PROSPECTIVE TEACHERS' ABILITY TO POSE WORD PROBLEMS

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ABSTRACT

The purpose of this study was to assess difference in prospective teachers' ability to pose word problems for mathematical expressions involving division before and after an instruction intervention. After the initial assessment of participants' ability to pose word problems for division expressions, the researcher introduced an alternative model i.e. rate or ratio to represent division involving whole numbers. After the intervention it was assessed whether the model helped the prospective teachers develop word problems for division of fractions. The findings revealed that the prospective teachers successfully posed word problems for division of whole numbers based on intuitive model of fair sharing and to some extent they used repeated subtraction model but failed to represent division of fractions. A considerable improvement was observed in participants' performance to pose word problems for division of fractions after going through instruction intervention. However the problems, which the prospective teachers posed, were, to a large extent, similar to the word problems, which the researcher used in the instruction.

THEORETICAL FRAMEWORK

Researchers in the field of mathematics education (e.g. Brown & Walter, 1993; English, 1998) have acknowledged the importance of problem posing skill in students ‘learning. However different mathematics educators have perceived different functions, which the activity of problem posing might involve in classrooms incorporated with the teaching of school curriculum. Students’ ability to identify and relate symbolic mathematical forms of knowledge to everyday life situations can be assessed and enhanced if the students are involved in the exercise of problem posing (English, 1998). Problem posing also provides a way to communicate mathematically. As Steinbring (1998) proposed the epistemological triangle of signifier (sign/symbol), reference context and signified (concept) for the establishment of successful communication in mathematics classroom. Problem posing provides students a reference context so the students could link signifier to signified and able to generate meaningful communication to facilitate their learning. Problem posing also fosters students to make connections among different concepts of the subjects (NCTM, 1991). Several researchers (Hadamard, 1945; Polya, 1954; Getzels & Csikszentmihalyi, 1976; Silver, 1994) perceived problem posing as a way to include all students in creative activities, as they did not consider that only genius or exceptional ones would engage in and accomplish creative tasks.

In the line with these researchers, and on the basis of my observations during my career as a teacher and teacher educator, I also claim that problem posing is important for students’ psychological and intellectual development. It helps students acquire positive attitude towards the subject by providing them sense of ownership of mathematical task. The activity of problem posing gives students a feeling that they are not the ones who have always been asked to work out the tasks which others (textbooks or teachers) design for them rather they can be in a position where they can design their own tasks or ask other to solve their questions. Intellectually problem posing is beneficial because it fosters students to think in multiple directions; integrate their learning from school, outside school, within the domain of the discipline and from different disciplines.

The skill of problem posing is to a greater extent required by the teachers, as they are required to continuously design and select task for their students in their teaching tenure. The task, which the teachers design and select for the students, has a great influence on students’ learning as well as on what they perceived about the subject (Crespo, 2003). These problems can also serve a purpose to assess the students at the *synthesis* level which is the second highest level of Bloom' taxonomy (1956) - which he created for categorizing level of abstraction of questions that commonly occur in educational settings as it gives students opportunities to use old ideas to: create new ones; combine several ideas; integrate different curriculum areas; modify the problems; rearrange the information given in the problems; and invent new situations.

The teachers also pose evocative word problems when they help students draw meanings from symbolic - mathematical representations, by relating these representations to their context. Posing word problems is a key indicator of a deep understanding because it is in this situation where numbers must be associated with the quantities and correct relationship between quantities must be expressed. However there are certain curriculum areas in primary and lower secondary mathematics where teachers faced problems in drawing meanings from symbolically representated mathematical content (Rubenstein & Thompson, 2001). Ultimately the teachers are not able to develop word problems for those symbolic representations. One of the areas in which teachers have shown difficulty in posing word problems is the area of division of fractions (Payne, 1976; Fendel, 1987; Greer 1992). Incapability to translate mathematical expressions, involving division of fractions, into word problems can be viewed as teachers' insufficient conceptual knowledge in this area. Contrary to this, the ability to successfully develop word problems is argued to be an indicator of a rich network of conceptual knowledge.

In the next section there is a brief account of research on teaching practice of the concept of division in primary and lower secondary classes.

