The Instructional Quality of Classroom Processes and Pupils’ Mathematical Attainment concerning Decimal Fractions

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Abstract. The objective of our study is to understand and analyse what significance cognitive and emotional networks of classroom processes have in mathematics learning. The subject of the study includes two classrooms from Year 5 of a teacher training school, their pupils (N_A=17, N_B=19) and student teachers (N=4). The course on decimals, which consisted of 17 lessons, was carried out in both classrooms. Research results are based on the analysis of 34 video-recorded lessons and the statistical analysis of pupils’ mathematic attainment before and after the course. The teaching in classroom A emphasised instructional coherence, cognitive activation and emotional support. Classroom B’s teaching represented lower levels of coherence, cognitive activation and emotional support on average. The pupils’ skills concerning decimal fractions developed in both classrooms, but in classroom A the average change in the score from the pre-test to the post-test was more powerful (p=.04). The change was particularly clear on pupils in the middle level (p=.00). According to the main findings, teaching courses including instructional coherence together with cognitive activation and emotionally supported students produces better mathematical attainment.

Keywords: Instructional quality; Instructional coherence; Cognitive activation; Teacher-pupil relations; Mathematical learning

1. Introduction

Background of the study

As researchers, we developed an interest in whether relatively stable, qualitative dimensions of teaching and group-specific dimensions were significant in pupils’ mathematical learning.¹ The connections between teaching and learning can be built logically and speculatively, but it is extremely difficult to examine them empirically. There are many methodological difficulties and sources of errors related to the research data and the analysis. Initially, we review some qualitative features of teaching that are potentially significant and which have been reported in the literature, especially in recent years. Then we present the study’s research task.

1) Instructional coherence

¹ The tentative findings of our research project – without rich data documentation and diagrammatic presentation such as in this article – have been published elsewhere in Finnish (Pitkäniemi & Häkkinen, 2012).
There is no clear consensus among researchers on the definition of instructional coherence in mathematics classrooms, probably due to the complex nature of the concept (see Cai, Ding & Wang, 2014). Instructional coherence is a concept that describes how teaching consists of related sequences or activities, the way in which aspects of the subject matter link together are realised, and how this is perceived by students in the teaching process. In an ideal teaching scenario, elements or sequences relate to each other and they will create a consistent whole from a comprehensive point of view (Fernandez, Yoshida & Stigler, 1992). The coherence within a lesson, and a coherent structure across consecutive lessons of the same subject, can be logically considered to be a central facilitating factor in student learning. One theory of classroom learning has presented that a teaching course that includes several situations should contain a common theme that supports the learning experiences of students (Nuthall & Alton-Lee, 1993). This theory does not use the concept of ‘instructional coherence’, but it highlights the importance of cognitive continuity between one teaching situation and another.

However, there are few empirical studies that focus on the micro-structure of lesson coherence. Chen and Li (2010) examined instructional coherence in mathematics teaching by analysing a sequence of four videotaped lessons on fraction division. Their analysis focused on the features of instructional coherence, both within and between individual lessons. Their findings suggest that a coherent curriculum, and the teacher’s perception of knowledge coherence, supported the teacher’s realisation of coherent instruction. However, the study did not examine pupils’ learning outcomes. In a recent study, Cai, Ding and Wang (2014) examined how exemplary mathematics teachers view instructional coherence in their classroom. Some Chinese teachers emphasised the underlying structures and knowledge connections embodied in these activities. In addition, they focused on challenging students’ thinking. They referred to the interconnected mathematical concepts and students’ coherent and gradually deepening thinking as real coherence. In this study the research findings rest only on an analysis of the teachers’ stated views, not on their actual practice. In summary, as instructional coherence has been identified as an important feature of instruction in mathematics classrooms, the empirical support linking teaching and student learning is scarce.

Helaakoski and Viiri (2011) studied the content structure of physics lessons and students’ learning gains in Finland, Germany and Switzerland. Their results show that the number of knowledge elements and the links between them positively correlated with student learning. A case study by Badreddine and Buty (2011) involved two classes in Year 7, and focused on a teaching sequence on the subject of electricity within the context of physics instruction. They reported a positive effect on the student learning process derived from making explicit links between the past, present and future of the content within the instructional discourse. A study by Seidel, Rimmmele and Prenzel (2005) rated video recordings of 13 physics classes on two topics. The analyses showed the positive effect of lessons with high-level goal clarity and coherence on the students’ perceptions of supportive learning environments, self-determined learning motivation and organisation of learning activities. The authors reported that over the course of a school year, high-level goal clarity and coherence resulted in positive development of pupil competence.

In summary, from the perspective of logic, and from descriptive studies, coherent processes could be justified in terms of instructional quality. However, few studies clearly link instructional coherence and student learning in mathematics. Verbal properties will partly construct a quality of coherence, but we can speculate that cognitive continuity will be an even more
essential feature of instructional coherence, that is, ideally there would be real continuity in the content and management of learning experiences.

