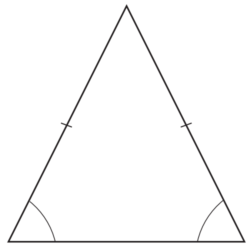
4 Trigonometry

4.1 Squares and Triangles

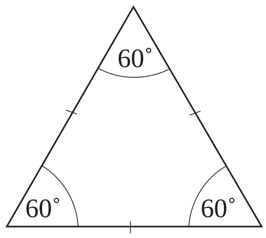
A *triangle* is a geometric shape with three sides and three angles. Some of the different types of triangles are described in this Unit.

A *square* is a four-sided geometric shape with all sides of equal length. All four angles are the same size.

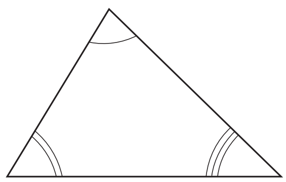
An *isosceles* triangle has two sides that are the same length, and the two base angles are equal.



All the sides of an *equilateral* triangle are the same length, and the angles are all $60°$.

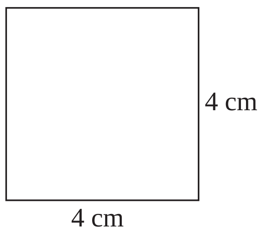


A *scalene* triangle has sides that all have different lengths, and has 3 different angles.



Worked Example 1

Find the area of the square shown in the diagram.



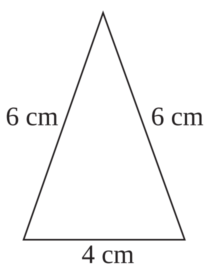
Solution

$$\begin{aligned}\text{Area} &= 4 \times 4 \\ &= 16 \text{ cm}^2\end{aligned}$$

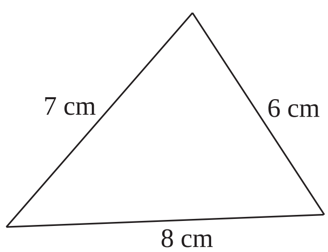
Exercises

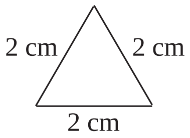
1. For each triangle below, state whether it is *scalene*, *isosceles* or *equilateral*.

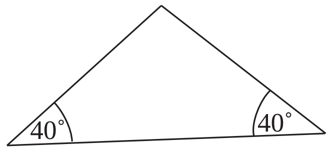
(a)

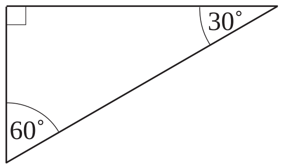


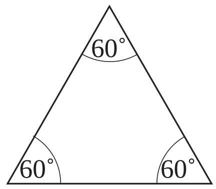
(b)

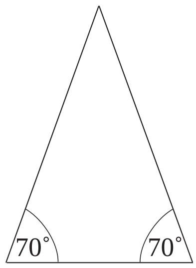


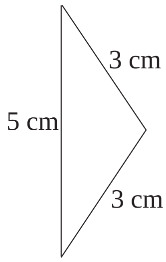
(c) 

(d) 

(e) 

(f) 

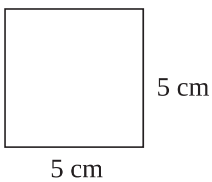
(g) 

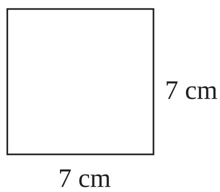
(h) 

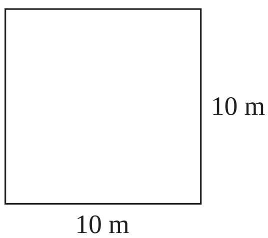
2. (a) When a square is cut in half diagonally, two triangles are obtained. Are these triangles scalene, isosceles or equilateral?

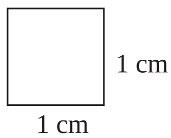
   (b) What type of triangle do you get if you cut a rectangle in half diagonally?

3. Find the area of each square below.

(a) 

(b) 

(c) 

(d) 

4. Find the areas of the squares with sides of length:

   (a) 2 m (b) 100 m (c) 15 cm (d) 17 cm

5. Find the lengths of the sides of a square that has an area of:

   (a) 9 cm^2 (b) 25 m^2 (c) 100 m^2
   (d) 64 cm^2 (e) 1 cm^2 (f) 400 m^2

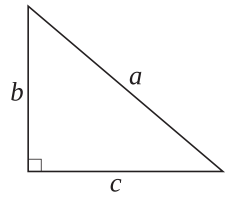
6. Two squares of side 4 cm are joined together to form a rectangle. What is the area of the rectangle?

7. A square of side 12 cm is cut in half to form two triangles. What is the area of each triangle?

8. A square of side 6 cm is cut into quarters to form 4 smaller squares. What is the area of each of these squares?

4.2 Pythagoras' Theorem

Pythagoras' Theorem gives a relationship between the lengths of the sides of a right angled triangle. For the triangle shown,

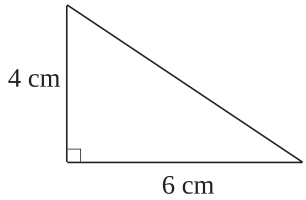


$$\boxed{a^2 = b^2 + c^2}$$

Note

The longest side of a right angled triangle is called the *hypotenuse*.

Worked Example 1



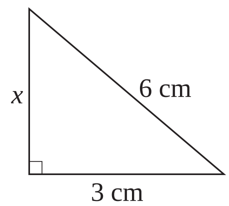
Find the length of the hypotenuse of the triangle shown in the diagram. Give your answer correct to 2 decimal places.

Solution

As this is a right angled triangle, Pythagoras' Theorem can be used. If the length of the hypotenuse is a, then $b = 4$ and $c = 6$.

So
$$\begin{aligned} a^2 &= b^2 + c^2 \\ a^2 &= 4^2 + 6^2 \\ a^2 &= 16 + 36 \\ a^2 &= 52 \\ a &= \sqrt{52} \\ &= 7.2 \text{ cm} \end{aligned}$$
(to one decimal place)

Worked Example 2



Find the length of the side of the triangle marked x in the diagram.

Solution

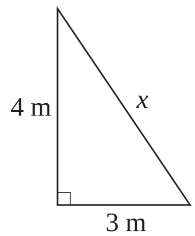
As this is a right angled triangle, Pythagoras' Theorem can be used. Here the length of the hypotenuse is 6 cm, so $a = 6$ cm and $c = 3$ cm with $b = x$.

So
$$\begin{aligned}
a^2 &= b^2 + c^2 \\
6^2 &= x^2 + 3^2 \\
36 &= x^2 + 9 \\
36 - 9 &= x^2 \\
36 - 9 &= x^2 \\
27 &= x^2 \\
\sqrt{27} &= x \\
x &= 5.2 \text{ cm} \qquad \text{(to one decimal place)}
\end{aligned}$$

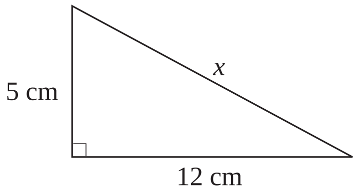
Exercises

1. Find the length of the side marked x in each triangle.

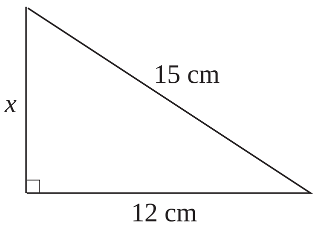
(a)



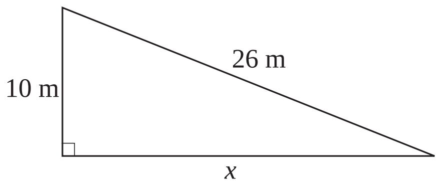
(b)



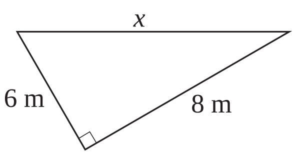
(c)



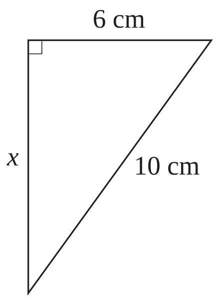
(d)



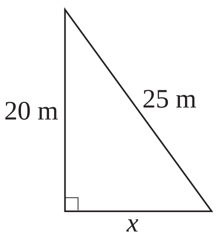
(e)



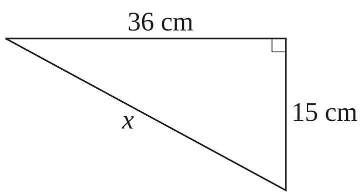
(f)



(g)

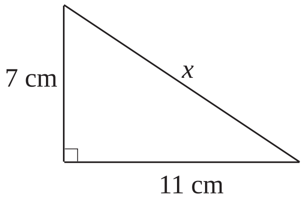


(h)

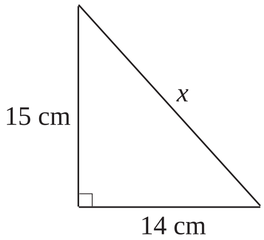


2. Find the length of the side marked x in each triangle. Give your answers correct to 2 decimal places.

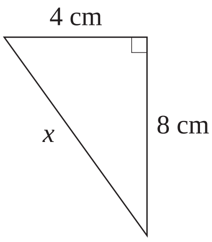
(a)



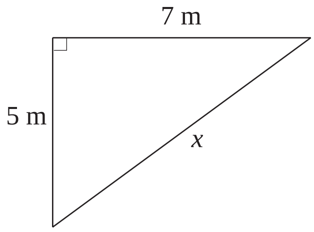
(b)



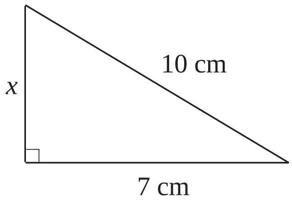
(c)



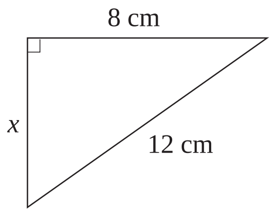
(d)



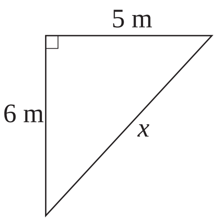
(e)



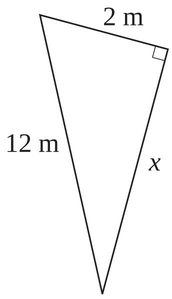
(f)



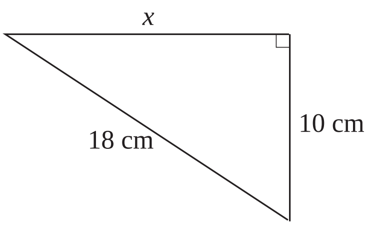
(g)



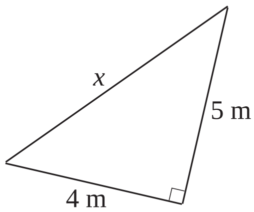
(h)



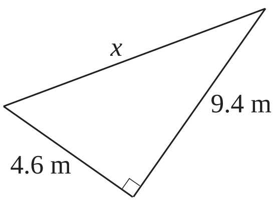
(i)



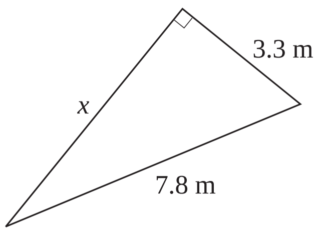
(j)



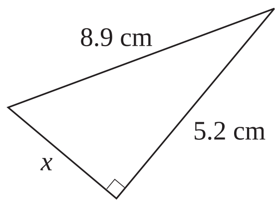
(k)



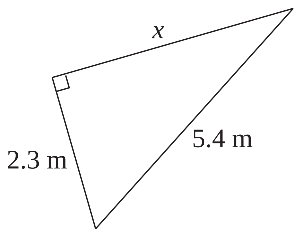
(l)



(m)



(n)

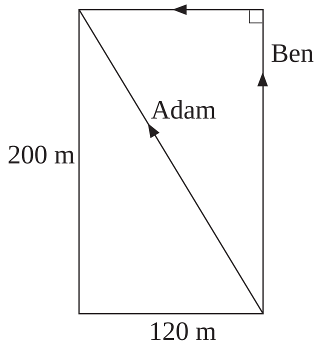


3. Adam runs diagonally across a school field, while Ben runs around the edge.

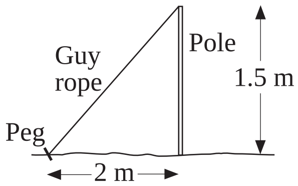
   (a) How far does Ben run?

   (b) How far does Adam run?

   (c) How much further does Ben run than Adam?



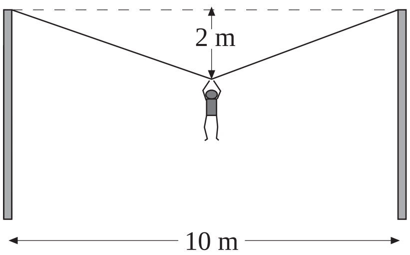
4. A guy rope is attached to the top of a tent pole, at a height of 1.5 metres above the ground, and to a tent peg 2 metres from the base of the pole. How long is the guy rope?



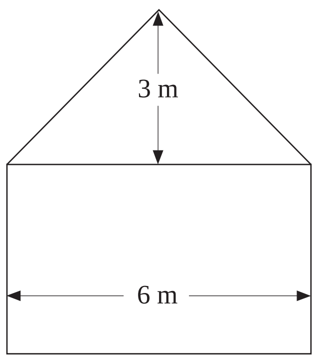
5. Farida is 1.4 metres tall. At a certain time her shadow is 2 metres long. What is the distance from the top of her head to the top of her shadow?