Concept of division

It has been reported (Bell, Swan & Taylor, 1981; Bell, Fischbein & Greer; 1984; Fischbein, Deri, Nello, & Marino, 1985) that the concept of division is usually taught in elementary schools by using models of fair sharing or/and repeated subtraction.

In fair sharing model, the dividend represents the size of the object or number of the objects, which are partitioned into a number of equal subgroups. The number of equal subgroups of the objects is represented by the divisor and the size or number of objects in each subgroup is the quotient - the answer of the division question. This model can only be successfully used when whole numbers are involved. Sharp mentioned (1998) "fair sharing is intuitive when the number of groups is a whole number, but imagining the division of an amount across two and half groups is perplexing to most learners" (p.23).

Another model represents division as repeated subtraction. In this model the dividend is the size or number of objects as in the sharing model, but the divisor is the size or number of one subgroup and the total number of the subgroup is represented by the quotient. The repeated subtraction model of division satisfactorily explains the situations when dividend and divisors are whole numbers, as well as when they are fractions. For example if a child interprets $6 \div 3$ as how many threes make six, she could generalise it to $6 \div \frac{1}{3}$, as how many one thirds make 6, or how many pieces of one third of a pie can be cut from six pies, or she can transfer this learning to $\frac{1}{2} \div \frac{1}{4}$ as how many one-fourth pieces can be cut from one half. However there is still a risk of overgeneralisation, as this model affects children's responses to expressions involving division of fractions when the dividend is smaller than the divisor (e.g. $\frac{1}{4} \div \frac{1}{2}$).

Some researchers (e.g. Sellke, Behr & Voelker, 1991; Brendefur & Pitingo, 1998) proposed an instructional strategy designed to provide students with a way to represent division problems. The rate or ratio model is based on establishing a multiplicative relationship between two similar quantities (ratio), or different (rate) quantities. It considers dividend and divisor as one pair of numbers from a set of infinite pairs of numbers that are related in the same proportion. If one of the quantities is unknown, knowing the multiplicative relationship between a pair of known quantities allows students to apply the same multiplication relation to determine the unknown quantity. This model works well to justify all mathematical expressions involving division irrespective of types of numbers involved.

CONTEXT AND BACKGROUND

This report is part of the study I undertook for my master dissertation on prospective teachers knowledge of division. I studied 17 volunteer prospective teachers from a cohort of lower secondary Bachelor of Education (BEd) from an Australian university. These prospective teachers had completed mathematics Curriculum Studies courses in their BEd.

In this paper I will report findings pertaining to prospective teachers' ability to represent mathematical expressions involving division in the form of word problems.

RESEARCH DESIGN

I carried out research in three into three stages.

In Stage 1(pre intervention) and Stage 3 (post intervention), I interviewed each participant individually. In these interviews the participants were asked to solve the worksheet (Appendix A) by working out on the paper, drawing illustrations or/ and thinking aloud in the response of each questions. I made efforts to minimise my intervention and limit it to paraphrasing and repeating of the written questions.

In stage 2 each participant individually went through a teaching intervention. Each stage lasted more than an hour (60 to 70 minutes). I also took great care to maintain consistency in my role in each interview.

I audiotaped all the responses, explanations and "thinking aloud" which participants produced in each phase and maintained records of their working on papers for further scrutiny. Professional transcribers first transcribed the tapes and then I also listened to each tape to double check the transcripts.

I analysed the data at two levels.

At the first level, I created individual Microsoft Excel files for each participant to record the data obtained from each phase. At the second level of analysis, I transferred the scores of each participant from the individual Excel files to a single SPSS file for comparative analysis.

Stage 1 (Pre intervention)

In this stage, I explored if the prospective teachers were able to pose word problems for the mathematical expressions involving division given in the worksheet and write mathematical expressions with division sign for the word problems based on different models of division – fair sharing model, repeated subtraction model, rate and ratio model (see Appendix A).

First, I observed the participants' preference for the models for representation of division of fractions and its impact on their problem posing performance.

Later I explored the participants' conceptual knowledge was explored in term of their ability to represent and solve word problems based on different models of division – fair sharing model, repeated subtraction model, rate and ratio model given in the worksheet. The reason to include this variable in the paper is to consider problem posing for a mathematical expression and writing a mathematical expression for a problem as two sides of a coin (Silver and Cai, 1996). It can be argued that if one cannot write a division expression for a situation - where it could be done by relating the situation to a certain model of division - might not able to pose a word problem (create a situation) linking to that mathematical expression.