2) Cognitive activation, student engagement and the affective side of instructional quality

Based on previous research, cognitive activation, that is, challenging tasks and the use of student knowledge in the teaching process (Klieme, Pauli & Reusser, 2009), can be regarded as a prominent factor in order to guide student activity in the classroom and in stimulating students’ learning processes. Cognitive activation occurs when a teacher attempts to assist her/his students’ engagement in higher-level thinking (Klieme et al., 2009; Lipowsky et al., 2009; see also Brophy, 2000; Hiebert & Grouws, 2007; Mayer, 2004; Reusser, 2006), and this highlights the concept of teaching for understanding (Cohen, 1993; Pauli, Reusser & Grob, 2007). In teaching situations, this refers to actions such as providing challenging tasks in zones of proximal development, using students’ existing knowledge, ideas and experiences, and asking stimulating questions. Cognitive activation integrates challenging tasks, the exploration of concepts, ideas, prior knowledge, and Socratic dialogue practice as key features (Lipowsky et al., 2009).

In recent studies, the teacher’s cognitive activation has been examined together with other factors, such as classroom climate and classroom management (e.g. Lipowsky et al., 2009; Hugener et al., 2009; Rakoczy, Klieme, Buergermeister & Harks, 2008; Klieme et al. 2009). Lipowsky et al. (2009) found two features of instructional quality – cognitive activation and classroom management – to have positive effects on mathematical learning. Cognitive activation in teaching had a direct effect on students’ performance. Lipowsky et al. concluded that not all students benefit to the same degree from cognitively activating instruction, and that the process has greater potential for those students who are more interested in mathematics.

Fauth, Decristan, Rieser, Klieme and Büttner (2014) explain the meaning of the concept of supportive climate. It is composed of specific aspects of the teacher-student relationship such as positive and constructive teacher feedback, a positive approach to student errors and misconceptions, and caring teacher behaviour (e.g. Klieme et al., 2009). A meta-analysis by Seidel and Shavelson (2007) concluded that there was no evidence that a supportive climate had any direct effect on students’ learning, but it did have an indirect effect via mathematics-related interest. In conclusion, more cognitively activating instruction and a more supportive classroom climate have the effect of activating students and transforming their existing interests towards mathematics learning.

Turner and Meyer (2004) focused on ‘challenge’ as a motivator in mathematics instruction. They argue that a combination of challenging instruction and positive affective support is necessary in order to cultivate motivation in classrooms. In a study by Crosnoe et al. (2010), children with differing maths skills prior to primary school showed different but parallel trajectories of maths learning throughout Year 5. When enrolled in classes with inference-based instruction, those who initially had the lowest skills narrowed the learning gap if they did not have a negative relationship with their teacher. They did not show the same progress if they were in classes that focused on basic skills instruction or if they were in inference-focused classes but had a negative relationship with their teacher. Buff, Reusser, Rakoczy and Pauli (2011) studied students’ affective experiences in mathematics instruction, their antecedents, their cognitive and motivational consequences and their effects on student learning. Students
who had positive affective experiences during the instruction were more cognitively activated, and they reported a better ability to deal with learning targets.

In a study by Hugener et al. (2009), a discovery pattern that afforded much cognitive autonomy to the students led to negative student emotions and the feeling of insufficient understanding of the instruction, whereas no significant effect was detected on self-determined motivation or on cognitive learning activity. Hugener et al. concluded that perhaps the students did not receive enough individual or collective systematic support from the teacher. Wentzel (2002) suggests that students who perceive their teachers to be supportive are more likely to promote the goals valued by teachers, such as engagement in classroom processes. Similarly, Crosnoe et al. (2010) concluded that a teacher who can offer encouragement, emotional support and comfort to a student is likely to be better able to make that higher order approach work (see also, Griffin, 2004).

Overall, the research findings above indicate that cognitive activation is a promising means of improving student learning in mathematics classrooms. However, several studies found that cognitive activation alone is not enough to achieve improved learning outcomes. According to these studies, a challenging learning environment can be cognitively successful if, at the same time, emotional support is provided for the pupil.

Previous research and the study task

A review of studies on the effects of mathematics teaching over the past decade provides a general view of the present situation (e.g. Hiebert & Grouws, 2007). However, in recent years, research on classroom processes has developed methodologically (Douglas, 2009; Jan-ik & Seidel, 2009), and a phenomenon of teaching and learning has emerged from a variety of conceptual perspectives. One prominent example has been instructional quality, which as a broad concept can consist of research into cognitive activation and emotional support, and in a few studies, instructional coherence. The main purpose of this study is to further develop instructional quality research, the key motive for which is to identify instructional factors that are essential for mathematical learning, and to analyse the relationship between these potential factors. It was our tentative idea to develop an experimental design, but as we noticed that explorative research functions offer us open questions, we decided to follow a naturalistic and comparative research design with a qualitative orientation.

We examined the relationship between the instructional quality of classroom teaching and student learning within the learning context of mathematics, taking the following aspects into consideration:

a) Cognitive and affective indicators of instructional quality;
b) Mutual interactions between effective factors in the classroom;
c) Variation in pupil readiness to engage with the teaching process, for their own learning, that is, what kind of students benefit most from the teaching.

2. Method

Design and participants
In this study we followed a 17-lesson teaching course on the subject of decimal fractions in two Year 5 classes at a single teacher training school. The study participants comprised the pupils of these two classes, A (N_A = 17) and B (N_B = 19), and student teachers (N_A = 2; N_B = 2) who were practising in those classes.

