A Mathematical Problem–Formulating Strategy

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Abstract. In this paper we propose a new thinking strategy directed to improve the mathematical problem–formulating process. Several specific strategies proposed by many authors are seen as techniques, related to the implementation of our strategy. The results have been applied in the Cuban mathematics teachers training.

Keywords: problem–posing, problem–formulating, problem–solving, teacher training.

INTRODUCTION

The finding of new problems is not only a higher qualitative stage in the process of solving problems, but also an efficient means to foster the learning of mathematics, as stated by outstanding figures in Mathematics Education such as Pòlya (1957), Freudenthal (1973) and Kilpatrick (1987). That is why the National Council of Teachers of Mathematics in its Principle and Standard for School Mathematics state: «… *a major goal of high school mathematics is to equip students with knowledge and tools that enable them to formulate, approach, and solve problems beyond those that they have studied.* […] *They should have opportunities to formulate and refine problems because problems that occur in real settings do not often arrive neatly packaged. Students need experience in identifying problems and articulating them clearly enough to determine when they have arrived at solutions*» (NCTM, 2000, p. 335).

Something similar is portrayed in the recent transformations of the methodological approach of the Cuban Mathematics Education since it is calling for «*the presentation and teaching of new contents starting from the posing and solving of practical problems which are characterized by being engaged to politics and ideology, economics and feasible actions, scientific and environmental issues, not*

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only from the logic of the subject» (Cuban Ministry of Education, 2000, p. 1; italics in the original).

On the other hand, although in the design of the curriculums students are encouraged to pose new problems, the theoretical basis existing today is too little. For instance, on the MATH–DI[1] data base, between 1975 and 2003 from 109511 summaries registered only 1257 include the basic index «problem posing» (1.15%). The following chart shows the amount of articles registered in the above–mentioned data base which are related to the posing and solving of problems. The statistics analysis shows a meaningful lineal correlation (r ≈ 0.91), which matches the assertion of many researchers about the close link that unites these fields of investigation.

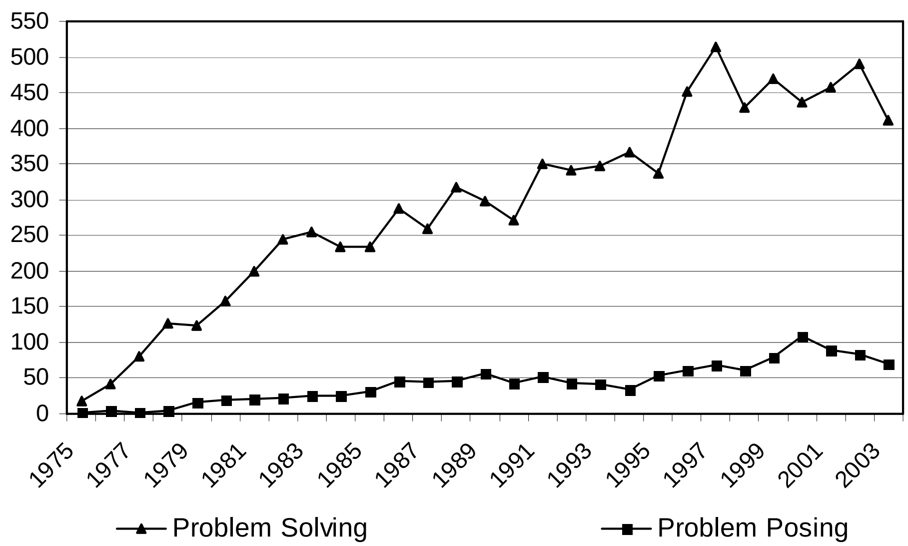

Figure 1. *The posing and problems solving in the MATH–DI data–base.*

Despite its importance, problem–posing has not been entirely treated as a part of mathematical curricula, neither the investigations related to it have been sufficiently systematic ones. In Cuba, as well as in the majority of the countries, there is a pedagogical problem of cyclic nature. This problem lies on having student involved in a traditional teaching, which does make emphasis on the problem posing issue. Students then get the shape of teachers, in an environment that portrays a more open vision of what mathematics certainly is. However, when they begin to work at schools, they are swallowed by the same traditional way of teaching.

