Strand B *ALGEBRA* Introduction

Introduction to Topic

Algebra is a system of mathematics that enables certain types of problems to be solved more quickly and in a more general way than by using arithmetic. Algebra is based on using unknown values called variables whereas arithmetic uses only known numbers. Using algebra, solutions of equations are shown to be shown to be true no matter which numbers are included. For example, we know that the solution of the linear equation

$$ax + b = 0$$

where a and b are any numbers (but not zero) is given by

$$x = -\frac{b}{a}$$

This is the formula that gives the solutions to any linear equation. This is powerful mathematics as it solves the problem for any values of a and b! You can make the analogy with how people move: where 'number' is equivalent to 'walking', 'algebra' is equivalent to 'running, jumping' etc! Walking is a crucial stage in the development of movement but it is just the basic stage of the process - a whole new dimension comes in when running and/or jumping are introduced. In mathematics, number is a crucial first stage but an extensive new world opens up with algebra.

It is unfortunate that, in this country, we tend to introduce algebra in secondary education and many learners have great difficulty in understanding agebraic concepts. In mathematically high-performing countries, the foundation for algebra is laid in primary education and learners become familiar with its use at this early stage of their mathematical development.

It is vitally important that we use opportunities in the primary sector to introduce algebraic notation and concepts wherever possible.

Even in Year 1, for example, the question

$$7 + \square = 10$$

can produce not only the answer $\square = 3$ but also can involve the reasoning, e.g.

$$7 + 3 = 10, \quad 10 - 7 = 3, \quad 10 - 3 = 7$$

Using seemingly simple examples like this can help to encourage algebraic or mathematical thinking.

Historical Background

The historical introduction to Algebra is given in Units B2 and B3 but we would encourage you to look carefully at the misconceptions section below. Learners often do not make progress in algebra as they do not understand the notation; a good understanding of this makes algebra a logical and straightforward topic.

Misconceptions

- Problems often occur with ''double' negative signs, e.g. $3 - (-5)$ given as -2 whereas it should be 2
- Multiplying throughout a bracket, e.g. $3(x + 6)$ given as $3x + 6$, instead of $3x + 18$ (correct)
- Realising that, for example, $2 \times 3x$ is, in fact, $2 \times 3 \times x = 6x$ and not $(3x)^2$ or $3x^2$
- Confusion between $x^2 (= x \times x)$ and $2x (= 2 \times x)$

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Misconceptions

- Writing $(x+a)^2 = x^2 + a^2$ instead of $x^2 + 2ax + a^2$ (correct)
- Writing $(2x^2)^2 = 4x^2$ whereas $(2 \times x \times x)^2 = (2 \times x \times x) \times (2 \times x \times x)$
  $= (2 \times 2) \times (x \times x \times x \times x)$
  $= 4x^4$
- Writing $\frac{x}{2} = 4 \Rightarrow x = 2$ (instead of $x = 8$)
- Writing $x + 3 = 5 \Rightarrow x = 5 + 3 = 8$ (instead of $x = 2$)
- Writing $x - (-4) = 0 \Rightarrow x = 4$ (instead of $x = -4$)