A teacher training course was conducted at the school during the study course, and the student teachers taught all the lessons during that mathematics course. The course was part of the third year of training for the student teachers, and its objective was to practise planning and implementing teaching over a longer course. There were two student teachers in each class and they were responsible for all the mathematics classes for approximately 4 to 5 weeks. They also taught some of the other subjects. The student teachers designed and carried out the whole teaching course, and also evaluated learning outcomes. Before the training course, the four student teachers had passed the obligatory mathematics course as part of their studies. In order to be able to plan their own teaching course, their teacher educator (one of the researchers) gave them a common guidance lesson about teaching decimal fractions.

Data source and measures

We collected a wide range of data on classroom teaching: we videotaped the 17 lessons of the decimal fraction topic in both classes, and collected all of the lesson plans and tasks given by the student teachers during these 34 lessons. The pupils’ learning progress was measured using two tests: a pre-test was conducted just before the teaching course began, and a post-test immediately after it had ended. The tests were designed by the researchers, and the student teachers were not informed of any of the test questions.

Every test question measured a separate element of understanding the concept of decimal fractions or decimal calculation skills. The latter were measured more precisely in the post-test because no fewer than 7–8 lessons were given to learning these skills in both classes. Approximately half of the questions were basic tasks such as conversion between fraction and decimal forms, setting decimals in order of magnitude, rounding tasks or mechanical calculations, and the other questions required reasoning or application of knowledge. In the reasoning tasks, pupils were required to continue a sequence of decimals by rule, or to figure out the decimal that fulfilled the stated conditions. In the application tasks, pupils applied their knowledge of decimals to measuring units, or to a real-world situation.

Every pre-test task had a corresponding post-test task. These counterparts were not exactly the same but were very similar, measuring the same element and requiring similar thinking. It was possible to divide the solution processes of these questions into separate stages, or the solutions included several separate elements. This meant that the evaluation of test performances was based on the proportion of correct stages or elements in the pupils’ solutions.

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2 To achieve validity, a sufficient quantity of lesson data is needed because some of the properties of the teaching will vary between the lessons, whereas other properties are quite stable. For instance, in their recent study Praetorius, Pauli, Reusser, Rakoczy and Klieme (2014) found that classroom management and personal learning support were stable across lessons, whereas cognitive activation showed high variability.

3 We are teacher educators and we decided to implement the study in our students’ teacher training school during a Year 5 decimal fraction teaching course. Two classroom teachers from the school volunteered their classes for the study. The student teachers of those classes were randomly chosen.
On the basis of the pre-test results, the pupils were divided into three groups: low ($N_A=3$, $N_B=4$), middle ($N_A=10$, $N_B=10$) or high ($N_A=4$, $N_B=5$). Pupils scoring in the lowest quartile in the pre-test formed the low learning level group, the top quartile formed the high group, and the rest were allocated to the middle group. Table 1 shows the division of pupils from Classes A and B into these learning level groups.

<table>
<thead>
<tr>
<th>Pre-test point score</th>
<th>Low level 0–3.5</th>
<th>Middle level 4–12</th>
<th>High level 12.5–23</th>
<th>Number of pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>Class A</td>
<td>3</td>
<td>10</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>Class B</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>19</td>
</tr>
</tbody>
</table>

**Data analysis**

The qualitative videotape analysis comprised four different stages: 1) video recordings of the lessons were transcribed; 2) themes that described the teaching and the pupils’ actions during the lesson were recorded; 3) based on this coding, a summary of each lesson was drawn up; 4) each student teacher’s performance was summarised on the basis of these lesson-specific assessments.

Some information on the videotapes was quantified, and a time axis was drawn up of all 34 lessons of the teaching course. The starting points of the different sequences of the lesson were marked on this time axis at one-minute intervals. A classification was made of the teaching methods employed (such as classroom discussion, individual work, teacher’s inquiry) and the ways in which they engaged the pupils (such as individual work, pair work, classroom teaching), and these categories were marked on the time axis. The tasks and questions asked during each lesson were also classified (such as conversions between fraction and decimal forms, basic questions like mechanical calculations or setting decimals in order of magnitude, reasoning and application tasks). The connections between different sequences were also determined: whether the same task was continuing from one sequence to another or whether an obvious connection (such as the same objective) could be found between two different sequences. The illustrations of the quantifications are presented later in Figure 1.

The pre- and post-test scores of each pupil were tabulated on a task-by-task basis, because each task measured a certain element of understanding the concept of decimal fractions or decimal calculation skills. Statistical analysis of this data used both descriptive and comparative methods. The Mann–Whitney U-test was used to measure the significance of differences between the two classes or between the three ability groups of these classes, because the groups were too small for other tests and the variables were not expected to be normally distributed.

