Abstract

Researchers (Ball, 2003; Ma, 1999; Schulman, 1986) have long investigated how a teacher’s subject matter and pedagogical content knowledge impact on students’ learning of mathematics. In an attempt to account for the relationship between teacher knowledge and student learning, this study examined four Romanian first grade teachers’ knowledge about place value concepts, and the relationship between this knowledge and their classroom practice. Findings reveal a direct relationship between teachers’ content and pedagogical content knowledge and their student learning of place value concepts. As such, the deeper understanding teachers had about place value concepts, the richer learning opportunities they created for their students and the better understanding their students had of place value concepts, as revealed by student test scores.
The Knowledge Needed for Teaching Students Mathematics

A decade ago, Ball & Bass (2000) stated that the divide between subject matter and pedagogy that has characterized the mathematics teaching and learning in the 20th century, was still present. Although researchers (Ball & Bass, 2000, 2003; Ball, Lubienski, & Mewborn, 2001; Schulman, 1986) have distinguished between content knowledge and pedagogical content knowledge, the understanding of how mathematics knowledge was used in practice was still weak. Content knowledge refers to the “amount and organization of knowledge per se in the mind of the teacher,” (Schulman, 1986, p.9), and it largely influences teachers’ ability to use curriculum effectively and to adopt teaching strategies that enhance learning (Ball, 2003). On the other hand, teachers need to know why students misinterpret certain topics, what topics might pose challenges to their students, and consequently they need to identify strategies to address such challenges. This knowledge is called pedagogical content knowledge, and it refers to the knowledge of learners, learning, and pedagogy.

While pedagogical content knowledge is acquired over time, as teachers teach the same topics to different groups of students, teachers acquire content knowledge in their teacher education classes. For the most part, however, teacher education classes do not provide a solid knowledge base for prospective mathematics teachers (Ball, Hill & Bass, 2005; Zeichner & Tabachnick, 1981), leaving them with a fragmented understanding of mathematics (Ma, 1999). Moreover, as students, prospective teachers were mostly exposed to traditional images of teaching. These experiences shaped their understanding of the mathematics they would teach (Lortie, 1975).

Ball & Bass (2003) discussed the interweaving of the knowledge of content (topics, procedures) and practice (mathematical processes ad skills) into teaching. The researchers
discussed the multiple facets of knowing mathematics for teaching, which include elements of mathematics found in the students’ curriculum, the ability to select definitions that students can use, as well as the ability to make connections across mathematical ideas. In discussing the multidimensionality of the knowledge of teaching Ball, Thames & Phelps (2008) identified the following domains pertaining to teacher knowledge: common content knowledge (knowledge and skills used in setting other than teaching); specialized content knowledge (knowledge and skills unique to teaching). As an extension to Schulman’s (1986) interpretation of pedagogical content knowledge the researchers added the following two domains: knowledge of content and students (knowing about students and knowing about mathematics), and knowledge of content and teaching (knowing about teaching and knowing about mathematics).

This knowledge is illustrated in a hypothetical second grade classroom by Cohen, Raudenbush & Ball (2003). In trying to engage students in solving a subtraction problem, the teacher instructs the students to use base 10 blocks to model the numbers and processes and to justify the answer. The teacher displayed both content and pedagogical content knowledge, knowing how to subtract, and how to model subtraction to her students. Moreover, being aware of the correspondence between numbers and their representation in base 10 blocks, the teacher enabled the students to discuss their work.

Ball et al., (2005) analyzed the relationship between teachers’ mathematical knowledge for teaching and student achievement. The mathematics knowledge for teaching was defined as teachers’ ability to perform certain computations, as well as their ability to explain and examine students’ work, analyzing their errors, and explaining in ways that students can understand. The researchers surveyed 700 first and third grade teachers on 250 items measuring their common mathematics knowledge (knowledge of mathematics that any adult should possess), and
specialized mathematics knowledge (knowledge that only the teachers need to know). Results indicate a close link between teacher knowledge and student learning, a significant finding being that the size of the effect of teachers’ knowledge for teaching was measureable to the size of the effect of SES on student gain scores. This finding indicates that teachers who possess the knowledge of teaching could stop the achievement gap between economically disadvantaged students. These teachers can be assisted to acquire this knowledge through professional development (Hill & Ball, 2004).

**Teacher Knowledge of Place Value Concepts**

Place value is a fundamental mathematical concept in the students’ mathematics learning, and it has long captured the attention of researchers. Deemed as “one of the most important arithmetic concepts to be learned by children in the early elementary grades,” (Sharma, 1993, p.3), place value refers to children’s flexibility with numbers and their ability to perform mental mathematics (Gersten & Chard, 2001). Kamii (2004, p.13) further explained place value as “the social conventional knowledge that in ‘333’ for example, the first 3 means three hundred (or 3 hundreds), the second 3 means thirty (or 3 tens), and the third 3 means three (or 3 ones).”

