Strand C *MEASUREMENT* Introduction

Introduction to Topics

This is another fundamental building block in mathematics for its use in practical problems. We have inherited our system of units from the past, when tools were not capable of great precision.

For example, the distance from the point of your elbow to the tip of your middle finger was called a 'cubit' (about 18 inches in length) and the distance from the end of your nose to the tip of your fingers when your arm is outstretched is almost exactly two cubits, which became about a yard in our Imperial system of units.

Now, of course, with today's technology, we need a precise system of measurement (and weight and time) and this is outlined in the following units of work.

Historical Context

Once a child – or a civilisation! – formalises the art of *counting*, it is but a short step to using whole numbers to quantify *measures*. The language and structures we use to refer to large positive integers (say up to low thousands) combine fairly naturally with the idea of a given *unit* to create a flexible and powerful way of quantifying amounts. Thus, for smallish **lengths** the ancient civilisations of the near east (Babylonian/Egyptian, c.1700 BC) used *cubits*; one *cubit* referred to the length of the forearm, or *ulna* (so is related to the later English unit – the *ell*). The ancient Greeks and Romans (400 BC to 600 AD) used *palms* – similar to our *hands*, still used for measuring the height of horses; for longer distances they used the *pes* (or foot), the *passus* (equal to 5 *pedes*), and the *stadium* (roughly a furlong). Many ancient cultures measured **volumes** of grain in *basketfuls*.

The most significant mathematical feature of these early *measures* is that although the units themselves may be inexact (What exactly is a *foot*? What is a *basket?*), the number of units is *absolutely exact* (because we are dealing with whole numbers).

The whole idea of using numbers to quantify amounts has two parts:

- The first part is the mathematical **idea** of choosing a fixed *unit* and then replicating that unit to match a given amount, which can then be assigned a certain number of units, or quantity. This **idea** is *abstract* and *exact*.
- The second part is the **practical implementation** of this scheme, by agreeing
  (a) how to realise the abstract idea of the fixed unit *in practice*; and
  (b) how to replicate the unit *reliably* and *fairly*.

  This **practical implementation** is inevitably approximate.

It is important to establish these two ideas (one exact, and one approximate) in pupils' minds as separate aspects of measurement.

Introducing partial units (halves and other fractions), raises a new source of approximation: it is tempting to think that *one complete basket* involves no approximation, whereas *two thirds of a basket* clearly involves a degree of estimation. This can add to the confusion as to what is exact and what is approximate – especially if one is unclear about the exact nature of fractions.

Units of length, weight, volume and currency developed locally, so only had to be sufficiently accurate for local needs. Trade between regions encouraged the development of common measures, but without the necessary political interest, change was inevitably slow. Moreover, units of measure could never be more accurate than the available technology allowed. The imposition of *standard units of measure* was at the mercy of political and technological developments. The most striking example is the spread of the *metric system*. The planning and introduction of the metric system (in France in the 1790s, and thence into other European countries conquered by Napoleon), was the result of a unique

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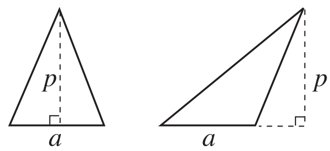
combination of events: namely the rise of a powerful Emperor (Napoleon), who happened to be scientifically educated (being a member of the *Académie des Sciences*, and a keen amateur mathematician) at a time when the necessary technological developments were in place for the first time. (For example, the definition of a *metre* as 'the distance between two marks on a specified platinum bar, stored at a fixed temperature' in a vault in Paris would have been unthinkable – for scientific, technological and political reasons – 100 years earlier.)

Many civilisations developed rules of thumb to find approximate areas and volumes of familiar everyday shapes (such as the area of special shaped fields, and the volume of special grain containers). However, since they usually felt no need to give precise definitions of those shapes, it is often impossible to tell how good their rules were. More detailed procedures – with some proofs – occur in the mathematics of ancient India, China and Japan, but it is hard to know exactly when their methods were developed. Again it was the ancient Greeks (around 300 BC) who probably first gave precise definitions, and set up a structure within which they could realise their insistence on *proving* that their rules were correct. They gave strict proofs for all the basic results we now know (area of rectangles, parallelograms, triangles, trapezia; volumes of prisms; area and perimeter of circles – including the amazing fact that the same number π appears in both formulas; volumes of cones and pyramids; volumes and surface areas of spheres).

Misconceptions

- $1 \text{ cm}^2 = 10 \text{ mm}^2$ is a misconception, as $1 \text{ cm}^2 = 10 \text{ mm} \times 10 \text{ mm} = 100 \text{ mm}^2$
- $1 \text{ m}^2 = 100 \text{ cm}^2$ is a misconception, as $1 \text{ m}^2 = 100 \text{ cm} \times 100 \text{ cm} = 10\,000 \text{ cm}^2$
- It is a misconception that triangles with the same base length and same perpendicular height have different areas (they are, in fact, equal in area).



- A common misconception is that the area of triangle $= \frac{1}{2}$ base $\times$ height. In fact, the area of a triangle is $= \frac{1}{2}$ base $\times$ perpendicular height)
- It is a misconception that a cube can have unequal sides (a cube, by definition has all its sides equal; if they are unequal, it is a cuboid)
- If you double the side lengths of a cube, its area is increased *fourfold* and its volume is increased *eightfold*. It is a misconception that doubling the side lengths of a cube results in a doubling of its area and volume.