Strand F *GEOMETRY*   Introduction

Introduction to Topics

This is another important topic in mathematics where we turn to what is sometimes called 'shape and space'. We start the strand by concentrating on angles, symmetry and circles, moving on to the concepts of similarity and congruence. These topics are brought together with the idea of coordinates and how they can be used to specify straight lines. Next comes the important topic of transformations that provides the basis of geometric analysis. Finally we introduce the main topic in trigonometry (not covered until Year 7 in the National Curriculum) but important to understanding how the topic is extended and built on in Key Stage 3.

Historical Context

The 'traditional' approach to geometry, which predominated all school geometry up to the 1960s, was based essentially on *Euclid's 'Elements'*, written about 300 BC. Euclid's theorems were based on five assumptions or postulates:

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1. For every point P, and for every point Q not equal to P, there exists a unique line, *l*, which passes through P and Q.

2. For every segment AB and for every segment CD there exists a unique point E such that B is between A and E and segment CD is congruent to segment BE.

3. For every point O and every point A not equal to O, there exists a circle with Centre O and radius OA.

4. All right angles are equal to each other.

5. If a straight line falls on two other straight lines to make the interior angles on the same side less than two right angles, then the two lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

Sum of angles > 180°
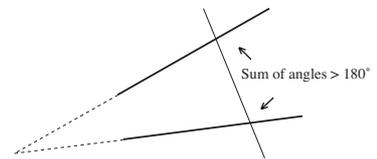
The basic tools were congruent triangles and parallel lines and the subject matter mainly concerned

<div align="center">triangles, parallelograms and circles</div>

and their properties.

However, in 1972, the German mathematician, *Felix Klein* (1849–1925), in his inaugural lecture as Professor of Mathematics at the University of Erlangen, gave a description of geometry as:

> *those properties of figures in space which remain unchanged under some fixed group of transformations.*

This influential address led directly to the *Erlangen Programme*, which has changed radically the style of geometry taught in schools today. It led to a shift in emphasis away from congruence as the fundamental idea.
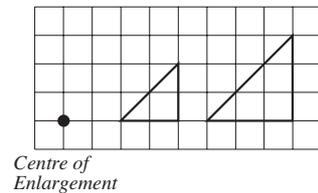
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To say that two plane figures are *congruent* means that one can be moved to fit exactly onto the other. Klein's approach would be to view this as a translation and (possible) rotation. For example, A and B as shown opposite are congruent but one shape can be obtained from the other by a translation, followed by a rotation or, indeed, by a single rotation about the centre of rotation as shown.

Centre of Rotation
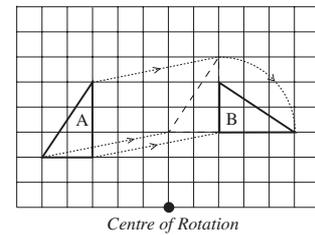
As another example, consider a parallelogram ABCD, as shown opposite. The triangles ABC and CDA are congruent. You can prove this either by *SAS* or *SSS* in the traditional way. Klein geometry, though, would consider rotating ADC through 180˚ about the midpoint, O, of the line AD and, in so doing, show that the triangle ADC fits exactly onto ABC. This is essentially the same mathematics but by a very different approach.
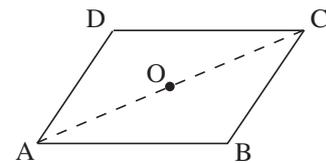
Also the traditional work on similarity can, in Klein geometry, be thought of as an enlargement with different scale factors (2 or $\frac{1}{2}$ shown opposite).


Centre of Enlargement

Some people may feel that geometry managed very well for more than 2000 years without transformations and that the introduction of transformation geometry is just a fad – but there are strong reasons for the use of transformations in school geometry. One reason is that rotation, reflection, etc. can be introduced in a practical way and so should be more accessible to some students than the more theoretical traditional geometry.