Despite the fact that students are introduced to place value concepts in the first grade, and then re-introduced in every grade of the elementary school, they face challenges when learning about numbers (Ball et al., 2001). Some of these difficulties are attributed to the written and spoken form of the English number system (Fuson & Briars, 1990). Other researchers attribute these difficulties to the inadequate teaching methodologies and materials used to teach numbers (Sharma, 1993). Since place value represents a secondary concept (Skemp, 1989), students need to acquire certain prerequisite skills, such as natural numbers, order, counting, unit object, sets of objects. Although a few researchers (Fuson, Grandau & Sygiyama, 2001; Thompson, 2000) have
investigated ways teachers approach place value concepts, and despite the numerous internet
tutorials existent for teaching children place value, the literature is scarce in discussing the
knowledge teachers need to possess to teach place value concepts. The review of literature below
briefly discusses these studies.

Thompson (2000) discussed the two stages in which mathematics teachers from Europe
teach place value concepts. In the first stage, children learn addition and subtraction up to 20, while in the second stage, students become familiarized with numbers 20-100. When students
learn place value concepts, they identify tens and units, groups of numbers, they construct
numbers using base 10-equipment, and represent numbers on an abacus. Thompson’s conclusion
is that students are taught place value concepts too early, as they do not need to use these for
mental or informal written calculations until year 4. This early introduction may lead to an
impoverished mental representation of two-digit numbers, which may explain the difficulties
students have in learning place value concepts (Ball et al., 2001). On the other hand, Fuson et al.
(2001) discussed the potential children of ages 3-7 have in learning about numbers, identifying
the need for teachers to build on the ideas students have about numbers when they enter school.

Cotter (2000) investigated the teaching of numbers in two first grade U.S. classrooms: a
control class, where the teacher based lessons on a traditional textbook, and an experimental
class, where the teacher developed alternative lesson materials. During the first 3 months in the
experimental class the teacher used the “Asian” method of teaching numbers (i.e. for 13, the
children were taught to say 1 ten 3). The students learned the finger combinations to ten, they
learned to enter quantities from one to ten on the abacus without counting, and they used base-10
cards for adding with trading. The teacher introduced the traditional number names only mid-
semester. Results show that at the end of the school year, the students in the experimental class
were more advanced than the students in the control class. For example, when given the number “3924”, 44% of the students in the experimental class, but only 7% of the students in the control class could circle the tens place. According to Cotter (2000) “teaching children to count initially with consistent counting words follows good teaching practice. Introducing the exceptions should occur only after the students understand the general rule” (p. 114). If students can understand patterns in the number system, they will search for more patterns, and this can lead to abstract thought.

Fuson, Smith and Lo Cicero (1997) discussed the 2-digit number acquisition in two low SES first grade classrooms. The researchers discussed the 3 conceptions demonstrated by US children in learning numbers: a unitary conception (counting a two digit number by ones), a sequence-tens conception (counting by tens and then by ones), and a separate tens-and-ones conception (the units of tens and the units of ones are counted separately). The researchers built activities that helped children construct the triads simultaneously. Children used ten-sticks and dots to record quantities, as opposed to base 10 blocks. Results show that students showed a preference for ten structured conceptions, being able to carry out a ten structured solution to two-digit addition and subtraction problems and to explain their regrouping, which is above what first grade USA students usually have an opportunity to reach. The implications of this study indicate that the achievement gap between Asian students and U.S. students (even disadvantaged students) may be narrowed: “doing so requires a more ambitious first grade curriculum and active teaching that supports the children’s construction of a web of multi unit conceptions in which number words are related to ten-structured quantities” (Fuson et. al, 1997, p.766).

The focus of this paper is to present the results from a study that attempted to shed more light into teachers’ knowledge of place value concepts and the way this knowledge played into
their classroom instruction.

**Methodology**

**Participants and Context**

The participants of this study were four first grade Romanian teachers and their students in a large urban school district in South East Romania, referred to as Tomis. Two general schools participated in this study, Iorga and Delavrancea (all names included in this study, of both schools and participants are pseudonyms). According to the students’ results in regional and national mathematics competitions, Iorga was identified a well performing school, while Delavrancea was identified an average performing school.

Founded in 1879 and functioning ever since, Iorga employed seventy-five licensed teachers to serve its 1,356 enrolled students. Delavrancea was founded in 1934 and it employed twenty-eight licensed teachers to teach the 350 students attending this school. Two elementary teachers from each school were selected, one veteran and one less experienced. Ms. Reiz and Ms. Ali were the less experienced teachers, and Ms. Ionescu and Ms. Popescu were the veteran teachers. Sixty-four students, of ages between 6 and a half and 7 and a half years old participated in this study: thirty-six students at Iorga (twenty-four in Ms. Ionescu’s class and twelve in Ms. Reiz’s class) and twenty-eight students at Delavrancea (eighteen in Ms. Popescu’s class and ten in Ms. Reiz’s class).

