The use of Mathematical Investigations in a Queensland Primary School and Implications for Professional Development

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Abstract

With the introduction of Ways of Working in 2008, Queensland teachers received professional development on using investigations to teach mathematics. This case study explores the extent to which teachers in one Queensland Primary School use this pedagogy. To determine teachers’ beliefs and teaching approaches, a five point Likert scale questionnaire was administered followed by open-ended interviews with three teachers: a beginning, an experienced and an expert teacher. The survey and interview results revealed that teachers generally held a balance of traditional and contemporary attitudes towards teaching maths. This was accompanied by a balance of traditional and contemporary teaching approaches. Teachers believed in the importance of investigations but preferred to spend time developing a sound mathematics knowledge base before allowing students to attempt an investigation. Interviews with the teachers revealed a need for ongoing structured professional development in four areas: approaches to collaborative learning in group work; strategies for teaching the language of mathematics; integration of investigative approaches, and scaffolding learning strategies.

Key words: mathematical investigations, beliefs, teaching approaches, professional development, elementary school

Introduction

With the introduction of the Queensland Mathematics Syllabus in 2004 there was an emphasis on thinking, reasoning and working mathematically which were described as ‘essential elements of learning’ for mathematics (Queensland Studies Authority, 2004, p1). This was to be achieved ‘through student engagement in mathematical investigations relevant to a range of (real life) situations’. This was followed in 2008 by the introduction of Ways of Working, as part of the Queensland Curriculum, Assessment and Reporting Framework, with the emphasis on Thinking and Reasoning, and, Communicating and Reflecting that now aligns with the Proficiency Strands (Understanding, Fluency, Problem Solving and Reasoning) of the Australian Curriculum: Mathematics (Australian Curriculum, Assessment and Reporting Authority, 2012). There is considerable overlap with Problem Solving being described as ‘the ability to make choices, interpret, formulate, model and investigate problem situations, and communicate solutions effectively’ (Australian Curriculum, Assessment and Reporting Authority, 2012, Organisation). Investigations are still an important part of learning.
In 2008 all Queensland teachers of mathematics, both primary and secondary received at least five hours of professional development on using investigations to teach mathematics. Today, Queensland teachers are expected to facilitate learning through investigations to address the proficiency strands of the *Australian Curriculum: Mathematics*. The tendency of teachers is to gradually take on board new ideas they agree with and make them work for their students in context. This study explores the current attitudes and teaching practices of some Queensland Primary teachers, to ascertain the longer term effect of this professional development, in terms of the degree of alignment with this investigative pedagogy and the professional development they feel they need.

**Background**

This section begins with mathematical investigations and how these can be used as a reform-orientated pedagogy. Mathematical language is more important in this type of classroom as students need to discuss their thinking with their peers, teacher and the whole class. Finally the apparent conflicts that can exist between teachers’ beliefs and actual practice are considered.

**Investigations**

Mathematical investigations are real-life or life-like learnings that are open ended and provide opportunities for students to use multiple pathways to investigate the situation/problem. There are parallels with a scientific inquiry in that an inquiry presents a question to be answered; however, to ‘enquire’ is not always to ‘investigate’. Investigations may be framed variously as: a problem to be solved; a question to be answered; a significant task to be completed, or an issue to be explored (Queensland Studies Authority, 2005). Effective investigations have multiple entry points allowing students to work at their own pace. Investigations are usually more involved than problem solving and can be extended tasks, substituting “an over-emphasis on routine technical exercises with more challenging complex tasks” (Swan, 2009, p1). These tasks are completed in a collaborative group setting, facilitating the process of discussion and contextual use of mathematical vocabulary.

The investigative approach can provide an ‘overarching organiser’ (Lovitt, 2000) or vehicle for mathematics to occur in a context, providing a balance between problem solving and skills-based activities and to engage students in deep mathematical learning (Boaler, 2008). Investigations provide students with the chance to ‘do’ mathematics, to ‘make sense’ of their world and to ‘be mathematicians’ (Marshman, Pendergast, & Brimmer, 2011) and hence offer an opportunity for students to learn mathematics through the investigation (QSA, 2005). The tasks aim “for students to make sense of a real-world use of mathematics, to get them involved in ‘problem formulation, problem solving, and mathematical reasoning’” (Battista, 1994, p. 463). Investigations are a broader category than problems. When problem solving is used in classrooms the problem is generally defined, whereas with investigations students may need to define the problem and are exploring possible options rather than finding a solution to a problem.

It is essential that both teachers and students are familiar with the ‘discourse’ of investigations (Morgan, 1998, as cited in Pimm & Wagner, 2003). Morgan separates this
discourse for teachers into: official (curriculum), practical (advice guides) and professional (teachers’ reflecting on the use of investigations). This is all based on the understanding that investigations involve students engaging in mathematics in a realistic context, hence, students need to be exposed to investigations on a regular basis by a teacher who incorporates them into their pedagogy in an authentic and realistic manner with transparent goals.

Investigations may vary in size and the time taken to complete them. Micro investigations can consist of simple open-ended questions that require thinking of multiple solutions, or it could mean finding the same solution using different pathways. Small investigations involve a similar open-ended question but in a more complex context that might involve working in pairs or groups for a whole lesson or the best part of one. Larger investigations incorporate a range of skills and concepts, possibly connecting several concepts previously taught as a culminating activity or as an ongoing teaching tool throughout the term that gradually scaffolds the students’ understanding.

