UNIT 1 *Logic* Activities

Activities

ACTIVITY 1.1 *Two Way Tables*

This is a whole-class or group activity.

You are going to divide up into groups according to *two* criteria. You need to either use the four corners of your classroom, or make sufficient space in one area of the room.

In each case, divide up into the appropriate corner or 'cell' according to the criteria given.

1.

	Boys	*Girls*
No sisters or brothers		
At least one sister or brother		

Add up the total number of pupils in each cell. What does this tell you?

2.

	No sisters	*One or more sisters*
No brothers		
One or more brothers		

What must the total number of pupils in each cell add up to?

3.

	No sisters	*One sister*
No brothers		
One brother		

What does the total number of pupils in each cell now add up to?

Describe the criteria for pupils who are not in any of the cells.

ACTIVITY 1.2 *Shapes in Two Way Tables*

This activity requires the use of a set of shapes (given on A 1.2a).

You will also need large (A3) 2×2 and 3×3 tables on which to place the shapes.

In each case, put the complete set of shapes, 40 in total, into the appropriate cell, according to the criteria given.

1. (a)

	No. of edges	
	All straight	Not all straight
Shaded		
Not shaded		

(b)

	No. of edges	
	2 or less	3 or more
Shaded		
Not shaded		

2.

	No. of edges		
	3 or less	4	5 or more
Shaded			
Not shaded			

3.

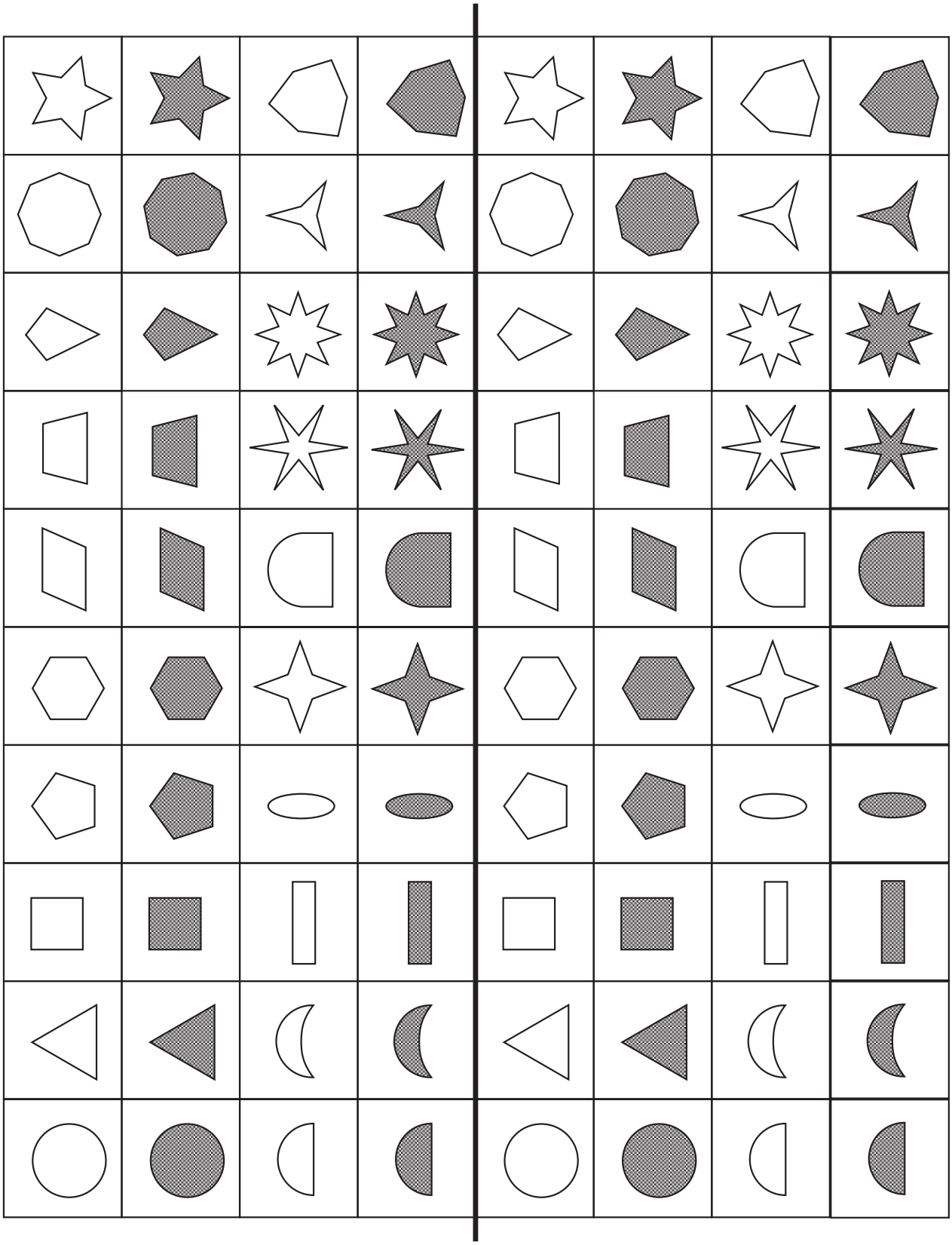
		No. of edges		
		3 or less	4	5 or more
Type of edge	All round			
	Some round and some straight			
	All straight			

