

UNIT 11 *Fractions and Percentages*

Activities

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11.1 Currency Exchange

11.2 Taxi Fares

11.3 VAT Problems

11.4 Premium Bonds

11.5 APR

Notes and Solutions (1 page)

ACTIVITY 11.1

Currency Exchange

The chart shows the exchange rate between the pound and various foreign currencies on Monday 10 March 1997.

- How many
 - \$ can you obtain for £10,
 - DM can you obtain for £5,
 - A\$ can you obtain for £20,
 - L can you obtain for £2.50?

You can also use the table for converting foreign currency to pounds.

- How many pounds can you obtain for
 - 24 F Fr
 - 47 DM
 - 101000 L
 - 100 \$
 - 4000Y?

THE £ ABROAD		
Australia	A \$	2.03
Canada	C \$	2.20
Denmark	D Kr	10.41
France	F Fr	9.20
Germany	DM	2.73
Hong Kong	HK \$	12.40
Ireland	IR£	1.03
Italy	L	2712.00
Japan	Y	194.91
Spain	Pes	231.40
Switzerland	S Fr	2.36
U.S.	\$	1.60

In practice, most currency exchanges charge commission, either a percentage or a fixed amount.

- Find how much you can obtain for £200 in
 - \$ with a commission charge of 2%,
 - DM with a commission charge of 1.5%,
 - F Fr with a fixed commission charge of £2.50.

Banks have different rates for buying and selling foreign currency, as well as commission charges.

MEP BANK		
	BUY	SELL
£1	2.90 DM	2.73 DM
Commission charge	3%	2%

- How many DM do you get for £1000 at the *MEP Bank*? (Use selling rates.)
- After changing your £1000 into DM, you find that your trip is cancelled.
 - How many pounds do you get back, using the buying and commission rates shown opposite?
 - How much money have you lost?
- Suppose you change £1000 to U.S.\$ at the *SELL* rate above, with 2% commission charge. To what level does the *BUY* rate of the pound have to fall in order to break even when you change back to pounds? (Assume a 3% commission charge.)

ACTIVITY 11.2

Taxi Fares

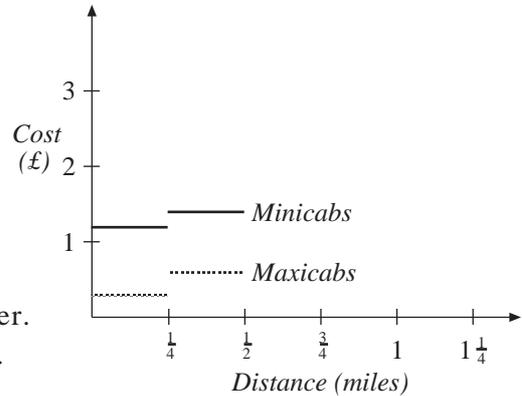
Two local taxi firms offer the rates shown.

<i>Minicabs</i>	£1 plus 20p for every $\frac{1}{4}$ mile
<i>Maxicabs</i>	30p for every $\frac{1}{4}$ mile

1. Find the cost for *both* firms of journeys of length
 - (a) $\frac{1}{2}$ mile
 - (b) 1 mile
 - (c) 2 miles

2. (a) Copy and complete the table below. (b) Display the information on a graph as shown below.

Distance (miles)	up to						
	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{4}$...	3
<i>Minicabs</i>	1.20	2.00	
<i>Maxicabs</i>	0.30	1.50	



3. When is it more economical to use *Minicabs*?
4. *Minicabs* also charge 25p for each extra passenger. If there are two people travelling, how does your answer to Question 3 change?

The current *London Taxi* fares are shown opposite.

5. How much will it cost for a journey of length
 - (a) $\frac{1}{2}$ mile
 - (b) 1 mile
 - (c) 2 miles
 using a *London Taxi*?

LONDON TAXI FARES		
£1	for the first	873 yards
25p	for every	291 yards

6. Illustrate the costs by drawing a cost/distance graph.

(1760 yards = 1 mile)

Extension

London Taxis also charge 40p for each extra person.

The *Underground* train fare for Inner London is £1.20 per person.

If 5 people are travelling together in London, at what distance does it become more expensive to use a *London Taxi* rather than the *Underground*?

ACTIVITY 11.3

VAT Problems

In the UK most articles are sold at the basic price plus

Value Added Tax (VAT) of $17\frac{1}{2}\%$.

Some goods such as cars and fuel, have an extra tax, whilst others such as food and children's clothes are exempt from VAT.

For example, a portable music centre of basic price £200, will also have VAT of

$$£200 \times \frac{17.5}{100} = £35$$

added, to give a total price of £235.

Finding the VAT to be charged using a calculator is relatively straightforward. However, even *without* a calculator, VAT is easy to find by calculating 10%, 5% and $2\frac{1}{2}\%$, and then adding them up.

So, for £200, we have	10% → £ 20	
	5% → £ 10	(divide by 2)
	$2\frac{1}{2}\%$ → £ 5	(divide again by 2)
	17½% → £ 35	(add up)

1. *Without* using a calculator, find the VAT to be added for articles at basic price
 - (a) £120
 - (b) £80
 - (c) £500.

Businessmen and companies can often claim back VAT. For example, if the total price is £235, they can claim back £35.