Through providing investigations, teachers can emulate the same processes used in the real world by mathematicians. This is not just professionals who work with mathematics as a ‘pure science’ as we all operate as mathematicians when we use mathematics to answer a question, for example, when measuring for a quote in the building industry or estimating costs or expenses in retail businesses or government departments. If educators are to prepare students for society, students need to be more aware of why they are studying mathematics. Primary schools have the potential to identify the need for ‘real world’ use of mathematics through investigations. This provides an underlying purpose for mathematics lessons and a context to attempt a challenging investigation of a question or to solve a difficult problem. When solving these challenging investigations or difficult problems students are engaged at the higher levels of thinking, such as ‘evaluation,’ and are looking to ‘create’ and ‘justify’ their own pathway to solutions in line with Krathwohl’s (2002) revision of Bloom’s Taxonomy. They would be evaluating the success (or failure) of their strategies. The SOLO Taxonomy (Biggs, 2011), extends this metacognitive activity further to include hypothesising and formulating a theory, which may involve primary students explaining a pattern they found in their answers and expressing their own general theory. For example, “Your class is organising the materials for garden beds along the front of the school. Each class may have one garden surrounded by wooden sleepers. (One sleeper goes along one side of the garden.) The pattern is shown in Figure 1 for one, two or three classes. How many sleepers would you need 1, 2 and 3 gardens? What if ten classes wanted a garden? Explain how you worked it out.

![Figure 1. The pattern of garden beds](image-url)
Learning through this approach has the potential to empower primary students to know that they are testing and proving their own theories to solve different problems and justifying and reflecting on their answers to ‘essential’ (over-arching) questions in an investigation.

Reform-orientated pedagogy

There has been a consensus since the 1980’s about the value of incorporating problem solving and to a lesser extent investigations and ‘working mathematically’, into the process strands of syllabus documents (Schoenfield, 1992). This is known as reform-orientated teaching. Jaworski (2006) not only supports the use of inquiry-based teaching as a tool to address mathematical tasks in a classroom, but sees inquiry as a philosophy or ‘fundamental theory’ adopted by the teacher for their own learning, engaging critically in key questions to be investigated. In so doing, teachers are then able to actively reflect on their own performance as professionals, ask questions about their own knowledge and skill areas and then investigate ways they can improve on them. This self-recognition that there is always room for improvement can potentially benefit their students and also encourage colleagues to do the same through indirect modelling and choice of professional development.

It is important for students to construct their own understanding, and the teacher must resist the urge to intervene, allowing the problem solving processes to unfold (Rowan & Bourne, 2001). These students are not afraid of taking risks or ‘being wrong’, thereby developing confidence and creativity. They would not hold the same insecurities as the students from the ‘traditional’ classroom due to their level of confidence in their ability to solve problems with familiar strategies they can apply to the unique context of a new investigation.

Students One way students can only develop this type of confidence is for their teachers to demonstrate their own level of confidence in their own ability to solve mathematical problems and to communicate to their students the method through which they arrived at the solution (McInerney & McInerney, 2010). Most teachers will not achieve this level of self-actualisation without a professional support network of their peers and mentor figures in administrative or coaching roles. Without this network, any professional development will have limited effect.

Using investigations and problem solving, students retain their knowledge longer (Dochy, Segers, Van Den Bossche, & Gijbels, 2003) due to the real-world context that students can relate to. But it is not merely problem solving that is the key to developing understanding. Taking students a step further and framing a question that needs to be investigated is a way of scaffolding the need to challenge what they already know, and to consolidate their own knowledge with their own data. This facilitates the development of multiple skills: sorting, analysing, evaluating and reflecting. Allowing students to arrive at a question they want to investigate themselves provides motivation for them to complete a complex task. Calder and Brough (2013) report that students who investigated real-life scenarios drawn from their questions were more highly engaged in mathematical learning. The measurement and geometric thinking these students participated in went beyond the outcomes required by the curriculum. Battista (1994) warns that taking problem solving and investigations away from students removes a key opportunity for students to make sense of mathematics. It is through
investigations that students can link theory to a practical context of particular interest to them. The results from the survey of New South Wales primary school teachers by Anderson and Bobis, (2005) showed that the ‘majority of these teachers support reform-oriented teaching approaches that promote working mathematically’ (Anderson & Bobis, 2005, p. 71) though this was not always reflected in their responses to open-ended questions about the teaching approaches they actually used in the classroom. This disjunct between teacher beliefs and teaching approaches is investigated in the present study.

Anderson, Sullivan and White (2004) found that teachers who endorse the focus on problem solving within the mathematics syllabus recognise it as an important life skill. They surveyed 162 primary school teachers to determine their beliefs and teaching approaches. They separated the teachers into three groups: those with a traditional teaching approach based on individual student work and a reliance on text books or worksheets; those with a contemporary teaching approach where concepts are to be explored and investigated; and those with mixed beliefs (a combination of both). Results showed that 70% of teachers of Years K – 2 were contemporary (with 4% traditional) and only 10% of teachers teaching Years 5 - 6 were contemporary (with 35% traditional) (Anderson, White, & Sullivan, 2004). This ‘dichotomy’ of traditional versus contemporary teacher beliefs and approaches will be explored further in the context of teaching through investigations.

An important component of a successful investigative or inquiry-based classroom is collaborative learning where students are working together to develop a common understanding or product, for example a solution. The important components of this are: a commitment to the learning of others by all students; a respect for the ideas of others; and accepted methods of communication and support (Boaler, 2008). This does not happen naturally in the classroom; the teacher needs to develop and nurture such a culture.

