12 Formulae

12.1 Substitution 1

In this section we practise substituting numbers for the letters in a formula: in other words, we replace the letters in formulae with their numerical values.

Example 1

If $a = 6$, $b = 3$ and $c = 7$, calculate the value of:

(a) $a + b$ (b) $a - b$ (c) $a + c$ (d) $c + b - a$

Solution

(a) $$\begin{aligned} a + b &= 6 + 3 \\ &= 9 \end{aligned}$$

(b) $$\begin{aligned} a - b &= 6 - 3 \\ &= 3 \end{aligned}$$

(c) $$\begin{aligned} a + c &= 6 + 7 \\ &= 13 \end{aligned}$$

(d) $$\begin{aligned} c + b - a &= 7 + 3 - 6 \\ &= 4 \end{aligned}$$

Reminders

We write: $2 \times a$ as $2a$

$a \times b$ as ab

$a \div 4$ as $\frac{a}{4}$

and $a \div b$ as $\frac{a}{b}$

Example 2

If $p = 6$, $q = 12$, $r = 4$ and $s = 3$, evaluate:

(a) rs (b) $4p$ (c) $2r + 3s$

(d) $\frac{s}{3}$ (e) $\frac{q}{s}$

Solution

(a) $rs = r \times s$

$= 4 \times 3$

$= 12$

(b) $4p = 4 \times p$

$= 4 \times 6$

$= 24$

(c) $2r + 3s = 2 \times r + 3 \times s$

$= 2 \times 4 + 3 \times 3$

$= 8 + 9$

$= 17$

(d) $\frac{s}{3} = \frac{3}{3}$

$= 1$

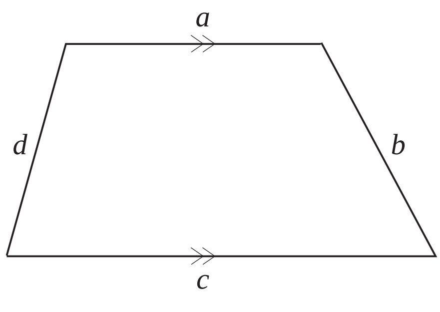
(e) $\frac{q}{s} = \frac{12}{3}$

$= 4$

Example 3

The perimeter of the trapezium shown is given by the formula

$$p = a + b + c + d$$



Calculate the perimeter if $a = 4$, $b = 6$, $c = 8$ and $d = 4$.

Solution

Perimeter $p = a + b + c + d$

$= 4 + 6 + 8 + 4$

$= 22$

Exercises

1. Calculate the values of the following expressions, if $x = 2$, $y = 5$ and $z = 9$:

(a) $x + y$	(b) $x + z$	(c) $y + z$
(d) $z - y$	(e) $y - x$	(f) $z - x$
(g) $x + y + z$	(h) $z + y - x$	(i) $z - y + x$

2. If $p = 7$, $q = 2$ and $r = 3$, evaluate the following expressions:

(a) $2p$	(b) $4r$	(c) $5q$
(d) $5p$	(e) $6r$	(f) $2q$
(g) $3p$	(h) $10p$	(i) $8r$

3. If $i = 6$, $j = 7$, $k = 3$ and $l = 4$, determine the values of the following expressions:

(a) $2i + 3k$	(b) $2l + 3i$	(c) $2j + 5l$
(d) $5j + 6k$	(e) $4i + 3l$	(f) $10j + 6l$
(g) $3i - j$	(h) $4k - i$	(i) $6l - 2k$
(j) $3i - 2j$	(k) $7k - 2i$	(l) $8l - 5k$

4. If $s = 10$, $t = 12$, $u = 15$ and $v = 20$, evaluate the following expressions:

(a) $\frac{s}{2}$	(b) $\frac{t}{3}$	(c) $\frac{u}{5}$
(d) $\frac{v}{10}$	(e) $\frac{v}{2}$	(f) $\frac{u}{3}$
(g) $\frac{t}{6}$	(h) $\frac{s}{10}$	(i) $\frac{u}{1}$

5. If $e = 10$, $f = 20$, $g = 5$ and $h = 4$, determine the values of the following expressions:

(a) eg	(b) gh	(c) ef
(d) eh	(e) $\frac{e}{g}$	(f) $\frac{f}{h}$
(g) $\frac{f}{g}$	(h) $\frac{f}{e}$	(i) $\frac{e}{g}$

