Imagining mathematics teaching via scripting tasks

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We discuss an innovative task used in teacher education – Lesson Play – that involves presenting a lesson in the form of an interaction between a teacher and students. We describe the motivation for the development of this task and, through specific examples, describe the iterative design process in which the task was refined and improved. We demonstrate how the task, initially designed considering mathematics, can be adapted and extended for different content areas.

Keywords: Lesson play, imagining teaching, iterative design, scripting tasks

As many of our colleagues, as researchers and teacher educators, we struggle with the question of how to better prepare teachers to teach mathematics in ways that support students’ conceptual understandings and achievement. We constantly seek out and refine experiences for prospective teachers that help them envision and practice ‘reform’ teaching strategies. In this paper, we discuss one of these experiences, which we refer to as Lesson Play.

Lesson Play

Lesson Play is a specific innovative task used in teacher education. The task was designed primarily for prospective teachers enrolled in ‘methods’ courses that focus on the teaching and learning of mathematics, and that are part of a teacher
certification program. A Lesson Play is an imagined dialogue between a teacher and students during a lesson (or part of a lesson). The task involves writing such a dialogue based on a given prompt.

As designers and users of the Lesson Play task for the past seven years, we have modified the task over time and designed alternate versions of it in order to better achieve our specific goals. Broadly, these goals are to provide prospective teachers with opportunities to imaginatively engage in mathematical discussions that will help promote student understanding. More particularly, we would like them to experiment with and practice pedagogical and mathematical moves that focus on students’ mathematical reasoning and communication.

Our research is situated within the extensive work that has been done on the role and usage of task in the context of teacher education (Mason, Watson and Zaslavsky, 2007; Zaslavsky & Sullivan, 2011). It contributes to the research literature by offering an innovative task that has been developed through iterative design and that can be used both by researchers and teacher educators.

We begin by introducing the notion of Lesson Play and the motivations for developing it. We then describe the design process that we undertook to refine the lesson play task in a variety of mathematical contexts, as well as the strategies for the task implementation. We conclude with suggestions for this task implementation in other STEM areas.

**Why Lesson Play**

The idea of a Lesson Play was developed out of personal frustration and our search of ideas that can be implemented within the structure of methods courses. We were dissatisfied with the ‘lesson plans’ written by our students, which, in many cases, had
all the desired features of a ‘good’ plan—starting with declaring objectives or learning outcomes, following with outlining activities for learners and ending with assessment. However, the format of the lesson plan did not provide an opportunity to think about some important aspects of teaching, such as the nature of interactions with students, support for students’ conceptual understanding, and the responses to students’ difficulties or mistakes.

Further, in our classroom discussions with prospective teachers, occasionally we asked them to explain how they would respond to a student claim or mistake. We found that they did so rather fluently by describing hypothetical moves such as “I would tell him...”, “I could show her...” or “I would ask them to...”. That is, they had no problem outlining their potential reaction. However, when they were invited to imagine a student and engage in role-playing—that is, actually interact with the student in the ways a potential approach was described—the task became considerably more challenging. The necessity of actually choosing the words that would be used to interact with a student became a real impediment—even in the absence of any real student.

This dissatisfaction occasioned an experience of “disturbance which starts things off” (Mason, 2002, p.10). We began to seek an approach that could help prospective teachers imagine “real teaching” before actually engaging in it, rather than simply discussing it. This goal echoes Watson and Mason’s (2007) epistemological standpoint that “the fundamental issue in working with teachers is to resonate with their experience so that they can imagine [our italics] themselves ‘doing something’ in their own situation” (p. 208). This approach is consistent with the participationist perspective that conceptualizes learning as becoming a participant in a certain activity (Lave, 1988; Lave & Wenger, 1991), rather than the acquisition of
knowledge. Writing a Lesson Play is a step in preparing for teaching, a step toward participatory activity, where the role-playing is carried out in one’s imagination.