Taking into account that the Teaching Training College[2] «José de la Luz y Caballero» is the institution that directs the education process in the Cuban province of Holguín, we carried out a pilot study in February, 2000. The application of a great variety of instruments showed that there were difficulties in posing new problems. The sample was made of 103 in–service mathematics teachers and 45 students (freshmen in the educational field, specialized in mathematics and computer science from our institution). As illustrative examples of the very many difficulties spotted we may pose:

1. High tendency towards the elaboration of simple exercises (86.7% of students and 82.5% of teachers). More information on the classification of the exercises used may be found in Cruz (2002).
2. Low tendency in the elaboration of exercises to demonstrate properties (75.6% of students and 85.4% of teachers).
3. The belief that it is in Geometry where most problems can be found.

These problems are due to many causes, one of them is the absence of an explicit teaching of problem posing in the formation of teachers–to–be. A strategy that paves the way for the teacher to lead the problem–posing during his pedagogical labor is shown in the present research work.

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[1] Corresponding to the ZDM magazine (*Zentralblatt für Didaktik der Mathematik*, available online at: http://www.emis.de/MATH/DI/).

[2] In Cuba it is named «Instituto Superior Pedagógico».

DEVELOPMENT

Nowadays there is no consensus in the use of verbs like «elaborate», «formulate» and «pose», when they are referred to new problems. As for us, problem elaboration will be referred to a complex cognitive activity that the teacher acts (at a macro level); problem formulation constitutes a substructure of such activity, which is at the same time made of several different actions (at a meso level), while the problem statement will be related to a final operation of the formulation (at a micro level). In general, the macrostructure of the process of elaboration of new problems, supposes the follow up of the three main stages. In the first stage, a question is asked, which is seed for a possible decision making exercise; in the intermediate stage, the teacher's capacity to answer his very same question is materialized in the didactical value of the exercise, changes are made in the level of complexity of the problem and the posing is bettered up, with the aim of making it fit with the real needs that brought about its elaboration. We have named all this process «metaproblem», and its complexity supports the statement that the formulation of a problem could be even more difficult than solving it.

Although the metaproblem integrates non–completely conscious psychological structures (habits), it is an essentially conscious activity and its level of mastery is a capacity. It is caused by a stage of solution, which is very complex by nature and makes it possible the presence of an «ignoramus». The development of the capacity to elaborate problems depends indirectly on how much and how good the formation and development of constituent habits and abilities are directly influenced upon. The individual will be more capable the more and more complex questions he is able to ask himself and work them out (qualitative aspect); or as well when he is able to pose himself many problems and solve a considerable amount of them in several different ways.

The development of such capacity is a reachable goal for the trainee teacher–to–be, not only because of the degree of systematization of the mathematics school contents that he encounters, but also for the flexibility he is able to show by abandoning a hypothesis and stating a new one, which is actually trained by his practice in problem–solving. So, in accordance with the intellectual development the Teacher Training College mathematics student acquires, it is feasible to conceptualize the former stages as procedures of the cognitive activity, which are related to partial reachable goals and are, at the same time, made by a system of actions and operations that make acting real (see Talizina, 1988, p. 201). Because of such reasons we could conclude that to elaborate a problem is a human activity made of three essential procedures: formulating, solving and improving.

This activity works for several needs of the teaching–learning process, related to the diagnosis, exemplification, systems of exercises, exams, problematic situations, etc. Each action should be considered in constant interaction with the rest, whose dialect seems to move in spiral when opening new enigmas after the clearing out of others. As an approximation to the modeling of the metaproblem, we propose the following diagram:

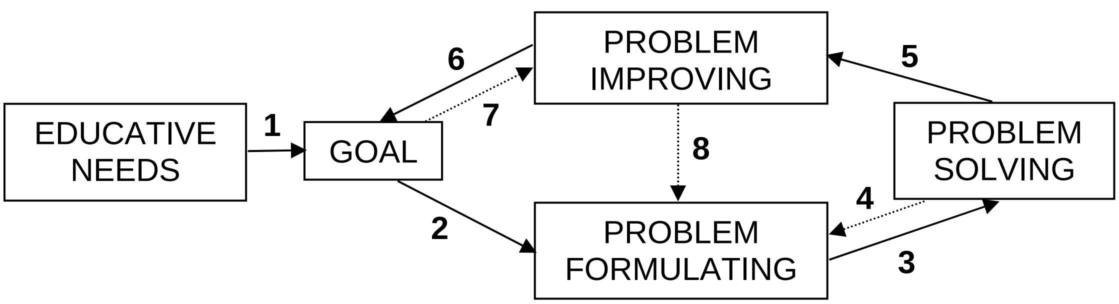


Figure 2. *The framework of the metaproblem*.