**3. Results**

In presenting the results, we will first describe and compare the teaching processes used in classrooms A and B. The main data set comprises the categorisation of the 34 mathematics lessons. Then we compare the learning outcomes of these teaching courses on the basis of the pre- and post-test data.
Teaching processes in classroom A

Lessons 1–11 in classroom A, taught by Agnes (all names are pseudonyms), had a clear and coherent sequence structure. This means that the educational objective of these sequences within one lesson was fundamentally identical and was realised by the pupils, but the sequences varied in form and approach. The material to be learned was described, practised, exemplified, etc. using several different teaching methods in separate lessons but also within a single lesson. The frequency and quality of pupils’ actual learning experiences was relatively high (cf. Nuthall 1999a, 1999b). This is also reflected in the lesson structure, in that it consisted of several short pedagogical periods connected to each other.

An example of the coherent sequence structure of Agnes’s teaching can be seen in a description of lesson 3, when four different approaches were used to achieve the learning goal. The progression of the lesson is shown in Table 2. The time axis of this lesson can be seen later in Figure 1.

Table 2. Description of lesson 3 in classroom A – teacher: Agnes

<table>
<thead>
<tr>
<th>Time m:s</th>
<th>Description of the teaching event</th>
<th>Tasks during the lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>0:38</td>
<td><strong>Checking the homework.</strong> The teacher asked who had not completed the homework, and concluded that she could ask the others to provide the answers. A message was hidden in the answers to the homework task, and one pupil was asked to say it and the others had to state whether the answer was correct or not.</td>
<td>4 conversions from one representation of a decimal to another form</td>
</tr>
<tr>
<td>2:55</td>
<td><strong>Announcement</strong> (concerning a lesson on another subject)</td>
<td>6 questions about decimal units or illustrating decimal fractions</td>
</tr>
<tr>
<td>3:37</td>
<td><strong>Discussion about the new content: thousandths in decimals.</strong> Pupils had to determine the subject of the lesson from the notes on the board. The concept of thousandths was taught by examining the decimal fraction on the board: it was converted as a fraction, pupils practised reading that number and finally it was illustrated with an abacus and decimal system tools. During the discussion, pupils actively asked questions and presented their own thoughts. The teacher either answered the questions directly or came up with new examples.</td>
<td>4 decimal fractions were marked on a continuum</td>
</tr>
<tr>
<td>14:22</td>
<td><strong>1. Classroom exercise.</strong> The teacher drew a continuum from 0 to 0.02 at intervals of 0.0002. She read a series of decimals, which pupils marked on the continuum. The continuum confused one pupil, and the exercise was interrupted twice to address his questions and to provide clarification. <em>The discussion about reading the continuum continued until the pupil was able to do it.</em></td>
<td>3 true/false statements about comparisons</td>
</tr>
<tr>
<td>24:43</td>
<td><strong>2. The teacher’s inquiry.</strong> The teacher had written 4 decimal fractions on the board and asked questions about them.</td>
<td>3 questions about the decimal units and 2 comparisons</td>
</tr>
<tr>
<td>31:19</td>
<td><strong>3. Individual work.</strong> The teacher had written 8 decimal fractions on the board. She described some properties and the pupils had to decide which of the decimals satisfied these conditions. While the pupils were thinking, the teacher distributed ‘true/false’ signs needed in the next exercise. Finally, they checked the exercise.</td>
<td>5 questions about comparing decimals and 1 about decimal units</td>
</tr>
<tr>
<td>41:21</td>
<td><strong>4. True/false statements.</strong> The teacher presented statements about decimal fractions and pupils had to decide whether each was true or false. They answered by raising either a true or a false sign.</td>
<td>3 true/false statements about comparisons</td>
</tr>
<tr>
<td>44:52</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The instructional coherence of Agnes’s teaching manifested in several ways: the explicit objective (the concept of a thousandth) connected all the sequences of this lesson; during the discussion, the learning objective was illustrated in several different ways, after which the class practised it using four different approaches. In the exercises, the connections between the ways of representing decimals were versatile and, furthermore, some of the tasks were connected to the subject of the subsequent lesson – decimal comparison – thereby also providing instructional coherence between the lessons.

Agnes’s teaching contained cognitive activation in several situations. The cognitive activation appeared, for example, in the many tasks and questions asked during a lesson, and which varied from the cognitive challenge level (for example, 28 questions were asked during lesson 3, including reasoning tasks). The cognitive activation frequently employed approaches that activated pupils, and by modelling thinking aloud. The cognitively active atmosphere in Class A can be described by the following conversation from lesson 9.

In this scenario, the pupils have two problems to solve; the first one has just been checked. Agnes is about to give the answer of problem two.

Pupil 1: Not yet, don’t give us the answer to number two, I may just work it out.
Agnes: We can wait for a moment. I can read problem two again. …
Agnes: Has anyone got this?
Pupil 2: If somebody gets this, he’ll be the Einstein of our class.
Pupil 3: Then I have to get it.
Pupil 4: I have the right answer!

Agnes’s emotional support is explicit, and the emotional climate of the class increases the feeling of comfortable study: the teacher tries to determine from the pupils’ faces, comments and responses whether the material that has already been taught is understood. She shows her caring feelings openly and, if needed, gives explicit support with further questions and examples, or by clarifying pupils’ thoughts. The emotional climate of the class gives pupils the courage to discuss their own learning and metacognitive thinking, as seen from the following excerpt from lesson 3.

In this excerpt, pupils place decimals along the continuum on the board – drawn from 0 to 0.02 at intervals of 0.0002. One pupil has already asked a question about something he does not understand about the intervals and the teacher has tried to explain it to him.