**Data Sources**

The researcher collected the data from teacher interviews, classroom observations and student tests, in an attempt to answer the following research questions: (1) What knowledge of place value concepts do the four Romanian teachers possess? (2) How does this knowledge

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1 In Romania, general schools house both elementary and middle school students.
transfer in their classroom practice? (3) Does teacher knowledge of content and pedagogy impact student learning?

1. The four teachers were each interviewed on two occasions, before and after teaching numbers 10-100. The seventeen pre-interview questions gathered information about teachers’ knowledge of numbers 10-100, the objectives they developed for the lesson and the strategies they planned to use to reach these objectives. The eleven post interview questions discussed whether or not the teachers reached their objectives.

2. Each teacher was observed twice while they taught numbers 10-100. The observations had been previously scheduled with the teacher, and they both lasted for 50 minutes (the entire duration of the mathematics class). The researcher collected the artifacts used during instruction (i.e. worksheets, games that were not included in the classroom textbook).

3. Each teacher administered a teacher-made test at the end of the unit, a limitation of this study being the fact that the teachers developed and graded the tests individually. The researcher compared the student scores in this final test, as this was the only consistent formal evaluation across the four classrooms. However, to reduce this limitation, the researcher considered only perfect tests for the grade of A.

Data Analysis

The researcher coded the data from the teacher interviews as strong and weak content knowledge, using the definition the teachers provided for base 10 numbers and the significance they placed on learning about these numbers. Teachers with a strong content knowledge (such as Ms. Reiz) defined base 10 numbers, as “single and double digits, from 10 up to an infinity all of them are formed with the help of these numbers from 1-9”, were also able to explain their

2 A comprehensive list of the interview questions is included in Appendix 1
significance for student mathematics learning, linking place value concepts to other mathematics concepts. For example, Ms Ionescu stated that “these concepts are important for their use in real life, as we don’t learn to stay at an abstract level but to apply what we learn,” while Ms. Popescu stated that “students need to learn these numbers because they will need them further for addition and subtraction and later on for multiplication and division”. On the other hand, Ms. Ali stated “learning about numbers was significant because mathematics itself was significant.”

Secondly, the researcher coded the types of exercises students were asked to solve in the test as higher order and lower order thinking exercises, according to the classification found in three U. S. First Grade mathematics: Everyday Math (2004), Investigations (2004), and Math Advantage (1998). According to these textbooks, the lower order exercises asked students to perform basic computations.

In this study, the lower order exercises in the tests asked students to find the tens and units of given numbers (13, 30, 32, 48); to write the numbers when given the tens and units (8 tens and 3 units, etc); to compare given numbers (ex. 12-16, 27-29); to compose and decompose numbers (46, 57, 16); to write numbers (count from 46-51); to order forward and backwards certain numbers (95, 8, 43, 17, 62); to find neighbors of given numbers (29-31; 69-70; 33; 57; 16, 10, etc).

Higher order thinking problems asked students to perform complex computations, such as to write all numbers between 30-100 where the units are equal to the tens; to find X, if X is higher than 10 and lower than 18; to discover the rule and continue the counting: 66, 67, …, …; 93, 92,

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3 See Appendix 2 for examples of exercises in these textbooks
... , 42, 44, ..., 80, 70, ..., ; to write all numbers made of two digit where the sum of the digits equals 10.\(^4\)

**Findings and Discussions**

The researcher conducted this study with the aim to learn more about the Romanian teachers’ content knowledge of place value concepts, how this knowledge transferred in their classroom instruction, and ultimately, whether or not teachers’ knowledge of place value concepts could be linked to student learning of place value concepts. The following sections discuss the findings in the light of the above research questions.

**Teachers’ Knowledge of Place Value Concepts**

Teachers’ knowledge of content is evident not only in teachers’ understanding of the facts and principles they teach, but also in their understanding of the organization of these concepts, the rules that govern them, as well as the degree of significance of each of these concepts (Ball et al, 2008; Schulman, 1986). The researcher interviewed teachers prior to instruction, asking them to define place value concepts and explain their significance for student learning. Moreover, the teachers discussed the objectives they set for the lesson.

The interview findings revealed that all four teachers had a good understanding of place value concepts. The veteran teachers defined place value concepts as “a group of numbers comprised between 10-100” (Ms Ionescu), “the numbers formed by adding a unit to each number, obtaining thus the next number” (Ms. Popescu). The less experienced teacher, Ms. Reiz, defined numbers as “both single and double digits from 10 up to an infinity all of them are formed with the help of the numbers from 1-9,” while Ms. Ali stated that “students learning

\(^4\) See Appendix 3 for the exercises contained in all four tests
about single digits with numbers 0-9, and now the passage from 10-100 is the formation of numbers from tens and units.”