**Pedagogy-Related mathematical tasks**

Several researchers have advanced the notion that teaching is a form of applied mathematics (Bass, 2005; Chick & Stacey, in press; Stylianides & Stylianides, 2010). In particular, Chick and Stacey (2013) claim that “teachers of mathematics act as applied mathematicians in applying mathematical knowledge to the resolution of teaching problems”. Similarly to applications of mathematics in fields such as finance or engineering, the problem arises from the field of application, the solution requires simultaneous application of knowledge from the field of knowledge and from the field of application, and the effectiveness of the solution is considered within the field of application.

For teachers of mathematics, the field of application is the teaching and learning of mathematics. The problem introduced by a prompt in a lesson play task is a student error (which might arise from a mistaken inference or a mistaken argument), and the effectiveness of the solution lies within the students’ learning and enhanced understanding of the mathematics involved.

Within this conception of teaching, Stylianides and Stylianides (2010) advocate opportunities for prospective teachers “to use mathematical content from the perspective of an adult who is preparing to become a teacher of mathematics” (p. 163). In particular, they propose the development and use of Pedagogy-Related mathematics tasks, which essentially involve attention to mathematical objects and immersion in a particular pedagogical space. As such, we began designing a task that involved the writing out of a Lesson Play, which we saw as a Pedagogy-Related
mathematics task. This work fits within the general extensive research on mathematical knowledge in teaching (e.g., Rowland & Ruthven, 2011).

**Why prompts: the need for careful design**

The development of the prompts that would provide some constraints for the task was a central component of task design. Prompts are needed because without them the task failed to achieve its purpose. In fact, in the first stages of task design there were no prompts. We simply asked prospective teachers to write a script for instructional interaction on the topic of their choice. The plays in this first iteration featured long monologues by teacher-characters, followed by several questions and short answers by student-characters.

In an attempt to direct the attention of the playwrights to student thinking, our next iteration of the task had an additional requirement that the play should address a particular student error, again, leaving the choice of the topic open. In the plays produced in response to this requirement, the teacher-character’s reaction in most cases was to ask for a “different answer” from another student. Further, the example space of embedded errors was very limited: most mistakes exemplified a buggy algorithm in adding fractions or in performing column subtraction. This choice of topics is hardly surprising as personal example spaces are “triggered by current tasks, cues, and environment, as well as by recent experience” (Watson & Mason, 2005, p. 76). Though prospective teachers had some exposure to classrooms in their practicum, prior to enrolling in our courses, it was clear that they had a very narrow sense of the kinds of erroneous reasoning that students can display across the range of topics in the curriculum. As such, we engaged in the development of prompts that introduce fictional situations of students’ claims.
Prompt development

In the design of prompts we were guided by our own experiences with learners, as well as by mathematics education research. We wanted to offer prompts that encouraged prospective teachers to think not only about the mathematical mistake that was being made but also about the complex origin of the mistake (how the reasoning might be productive in other situations, for example) and the reasoning that might need to be involved in order to correct it. We exemplify our approach with several prompts and explain how they were developed.

Errors that stem from productive inferences

One of our goals in teacher education is to promote the notion that student errors should not be though of as misconceptions that need to be eradicated (as Smith, di Sessa and Roschelle (1993) argue). In contrast, we would like prospective teachers to come to an appreciation of the way in which errors are often based on generalizations of very productive inferences, that is inferences that work well in other situations (usually, more specific ones). As such, the teacher cannot simply replace previous ways of thinking with new ones. This requires that the teacher come to his or her own understanding of the basis for the student error and, subsequently, the possible trajectories that might be taken to help the student work toward a correct solution that does not simply throw out previous, productive inferences.

The prompt that we developed for prime numbers relates to a frequently-observed error (see Zazkis, 2011) that students make when deciding whether numbers are prime. In particular, many students claim that 91 is a prime number; and they base their decision on the fact that 91 does not appear on the standard 12 by 12 multiplication table (since it is the product of 7 and 13). We find the error particularly
interesting because it is based on reasoning that can often produce correct answers (especially in the context of the examples offered in a typical classroom) and that draws on the very prevalent and powerful use of the multiplication chart. It would thus be unhelpful for a teacher to tell students to ignore the chart altogether. And it would be limiting to miss the opportunity for students to understand when and why the multiplication chart can be used to help determine whether a number is composite (and hence, not prime).

