Important Prerequisites to Educational Success in Mathematics in Lower Secondary School

Joakim Samuelsson
Linköpings Universitet/IBL
581 83 Linköping
Sweden

E-mail: Joakim.Samuelsson@liu.se

Department of Behavioural Sciences and Learning, Linköping University, Sweden
Abstract
In this study, we investigated to what extent arithmetic ability and self-regulated learning skills in the beginning of lower secondary school predicts measures of students’ performance in mathematics at the end of lower secondary school. Arithmetic ability and self-regulated learning skills were tested the first two weeks in lower secondary school. Post-tests were performed the last two months in lower secondary school. All testing was performed by the class teacher. A total of 219 students attending 10 different classes in ten different schools were included in the study. The 108 female students and 111 male students were all between 12-13 years old when they started lower secondary school, and 15-16 years old when they left lower secondary school. The present study shows that arithmetical knowledge is an important predictor to all competencies tested in the national test at the end of lower secondary school. The regression equation demonstrates that operation with numbers is a stronger predictor than concept of numbers to all competencies measured in the national test except complex problem solving. This study also highlights the importance of taking affective factors into account in discussions about the results of mathematics teaching and learning. It is widely reported that students self-regulated learning skills have a strong relation to achievement in mathematics which is also proven in this study. The strong correlation between operation with numbers and achievement, affective factors and achievement in mathematics helps us to identify some weaknesses in the Swedish education system.

Keywords: Mathematics; self-regulated learning skills; development; lower secondary school
Important prerequisites to educational success in mathematics in lower secondary school

Introduction

The Swedish compulsory school system is characterized by compulsory schooling for all children between 7 and 16 years of age. There is a national curriculum for classes taken in a fixed order with the same classmates together for at least the first six years. This curriculum aims to provide the same standard of education throughout the country and establish a platform for further studies. For the first six years (age 7-12) students are often taught by the same teacher who teaches most subjects. After these six years students attend the equivalent of lower secondary school (age 13-16) where teachers teach in only two or three specific subjects, for example, mathematics and physics.

A common teaching method in Swedish classrooms is characterized by teachers instructing or imparting knowledge with the students then practicing their skills to operate with numbers (Lundgren, 1972; Neuman, 1987; Magne, 1998; Engström, 1993; Lindqvist, Emanuelsson, Lindström & Rönnberg, 2003, Samuelsson, 2003, NU, 2003). This teaching method encourages a dependence on a teacher’s confirmation, text books that drill, and a key that gives the right answers (Samuelsson, 2007). National testing of learning is overseen by The National Agency of Education three times during compulsory schooling, in grade 3, 5 and 9 (age 9, 11 and 15).

Literature review

The field of mathematics is extremely complex, including areas such as arithmetic, and geometry with each of these areas consisting of several subdomains and encompassing many cognitive processes (Kilpatrick et. al., 2001). The mathematics curriculum during elementary
and middle school in Sweden has many components, but there is a strong emphasis on concepts of numbers and operations with numbers. From an international perspective, mathematics knowledge is defined as something more complex than the concept of numbers and operations with numbers. Kilpatrick et al. (2001) argue for five strands which together build students’ mathematical proficiency. The five strands provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency. In their report they discuss,

1. **Conceptual understanding** is about comprehension of mathematical concepts, operations, and relationships. Students with conceptual understanding know more than isolated facts and methods. Items measuring conceptual understanding are for instance: “Your number is 123.45. Change the hundreds and the tenths. What is your new number?

2. **Procedural fluency** refers to skills in carrying out procedures flexibly, accurately, efficiently, and appropriately. Students need to be efficient in performing basic computations with whole numbers (e.g., 6+7, 17–9, 8×4) without always having to refer to tables or other aids.

3. **Strategic competence** is the ability to formulate, represent, and solve mathematical problems. Kilpatrick et al. (2001, p. 126) give the following example of item testing strategic competence: “A cycle shop has a total of 36 bicycles and tricycles in stock. Collectively there are 80 wheels. How many bikes and how many tricycles are there?”

4. **Adaptive reasoning** refers to the capacity for logical thought, reflection, explanation, and justification. Kilpatrick et al. (2001) gives the following example where students can use their adaptive reasoning; “Through a carefully constructed sequence of activities about adding and removing marbles from a bag containing many marbles, second graders can reason that 5+(−6)=−1. In the context of cutting short bows from a
12-meter package of ribbon and using physical models to calculate that 12 divided by 1/3 is 36, fifth graders can reason that 12 divided by 2/3 cannot be 72 because that would mean getting more bows from a package when the individual bow is larger, which does not make sense” (p.130).

5. “Productive disposition is the habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Kilpatrick et al., 2001, p.5). Items measuring productive disposition are for instance: “How confident are you in the following situations? When you count 8-1=___+3 (completely confident, confident, fairly confident, not at all confident).”