6. A rope of length 10 metres is stretched from the top of a pole 3 metres high until it reaches ground level. How far is the end of the line from the base of the pole?

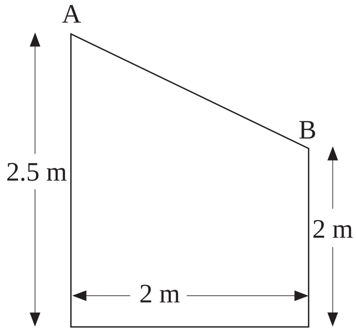
7. A rope is fixed between two trees that are 10 metres apart. When a child hangs on to the centre of the rope, it sags so that the centre is 2 metres below the level of the ends. Find the length of the rope.



8. The roof on a house that is 6 metres wide peaks at a height of 3 metres above the top of the walls. Find the length of the sloping side of the roof.



9. The picture shows a garden shed. Find the length, AB, of the roof.



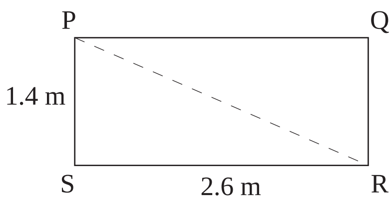
10. Miles walks 3 km east and then 10 km north.

    (a) How far is he from his starting point?

    (b) He then walks east until he is 20 km from his starting point. How much further east has he walked?

11. Ali is building a shed. It should be rectangular with sides of length 3 metres and 6 metres. He measures the diagonal of the base of the shed before he starts to put up the walls. How long should the diagonal be?

12. Pauline is building a greenhouse. The base PQRS of the greenhouse should be a rectangle measuring 2.6 metres by 1.4 metres.



To check the base is rectangular Pauline has to measure the diagonal PR.

    (a) Calculate the length of PR when the base is rectangular. You *must* show all your working.

    (b) When building the greenhouse Pauline finds angle $\text{PSR} > 90°$. She measures PR. Which of the following statements is *true*?

        X: PR is greater than it should be.

        Y: PR is less than it should be.

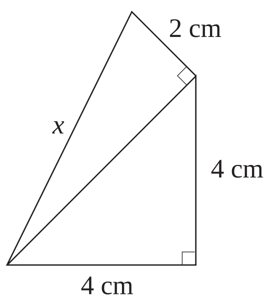
        Z: PR is the right length.

(SEG)

4.3 Further Work with Pythagoras' Theorem

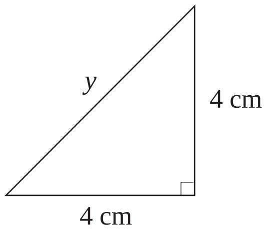
Worked Example 1

Find the length of the side marked *x* in the diagram.



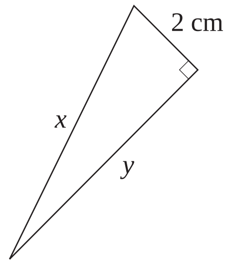
Solution

First consider the lower triangle. The unknown length of the hypotenuse has been marked y.



$$y^2 = 4^2 + 4^2$$
$$y^2 = 16 + 16$$
$$y^2 = 32 \text{ cm}$$

The upper triangle can now be considered, using the value for y^2.



From the triangle, $x^2 = y^2 + 2^2$, and using $y^2 = 32$ gives

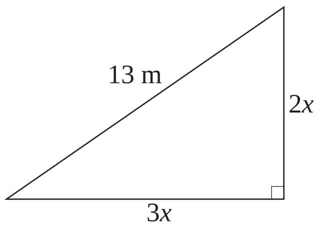
$$x^2 = 32 + 4$$
$$x^2 = 36$$
$$x = \sqrt{36}$$
$$x = 6 \text{ cm}$$

Note

When finding the side x, it is not necessary to find $\sqrt{32}$, but to simply use $y^2 = 32$.

Worked Example 2

Find the value of x as shown on the diagram, and state the lengths of the two unknown sides.



Solution

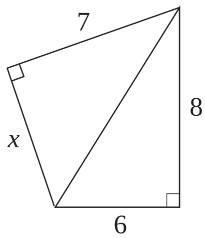
Using Pythagoras' Theorem gives

$$13^2 = (2x)^2 + (3x)^2$$
$$169 = 4x^2 + 9x^2 \qquad (\text{since } (2x)^2 = 2^2 x^2 = 4x^2)$$
$$169 = 13x^2$$
$$13 = x^2$$
$$x = \sqrt{13}$$
$$= 3.61 \text{ cm} \qquad (\text{to 2 decimal places})$$

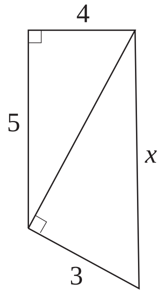
Exercises

1. Find the length of the side marked x in each diagram.

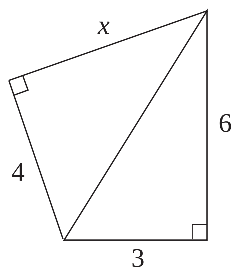
(a)



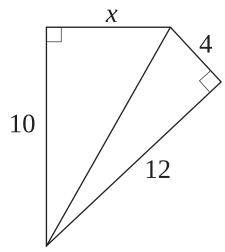
(b)



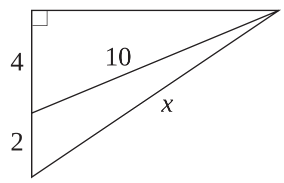
(c)



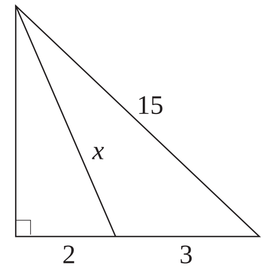
(d)



(e)

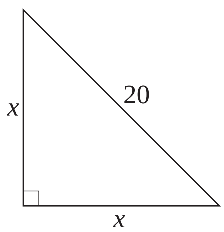


(f)

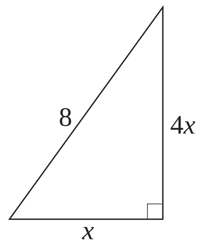


2. Find the length of the side marked *x* in the following situations.

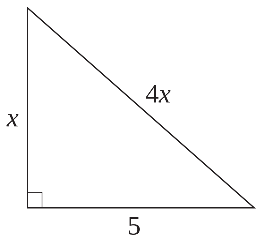
(a)



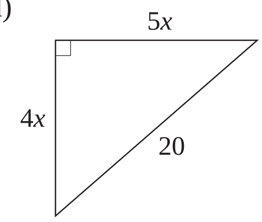
(b)



(c)

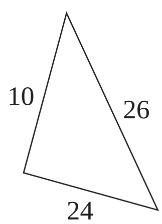


(d)

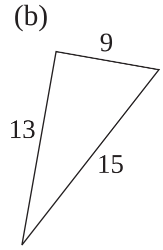


3. Which of the following triangles are right angled triangles?

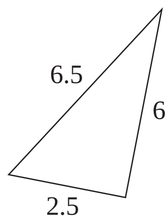
(a)



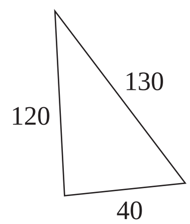
(b)



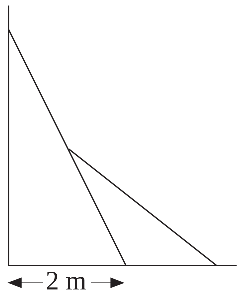
(c)



(d)



4. A ladder of length 4 metres leans against a vertical wall. The foot of the ladder is 2 metres from the wall. A plank that has a length of 5 metres rests on the ladder, so that one end is halfway up the ladder.

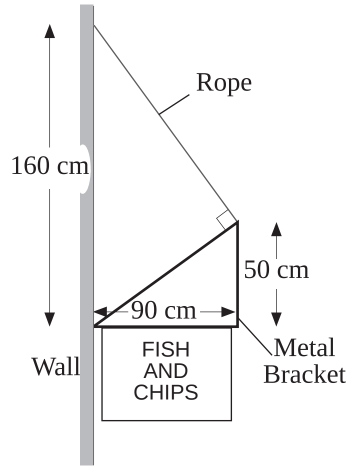


(a) How high is the top of the ladder?

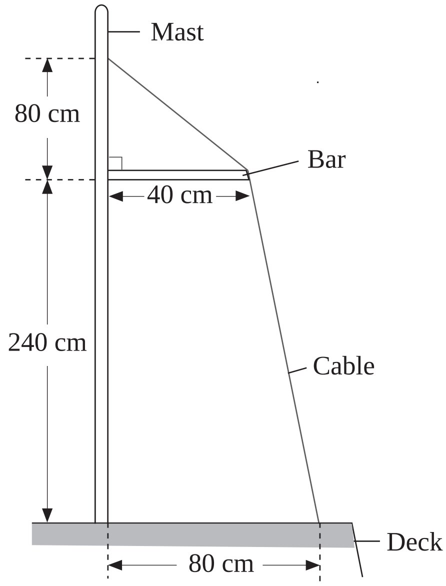
(b) How high is the top of the plank?

(c) How far is the bottom of the plank from the wall?

5. The diagram shows how the sign that hangs over a Fish and Chip shop is suspended by a rope and a triangular metal bracket. Find the length of the rope.



6. The diagram shows how a cable is attached to the mast of a sailing dingy. A bar pushes the cable out away from the mast. Find the total length of the cable.

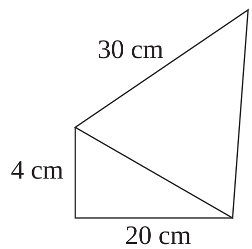


7. A helicopter flies in a straight line until it reaches a point 20 km east and 15 km north of its starting point. It then turns through $90°$ and travels a further 10 km.

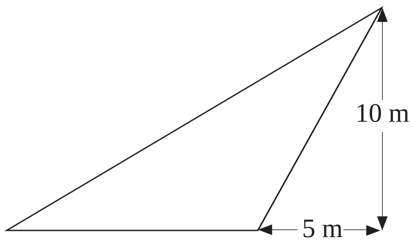
   (a) How far is the helicopter from its starting point?

   (b) If the helicopter turned $90°$ the other way, how far would it end up from its starting point?

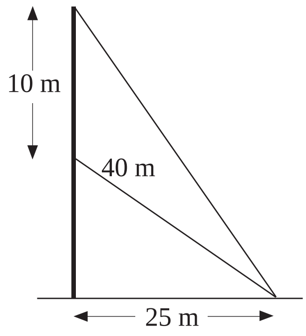
8. A cone is placed on a wedge. The dimensions of the wedge are shown in the diagram. The cone has a slant height of 30 cm. Find the height of the cone.



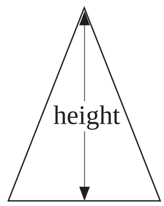
9. A simple crane is to be constructed using an isosceles triangular metal frame. The top of the frame is to be 10 metres above ground level and 5 metres away from the base of the crane, as shown in the diagram. Find the length of each side of the triangle.



10. A thin steel tower is supported on one side by two cables. Find the height of the tower and the length of the longer cable.



11. An isosceles triangle has two sides of length 8 cm and one of length 4 cm. Find the height of the triangle and its area.

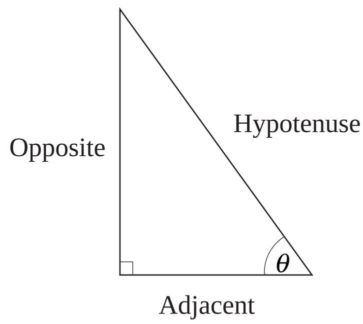


12. Find the area of each the equilateral triangles that have sides of lengths

(a) 8 cm (b) 20 cm (c) 2 cm

4.4 Sine, Cosine and Tangent

When working in a right angled triangle, the longest side is known as the *hypotenuse*. The other two sides are known as the *opposite* and the *adjacent*. The adjacent is the side next to a marked angle, and the opposite side is opposite this angle.



For a right angled triangle, the *sine*, *cosine* and *tangent* of the angle θ are defined as:

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

Sin θ will always have the same value for any particular angle, regardless of the size of the triangle. The same is true for $\cos\theta$ and $\tan\theta$.

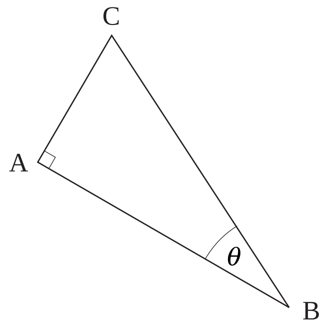
Worked Example 1

For the triangle shown, state which sides are:

(a) the hypotenuse

(b) the adjacent

(c) the opposite



Solution

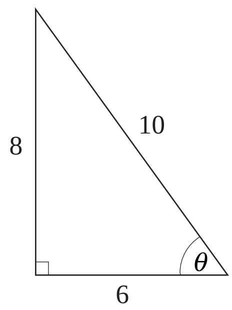
(a) The hypotenuse is the longest side, which for this triangle is CB.

(b) The adjacent is the side that is next to the angle θ, which for this triangle is AB.

(c) The opposite side is the side that is opposite the angle θ, which for this triangle is AC.

Worked Example 2

Write down the values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ for the triangle shown. Then use a calculator to find the angle in each case.



Solution

First, opposite = 8

adjacent = 6

hypotenuse = 10

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{8}{10} = 0.8$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{6}{10} = 0.6$$

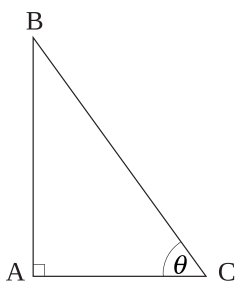
$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}} = \frac{8}{6} = \frac{4}{3}$$

Using a calculator gives $\theta = 53.1°$ (correct to 1 decimal place) in each case.