Understanding the “what” is only a part of content knowledge, however, as the teachers also need to discern why some topics are pivotal to mathematics concepts (Schulman, 1986). Only three of the teachers could link students’ acquisition of place value concepts to subsequent mathematics concepts. The veteran teacher at Iorga, Ms. Ionescu stated that these concepts were important “for their use in real life, as we don’t learn to stay at an abstract level but to apply what we learn,” while Ms. Popescu, the veteran teacher at Delavrancea, stated that “students need to learn these numbers because they will need them further for addition and subtraction and later on for multiplication and division.” Ms. Popecu’s less experienced colleague, Ms. Reiz saw learning about these concepts important as they further related to other mathematics concepts. On the other hand, the less experienced teacher at Iorga, Ms. Ali, did not link the learning of base 10 numbers with concepts that were going to be studied further on, as she stated that “learning about numbers was significant because mathematics itself was significant.”

Discrepancies in teacher knowledge of the content are also evident in the lesson objectives. While all teachers stated the school curriculum dictated the objectives they set for the lessons, they had flexibility to exceed these standards. This is the case of the two teachers at Iorga, who used the same mathematics textbook. However, the veteran teacher, Ms. Ionescu, stated that she followed the curriculum requirements, but she supplemented the textbook with many other activities, as she wanted her students to:

Understand that these numbers correspond to a group of elements that have as many elements as these elements, to read, write, compare, order and eventually for some kids who have a more developed intellectual ability and are better thinkers than others, to
discover numbers from tens and units respecting certain requirements: given the ten and unit, the sum of numbers is a certain number.

Ms. Ionescu’s statement shows her preoccupation with the different learning styles of her students and the desire to address these differences in learning by providing her students with a challenge appropriate for their individual rhythm of development, which is key to learning (Greenes, 1995).

Ms. Ionescu’s less experienced colleague, Ms. Ali, stated that she closely followed the curriculum while developing her objectives, Ms. Ali wanted her students to “understand ten concepts, the formation of base 10 numbers, 10-20, 11, 12, etc., decomposition, ordering…to know how to represent the tens, to order backwards and forwards, even and uneven numbers.” Although Ms. Ali stated in her interview that some of her students were strong independent learners, she did not create additional challenges for these students in her lesson.

Both teachers at Delavrancea had objectives dictated by the curriculum and they followed these objectives entirely, exposing their students to simple computations, as they wanted their students to master these concepts before moving on to more complex notions. For example, the veteran teacher, Ms. Popescu wanted her students “to know how to count backwards and forwards, composition and decomposition of numbers, ordering and comparing numbers.” Similarly, Ms. Reiz focused more on less significant aspects, like number pronunciation for example: “to know how to count correctly from 10-100 and especially to pronounce correctly the numbers 10-20. The second objective is their formation, composition and decomposition.”

Teacher content knowledge is also reflected in the ways teachers used what they had learned from their past experience teaching the base 10 numbers lesson. At Iorga, Ms. Ionescu stated that her past experience taught her the importance of treating students differently, and she
implemented differentiate worksheets, “developmental sheets for some and improvement and catching-up sheets for others.” The teacher also diversified her use of manipulatives in class, adding to the usual abacus, sticks and slide rules, more hands-on objects (cards with numbers, logical game with group-diagrams, charts with groups of elements, and chips with numbers) to engage her students in learning numbers hands-on:

I learn new things from one series of students to the other, so I can always improve my activities and I can say I learn from experience. One can better adjust the manipulatives to a new series of students, I would say that knowing this series I managed to select the best manipulatives and activities for them.

Ms. Ali stated in her initial interview that what she had learned from teaching the same lesson in the past was mostly the base 10 concept. In her present teaching, the teacher stated that she made a point of planning the lesson and teaching with her current students, as she believed this would help them understand better base 10 numbers. The teacher stated she used the same manipulatives from one series to the next (i.e. the slide rules and sticks to count numbers and form groups of tens and units).

At Delavrancea, Ms. Popescu stated that she had learned that her students mastered counting when they could count up to 10 and skip count by 10 to 100. The teacher indentified as a possible challenge going over the threshold and to avoid this challenge, the teacher spent a lot of time on numbers 0-9 to make sure her students had a solid understanding of these numbers. Ms Popescu also created opportunities to learn by playing, yet she continued to use the same manipulatives she had used in the past: sticks, abacus, the shapes, as well as the big axis with numbers from their textbook. The teacher believed that these concrete materials would provide
her students with the support needed to understand more complex notions. However, Ms. Popescu did not address any complex notions in her lessons.

Ms. Reiz, stated that in planning the present lesson, she took into account the mistakes or misunderstandings she discovered at her older students. This made her think of better ways to explain number formation and writing to her current students, such as creating opportunities for her students to be engaged in learning through discovery. The teacher increased her use of manipulatives, adding more concrete materials, like sticks, pens, groups of elements as she taught the lesson with the goal to have students understand that ten units form a ten.

