2-step WP, there should be two question marks to indicate two unknowns to be found. However, sometimes only one of the question marks was indicated in the diagram. In some instances, both question marks were not indicated. (iii) Given quantity not labelled, (iv) Indicate a given quantity in the wrong rectangle.
(i) places the question mark for the unknown values to be evaluated in the wrong place

Problem 3 M4

(ii) Did not indicate the unknown values to be evaluated with a question mark.

Problem 5 H7

(iii) Given quantity not labelled.

Problem 5 M5

(iv) Indicate a given quantity in the wrong rectangle.

Problem 5 H7

Figure 8. Labelling of diagrams.

Incorrect diagrams

Correct model drawings were underpinned by sound conceptual knowledge (Ng and Lee, 2009). However, samples of students’ work also showed that some students arrived at the correct answer using the wrong model diagram, that is, the diagrams did not represent the problem situation and the algebraic expressions (see Figure 9).

Problem 5 H7

Figure 9. Diagrams did not represent the problem situation.
Conclusion and Implications for Teaching

An assortment of errors made by Primary Three students while solving 1-step and 2-step WPs are presented in this study. The quantitative data showed the frequency of errors made from computation error, clerical/transfer errors, wrong operations used. Like Koay, Chang & Ghani (2012), the errors in solving WPs can be discussed in 3 areas: semantic structures, computational proficiency and choice of operations by students. The qualitative data from samples of students’ work, in particular, the students’ drawings helped us gain greater understanding on students’ errors in solving WPs. The results from the study provide some insight into the planning of an intervention programme to address the errors in Phase 3 of the professional development programme.

Problem 1(Combine problem), Problem 2 (Change problem) and Problem 3 (Compare problem) was structurally similar to CB1, CH1 and CP3 respectively. CH1 and CB1 were considered to the easiest problem while CP3 was considered to be of intermediate difficulty in Riley, Greeno and Heller (1983). In this study, however, Problem 2 (equivalent to CH1) posed the greatest challenge. This could be due to students having difficulty with the “action” in the problem situation and this difficulty can be overcome by incorporating more situations for students to ‘act-out’ the problem situation in Change problems so that students are able to experience the problem situation.

Generally, the students performed better in 1-step addition and subtraction WP compared to 2-step word problem. This result aligns with Yeap & Kuar (2001) and Koay, Chang & Ghani (2012, p. 46) that “problems more than one semantic relation yielded poorer performance than those of common semantic relation (single-relation)”. M4, M5 and M6 perform well in the 2-step combine-combine problem (Problem 4) compared to the 1-step change problem and 1-step compare problem because Problem 4 belongs to the same subtype (total set unknown) in Problem 1 (Combine problem). Problem 4 builds from the semantic structure of Problem 1 and helps students transit from 1-step to 2-step word problem. This draws the importance of proper scaffolding for students when they transit from 1-step to 2 step word problems. If classroom teachers are aware of students’ pre-requisites for 1-step addition and subtraction WPs, perhaps then, the teacher can better design classroom instructions for solving 2-step word problems that tailored to the learning needs of the students.

The 3-stage teaching by analogy by Cockburn & Littler (2008, p. 65) may be a powerful strategy to remediate such errors in WPs. In Stage 1, an anchoring task or task designed to elicit correct response can be introduced to the students. For example, the Steven spends $240 on transport each month. Jane spends $80 on the MRT and $35 on buses each month. (a) How much does Jane spend on transport each month? In Stage 2, the bridging task or similar task in which a factor was identified as misleading can then be introduced. For example, (b) How much more does Steven spend on transport each month? Lastly, in Stage 3, the target task or task likely to elicit incorrect response is finally introduced e.g. Steven spends $240 on transport each month. Jane spends $80 on the MRT and $35 on buses each month. How much more does Steven spend on transport each month? If such learning progression is carefully mapped out for students who are likely to experience difficulty with multi-step WPs, perhaps then students will more likely to appreciate the WPs in their classrooms.