The first aim of this study is to investigate to what extent arithmetic skills as conceptual understanding (concept of numbers) and procedural fluency (operation with numbers) in the beginning of lower secondary school predict four of the strands of mathematical proficiency (conceptual understanding, procedural fluency, strategic competence, adaptive reasoning) and achievement (grade) at the end of lower secondary school. By expanding the knowledge base of the extent to which different aspects are important to mathematics cognitive development in predicting educational success in mathematics will lead to better ways to foster mathematics development as well as to detect, ameliorate and prevent problems.

In discussions about the effect of teaching in school, cognitive outcomes often receive a lot of the attention. In OECD (2004) the authors argue that another important component in mathematics teaching is self-regulated learning skills. Self-regulated learning skills are important goals in the Swedish national curriculum as well as individually related to students performance in mathematics (Törnroos et. al., 2006, Samuelsson, 2010). As people are not only hosts of internal mechanism but rather agents of experiences, investigations of affective
factors such as self-regulated learning skills attest to the importance of one’s perceived self for successful functioning and adaption across many different domains (Bandura, 2001). Bandura (2001) also showed that the human intellect is not just random but rather generative, creative, and proactive. Self-construct that provides students with a sense of agency to motivate their learning through use of such self-regulatory processes are for instance self-monitoring and self-evaluation (Bandura, 1997). Students in grade 7-9 (age 13-15) are typically in early adolescence, which appears to be a critical stage in their affective development. In mathematics education, evidence points to a decline in levels of positive affect as student progress school (Fredricks & Eccles, 2002, Samuelsson, 2010) with such levels reaching a minimum in year 9 (Watt, 2004). A better understanding of these issues is important, since it can help schools to draw attention to these aspects in the classroom, and find ways of helping every student to reach his or her highest and develop positive self-regulated learning skills. The second aim of this study is to investigate to what extent self-regulated learning skills in the beginning of lower secondary school predict mathematical proficiency with respect to arithmetic ability (concept of numbers and operation with numbers), strategic competence (problem solving, complex problem solving), adaptive reasoning (oral communicative ability) and grade at the end of lower secondary school.

**Arithmetic and mathematics performance**

Understanding concepts of numbers and operating with numbers are necessary components of proficiency in arithmetic and are needed to transfer previously learned procedures to solve everyday problems (U.S. Department of Education, 2008). Weaknesses in arithmetic will affect the development of other areas such as algebra and problem solving (U.S. Department of Education, 2008). It is well known that individual differences in arithmetical performance are marked in both children and young adolescence. An average British class of 11-years-olds
Arithmetic ability is sometimes discussed as a single entity. However, there is extensive and increasing evidence that there is no such thing as arithmetic ability: only arithmetic abilities. Important evidence can be found in several factor analytic studies of arithmetic involving identification of tests that cluster together. The hypothesis is that if a number of different clusters are obtained, these represent different domains of arithmetic (Werdelin, 1961). Researchers have proposed different ways of how to conceptualize different aspects of arithmetic abilities. Ginsburg (1972) suggests four cognitive systems, a system for calculation procedures, a system for estimation, a system of formal knowledge (involving arithmetical facts and concepts such as place value) and a system of informal knowledge. Other researchers such as Levine, Lindsay and Reed (1992) propose sixteen subcomponents to arithmetic ability. Most frequently arithmetic is divided into four subcomponents, factual knowledge, procedural knowledge, conceptual knowledge and applied arithmetic (problem solving) (Dowker, 2005).

Arithmetic tends often to be taught in a hierarchical way (Dowker, 2005). This means that if children have missed or failed to learn early on, they may have difficulty with later lessons based on attempts to build on earlier information. There are relatively few studies trying to predict educational success in mathematics across lower secondary school. Thus, some researchers have tried to find trajectories predicting math achievement in early years of elementary school. Jordan et. al (2006) found that number sense (concept of numbers and operation with numbers) growth in kindergarten through first grade, accounted for 66 percent
of variance in first grade math achievement. In another study Baker et. al (2006) reported that their test battery on conceptual understanding (concept of numbers) and procedural fluency (operation with numbers) in pre-school predicted mathematics achievement in school year 1 ($r=0.47$ and $r=0.72$). Thus, arithmetic abilities seem to be important predictors to the development of a mathematical proficiency in kindergarten and elementary school.

**Self-regulated learning skills and mathematics performance**

Within the framework of PISA 2003 (OECD, 2004) self-regulated learning skills are internal motivation (student’s interest in and enjoyment of mathematics), instrumental motivation in mathematics, self-concept in mathematics and anxiety in mathematics.