Pupil: I don’t understand that.
Agnes: What is it that you don’t understand?
Pupil: Well, with those below (the marked numbers of the continuum: 0, 0.002, 0.004,...), you have drawn there 2, 4, 6 …
Agnes: In the middle there is always the one thousandth.
Pupil: Oh, well, now I understand.
(The discussion about the intervals of the continuum continues, and the teacher tries to explain how the continuum can be read.)
Pupil: Why did you write 0 to 0.02 there?
Agnes: Because our continuum is in that (decimal) range.
(The discussion continues...)  
Agnes: Can we move on? (Waits for the pupils’ reactions.) Then we will move on.

This situation demonstrates how the teacher does not progress with the lesson until she is convinced that the subject is fully understood.

The remaining lessons (12–17) were taught by Ann-Marie. These also demonstrate a clear coherent sequence structure from the learner’s point of view. This feature is similar to Agnes’s lessons, but in Ann-Marie’s case the sequences are longer-term in the inner structure of the lessons, hence their frequency per lesson is lower. In Ann-Marie’s lessons cognitive activation was also present, but in a different way to Agnes’s. During the lessons, the pupils’ thoughts and existing knowledge related to the subject were actively used in developing the learning situations. In this way, a cognitive continuum is constructed from previous knowledge to the present teaching event, and to future lessons. The tempo of the teaching is more relaxed than that of Agnes’s. However, there are clear patterns in teaching events, which structure the separate approaches and transitions for the learner.

The nonverbal and emotional support is less observable in these lessons; the teacher’s feelings do not appear as strongly as with Agnes. Even though the lesson structure differs from that of Agnes, the pupils’ ability to concentrate remains good throughout all the lessons. The pupils have an idea of the lessons’ objectives during its separate sequences and approaches. In the lessons, there is a calm and motivated atmosphere that supports learning.

**Teaching processes in classroom B**

In classroom B, the lessons given by Birgit (1–8) were loose relative to the cognitive internal continuity, which made it more difficult for the pupils to concentrate. In most lessons, the teaching objective was difficult to detect, either explicitly or implicitly, because the inner sequence structure of the lessons was usually non-uniform and fragmented. A sequence may have begun with a question which the pupils were expected to answer. However, the situation could become fragmented, as a result of which the pupils’ interest and commitment to the subject disappeared. There were shortcomings in the logical and cognitive continuity of the teaching, of which the following situation from Birgit’s fifth lesson is an example.

The pupils have to conclude which decimal fraction the teacher is describing.

Birgit: And then we will go to the next one. It has 0 hundredths. Jim, are you using your pen? Are you writing things down for yourself? … In it, there are three numbers after the decimal point. At this point, how many numbers can you write? Do you want a few more hints?  
Pupil 1: Yes, I do.  
Birgit: Did you write any numbers already?  
Pupils: Yes, ... Not yet.  
Birgit: OK, don’t rub them out; let us see them. In the number there is the number 2 in the place of wholes and thousandths. Bill, listen, (work) with your partner.
There are breaks in the teacher’s speech and her sentences are incomplete, and sometimes she poses several questions consecutively. This makes it challenging for the pupils to receive information as they are experiencing cognitively non-uniform teaching.

There was minor cognitive activation during the lessons, leading to poor concentration among the pupils. The cognitive activation was mainly performed with the help of questions, but their cognitive level was often low, controlling or testing. Furthermore, only a few mathematical processes were introduced per lesson. The teacher tried to activate the pupils’ thinking by asking them to estimate the validity of their own answers or to justify their solutions. However, she neither summarised the pupils’ justifications nor explained the solutions by using the concepts of decimal fractions and decimal units.

There was no pedagogical or focused emotional support for the learner’s studying situations. However, in certain situations, the teacher attempted to introduce some variation or relaxation to the lessons with help of playfulness and humour.

Table 3 describes lesson 3 in classroom B as an example of Birgit’s teaching. The time axis of this lesson is presented later in Figure 1.

<table>
<thead>
<tr>
<th>Time m:s</th>
<th>Description of the teaching event</th>
<th>Tasks during the lesson</th>
</tr>
</thead>
<tbody>
<tr>
<td>0:00</td>
<td>Checking the homework. Initially, there was a self-assessment: each pupil showed (using their thumb) whether their answers were correct in their opinion. The pupils then checked, in pairs, whether they got the same answers. Finally, any exercises that had caused difficulties were reviewed in more detail.</td>
<td>2 questions about decimal units</td>
</tr>
<tr>
<td>12:11</td>
<td>Mental arithmetic. The teacher read questions and pupils wrote the answers in their notebooks. When pupils had answered these questions, they estimated whether their answers were correct and the answers were then checked. Two decimals from these mental arithmetic exercises were illustrated using an abacus.</td>
<td>2 conversions between the presentations of decimals (in one task only tenths)</td>
</tr>
<tr>
<td>23:09</td>
<td>Pair work. The pupils had to decide which of two solutions given in answer to a certain exercise was correct, and were asked to justify their decision. The pupils considered the task in pairs. When they were ready, the teacher asked each pair for their answer and their justification. (The teacher neither shared her thoughts on the presented justifications nor referred to the key concept: the decimal units.) Lastly, board magnets were placed on the continuum to illustrate the decimal fractions of the task.</td>
<td>1 comparison</td>
</tr>
<tr>
<td>47:14</td>
<td>The second exercise was handled in the same way and, finally, the decimals in it were illustrated using an abacus.</td>
<td>1 conversion of fraction into a decimal (in both tasks only tenths and hundredths)</td>
</tr>
</tbody>
</table>