The teachers’ flexibility to learn from the past experiences, their ability to use this knowledge in setting complex objectives, and designing a better lesson for their present students, were factors that reflected the teachers’ content knowledge. Knowledge of content is essential to student learning, as its presence or absence affects the learning opportunities the teachers provide for their students (Cohen at al., 2003). Effective teachers need not only possess this knowledge, they need to be able to represent this knowledge in class in ways that make it comprehensible to their students (Ball et al., 2005; Schulman, 1986).

**Teacher Pedagogical Content Knowledge**

In the best learning environment, teachers’ classroom practice is informed by the teachers’ content knowledge. Pedagogical content knowledge, or teachers’ ability to convey the information in ways that are accessible to students, represents a mixture of both knowledge of content and pedagogy (Ball et al., 2008; Schulman, 1986). In order to account for teachers’ pedagogical content knowledge, the researcher observed the teachers on two separate occasions, taking extensive field notes and collecting the artifacts used by the teachers during their instruction (games, worksheets). The observations revealed that the teachers possessed different
degrees of pedagogical content knowledge, as evidenced by the activities teachers planned, the
manipulatives they used, the way they made use of the textbook, and ways they reinforced the
more challenging topics.

In analyzing the classroom activities implemented by the teachers, the researcher found
that at Iorga, Ms. Ionescu assessed student understanding in different ways. For example, her
students filled out independent sheets at the end of each class, which provided the teacher
with feedback for the lesson, enabling her to spend more time with the students who needed her
help. The teacher also played games with her students, she had them recognize the numbers
written on cards and raise the cards with the appropriate number, played logical games with
group diagrams, used charts with groups on which students could group the elements by tens.

Moreover, Ms. Ionescu engaged her students in discovery, having them working in small
groups trying to solve problems, figuring out the rules by themselves. Students were engaged in
numerous student-student interactions (at least five per observation on average), and engaged in
small groups 23% of the instructional time (average time per observation). As a facilitator, the
teacher monitored group/pair work and provided support and explanations when necessary,
asked students for multiple solutions for the same problems and guided learning by discovery.

In order to supplement the classroom textbook, the teacher introduced fifty-three
additional exercises of both lower and higher order thinking, and she engaged all her students in
solving these exercises. In addition, the teacher challenged her stronger students with exercises
that required them to discover certain rules to continue counting or discover two-digit numbers
respecting certain requirements. The teacher spent time to make sure her students mastered the
easier concepts before increasing the difficulty level (Ma, 1999), and she reinforced all types of
exercises introduced in class with homework assignments and she probed her student
understanding of the same topics in the final evaluation. Overall Ms. Ionescu’s students seemed to have benefited from her choice of instructional strategies, as reflected by the high percentage of students who scored A in the final tests.

Ms. Ali, the less experienced teacher, assessed student understanding both in writing (through worksheets) and orally, asking students to take turns going to the board to solve problems. Ms. Ali’s students formed numbers using sticks and slide rules, they composed and decomposed numbers. Ms. Ali created more limited opportunities for student-student interactions, although she had previously stated that this interaction seemed to benefit her weaker students, and she drilled her students more in front of the classroom. Observation notes reveal that all additional extra-curriculum materials used by the teacher (a total of twenty-five exercises) were of reduced difficulty level as all the exercises solved from the classroom textbook.

At Delavrancea, Ms. Popescu also implemented multiple activities to assess student understanding but all these activities were based on the classroom textbook, as the teacher used no extra-curricular materials. Moreover, not all the concepts covered in class was reinforced at home, or assigned in the final evaluation. All the classroom activities offered a minimum degree of challenge, having students counting and showing the place of tens and units with the use of sticks, shapes, and the axis with numbers. Student assessment was both formal (through worksheets and the final test) and informal (asking students to come to the board and write numbers after dictation). Ms. Popescu also created a few opportunities for her students to learn through games, by asking a lot of questions and encouraging her students to help one another. In small groups, students had to find the neighbors to certain given numbers and skip count backwards and forwards. Ms. Popescu’s students understood base 10 numbers, yet they
performed at the same level with Ms. Ionescu’s students, who were assessed on more difficult concepts.

Ms. Reiz stated in the pre-interview that her students would learn through answering her questions, coming to the board and learning by discovery. The teacher provided ample opportunities for her students to work individually at the board and on workbooks, and asked them questions to check their understanding, but she did not engage her students in learning by doing. Students counted tens and units using different class objects, sticks and the abacus, and they used groups of elements for numbers 10-100, but they did not experience any type of hands-on activities, games or group/pair work. The teacher walked among her students checking their answers, and asking students to go to the board. When the student at the board made a mistake, Ms. Reiz called on another student to answer. Despite the fact that the teacher introduced seventeen new exercises in class, only one exercise was of a more complex difficulty level, requiring students to discover the rule and continue the counting. Moreover, some of the concepts addressed in class were not reinforced with homework, yet students were tested on these concepts, which may have lead to the weaker understanding of Ms. Reiz’s students as indicated by her student test scores.