*Internal motivation in mathematics*

Internal motivation means that your motivation to accomplish your goal comes from within you. Your motivation is from you. It is determined by your own values and goals. Thus, internal motivation to learn is characterised by an intention to engage in learning activities because it is considered interesting, exciting, challenging etc (OECD, 2004). Lately, interest has been differentiated in two forms in the literature, situational interest and individual interest (Hidi & Harackiewicz, 2000; Krapp, 2000; Schraw & Lehman, 2001). Situational interest can be characterised as transient, context-dependent enjoyment trigged by environmental factors. This type of interest is often a necessary first step in the development of more stable individual interest (Hidi & Renninger, 2006). Several researchers argue that individual interest is an important prerequisite to attend to certain objects and activities (Hidi & Ainley, 2002; Köller, Baumert, & Schnabel, 2001; Krapp, 2000; Schraw & Lehman, 2001). In interest-driven activities, the activity is associated with favourable learning outcomes. Students experience competence and personal control, feelings of autonomy, by an experience
of flow in which the student and the object of interest combine (Csikszentmihalyi & Schiefele, 1993).

Törnroos et. al. (2006) used hierarchical linear models to explore connections between self-regulated learning skills and students’ performances in mathematics. They reported that interest itself was positively associated with students’ results on the test but combined with other factors the coefficient became negative. They explain the result by the fact that several students with high self-concept and good results are not interested in mathematics and that students with problems learning mathematics are interested in the subject.

*Instrumental motivation in mathematics*

Instrumental motivation, in contrast means that your motivation to attain your goal comes from a source outside yourself. This type of motivation is characterised by the desire to engage in a learning process because it has positive outcomes or can help you avoid negative outcomes (OECD, 2004). With instrumental motivation the purpose of mathematics attainment is more practical. Students wants to learn mathematics because it will help them in future work, it will help them with the subject that they want to study further in school, they need it for what they want to study later or it will help them get a job. In the hierarchical linear models presented by Törnroos et. al. (2006) instrumental motivation had a small but statistical significant relation to students’ performance in mathematics in Finland, Norway, Sweden and OECD but not in Denmark and Iceland.

In contrast to the mixed degree to which students find mathematics interesting or enjoyable, the overwhelming majority of students in OECD understand that it is important to study mathematics because of the future benefits it will bring (OECD, 2004). Boys are somewhat more likely to do so than girls, which is not surprising given that boys have a greater tendency
to go on to further studies in disciplines that demand an understanding of mathematics. Yet in the Nordic countries students do not conform to a stereotype in which girls think that mathematics is irrelevant to their future (Törnroos et al, 2006).

Samuelsson and Granström (2007) showed that very high levels of instrumental motivation in mathematics could be experienced as unreasonable demands. Demands they found were counter-productive to students learning of mathematics. A similar result was found in a study by Artelt (2005), but in that case student’s instrumental motivation was negative correlated to reading literacy. Thus, instrumental motivation seems to correlate with performance in different subjects in a positive way as well as in a negative way.

**Self-concept in mathematics**

Self-concept is defined as self-perceptions about one’s abilities and competences that influence the possibility of success (Byrne & Shavelson, 1986) in for instance mathematics. Students with a positive self-concept show more motivated behaviour and greater persistence with challenging tasks (Stipek, 1998). Many researchers have viewed self-concept as an explanatory variable for student’s varying performance in school; whereas other researchers insist that self-concept is a consequence and not a cause of students’ achievement (Bong & Clark, 1999).

Skaalvik and Hagtvet (1990) argue that the causality between academic self-concept and results mainly goes in the direction from self-concept to achievement for older students (aged 13, when they start lower secondary school in Sweden) when they are subjected to greater
educational demands. Another explanation is that children’s self-concept becomes increasingly differentiated as they grow older and as the ability to judge their overall self-worth increases (Harter, 1998; 1999). Thus, as children become adolescents, when they begin lower secondary school, previously confident children experience an increasing awareness of peers and their relative ability, they become more aware of their own competence, more realistic about task demands, and more sensitive to social comparisons (Harter, 1998).

Explanations to why student’s self-concept may fall in lower secondary school are less personalised instruction, and perceptions of increased pressures (Pajares & Cheong, 2003). In other studies researchers have found strong relations between self-concept and achievement established within two years of starting school (Chapman, Tummer, & Prochnow, 2000).

Self-concept is positively related to mathematics achievement in a variety of settings across school and across countries (Dermitzaki, Leondari, & Goudas, 2009; Ireson & Hallam, 2009). When Törnroos et al. (2006) used hierarchical linear models in the Nordic countries in PISA they found that self-concept was a stronger predictor to student achievement in mathematics than internal motivation, instrumental motivation and anxiety in mathematics. That result differed from average results in all OECD countries. In OECD students’ self-concept was not as strong a predictor to students’ performance in mathematics as in the Nordic countries.