In this lesson, the objective remained unclear, and there were no connections between the sequences. The lesson was based on separate consecutive tasks, resulting in shortcomings in instructional coherence. Minor cognitive activation was revealed by the tasks during the lesson: only ten questions with rather low cognitive levels were asked. It should be noted that one objective in the Year 5 decimals teaching course is to learn the concept of thousandths;
however, of the decimals in these ten questions, thousandths were missing in seven cases, so they would be categorised as Year 4 questions.

The remaining lessons (9–17) were taught by Bruno, and displayed different levels of coherence: the sequence structure of the lessons was clear and logical, and each lesson had a particular scheme for developing the pupils’ mathematical thinking. This scheme connected the sequences to each other, but the approaches showed little variation. Especially at the end of the course, the lessons were quite monotonous, and included long sequences with little variety. An aspiration to introduce cognitive activation was present during Bruno’s lessons: there was a lot of reliance on using pupils’ own thinking; the rules for calculation were not given, but an attempt was made to lead them; The teacher provided hints to support the pupils’ reasoning and he often used error spotting as a pedagogical idea. The teacher also took advantage of metacognitive speech and modelling of thinking in his teaching. The methodological solutions of Bruno’s teaching had positive effects on the pupils’ ability to concentrate. However, sometimes the attempts to lead the calculation rules turned out to be too challenging for them, and so the teacher finally had to tell pupils the algorithm. This prolonged the sequences, meaning that pupils’ concentration and motivation diminished, and some pupils became distracted by other activities.

There was evidence of emotional support during Bruno’s lessons but its nature was implicit rather than explicit: He cared whether the pupils learned the subject matter and that they kept on trying, and he demonstrated this by asking ‘Did you understand?’ or ‘Was it difficult?’, or by saying ‘Don’t give up’. However, he did not explicitly offer support like Agnes. The teacher’s enthusiasm and nonverbal affirmative communication improved the working climate in the classroom. He openly expressed his satisfaction or dissatisfaction with the pupils’ work and effort: after checking the homework in lesson 14, he praised the children: ‘I’m pleased with you now, almost everybody has done their homework and no one has left their things at home.’ In contrast, in lesson 17, the pupils failed to figure out how to divide by 10, 100 and 1000, the sequence was prolonged and some pupils turned to other activities, which made Bruno impatient: ‘Dan, you do not follow at all. You don’t follow because you’re chatting all the time. But you do that for yourself. Don’t be surprised then if you don’t know.’ In these kinds of situations, the classroom climate became slightly negative.

**Summary of teaching in Classes A and B**

To conclude the comparison of the teaching processes in these two classes, in Figure 1 we present two time axes for Classes A and B, both taken from lesson 3. Each time axis is divided into the separate sequences of the lesson, shown as rectangles. The acronyms indicate the content or teaching method of each sequence, and below these, the patterns illustrate the involvement of these sequences (see the explanations in the figure). Below the rectangle you can see the time and polygons depicting the exercises during the sequences: the shape of the polygon expresses the level of cognitive challenge posed by these exercises – the more angles in the polygon, the greater the cognitive challenge – and the number within the polygon indicates the count of that kind of exercise. Above the rectangle, the continuous arrows show the connections between two separate sequences, and dashed arrows show the connections between separate parts of the same task.
Lesson 3 in classroom A. The lesson objective was introduced through classroom discussion and practiced using classroom exercises (CLE), teacher’s inquiry, individual work, and true/false statements (T/F). The exercises were numerous and varied in terms of their cognitive challenge level.

Lesson 3 in classroom B. After checking the homework, there was some mental arithmetic and two exercises in pairs. There was no connection between the different tasks. The exercises were few and their cognitive level was low.

EXPLANATIONS OF ILLUSTRATIONS USED

Connection between two different sequences

Connection within the same task

HW = homework
O = other activity
CH = checking
? = clarifying pupils’ questions
INFO = briefing
S-A = self-assessment

Checking exercises
Individual work
Classroom teaching
Clarifying pupils’ questions
No involvement

More angles = more cognitive challenge in exercises
4 conversions
2 basic questions
1 reasoning task

Figure 1. The time axis of lesson 3 in classes A and B. Each axis represents the sequences and methods of the lesson, the connections between sequences, the type of involvement and the level and number of exercises during each sequence. The explanations of the illustrations used are presented below the time axes.
A comparison between these two time axes indicates higher instructional coherence in classroom A, because all the separate sequences are connected by the lesson objective, whereas in classroom B there were only connections within the same task. When focusing on the exercises, the level of cognitive activation is also higher in classroom A: there are many exercises and variations in terms of the level of cognitive challenge, whereas in classroom B the situation is the opposite – there were few exercises, mainly at a low cognitive level.