Mathematical Language

Teachers often lack the necessary skills to develop the students’ writing skills in mathematics, and students are unable to communicate their mathematical ideas adequately (Pimm & Wagner, 2003). Mathematical vocabulary knowledge in particular has been identified as a neglected area of mathematics students’ instruction, which impacts comprehension and written expression of mathematical concepts. Some instructional strategies for fostering vocabulary development in mathematics have been developed (Kovarik, 2010). A mixture of direct and indirect instructional strategies is required for optimal vocabulary development (Partnership for Reading, 2006; Thompson & Rubenstein, 2000). Indirect instruction provides opportunities to discover the meanings of words from context through authentic exercises involving peer conversations and writing activities during investigations, while direct instruction allows students to learn words through specific vocabulary instruction and to master word-learning strategies that involve, for example, prediction of meaning based on how words are “built” morphologically (prefixes, roots and suffixes). Teaching vocabulary and written expression in these ways could provide some direction for many Queensland teachers in how to embed the AITSL requirement for language and literacy across the curriculum within investigative tasks.

Teacher Beliefs
For teachers to shift to this reform orientated pedagogy generally requires a shift in their belief system, especially their ideas on the nature of mathematics and its learning and teaching (Ernst, 1988). Raymond (1997) defined mathematics beliefs “as personal judgements about mathematics formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics and teaching mathematics” (p.552). Teachers may not be aware of their beliefs but hold them in their subconscious (Cross, 2009). Ernst (1988) identified three different views of mathematics: the instrumentalist view where mathematics is seen as useful but an unconnected set of rules and facts; the Platonist view, a definite, unified, unchanging body of knowledge that is discovered; and the problem solving view which sees mathematics as an ever evolving construction which is socially and contextually situated (Ernst, 1988).

Much has been written about teachers’ mathematics beliefs and teaching practices. Research into teacher beliefs has varied findings. Some studies (e.g., Raymond, 1997; Thompson, 1984; Handal & Herrington, 2003) have found that teacher beliefs were not always consistent with their teaching practice; however, in contrast, Cross (2009) found a link between teaching beliefs and practice. For teachers to change their pedagogy they need to engage in reflection and consider their views and the assumptions these are built on. They need to be aware of other feasible options that they can possibly assimilate into their classroom practice. They also need a social environment that will allow these changes to occur (Ernst, 1988).

A major influence on mathematics teaching practices is the teachers’ mathematics beliefs and the immediate classroom environment, including the behaviour, abilities and attitudes of the students; the mathematics topic; and time restrictions. Social teaching norms, including the school’s philosophy, curriculum, textbook, resources, the presence of standardised tests and other teachers, their teacher education program and their life outside school may also have some impact on how they teach (Raymond, 1997). Teacher mathematical beliefs about the nature and pedagogy of mathematics are generally formed during their prior school experiences with perhaps some impact by teacher education programs and family environment (Raymond, 1997). Teachers have a wide range of beliefs and if these are not integrated into a coherent conceptual system the person may have incongruent and conflating beliefs leading to their practice being different from their beliefs (Thompson, 1984). Handal and Herrington (2003) and Ernst (1989) claim that change in pedagogy will only occur when the new pedagogy is accepted into the teachers’ belief system.

Hoyle (1992) believes that beliefs are situated and that teachers develop them within the context and culture of their classroom and so the inconsistencies in beliefs become irrelevant. Skott (2001) introduced the idea of school mathematics images to describe the apparent inconsistencies between the teachers’ beliefs about mathematics teaching and learning and about mathematics and their teaching practices. Skott (2001) believed that teachers’ main concern was educational priorities such as behaviour management and student confidence building.
Study Design

A single case study design was used to explore the problem of why teachers tend not to include investigations in mathematics in their teaching practice in the primary school context by examining the case of a particular primary school. A mixed methods approach was used to determine the beliefs and approaches to teaching mathematics through investigations of a group of primary school teachers in this convenience sample. The teachers worked in a large, regional Prep to Year 7 Primary School in Queensland. Most of these teachers had attended a professional development session on teaching mathematics through investigations. Quantitative data were collected with a survey to ascertain beliefs about and the frequency of use of investigative teaching approaches. The guiding question was ‘What perspectives are held by this group of teachers in relation to teaching mathematics through investigations?’ ‘Problem solving’ and ‘investigations’ are similar mathematical processes, so the validated survey developed by Anderson, Sullivan and White (2005) was used in this study to analyse and compare teachers’ attitudes to teaching mathematics through investigation.

It is important to look at both teachers’ beliefs and approaches to teaching, actual practice as self-reported, as there is often a lack of alignment between teachers’ professional expectations and what is happening in practice. In Raymond’s study (1997) elementary teachers demonstrated a strong focus on mathematical content knowledge and insufficient focus on pedagogical knowledge due to beliefs being shaped by their own early experience as a student rather than on university learning.

Qualitative data were collected in the form of semi-structured interviews which were undertaken with a sub-set of three teachers who were selected on the basis of their level of experience with teaching. These teachers came from a mix of lower, middle and upper primary years, and expressed an interest in teaching mathematics. Their proficiency levels are based on Hattie’s (2003) categorisation of novice, experienced and expert. Their teaching experience also aligned with the categories outlined in the National Professional Standards for Teachers: Foundational, Proficient, and Highly Accomplished/Lead (http://www.teacherstandards.aiitsl.edu.au). For the purpose of this study, they will be described as a beginning teacher (a first year graduate), an experienced teacher (of about fifteen years) and an expert teacher (with almost forty year’s experience). The teachers brought student samples to look for evidence of successful strategies that engaged students in higher-order thinking, as well as the complexity of investigation needed by the students to complete the task.