For example, suppose that the teacher has a need to establish a problematic situation, directed to the introduction of the concept «derivative». This poses the goal to elaborate a mathematical problem (practical or not), related with this concept (relations 1 and 2). In the first stage, the teacher formulates a problem and figures out how to solve it (interaction 3–4). At this moment there may occur regressions, changes of hypothesis, even the teacher may select new objects for his analysis. So, it is possible to start from the scanning of a function whose graph motivates the drawing of the tangent at a certain point. Nevertheless, after a fruitless work, a flexible thought may begin the searching of a practical situation that demands the calculation of a certain limit of ratio of change. Once the problem has been solved it is necessary to appeal to a second stage (relation 5), which includes the problem correction and the variation of complexity degree. At this time, a new interaction arises (6–7), where the teacher analyzes if his elaborated problem satisfies the outlined goal. In the negative case, two variants subsist: to carry out pertinent transformations without varying the essence of the phenomenon (e.g. to select more comfortable data to calculate), or to discard the whole result and begin again (relation 8).

An objective scanning in the process of problem–posing demands to clear out not only its shape, but its content as well. Such clearing out is possible if the problem–formulating act is seen as a problem itself. According to Pehkonen (1995, p. 56) it is about an open problem, so that it starts from an initial situation which can be a precise one or not; and it also deals with knowing the final goal, which is essentially not precise, as well as of the certainty that the process of obtaining the new problem is unknown a priori.

On the basis of the ideas from the historical and cultural school, Campistrous & Rizo have brought to concepts the solutions of problems. A strategy for the solving of problems according to them is but «*a generalized procedure that is made up by schemes of actions whose content is not a specific one, but a general content, applicable in situations of different contents, which someone uses to orient himself to situations in which he or she does not have an 'ad hoc' procedure and on its bases he or she decides and controls the course of the action of finding the solution*» (2000, p. 8).

Particularly, starting from an idea developed by Brown & Walter, a new strategy to ease the posing of new problems, and the interpretation of other simpler and more specific strategies which are called techniques is proposed here. These authors propose five levels directed to the new problem creation act: (I) Choosing a starting point, (II) Listing attributes, (III) What–if–not, (IV) Question asking or problem posing, and (V) Analyzing the problem (Brown & Walter, 1983/1990, p. 61). Particularly, the last level is closely related to Pòlya's «Looking Back» stage, which confirms the problem formulating conception as a problem itself.

In order to embed this statement into the metaproblem framework, we have carried out certain variations. Virtually, taking into account the Campistrous & Rizo's point of view, it is possible to conceive the I, II, IV, and V levels as actions of one strategy, while the III level can be seen as a complex technique. Overall, generalization, particularization, the use of analogies, considering as unknown some elements of a problem, applying an algorithmic procedure, etc. may be found among the possible techniques. Our strategy (see figure 3) is composed by six non–lineal connected actions. Each action contains itself a set of more elemental actions. In advance, we will see how to organize these techniques according to a typology proposed by us. The foundation of this thinking strategy can be seen in Cruz (2002).

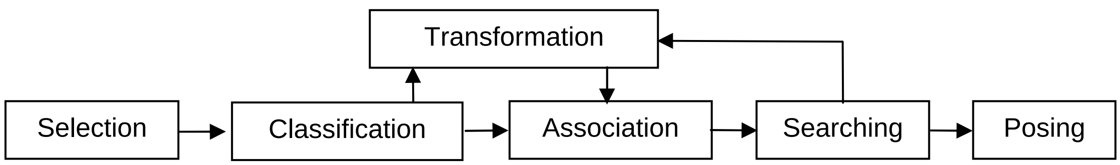


Figure 3. *A mathematical problem–posing strategy*.