Overall findings indicate that when teachers supplemented the textbook with more challenging topics, their students’ learning improved (Ms. Ionescu). The teacher as a facilitator seemed to have a better impact on student learning than the teacher as information giver. When teachers (Ms. Ionescu, Ms. Popescu) asked for multiple solutions to the problems, raised questions, provided explanations, student learning of place value concepts increased.

**Teacher Knowledge and Student Learning**
What teachers know and how they teach influences how much their students learn (Ball & Bass, 2000). If this is the case, the Romanian teachers’ knowledge of place value concepts and their classroom practice can be linked to their students’ learning. In an attempt to answer the third research question, namely whether or not teacher knowledge influences how much students learn, the researcher analyzed student test scores. Within school comparisons reveal differences in student test scores: at Iorga, 65% of all Ms. Ionescu’s students and only 50% of all Ms. Ali’s students scored A in the final test. Noteworthy is the fact that Ms. Ionescu’s test checked student understanding of both easier and more complex topics, while Ms. Ali tested her students only on less complex notions. Differences also appear in students’ test scores at Delavrancea, where 65% of students in Ms. Popescu’s class scored A in the final test, as opposed to 30% students in Ms. Reiz’s class.

Across-school comparisons show similarities in veteran teacher understanding of place value concepts as well as their student results: if teachers possessed both a procedural and conceptual understanding of the place value concepts, they provided richer learning opportunities for their students (Ma, 1999). On the other hand, if teachers possessed the procedural understanding but lacked the conceptual understanding of place value concepts (Ball 1990), their students were exposed to a limited understanding of the concepts, as is the case of the students in Ms. Ali’s and Ms. Reiz’s classes.

Conclusions

This study is warranting three conclusions, which support previous research findings: firstly, teachers’ knowledge of content is an important factor in student learning, influencing the way teachers plan and implement instruction (Ma, 1999). Most importantly, content knowledge
goes beyond teachers’ mastery of the concepts and facts of a subject, as the teachers need to understand the rules and principles on which particular concepts are grounded (Schulman, 1986).

As reflected by the above findings, despite the fact that all teachers provided a good definition of place value concepts, not all of them were able to link place value concepts to further mathematical concepts (Ms. Ali). Moreover, teachers set different objectives for their students, and some teachers challenged their students more (Ms. Ionescu), while others planned for a uniform instruction (Ms. Ali, Ms. Popescu). Findings show that when teachers set more complex objectives for their class, and engaged their students in both higher order and lower order thinking problems, student knowledge increased. The fact that teachers’ content knowledge goes beyond their familiarity with the facts they teach (Ball & Bass, 2003; Grossman, Wilson & Schulman, 1989) is reflected in teachers’ use of the curriculum. Even if Ms. Ionescu and Ms. Ali used the same curriculum, because Ms. Ionescu used curriculum as a starting point, exceeding its requirements, and challenging their students with additional problems, her students’ understanding of base 10 numbers increased.

The second conclusion of the study is that while necessary, teacher content knowledge represents only one part of the knowledge needed for teaching (Ball et al., 2008). Effective teachers need to be able to implement the content in ways that make sense to their students (Ball et al., 2005; Ball et al., 2001), “they need to adjust instruction within and among students on the basis of their estimates of students’ capabilities,” (Cohen et. al, 2003), making use of different resources. In the current study, teachers (Ms. Ionescu) who displayed a strong knowledge of their students, using their past experience with teaching that lesson to design activities that addressed their students’ individual needs, had students who performed well in the final test. On the other hand, if teachers (Ms. Ali, Ms. Popescu) acknowledged the fact that they had strong learners and
students who struggled but maintained a lower standard of instruction, they did a disservice to their students.

Thirdly, even if teachers have a strong knowledge of content, students, and teaching (Ball et al., 2008), students may still perform at or below average levels in a final test. In Ms Ionescu class 35% of their students still performed at or below average levels in the final tests. This finding indicates the fact that although significant, teacher knowledge is not the only factor influencing student learning. External factors, such as parental teaching strategies, students’ socio economic status, as well as the quantity and quality of home-school collaboration may be equally responsible for student mathematics learning, (Cai, 2000; Huntsinger, Jose, Larson, Krieg, & Shaligram, 2000; LeFevre, Polyzois, Skwarchuk, Fast, & Sowinski, 2010; Pan, Gauvain, Liu, & Cheng, 2006). In order to have a deeper understanding of student learning, future research studies need to look at how schooling and non-schooling factors interact and influence student mathematics learning.

**Implications**

This study raises the issue of the multidimensionality of teacher knowledge, trying to explain the achievement gap from the perspective of what teachers know about place value concepts and how they teach these concepts. The following implications emerge, for both teachers and teacher educators.