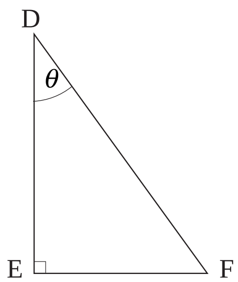
Exercises

1. For each triangle, state which side is the hypotenuse, the adjacent and the opposite.

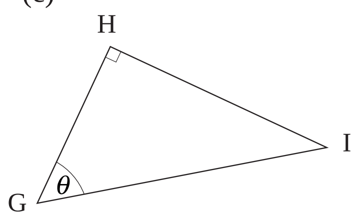
(a)



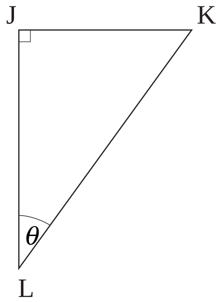
(b)



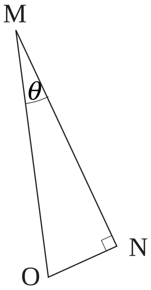
(c)



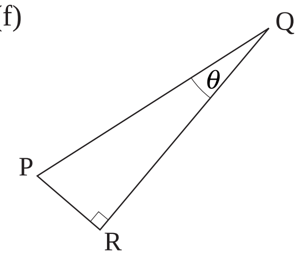
(d)



(e)

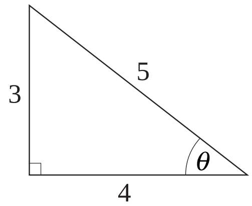


(f)

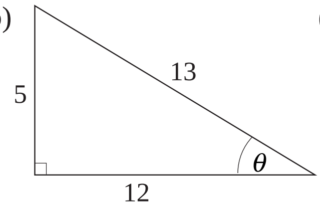


2. For each triangle, write $\sin\theta$, $\cos\theta$ and $\tan\theta$ as fractions.

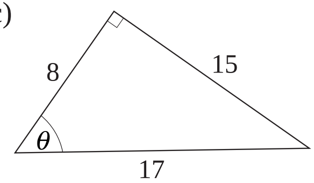
(a)



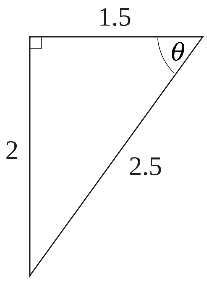
(b)



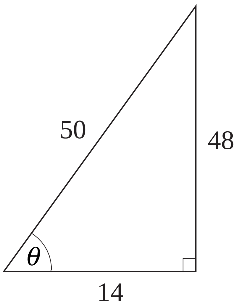
(c)



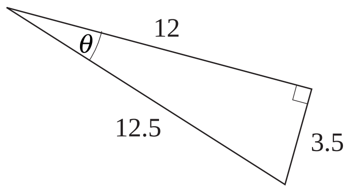
(d)



(e)



(f)



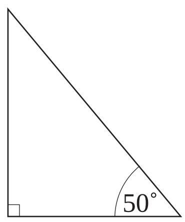
3. Use a calculator to find the following. Give your answers correct to 3 decimal places.

(a)	$\sin 30°$	(b)	$\tan 75°$	(c)	$\tan 52.6°$
(d)	$\cos 66°$	(e)	$\tan 33°$	(f)	$\tan 45°$
(g)	$\tan 37°$	(h)	$\sin 88.2°$	(i)	$\cos 45°$
(j)	$\cos 48°$	(k)	$\cos 46.7°$	(l)	$\sin 45°$

4. Use a calculator to find θ in each case. Give your answers correct to 1 decimal place.

(a)	$\cos\theta = 0.5$	(b)	$\sin\theta = 1$	(c)	$\tan\theta = 0.45$
(d)	$\sin\theta = 0.821$	(e)	$\sin\theta = 0.75$	(f)	$\cos\theta = 0.92$
(g)	$\tan\theta = 1$	(h)	$\sin\theta = 0.5$	(i)	$\tan\theta = 2$
(j)	$\cos\theta = 0.14$	(k)	$\sin\theta = 0.26$	(l)	$\tan\theta = 5.25$

5. (a) Draw a triangle with an angle of $50°$ as shown in the diagram, and measure the length of each side.



(b) Using

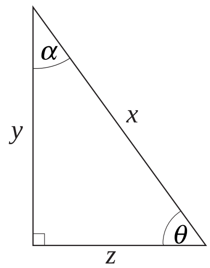
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

and the lengths of the sides of your triangle, find $\sin 50°$, $\cos 50°$ and $\tan 50°$.

(c) Use your calculator to find $\sin 50°$, $\cos 50°$ and $\tan 50°$.

(d) Compare your results to (b) and (c).

6. For the triangle shown, write down expressions for:



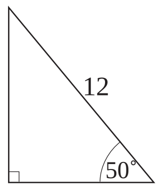
(a) $\cos\theta$ (b) $\sin\alpha$

(c) $\tan\theta$ (d) $\cos\alpha$

(e) $\sin\theta$ (f) $\tan\alpha$

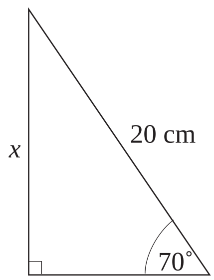
4.5 Finding Lengths in Right Angled Triangles

When one angle and the length of one side are known, it is possible to find the lengths of other sides in the same triangle, by using sine, cosine or tangent.



Worked Example 1

Find the length of the side marked x in the triangle shown.



Solution

In this triangle, hypotenuse $= 20$

opposite $= x$

Choose sine because it involves hypotenuse and opposite.

Using $$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

gives $$\sin 70° = \frac{x}{20}$$

To obtain x, multiply both sides of this equation by 20, which gives

$$20\sin 70° = x$$

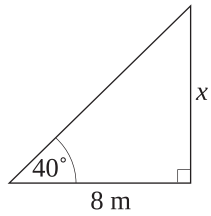
or

$$x = 20\sin 70°$$
$$= 18.8 \text{ cm} \qquad \text{(to 1 d.p.)}$$

This value is obtained using a calculator.

Worked Example 2

Find the length of the side marked x in the triangle.



Solution

In this triangle, opposite $= x$

adjacent $= 8$ metres

Use tangent because it involves the opposite and adjacent.

Using $$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

gives $$\tan 40° = \frac{x}{8}$$

Multiplying both sides by 8 gives

$$8\tan 40° = x$$

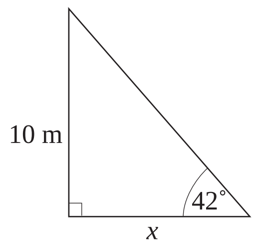
or

$$x = 8\tan 40°$$

$$= 6.7 \text{ metres} \qquad \text{(to 1 decimal place)}$$

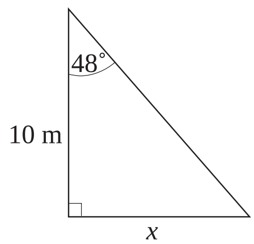
Worked Example 3

Find the length marked x in the triangle.



Solution

This problem will involve tangent, so use the other angle which is $90° - 42° = 48°$, so that x is the opposite.



Then opposite $= x$

adjacent $= 10$ metres

and using $$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

gives $$\tan 48° = \frac{x}{10}$$

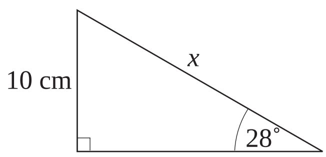
Multiplying both sides by 10 gives

$$x = 10 \times \tan 48°$$

$$= 11.1 \text{ metres} \qquad \text{(to 1 decimal place)}$$

Worked Example 4

Find the length of the hypotenuse, marked x, in the triangle.



Solution

In this triangle, hypotenuse $= x$

opposite $= 10$ cm

Use sine because it involves hypotenuse and opposite.

Using $$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

gives $$\sin 28° = \frac{10}{x}$$

where x is the length of the hypotenuse.

Multiplying both sides by x gives

$$x \sin 28° = 10,$$

then dividing both sides by $\sin 28°$ gives

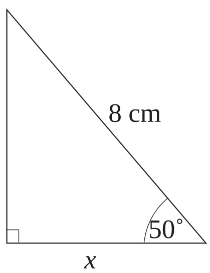
$$x = \frac{10}{\sin 28°}$$

$$= 21.3 \text{ cm} \qquad \text{(to 1 decimal place)}$$

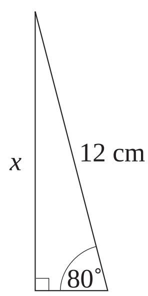
Exercises

1. Find the length of the side marked x in each triangle.

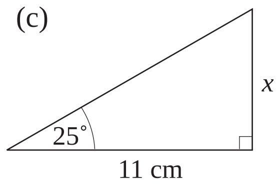
(a)



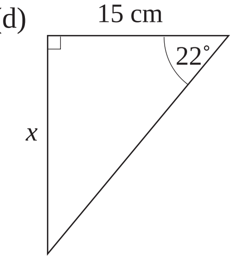
(b)



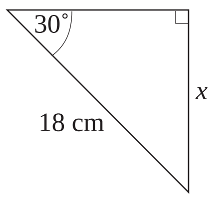
(c)



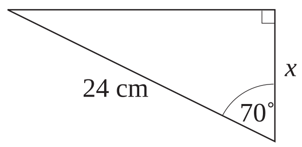
(d)



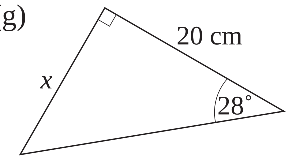
(e)



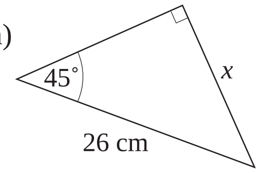
(f)



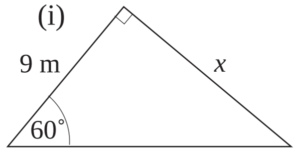
(g)



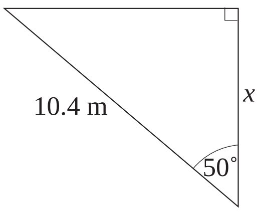
(h)



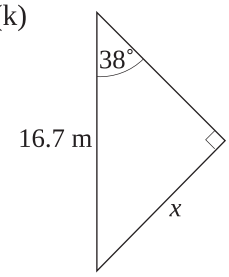
(i)



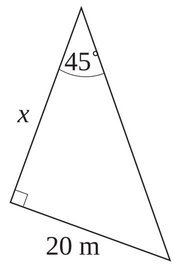
(j)



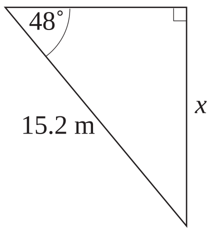
(k)



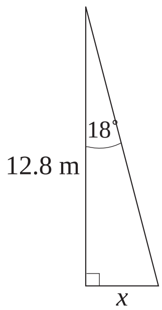
(l)



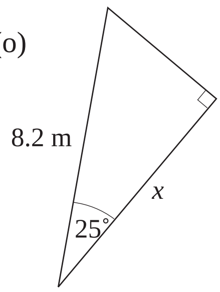
(m)



(n)



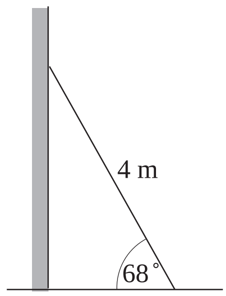
(o)



2. A ladder leans against a wall as shown in the diagram.

   (a) How far is the top of the ladder from the ground?

   (b) How far is the bottom of the ladder from the wall?

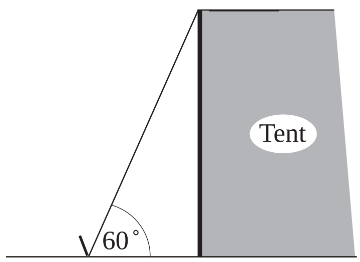


3. A guy rope is attached to a tent peg and the top of a tent pole so that the angle between the peg and the bottom of the pole is $60°$.

   (a) Find the height of the pole if the peg is 1 metre from the bottom of the pole.

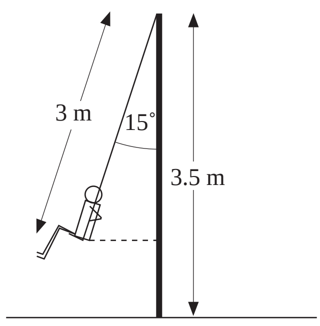
   (b) If the length of the rope is 1.4 metres, find the height of the pole.

   (c) Find the distance of the peg from the base of the pole if the length of the guy rope is 2 metres.



4. A child is on a swing in a park. The highest position that she reaches is as shown.

   Find the height of the swing seat above the ground in this position.



5. A laser beam shines on the side of a building. The side of the building is 500 metres from the source of the beam, which is at an angle of $16°$ above the horizontal. Find the height of the point where the beam hits the building.

6. A ship sails 400 km on a bearing of $075°$.

   (a) How far east has the ship sailed?

   (b) How far north has the ship sailed?

7. An aeroplane flies 120 km on a bearing of $210°$.

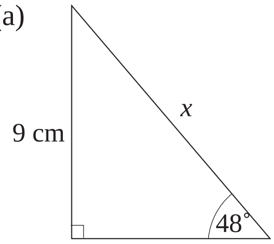
   (a) How far south has the aeroplane flown?

   (b) How far west has the aeroplane flown?

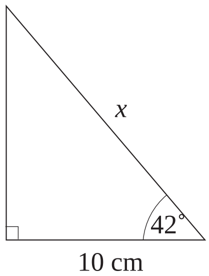
8. A kite has a string of length 60 metres. On a windy day all the string is let out and makes an angle of between $20°$ and $36°$ with the ground. Find the minimum and maximum heights of the kite.