The **object selection** is the first action. Its happenings are conditioned by necessities of pedagogical order such as how to evaluate, motivate, exemplify, etc., so that it depends on a conscious objective. The person analyses what kind of mathematical objects are appropriate ones so as to compare them, having the final sphere, so, taking decisions is frequently conditioned by wishes and interests of the person. Once the object has been selected the **classification of components** takes place. In this action the procedure to be followed is to take its components apart (analysis), and then the obtained information is organized and compared according to certain criteria. In a parallel way, there is a respective integration of components (synthesis) in a way that they can make up some other more complex components of the object. A flat figure can be taken as an example, which is made up by segments, forming triangles by certain trios which are part of the figure too.

The following action is but the **object transformation**, which can be total, partial or identical. These changes may take place after the generalization of certain emerging elements during classification. Such logical operation is very complex and may have a synthetic or analytic nature. Essentially, the generalization facilitates the change from a specific concept to a generic one by removing from its content the clues specifying it (it lessens the content and increases the volume of the concept). It is also possible to transform the object by using analogies, in this case it is about a reasoning on the pertaining of a given sign with another object (property or relation) departing from the homology of substantial sign with another object.

According to the character of the information transferred from the modal to the prototype, the analogy can be of properties or relations. After having transformed or not the object, the subsequent action comprises the **association of concepts**. Therewith, the elements obtained during the classification are separated by abstraction and then move on to be related with a joint of the mathematical concepts. Such elements may linked with certain properties (area, perimeter, monotony …) or with a certain relationship (similarity, parallelism, congruence …).

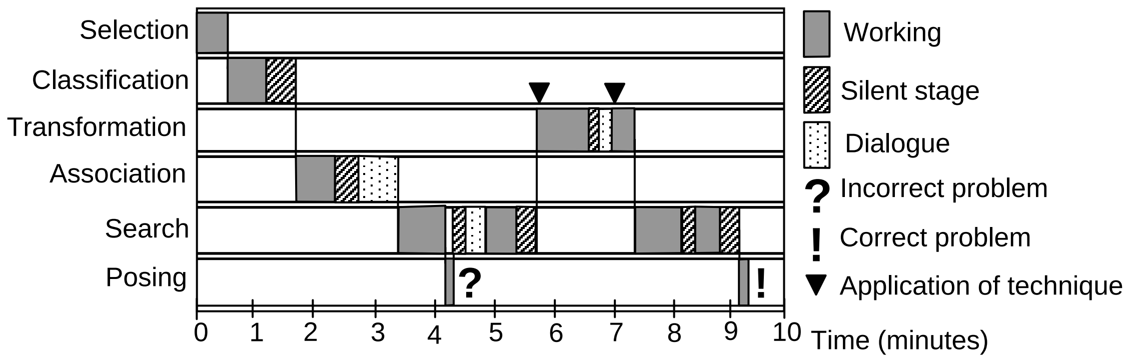
Decision making is again necessary since the subject must select a subset of such associated concepts. At this moment several interrogations may arise naturally; however some of them may be senseless. It is for such reason that we consider a last but one phase related to the **search for dependencies** where existing relations among the properties that have been associated are analyzed. Finally, all the information is synthesized and the immanent questions are valued with the aim of selecting one or several of them. With this decision taking ends the **posing of the question**, taking place, then the subsequent phase of the metaproblem.[3]

At the formulation phase the analysis of the developed strategies is possible, a way of doing it is by applying the well known diagrams of Schoenfeld (1985, compare whit figure 4) or «graphical episodes». In this case we have made an adaptation in which we take the Cartesian product among the action marked on the previous graphic and the time elapsed.[4] Departing from a situation linked to an object or

[3] From Brown & Walter’s perspective (1983/1990), corresponds with the level IV: «Analyzing the Problem».

[4] Schoenfeld does it taking into consideration the enriched Pòlya’s model and the time.

phenomenon, we orient the student to formulate an associated problem, emphasizing on the possibility he or she has to develop transformations. The activity will be recorded, which favors a higher exactitude on the later analysis, and a favorable environment must be created in every moment where the student can express his ideas orally. The researcher will take note of his observations, which will be complemented with the written information and the recording, he will also be able to interact with the student as long as he deems necessary.