A possible prompt could have been limited to the teacher-character question in prompt PN1 (Prime Numbers prompt #1), which clearly indicates that a student has erroneously claimed that 143 is prime. However, as a result of our more open-ended prompts used in the previous phase of design, we were concerned that the resulting lesson plays would not take into account the role of the multiplication table in students’ understanding of primality, and would focus more narrowly on correct definitions of primality. Therefore, in order to draw prospective teachers’ attention to the complex nature of student’s reasoning around the identification of prime number, the prompt PN1 explicitly refers to the “times tables.” And since the number 91 had been discussed in the methods course, the number 143 (the product of 11 and 13) was chosen. Note that the prompt plays the double role of making the prospective teachers aware that students might use the multiplication table to identify prime numbers and of providing an opportunity to think about how they might contend with such an inference.

PN1  Teacher: Why do you say 143 is prime?
       Student: Because it is not on our times tables
Research has also shown that in testing for primality students often check divisibility by small numbers, often relying entirely on familiar divisibility rules (Zazkis & Liljedahl, 2004). Again, this is a productive strategy for many of the examples that students encounter. We therefore designed prompt PN2 (Prime Numbers prompt #2) in order to play the double role described above for PN1.

PN2  Teacher: Why do you say 143 is prime?
   Student: Because 2, 3, 4, 5, 6, 7, 8 and 9 don’t go into it.

These two prompts address different errors that students are known to make in identifying prime numbers. And while we hope to help prospective teachers become aware of the kinds of errors students might make, our primary goal is to provide them with specific contexts in which they have to interact with a student who has produced an erroneous answer using reasoning that has also been effective. In this sense, the particularity of the prompt is important, even though more general teaching moves can be used for both prompts.

As with primality, prior research has shown that students often mis-generalise divisibility properties (Zazkis & Campbell, 2006; Zazkis, 2011). In the following prompt D1 (Divisibility prompt #1), the student-character generalizes the divisibility property for 3 to divisibility by 4. The response of the student-character thus draws the prospective teacher’s attention to the specific kind of erroneous inference that a student might make, which will require a follow-up that is more complex than merely re-stating the divisibility rule for 4.
Teacher: Why do you say that 462 is divisible by 4?

Student: Because the sum of the digits is divisible by 4.

The analysis of the lesson plays produced by the prospective teachers provides us with insight into what the playwrights attend to in describing their fictional interactions with students. It helps us recognize how teachers’ knowledge should be strengthened, and how pedagogical approaches might be shifted or enhanced. These insights can be used as part of follow-up activities in the methods course, and can even set the stage for the writing of revised lesson plays. However, our analysis of the lesson plays produced from the prompts led us to further revise the prompts themselves, in order to better achieve goals that are complementary to the ones articulated above. We provide some examples of this below.

**Focus on diagnosis**

While the prompts above achieve an important goal of drawing attention to the kinds of well-documented errors that students make, they do not give prospective teachers the opportunity to develop their own skills at diagnosing errors. This skill is indispensable in the classroom, particularly for new teachers who have not yet become familiar with common errors. Diagnosis involves trying to understand the basis for a student’s inference and being able to analyse the validity of student reasoning. We therefore refined the Lesson Play task by asking the playwrights to provide a “diagnosis,” in which they analysed what could have contributed to a student mistake. Since prompts PN1 and PN2 already point to the source of the mistake, we decided to introduce a mistake in PN3 (Prime Numbers prompt #3), but leave it to the playwright to invent a reason for it.
PN3  Teacher:  Why do you say 143 is prime?
  Student:  Because …

This resulted in rather creative responses, such as, “143 is prime because 43 is prime”.

Similarly to the case of prime numbers, we occasionally felt that we restricted the imagination of prospective teachers by providing the reason for the error in a student’s response. Therefore, we introduced an amended prompt D2 (Divisibility prompt #2).