4.

		No. of edges		
		3 or less	4	5 or more
No. of vertices	None or one			
	Two or three			
	Four or more			

Do all the shapes belong in a cell?

ACTIVITY 1.2 a *Shapes*



ACTIVITY 1.2 b *Numbers*

10	20	30	40	10	20	30	40
9	19	29	39	9	19	29	39
8	18	28	38	8	18	28	38
7	17	27	37	7	17	27	37
6	16	26	36	6	16	26	36
5	15	25	35	5	15	25	35
4	14	24	34	4	14	24	34
3	13	23	33	3	13	23	33
2	12	22	32	2	12	22	32
1	11	21	31	1	11	21	31

ACTIVITY 1.2 c *Letters*

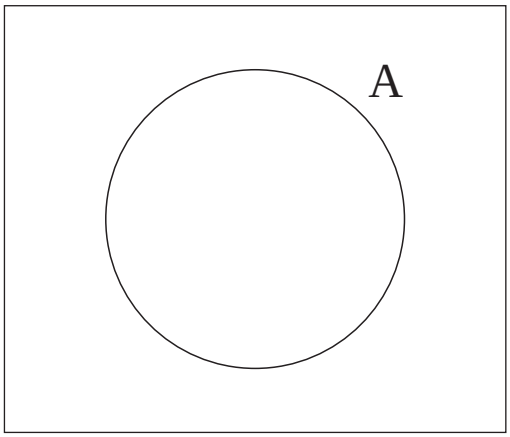
ACTIVITY 1.3 *Venn Diagrams*

This a whole-class or group activity.

You are going to be placed in sets according to the given criteria, and you must find your appropriate place on a Venn diagram of the type shown below, which will be indicated on the classroom floor.

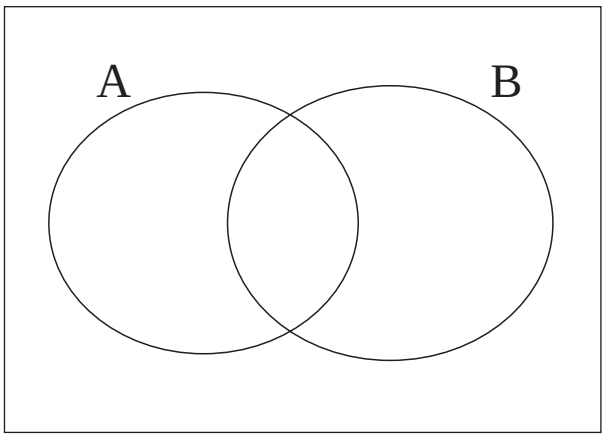
In all cases, the complete set consists of *all* the pupils in the class.

Type 1



or

Type 2



1. Using *Type 1*, A = set of boys.
   Describe the set of girls in terms of A.

2. Using *Type 1*, A = set of pupils who are 11 years old.
   Describe pupils in the *complement* of A.

3. Using *Type 2*, A = boys, B = girls.
   What can you say about
   (i) the *union* of A and B,
   (ii) the *intersection* of A and B,
   (iii) the *complement* of A and B?

4. Using *Type 2*, A = pupils with at least one sister,
   B = pupils with at least one brother.
   State in words, what is shown by
   (i) the *intersection* of A and B,
   (ii) the *complement* of the *union* of A and B.

ACTIVITY 1.4 *Numbers in Venn Diagrams*

This activity requires the use of a set of numbers (given on A 1.2b).

You will also need a large (A3) size Venn diagram (you could enlarge the diagram given on A 1.4a) on which to place the numbers.