<u>Figure 4.</u> *A student graphical episode in a mathematical problem–posing*.

These diagrams will allow us to better understand the logic of the strategies as well as the difficulties faced by the students during its execution. For example, the previous graphic illustrated (see figure 4) a student who did not find great obstacles in the course of the first two phases but during association he began to experiment small difficulties with decision taking. When interacting with him he could pass to look for more convenient elections, but an inadequate synthesis of the information drove him to state a nonsense question. After a few seconds it was possible to observe the absence of a retrospective examination which was proved on a new dialogue. When continuing the search and not finding relations he decided by himself, to transform the given object showing abilities on the matter since the new search drove him to the posing of a problem after a couple of minutes.

Particularly, the total amount of time corresponds with the simplicity of the proposed object and the analyzed relations. Simplicity was stressed further with the transformation that took place, which was originated by the double implication of the logical technique of particularization. These diagrams are a sample showing that the proposed strategy is not constrained to a rigid order of its functions. The form in which it is carried out varies from one subject to another, even one individual may behave differently according to a same given situation. Besides, in spite of the fact that the selection of the techniques shows certain changes in the content, this is not motive to go on considering the strategy as such since these changes do not alter the joint of the actions. It is necessary to point out that even though we do without the recording, it is possible to gather valuable information. In this case, the instrument coincides simply with the well known technique of «thinking aloud».

It is important not to confuse the trivial realization of a question with the procedure of formulating a problem which constitutes the focus of this investigation. A question is a materialized expression of the formulation and appears as a result of dissatisfaction, of an internal conflict combined with, both, the analyzed object or phenomenon. However, the typology of exercises that we have assumed exists with respect to a certain type of questions. In fact, specific questions (whose answers are «yes» or «no») correspond to decision exercises.

Thus, the question «is $f(x)$ derivable in $x = x_0$?» there could be two related possible exercises «prove or refute that $f(x)$ is derivable in $x = x_0$». On the other hand, the complementary questions (headed by operators what?, why?, which? …) correspond to determination exercises. For example, the question «which are the roots of the polynomial $P(x) = x^3 - 1$?» is related to the exercise «find the roots of $P(x) = x^3 - 1$». It is important to point out that this last case starts from the assumption that the mathematical object to be found exists, that reverts to a specific question. That is to say the previous mentioned example precedes, in a logical order, the question «does the polynomial P(x) admit roots?» which is direct by nature.

The primary absence of the question is justified by the presence of enthymeme «at least exists a real root of P(x), since its grade is odd», where the following long premise has been omitted «every polynomial with an odd grade admits, at least, a real root». This abbreviated categorical syllogism is not a logical fact but a psychological one, which evidence the assimilated character of the action. Reciprocally, behind a specific question there can be a complementary one. For example, knowing previously that $f(x)$ is derivable in $x = x_0$ we can ask the question «what is the value of $f'(x_0)$?», which is indirect by nature.

One important aspect consists in the establishment of the hypothesis during the development of the action, whose verification or refutation states the problem in an immediate form. Such hypothesis can respond to simple statements, which can be attributive (triangle ABC equilateral, $f(x)$ continue …), relational (every triangle is similar to its pedal triangle, $\forall x \in \mathbf{R}$: $g(x) \leq h(x)$ …) or existential (exist a limit of $f(x)$ when $x \to x_0$, exist infinity prime numbers of the form 6k + 1 …); or composite statements which are formed by logical connectors of conjunction, disjunction, implication or equivalence (if *n* is a perfect square then it has an odd number of positive integers divisors, if two lines are parallel and one of them is perpendicular to a plane, then the same applies to the other one …).

The hypothesis about the value of veracity of a certain statement can occur spontaneously, as well as during the initial transformation of the object. This is explained by the level of systematization in the execution of the action, in a manner that same included operations can be realized unconsciously. The formulation cannot be seen outside the interrelation subject–object. Furthermore, taking into account the form of discovering the essence of the phenomenon, this mental activity can be predominately analytic or synthetic.

