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ABSTRACT

The purpose of this exploratory quasi-experimental study was used to see if second grade students could learn to transform visual, aural, and kinesthetic rhythm experiences into mathematical symbols in order to equate and add fractions with unlike denominators. Forty-three second graders (n=22 experimental group and n=21 control group) from a suburban/rural public school in central New York State completed a researcher-designed test (pretest/posttest) on fractions. Results of the gain scores from the experimental group were significantly higher than the gain scores of the control group. Discussion focuses on the possibility that music may have a symbolic language that lends itself to mathematical transformation and that this connection may help expedite developmental learning levels when teachers in different content areas work together.

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Elementary general music teachers often are asked to help facilitate student learning in academic areas outside of music. This reality is enhanced further by recommendations in the eighth voluntary national standard for music education that advocates students learn to understand relationships between music, the arts, and disciplines outside of the arts (MENC, 1994). The reasoning for encouraging this kind of collaboration often resides in the belief that students will learn more about a particular subject if they can discover how the subject relates to other areas of learning and life, both in and out of school. This is an intriguing notion, which should be examined through controlled investigation. In order for this type of research to be considered meaningful, the impetus of examination might begin by identifying areas or concepts within subjects or disciplines that are particularly problematic for students to learn.

The Problem

There is a documented trend showing the difficulty students have at learning initial fractions in mathematics. Results of multiple assessments of the National Assessment of Educational Progress (NAEP) dating from 1978 to 1997 have shown that many children do not seem to possess basic fractional understanding (Carpenter, Coburn, Reys, Wilson, & Corbitt, 1978; Dossey, Mullins, Lindquist, & Chambers, 1988; Kuba, Zawojewski, & Strutchens, 1997). Although teaching and learning initial fractions is a multifaceted process, leaders in both education and government are becoming increasingly concerned with these immutable results and might welcome suggestions in teaching approaches that could help reverse these trends.

Problems seem to stem from students not possessing deep concept knowledge of fractions necessary for understanding and applying fraction principles. Students routinely
miss questions such as “how much is 3/4 plus 1/2?” and “which of two amounts is greater, 5/8 or 2/4?” on standardized math tests. Bezuk and Cramer (1989) suggested that this kind of misunderstanding comes from teachers possibly trying to do too much too soon and not spending enough time developing basic concepts with students. They stated that an increase in instructional time is necessary at all grade levels so that students at each level understand rudimentary fractional concepts before proceeding to subsequent levels. Perhaps what is needed in addition to an increase in time-on-task specifications are more and varied examples of concrete fractions, or what Bezuk and Cramer call physical models. Students possessing this background might be more successful with fractions due to the increased array of problem-solving tools and mental schemas of fractions at their disposal.

Since numerical fractions (e.g. 1/2, 3/4, etc.) are abstract representations of a physical construct, students need many primary experiences with physical models of fractions in order for the numeral representations to be meaningful. In the document Principles and Standards for School Mathematics (NCTM, 2000), it was suggested that building number sense requires multiple uses of concrete models that “can help bring meaning to students’ use of written symbols” (p.80). Indeed, Kato, Kamii, Ozaki, and Nagahiro (2002) found that students’ deep understanding about quantities are necessary in order for them to be able to represent those quantities with numerals; simply knowing numerals does not necessarily translate to students’ understanding that the numerals stand for specific quantities. It seems clear that before students can use abstractions, they need to understand the concepts underlying the representations of those abstractions.
The acquisition of fraction conceptualization is complex. Verbally identifying a half loaf of bread is different than using the numerical symbol 1/2 to express the same half loaf of bread. Students must traverse through logical stages in order to achieve mastery of using mathematical symbols to communicate fractional understanding; however, an important question remains as to what grade level is appropriate for students to learn this abstract system. There are various opinions as to how early students can learn to use abstract numbers to express fractions. Most sources suggest that this is best attempted with students above third grade. Ellerbruch and Payne (1978) targeted fourth grade as the optimal time to teach addition of unlike fractions (fractions with different denominators), while Bezuk and Cramer (1989) suggested “postponing most operations with fractions at the symbolic level until grade six and using instructional time in grades four and five to develop fraction concepts and the ideas of order and equivalence” and giving children in the primary grades only those “experiences that will allow them to develop strong mental images of fractions” (p.158). Authors of the Principles and Standards for School Mathematics (NCTM, 2000) stated that although students at the end of second grade are expected to be knowledgeable about traditional digit notation, “fractions are not a topic for major emphasis for preK-2 students.... At this level, it is more important for students to recognize when things are divided into equal parts than to focus on fraction notation” (p.82-83).