The guiding questions underpinning this research are:
1. What perspectives are held by this group of teachers in relation to teaching mathematics through investigations?
2. To what degree is this reflected in the teaching approach that they report using?
3. What are the implications of the findings for teachers in Queensland Primary schools?

Survey findings and discussion

Twenty teachers ranging from Prep to Year Seven, working in the same Queensland primary school, returned surveys on their beliefs and teaching approaches regarding the use of investigations when teaching mathematics.
**Belief statements**

Overall, the teachers responded to the belief statements conservatively, showing a limited number of strong responses when disagreeing or agreeing. Table 1 summarises the responses of the participants’ beliefs grouped by Anderson, Sullivan and White’s (2005) categories of ‘traditional’ and ‘contemporary’ teacher beliefs. ‘Traditional’ refers to a more teacher-directed style, while ‘contemporary’ refers to a more student-centred inquiry approach that allows for more discussion and investigation.

From Table 1 it appears that the teachers in this school held an overall balance of traditional and contemporary beliefs. This was shown using a Wilcoxon signed ranks test where there was no significant difference between the sum of Likert scale traditional scores (sum=156) and contemporary scores (sum=160).

**Table 1. Summary of Responses: Belief statements**
Scores are the number of teachers who responded to each point on the scale (N=20).

<table>
<thead>
<tr>
<th></th>
<th>Strongly disagree</th>
<th>Disagree</th>
<th>Agree</th>
<th>Strongly agree</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traditional teacher beliefs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. Students should successfully complete knowledge-based questions before they apply themselves to a mathematical investigation.</td>
<td>1</td>
<td>5</td>
<td>11</td>
<td>3</td>
<td>n.s. p&gt;0.05</td>
</tr>
<tr>
<td>2. Application through investigative tasks is best left to the end of the topic in mathematics.</td>
<td>2</td>
<td>11</td>
<td>6</td>
<td>1</td>
<td>n.s. p&gt;0.05</td>
</tr>
<tr>
<td>3. Mathematics lessons should focus on practising skills.</td>
<td>1</td>
<td>5</td>
<td>13</td>
<td>1</td>
<td>n.s. p&gt;0.05</td>
</tr>
<tr>
<td><strong>Contemporary teacher beliefs</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. Mathematics lessons should be taught in the context of an investigation rather than through practice of algorithms.</td>
<td>1</td>
<td>6</td>
<td>12</td>
<td>1</td>
<td>n.s. p&gt;0.05</td>
</tr>
<tr>
<td>5. Students can learn most mathematical concepts by working in the context of a mathematical investigation.</td>
<td>1</td>
<td>4</td>
<td>12</td>
<td>3</td>
<td>.025</td>
</tr>
<tr>
<td>6. It is essential for students to explore their own strategies before being shown the teacher’s methods.</td>
<td>0</td>
<td>10</td>
<td>10</td>
<td>0</td>
<td>n.s. p&gt;0.05</td>
</tr>
</tbody>
</table>

Note: Significant Chi-square p-values are presented for differences between proportions of grouped agree (strongly agree + agree) and disagree (strongly disagree + disagree) responses for each question.
70% of participants agreed with the importance of students completing knowledge-based questions before applying themselves to a mathematical investigation (traditional belief), and 70% of teachers felt that mathematics lessons should focus on practising skills. However, approximately 65% of participants disagreed with the belief that investigations are best left towards the end of the unit of work in mathematics. Both of these findings failed to reach statistical significance (p>.05).

These teachers may believe that students require basic knowledge before solving investigations, which is in agreement with Lovitt (2000) who states that mid-way through the intervention process students need to draw upon “the acquired background set of basic skills — algorithms, graphing techniques, algebraic modelling, solution methods for equations, etc. The greater this background set of skills, the greater the probability of making progress in the problem”. Most teachers felt that mathematics lessons should focus on practising skills, possibly suggesting a belief that skills are a higher priority than strategies to investigate a problem. This priority could be the result of the ‘wash back effect’ (Klenowski & Wyatt-Smith, 2012; Ward, 2012) of standardised testing. ‘Wash back’ is the influence that assessment exerts on teacher and learner behaviour, which sometimes results in ‘teaching to the test’.

Interestingly, 65% of teachers agreed (p>.05) that teaching mathematics in the context of an investigation is preferable to practising algorithms (contemporary belief). This seems to contradict the statement that mathematics lessons should focus on practising skills as investigations are usually viewed as a different approach to practising skills such as using algorithms. A significant number of teachers (75%; p=.025) also agreed that students could learn most maths concepts within the context of an investigation (contemporary belief). This may reflect a deeper belief in the value of investigations as a vehicle for learning, as this contemporary belief statement showed the highest consensus of agreement. This suggests they feel confident in the ability of students to learn within the context of an investigation as opposed to learning skills in isolation in a more traditional manner. This appears to be in conflict with the traditional beliefs statements that they had supported earlier that: students should successfully complete knowledge-based questions before they apply themselves to a mathematical investigation; and investigative tasks are best left to the end of the topic. This is consistent with the research indicating the mismatch between teachers’ beliefs and practices (Raymond, 1997; Thompson, 1984; Handal & Herrington, 2003) and may be due to the teachers believing it is more important to build students’ confidence and manage behaviour (Skott, 2001). This indicates that teachers are unclear on how to best combine traditional skills within investigations and would benefit from some professional development on integrating skills within mathematical investigations.