Stage 2 (Intervention stage)

In this stage each participant individually went through a brief teaching session where I introduced them to rate or ratio model for solving division problems using schematic diagram (Fig 1A) proposed by Vergnand's (1994). Vergnaud's schematic diagram that was used to solve word problems involving whole numbers is as follows:

> If 12 apples are to be distributed equally among 3 girls, how many apples will each girl get?

First I translated the question into mathematical form, and helped them see division expressions in terms of rate and ratio between dividend and divisor, with the following protocol.

"This is a typical division problem, which can be written as 12 ÷ 3. Division problems can be written and solved using the concept of rate/ratio. In a division expression, the dividend and the divisor show a multiplicative relationship between two quantities. This shows that a particular value of a quantity (dividend indicated) is multiplicatively associated with a particular value of another quantity (divisor indicated). To find the answer for a division problem is to figure out how many times one quantity (divisor indicated), reduces or increases to get 1. And then the other quantity (dividend indicated), is increased or decreased by the same number of times in order to keep the same multiplicative relationship between both the quantities. For example, if quantity one is reduced by 3 times, the other quantity also has to be reduced by 3 times".

I further explained,

"In Question 1, mentioned above**,** two quantities are involved. Quantity no. 1 is the number of apples and Quantity no. 2 is the number of girls".

I further told the participants,

"The first part of the question shows the relationship between both these quantities. 12 of quantity one is set up for 3 of quantity two. In the next step the number of girls is reduced from 3 to 1, so 3 will represent the divisor. This means that one quantity is reduced by three times. So, the other quantity will also be reduced by three times. To find the share for each girl, one has to reduce the number of apples by three times. This can be shown in the form of a schematic diagram".

The researcher drew the Vergnand' s schematic diagram as shown below (Figure 1A).

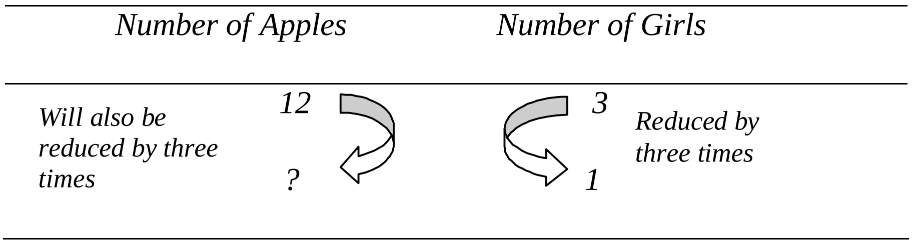

Figure 1A
Problem Solving Using The Schematic Diagram

I also explained the purpose of the arrows in the figure as,
"The vertical arrows in the above figure, linking the numbers written in a column, function to show the relationship between the values of quantity of one type. This relationship will be the same as the relationship between the values of quantity of the other type".

I asked the participants to check the answer by viewing the schematic diagram in another way, which is shown in Figure 1B.

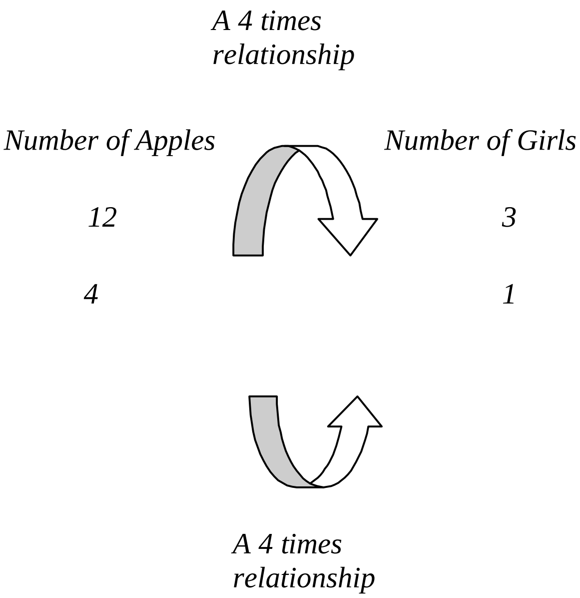

Figure 1 B
Problem Solving Using The Schematic Diagram

I described,

"The arrows in Figure 1B link the numbers written in a row function, to show the relationship between two types of quantities. This relationship will be the same as the relationship between the quantities in the other row".