Controlled investigations of initial fraction learning with traditional notation also have focused on students who were in at least the 3rd grade or above (Coburn, 1973; Cramer, Post, & delMas, 2002; Muangnapoe, 1975). But is it possible for students below third grade to understand fractions using traditional number symbols? Do alternate
concrete/physical models exist that might help younger students make the transition from concrete association to abstract symbolization? Is the lack of fractional competence in students a result of too much too soon or could it be from a lack of quality teaching-learning experiences that have failed to maintain a sustained effort to show connections between fractions in mathematics and fractions in other phases of school and life?

**Drawing Research Questions from a Hypothesis of Natural Connection**

As stated in *Principles and Standards for School Mathematics* (NCTM, 2000), helping preK-2 students form connections between mathematics and contexts outside of mathematics is extremely important. The authors of that document maintain, “the most important connection for early mathematics development is between the intuitive, informal mathematics that students have learned through their own experiences and the mathematics they are learning in school” (p.132). In most elementary schools and homes, children have access to music. In many of these situations, children’s involvement with music goes well beyond trivial association to more persistent, concentrated, and sustained engagement. Many children are instructed in music both inside and outside school. When immersed in music instruction, particularly in rhythm, children, inevitably, engage in an aural (and, often, oral) form of fractional communication and understanding. Musical rhythm is abundant with fraction terminology. In fact, students cannot learn rhythm without using the language and/or mental operations of fractions. When preschool and early elementary students receive musical training from music teachers, they often hear and learn about whole, half, quarter, and eighth (and, sometimes, sixteenth) notes. Rhythm is a physical model of fractions in aural form. Furthermore, when music teachers, adhering to content standard #5 (reading and notating music) from the national
standards of music education (MENC, 1994), expose students to what these sounds look like (in the form of music notation) they are showing students another visual representation of a concrete fractional model, this time in the form of notes rather than numbers. The musical notation of fractions is not unlike the numerical representation of fractions in mathematics; it is merely another symbol system. Finally, when students clap, sing, and play musical instruments corresponding to short and long durations, they feel and hear fractions, yet another example of an alternate physical model. This time, however, the physical model of fractions is kinesthetic as well as aural. If students below third grade can learn the symbol system of fractions in music (i.e., rhythm symbols), then can those symbols be used to teach these students the numerical system of fractions in mathematics? DeLoache (1991) asserted “experience with a task that activates one symbolic system that children understand makes them more receptive to learning a new type of symbolic relation” (p. 747). It would be interesting and informative to test this assertion experimentally. Given the aural, oral, and visual nature of fractions in music (via rhythm), the purpose of this investigation was to explore the possibility of teaching students to use traditional mathematical symbols to equate and add unlike fractions (with different denominators) through interdisciplinary instruction with musical rhythm. By providing students with an alternate concrete model of learning fractions, rich with its own symbolic representation, they might be able to use the model to learn the traditional mathematical symbols of fractions. A concurrent hypothesis was to see if it was possible to accomplish this with students at second grade level.
Existing Data of Using Music to Teach Other Subjects

Researchers have suggested that there may be positive links between music participation and scholastic achievement outside of music. Results from investigations have revealed associations between music participation and improved scores in mathematics, reading and language, and memory retention of mentally challenged students (Krebs, 1978), improved mathematics achievement of typical third grade students (Gregory, 1988), and second graders’ improved comprehension of geometry, measurement, and money (Benes-Lafferty, 1995). Other studies have been conducted to examine the effects of arts infusion approaches on mathematics achievement. Arts infusion is the use of multiple arts disciplines such as dance, music, theater, and visual art to teach students in disciplines outside of the arts. Researchers have suggested a link between this teaching approach and improved achievement in double-digit subtraction with second graders (Omniewski & Habursky, 1998), improved scores in reading, social studies, and math over a three-year period (Catterall, 1995), and improved overall academic performance over a four-year period in the Greater Augusta Georgia Area Schools (Greater Augusta Arts Council, 1995). Despite the existence of these studies, none were found that examined the effect of rhythm instruction on fraction learning.