Anxiety in mathematics

Anxiety in mathematics has been defined in different ways: a feeling of tension, apprehension or fear that interferes with mathematics performance (Richardson & Shuinn, 1972; Ashcraft, 2002); a state of discomfort, which occurs in situations involving mathematics (Trujillo &
Hadfield, 1999). The first definition focuses on the impact of anxiety on cognitive outcomes, while the latter definition highlights the impact on self-esteem.

Aschcraft (2002) summarizes the extensive literature on personal and educational consequences of anxiety in mathematics. One unfortunate consequence is that people with anxiety in mathematics have a tendency to avoid mathematics. They take fewer and easier courses and when they take mathematics courses they receive lower grades. Students with anxiety in mathematics also seem to adopt negative attitudes and negative self-concept. One explanation to why people with high anxiety in mathematics end up with lower grades is their avoidance behaviour. They are exposed to less mathematics in school and learn less of what they are exposed to (Fennema, 1989).

Researchers have found three variables affecting maths anxiety: environmental, personality and intellectual. Environmental factors contain classroom issues, parental pressure and the perception of mathematics as a difficult set of rules. Personality refers to reluctance to ask questions in class and self-esteem; intellectual include a mismatch of learning styles and self-doubt (Hadfield & Mc Neil, 1994).

In the OECD model anxiety has stronger negative effect on student performance than in the Nordic countries (Törnroos et. al. 2006). It was also shown that students in the Nordic countries had the least anxiety towards mathematics (OECD, 2004).

Method

Participants
A total of 219 students attending 10 different classes in ten different schools in mathematics were included in the study. They were all between 12-13 years old when they started in lower secondary school, 108 female students and 111 male students, and they were 15-16 years old when they left lower secondary school. They all attended ten schools, all schools mainly recruit students from a part of Sweden with average socio-economic status. Their performances on standardised national tests in 5th grade in language and mathematics were representative for Swedish students according to the National Agency of Education. Thus, there were 10 groups of mathematic students attending a new school at grade 7 in mid-August 2006. Pre-testing (arithmetic and self-regulated learning skills test) was performed during the first two weeks in school. This testing was performed by the math class teacher. The second test (national test) measured arithmetic ability (concept of numbers and operation with numbers), strategic competence (problem solving, complex problem solving) and adaptive reasoning (oral reasoning) and was performed the last two months in grade 9 in April-May 2009. Post testing was also performed by the math class teacher.

Table 1  *Types and timing of tests and number of participants (N)*

<table>
<thead>
<tr>
<th>Test</th>
<th>August 2006</th>
<th>April-May 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arithmetic pre-test</td>
<td>219</td>
<td>National test (arithmetic ability,</td>
</tr>
<tr>
<td>Self-regulated test</td>
<td></td>
<td>strategic competence and adaptive</td>
</tr>
<tr>
<td></td>
<td></td>
<td>reasoning)</td>
</tr>
<tr>
<td>(N)</td>
<td>219</td>
<td>204</td>
</tr>
</tbody>
</table>

The arithmetic pre-test

The pre-test employed in this study was developed by six teachers in mathematics and three teacher educators. The teacher educators are all involved in textbook writing in mathematics.
and are also part of the committee working on the national mathematics tests in Sweden. One test covering arithmetic ability, operation with numbers (15 item), and concepts of numbers (15 item) was developed. The calculation items required the student to perform addition, subtraction, multiplication and division (19 + 9 =; \frac{450}{9} =; 1 - \frac{3}{5} =; 2.1 + 0.7 =; 13.1 + 0.01 =; 10 - 0.3 =; 21 \cdot 10 =; 10 \cdot 2.45 =; 420/10 =; 141/6 =; 567 + 273 =; 5 \cdot 6.4 =; 5 \cdot 9.6 =).

The conceptual understanding item required the student to solve problems related to knowledge about positions system and magnitude of numbers. For instance:

a) Draw a line under the largest number 1.49 1.499 1.5 1.099.

b) Your number is 123.45. Change the hundreds and the tenth. What is your new number?

c) What numbers should be in the squares [ ]?

a) 10 20 30 [ ] [ ]

b) 8.4 8.6 8.8 [ ] [ ]

c) 14, 34 14,32 14,30 [ ] [ ]

A reliability test (Cronbach \( \alpha \)) was carried out on each part of the test giving \( \alpha = .87 \) for conceptual understanding and \( \alpha = .82 \) for procedural fluency demonstrating high levels of relability.