I also asked the participants to look at one more representation shown in Figure 1C.

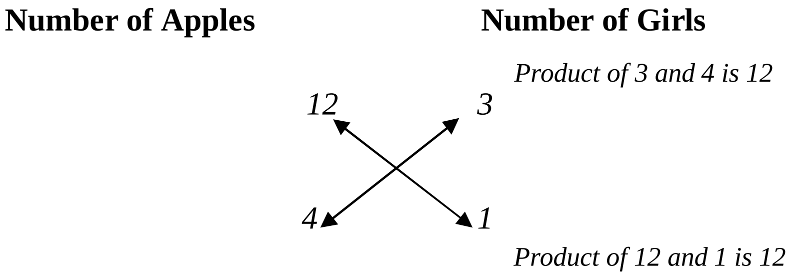

Figure 1 C
Problem Solving Using The Schematic Diagram

"The arrows in the above figure, linking numbers diagonally, function to show that the product of the value of one quantity in one instance and the value of the other quantity in another instance will always be the same in a division situation. In other words, the product of the quotient and the divisor will always be the same as the product of the dividend and 1. This representation emerges out of the idea of division as the inverse of multiplication".

After this demonstration I solved three more questions in the similar manner. Then I checked the participants' understanding about the rate and ratio model by asking them to represent and solve three division problems in the same way.

In the teaching session overall seven different division problems were solved and all the problems contained only whole numbers (other than zero).

I gradually minimised my assistance to require the student to apply the newly learnt method independently. However the level of assistance varied from person to person. Generally I solved first three questions with detailed explanation. In the fourth question I partially transferred the problem solving responsibility to the participants. I helped them identify the quantities involved and helped them in setting up a rectangular array then I asked them to solve the rest of the problem themselves. I also asked them to check each of three relationships between the sets of the dividends and the divisors. For the fifth question I reduced my role to a minimum level. For this question I intervened only if the participant expected me to do so. Finally the participants solved the last two independently.

At the end of the teaching session, all the participants were able to represent and solve mathematical expressions and word problems involving division of whole numbers by setting up the rectangular array using the schematic diagram. They also found each of three relationships between the sets of the dividends and the divisors as described by the researcher in Question 1 of the teaching session.

Stage 3 (Post intervention)

In Phase 3, I assessed whether the new understanding of division (using a rate or ratio model) enabled the participants to represent division of fractions in terms of real life situations. I found out what quantities (whether the participants had been able to pose problems) and qualitative (which model of division the participants use in problem posing) changes occurred in participants' ability to pose word problems for division expressions involving fractions given in the worksheet.

Finding of Phase 1

The section describes the participants' ability to pose problems for the expression:
Where the divisor and the dividend were whole numbers and the dividend was bigger than the divisor (108 ÷ 4)

All the participants successfully framed the word problems, but every problem was based on a sharing model where a number (equal to the number given as the dividend) of concrete objects (countable objects such as pens, lollies etc), were being distributed equally among a number (equal to number given as the divisor) of people. No students produced any problems related to repeated subtraction model - a model of where the divisor represents size of the share- or rate and ratio model where the divisor and the dividend linked proportionally. Neither did they use any uncountable quantity (such as measurement like liters or kilograms). A typical problem posed by the participants was as follows:

> *"There are one hundred and eight students in a classroom, and you've got to be separated into four groups, how many people would be in a group"*
>
> (Participant F).

or

> *"John has a hundred and eight apples and he has four friends and he wants to divide the a hundred and eight apples equally to four of his friends"*
>
> (Participant I).