9. Find the length of the side marked *x* in each triangle.

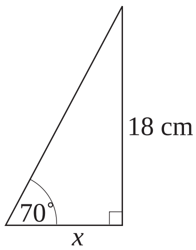
(a)



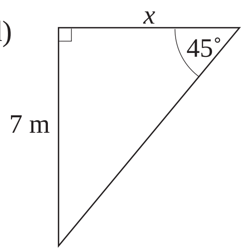
(b)



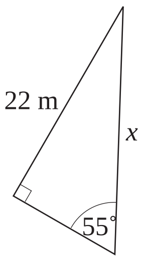
(c)



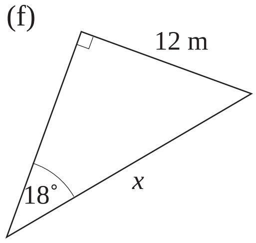
(d)



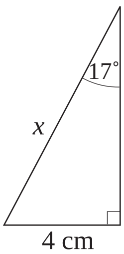
(e)



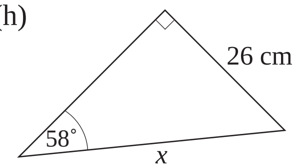
(f)



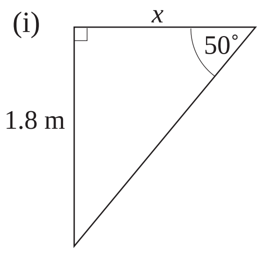
(g)



(h)



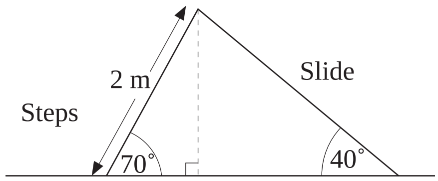
(i)



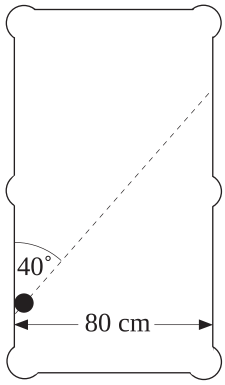
10. The diagram shows a slide.

(a) Find the height of the top of the slide.

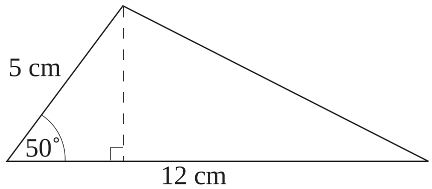
(b) Find the length of the slide.



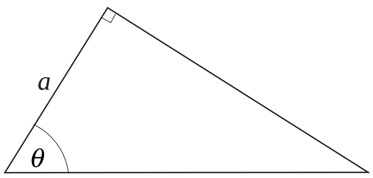
11. A snooker ball rests against the side cushion of a snooker table. It is hit so that it moves at $40°$ to the side of the table. How far does the ball travel before it hits the cushion on the other side of the table?



12. (a) Find the length of the dotted line and the area of this triangle.

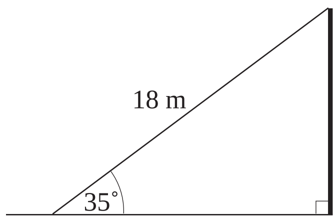


(b) Find the height of the triangle below and then find a formula for its area in terms of a and θ.



13. A wire 18 metres long runs from the top of a pole to the ground as shown in the diagram. The wire makes an angle of $35°$ with the ground. Calculate the height of the pole. Give your answer to a reasonable degree of accuracy.

(NEAB)

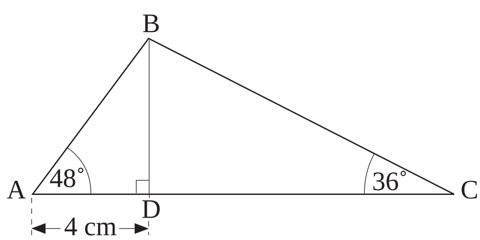


14. In the figure shown, calculate

(a) the length of BD.

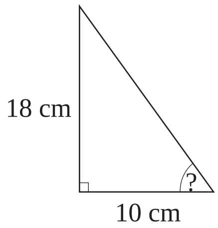
(b) the length of BC.

(NEAB)



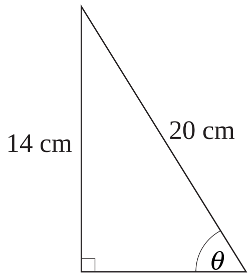
4.6 Finding Angles in Right Angled Triangles

If the lengths of any two sides of a right angled triangle are known, then sine, cosine and tangent can be used to find the angles of the triangle.



Worked Example 1

Find the angle marked θ in the triangle shown.



Solution

In this triangle, hypotenuse $= 20$ cm
opposite $= 14$ cm

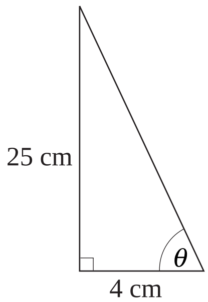
Using $$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

gives $$\sin\theta = \frac{14}{20} = 0.7$$

Then using the SHIFT and SIN buttons on a calculator gives $\theta = 44.4°$ (to 1 d.p.)

Worked Example 2

Find the angle marked θ in the triangle shown.



Solution

In this triangle, opposite $= 25$ cm
adjacent $= 4$ cm

Using $$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

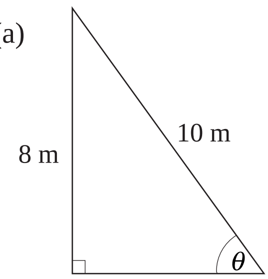
gives $$\tan\theta = \frac{25}{4} = 6.25$$

Using a calculator gives $\theta = 80.9°$ (to 1 d.p.)

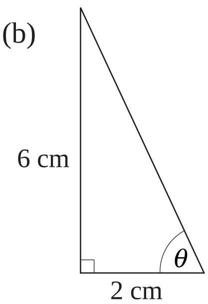
Exercises

1. Find the angle θ of:

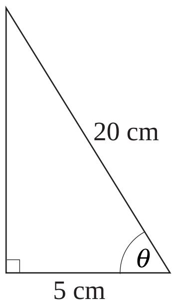
(a)



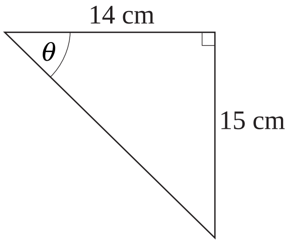
(b)



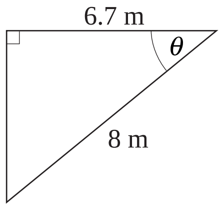
(c)



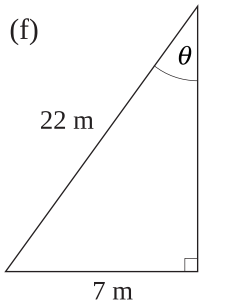
(d)



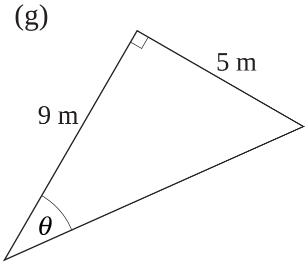
(e)



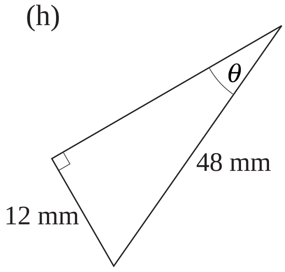
(f)



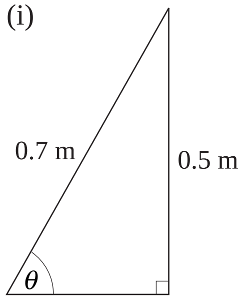
(g)



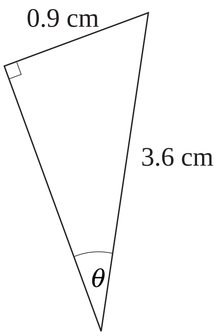
(h)



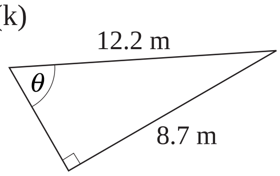
(i)



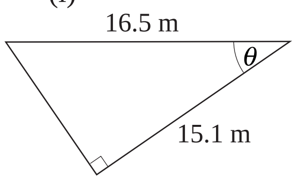
(j)



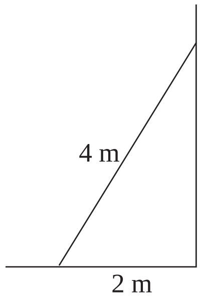
(k)



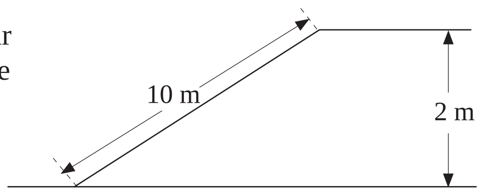
(l)



2. A ladder leans against a wall. The length of the ladder is 4 metres and the base is 2 metres from the wall. Find the angle between the ladder and the ground.



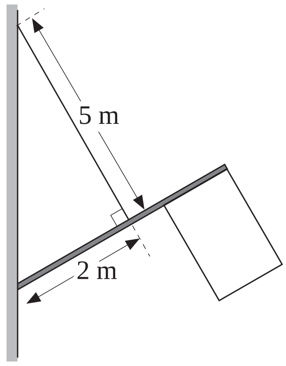
3. As cars drive up a ramp at a multi-storey car park, they go up 2 metres. The length of the ramp is 10 metres. Find the angle between the ramp and the horizontal.



4. A flag pole is fixed to a wall and supported by a rope, as shown. Find the angle between

   (a) the rope and the wall

   (b) the pole and the wall.



5. The mast on a yacht is supported by a number of wire ropes. One, which has a length of 15 metres, goes from the top of the mast at a height of 10 metres, to the front of the boat.

   (a) Find the angle between the wire rope and the mast.

   (b) Find the distance between the base of the mast and the front of the boat.

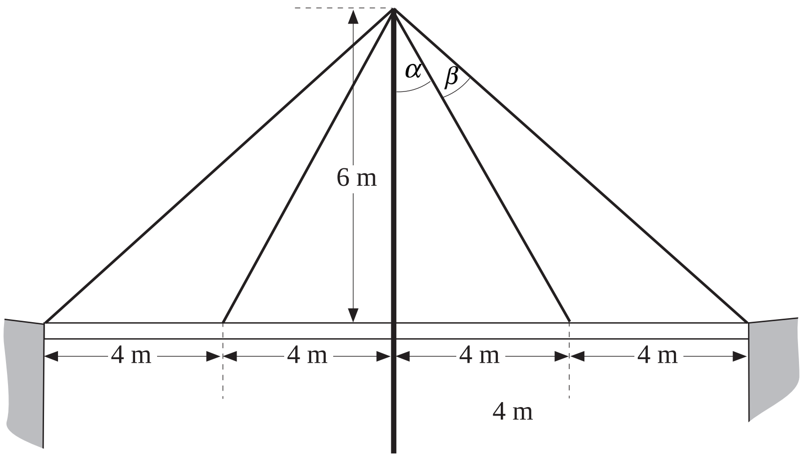
6. A marine runs 500 metres east and then 600 metres north. If he had run directly from his starting point to his final position, what bearing should he have run on?

7. A ship is 50 km south and 70 km west of the port that it is heading for. What bearing should it sail on to reach the port?

8. The diagram shows a simple bridge, which is supported by four steel cables.

   (a) Find the angles at α and β.

   (b) Find the length of each cable.



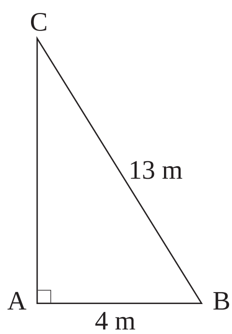
9. A rope has a length of 20 metres. When a boy hangs at the centre of the rope, the centre is 1 metre below its normal horizontal position. Find the angle between the rope and the horizontal in this position.

10. ABC is a right angled triangle. AB is of length 4 metres and BC is of length 13 metres.

   (a) Calculate the length of AC.

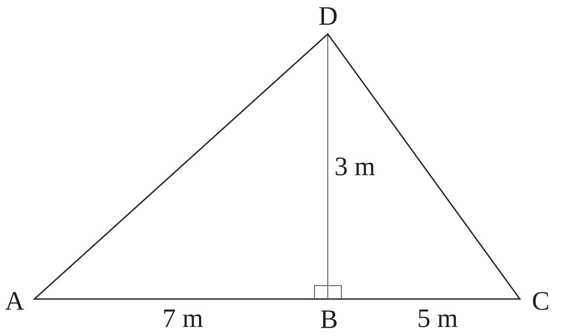
   (b) Calculate the size of angle ABC.

   (LON)



11. The diagram shows a roofing frame ABCD.

AB = 7 m, BC = 5 m, angle ABD = angle DBC = 90°



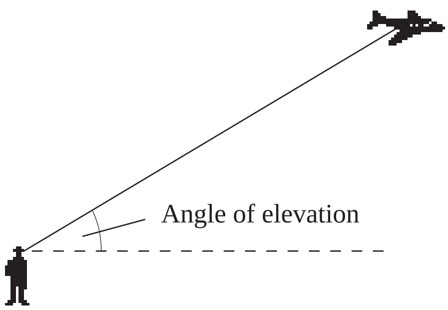
(a) Calculate the length of AD.

(b) Calculate the size of angle DCB.

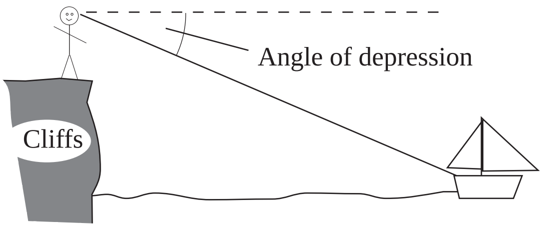
(MEG)

4.7 Mixed Problems with Trigonometry

When you look *up* at something, such as an aeroplane, the angle between your line of sight and the horizontal is called the *angle of elevation*.