(j) efg (k) gfe (l) feg

(m) heg (n) $\frac{gh}{f}$ (o) $\frac{ef}{gh}$

6. In a sweet shop you can buy packets of mints for 20p each and bars of chocolate for 30p each. The total cost of m packets of mints and c bars of chocolate is given by the formula

$$T = 20m + 30c$$

Use this formula to calculate the total cost if:

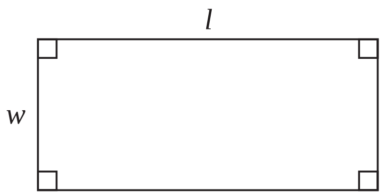
(a) $m = 2$ and $c = 1$ (b) $m = 8$ and $c = 0$

(c) $m = 3$ and $c = 3$ (d) $m = 5$ and $c = 4$

(e) $m = 1$ and $c = 10$ (f) $m = 2$ and $c = 3$

7. The perimeter of the rectangle shown is given by the formula

$$p = 2l + 2w$$



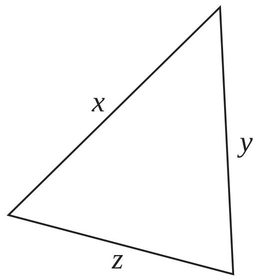
Calculate the perimeter of rectangles for which:

(a) $l = 2,\ w = 1$ (b) $l = 8,\ w = 2$

(c) $l = 10,\ w = 9$ (d) $l = 10,\ w = 3$

8. The perimeter of the triangle shown is given by the formula

$$p = x + y + z$$



Determine p if:

(a) $x = 4,\ y = 8$ and $z = 6$ (b) $x = 2,\ y = 3$ and $z = 4$

(c) $x = 10,\ y = 17$ and $z = 20$ (d) $x = 9,\ y = 14$ and $z = 15$

9. The cost of entry to a leisure park for an adult is £5 and for a child is £4. The total cost in pounds for a adults and c children is given by the formula

$$T = 5a + 4c$$

Calculate the cost if:

(a) $a = 2$ and $c = 4$ (b) $a = 7$ and $c = 1$

(c) $a = 1$ and $c = 5$ (d) $a = 2$ and $c = 3$

(e) $a = 3$ and $c = 8$ (f) $a = 10$ and $c = 30$

10. The time, T hours, taken to drive D kilometres along a motorway at a speed of S kilometres per hour is calculated using the formula

$$T = \frac{D}{S}$$

Calculate the time taken if:

(a) $D = 200$ and $S = 100$

(b) $D = 160$ and $S = 80$

(c) $D = 360$ and $S = 60$

(d) $D = 5$ and $S = 10$

12.2 Substitution 2

In this section we look at substituting *positive* and *negative* values; we also look at more complex equations.

Reminder

B rackets

O

D ivision

M ultiplication

A ddition

S ubtraction

BODMAS can be used to remember the order in which to carry out operations

Example 1

If $p = 2(x + y)$, calculate the value of p when $x = 3$ and $y = 5$.

Solution

$$\begin{aligned} p &= 2(x + y) \\ &= 2(3 + 5) \\ &= 2 \times 8 \\ &= 16 \end{aligned}$$

Example 2

A formula states that $Q = uv - \frac{v}{4}$.

Determine the value of Q if $u = 8$ and $v = 12$.

Solution

$$\begin{aligned} Q &= uv - \frac{v}{4} \\ &= 8 \times 12 - \frac{12}{4} \\ &= 96 - 3 \\ &= 93 \end{aligned}$$

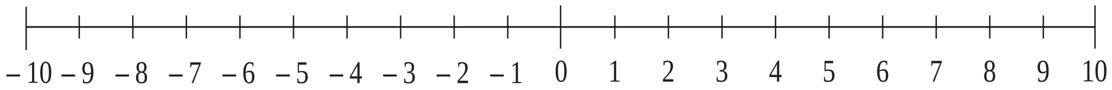
Reminders on adding and subtracting negative numbers

To *add a positive* number, *move to the right* on a number line.

To *add a negative* number, *move to the left* on a number line.