Opinion was equally divided on whether it is essential for students to explore their own strategies (when investigating solutions) before being shown the teacher’s methods. This could reflect two types of beliefs: (a) students should be given the opportunity to explore their strategies by taking risks and making mistakes in a safe environment, or (b) teachers must model a strategy first before asking students to use it independently. Interestingly, no one felt strongly about this statement, possibly suggesting participants were uncertain or had not reflected deeply on this idea prior to the survey. Another possibility could be that the
teachers felt that there are situations where they believe it is appropriate to give students freedom to explore, and other times when it may damage the students’ confidence by leaving them frustrated when they do not have adequate strategies. It also may mean that some teachers believe there is only one right way to solve a problem.

**Teaching Approaches**

The survey responses to statements regarding teaching approaches (Table 2) indicate that teachers prefer to balance ‘contemporary’ and ‘traditional’ teaching approaches within mathematics investigations. 80% (p=.007) of the teachers encourage the students to work in small cooperative groups and 70% (p>.05) of respondents encourage students to record their own procedures and methods of solving problems. Both are contemporary approaches.

Table 2. Summary of Responses: Teaching Approaches.
Scores are the number of teachers who responded to each point on the scale (N=20).

<table>
<thead>
<tr>
<th></th>
<th>Hardly ever</th>
<th>Sometimes</th>
<th>Often</th>
<th>Almost always</th>
<th>P-value</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Traditional Teacher approaches</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1. You ensure that students work alone</td>
<td>1</td>
<td>14</td>
<td>5</td>
<td>0</td>
<td>.025</td>
<td></td>
</tr>
<tr>
<td>2. You explain in detail what the students have to do prior to a mathematical investigation</td>
<td>0</td>
<td>3</td>
<td>9</td>
<td>8</td>
<td>.007</td>
<td></td>
</tr>
<tr>
<td>3. You set exercises to allow the students to practise their skills</td>
<td>0</td>
<td>4</td>
<td>11</td>
<td>5</td>
<td>.007</td>
<td></td>
</tr>
<tr>
<td><strong>Contemporary teacher approaches</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4. You encourage the students to work in small cooperative groups</td>
<td>1</td>
<td>3</td>
<td>15</td>
<td>1</td>
<td>.007</td>
<td></td>
</tr>
<tr>
<td>5. You present unfamiliar investigations to the class deliberately with very little indication of how to solve them</td>
<td>10</td>
<td>8</td>
<td>2</td>
<td>0</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>6. You encourage students to record their own procedures and methods of solving problems</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>4</td>
<td>n.s.</td>
<td>p&gt;0.05</td>
</tr>
<tr>
<td>7. You pose open-ended problems to develop students’ ability to explore mathematical investigations for themselves</td>
<td>2</td>
<td>10</td>
<td>7</td>
<td>1</td>
<td>n.s.</td>
<td>p&gt;0.05</td>
</tr>
</tbody>
</table>

Note: Significant Chi-square p-values are presented for differences between proportions of grouped (hardly ever + sometimes) and (often + almost always) responses for each question.
In support of a more ‘traditional’ teaching approach, 85% (p=.007) of respondents often/almost always explain in detail what the students have to do prior to a mathematical investigation. This contradicted the result that half of the respondents agreed with the belief statement that it was essential for students to explore their own strategies before seeing the teacher’s methods. A possible interpretation of this could be that they understand the term ‘investigation’ refers to a teaching approach, and is not a heuristic strategy like ‘guess and check’ or ‘working backwards’, which would have to be scaffolded by the teacher using ‘guided practice’ before it can be used independently.

Approximately 80% (p=.007) of respondents indicated that they often/almost always set exercises to allow the students to practise their skills. ‘Exercises’ are associated with traditional learning from a textbook so these teachers still prefer to use this method in their lessons. This supports teachers’ beliefs that skills are important.

90% (p=0.000) had reservations about presenting unfamiliar investigations to the class deliberately, with very little indication of how to solve them, indicating a stronger consensus for a traditional approach. This supports statement two where teachers ensure students have received proper scaffolding (and have the necessary skills) before independently attempting an investigation. It appears that teachers are not comfortable with students taking risks without first achieving what is perceived as an acceptable level of proficiency in skills.

Only 40% responded favourably to the strategy of open-ended problems to develop students’ ability to explore mathematical investigations for themselves. As open-ended questions are a recognised strategy within the investigative approach and for developing higher order thinking, this is a salient finding requiring further investigation. If the participants believe in using investigations and group work, why are most finding it difficult to use open-ended questions? Perhaps this is another area for professional development.

75% of teachers indicated that they believed that most mathematical concepts could be taught through investigations; however, in practice they are being used in a limited capacity. This may indicate a fear of investigations, which could be assisted by professional development.

To unearth further detail regarding these survey responses on mathematical investigations, semi-structured interviews were conducted with three teachers representing a range of experience using Hattie’s (2002) categorisation, across the whole spectrum from a beginning (novice) teacher, to one that has taught for fifteen years (experienced), to a teacher with almost forty years’ experience who was nearing retirement (expert).

Interview responses and discussion.

Several themes emerged from an analysis of the recorded transcripts of the interviews: scaffolding of knowledge and skills; collaborative learning; developing mathematical language; an investigative approach to teaching, and professional development.