Table 4 presents a summary depicting the main features of instructional coherence, cognitive activation and emotional support in teaching demonstrated by the four student teachers, and comparing the quality of teaching in Classes A and B.

**Table 4. Comparison of the main features of instructional quality in Classes A and B**

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Instructional coherence</th>
<th>Cognitive activation</th>
<th>Emotional support</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Class A</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agnes</td>
<td>coherent sequence structure – lots of variation</td>
<td>many exercises, variation in cognitive challenge</td>
<td>caring teacher behaviour, explicit support</td>
</tr>
<tr>
<td>Ann-Marie</td>
<td>coherent sequence structure – slower tempo</td>
<td>use of pupils’ knowledge in teaching process</td>
<td>emotional support less apparent, calm and motivated atmosphere</td>
</tr>
<tr>
<td><strong>Class B</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Birgit</td>
<td>fragmented sequence structure</td>
<td>few exercises, at a low cognitive level</td>
<td>no pedagogical or focused emotional support</td>
</tr>
<tr>
<td>Bruno</td>
<td>variation in instructional coherence</td>
<td>use of pupils’ own thinking and attempts to explore, but they failed as they were too challenging</td>
<td>implicit emotional support, positive and negative feedback</td>
</tr>
</tbody>
</table>

**Learning in classrooms A and B during the teaching course**

Next, we review the impacts of these teaching processes in classrooms A and B on the understanding of the concept of decimal fractions and the pupils’ skills in calculating decimals. Table 5 compares the pre- and post-test scores between the two classrooms. In the pre-test, both A (N = 17) and B (N = 19) performed equally, with an average score of 8.8 points in Class A, and 8.3 in Class B (maximum 23). In the post-test, the pupils in Class A (N = 17) performed better than those in Class B (N = 19), with an average of 17.3 in Class A and 13.3 in Class B (maximum score 27). The change in the score from the pre-test to the post-test was significantly greater in classroom A (p = 0.04).
Next, we compare the three learning level groups between Classes A and B. In the pre-test, each of the three groups scored equally compared to the equivalent group in the other classroom. In the post-test, there was no significant difference between the low learning level groups of Classes A and B, or between the high learning level groups. The comparison between the middle-level groups of Classes A and B is presented in Table 6.

**Table 6. Comparison of scores between Classes A and B for the middle-level group**

<table>
<thead>
<tr>
<th></th>
<th>Class A middle level</th>
<th>Class B middle level</th>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$X_1$</td>
<td>$X_2$</td>
<td></td>
</tr>
<tr>
<td>Pre-test</td>
<td>8.15</td>
<td>7.35</td>
<td>0.53</td>
</tr>
<tr>
<td>Post-test</td>
<td>20.50</td>
<td>12.05</td>
<td>0.00</td>
</tr>
<tr>
<td>Change</td>
<td>12.35</td>
<td>4.70</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Table 6 shows that the middle-level group in Class A scored better in the post-test ($p = 0.00$), and the difference between these two middle-level groups was highly significant in the change in scores from the pre-test to the post-test ($p = 0.00$). These comparisons show that Class A benefited most from the variety of teaching offered, especially those in the middle-level group.

4. Discussion

Few empirical studies have previously examined lesson coherence. The study by Chen and Li (2010) in the context of mathematics instruction is one of the most recent studies of lesson coherence, but it does not report on student learning gains. Few older studies relate the coherence of lessons to student comprehension of mathematics (Baranes, 1990; Fernandez, Yoshida & Stigle, 1992). According to this study, *instructional coherence together with cognitive activation produces better learning results* than teaching that lacks these properties.

But which psychological learning mechanism can that result be based on? We believe that instructional coherence is likely to have at least two positive tasks: (1) it supports the learner’s long-term working memory, which could benefit permanent memory and transfer in real learning; in other words, the learning can be utilised in new situations (see also Nuthall, 1999a, 1999b; Nuthall & Alton-Lee, 1993); (2) High-level coherence makes it possible to move teaching towards a cognitively higher level, that is, to the challenging tasks of learning, because the learners and the teacher have achieved a ‘shared cognition’ through their working history in the classroom, which opens up opportunities in the cognitive processing of the same area – both intentionally and unintentionally.
However, the coherent sequence does not automatically maximise student development. We are not aware of any previous studies in which both instructional coherence and cognitive activation have been studied (cf. Rakoczy et al., 2007). Furthermore, in earlier studies, the evaluation of the preconditions for cognitive activation was somewhat incomplete. According to the present study, the high coherence of the teaching process is one factor that appears to facilitate the preconditions for cognitive activation. This connection is not absolute (that is, a coherent sequence does not necessarily lead to cognitive activation), but instructional sequences that are related to each other with intentions and realisations make it possible to proceed incrementally to higher cognitive levels and – even within a lesson or at least during a sequence of a couple of lessons – to cognitively challenging demands in the learning of mathematics. Demanding cognitive processes would be anticipated in this way in classroom teaching. In addition, a student’s cognitive history together with their learning experiences will support them in encountering challenging situations.