To *subtract a positive* number, *move to the left* on a number line.

To *subtract a negative* number, *move to the right* on a number line.



For example: $(-3) + 8 = +5$ (more usually written as 5)

$7 + (-4) = 3$

$(-2) - 6 = -8$

and $(-3) - (-9) = 6$

Reminders on multiplying and dividing negative numbers

The table shows what happens to the sign of the answer when *positive* and *negative* numbers are *multiplied* or *divided*.

$\times$ or $\div$	$+$	$-$
$+$	$+$	$-$
$-$	$-$	$+$

For example:

$$3 \times (-7) = -21$$

$$(-8) \times (-11) = 88$$

$$12 \div (-6) = -2$$

and

$$(-15) \div (-3) = 5$$

Example 3

If $p = 9$, $q = -4$, $r = 2$ and $s = -7$, determine the values of the following expressions:

(a) $p + q$ (b) $p - q$ (c) $q - r + s$

(d) ps (e) $\frac{q}{r}$ (f) $\frac{r}{q}$

Solution

(a) $p + q = 9 + (-4)$

$= 5$

(b) $p - q = 9 - (-4)$

$= 13$

(c) $q - r + s = (-4) - 2 + (-7)$

$= -13$

(d) $ps = 9 \times (-7)$

$= -63$

(e) $\dfrac{q}{r} = \dfrac{-4}{2}$

$= -2$

(f) $\dfrac{r}{q} = \dfrac{2}{-4}$

$= -\dfrac{1}{2}$

Example 4

If $a = 6$, $b = 5$ and $c = -2$, determine the value of:

(a) abc (b) $a(b+c)$ (c) $ab - bc$

(d) $\sqrt{a(b+1)}$ (e) $\dfrac{ab}{2} + bc^2$

Solution

(a) $abc = 6 \times 5 \times (-2)$

$= -60$

(b) $a(b+c) = 6\,(5 + (-2))$

$= 6 \times 3$

$= 18$

(c) $ab - bc = 6 \times 5 - 5 \times (-2)$

$= 30 - (-10)$

$= 30 + 10$

$= 40$

(d) $\sqrt{a(b+1)} = \sqrt{6 \times (5+1)}$

$= \sqrt{6 \times 6}$

$= \sqrt{36}$

$= 6$

(e) $\dfrac{ab}{2} + bc^2 = \dfrac{6 \times 5}{2} + 5 \times (-2)^2$

$= \dfrac{30}{2} + 5 \times 4$

$= 15 + 20$

$= 35$

Exercises

1. Calculate:

(a) $6 + (-2)$ (b) $(-3) + 5$ (c) $(-4) + (-2)$

(d) $2 - 4$ (e) $3 - (-2)$ (f) $(-7) - (-4)$

(g) $2 \times (-6)$ (h) $(-10) \times 5$ (i) $(-12) \times (-4)$

(j) $(-8) \div 4$ (k) $14 \div (-7)$ (l) $(-25) \div (-5)$

(m) $(-3)^2$ (n) $(-5)^2 \times (-2)$ (o) $(4 \times 5) + (-2)$

(p) $(-3) \times (-4) \div 6$ (q) $(-3) \times (-8) + (-7)$

(r) $\dfrac{(-6) \times (-4)}{(-12)}$ (s) $\dfrac{(-10)^2}{4}$

(t) $(-3) \times (-5) \times (-9)$ (u) $(-5)^2 + (-6)^2$

2. If $a = 6$, $b = 3$ and $c = 7$, calculate:

(a) ab (b) $b + c$ (c) $c - a$

(d) $4b + 6c$ (e) $4c - 2b$ (f) $6a - 2c$

(g) abc (h) $ab - bc$ (i) $2bc + ac$

(j) b^2 (k) $a^2 - b^2$ (l) $a^2 + b^2 - c^2$

3. If $a = 2$, $b = -4$ and $c = -5$, evaluate:

(a) $a^2 + b^2$ (b) ab (c) bc

(d) $a - b$ (e) $c - b$ (f) $3a + 2c$

(g) $2a - 4c$ (h) $3a + 2b$ (i) $ab - ac$

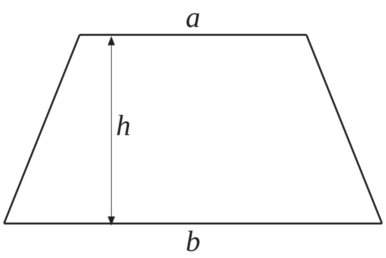
4. Calculate $\sqrt{a + bc}$ when $a = 15$, $b = 2$ and $c = -3$.