In each case, put the given set of numbers into the appropriate region on the Venn diagram, according to the definition given for each set.

In all cases, the complete set is the set of whole numbers 1 to 40.

1. A = even numbers

   B = numbers divisible by 3

   What is in (i) the *intersection* of A and B,

   (ii) the *complement* of A and B?

2. A = single-digit, even numbers

   B = even numbers

   Describe the sets (i) the *intersection* of A with the *complement* of B,

   (ii) the *union* of A and B,

   (iii) the *complement* of the *union* of A and B.

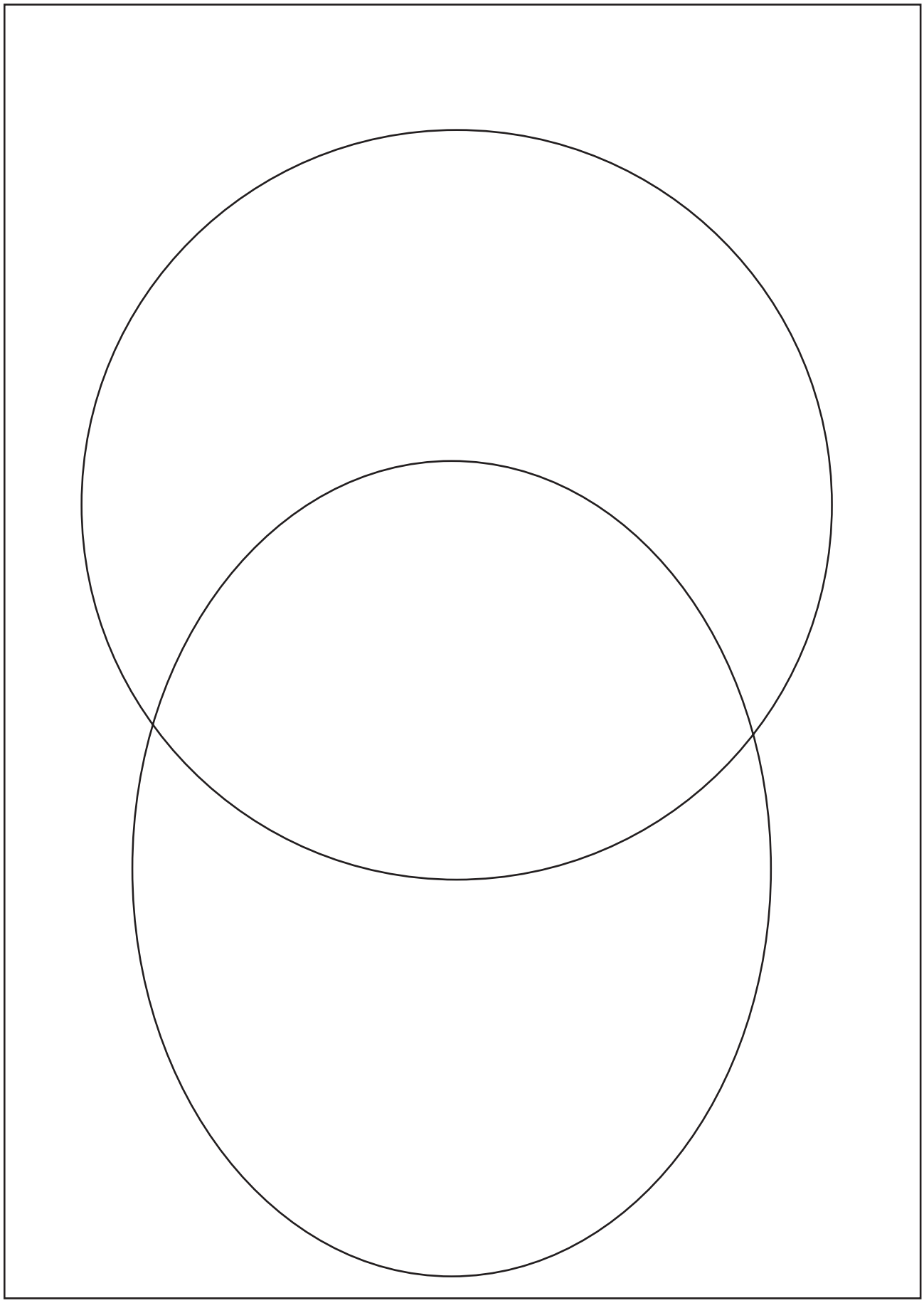
3. A = numbers divisible by 4

   B = numbers divisible by 3

   What is in (i) the *intersection* of A and B?