Method

Unlike most of the studies cited above, the present investigation was not designed to test one teaching methodology against another. Due to the lack of existing literature on the present hypothesis, and given the lack of attention on fractional understanding using numerals among second graders, this exploratory quasi-experimental study was designed
to see whether it was at all possible for students in second grade to learn to equate and add unlike fractions using mathematical symbols by way of rhythmic association. The strategy chosen was a pretest-posttest nonequivalent control group design (Huck & Cormier, 1996). Two groups of second-grade students (n=22 experimental and n=21 control) were selected from intact classes within a public school system. This plan was considered over a one-group pretest-posttest design in order to control for most of the threats to internal validity, such as history, maturation, testing, instrumentation, and regression. Participant mortality was preserved throughout the entire study; every person who began the study finished it.

School and Participants

The study was conducted in a public elementary school in central New York State that was attended by students from primarily rural/suburban backgrounds. The total number of students in the K-4 building, in the 2001-2002 school year, was 479. Students were grouped heterogeneously, and the school espoused a full inclusion philosophy. There were a total of 88 second-grade students separated into four classes, and the two classes chosen for this investigation were done so randomly. Although data were not available on the mathematics achievement level of the two classes, 100% of the third graders in the school scored above the state average on the mathematics portion of the New York State Pupil Evaluation Program (PEP) test. The mean mathematics score for the school’s fourth graders on the PEP test was 659 with 74% of the students obtaining a level three on the four-level assessment rubric. Level three states that students at that level basically meet state standards. The percentage of children receiving reduced lunch was 4.8 while the percentage receiving free lunch was 6.5. The racial makeup of the
elementary school was 95% white, 3.1% Asian, 1.7% black, and .2% Hispanic. The annual attendance rate for the school was 96%. The experimental group was comprised of 11 boys and 11 girls, while the control group contained 10 boys and 11 girls. Every class in the school received music instruction from a certified music education specialist twice per week for 30 minutes over an academic year.

The school was selected on two criteria. First, it employed a full-time music specialist who taught a comprehensive and sequential music education program based on the National Standards for Music Education to all students. This meant that by second grade, students were familiar with both the aural and visual representation of rhythms; that is, they knew how quarter notes, eighth notes, etc. looked and sounded. Second, classroom teachers had to verify that they followed the Principles and Standards for School Mathematics for second graders, which excluded teaching fractions using numerical symbols; therefore, neither group had been exposed to fractions containing numerals.

Treatment

Before the experimental group received the interdisciplinary instructional technique, both groups (experimental and control) were administered a researcher-designed pretest on fractions in order to provide baseline data. The logic regarding the format and content of the test was patterned after existing tests on beginning fractions (Figure 1). Students were informed that the spaces between each proportion were to be considered “working areas,” if necessary, and that they were to provide numbers for the question marks that would make the amount to the right of the working space equal to the
amount on the left. Both groups completed the pretest during their regularly scheduled music period.

Following the pretest, the experimental group received four 30 minute integrated lessons (over a two-week period) from their music teacher who used musical rhythm experiences involving oral, aural, and visual representations of the rhythms to associate with the mathematical symbols. Students in the experimental group received treatment during their regularly scheduled music period. The control group received alternate music lessons during this time and did not receive instruction illustrating the transference of rhythm symbols to mathematical symbols. The procedure used in the teaching sequence with the experimental group followed basically the suggestions offered by Ellerbruch and Payne (1978), who identified four sequential steps that successful mathematics teachers typically follow when teaching initial fractions. They stated that these steps help students reach the “ultimate goal” of fraction competence: adding fractions with unlike denominators (p.131). The major steps in Ellerbruch and Payne’s teaching sequence include:

1. using concrete objects to make equal size partitions.
2. recognizing and using the oral names for the various-size parts.
3. drawing diagrams of the concrete objects and attaching the oral names to the parts.
4. using the concrete objects and diagrams together with the oral names to write the fraction symbols.

Using the basic premise of these steps, the music teacher and I planned the four instructional periods, which employed the following teaching techniques. First of all, the
individual and grouped rhythms found in the rhythm pyramid (Figure 2) were considered the “concrete objects” used for the transition to the mathematical symbols. Although these rhythms can be seen, they also can be heard when put into sound and felt when performed on instruments or put into bodily movement. Therefore, the first two classes of treatment involved students clapping, walking, and playing instruments corresponding to the duration of the rhythms (e.g., one group would repeatedly clap, walk, or play four quarter notes while another group would do the same for one whole note. This same procedure would be repeated with two eighth notes sounding for every quarter note, etc.). After hearing and feeling how these “aural and kinesthetic fractions” functioned together and in different combinations, students were taught to use the fractional names associated with each of the corresponding rhythms (e.g., students verbalized sixteenth, eighth, quarter, half, and whole notes by those terms).