Similarly, if you look *down* at something, then the angle between your line of sight and the horizontal is called the *angle of depression.*

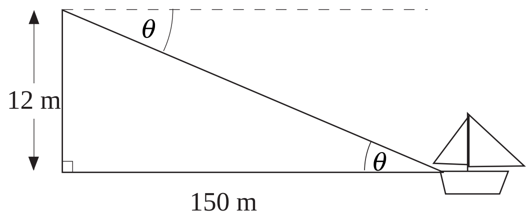


Worked Example 1

A man looks out to sea from a cliff top at a height of 12 metres. He sees a boat that is 150 metres from the cliffs. What is the angle of depression?

Solution

The situation can be represented by the triangle shown in the diagram, where θ is the angle of depression.



In this triangle,

$$\text{opposite} = 12 \text{ m}$$
$$\text{adjacent} = 150 \text{ m}$$

Using

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

gives

$$\tan\theta = \frac{12}{150}$$
$$= 0.08$$

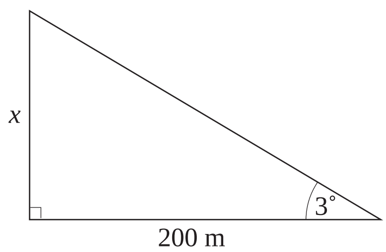
Using a calculator gives $\theta = 4.6°$ (to 1 d.p.)

Worked Example 2

A person walking on open moorland can see the top of a radio mast. The person is 200 metres from the mast. The angle of elevation of the top of the mast is $3°$. What is the height of the mast?

Solution

The triangle illustrates the situation described.



In this triangle,

$$\text{opposite} = x$$
$$\text{adjacent} = 200 \text{ m}$$

Using

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

gives

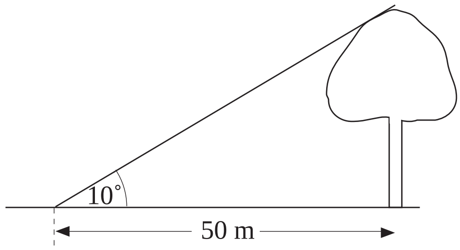
$$\tan 3° = \frac{x}{200}$$

Multiplying both sides by 200 gives

$$x = 200 \times \tan 3°$$
$$= 10.5 \text{ metres} \qquad \text{(to 1 d.p.)}$$

Exercises

1. In order to find the height of a tree, some children walk 50 metres from the base of the tree and measure the angle of elevation as $10°$. Find the height of the tree.



2. From a distance of 20 metres from its base, the angle of elevation of the top of a pylon is $32°$. Find the height of the pylon.

3. The height of a church tower is 15 metres. A man looks at the tower from a distance of 120 metres. What is the angle of elevation of the top of the tower from the man?

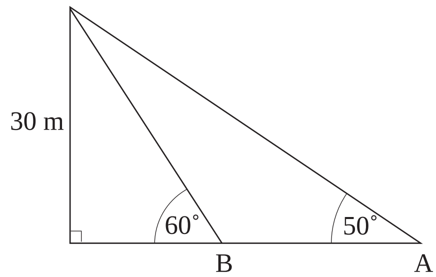
4. A coastguard looks out from an observation tower of height 9 metres and sees a boat in distress at a distance of 500 metres from the tower. What is the angle of depression of the boat from the tower?

5. A lighthouse is 20 metres high. A life-raft is drifting and one of its occupants estimates the angle of elevation of the top of the lighthouse as $3°$.

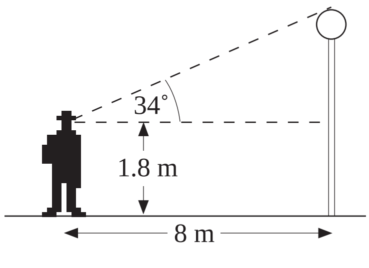
   (a) Use the estimated angle to find the distance of the life-raft from the lighthouse.

   (b) If the life-raft is in fact 600 metres from the lighthouse, find the correct angle of elevation.

6. A radio mast is supported by two cables as shown. Find the distance between the two points A and B.

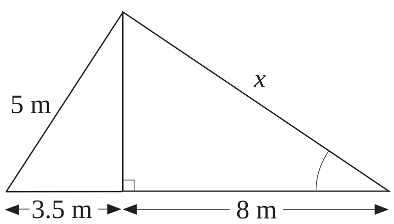


7. A man stands at a distance of 8 metres from a lamppost. When standing as shown, he measures the angle of elevation as $34°$. Find the height of the lamppost.

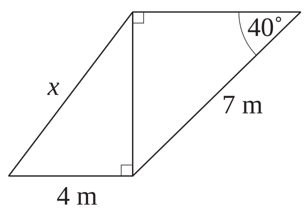


8. Find the unknown length (x) in each diagram.

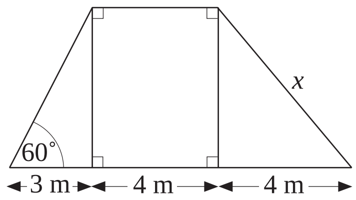
   (a)



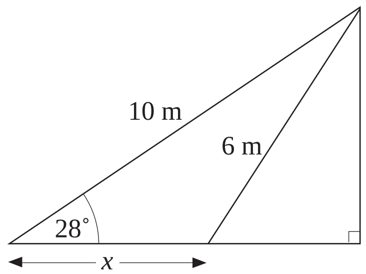
   (b)



(c)



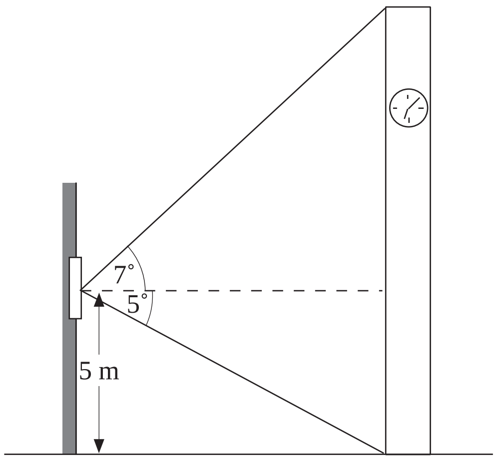
(d)



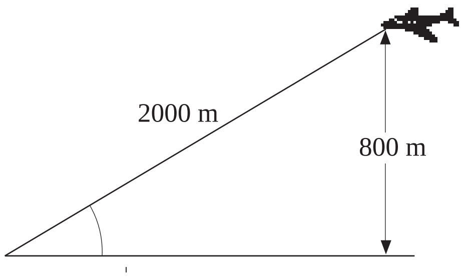
9. From his hotel window a tourist has a clear view of a clock tower. The window is 5 metres above ground level. The angle of depression of the bottom of the tower is $5°$ and the angle of elevation of the top of the tower is $7°$.

   (a) How far is the hotel from the tower?

   (b) What is the height of the tower?



10. A radar operator notes that an aeroplane is at a distance of 2000 metres and at a height of 800 metres. Find the angle of elevation. A little while later the distance has reduced to 1200 metres, but the height remains 800 metres. How far has the aeroplane moved?

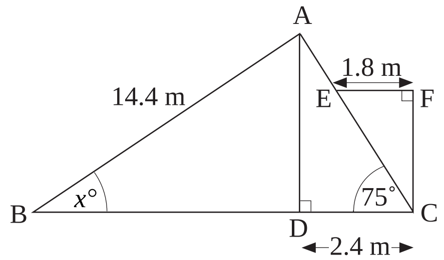


11. The diagram represents a triangular roof frame ABC with a window frame EFC. BDC and EF are horizontal and AD and FC are vertical.

   (a) Calculate the height AD.

   (b) Calculate the size of the angle marked $x°$ in the diagram.

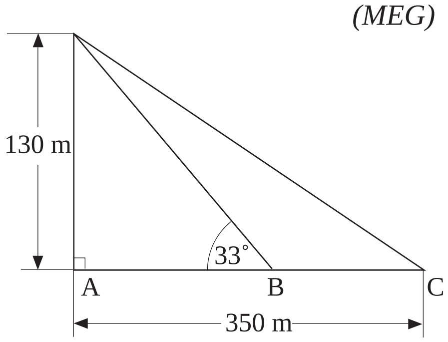
   (c) Calculate FC.



(MEG)

12. Two ships B and C are both due east of a point A at the base of a vertical cliff. The cliff is 130 metres high. The ship at C is 350 metres from the bottom of the cliff.

   (a) (i) Calculate the distance from the top of the cliff to the ship at C.



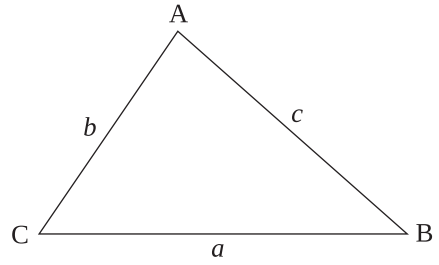
(ii) Calculate the angle of depression from the top of the cliff to the ship at C.

(b) The angle of elevation of the top of the cliff from the ship at B is $33°$. Calculate the distance AB.

(SEG)

4.8 Sine and Cosine Rules

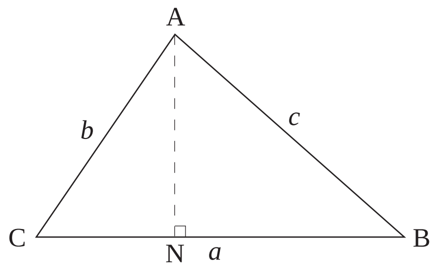
In the triangle ABC, the side opposite angle A has length *a*, the side opposite angle B has length *b* and the side opposite angle C has length *c*.



The *sine rule* states

$$\boxed{\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}}$$

Proof of Sine Rule



If you construct the perpendicular from vertex A to meet side CB at N, then

$$\begin{aligned} AN &= c\sin B \quad &&(\text{from } \Delta \text{ ABN}) \\ &= b\sin C \quad &&(\text{from } \Delta \text{ ACN}) \end{aligned}$$

Hence

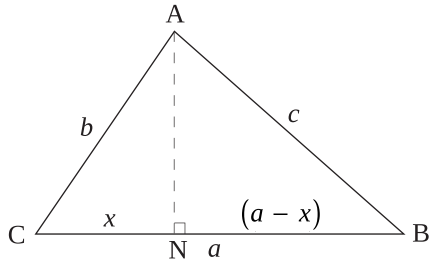
$$c\sin B = b\sin C \Rightarrow \frac{\sin B}{b} = \frac{\sin C}{c}$$

similarly for $\frac{\sin A}{a}$.

The *cosine rule* states

$$\boxed{\begin{aligned} a^2 &= b^2 + c^2 - 2bc\cos A \\ b^2 &= c^2 + a^2 - 2ca\cos B \\ c^2 &= a^2 + b^2 - 2ab\cos C \end{aligned}}$$

Proof of Cosine Rule



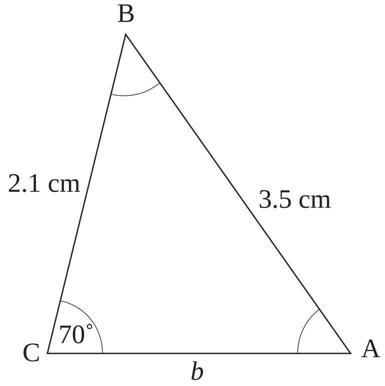
If $CN = x$, then $NB = a - x$ and

$$\begin{aligned} c^2 &= AN^2 + (a-x)^2 \quad \text{when } x = CN \\ &= (b\sin C)^2 + (a - b\cos C)^2, \quad \text{since } x = b\cos C \\ &= b^2\sin^2 C + b^2\cos^2 C - 2ab\cos C + a^2 \\ &= b^2\left(\sin^2 C + \cos^2 C\right) + a^2 - 2ab\cos C \end{aligned}$$

i.e. $c^2 = b^2 + a^2 - 2ab\cos C$, since $\sin^2 C + \cos^2 C = 1$

Worked Example 1

Find the unknown angles and side length of the triangle shown.



Solution

Using the sine rule,

$$\frac{\sin A}{2.1} = \frac{\sin 70°}{3.5} = \frac{\sin B}{b}$$

From the first equality,

$$\sin A = \frac{2.1 \times \sin 70°}{3.5} = 0.5638$$

$$A = 34.32°$$

Since angles in a triangle add up to $180°$,

$$B = 180° - 70° - A = 75.68°$$

From the sine rule,

$$\frac{\sin 70°}{3.5} = \frac{\sin B}{b}$$

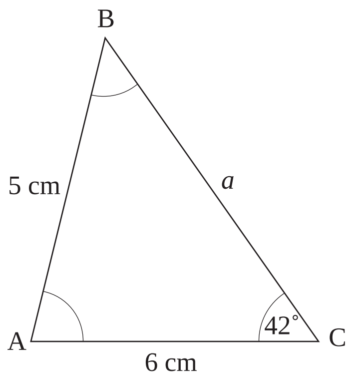
$$b = \frac{3.5 \times \sin B}{\sin 70°}$$

$$= \frac{3.5 \times \sin 75.68°}{\sin 70°}$$

$$= 3.61 \text{ cm}$$

Worked Example 2

Find two solutions for the unknown angles and side of the triangle shown.



Solution

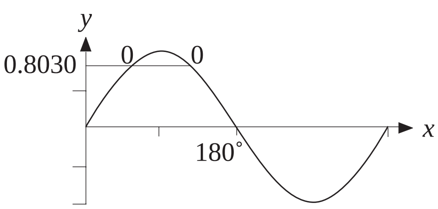
Using the sine rule,

$$\frac{\sin A}{a} = \frac{\sin B}{6} = \frac{\sin 42°}{5}$$

From the second equality,

$$\sin B = \frac{6 \times \sin 42°}{5} = 0.8030$$

A graph of $\sin x$ shows that between $0°$ and $180°$ there are two solutions for B.