   (ii) the *intersection* of the *complement* of A with the *complement* of B?

ACTIVITY 1.4 a *Venn Diagram*



ACTIVITY 1.5 *Plane Passengers*

In a plane flying to London, five passengers are seated in a row next to each other.

Their professions are

journalist, singer, teacher, sailor, engineer;

and their nationalities, in any order, are:

English, French, German, Italian, Dutch.

Their ages are

21 years, 24 years, 32 years, 40 years, 52 years,

and each plays a different one of the following sports

handball, swimming, volleyball, athletics, football.

They will each travel from London to a different destination

Liverpool, Birmingham, Manchester, Newcastle or *Plymouth.*

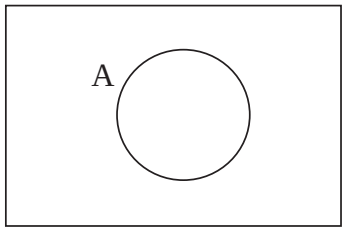
Clue 1	The engineer is seated on the extreme left.
Clue 2	The volleyball player is seated in the middle.
Clue 3	The Englishman is a journalist.
Clue 4	The singer is 21 years old.
Clue 5	The teacher's sport is swimming.
Clue 6	The sailor is travelling to Plymouth.
Clue 7	The handball player is French.
Clue 8	The passenger from Holland is bound for Birmingham.
Clue 9	The passenger bound for Liverpool is 32 years old.
Clue 10	The athlete is bound for Newcastle.
Clue 11	The French passenger is seated next to the German.
Clue 12	The 52-year-old passenger is seated next to the passenger who is bound for Manchester.
Clue 13	The 24-year-old passenger is seated next to the passenger who is travelling to Birmingham.
Clue 14	The engineer is seated next to the Italian.

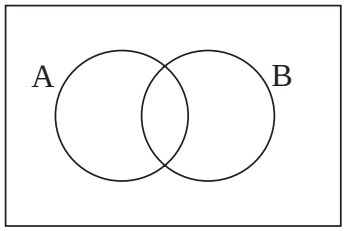
Use the clues to deduce the answers to these questions:

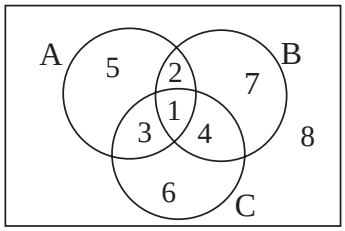
1. How old is the sailor?
2. What is the nationality of the football player?

ACTIVITY 1.6 *Representing Sets by Venn Diagrams*

Introduced by the English mathematician, *John Venn* (1834-1923), so called Venn diagrams are a convenient way of representing subsets of a universal set. For one, two or three subsets, it is easy to see that these diagrams can represent all the probabilities.







For example, for three subsets, A, B and C, you need separate regions to represent all the combinations that can exist.

1. Complete the table below and check that all the combinations are represented on the Venn diagram by the numbered regions.

A	B	C	On diagram
✓	✓	✓	✓ (1)
✓	✓	×	✓ (2)
✓	×	✓	✓ (3)
...	...	...	
...	...	...	
...	...	...	
...	...	...	
...	...	...	

2. Try to represent four subsets, A, B, C and D in a similar way. Check that, in your proposed diagram, all the possibilities are represented.

ACTIVITIES 1.1 - 1.3 *Notes and Solutions*

Notes and solutions are only given where appropriate.

1.1 You must ensure that there is plenty of room for this activity, and that it is relatively easy or pupils to move around.

1/2. The total will be the total number in the class, but this may not be the case in questions 3 and 4.

3. Any cell can be classified as 'having more than one sister or more than one brother (or both)'.

1.2 This is again a teacher-led activity. You will need to supply the shapes (preferably already cut out!), and might find it helpful to use the large 2×2 and 3×3 tables, photocopied onto A3 (OS 1.17 and OS 1.18).

Problems 1-3 should not cause difficulties, although the number of edges and vertices for the ∩ shape depends on how you define a vertex.

e.g.

2 vertices and 2 edges

4 vertices and 4 edges

There is also a potential problem with the circle for problem 4, although the entry 'none or one' for the number of vertices should sort this out.

These problems can be made:

(i) easier, by restricting the number of shapes used;

(ii) more complex, by having more criteria and hence more cells.