5. A formula for the perimeter of a triangle is $p = x + y + z$, where x, y and z are the lengths of the three sides. Calculate the value of p when $x = 1\frac{1}{2}$ cm, $y = 2\frac{1}{2}$ cm and $z = 3\frac{1}{2}$ cm.

6. The area of a trapezium is given by the formula

$$A = \frac{1}{2}(a + b)h$$

Calculate the area of the trapezium for which $a = 3$ cm, $b = 3.6$ cm and $h = 2.2$ cm.



7. The length of one side of a right-angled triangle is given by the following formula:

$$l = \sqrt{h^2 - x^2}$$

Calculate the length l, if $h = 13$ cm and $x = 12$ cm.

8. The following formula can be used to convert temperatures from degrees Celsius (C) to degrees Fahrenheit (F):

$$F = 32 + \frac{9C}{5}$$

Calculate the value of F, if:

(a) $C = 100$

(b) $C = 20$

(c) $C = -10$

(d) $C = -20$

9. A formula states that

$$s = \frac{1}{2}(u + v)t$$

Calculate the value of s, if:

(a) $u = 3$, $v = 6$ and $t = 10$

(b) $u = -2$, $v = 4$ and $t = 2$

(c) $u = -10$, $v = -6$ and $t = 3$

(d) $u = -20$, $v = -40$ and $t = 3$

10. A formula states that:

$$f = \frac{u + v}{uv}$$

(i) Calculate the value of f, if:

(a) $u = 10$ and $v = 5$ (b) $u = 2$ and $v = 5$

(c) $u = 20$ and $v = 10$

(ii) Show that you obtain the same value in each case using the formula

$$f = \frac{1}{v} + \frac{1}{u}$$

Why does this happen?

11. Alan knows that $x = 2$, $y = -6$ and $z = -4$. He calculates that $Q = \frac{1}{4}$.

Which of the formulae below could he have used?

Formula A $Q = \frac{xy + yz}{xyz}$

Formula B $Q = \frac{yz - xy}{xyz}$

Formula C $Q = \frac{1}{x} - \frac{1}{z}$

Formula D $Q = \frac{1}{z} + \frac{1}{x}$

Formula E $Q = \frac{1}{x} + \frac{1}{y}$

12.3 Linear Equations 1

In this section we revise the solution of simple equations.

> *Reminder*
>
> Whatever you do to *one side* of an equation you must do to the *other side*: it is like keeping a set of scales balanced.
>
> It is conventional to give the solution to an equation with the *unknown value* on the *left hand side*, and its *value* on the *right hand side*, e.g. $x = 4$ not $4 = x$.

Example 1

Solve the following equations:

(a) $x + 3 = 7$

(b) $13 = 5 + a$

(c) $y - 3 = 8$

(d) $11 = p - 4$

Solution

(a) $x + 3 = 7$

[Subtract 3 from both sides] $x = 7 - 3$

$x = 4$

(b) $13 = 5 + a$

[Subtract 5 from both sides] $13 - 5 = a$

$8 = a$

$a = 8$

(c) $y - 3 = 8$

[Add 3 to both sides] $y = 8 + 3$

$y = 11$

(d) $11 = p - 4$

[Add 4 to both sides] $11 + 4 = p$

$15 = p$

$p = 15$

Example 2

Solve the following equations:

(a) $6x = 24$ (b) $15 = 3t$ (c) $\frac{w}{2} = 9$

Solution

(a) $6x = 24$

[Divide both sides by 6] $x = \frac{24}{6}$

$x = 4$

(b) $15 = 3t$

[Divide both sides by 3] $\frac{15}{3} = t$

$5 = t$

$t = 5$

(c) $\frac{w}{2} = 9$

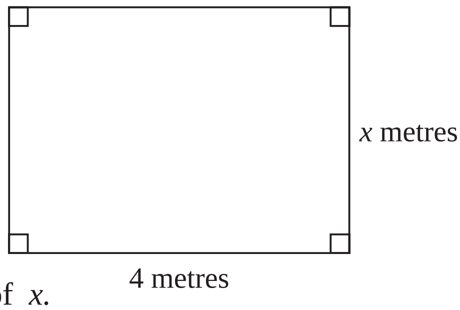
[Multiply both sides by 2] $w = 9 \times 2$

$w = 18$

Example 3

The length of the rectangle shown is 4 metres, and its width is x metres.