Where the divisor and the dividend are whole numbers and the divisor is bigger than the dividend (5 ÷ 7)

When asked to pose word problems for the expression where the divisor and the dividend are whole number and the divisor is bigger than the dividend, all participants posed a sharing type division problem where they used concrete objects such apples and pencils. Many participants (12 out of 17) could not articulate the question part of the story (word problem) and just described the information part of the question. For example participant A noted *"there are five apples and I want to divide it into seven people.* At this point the researcher intervened by "*what do you want to know.*" Then the participants could structure the question part of the problem as "*how much will be the share of each person".*

Where the dividend was a fraction and the divisor was a whole number ($\frac{1}{2}$ ÷3)

Generally all the participants showed reservations with expressions involving fractions. They noted that the fractions were very scary and that division of fractions was even scarier. But after the researcher's reassurance, they pondered and came up with some interpretations. Eleven participants attempted to pose fair sharing type word problem. The rest could not give any answer and said that they had last done those problems in their primary schools and ever since they had not encountered them again and so needed to learn the concepts before beginning their teaching careers.

The participants used a pie or pizza and divided it into a number of people. None of the participants posed any problem related to repeated subtraction, or to any other model. As was the case with the earlier problems, eight participants had the same difficulty in developing the problems. They were able to represent the situation as half of anything (cake or pizza) is distributed among three people, but they could not tell what was to be found out in the problem.

No representation was based on the rate or ratio model.

Where the divisor was a fraction but the dividend was a whole number (4 ÷$\frac{1}{3}$)

No participants were able to pose an appropriate word problem for the expression, though they made some attempts by interpreting the expression differently, such as considering the division expression as a multiplication expression, or considering the divisor as a whole number rather than a fraction.

Where the divisor and the dividend both were fractions and the dividend was bigger than the divisor ($\frac{1}{2} \div \frac{1}{4}$ and $\frac{1}{3} \div \frac{1}{5}$)

The participants' ability to pose problems was assessed for two of these expressions. In one expression the denominator of the divisor was a multiple of the denominator of the dividend and in the other expression the denominators of the dividend and divisor were not multiples of each other.

No participant could produce any type of word problems for the above expressions. They repeatedly said that they had never made word problems for these expression and they never thought of relating these expressions to real life situations.
The following conversation between the researcher and the participants shows the participants' view in this regard.

Interviewer: *Can you make any story problem?*

Interviewee: *I guess I can't really see how I would use a fraction in a story when I'm dividing. I would only be able to see it as a multiplication. So I guess I can't really see how I would use that division.* (Participant Q)

Where the divisor and the dividend both were fractions and the dividend was smaller than the divisor ($\frac{1}{4} \div \frac{1}{2}$ and $\frac{1}{5} \div \frac{1}{2}$)

As in the previous task, the participants' understanding was assessed for two expressions. In the first expression the denominator of the dividend was the multiple of the denominator of the dividend and in the second expression the denominators of the dividend and the divisor were not multiples of each other. The participants did not pose any problem for these expressions.

The participants' ability to translate the word problems into mathematical expressions

For the word problems they were asked to write mathematical expressions. No participants wrote division expressions where they were to write a fraction for the divisor for instance in the question: *"If a person's income in half a month is $ 400 how much can he earn in a month?"* whereas they wrote a division expression for the question: *"A man earns $ 240 in 3 weeks. How much is his weekly salary?"*

The above observation reaffirmed Tirosh's (2000) views that people intuitively thought that division made things smaller. That is why for the question where they knew that the answer would be bigger, they had not selected division as a correct operation for the problem.

No participant wrote division expressions for the problems based on the concept of ratio and proportion for instance for the question:

"Jessica and Shane had a recipe to make a cake. In this recipe $\frac{1}{2}$ *kg of flour is mixed with* $\frac{1}{4}$ *kg of sugar. Shane wants to use 1 kg of flour. How much* <u>*sugar*</u> *should he use to have the same taste?"*

However, they understood the proportional relationship between the quantities such as amount of ingredients in a recipe; distance and time. This suggests that the participants were unable to identify or activate the connections between the concept of rate and ratio and the concept of division and could not think that similar proportional relations existed between a pair of divisor and dividend.

Only seven participants considered that division was the appropriate operation for the repeated subtraction type word problem and five of them wrote correct division expression. On the other side, all the participants wrote a correct division expression for a sharing type word problem. This provides another piece of evidence that the participants' understanding of division was, to a large extent, embedded strongly in the fair sharing model**,** and that their problem solving performance declined where the problem situation did not allow them to use the fair sharing model.