Scaffolding of knowledge and skills

One of the stronger themes from the interview data was the support for the scaffolding of knowledge and skills before an investigation, which is in keeping with the beliefs of Lovitt (2000). Comments like ‘building up a knowledge base’, ‘put the building blocks in place’,
‘help them form a foundation’ and giving students a ‘general basic knowledge of maths concepts’ support the idea of scaffolding, enabling the students to build the confidence needed to attempt a mathematics investigation. The experienced teacher clarifies this by emphasising that it ‘makes them confident in the use of their strategies and approaches ... I just think confidence is the key. Teaching them different strategies they can then go and use’. The novice teacher always likes to explain exactly what the students are going to do and what her expectations are for the students by the end of the investigation, so that they are not wondering ‘where is this leading, or why am I even doing this?’ and so it makes sense to them in a context. While the expert teacher agreed this is a good idea normally, it would also depend on the situation. ‘There might be some times where you deliberately don’t do a lot of explanations and leave the task a little bit open.’ He also stressed that scaffolding mathematical thinking strategies over time is effective until students can start choosing strategies for themselves. Similarly, the experienced teacher felt more comfortable explaining the task clearly prior to facilitating maths investigations using the ‘iMaths’ program, stressing that she prefers to scaffold their prior knowledge and explain the relevant vocabulary. The ‘iMaths’ is a program of real-life investigations that are linked to the Mathematics Curriculum and have been written for Prep to Year Six. \url{http://www.fireflyeducation.com.au/imaths}). She usually suggests a strategy to the students that is relevant to the investigation being attempted. The importance of ‘why they are doing this’ was also made clear, particularly with younger students such as her Year Three class. This teacher felt that developing the students’ strategies to the point where they could attempt un-scaffolded investigations was indeed important and desirable, but felt this was beyond realistic expectations. In contrast to this view, Boaler (2008) has shown that deep mathematical learning is possible within investigations, even within the mixed ability classroom. Students need to be given the opportunity to experience investigations, where they need to ‘wrestle with the ideas’, so that they will have the opportunity to develop perseverance and be mathematicians. Perhaps teachers need professional development to develop ways of scaffolding learning through the investigation to ease the transition for their students.

**Collaborative learning**

A theme that encouraged much discussion was that of students learning from their peers in various combinations of collaborative learning groups. The experienced teacher had been working for about 15 years, and varies her approach to student grouping during mathematics. Sometimes, she groups them according to ability. At other times she has mixed ability groups because the students who are more capable of finding solutions can help those who are not. She explains that ‘kids are more willing to offer ideas to each other because it is a secure situation for them. They’re definitely more comfortable with their peers.’ This is in agreement with Boaler (2008) who states the need for collaborative groups to be committed to each other’s learning to deepen mathematical understanding.

This view is also supported by the novice teacher, who describes mathematics group work as a form of peer mentoring, and that her students learn quite well from each other. Because it is not always possible to physically move around and help each student who is
experiencing difficulty or has a question to ask, this teacher often relies on ‘a small group of kids that are quite switched on to maths, are high achievers and are usually finished their task before the rest of the class’ to help those needing support.

The expert teacher also sees value in using the group situation of mathematics investigations to allow students to learn from their peers and to provide an opportunity to share ideas. These groups vary in size and are used for different types of lessons, depending on whether it is an investigation, problem solving, measurement activities or thinking exercises. The common ingredient is always ‘lots of discussion, whether it be in pairs, small groups or otherwise’. The only reservation was that if the teacher wanted to assess how well individuals can do things by themselves, group work might not always be the best approach.

**Developing mathematical language**

The importance of mathematical language and the use of relevant vocabulary emerged as a theme in the interview data. The novice teacher approaches this by sometimes making a ‘word-web’ chart when introducing a topic to capture the ‘words that belong’ with that concept (for example with money; ‘change’, ‘expensive’, ‘cheaper’). This is done by relating the concept to the real world contexts the Year Two students can relate to, as well as the context being used for the investigation she is preparing the students for.

Similarly, the experienced teacher stresses the importance of the ‘language of mathematics’, preferring to introduce the ‘maths words’ with her class before beginning to teach her Year Three class about a new concept. This teacher also felt that investigations help students to learn new vocabulary because of their preference for hands-on, kinaesthetic learning, ‘especially if you take them outside and physically show them’ a mathematical skill like measuring’.

The expert teacher felt strongly about the importance of reinforcing relevant vocabulary all the time, explaining that ‘you’ve got to keep on revising it, explaining it, using it, and then kids start to become more familiar and remember those terms and when to use them’. This teacher likes to approach vocabulary with his Year Six students by asking them to write a reflection after an investigation, either individually or in pairs. This was felt to be very valuable because it gives the teacher a good insight into the students’ understanding and is a useful way to assess or evaluate how well the students are going.

This idea has been developed by Thompson and Rubenstein (2000, p. 573) ‘[E]nculturating students to the vocabulary, phrasings, and meanings of mathematical language is a dimension of instruction that needs specific attention.’ This then gives a common understanding of the language students will use to communicate and support each other as they work collaboratively (Boaler, 2008).

**An investigative approach to teaching**

Mathematical investigations are framed by an investigative or inquiry based approach to teaching. All three teachers emphasised the development of mathematical strategies when teaching through investigations. The beginning teacher makes a point of asking her Year Two students, ‘How did you get there?’ after they complete a set of mental maths questions. Other useful questions included ‘Why did that strategy work?’, ‘How easy was that strategy to use?’
and ‘Would you continue to use that strategy?’ This teacher is confident her students are getting better at explaining in more detail how they got their answer through the practice of this routine of asking these sorts of questions. Modelling a high level of enthusiasm during an investigation is also seen as an important teaching approach, even to the point of being dramatic and using role-play when working with students from Prep to Year Three in particular.