D2  Teacher:  Why do you say that 354 is divisible by 4?
  Student:  Because …

Note that not only did we omit the reason for the error, we also changed the number, from 462 to 354. This was an intentional choice, to allow a potential focus on the last digit and potential misapplication of yet a different divisibility property. To our surprise, in about half of the plays that responded to this prompt, divisibility properties was not mentioned and the scenarios attributed the student’s error to careless carrying out of long division. Several other scenarios featured a logical error in having a student character notice that 354 is an even number and claim that this is the reason for divisibility by 4.

We note that the quality of a diagnosis was not always related to the quality of the dialogue in the lesson play. In other words, some prospective teachers provided very mathematically sophisticated analyses of the student’s error, but in their dialogues, they heavy-handedly directed the student to the correct solution. In other
cases, the diagnosis was weaker, but the interaction in the lesson play featured more attentive consideration of the student’s explanation.

Shift from “right answer” to reasoning

In examining the lesson plays produced in response to PN1 and PN2, we noted that many playwrights were more focused on guiding students toward a correct answer, rather than on the appropriateness of a student’s reasoning. This focus on the “right answer” is well documented by researchers and relates at least in part to teachers’ conceptions of mathematics (Stipek et al., 2001). In order to focus on reasoning, rather than on the correct answer, we offered the following prompt PN4 (Prime Number prompt #4) in which the student’s answer is correct, but the reason given is insufficient. Note that we replaced the number 143 used in PN1 and PN2 with the number 37 to allow for an exhaustive a list of “potential divisors” smaller than 10.

PN4  Teacher:  Why do you say 37 is prime?
Student:  Because 2,3,4,5,6,7,8 and 9 don’t go into it.

The playwrights found this prompt much more challenging. This has led us to design several other prompts in which the solution is correct, but where the reasoning can be improved in terms of efficiency. Prompt SUB1 (Subtraction prompt #1) exemplifies this for the context of addition and subtraction of mixed numbers.

SUB1  Teacher:  How do you subtract of $5 \frac{1}{4}$ from $7 \frac{1}{2}$ ?
Student:  $7 \frac{1}{2} - 5 \frac{1}{4} = \frac{15}{2} - \frac{21}{4} = \frac{30}{4} - \frac{21}{4} = \frac{30-21}{4} = \frac{9}{4} = 2 \frac{1}{4}$
Turning teacher errors into prompts

In examining the lesson plays we occasionally find mistakes that are either attributed to teacher-character, or mistakes attributed to a student-character, which are accepted by the teacher as correct. For example, a mistake found in a play that was based on prompt R1 (Repeating pattern prompt #1) led us to develop R2 (Repeating pattern prompt #2).

The prompt R1 was triggered by various studies on interpretation of repeating patterns (e.g., Threlfall, 1999) showing that young learners have difficulty recognizing a unit of repeat. This finding was confirmed working with prospective teachers, who often rely on extraneous features of a repeating pattern rather than identifying a repeating cycle (Zazkis & Liljedahl, 2002). This led to the design of the following prompt:

R1    Students are working on the following problem:

A toy train has 100 cars. The first car is red, the second is blue, the third is yellow, the fourth is red, the fifth is blue and sixth is yellow and so on.

What is the colour of the 80th car?

The teacher is moving through the room observing how the students are progressing. S/he stops and points at one student’s work.

Teacher: Why is the 80th car red?

Student: Because the 4th car is red, and 80 is a multiple of 4.
However, in a play that followed R1, in exploring repeating pattern in which the length of unit of repeat was 4, a student-character suggested that the 15th car should have the same colour as the third car, because 15 is a multiple of 3. In several plays, the teacher’s response acknowledged with praise the fact that the student attended to multiples, but ignored the coincidence that this particular multiple of 3 is one less than a multiple of 4. To direct prospective teachers’ attention to this way of thinking, we designed a prompt R2, which introduces a correct answer followed by erroneous reasoning that we adopted from a previously written play.