Prospective teachers learn their content knowledge in their teacher education classes, and for the most part they leave the teacher education program with a fragmented knowledge (Ball et al., 2005; Ma, 1999). In order to increase this content knowledge, teacher educators need to “know more, and different mathematics” (Ball et al., 2008, p. 396), because “teachers cannot teach what they have not learned (National Advisory Mathematics Panel, 2008). Therefore
mathematics teacher educators need to identify the big mathematics ideas in their courses and teach them to their students (Graeber, 1999; Greenes, 1995), enabling their students to possess both conceptual and procedural knowledge (Ma, 1999).

On the other hand, if teachers have left their teacher education programs with a more fragmented mathematics knowledge, they need to be helped to enhance their knowledge of content and pedagogy as they engage in professional development opportunities. Teachers can benefit from professional development (Heaton, 2000; Sowder et al., 1998), that engages them in activities that connect the teaching of mathematics with the content of mathematics (i.e. analysis and discussions of video tapes of mathematics lessons, etc.). If the ultimate goal of education is to improve student learning, teacher education programs have the responsibility to prepare teachers who know what and how to teach in ways that reach all their students, who learn with and from their students. It all starts with the teacher.

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**Author Biography**

Madalina Tanase received a PhD in Teacher Education at the University of Nevada, Las Vegas. She is currently an Assistant Professor in the Department of Foundations and Secondary education at the University of North Florida, where she teaches undergraduates methods courses (i.e. teaching strategies, assessment and classroom management). Dr. Tanase’s research areas include the impact of national language on mathematics learning, teacher epistemological beliefs and their impact on teachers’ classroom practice, as well as comparative and international approaches to teaching and learning mathematics.
APPENDIX 1: Pre-teaching Interview Questions

1. Could you briefly define place value?

2. Did you teach this lesson before? How many times did you teach it?

3. What resources did you use for teaching this lesson before?

4. What did you learn from teaching this lesson in the past about the content of this lesson that helped you prepare the present lesson?

5. What are the common misunderstandings that your former students had about place value? How did you learn about these? How are you going to cope with such situations?

6. What are your objectives for this lesson on place value? How did you come up with these objectives? Why do you think these objectives are necessary?

7. Could you briefly describe how you are going to teach this lesson on place value? What examples are you going to use to teach your students and why?

8. What materials, including textbook, did you use to plan this lesson?

9. How much time did you spend preparing for the lesson you are going to teach today?

10. What will your students be doing during this lesson? Why?

11. Did you discuss the lesson with anyone in the school and what did you talk about?

12. Why do you think it is important for the students to develop an understanding of the place value concepts?

13. What would you say students need to be able to understand or be able to do before they could start learning about place value/number naming systems and why?

14. What do you anticipate will be the most difficult concepts your students will struggle with and why do you think this will be the case?

15. How will you approach these difficult concepts? Why?
16. How will you be assessing your students’ understanding of place value and why do you think these assessments are useful with this particular lesson?

17. What concrete materials do you mostly use to teach these concepts and why?

Post-Teaching Interview Questions

1. Can you tell me three important things that you learned about teaching this lesson on place value and how did you learn these?

2. What major problems, if any, did you face while teaching this lesson?

3. How did your students count or estimate quantities? Did they spontaneously use sets of tens? What is your evidence for that?

4. How flexible were children with their thinking about numbers? Could they take them apart and combine them in ways that reflect an understanding of ones and tens? What is your evidence for that?

5. What materials did you use to represent one and tens in your classroom instruction?

6. When materials were already arranged in groups of tens, did students use these structures to tell how many? What is your evidence for that?

7. How does understanding of place value help students develop skills in reading and writing numbers?

8. How did you help your students discover the relationship between tens and ones? What is your evidence for that?

9. To what extent do you think your students have reached the goals and objectives that you set up for this lesson?
10. What did you think about the lesson procedures that you developed in this lesson? To what extent did you think the major procedures that you used in your teaching were useful for your student learning in this lesson?

11. If you are going to teach this lesson again, are you going to use the same examples and/or assessment that you used in this lesson and why and why not?
## Appendix 2: Mathematics Exercises Found in U.S. Textbooks