The experienced teacher employs a similar approach when playing maths games by asking the students to write down what strategy worked best to win the game and why. The students are not always told what strategy is the best to use for each game, so there is rich discussion needed among the players to identify this.

The expert teacher prefers to do a lot of work getting Year Six students to use metacognition by emphasising the importance of students showing their working out and demonstrating their strategies used, with as much detail as possible. This is done through the routine of students setting out their work in their notebook, where the largest column is devoted to detailing strategies used, neatly showing calculations and operations, even including artistic diagrams. These students are encouraged to think of different ways to find a solution, as well as different possible answers through exploration in this notebook. Positive feedback is given to the students who derive different solutions from the obvious ones, and they are asked to share their solutions with the group or whole class.

The expert teacher is quite passionate about teaching through inquiry-based learning. He explains, ‘I think letting kids think of the different ways they can go to solve things, exploring things, and the different ... range of possible answers ... where it might lead to ... I think it is good for them to be aware of that.’ The expert teacher uses this particular approach to stimulate students who demonstrate a high level of mathematical ability by ‘not giving them too much development of the problem just to see how they operate, without giving them clues or pointing them in directions. I believe in that.’ This teacher sees this as a way of developing lateral thinking, with the students getting better at thinking of categories to narrow down possible solutions. ‘Kids normally really like that sort of thing’.

The interviews revealed a general belief in the benefit of hands-on skills for learning from all three interviewees. The novice teacher noted that she believed the students found maths more fun when involved in hands-on experiences, with some of them asking her questions like ‘Is this maths? Are we doing maths?’ when cutting and pasting or constructing. She found this was important when her class showed signs of being lethargic during mathematics lessons. Both the novice and expert teacher showed a clear preference for using smaller mathematical investigations as an effective teaching strategy. The novice teacher referred to the benefits of mathematics becoming more hands-on, even utilising opportunities outdoors and referring to the benefits of integrating a recent maths investigation with science while collecting data on cassowaries and graphing this with her Year Two class. This was an effective way to consolidate concepts taught at the end of the unit.

The expert teacher (with a Year Six class) was enthusiastic about the use of smaller mathematical investigations, emphasising that their value lies in the way they can expose students to mathematical thinking without being too complex, involved or time-consuming, thereby increasing the students’ chances of experiencing some success and developing a more positive attitude towards mathematics. He describes them as being ‘very teacher-friendly and
kid-friendly’. While admitting to finding the bigger investigations valuable because they enabled opportunities to go deeper into concepts, the experienced teacher also stressed that they were too time consuming, ‘with time becoming a real issue for teachers these days.’

As a recent graduate using maths investigations with her students on a regular basis, the novice teacher expressed her confidence had increased as time passed, and generally she felt positive towards this approach to teaching mathematics. However, she was concerned that the tasks set for her students in the ‘I Maths’ program were at a level that was too challenging for her students at times, and this discouraged some of her struggling students. She tried to avoid this by modifying the activity by reducing the amount of work to be done so these students could experience some success.

This teacher also felt a bit frustrated that, after teaching with the investigative approach and using group work and discussion strategies, she was forced to use a ‘common assessment task’ used by other teachers in her grade level (for end of semester reports), which lacked a context. She felt this compromised their interest and motivation to do well, and was therefore not a true indication of their ability. Rather, the students were keen to just get the tests over with, so that they could return to ‘the more interesting hands-on stuff again’.

The experienced teacher has found through her experience that when students are just told the answer, ‘they are not as successful at retaining the new knowledge. They can’t transfer the skills learnt to a new situation’. She also believes that mathematics should be practical, ‘and students need to know that it is practical and that it is something they are going to use’. There was also mention of the importance of her students being able to make discoveries about maths for themselves before being told the answers to questions.

Adding to this, the expert teacher believed it was the ‘real practical experiences with things like measuring and building their knowledge base that helps them to experience some success with carrying out maths investigations’. He also felt this feeling of success can be generated by ‘developing kids’ interests by initially doing some type of investigation task where they just get into (the concept) straight away. While clarifying that he was uncomfortable to leave the question open for too long at the beginning, the timing of the investigation depends on the situation. This teacher felt that it could also be placed at the end of the unit of work on a concept, as a means of consolidation, or even assessment.

**Professional development**

The interviewed teachers stated that they had experienced some difficulty when using the investigative approach to teaching mathematics. Being a first year graduate, the novice teacher was not confident with practising inquiry-based learning during mathematics lessons. However, this emerged as a theme as the interviews unfolded because she felt the open-ended nature of the ‘I Maths’ investigations she used was ‘what really draws the students in’. Furthermore, she explained ‘it excites their interest and gives them opportunities to explore other paths or avenues for how to arrive at a solution’. There was also mention of some surprises from students on discovering some real-world connections. The experienced teacher was not really confident with teaching mathematics on the whole, worrying that she was not often able to think of exciting or challenging investigations for her students to attempt. Furthermore, this teacher felt it was important for her to receive professional development in this area, but was not sure where to look for this, suggesting this was something she did not
have the energy to pursue. She did say that physical resources were not an issue. Rather, it was her own mathematical knowledge that was holding her back. The expert teacher did not feel he was having any difficulty at all with this approach, apart from not always having the time to teach concepts as deeply as he would like.