_The self-regulated learning skills questionnaire_

Self-regulated learning skills were assessed with a questionnaire originally designed and used in PISA 2003 (OECD, 2004). However, in this study a ten-point scale was employed (don’t agree = 1 to totally agree = 10) instead of a six-point scale used in the PISA study. The first four statements in the questionnaire were related to _internal motivation_, that is, (a) I enjoy
reading about mathematics, (b) I look forward to my mathematics lessons, (c) I do mathematics because I enjoy it, and (d) I am interested in the things I learn in mathematics. Another four statements in the questionnaire were employed to measure *instrumental motivation*. These statements were as follows: (a) Making an effort in mathematics is worth it because it will help me in the work that I want to do later, (b) Learning mathematics is important because it will help me with the subject that I will study further on in school, (c) Mathematics is an important subject for me because I need it for what I want to study later on, and (d) I will learn many things in mathematics that will help me get a job. *Self-concept* was measured by 5 different statements: (a) I am good at mathematics; (b) I get good grades; (c) I learn mathematics quickly; (d) I have always believed that mathematics is one of my best subjects; and (e) In my mathematics class, I understand even the most difficult work. Finally, the last five statements focused on *anxiety* about mathematics: (a) I often worry that it will be difficult for me in mathematics classes, (b) I get very tense when I have to do mathematics homework, (c) I get nervous doing mathematics problems, (d) I feel helpless when doing mathematics problems, and (e) I worry that I will get poor grades in mathematics. A reliability test (Cronbach $\alpha$) was carried out on each factor in test giving, $\alpha=.94$ for internal motivation, $\alpha=.90$ for instrumental motivation, $\alpha=.94$self-concept and $\alpha=.85$ anxiety demonstrating high levels of reliability.

*Student’s achievement (post-test)*

The Swedish school system is goal-directed, which means that the education is governed by objectives. The students’ achievements measured by grades are to be related to these objectives, which reflect competencies considered important in mathematics. Achievement data used in this study are students’ grades in mathematics: from failed (1), passed (2), passed with distinction (3) to superior (4).
And the national test at the end of lower secondary school contains four subtests. *Oral tasks testing student’s adaptive reasoning:* Testing: To what extent they show that they have understood the question, the concepts and relationships between them; How clear their explanation is, and how well they use mathematical language; To what extent they participate in the discussion, can argue for their ideas and respond to the explanations of other students (maximum score: 8). *Problem solving tasks testing student’s strategic competence:* Problem solving tasks are divided into two parts, problem solving (maximum score: 34) and complex problem solving (maximum score: 11). Testing: What mathematical knowledge students have shown; What conclusions they have made; How well they have presented their work and carried out their calculations. *Arithmetic tasks testing student’s arithmetic ability:* Testing: Students concept of numbers and operation with numbers (maximum score: 18).

**Data analyses**

To be able to answer the research questions, to what extent arithmetic ability and self-regulated learning skills predict mathematical proficiency with respect to arithmetic ability (conceptual understanding and procedural fluency), strategic competence (problem solving, complex problem solving), adaptive reasoning (oral communicative ability) and grade at the end of lower secondary, Pearson’s product moment correlation test and a multiple regression analyses were carried out. There were two regression equations set up a) *concept of numbers and operation with numbers* and b) *internal motivation, instrumental motivation, self-concept and anxiety* was used as an independent variable in the regression equations. The dependent variable was grade in mathematics and arithmetic ability (concept of numbers and operation
with numbers), strategic competence (problem solving, complex problem solving), adaptive reasoning (oral reasoning) at the end of lower secondary school.

Results

Arithmetic skills as predictors to educational success in mathematics

The first aim of this study is to investigate to what extent arithmetic skills as conceptual understanding (concept of numbers) and procedural fluency (operation with numbers) in the beginning of lower secondary school predict mathematical proficiency with respect to arithmetic ability (concept of numbers and operation with numbers), strategic competence (problem solving, complex problem solving), adaptive reasoning (oral ability) and grade at the end of lower secondary school.

The statistical analyses concerned the relationship between concept of numbers, operation with numbers in the beginning of lower secondary school and grade in mathematics, arithmetic ability, strategic competence and adaptive reasoning at the end of lower secondary. Analyzed individually both factors (concept of numbers, operation with numbers) were strongly related to measured abilities and grades at the end of lower secondary school.

Table 2. Correlate arithmetic ability (AA), strategic competence (SC), strategic competence complex (SCC), adaptive reasoning (AR) and grade

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>SC</th>
<th>SCC</th>
<th>AR</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concept of numbers</td>
<td>.683**</td>
<td>.619**</td>
<td>.630**</td>
<td>.499**</td>
<td>.618**</td>
</tr>
<tr>
<td>Operation with numbers</td>
<td>.704**</td>
<td>.632**</td>
<td>.552**</td>
<td>.509**</td>
<td>.675**</td>
</tr>
</tbody>
</table>

*p<.05, **p<.01
The strongest correlation to test scores measured at the end of secondary school was student ability to operate with numbers in the beginning of lower secondary school. There was only one relation where student’s concept of number correlated more than students operation with numbers, the correlation to complex problem solving ability. In the next step a regression model was specified with math grade, arithmetic ability, strategic competence, adaptive reasoning as dependent variable and arithmetic skills (operation with numbers, concept of numbers) in the beginning of lower secondary school as independent variable. All standardized regression coefficients (Beta) for the equation are shown in table 2.
Table 3. Regressing arithmetic skills (Concept of numbers (CN), operation with numbers (ON)) with arithmetic ability (AA), strategic competence (SC), strategic competence complex (SCC), adaptive reasoning (AR) and grade