R2 Students are working on the following problem:

*A toy train has 100 cars. The first car is red, the second is blue, the third is yellow, the fourth is green, the fifth is red and sixth is blue, seventh is yellow, eighth is green and so on. What is the colour of the 39th car?*

The teacher is moving through the room observing how the students are progressing. S/he stops and points at one student’s work.

**Teacher:** Why is the 39th car yellow?

**Student:** Because the 3rd car is yellow, and 39 is a multiple of 3.

We keep on developing new prompts, as well as revising and altering the previously developed ones. In our future stages of the development of prompts, we plan to focus mostly on cases where the error is introduced but the reason for the error is not provided, such as D2 and PN3, rather than indicating the source of the error in the prompt, such as in PN1 and PN2. Further, we are interested in investigating cases where a correct answer is followed by either erroneous explanation, as in R2, or an
explanation that is not erroneous but has undesirable features, such as PN4 and SUB1. We have found that both these types of prompts lead to a greater variety of diagnoses. They in turn provide for richer classroom discussion in which prospective teachers can become more aware of the range of possible student responses, and the related teacher moves that could effectively support learning.

**Impact of lesson play writing on prospective teachers**

For prospective teachers writing a play serves as a training device, a simulator. Like in a flight simulator, one acts as one would in reality. However, unlike a flight simulator, there is an opportunity to pose, to think and to examine alternatives. That is, a lesson play creates an opportunity to imagine a classroom, to imagine learners’ reactions and their possible explanations. Further, it frees a teacher from the need to respond in ‘real time’ and, as such, it enables that teacher to examine personal responses to students’ ideas. We believe that this experience is a valuable one in preparing for “real” teaching that values classroom communication and effectively connects content to pedagogy.

Experience in playwriting creates and extends a repertoire of responses and of general strategies that can be used in future situations, in more improvisational ways. It also creates an opportunity to revisit personal mathematical understanding, enhance it if needed, and unpack it in a way that is accessible for learners. For prospective teachers, Lesson Plays create a bridge between a plan for action and its implementation.

**Conclusion**

In this paper we discussed the Lesson Play task, which we consider to be an example of a Pedagogy-Related mathematics task. Extended excerpts of the lesson plays
written by prospective teachers and detailed analysis of the plays is found in Zazkis, Sinclair and Liljedahl (2013). In this paper our focus is solely on describing the iterative design of the task.

Initially, the task assigned to prospective teachers was to present a lesson, or part of it, as a script for interaction between a teacher and students. Subsequent implementations included particular prompts that introduced students’ erroneous decisions or inadequate reasoning, and the task for prospective teachers was to write a play that followed the prompt, that is, addressed the error and guided students toward resolution.

Along with the variations in prompts presented to prospective teachers, the task also evolved in terms of its implementation in the methods course. Initially, the Lesson Play assignment was the final project in the course. This served as a lens for us, as researchers, to analyse teachers’ knowledge, either mathematical or pedagogical. However, this left us, as teacher-educators, dissatisfied as many issues that surfaced in the plays could have been and should have been addressed in our courses. As such, we started to use Lesson Play also in the first weeks of the course as a formative assessment that guided our choices of emphases in our subsequent work with prospective teachers.

**Extensions**

While our main experience with Lesson Plays concerns elementary school teachers (Zazkis, Sinclair & Liljedahl, 2013), we are now, along with our colleagues working on extending the ideas of script-writing to other contexts. For example, script-writing was recently used in studying secondary school teachers’ understanding of several proofs (Koichu & Zazkis, 2013; D. Zazkis, 2013). Additionally, the method
was used in examining prospective teachers’ ideas related to number structure, based on prompts that presented conversation starters between characters who disagree on a number property (Zazkis & D. Zazkis, 2013).

We invite our colleagues in teacher education to design prompts for Lesson Plays in their respective areas. They serve as a valuable tool in teacher development as well as in research studies, while also aiming to enhance, effective pedagogical approaches.

References