The findings of some previous studies show that emotional support and positive affective experiences are a precondition for successful cognitive activation or high quality teaching (see Buff et al., 2011; Crosnoe et al., 2010; Greenberg et al., 2003; Griffin, 2004; Howes, 2000; Pianta & Hamre, 2009; Turner & Meyer, 2004). Our findings are in accordance with these research results, but in our study, the emotional support given by the student teachers did not vary greatly enough between two classes, that is, it is difficult to detect the potential influence of emotional support on student studying and learning processes. Further, it is even possible to discuss the synergy of teaching between its cognitive and emotional-affective sides. A recent study by Sakiz, Pape and Woolfolk Hoy (2012) indicates the significance of the affective properties of teaching. The research results show that students who perceived higher teacher affective support were likely to more often report a sense of belonging, higher academic enjoyment, lower academic hopelessness, greater self-efficacy and greater effort within an academic context.

5. Conclusion and future directions

We can present the conclusion as a summary of previous studies, supplemented with the findings of the present study: that instructional coherence and emotional support – as well as the student-teacher relationship in general – in the instructional process will support the teacher’s capability to utilise cognitive activation; to cultivate challenges in teaching; and – most importantly – to provide high quality teaching. In this ideal scenario, the different instructional factors function interactively, supporting each other. This kind of methodological thinking, based on the conception of teaching as an interactive system relating to the learning of the pupil (Hiebert & Grouws, 2007; Walshaw & Anthony, 2008), is not a simple one. However, that does not mean that teaching and teachers do not have an effect. On the contrary, recent studies have noted that teachers and classroom processes have an essential effect on how actively pupils work and concentrate in the classroom (Pianta & Hamre, 2009). Palardy and Rumberger (2008) investigated the importance of three aspects of teacher effects on student learning, including achievement in maths: the teacher’s qualifications, their attitudes and their instructional practices. They express the main results in the following way: ‘Rather than the qualifications teachers bring into the classroom, it is aspects of their teaching – practices, attitudes and beliefs – that are most relevant to their effectiveness in first grade.’ (Palardy & Rumber, 2008, p. 130)
Kistner et al. (2010) reported that self-regulated learning is mainly promoted by implicit instruction. On the other hand, their results suggest that teaching certain kinds of strategies (organisation) and arranging a supportive learning environment (constructivism, transfer) are positively related to student learning in mathematics. Certain features of the learning environment, such as constructivism and transfer, are connected to the qualitative features of teaching, such as instructional coherence and the pupil’s cognitive activation by the teacher. This comparison enables us to see why coherence and cognitive activation support pupils’ learning results.

The teaching method as such probably does not have a direct impact on the quality of pupils’ learning, but active approaches are usually effective (see also Hugener et al., 2009). Similarly, Watson and De Geest (2005, p. 209) present their conclusion thus: ‘However, this is not a study which shows how certain methods lead to better results’; and ‘Overt methods were less important than the collection of beliefs and commitments which underpinned teachers’ choices.’ We conclude that enough long-lasting sequences in the teaching-learning process for the methodology are needed in order to uncover essential interactive mechanisms in the relationship between teaching and learning (see Schmitz, 2006). By applying the same methodological principle, Nuthall (1999a, 1999b) conducted numerous studies in the context of classroom learning. In other words, the structure of the research design has significance in terms of how valid a study can be considered.

In addition to strategic plans concerning teaching, recent studies showed that the teacher’s knowledge of pedagogical content was an important requirement in a high quality teaching process (Baumert et al., 2010; Hill, Rowan & Ball, 2005; Hill, Ball, Blunk, Goffney & Rowan, 2007; Hill et al., 2008; Kunter et al., 2007; Tchoshanov, 2011). Pedagogical content knowledge is connected to cognitive activation, which is manifested in the quality of the instructional process and its cognitive structure (see Baumert et al., 2010). Similarly, findings from the German COACTIV project indicate that the level of cognitive activation depends on the teacher’s pedagogical content knowledge (Kunter et al., 2007). A video study of 10 teachers (Hill et al., 2007) presented qualitative data indicating that mathematical knowledge for teaching is associated with the mathematical quality of instruction, that is, rich representations, explanations, reasoning and meaning. In conclusion, it seems that pedagogical content knowledge is essential to a teacher’s cognitive activation, and to the quality of classroom instruction as a whole. As we did not investigate the significance of pedagogical content knowledge in this study, it will be one potential direction for future investigations, that is, ‘going backwards’ from instructional quality in order to analyse explanatory factors.

This study was based on a comparative research design, examining the instructional processes of two classrooms and pupils’ learning outcomes, including pupils of differing levels of learning. Our future work aims to examine more profoundly, through a few ‘target pupils’, how their learning processes proceed in the long term. In addition to video data on classroom lessons and pre-/post-test data on mathematics learning, we intend to utilise interviews and focused classroom observations of pupils selected on the basis of their learning levels. In other words, the findings of this study motivate us to utilise in-depth data we have already collected, to analyse which factors in teaching and student activity are essential from the point of view of learning; and the ways in which pupils of contrasting cognitive backgrounds differ from each other in these characteristics. Using this additional research data, we aim to understand, above all, how certain qualitative features of teaching – already detected in this study – operate when focusing on a variety of learning levels.
References