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>SC</th>
<th>SCC</th>
<th>AR</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Beta</td>
<td>t-value</td>
<td>Beta</td>
<td>t-value</td>
<td>Beta</td>
</tr>
<tr>
<td>CN</td>
<td>.34</td>
<td>4.53***</td>
<td>.32</td>
<td>3.86***</td>
<td>.51</td>
</tr>
<tr>
<td>ON</td>
<td>.44</td>
<td>5.74***</td>
<td>.38</td>
<td>4.50***</td>
<td>.16</td>
</tr>
</tbody>
</table>

* p<.05, ** p<.01, *** p<.001

As expected arithmetic skills (concept of numbers and operation with numbers) in the beginning of lower secondary school predicted all dependent variables at the end of lower secondary school. The multiple regression coefficients were significant for all measurement: Arithmetic ability $R=.74$, $F(2,195)=116.4$, $p<.001$, strategic competence $R=.66$, $F(2,195)=77.15$, $p<.001$, strategic competence (complex) $R=.64$, $F(2,195)=67.06$, $p<.001$, adaptive reasoning $R=.54$, $F(2,195)=39.28$, $p<.001$, and grade $R=.69$, $F(2,195)=89.44$, $p<.001$. This implies that there is a relationship between arithmetic skills in the beginning of secondary school and arithmetic skills, strategic competence, adaptive reasoning and grade at the end of lower secondary school. The regression model involving arithmetic skills, explain between 29 and 54 percent of the results students achieve in mathematics before they leave lower secondary school. Student’s ability to operate with numbers seems to explain slightly more than student’s concept of numbers in all cases except when regressing to complex problem solving. In that case student’s concept of numbers (.51) explains three times as much as student ability to operate with numbers (.16). Another interesting finding is that students ability to operate with numbers in the beginning of secondary school predict grades in
mathematics twice as much as students’ concept of numbers. This implies that skills in carrying out procedures flexibly, accurately, efficiently, and appropriately (.49) have a stronger impact on grades at the end of lower secondary school than student’s comprehension of mathematical concepts, operations, and relationships (.24).

*Self-regulated learning skills as predictor to educational success in mathematics*

The second aim of this study is to investigate to what extent self-regulated learning skills in the beginning of lower secondary school predict mathematical proficiency with respect to arithmetic ability (concept of numbers and operation with numbers), strategic competence (problem solving, complex problem solving), adaptive reasoning (oral communicative ability) and grade at the end of lower secondary school.

The statistical analyses concerned the relationship between self-regulated learning skills (internal motivation, instrumental motivation, self-concept, anxiety) in the beginning of lower secondary school and grade in mathematics, arithmetic ability, strategic competence, adaptive reasoning at the end of lower secondary. Analyzed individually, three factors (internal motivation, self-concept, anxiety) were strongly related to almost all measured abilities and grades at the end of lower secondary school. Student’s interest and enjoyment in mathematics did not correlate with complex problem solving (strategic competence complex). Instrumental motivation in the beginning of lower secondary school had no significant correlation to grade in mathematics, arithmetic ability, strategic competence, adaptive reasoning at the end of lower secondary.

Table 4. Correlate self-regulated learning with arithmetic ability (AA), strategic competence (SC), strategic competence complex (SCC), adaptive reasoning (AR) and grade
In the next step a regression model was specified with math grade, arithmetic ability, strategic competence, adaptive reasoning as dependent variable and self-regulated learning skills in the beginning of lower secondary school as independent. All standardized regression coefficients (Beta) for the equation are shown in Table 5. The multiple regression coefficients were significant for all measurement: Arithmetic ability $R=.57$, $F(4,193)=23.48$, $p<.001$, strategic competence $R=.60$, $F(4,193)=27.12$, $p<.001$, strategic competence (complex) $R=.48$, $F(4,193)=14.09$, $p<.001$, adaptive reasoning $R=.51$, $F(4,193)=16.88$, $p<.001$, and grade $R=.56$, $F(4,193)=21.77$, $p<.001$. This implies that there is a relationship between self-regulated learning skills in the beginning of lower secondary school and arithmetic skills, strategic competence, adaptive reasoning and grade at the end of lower secondary school. The regression model involving self-regulated learning skills, explain between 23 and 36 percent of the results students achieve in mathematics before they leave lower secondary school.