These solutions are $B = 53.41°$ and, by symmetry, $B = 180 - 53.41$

$$= 126.59°$$

Solving for angle A we have

$$A = 180° - 42° - B$$

$$\text{when } B = 53.41°, \quad A = 84.59°$$
$$\text{when } B = 126.59°, \quad A = 11.41°$$

From the sine rule,

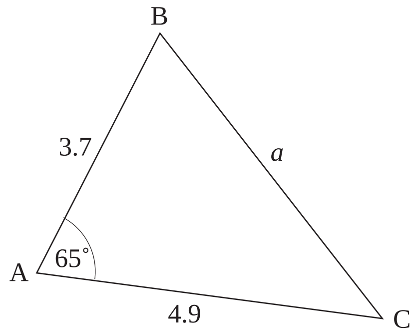
$$a = \frac{6 \times \sin A}{\sin B}$$

For $A = 84.59°$, $B = 53.41°$, $a = 7.44$ cm

For $A = 11.41°$, $B = 126.59°$, $a = 1.48$ cm

Worked Example 3

Find the unknown side and angles of the triangle shown.



Solution

To find a, use the cosine rule:

$$a^2 = 3.7^2 + 4.9^2 - 2 \times 3.7 \times 4.9 \times \cos 65°$$
$$a^2 = 22.3759$$
$$a = 4.73 \quad \text{(to 2 d.p.)}$$

To find the angles, use the sine rule:

$$\frac{\sin 65°}{a} = \frac{\sin B}{4.9} = \frac{\sin C}{3.7}$$

$$\sin B = \frac{4.9 \times \sin 65°}{a} = \frac{4.9 \times \sin 65°}{4.73} = 0.9389$$
$$B = 69.86°$$

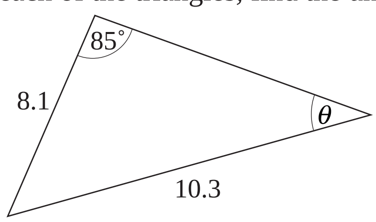
$$\sin C = \frac{3.7 \times \sin 65°}{a} = \frac{3.7 \times \sin 65°}{4.73} = 0.7090$$
$$C = 45.15° \quad \text{(alternatively, use } A + B + C = 180° \text{ to find C)}$$

Checking, $A + B + C = 65° + 69.86° + 45.15° = 180.01°$. The three angles should add to $180°$; the extra $0.01°$ is due to rounding errors.

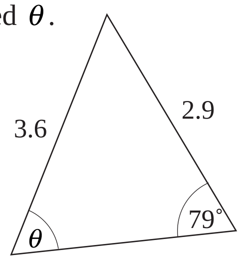
Exercises

1. For each of the triangles, find the unknown angle marked θ.

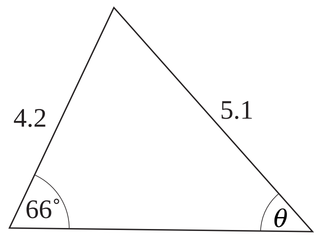
(a)



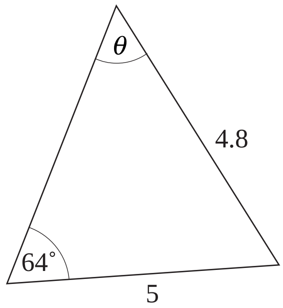
(b)



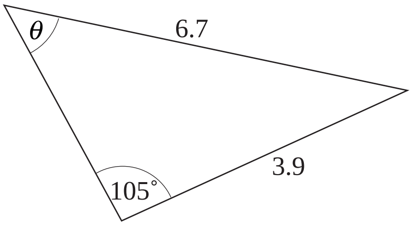
(c)



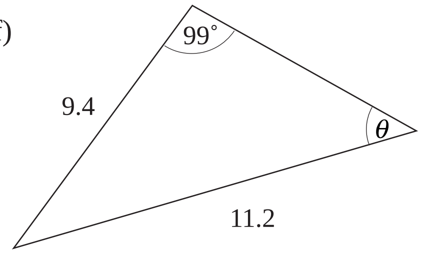
(d)



(e)

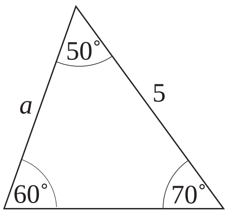


(f)

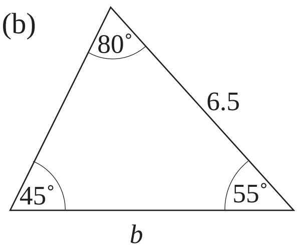


2. For each triangle, find the unknown side marked *a*, *b* or *c*.

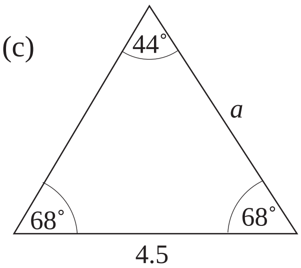
(a)



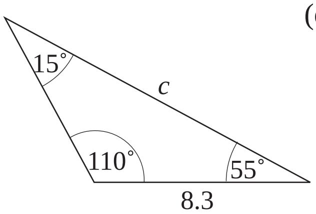
(b)



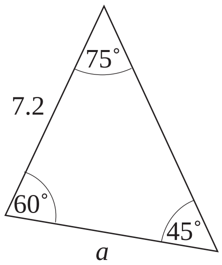
(c)



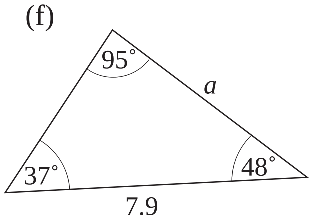
(d)



(e)

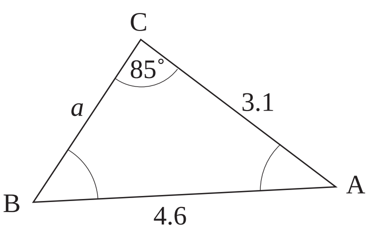


(f)

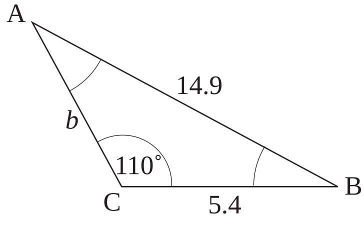


3. For each of the triangles, find the unknown angles and sides.

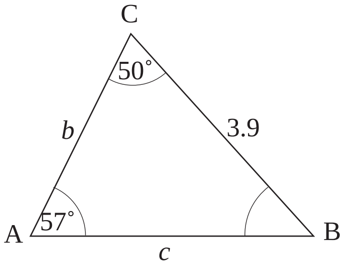
(a)



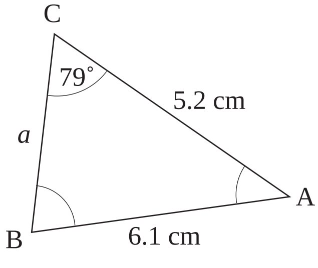
(b)



(c)

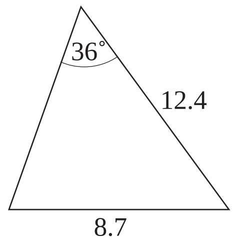


(d)

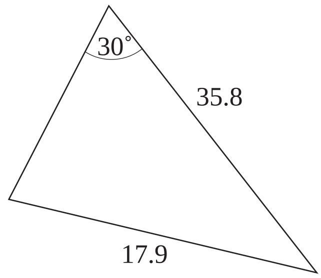


4. Which of the following triangles could have *two* solutions?

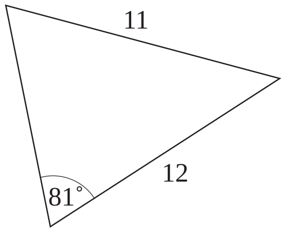
(a)



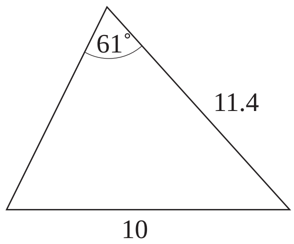
(b)



(c)



(d)

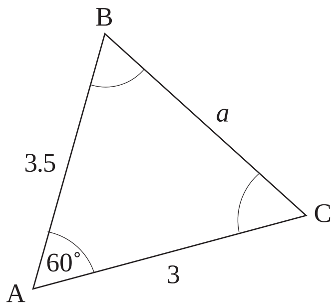


5. Find the remaining angles and sides of the triangle ABC if A = 67°, $a = 125$ and $c = 100$.

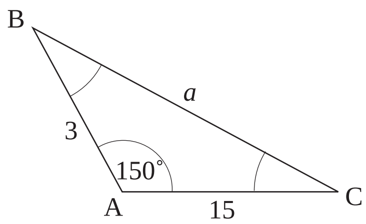
6. Find the remaining angles and sides of the triangle ABC if B = 81°, $b = 12$ and $c = 11$.

7. For each of the following triangles, find the unknown angles and sides.

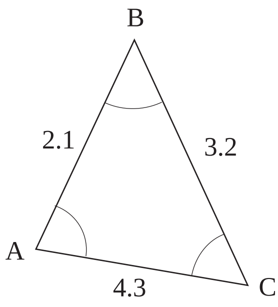
(a)



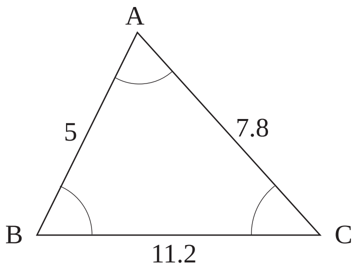
(b)



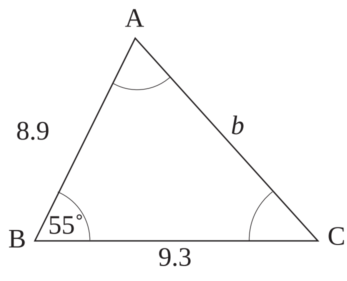
(c)



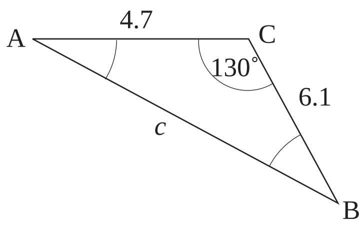
(d)



(e)



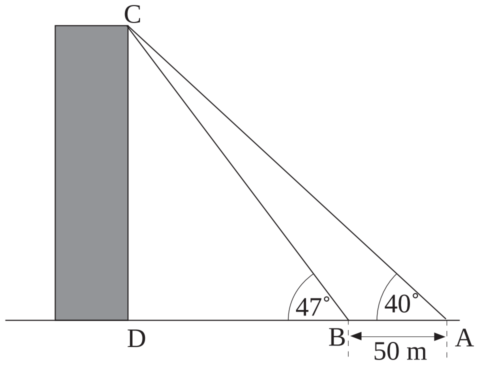
(f)

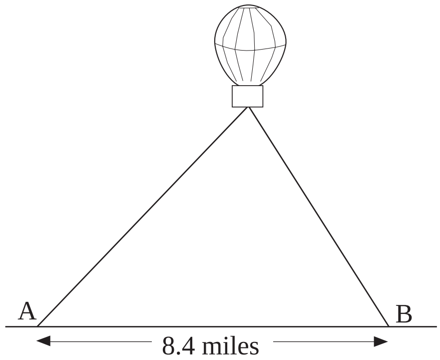


8. To calculate the height of a church tower, a surveyor measures the angle of elevation of the top of the tower from two points 50 metres apart. The angles are shown in the diagram.

   (a) Calculate the distance BC.

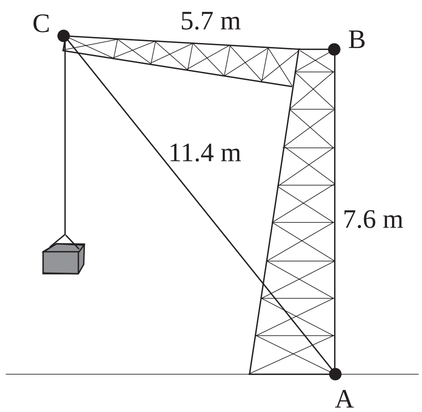
   (b) Hence calculate the height of the tower CD.





9. The angles of elevation of a hot air balloon from two points, A and B, on level ground, are $24.2°$ and $46.8°$, respectively.

   The points A and B are 8.4 miles apart, and the balloon is between the points in the same vertical plane. Find the height of the balloon above the ground.

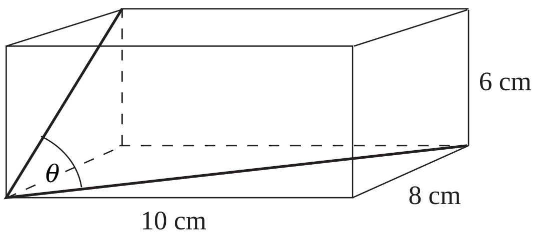


10. The diagram shows a crane working on a wharf. AB is vertical.

    (a) Find the size of angle ABC.

    (b) Find the height of point C above the wharf.

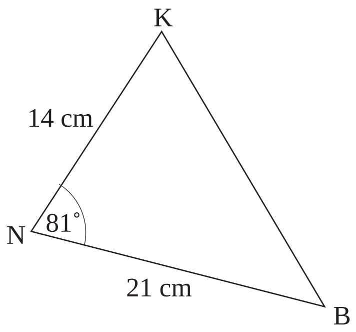
11. The rectangular box shown in the diagram has dimensions 10 cm by 8 cm by 6 cm. Find the angle θ formed by a diagonal of the base and a diagonal of the 8 cm by 6 cm side.