Misconceptions

Angles

- Learners often omit the degrees sign $(°)$, writing, for example, 60 instead of $60°$.

- Angles in a triangle sum to exactly $180°$ - learners sometimes say that they add to *about* $180°$.

- There is a common misconception that the bearing of A from B is the same as the bearing of B from A. This is incorrect as can be seen from the diagrams below.

Bearing of B from A is θ

Bearing of A from B is $\theta + 180°$
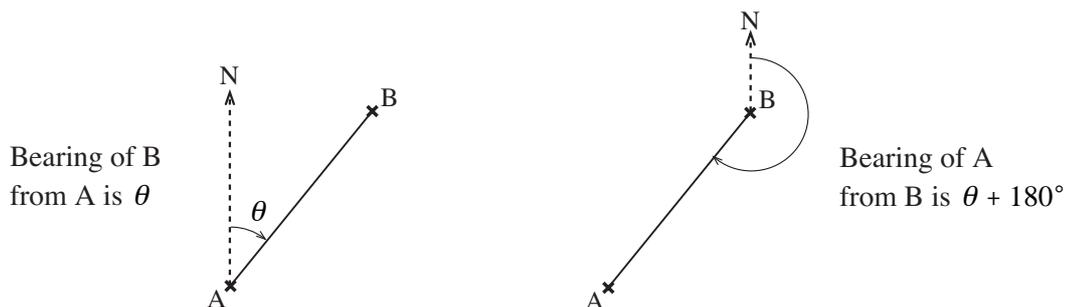
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Misconceptions

Bearings are always measured clockwise from North, so θ in the diagram is the bearing of B from A, whilst the bearing of A from B is $\theta + 180°$.

Shapes

- There is often a misconception that a square is not a rectangle. This is untrue as a square conforms to all the definitions of a rectangle. However, not all rectangles are squares.

- 'A parallelogram is not a quadrilateral.' Again, this is not true as a parallelogram meets the definition of a quadrilateral. In fact, all squares, rectangles, parallelograms, trapeziums, rhombuses, etc. are quadrilaterals.

Coordinates

- It is a misconception to say that the point $(2, 3)$ is 2 units in the positive *y*-direction and 3 units in the positive *x*-direction. In fact, the opposite is true: the point $(2, 3)$ is 2 units in the positive *x*-direction and 3 units in the positive *y*-direction - see diagram.
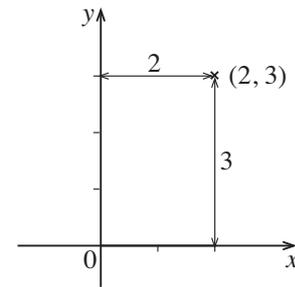
- Learners often mistakenly think that the $y = 0$ line is vertical. It is, in fact, horizontal - see diagram.
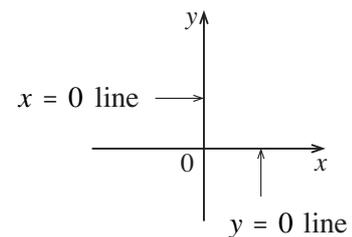
Translations

- It is incorrect to say that rotations are always about the origin; they can be about any point.

- A clockwise $180°$ rotation about a point is not the same as an anti-clockwise rotation about the same point. In fact, these two rotations are equivalent, i.e. they have the same result.

- Enlargement of scale factor of $\frac{1}{2}$ increases lengths by 2. This is incorrect: in fact, enlargement by $\frac{1}{2}$ decreases lengths by $\frac{1}{2}$ and is a reduction.

Trigonometry

- There is often confusion in identifying the 'adjacent', 'opposite' and 'hypotenuse' in a right-angled triangle. The *hypotenuse* is opposite the right angle; the *opposite* is the side opposite the marked angle; the *adjacent* side is next to the marked angle.

- Solving for a smaller side in a right-angled triangle can cause problems ($a^2 + b^2 = c^2$ does <u>not</u> imply $a^2 = c^2 + b^2$).