Table 5. Regressing self-regulated learning skills with arithmetic ability (AA), strategic competence (SC), strategic competence complex (SCC), adaptive reasoning (AR) and grade

<table>
<thead>
<tr>
<th></th>
<th>AA</th>
<th>SC</th>
<th>SCC</th>
<th>AR</th>
<th>Grade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal motivation</td>
<td>.184**</td>
<td>.247**</td>
<td>No sig.</td>
<td>.237**</td>
<td>.195**</td>
</tr>
<tr>
<td>Self-concept</td>
<td>.511**</td>
<td>.531**</td>
<td>.394**</td>
<td>.479**</td>
<td>.480**</td>
</tr>
<tr>
<td>Anxiety</td>
<td>-.377**</td>
<td>-.482**</td>
<td>-.330**</td>
<td>-.377**</td>
<td>-.449**</td>
</tr>
</tbody>
</table>

* $p<.05$, ** $p<.01$
<table>
<thead>
<tr>
<th></th>
<th>Beta</th>
<th>t-value</th>
<th>Beta</th>
<th>t-value</th>
<th>Beta</th>
<th>t-value</th>
<th>Beta</th>
<th>t-value</th>
<th>Beta</th>
<th>t-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Int.m</td>
<td>-.15</td>
<td>-1.74</td>
<td>-.05</td>
<td>-.61</td>
<td>-.15</td>
<td>-1.58</td>
<td>-.06</td>
<td>-.9</td>
<td>-.09</td>
<td>-1.0</td>
</tr>
<tr>
<td>Inst.m</td>
<td>-.18</td>
<td>-2.57*</td>
<td>-.15</td>
<td>-2.55*</td>
<td>-.18</td>
<td>-2.39*</td>
<td>-.11</td>
<td>-1.49</td>
<td>-.17</td>
<td>-2.38*</td>
</tr>
<tr>
<td>Self-c</td>
<td>.63</td>
<td>6.85***</td>
<td>.49</td>
<td>5.5***</td>
<td>.49</td>
<td>4.95***</td>
<td>.48</td>
<td>5.0***</td>
<td>.47</td>
<td>5.0***</td>
</tr>
<tr>
<td>Anxie</td>
<td>-.09</td>
<td>-1.23</td>
<td>-.24</td>
<td>-3.4**</td>
<td>-.12</td>
<td>-1.58</td>
<td>-.13</td>
<td>-1.73</td>
<td>-.24</td>
<td>-3.2**</td>
</tr>
</tbody>
</table>

* p<.05, ** p<.01, *** p<.001

Internal motivation and student’s interest and enjoyment in mathematics are not significant predictors to any of the dependent variables in the regression model. Grades and strategic competence are predicted by instrumental motivation, self-concept and anxiety. However, high instrumental motivation and student’s high anxiety seem not to support grades and strategic competence, but, rather, seem to promote the opposite. The relationship between complex strategic competence and arithmetic skills is explained by self-concept and instrumental motivation. Anxiety in the beginning of lower secondary school does not seem to have any significant relation to complex strategic competence and arithmetic skills in the regression model. Oral adaptive reasoning is only explained by self-concept in the model.

The standardized beta coefficient in the regression model shows that students’ self-concept in the beginning of lower secondary school is the strongest predictor to arithmetic skills, strategic competence, adaptive reasoning and grade at the end of lower secondary school. Student’s self-concept explain results three times (0.47-.63) as much as instrumental motivation (0.17) and anxiety (0.24) with respect to strategic competence and grade.

**Discussion**
The purpose of the present study was to scrutinise to what extent arithmetic and self-regulated learning skills in the beginning of lower secondary school predict arithmetic ability, strategic competence, adaptive reasoning and grades at the end of lower secondary school.

The results expanding the knowledge base about to what extent different aspects important to mathematics cognitive development predicts educational success in mathematics. The results will, hopefully, lead to better ways to foster mathematics development as well as to detect, ameliorate and prevent problems.

**Arithmetic skills as predictor**

Arithmetic is an important part of mathematics during compulsory school in Sweden. The first six years a lot of mathematics lessons focus on aspects of arithmetic such as concept of numbers and operations of numbers. The present study shows that arithmetical ability is an important predictor to all competencies tested in the national test at the end of lower secondary school, a result consistent with result presented by Baker et. Al, (2006) who found that pre-school children’s competence in operation with numbers as well as their concept of numbers correlated with math achievement in first grade. This study also shows that concept of numbers correlates with math grades just as much as operation with numbers correlate with math grades. Both aspects seem to be of equal importance to achievement in mathematics. Thus, the regression equation demonstrates that operation with numbers is a stronger predictor to all competencies measured on national test except complex problem solving. One interpretation is that the national test items often (and sometimes only) claim students to recall a solution algorithm which also is a common teaching method in Swedish schools (Lundgren, 1972; Neuman, 1987; Magne, 1998; Engström, 1993; Lindqvist, Emanuelsson, Lindström & Rönnberg, 2003, Samuelsson, 2003, NU, 2003). Only when they are going to solve a more
complex task concept of numbers seem to predict their achievement more than operation with numbers. One explanation is that when they ought to solve complex problem they need a deeper understanding of the mathematics, a more analytical and conceptual process is necessary.