The previous considerations allow us explain what strategies have been used during the formulation of a problems, which approximate to the solution of an important open problem, which was declared by Kilpatrick (1987, p. 142) about a decade ago and confirmed recently by Silver (1996) and English (1997 and 1998). Particularly, our theoretic model of figure 3 can be adjusted to various strategies, which have been isolated and studied by other authors. For example, Pòlya (1957) considers the interchange done between the data and the unknown, the variations of certain elements, breaking down and rebuilding the solved exercise, the generalization, the particularization, and the use of analogies. All these are integrated in a general strategy, revealed by Silver (1994, p. 19), and this consists in obtaining a new problem from an already solved one.

On the other hand, Brown & Walter (1983/1990) proposed two wide strategies: «accepting» and the well known «what–if–not». Finally, Kilpatrick (1987, pp. 136–139) reconsiders the analogies, the generalization and the contradiction (that includes the «what–if–not» and the variant «what–if–more» communicated by Kaput in his personal correspondence). Again this author adds the association (combining it to the idea of Novak and Gowin about the concept maps as a cognitive process) and «other processes» (taking characteristics of two concepts and forming their Cartesian product or their intersection, asking how would X think about this problem, etc.). To deepen in to each one of these strategies surpasses the frontiers of this work. However, a detailed study has been realized by Pòlya (1981), Brown & Walter (1993), Cruz (2001 and 2002), and Cruz & Álvarez (2002).

It is not common that the above mentioned techniques be seen in isolated form; in general, they occur combined, although one can predominate over the others. As an example, we are going to describe how Pòlya's second technique can be applied (considering as unknown some elements of the exercise) following our strategy. In this case we suppose that the selected object consists of three circumferences exterior tangents two in two (see figure 5). Two of them have a unit radius and inscribed, respectively, in each of the angles of the isosceles triangle with base b = 5. In a solved problem it was asked to compute the radius R of the third circumference. During the classification we separated the components (circumferences, sides, angles, regions …) and then, the strategy «what–if–not» makes sense in renouncing the object and making transformations. The new object can result from the immediate generalization, where the length of the base is a variable magnitude.

Now we associate various concepts, particularly we correspond the base to the concepts «tangents», «length», etc. Similarly with the original question it is possible to analyze the dependence that exists between b and R. Particularly, finding the domain of R = R(b) makes us state the problem as the geometrical interpretation: «for what values of b is it possible to inscribe the three circumferences?». Similarly, other queries can arise such as «is R a one to one function?», «what is the minimum value that can be taken as the area of the interior regions to the triangle and exterior to the circumferences», etc.

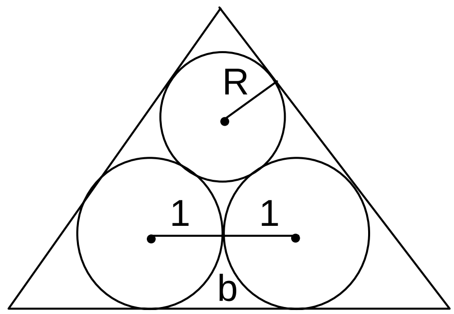


Figure 5. *Combined implementation of Pòlya's techniques in our strategy*.

Taking into account the relation that exists between strategies and the manner in which the process of formulation occurs, it is possible to conceive trichotomous typologies, made up of **algorithmic**, **logical** and **heuristic** strategies. The first were structured by fixed successions of operations in an univocal manner. Its use is convenient if we need to elaborate problem classes where the formal representation of the solution is clearly predetermined. The logical strategies are those that make abstraction of objects or phenomena, transforming them according to laws of Formal Logic, as the generalization, particularization, formation of reciprocals, finding of equivalent propositions, negation of a quantified proposition, etc.

Finally, the heuristic strategies are those that by nature combine with the finding or discovering act. Its use is common when we need to explore intrinsic properties of objects and phenomena, and the underlying relationship among these and other not necessarily well–known. Take as example analogies, contradiction, variation of some elements within certain range, association (associate relative problems to function concept when studying the derivative concept, associate isosceles triangles to the 2–by–2 matrices …), and to form the intersection between the characteristics of two concepts (what is common in complex numbers and circumferences?, what is common in sets $\mathbf{Z}^2$ and **Q**? …). It is possible to include here the combination of several results from the same or different domains (see an excellent example in Bairac, 2005).