Following the third step in Ellerbruch and Payne’s sequence, the students and teacher drew pictures of the various rhythms on the board and on individual papers. Part of this process included equating various rhythms with each other (e.g., drawing two quarter notes to equal one half note, two half notes to equal one whole note, etc.) and adding various rhythms together to equal other sums of rhythms (e.g., drawing a quarter note to show that it represents the sum of one eighth note and two sixteenth notes, etc.). Once again, the students were encouraged to verbalize the fractional terms associated with each rhythm and group of rhythms during this step, which, of course, are symbiotic of both musical and mathematical disciplinary content.

Another instructional technique employed during this step involved having students become part of a type of human rhythm pyramid. Four students (with quarter
note symbol cutouts taped to their chests, and one eighth note symbol cutout taped to each leg) volunteered to stand with both legs placed on one chair (with the legs of each chair representing four sixteenth notes). Each group of two students then held the same half note symbol (a yard stick fashioned with a half-note head), while positioning themselves under a hoola-hoop (representing a whole note) that hung from the ceiling of the room symbolizing the original unit of measure (Figure 3). With this visual representation positioned in front of the room, the rest of the class was encouraged to ask various questions (both hypothetical and actual) concerning the equivalence and sum of various patterns of fractions before them. For example, upon examining the human pyramid, a student would ask the class, “how many whole notes equals sixteen sixteenth notes?” The class would look at the four legs of each chair (representing four sixteenth notes on each chair) and see that one hoola-hoop was supported by this amount thus giving the answer that one whole equals sixteen sixteenths. Both the teacher and the students continued asking questions such as this utilizing all possible combinations of rhythmic equivalence. Students also expanded their thinking to include hypothetical questions such as “how many eighth notes will equal three half notes” and “how many whole notes will equal two half notes plus four quarter notes?” In order for students to answer these latter questions, they had to mentally expand the human pyramid before them and calculate the fraction problem hypothetically.

Finally, the last step of the process (as adapted from Ellerbruch and Payne) involved showing students how to write these rhythm representations as fractions using numbers (the top number representing the number of rhythms and the bottom number representing the kind of rhythm). For example, two quarter notes were translated into 2/4
and four sixteenth notes were shown to look like 4/16, etc. The last two instructional periods with the experimental group were devoted to this transfer exercise (Figure 4). Subsequent to these four instructional class periods with the experimental group, both groups were administered a posttest, which was the same as the pretest. The amount of time between pretest and posttest was about two and one half weeks.

**Test Instrument**

To test the internal consistency of the test instrument, the four tests (pre/post experimental and pre/post control) were analyzed individually via the Kuder-Richardson 20 formula. Reliabilities were .70 and .68 for the experimental group’s pretest/posttest scores and .72 and .90 for the control group’s pretest/posttest scores respectively.

**Results**

Given the ordinal nature of the data collected and the relatively small and unequal sample sizes employed, the gain scores of the two groups were compared using the non-parametric Mann-Whitney $U (Z)$ test. Results showed that the experimental group’s gain scores ranked significantly higher than the gain scores for the control group ($U = 58, p < .0001$, one-tailed). Since both sample sizes were larger than 20, the U value was converted into a z-score of -4.20 with a critical value of 99.9987%. Raw gain scores for both groups can be found in Table 1.

**Discussion**

Although the two research questions proposed in this investigation were supported positively by the data, they should be interpreted cautiously. Even though the
majority of the students in the experimental group seemed to learn to add and equate fractions using numerical symbols on at least some of the items, there was not consistent performance among most individuals across the eight questions on the posttest. Only 6 of the 22 students in the experimental group made gain scores of between six and eight. Only three more students had a gain score of five; therefore, more than half of the students in the experimental group had gain scores of four or less on the posttest. Eight students acquired gain scores of between zero and two, which was similar to the performance of the majority in the control group. Previous scholars suggested that children younger than third grade should not engage in proportional calculation using traditional notation due to their lack of maturity and previous experience (Bezuk & Cramer, 1989; Coburn, 1973; Cramer, Post, & delMas, 2002; Ellerbruch & Payne, 1978; Muangnapoe, 1975; NCTM, 2000), and the results of this study do not provide an abundance of evidence to contradict this notion. However, the experimental group, on the whole, achieved gain scores that were statistically significant over the control group, thus providing evidence that something in the way of success was apparent. Since neither group of students had been exposed to fractions containing traditional notation before this study was undertaken, the success of some students in learning to perform such calculations was notable.