<table>
<thead>
<tr>
<th>Types of Exercises</th>
<th>Lower order thinking problems (called “Practice Items” or “Basic Computations”)</th>
<th>Higher order thinking problems (called “Problem of the Day” or Challenging Problems”)</th>
</tr>
</thead>
</table>
| **T/U** (tens and units) | 1. Given the numbers 17, 18, 19, 20, 14, write in 2 columns how many tens and units you have (Math Advantage, 1998, p.205 A).  
2. Students work with partners in an activity called “Copying cubes.” Each builds something with 10-15 interlocking cubes. They exchange and try to make a copy of each other’s objects. When both students have finished, they check that the copies are identical. (Investigations, 2004, p.14)  
3. Given the tens and units, guess what number am I: 3 Us and 2 Ts; 5 Us and 6 Ts; 2 Ts and 7 Us; 4 Us and 6 Ts (Everyday Mathematics, 2004, p.336). | 1. Write problems involving numbers 10-20. Then exchange your problem with your partner’s and solve each other’s problems (Math Advantage, 1998, p.206 A).  
2. Given the tens and units, what number am I: 3 Ts and 19 Us (Everyday Mathematics, 2004, p.338). |
| **Counting** | 8..., 9, 10, 11, ..., 15, 16, 17, ..., 19, 20 (Math Advantage, 1998, p.204).  
2. Given the 100 numbers chart, find the following numbers on the chart: 10, 20, 50, 15, 63 (Investigations, 2004, p.83).  
4. Write one more: 36, ..., 45, ..., 61, ..., 83, ... (Everyday Mathematics, 2004, p.337). | 1. Given the following groups of 10 elements, color groups of tens to show the numbers: 30, 60, 50, 20 (Math Advantage, 1998, p.204).  
4. Continue the sequence, counting by 2: 17, ..., 21, ..., 25, ..., 27.  
   Same for 5: 20, ..., 50.  
10, 20, ... 40  
60, 65, ..., 75  
70, 80, ... 100 (Math Advantage, 1998, p.237).  
| **CP (comparison)** | 1. Compare numbers 11 and 19, circle the greater and explain why it is greater (Math Advantage, 1998, p.221 A).  
2. Which number is greater, 14 or 41, 47 or 57, 43 or 34, 86 or 68 (Math Advantage, 1998, p. 227)?  
2. Playing the double compare dots game, players determine the total number of dots on both cards and player with the higher number of dots says “me” (Investigations, 2004, p.9).  
3. Fill in the numbers to make these correct: a number lower than 46; a number lower than 155 (Everyday Mathematics, 2004, p.363). |
| **N (neighbors)** | 1. Given the following numbers, find the numbers before and after them: 47, 98, 32, 20, 26, 61, 84, 90 (Math Advantage, 1998, p.225). | 1. Guess the number: it is between 70 and 90, it is greater than 80, it has 8 ones (Math Advantage, 1998, p.225). |
2. Which number comes before 72, which number comes after 72, which number comes between 75 and 77, and which number comes between 78 and 79 (Math Advantage, 1998, p.226)?
3. On the 100 number chart, what is the number coming after 18, before 10, between 14-16, 85-87 (Investigations, 2004, p.84).
4. Find all numbers that come before and after the following numbers: 14; 49; 71; 88 (Everyday Mathematics, 2004, p.381).

C/D (Composition/Decomposition)

1. Suppose I have 12 pets. How many cats and how many dogs can I have (Investigations, 2004, p.38).

CB/CF (counting backwards/forwards)

1. Order the following numbers from lowest to greatest: 86, 17, 21, 5, 43 (Math Advantage, 1998, p.228).
2. What is wrong with the following counting sequences?
   a. 9, 10, 11, 12, 13, 14, 16, 17, 18, 19?
   b. 9, 10, 11, 12, 13, 41, 15, 16, 17, 18, 19?

2. On the 100 number chart, remove consecutive numbers and ask students to identify numbers in the middle of a set of empty spaces (Investigations, 2004, p. 84).

1. Make your own problem in which you have two different kinds of things (Investigations, 2004, p. 40).
1. Write numbers 1-8. Cross out the 2 numbers that come before 3. Cross out the number that comes before 7. Cross out the numbers between 3-7. Cross out the number that is less than 5. What number is left (Math Advantage, 1998, p.227)?
## Appendix 3: Sample Exercises Found in Tests in the Four Classes

<table>
<thead>
<tr>
<th>Teachers</th>
<th>Low Order</th>
<th>LO</th>
<th>LO</th>
<th>LO</th>
<th>LO</th>
<th>High Order</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ms. Ionescu</strong></td>
<td>T/U</td>
<td>C</td>
<td>CP</td>
<td>C/D</td>
<td>C backwards/forwards</td>
<td>N/A</td>
</tr>
<tr>
<td></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Order forwards (95, 8, 43, 17, 62) and backwards (70, 89, 63, 85, 72).</td>
<td>Find the neighbors of the numbers: 29-31; 69-70; 33; 57.</td>
<td>Discover the rule and count: 57, 59…; 31, 34, 37…; 95, 90, 85…</td>
</tr>
<tr>
<td><strong>Ms. Ali</strong></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Order forwards /backwards: 19, 15, 18, 13, 12, 10, 17.</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Ms. Popescu</strong></td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
<td>Decompose the numbers: 26, 29, 15, 13.</td>
<td>N/A</td>
</tr>
<tr>
<td><strong>Ms. Reiz</strong></td>
<td>Write numbers 45-51.</td>
<td>Compare the numbers: 7-75; 27-29; 56-46; 20-50.</td>
<td>N/A</td>
<td>Order forwards: 78, 37, 19, 90, 28, 34, 85, 43.</td>
<td>N/A</td>
<td>Write the neighbors: 46-48; 38-40; 50-52; 47-49; 74-72.</td>
</tr>
</tbody>
</table>