Considering an example, relative to the use of analogies (see the following figure). Taking a unit square ABCD, with $P \in AB$, $Q \in BC$, $\angle PDQ = 45°$, calculate the perimeter of ΔPBQ. It is easy to demonstrate that the unknown is constant for all positions admitted by the angle. Taking this problem as a prototype it is possible to consider other similarities, taking into account its properties and again its relations.

Our colleague Ochoa (1998, see figure 6) studied the possibility that in an equilateral triangle ABC there will exist a similar phenomenon. After putting the vertex of the angle PDQ in several positions, he discovered that if this coincide with middle point of side AC and the amplitude was fixed at 60°, then the perimeter of triangle PBQ was also constant. In this way it remains stated a similar problem, for its properties as well as its internal relations.

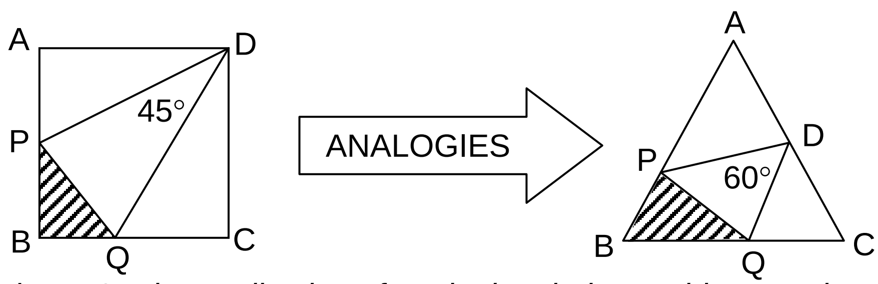


Figure 6. *The application of analogies during problem–posing*.

Generally, the above mentioned typology does not bound each strategy into its particular group, because in some occasions it is possible to understand its logical nature as algorithmic or heuristic and vice versa. This tells us of a relative character of such strategies. On the other hand, the problem formulation making use of a certain strategy does not presuppose a unique way, there even subsists the possibility that different people get identical results making use of strategies of a different nature. For example, analogy can take us from calculation problems of the diagonal of the square to the calculation of the diagonal of a regular hexagon (both are regular polygons, in both cases we are dealing with the length of the diameter of a circumscribed circumference …), however similar results can be obtained from generalization–particularization reasoning (consider a convex regular n–sided polygon and further analyze a particular case n = 6).

On the other hand, with the aim of materializing the implementation of the strategy in the mathematics teacher training we have conceptualized an environment of learning that facilitates the teaching of it and its respective techniques in the teaching studies. Therefore, the carrying out of an experiment has shown us the advantages and disadvantages of the metacognitive strategy. It is for such reason that we also elaborate a methodology that allows us to characterize the process of formulation in a qualitative way, departing from three substantive indicators: the metacognition, the strategy and the formulated problem (see Cruz, 2002). We now show the results obtained after the experiment.

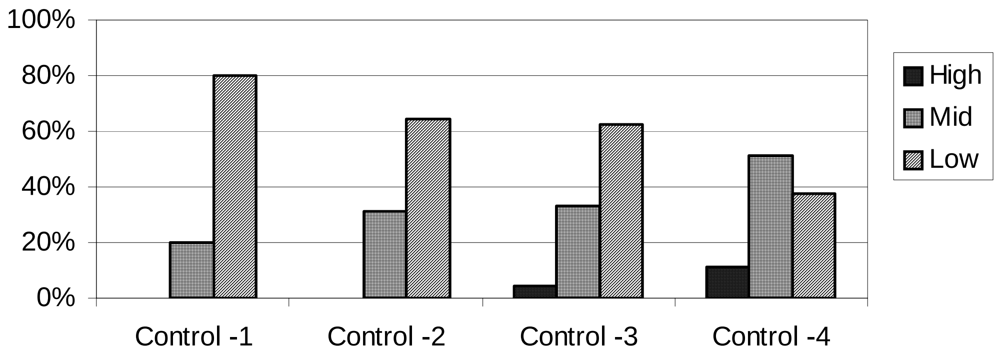


Figure 7. *Develop of problem formulating process, through the learning of our strategy*.