Summery of Phase 1

No participants were able to pose any sort of word problems for the expressions where the divisors were fractions. Participants posed only sharing type word problems for the expressions where the divisor was a whole number, though they could also have produced repeated subtraction type word problems. It is suggested that the participants' inability to pose word problems for the expressions where the divisors were fractions, was because of their dependence on the sharing model. This hindered them in thinking about real life situations where these expressions could be valid. Although 12 participants showed that they possessed the repeated subtraction models in their semantic (by saying) and iconic (by developing illustrations) representations, no participants posed any word problem based on the repeated subtraction model for any division expression.

In this research the prospective teachers' inability to pose problems for the expressions where the divisors were fractional numbers shows that they found it very difficult to make sense of the expressions in term of real life situations. This observation confirms Greer's (1992) argument that people often solve sums of division of fraction written mathematically but they cannot represent them in the form of real life situations.

The findings with respect to repeated subtraction also indicate that the different forms of representation examined in this study are of practical significance. The requirement to develop a word problem representation appears to involve greater complexity than that associated with the semantic and iconic representations, as the participants' performance was better in developing semantic and iconic representations. This suggests that this problem-posing task could be used as an indicator of a better quality understanding in students.

Looking at the participants' performance in Phase 1 reaffirmed that the prospective teachers did face difficulties in developing successful representations- in term of posing problem - of division of fractions. These difficulties were mainly because of their total reliance on intuitive sharing and repeated subtraction models. This suggested that it would be important to investigate how the participants could be assisted to develop a more adequate understanding of division – as per an alternative rate and ratio model (Vergnaud's schematic diagram).

Finding of Phase 3

The participants were asked to pose word problems for the expressions where the divisors were fraction such as $4 \div \frac{1}{3}$, $\frac{1}{3} \div \frac{1}{5}$, and $\frac{1}{4} \div \frac{1}{2}$. The same task had also been given in Phase I where the participants' performance was very imitated. None of the participants could pose word problems for these expressions (for the other expressions their performance of posing problems were satisfactory). They generally said that they had not thought whether it would ever be possible to express those expressions in word problems: One noted, *"For us they are symbols – just Maths symbols. We don't associate it with every day life"* (Subject E).

However, after going through the instructional intervention all participants managed to pose word problems. Eleven participants posed problems for all the expressions, five posed problems for two and one posed problems for one of the expressions. There were 77 total word problem posed by the participants.

Looking at the large number of problem posed it could be interpreted that the instruction intervention brought improvement at the level, which Silver (1997) considered as *fluency*. However there was not enough evidence to prove *flexibility* and *novelty* in participants' problem posing activity. As the problems were either very similar to the problems which were posed by the researcher in the instruction intervention or they were similar to the problems which the participants were to translate into mathematical expressions in Phase 1. In this phase out of 77 word problems posed by the participants 62 were related to masses of ingredients, or masses of chemicals in a certain combination. Usually they developed word problems for different expressions using the same context but changing the numbers. These are the example of some of the questions, which the participants posed.

(Subject Q posed the word problem for the expression $2 \div \frac{1}{3}$)

"So I guess if you were using, mixing chemicals and if you're using two gram of sodium with half a gram of acid or something, and you want to see how much you would use if you were using one gram of acid, then how much sodium would you use, and so that would be the same as two dividing by the half".

(Subject M posed the problem for the expression $\frac{1}{5} \div \frac{1}{3}$)

"One third of a glass can fill one fifth of a bottle. How many bottles can you fill with one glass"?

(Subject B posed the problem for the expression $\frac{1}{4} \div \frac{1}{2}$).

"Half the cost of any project is a quarter million. What is the worth of full project ..."?

DISCUSSION

The study was aimed to explore participants' ability to pose word problem relating to division of fractions. It added up to our knowledge about one more aspect of the limitation of fair sharing and repeated subtraction models (the limitation of the models has already been well reported by Bell, Swan & Taylor, 1981; Bell, Fischbein & Greer; 1984; Fischbein, Deri, Nello, & Marino, 1985) as the models render the learners to not to come up with real life situations for division of fractions. It suggests teachers, teacher educators and textbooks writers to revisit the notion of "division" by moving ahead from its intuitive meanings and developing a more mathematical connotation of the word.