The regression equation for operation of numbers, concept of numbers and grade show that operation of numbers (Beta=.49) in the beginning of lower secondary school predict math grades twice as much as concept of numbers (Beta=.24). Swedish mathematics teaching often receives criticism because too many teachers draw attention to operations with numbers which do not help children to understand mathematics in a deeper sense. The results from this study support teachers’ intention for pupils to perform as successfully as possible on national tests. In order to change the teaching practice, the national test may need to involve more creative thinking, analytical and conceptual thinking than algorithmic reasoning where students only need to recall an algorithm.

*Self-regulated-learning skills as predictor*

In discussions about the effect of mathematics teaching in schools, cognitive outcomes receive a lot of the attention. This study highlights the importance of also taking affective factors into account in discussions about the results of mathematics teaching and learning. It is widely reported that students self-regulated learning skills have a strong relation to achievement in mathematics (Törnroos, et al., 2006, OECD, 2004). The strong correlation between affective factors and achievement in mathematics helps us to identify some weaknesses in the Swedish education system. The results show that students with low interest and enjoyment in mathematics (e.g. Törnroos, et al., 2006), with low self-concept (e.g. Dermitzaki, Leondari, & Goudas, 2009; Iresson & Hallam, 2009) and high anxiety (e.g.
Richardson & Shuinn, 1972; Ashcraft, 2002) receive lower grades. These results are significant with earlier research on the relation between affective factors and performance in mathematics. The correlation for instrumental motivation was certainly unexpected as well as the beta-value in the regression equation. In the regression model instrumental motivation was a negative predictor and it didn’t correlate to grades itself. One explanation could be that students with high grades and good results in mathematics do not see knowledge in mathematics as important in their future studies or working life. The student answered the questionnaire after they received their results on national tests as well as what study program they were going to attend in higher secondary school. Sweden will have a great problem if students with high grades in mathematics do not see the subject as an important competence in their future life. Too many students with strong mathematics ability will chose to complete programs with low demands on their mathematics knowledge. A second interpretation is that too high instrumental motivation in mathematics could be experienced as unreasonable demands which Samuelsson and Granström (2007) as well as Artelt (2005) found were counter-productive to students learning.

A third explanation why the factor, instrumental motivation in the beginning of lower secondary school, doesn’t predict results at the end of lower secondary in the regression model is that many students who think mathematics is easy do not think the subject is important. On the other hand, some students think it is very important but have great difficulty learning it (e.g. Törnroos, et al., 2006).

The strongest predictor for educational success in mathematics with respect to affective factors is student’s self-concept. These results support earlier research focusing the relation between self-regulated learning skills and mathematics achievement (e.g. Törnroos et. al.
The connection between self-concept and learning outcome helps us to identify some plausible explanation for low grades in mathematics. If there is causality between academic self-concept in the direction from self-concept in lower secondary school (e.g. Skaalvik & Hagtvet, 1990) teachers must help students to develop their self-concept in mathematics.

Not surprisingly anxiety predicts mathematics achievement in a negative way (e.g. Törnroos et. al. 2006). Ashcraft (2002) argues that one explanation is that pupils with anxiety tend to avoid mathematics. They work less with mathematics and therefore they obtain lower grades in mathematics.

Didactical implications

The results in this study draw the intention to the importance of the first six years of mathematics learning. Cognitive competences as procedural fluency (operation with numbers) and concept of numbers (conceptual understanding) as well as affective outcomes, internal motivation, instrumental motivation, self-concept and anxiety learned the first six years predict learning outcomes in different ways. The cognitive competences explain between 29 and 54 percent of the achievement on national test at the end of lower secondary and the self-regulated learning skills explain between 23 to 36 percent of outcome at the end of lower secondary school. One important question to consider is how teachers can help students to develop their self-regulated learning skills. Törnroos et al. (2006) proposed that teachers should help students to see the fascination and importance of mathematics in our world. Some possible solutions are for teachers to—discuss demands across year groups, for example lower secondary teachers with teachers in middle school to help students when they change schools (Pajares & Cheong, 2003)—develop a more individual and personalised teaching environment (Pajares & Cheong, 2003)—help students manage different types of pressure, for instance
parents, school, peers (Pajares & Cheong, 2003; Samuelsson & Granström, 2007). Thus, the teacher of mathematics plays a critical role in encouraging students to sustain positive attitudes. How a teacher views mathematics and its learning affects students view about themselves as mathematics learners. When norms allow students to be comfortable in doing mathematics and sharing their ideas with others, they see themselves as capable of understanding (Boaler, 1999; Samuelsson, 2008). Further research is needed of how to develop self-regulated learning skills in mathematics in a positive way.
References