All three teachers indicated they believe in the value of working in the context of an investigation. The novice teacher likes the fact that investigations are ‘more hands-on’, the students get more involved than in teacher-directed lessons and also believes in the potential for investigations to bring together a number of concepts into a context, demonstrating the connections within mathematics. The expert teacher believes quite strongly in the value of investigations as a means of putting mathematics into a real context, while the experienced teacher believes the value lies in the ‘fun’ factor for her students when working on an investigation, motivating them and making them ‘feel part of the lesson’.

The other main belief was their support for developing the metacognitive process in students by encouraging them to reflect on the strategies they have used. The experienced teacher not only believes in students explaining to her how they solve a problem or answer a question, but also actively encourages them to explain this to their peers, as she believes this process encourages reflection in a secure, supportive setting. She elaborates further along this line of thought by explaining: ‘I think students like to believe they have some control over their choice of strategies. Investigations are a good way to let me know their background knowledge and what level of language they use while finding solutions’. This aligns with her teaching approach, which emphasises their use of maths vocabulary. The novice teacher also showed support for the belief that students should reflect on their ideas and strategies used, confirming if these strategies worked in finding a solution and analysing why or why not. ‘I think it is good for students to come up with their own way.’

The expert teacher prefers to see students exploring a variety of strategies, and believes it is up to the teacher to not only expose the students to this variety but also to actively demonstrate ideas and methods until they become familiar with the strategies needed for that type of problem. Questioning was felt to be an effective technique to encourage students to reflect on strategies used. This teacher suggested that teachers need more professional development in developing their own knowledge base of mathematical strategies in order to do this. He recalls what originally gave him confidence in this area:

\textit{When I started teaching in the 1970’s, I had to do a five-week session of professional development and it was all to do with mathematics ... and that sort of experience, developing that knowledge base was fantastic. It was far more involved (more useful) than what I did in courses at Teachers’ College because it was so intense. There was so much covered at such a high level, and there was such a variety of experiences happening in mathematics. It really emphasised the fun part of mathematics as well.}

He explains further what has maintained that level of interest during his long career:

\textit{I’ve been involved in a number of PD workshops where there have been some good problem solving situations, small investigations, those sorts of things. Some of them were very hands-on and practical; some of them were looking at number patterns and number sense, thinking mathematically and all that ... they’ve always been fun and interesting.}
Implications and Conclusions

The interview results suggested that some teachers recognise a shortfall in their own professional understanding of the best way to use mathematical investigations. This type of professed ‘insecurity’ that teachers feel towards teaching mathematics through investigations provides insights for professional development planning for school administrators trying to ensure that the Proficiency Strands of the Australian Curriculum do not become subverted to the development of concepts and skills. In the same way that some teachers refer to the need to develop students’ basic skills and knowledge before they attempt any problem solving or investigations, so too could we approach the professional development of teachers with this style of pedagogy, particularly if teachers are more comfortable with this.

The expert teacher mentioned that the focus of teachers’ mathematics professional development in the 1970s was intense and involved being immersed in mathematics. This helped him see the connections between concepts and to ‘do the maths’ by attempting a range of hands-on problem solving scenarios in a group situation, making his experience as a beginning teacher all the more enjoyable and motivational. He reflected that this experience gave him the confidence he needed to teach mathematics in the classroom at the commencement of his career as a primary teacher in Queensland. The practice of teachers ‘doing the maths’ and looking for opportunities for misunderstanding, and just having a deep understanding of mathematics can eliminate the insecurity that some teachers refer to. This may also be part of the reason for the conflict between teachers’ beliefs and their actual classroom practices teaching mathematics. In addition, it can provide a platform for exploration and risk-taking with their own teaching pedagogy away from more traditional styles and more towards an investigative style.

Teachers also expressed concerns that there was a mismatch between teaching through investigation and the more traditional style of common assessment tasks being used in this school for each grade level for reporting purposes. If teachers are focused on this type of assessment, the wash back effect results in devoting less time to thinking skills such as analysing, evaluating and reflecting. The teachers within this school therefore need to have a discussion about the types of assessment that reflect the pedagogy that they believe will best suit their students.

The school participating in this study underwent professional development in mathematics investigations in 2008, however the impact of this professional development has not been long lasting for some teachers, and other teachers have arrived at the school since then without undergoing this training. This scenario could be similar in many Queensland schools, and has implications for the delivery style of professional development. One-off style workshops are the cheapest method but do not appear to have a long lasting effect compared to the ‘Coaching Model’ (West & Staub, 2003). This naturally has the added cost of employing a mathematics coach, but in Queensland it may be more practical to share coaches between schools. Federal funding has already led to intensive, targeted, short term employment of ‘numeracy coaches’ in a small number of disadvantaged schools.
Alternatively, peer mentoring has been shown to be a cost efficient form of professional development (Kensington-Miller, 2012).

The implications for professional development are two-fold. Firstly, teachers need professional development on collaborative learning in group work; strategies for teaching the language of mathematics, and scaffolding learning strategies within an investigative pedagogy. Secondly, teachers need support to raise their confidence levels in teaching mathematics through investigations, including allocation of resources and the opportunity to design and plan for the best use of these resources to support the teaching of mathematics. Part of this professional development needs to allow “them to see the social and constructive aspects of the discipline” (Cross, 2009, p. 342)

Teachers also need ongoing support in the four identified areas above as they implement these changes in their classrooms to avoid the common circumstances quoted below from the Anderson et al. study:

“It appears from these responses that the knowledge, advice and curriculum efforts to date have not provided enough support for teachers. It is possible that the support has not challenged teachers’ beliefs enough for them to reconsider their current practice in order to fully embrace reform recommendations” (Anderson, White, & Sullivan, 2004, p.26).

References