2. What is the VAT when the total price is (a) £117.50 (b) £470 ?
3. If £17.50 is the VAT part of a total price of £117.50,
 - (a) what is the VAT part of a total price of £1.00 ?
 - (b) what is the VAT part of a total price of £ x ?
4. (a) Explain why dividing the total price by 6.71 approximately gives the VAT .
 (b) Give the values of the divisor (correct to 5 significant figures), which should be used for more accurate calculations.
5. (a) Suppose VAT is increased to 19%. What divisor is now needed to find the VAT part of the total price?
 (b) Use your divisor to find the VAT included in these total prices:
 - (i) £119
 - (ii) £50
 - (iii) £80.

Extension

Generalise the formulae for the divisor for VAT at a rate of $r\%$

ACTIVITY 11.4a

Premium Bonds

Despite the success of the National Lottery, the public continue to invest in £1 *National Savings Premium Bonds*. These are safer than the lottery as you do not lose your initial investment. In 1996, a £1 million monthly winner was introduced, as well as 349 999 lower value wins. The prize breakdown in April 1997 is given below.

£1m	£100 000	£50 000	£25 000	£10 000	£5000	£1000	£500	£100	£50
1	4	9	17	44	85	1882	5646	128 468	213 844

1. Check that there are 350 000 winners in total.
 2. What was the total prize money given out in April 1997?
- 4.75% of the total money invested in *Premium Bonds* is given out each year in 12 monthly prize draws.
3. Assuming the same amount of prize money is given out each month, what is the total prize money given out in 1997? Estimate the total amount of money invested in *Premium Bonds* in 1997.
 4. How many prizes are given in total in a year?
 5. What is the probability of winning once a year if you hold one £1 Premium Bond?
 6. How many Premium Bonds do you need to hold to expect to win one prize in a year?

Extension

If you invest in 10 000 £1 Premium Bonds, on average how many wins will you expect in a year?

ACTIVITY 11.4b

Premium Bonds

Currently, the actual number of prizes in each category is worked out along the following lines.

<i>Prize Value (£)</i>			
1 000 000 100 000 50 000 25 000 10 000 5 000	} } } } } }	'Higher' value prizes 10% of monthly prize fund	{ { { { { {
<i>One £1 million prize and the remaining shared out so that, in each category, the same amount of money is paid out.</i>			

1000 500	} }	'Medium' value prizes 15% of monthly prize fund	{ { {
<i>Allocated in the ratio of one £1000 prize for every three £500 prizes.</i>			

100 50	} }	'Lower' value prizes 75% of monthly prize fund	{ { {
<i>Allocated so that there are the maximum number of £100 prizes consistent with a total of 350 000 prizes.</i>			

1. If there is a £25 million monthly prize fund, how much is allocated to 'Higher', 'Medium' and 'Lower' value prizes?
2. How much is allocated each month to each category of the 'Higher' value prizes (after the £1 million prize has been allocated)?
How many of each of the 'Higher' value prizes will be given out?
3. How many £1000 and £500 prizes will there be each month?

Extension

Let x = number of £100 prizes, y = number of £50 prizes. Find two simultaneous equations satisfied by x and y . Hence solve for x and y .

ACTIVITY 11.5

APR

Interest on investment money, and interest on loans such as credit cards, can be offered in a confusing variety of ways (see opposite). To help compare different accounts or loans, the concept of

Annual Percentage Rate (APR)

was developed.


MEP Building Society


Investment Accounts

Choose the account to suit **you!**

9.25% p.a.	paid yearly
8.88% p.a.	paid monthly
8.83% p.a.	paid daily

For an account paying interest at $r\%$ per annum once a year, the APR is $r\%$. But for an account paying interest of, say, 10% per annum, every six months, the APR will be more than 10%.

Suppose you invest £100 at the beginning of the year. After six months, the interest payable is

$$£100 \times \left(10 \times \frac{1}{2}\right)\% = £100 \times \frac{5}{100} = £5$$

This is added to the account, to give a balance after 6 months of £105. After the next 6 months, i.e. after one year from the start of the investment, the interest payable is

$$£105 \times \left(10 \times \frac{1}{2}\right)\% = £105 \times \frac{5}{100} = £5.25$$

giving an end of year balance of £110.25. So the APR is 10.25%. This is not much more than the advertised yearly rate, but significant if you are borrowing or investing a large sum of money.

1. Find the APR for the following investment accounts:
 - (a) 9% p.a. paid monthly (the monthly rate will be $\frac{1}{12} \times 9\%$)
 - (b) 12% p.a. paid monthly.

The method of finding the APR generalises to accounts paying interest monthly, weekly or even daily.

2. Find the APR for the following investment account:
 - (a) 20% p.a. paid monthly
 - (b) 19% p.a. paid weekly
 - (c) 18.5% p.a. paid daily.
3. One savings account advertises a yearly rate of interest but states that it is *paid daily*. They claim this gives an APR of 10%. What rate is given in the advertisement?
4. Go back to the advertisement at the top of this sheet. Does it matter which account you choose to invest in?

Extension

The table opposite gives three ways of buying an *MEP GTi*.

Explain how the APR values quoted are calculated.

<i>MEP GTi</i>	on the road price £10 156		
Payment over	24 months	36 months	48 months
Flat rate	0 %	3.4%	4.9 %
APR	0 %	7.7%	9.6 %
Initial deposit	50 %	20 %	10 %
Initial payment	£5078.00	£2031.20	£1015.60
Monthly payments	£211.57	£252.04	£227.75
Admin. fee	NIL	£35.25	£35.25
Total Payment	£10 156.00	£11 139.89	£11 982.85