Computation skills and concepts are usually presented first in the mathematics textbooks in Singapore before the word problems. One of the concepts taught in the lower primary is number bonds. In fact, the lower primary mathematics curriculum places much emphasis on
the role of splitting (decomposing and composing) quantities in gaining insight into numerical relations and in developing number sense. Number sense as discussed in the literature (Hunting, 2003; Steffe, Cobb, & Von Glasersfeld, 1988) plays a significant role in improving students’ performance in WPs for errors that are due to computation errors. However, the results from this study show several situations where students were unable to pick up “senseless” solutions using their understanding of number relations. Perhaps, students’ ability to apply decomposition and composing of numbers into WPs should not be taken as a “matter-of-fact” that ‘they-should-know-by themselves’. The application and connection of these skills and concepts should be pointed out to students explicitly. In addition, sensitizing students to think more deeply about numerical relations may also help students to develop greater number sense. In the same manner, one of the key features of the Singapore curriculum is the spiral curriculum. Topics and skills are spiralled. However, if effort is not made to point out the connections and application of each of the topic, subtopic, skills and subskills, the students might not be aware of this “matter-of-fact” resulting in possible errors in their solutions.

Diagrams need not be used to solve a WP. Those who used diagrams in this study used the model diagrams. Model diagrams are used more frequently to solve 2-step than 1-step WP. Those who used diagrams mostly arrived at the correct answer. Several types of errors were identified in the drawings of the models. Some model diagrams did not make sense and illustrated the inability of students to “integrate reading, interpreting the problem, and computation into the total solution of the problem” (Ballew & Cullingham, 1982, p. 203). Another error was incorrectly identifying the unknown to be evaluated. In this case, the diagram reflected an incorrect unknown, resulting in the incorrect choice of operations to the problem situation. This error could be due to incorrect interpretation of the problem situation or a situation when the misrepresentation of a single piece of information may cause an incorrect solution (Ng & Lee, 2009). Another error was not solving the problem fully in 2-step WPs, that is, correctly stating only one of the two steps. There are several possible reasons for the errors, for example, students may not fully understand the WP, or recognize the relationships between text (Nortvedt, 2010). Unfamiliar term or context (Caldwell & Goldin, 1979) e.g. transport in Problem 5 may have also caused some difficulty in understanding the problem situation. To address errors due to the misinterpretation of the questions, question prompts, e.g. from the STARTUP metacognitive-based scheme (Lee, Yeo & Hong (2014) may be used to help students gain greater understanding of the problem. The STARTUP scheme emphasises on understanding and planning in Pólya’s (1971) approach and is a possible approach to help students become more aware of the given conditions of the problems, and the information that is needed to solve the problem.

In this study, some students were able to obtain the correct solutions despite their inaccurate diagrams. Several variations to the diagrams that yield the correct solutions were also found. Drawing disproportionately sized rectangles were shown to result in the wrong operation used in one problem because the model diagram did not reflect the problem situation correctly. The concept of proportionately sized rectangles is thus important in model diagrams. Even though “care had to be exercised” (Ng & Lee, 2009, p. 296) in the construction of related models so that the entire model drawing was meaningful, “extreme emphasis on the precision” on the diagrams may “interfere with the solution process which may in turn results in poor problem solving performance (Diezmann, 2000; Larkin & Simon, 1987). Furthermore, a view on model diagrams is that they are sketchpad for the students’ thinking and some students do not need the sketchpad to be in ‘perfect precision’ to identify the associate arithmetic equation to solve the problem.

20
Some correct solutions contain model diagrams that do not describe the problem situation. It is unclear how students used the diagrams to arrive at the correct solution. In the classroom situations, the teacher may want to find out from the students how the diagrams are developed and how the diagrams are connected with the solutions and the operations. This is to ensure that the student who used the model diagrams understand the value of the tool. If the problem solver does not know what the rectangles mean in the model diagram, the importance of drawing the rectangles proportionately and defining the rectangles clearly, it may also pose a challenge for the students to transit from the model to letter symbolic algebra when solving algebraic WPs.