You can also design similar activities using numbers (A 1.2b) or letters (A 1.2c), where you could, for example, use the criteria

- vowels or consonants,
- no. of lines of symmetry.

1.3 This is a teacher-led activity, and will need careful planning to ensure that the Venn diagram is physically large enough (you could perhaps borrow some ropes from the PE department!). Of course, the sets will need modifying for single sex schools!

1. The set of girls is the complement of A.

2. Pupils in the complement of A are either older than 11 or younger than 11.

3. (i) all pupils in the class, (ii) the empty set, (iii) the empty set.

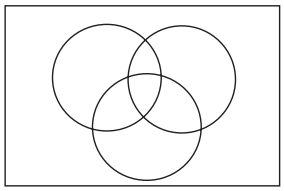
4. (i) pupils with both at least one sister and at least one brother,
(ii) pupils with no brother or sister.

ACTIVITIES 1.4 - 1.5 *Notes and Solutions*

1.4 This is a teacher-led activity, and you will need to supply numbers (A 1.2b). This problem can be made:

(i) easier by restricting the numbers used (e.g. 1 – 20);

(ii) more complex with different definitions of the sets (e.g. prime numbers or square numbers), or even using three subsets on a sheet.



You can also design similar problems using shapes (A 1.2a) or letters (A 1.2c), where you could classify according to lines of symmetry and rotational symmetry.

1. (i) { 6, 12, 18, 24, 30, 36 }

   (ii) { 1, 5, 7, 11, 13, 17, 19, 23, 25, 29, 31, 35, 37 }

2. (i) empty set (ii) even numbers (iii) odd numbers

3. (i) { 12, 24, 36 }

   (ii) { 1, 2, 5, 6, 7, 10, 11, 13, 14, 17, 18, 19, 23, 25, 26, 29, 30, 31, 34, 35, 37, 38 }

1.5 There are many ways of tackling this problem, ranging from trial and error to more systematic methods. For example, you could make 25 small squares with one piece of information on each square, and then combine them according to the constraints.

Another method would be to use transparent strips, with each of the given combinations written in the appropriate columns on each strip. The strips can be placed on top of one another without obscuring any information, and related pieces of information will be kept together.

The final solution is:

Engineer	*Teacher*	*Journalist*	*Sailor*	*Singer*
Dutch	Italian	English	French	German
52 years old	24 years old	32 years old	40 years old	21 years old
Birmingham	Manchester	Liverpool	Plymouth	Newcastle
Football	Swimming	Volleyball	Handball	Athletics

giving the answers:

1. 40 years old
2. Dutch

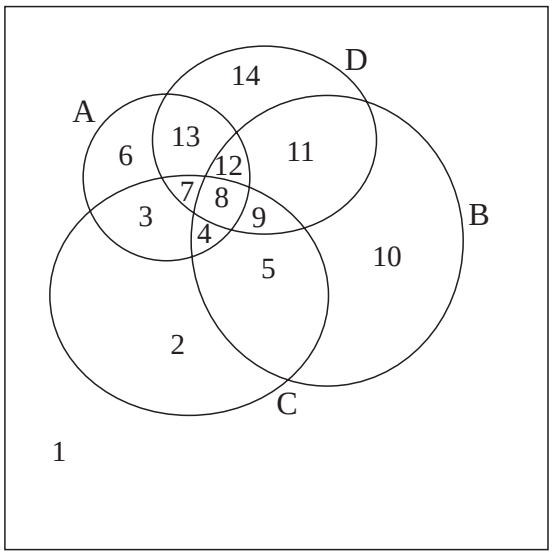
ACTIVITIES 1.6 *Notes and Solutions*

1.6 This is not an easy problem, and it doesn't even have a satisfactory answer – but it is a fascinating problem for really talented and creative pupils.

1. Yes, they are all represented.

A	B	C	On diagram
✓	✓	✓	✓ (no.1)
✓	✓	×	✓ (no.2)
✓	×	✓	✓ (no.3)
×	✓	✓	✓ (no.4)
×	×	✓	✓ (no.5)
×	✓	×	✓ (no.6)
✓	×	×	✓ (no.7)
×	×	×	✓ (no.6)

2. The obvious extension does not work.



There are 14 regions here, but you need, in fact, 16 distinct regions for every combination to be represented: the two opposite are not included.

A	B	C	D
×	×	✓	✓
✓	✓	×	×