As can be observed in the previous figure, discrete improvements can be noticed. This was expected for the experiment was carried out with a group a freshmen. The analysis of the mean ranks showed a high significance level, according to the Friedman Test ($\chi^2_p(3) = 45,3$; $p < 0.001$). What is more, from 36 students whose process was characterized as low in the initial control, 19 went up to the intermediate level (52.8%), meanwhile from 9 in the intermediate level, 5 went up to the high level (55.6%). In neither case involution was observed which, together with the previous observation, constitutes a sample of uniform development. Once finished the experimenting period, we proceeded to the application of a post hoc test three months later. The objective consisted in determining the fixing level of the learned strategy. The results are shown below.

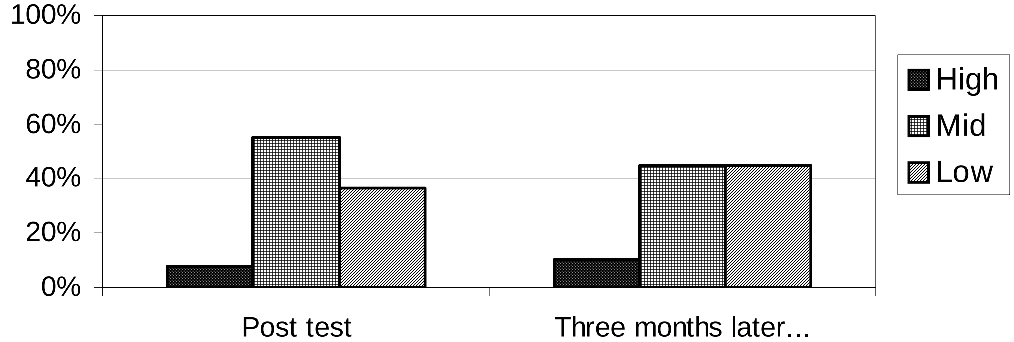


Figure 8. *The formulation process at post test level vs. a post hoc test*.

As is shown, there are differences between the final control and the post hoc, however, the Wilcoxon Signed Ranks Test shows that this change was not meaningful (Z_p = –0.816; p = 0.414 > 0.05). Likewise, it meaningful changes were not observed in any of the indicators, especially in the problem (Z_p = –0.333; p = 0.739 >> 0.05). This leads to the conclusion that the learning was solid, but predicts the need to continue the development of the process.

With the purpose of comparing the obtained results in this investigation with others, we proceeded to calculate the non–parametric correlation between the problem–solving process (at the beginning of the experiment) and the problem formulation process. The application of the Spearman Test revealed that as long as the formulation process was getting better, the correlation with the resolution process tended to grow in the group of controls (r_s = 0, 628; 0,654; 0,635 y 0,734). This confirms a thesis defended by many authors, it refers to the close interrelation between both processes.

CONCLUSIONS

Through this investigation we have proposed a new thinking strategy, which makes easier for the mathematics teachers the formulation of problems in their daily activities as educator. This strategy is conformed by a system of actions and these, at the same time, are compounded by a group of basic operations. In order to prove the practical value of this proposal, it has been experimented its teaching in the formation of this professional. The results reveal that the learning of this strategy, taking into account specific techniques, favors the problems–formulation process.

It is necessary to point out that the development of this investigation has revealed a multiplicity of open problems, e. g. the need to carry out a simultaneous control of the «problem–posing» and «problem–solving» variables so that its interdependence could be cleared up better. It is also necessary to deepen in the way this strategy works. An example is shown, it was not possible to extend the Schoenfeld's diagrams to the relation of control of the experiment (how it often happens, this occurs as a posteriori result of the investigation).

From this way we could analyze the frequency of change between the actions, the average time of them, the location of the techniques within the strategy, etc. However, perhaps the most interesting of the opened problems consists of bringing about the implementation of non–mathematic problems; just like physical problems, chemistry, biological, geographical, etc. It opens a fascinating scope of investigation for the anyway coming future.

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