12. (a) Calculate the length KB.

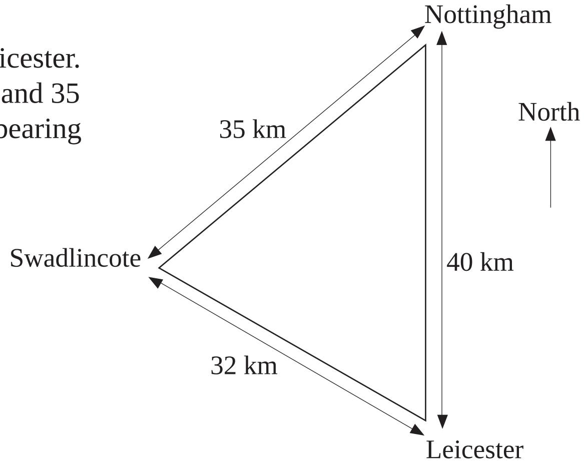
(b) Calculate the size of the angle NKB.

(LON)



13. Nottingham is 40 km due north of Leicester. Swadlincote is 32 km from Leicester and 35 km from Nottingham. Calculate the bearing of Swadlincote from Leicester.

(MEG)

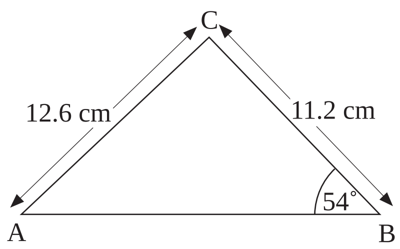


14. In triangle ABC, $AC = 12.6$ cm, $BC = 11.2$ cm and angle $B = 54°$. The lengths AC and BC are correct to the nearest millimetre and angle B is correct to the nearest degree. Use the sine rule

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

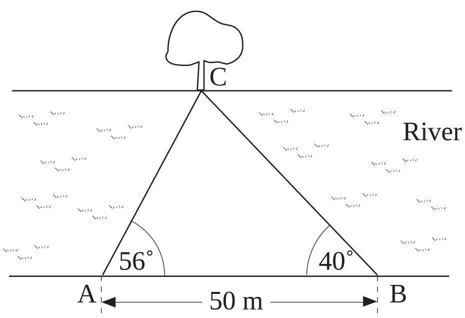
to calculate the smallest possible value of angle A.

(MEG)

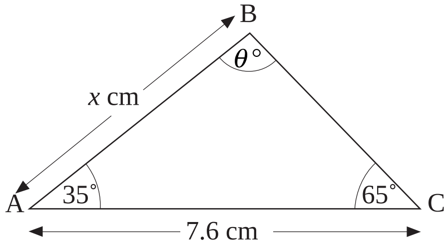


15. The banks of a river are straight and parallel. To find the width of the river, two points, A and B, are chosen 50 metres apart. The angles made with a tree at C on the opposite bank are measured as angle $CAB = 56°$, angle $CBA = 40°$. Calculate the width of the river.

(SEG)



16. In triangle ABC, $AC = 7.6$ cm, angle $BAC = 35°$, angle $ACB = 65°$. The length of AB is x cm. The size of angle ABC is $\theta°$.



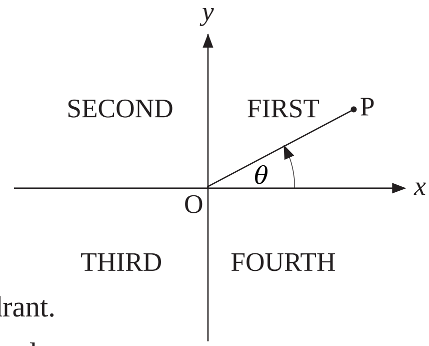
(a) Write down the value of θ.

(b) Hence calculate the value of x.

(NEAB)

4.9 Angles Larger than 90°

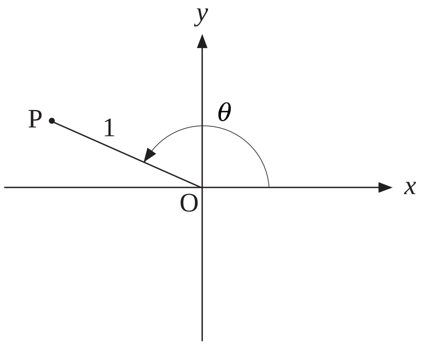
The x - y plane is divided into four quadrants by the x and y axes. The angle θ that a line OP makes with the positive x-axis lies between $0°$ and $360°$.



Angles between 0 and $90°$ are in the *first* quadrant.
Angles between $90°$ and $180°$ are in the *second* quadrant.
Angles between $180°$ and $270°$ are in the *third* quadrant.
Angles between $270°$ and $360°$ are in the *fourth* quadrant.

Angles bigger than $360°$ can be reduced to lie between $0°$ and $360°$ by subtracting multiples of $360°$.

The trigonometric formulae, $\cos\theta$ and $\sin\theta$, are defined for all angles between $0°$ and $360°$ as the coordinates of a point, P, where OP is a line of length 1, making an angle θ with the positive x-axis.



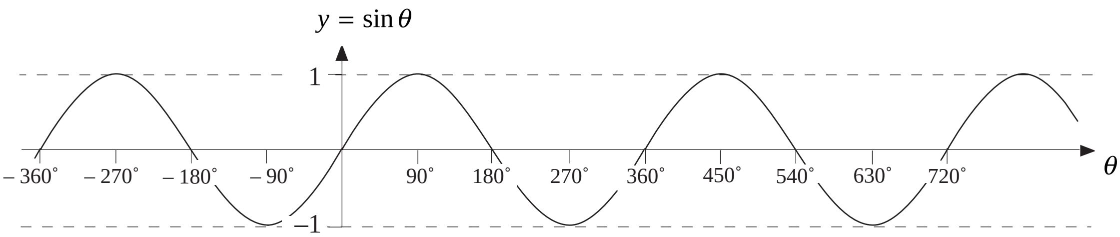
Information

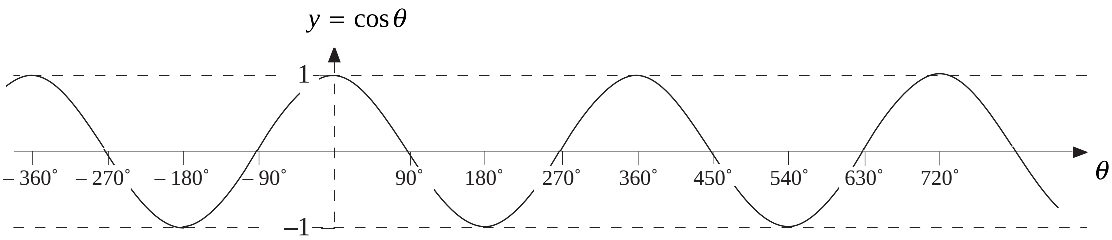
The Greeks, *(in their analysis of the arcs of circles) were the first to establish the relationships or ratios between the sides and the angles of a right angled triangle.*
The Chinese *also recognised the ratios of sides in a right angled triangle and some survey problems involving such ratios were quoted in Zhou Bi Suan Jing.*
It is interesting to note that sound waves are related to the sine curve. This discovery by Joseph Fourier, *a French mathematician, is the essence of the electronic musical instrument developments today.*

Some important values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ are shown in this table.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$0°$	0	1	0
$30°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$45°$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
$60°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$
$90°$	1	0	infinite

The graphs of $\sin\theta$ and $\cos\theta$ for any angle are shown in the following diagrams.





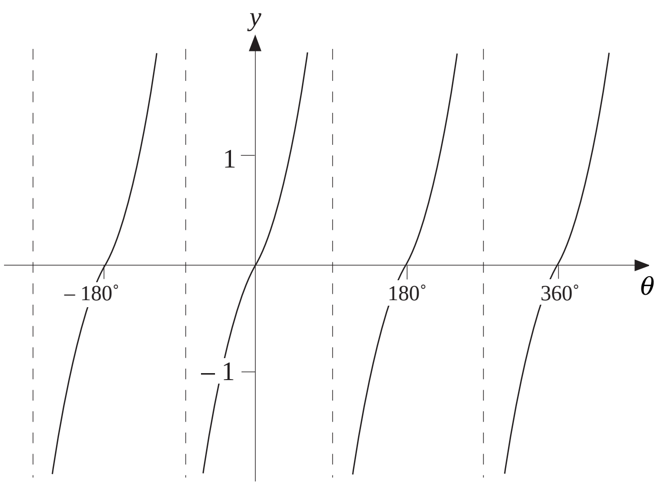
The graphs are examples of *periodic functions.* Each basic pattern repeats itself every $360°$. We say that the *period* is $360°$.

Sin θ and $\cos\theta$ are often called *sinusoidal functions*.

Note

For any angle, note that

$\sin(\theta - 90°) = \cos\theta$.



The graph of $\tan\theta$ has period $180°$.

It is an example of a *discontinuous graph.*

The trigonometric equations $\sin\theta = a$, $\cos\theta = b$ and $\tan\theta = c$ can have many solutions. The inverse trigonometric keys on a calculator $(\sin^{-1}, \cos^{-2}, \tan^{-1})$, give the *principal value solution.*

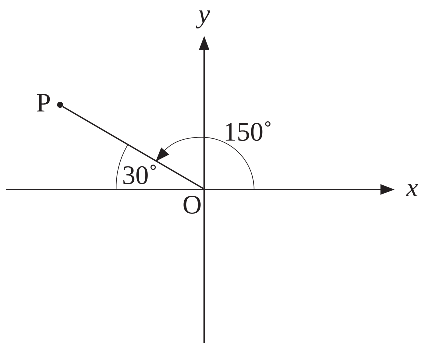
For $\sin\theta = a$ and $\tan\theta = c$, the principal value solution is in the range $-90° < \theta \le 90°$.

For $\cos\theta = b$, the principal value solution is in the range $0 \le \theta < 180°$.

Worked Example 1

Find $\cos 150°$, $\sin 240°$, $\cos 315°$ and $\sin 270°$.

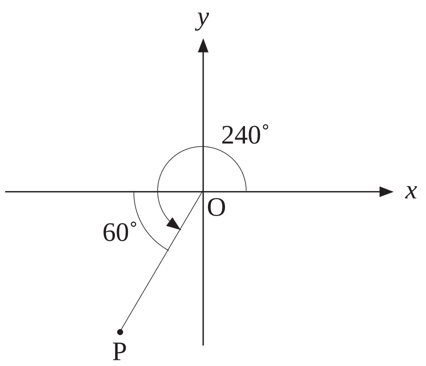
Solution



$150°$ is in the second quadrant.
The coordinates of point P are $(-\cos 30°,\ \sin 30°)$.

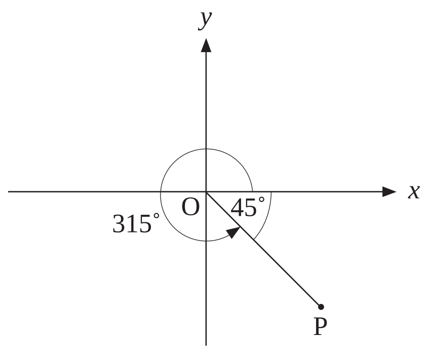
Hence $\cos 150° = -\cos 30° = -\dfrac{\sqrt{3}}{2}$.

(Note also that $\sin 150° = \sin 30° = \dfrac{1}{2}$.)



$240°$ is in the third quadrant.
The coordinates of point P are $(-\cos 60°,\ -\sin 60°)$.

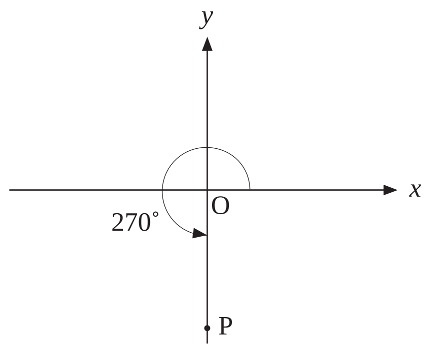
Hence $\sin 240° = -\sin 60° = -\dfrac{\sqrt{3}}{2}$.



$315°$ is in the fourth quadrant.
The coordinates of point P are $(\cos 45°,\ -\sin 45°)$.

Hence $\cos 315° = \cos 45° = \dfrac{1}{\sqrt{2}}$.

$270°$ lies on the *y* axis. Sin $270° = -1$ since the coordinates of P are $(0, -1)$.

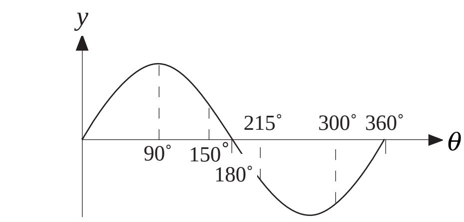


Worked Example 2

Sketch a graph of $\sin\theta$ for $0 \le \theta \le 360°$. From the graph, deduce the values of $\sin 150°$, $\sin 215°$, $\sin 300°$.

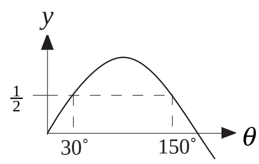
Solution

A sketch of the graph of $\sin\theta$ looks like this.