The hypothesis that integrated experiences in psychomotor and visual rhythm activities might help second graders learn to calculate traditional fractions, was similarly tenable, yet circumspect. For example, it might be just as possible that students learned to give the rhythm symbols new names that happened to look like the symbols of fractions rather than learning to think mathematically. Future researchers might employ qualitative
techniques and analysis in order to isolate students’ verbalizations regarding their thought processes when engaging in proportional calculations with traditional mathematical symbols. Future investigators also might explore the longitudinal implications of students achieving early success with fractions using traditional symbols. When students learn to understand fractions using traditional symbols when they are younger, will their test scores on fractions improve when they are older? Since previous investigators of initial fraction learning found that students seem to lack a functional understanding of fractions even as late as sixth grade (Kuba, Zawojewski, & Strutchens, 1997), it might be helpful for teachers to explore pedagogical avenues, such as the one investigated in this study, earlier in students’ education in order to bolster this understanding later. Clearly, more research is needed in order to determine if this is possible.

This study should not be interpreted as lending support to unsubstantiated claims that instruction in some disciplines will inevitably produce positive effects in the achievement of others. The significance of this quasi-experimental exploratory investigation lies less in the provision of definitive conclusions regarding the possibility of students in grades lower than third grade to understand fractions using traditional mathematical symbols than it does in providing possibilities regarding the cooperation of teachers in sharing the responsibilities of mathematics instruction. When teachers with various backgrounds and expertise in pedagogical content work together, some students might become beneficiaries of learning. Additionally, it should not be interpreted to mean that other methods of teaching fractions to children are inferior to the approach used here. Just as direct teaching within the discipline of music is necessary in order to produce quality learning in music, math educators also need to use direct teaching in math (e.g.,
fractions) in order to achieve results in student understanding. However, this investigation does seem to illustrate that teachers who have alternate or multiple teaching methodologies at their disposal, whether from within a specific discipline or from disciplines outside those targeted for learning, might increase or expedite student achievement because of the various strategies involved in the teaching process and the learning styles that students bring into the classroom. Interested researchers might seek further evidence of students’ ability to grasp mathematics concepts at developmental levels previously believed to be premature. Furthermore, it also would be interesting to determine how alternate forms of instruction, such as the one utilized in this study, affect proportional understanding beyond fractions with denominators of 1, 2, 4, 8, and 16.

References


Name _______________________ Date _____________

Teacher ______________________

1) \( \frac{2}{2} \) | \( \_? \) |
   | \( \frac{1}{1} \)

2) \( \frac{4}{4} \) | \( \_? \) |
   | \( \frac{8}{8} \)

3) \( \frac{8}{8} \) | \( \_? \) |
   | \( \frac{16}{16} \)

4) \( \frac{1}{2} \) | \( \_? \) |
   | \( \frac{4}{4} \)

5) \( \frac{8}{16} \) | \( \_? \) |
   | \( \frac{2}{2} \)

6) \( \frac{1}{4} + \frac{4}{8} \) | \( \_? \) |
   | \( \frac{4}{4} \)

7) \( \frac{1}{2} + \frac{2}{4} \) | \( \_? \) |
   | \( \frac{1}{1} \)

8) \( \frac{4}{16} + \frac{1}{4} \) | \( \_? \) |
   | \( \frac{2}{2} \)

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Figure 1. Pretest and Posttest items
Figure 2. Rhythm Pyramid
Figure 3. Human Pyramid
Figure 4. Rhythm pyramid with fraction equivalents

1/1  =  \( \boxed{} \)

2/2  =  \( \boxed{} \)  \( \boxed{} \)

4/4  =  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)

8/8  =  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)

16/16 =  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)  \( \boxed{} \)
Table 1
Raw Gain Scores* with a Possible Range of -8 to 8

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* Gain scores = posttest scores minus pretest scores