In the classroom instruction, meaningfulness of using model drawing as a tool for problem solving can be highlighted so that problem solvers can appreciate and fully utilise the model drawing tools during WP solving. Furthermore, if students and teachers are able to view and appreciate the model diagram as a tool for solving WPs (a mean to an end), care in constructing the models will help students achieve greater clarity in communicating their mathematical thinking through the diagrams. The clarity in drawing the models can also enhance teachers’ efforts to diagnose students’ difficulty in solving WPs.

In the analysis of the students’ solutions, conscientious effort to check the final solutions (e.g. written evidence in the working column to check the calculations, fitting the solutions back into the problem, alternative method etc.) were not found. Mathematics teachers in Singapore are encouraged to expose students to Pólya’s (1971) approach towards problem solving (Ministry of Education, 2006). In the fourth stage of Pólya’s approach, “students look back at their solution to determine if their answer is reasonable within the problem context, and that they have indeed solved the problem by finding what they are required to find” (Lee, Yeo & Hong, 2014, p.466). Perhaps, if checking of final solutions became part of the students’ mathematical habits, then errors such as solutions that do not make any numerical sense or solutions that do not make sense when fitted back into the problem will prompt students to re-examine their approach to the problem. In the checking stage, students can also be promoted to check if their correct mathematical equations are written correctly.

In this study, the WPs administered to the students is a small subset of the list of problems in Riley, Greeno & Heller (1983, p. 165). The findings suggest many ideas for the intervention programme in the next phase of the study. In order to gain a more comprehensive view of students’ errors in 1-step addition and subtraction WPs, all the specific features within each semantic structure in the addition and subtraction problems can be taken into consideration in future studies.
References


## Appendix A

### Table 10

*The Number of Incorrect, Correct Answers and the Frequency of Number Sentence Type for Combine Structure Problem*

<table>
<thead>
<tr>
<th>Problem 1</th>
<th>Jim has 132 red marbles and 45 blue marbles. How many marbles does he have altogether? (Combine)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Correct answer</td>
<td>7</td>
</tr>
<tr>
<td>177 marbles</td>
<td></td>
</tr>
<tr>
<td>Correct operations</td>
<td>→ add incorrectly</td>
</tr>
<tr>
<td>e.g. 132 + 45 = 413</td>
<td></td>
</tr>
<tr>
<td>Correct operations</td>
<td>→ transfer errors</td>
</tr>
<tr>
<td>e.g. 122 + 45 = 572</td>
<td>-</td>
</tr>
<tr>
<td>Correct operations</td>
<td>→ transfer errors</td>
</tr>
<tr>
<td>e.g. 132 + 40 = 172</td>
<td></td>
</tr>
<tr>
<td>Incorrect operations</td>
<td>→ subtract correctly</td>
</tr>
<tr>
<td>e.g. 132 - 45 = 87</td>
<td></td>
</tr>
<tr>
<td>Incorrect operations</td>
<td>→ subtract incorrectly</td>
</tr>
<tr>
<td>e.g. 132 - 45 = 101</td>
<td></td>
</tr>
<tr>
<td>Special case:</td>
<td>Correct operations</td>
</tr>
<tr>
<td>e.g. 132+45 = 87</td>
<td></td>
</tr>
</tbody>
</table>
Table 9
Correct Model Diagrams Found in Students’ Drawing for 2-step Word Problems

<table>
<thead>
<tr>
<th>Part-whole model</th>
<th>Comparison Model</th>
<th>Part-whole model and comparison model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Problem 4 M4</td>
<td>Problem 4 M4</td>
<td>Problem 5 H8</td>
</tr>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
</tr>
<tr>
<td>Problem 5 H8</td>
<td>Problem 6 H8</td>
<td></td>
</tr>
<tr>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
<td></td>
</tr>
<tr>
<td>Problem 4 M5</td>
<td>Problem 5 M4</td>
<td>Problem 5 H7</td>
</tr>
<tr>
<td><img src="image6" alt="Diagram" /></td>
<td><img src="image7" alt="Diagram" /></td>
<td><img src="image8" alt="Diagram" /></td>
</tr>
</tbody>
</table>

27