It is obvious that the existing knowledge, which the prospective teachers possessed, does not help them pose problems for division of fractions. It was also noticed that the teachers had never tried and they had have never been asked to pose problems related to division of fractions in their academic and professional lives. It is predicted that if they would be asked to do so with the help of probing questions in teacher education courses they might realize: the missing connections between the concepts of division of whole numbers and division of fractions; their own lack of knowledge in this area; and limitations of the models which they use to represent concept of division. The process of generalizing division of whole number and division of fractions may help them come up with certain ways to fabricate the missing connections in their knowledge representation.

In the paper, generally, posing word problems is viewed as the ability to link specific mathematical content to real life situation; it can also raise similar epistemological issues for other content areas of secondary and lower secondary school mathematics. In this connection there could be even more basic questions: Is it important that we expect teachers and students to relate all mathematical content with real life as suggested by NCTM. (1991)? Or is it possible to do so?

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APPENDIX A

Phase I

Work Sheet A

Q1 (a) In how many ways can you interpret the following expressions? Write down your interpretations.

(i) $108 \div 4$ (ii) $\frac{1}{2} \div 3$ (iii) $\frac{1}{2} \div \frac{1}{4}$ (iv) $\frac{1}{3}$ f$\div \frac{1}{5}$ (v) $4 \div \frac{1}{3}$

(vi) $5 \div 7$ (vii) $\frac{1}{4} \div \frac{1}{2}$ (viii) $\frac{1}{5} \div \frac{1}{2}$

(b) Draw or describe diagrams to represent each of the division situation Q1a (i) to Q1 (viii).

(c) Solve Q1a (i) to Q1a(viii).

(d) Try to write down a word problem for each expression in Q 1(i) to Q1 (viii).

Q2 (a) If a person' s income in half a month is $ 400 how much can he earn in a month?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only <u>numbers which are given in the problem</u> and <u>mathematical symbols</u>.
- What is the answer?
- Explain how you worked out the answer?

(b) A man earns $ 240 in 3 weeks. How much is his weekly salary?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only <u>numbers which are given in the problem</u> and <u>mathematical symbols</u>.
- What is the answer?
- Explain how you worked out the answer?

Q3 (a) Jessica and Shane had a recipe to make a cake. In this recipe $\frac{1}{2}$ kg of flour is mixed with $\frac{1}{4}$ kg of sugar. Shane wants to use 1 kg of flour. How much sugar should he use to have the same taste?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

(b) Jessica also used the same recipe in which $\frac{1}{2}$ kg of flour is mixed with $\frac{1}{4}$ kg of sugar. Because she bought 1kg of sugar she wants to use all the sugar. How much flour should she use?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

Q4 (a)) A girl walks $\frac{1}{2}$ km in $\frac{1}{3}$ of an hour. How much distance can she cover in one hour if she keeps her speed constant?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

(b) A girl walks $\frac{1}{2}$ km in $\frac{1}{3}$ of an hour. How much time is required to travel a distance of 1 km if she walks with the same speed?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

Q5 (a) $8\frac{1}{4}$ cm is approximately equal to $3\frac{1}{2}$ inches. About how many cms are in an inch?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

(b) $8\frac{1}{4}$ cm is approximately equal to $3\frac{1}{2}$ inches. What part of an inch is equal to one cm?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

Q6 (a) A boat moved 4 metres in a second with constant speed. How much time will it take to move to 12 metres?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is the answer?
- Explain how you worked out the answer?

(b) A boat moves at a constant speed of 4 metres per second. How far does it move in 3 seconds?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.
- What is answer?
- Explain how you worked out the answer?

(c)A boat moved 12 metres in 3 seconds at a constant speed. How far can it move in a second?

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only numbers and mathematical symbols.

- What is the answer?
- Explain how you worked out the answer?

Q7 It takes $\frac{3}{8}$ of a bottle of milk to fill a large glass. How many of these glasses can be filled with 40 bottles of milk.

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only <u>numbers</u> and <u>mathematical symbols</u>.
- What is the answer?
- Explain how you worked out the answer?

Q 8 An ant moved exactly the same distance each day. After five days it had moved a distance of $\frac{3}{4}$ km. how much did it moved each day.

- Write mathematical expressions for the above situation (as many as you can). It means you are asked to express the above problems by using only <u>numbers</u> and <u>mathematical symbols</u>.
- What is the answer?
- Explain how you worked out the answer?