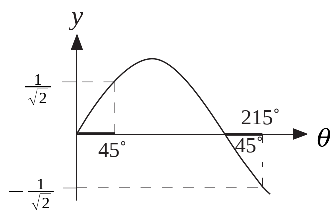
From the symmetry of the curve we can deduce that

$$\sin 150° = \sin 30° = \frac{1}{2}$$

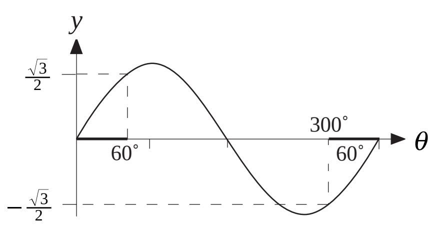


$$\sin 180° = 0$$

$$\sin 215° = -\sin 45° = -\frac{1}{\sqrt{2}}$$



$$\sin 300° = -\sin 60° = -\frac{\sqrt{3}}{2}$$



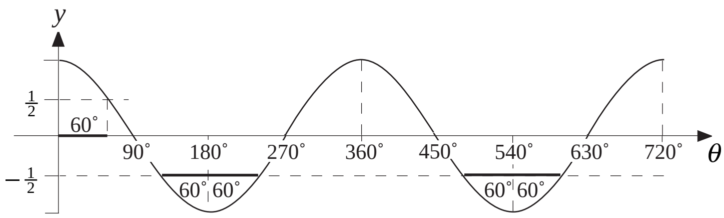
Worked Example 3

If $\cos\theta = -\dfrac{1}{2}$, how many values of the angle θ are possible for $0 \le \theta \le 720°$?

Find these values for θ.

Solution

A graph of $\cos\theta$ shows that there are four possible values for θ.



Using the symmetry of the graph, the values of θ are

$$\theta = 120°,\ 240°,\ 480°,\ 600°$$

The solution in the range $0 \le \theta < 180°$, $\theta = 120°$, is called the *principal value.*

Worked Example 4

Use a calculator to solve the equation $\sin\theta = -0.2$.

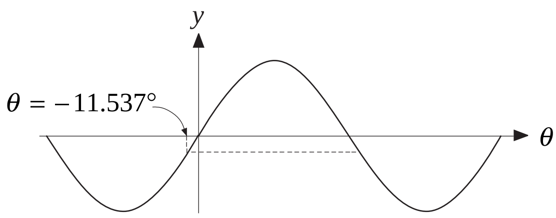
Sketch the sine graph to show this solution. Give the principal value solution.

Solution

Using the $\sin^{-1}$ key on a calculator gives

$$\theta = \sin^{-1}(-0.2) = -11.537°$$

A sketch of the graph of $\sin\theta$ shows why θ is negative.



The principal value solution is $-11.537°$.

Worked Example 5

An angle θ is such that $\cos\theta = -0.6$, $\sin\theta = -0.8$ and $0 \le \theta \le 360°$.
Deduce in which quadrant the angle lies,

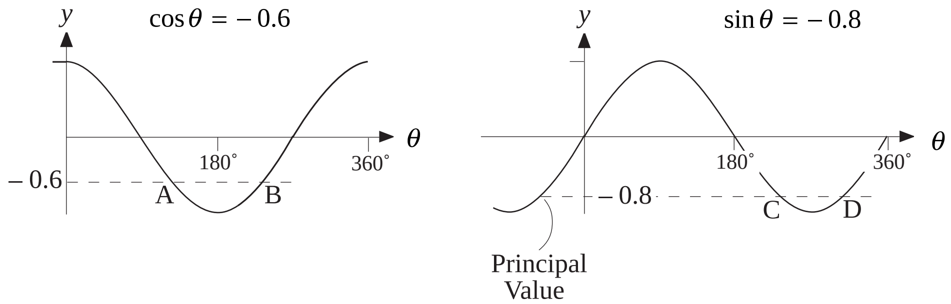
(a) from the graphs of $\sin\theta$ and $\cos\theta$, and

(b) from the quadrant definition of the point $(\cos\theta, \sin\theta)$.

Hence, using a calculator, find the value of θ.

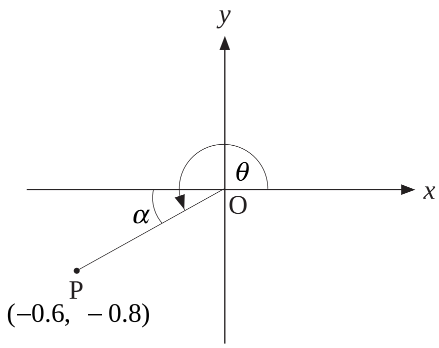
Solution

(a) The following graphs show the possible solutions for θ between $0°$ and $360°$.



From the graphs we deduce that the value of θ for which $\cos\theta = -0.6$ and $\sin\theta = -0.8$ must lie between $180°$ and $270°$, i.e. at point B on the cosine curve and at point C on the sine curve.

(b) From the quadrant definition of the point $(\cos\theta, \sin\theta)$ we see that the point P lies in the third quadrant for which $180° \le \theta \le 270°$.



The $\cos^{-1}$ and $\sin^{-1}$ keys on a calculator give the principal values

$$\theta = \cos^{-1}(-0.6) = 126.87°$$

$$\theta = \sin^{-1}(-0.8) = -53.13°$$

From the graph of $\sin\theta$, for point C we deduce that

$$\begin{aligned}\theta &= 180° + 53.13° \\ &= 233.13°\end{aligned}$$

From the quadrant approach we calculate $\tan\alpha$ using the coordinates of P.

$$\tan\alpha = \frac{0.6}{0.8} = 0.75 \quad \text{so} \quad \alpha = \tan^{-1} 0.75 = 53.13°,$$

and hence $\theta = 180° + 53.13° = 233.13°$.

Exercises

1. Without using a calculator, apply the quadrant definition to find the values of:

(a) $\sin 60°$ (b) $\sin 210°$ (c) $\cos 135°$

(d) $\cos 240°$ (e) $\sin 315°$ (f) $\cos 180°$

(g) $\cos 300°$ (h) $\sin 120°$ (i) $\sin 495°$

(j) $\sin 660°$ (k) $\cos 540°$ (l) $\cos 600°$

2. Sketch graphs of $\sin\theta$ and $\cos\theta$ for $0 \le \theta \le 720$. Without using a calculator, use the symmetry of the graphs to find the values of the $\sin\theta$ and $\cos\theta$ in problem 1.

Now check your answers with a calculator.

3. Use a calculator to find the values of the following. In each case show the answer on sketch graphs of $\sin\theta$ or $\cos\theta$.

(a) $\sin 130°$ (b) $\sin 235°$ (c) $\sin 310°$

(d) $\sin 400°$ (e) $\sin 830°$ (f) $\sin 1310°$

(g) $\cos 170°$ (h) $\cos 190°$ (i) $\cos 255°$

(j) $\cos 350°$ (k) $\cos 765°$ (l) $\cos 940°$

4. Sketch a graph of $y = \sin\theta$ for $-360° \le \theta \le 720°$. For this domain of θ, how many solutions are there of the equation $\sin\theta = -\frac{1}{\sqrt{2}}$?

Use the symmetry of the graph to deduce these solutions. What is the principal value?

5. Sketch a graph of $y = \cos\theta$ for $-360° \le \theta \le 720°$. For this domain of θ, how many solutions are there of the equation $\cos\theta = \frac{1}{2}$?

Use the symmetry of the graph to deduce these solutions. What is the principal value?

6. Using a calculator and sketch graphs, find all the solutions of the following equations for $-360 \le \theta \le 360°$.

(a) $\sin\theta = 0.7$ (b) $\sin\theta = -0.4$ (c) $\sin\theta = -1$

(d) $\cos\theta = 0.6$ (e) $\cos\theta = -0.4$ (f) $\cos\theta = -1$

7. Use a calculator and a sketch graph of $y = \tan\theta$ to solve the equation for $0 \le \theta \le 720°$.

(a) $\tan\theta = 0.25$ (b) $\tan\theta = 1$ (c) $\tan\theta = -0.5$

8. In each of the following problems find the value of θ in the range 0 to $360°$ that satisfies both equations.

   (a) $\cos\theta = 0.6$ and $\sin\theta = -0.8$

   (b) $\cos\theta = -0.8$ and $\sin\theta = 0.6$

   (c) $\sin\theta = -0.6428$ and $\cos\theta = -0.7660$ (each correct to 4 d.p.)

   (d) $\sin\theta = -1$ and $\cos\theta = 0$

9. Use a graphic calculator or computer software for this problem.

   (a) Draw a graph of $y = \sin 2x$ for values of x between $-360°$ and $360°$.

   (b) Compare your graph with $y = \sin x$. What is the period of the function $\sin 2x$?

   (c) Repeat parts (a) and (b) for $y = \sin 3x$ and $y = \sin\frac{1}{2}x$.

   (d) Use your answers to sketch a graph of $y = \sin ax$.

   (e) Draw a graph of $y = 2\sin x$ for values of x between $-360°$ and $360°$. What is the relationship between the graphs of $y = 2\sin x$ and $\sin x$?

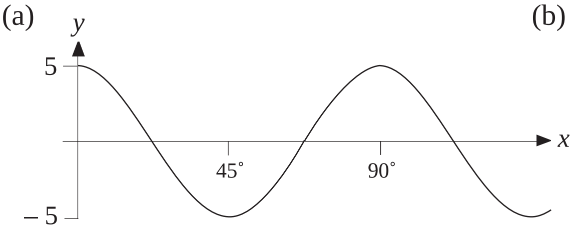
   (f) Repeat part (e) for $y = 3\sin x$ and $\frac{1}{2}\sin x$.

   (g) Use your answers in parts (e) and (f) to sketch a graph of $b\sin x$.

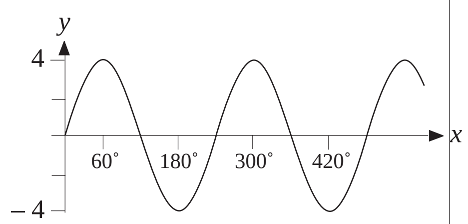
   (h) Sketch a graph of $y = b\sin ax$.

10. Find formulae in terms of sine or cosine for the following graphs.

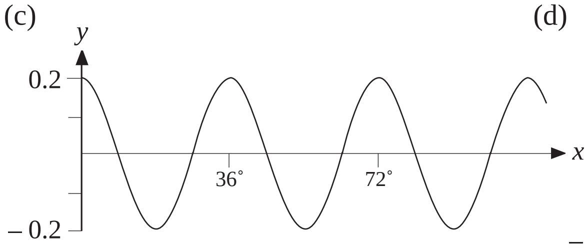
(a)



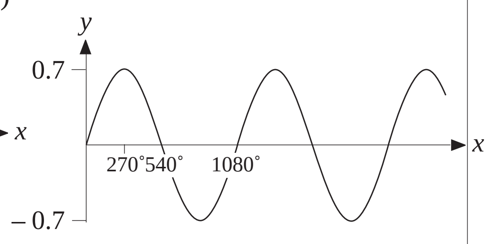
(b)



(c)



(d)



11. Draw graphs of the following.

    (a) $y = 1 + \cos x$ (b) $y = 3 + \cos x$ (c) $y = \cos x - 2$

    What is the relationship between these graphs and the graph of $y = \cos x$?

12. At a time t hours after midnight, the depth of water, d, in metres, in a harbour is given by

$$d = 8 + 5\sin(30t)°$$

    Draw up a table to show the depth of water in the harbour on each hour of the day.

13. The mean monthly temperature in Crapstone, Devon, in August is $21°\text{C}$ and the minimum temperature in February is $0°\text{C}$. Assuming that the variation in temperature is periodic satisfying a sine function, obtain a mathematical model to represent the mean monthly temperature. Use your model to predict the mean monthly temperature in June and January.

14. The variation in body temperature is an example of a biological process that repeats itself approximately every 24 hours, and is called a *circadian rhythm.* Body temperature is highest ($98.9°\text{C}$) around 5 pm (1700 hours) and lowest ($98.3°\text{C}$) around 5 am (0500 hours). Let T be the body temperature in $°\text{C}$ and t be the time in hours.

    (a) Sketch a curve of the body temperature against time, using the given information.

    (b) Choosing $t = 0$ so that the model of temperature is a cosine function, find a formula of the form that fits the given information.

15. This question is about angles between $0°$ and $360°$.

    (a) Find the *two* solutions of the equation

$$\cos x = 0.5$$

    (b) Angle p satisfies the equation

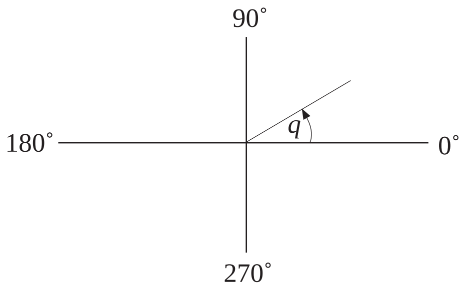
$$\sin p = \sin 210°$$

    Angle p is not equal to $210°$.

    Find the value of p.

    (c) Sketch the graph of $y = 5\sin x$.

(d) Angle q is shown in the diagram.



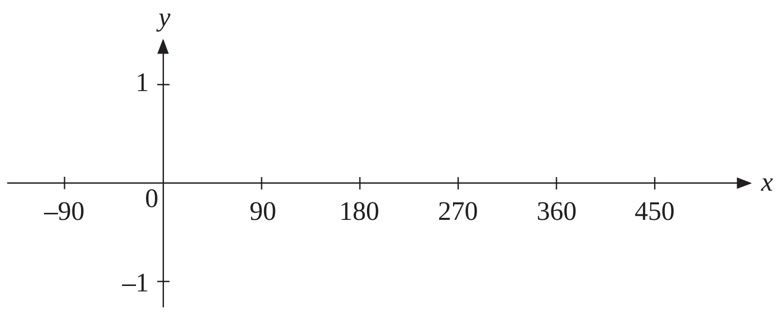
Angles q and r are connected by the equation

$$\tan q = \tan r$$

Copy the diagram and mark clearly the angle r.

(SEG)

16. (a) Sketch the graphs of $y = \cos x°$ on axes similar to those below.



(b) Use your calculator to find the value of x between 0 and 90 for which $\cos x° = 0.5$.

(c) Using your graph and the answer to part (b), find two more solutions in the range $-90 \le x \le 450$ for which $\cos x° = 0.5$.

(MEG)