The area of the rectangle is 8 m^2.



(a) Use this information to write down an equation involving x.

(b) Solve the equation to determine the value of x.

(c) What is the width of the rectangle in cm?

Solution

(a) The area of the rectangle is $(4 \times x) \text{ m}^2$, and we are told that this equals 8 m^2.

So the equation is $4 \times x = 8$, which we write as $4x = 8$.

(b) $4x = 8$

[Divide both sides by 4] $x = \frac{8}{4}$

$x = 2 \text{ m}$

(c) The width of the rectangle is 200 cm.

Exercises

1. Solve the following equations:

(a) $x + 5 = 9$ (b) $x + 11 = 12$ (c) $7 + x = 9$

(d) $x + 2 = 17$ (e) $14 = x + 6$ (f) $x - 2 = 10$

(g) $x - 6 = 5$ (h) $2 = x - 9$ (i) $x + 3 = 0$

(j) $x - 7 = 7$ (k) $x + 12 = 7$ (l) $x - 6 = -10$

2. Solve the following equations:

(a) $2x = 12$ (b) $3x = 18$ (c) $5x = 20$

(d) $7x = 21$ (e) $36 = 9x$ (f) $5x = 0$

(g) $80 = 10x$ (h) $\frac{x}{2} = 5$ (i) $\frac{x}{3} = 6$

(j) $9 = \frac{x}{4}$ (k) $\frac{x}{2} = 22$ (l) $\frac{x}{7} = 4$

(m) $4x = 2$ (n) $\frac{x}{2} = 6$ (o) $\frac{x}{10} = 0$

3. Solve the following equations:

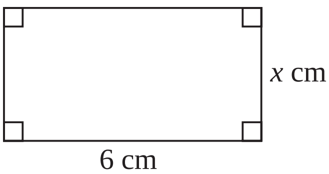
(a) $x + 7 = 9$ (b) $x - 6 = 8$ (c) $3x = 33$

(d) $\frac{x}{5} = 2$ (e) $x + 2 = 13$ (f) $5x = 35$

(g) $4 + x = 15$ (h) $7 = y - 9$ (i) $42 = 6p$

(j) $90 = \frac{q}{9}$ (k) $5r = -10$ (l) $-4 = \frac{s}{8}$

4. The area of the rectangle shown is 18 cm^2.

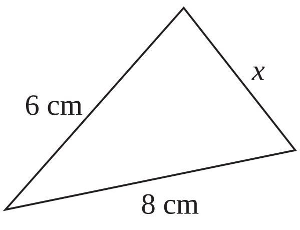


(a) Write down an equation involving x.

(b) Solve your equation.

(c) Write down the width of the rectangle.

5. The perimeter of the triangle shown is 17 cm.



(a) Write down an equation and solve it for x.

(b) Write down the length of the side marked x.

12.4 Linear Equations 2

In this section we solve linear equations where more than one step is needed to reach the solution. There are no simple rules here, since methods of solution vary from one equation to another.

Example 1

Solve the equation,

$$5x + 2 = 17$$

Solution

The first step is to subtract 2 from both sides, giving

$$5x = 15$$

Secondly, divide both sides by 5, to give the solution

$$x = 3$$

Example 2

Solve the equation,

$$4x - 7 = 17$$

Solution

[Add 7 to both sides] $4x = 24$

[Divide both sides by 4] $x = 6$

Example 3

Solve the equation,

$$5(7 + 2x) = 65$$

Solution

EITHER

[Multiply out the brackets]

$$35 + 10x = 65$$

[Subtract 35 from both sides]

$$10x = 30$$

[Divide both sides by 3] $x = 3$

OR

[Divide both sides by 5]

$$7 + 2x = 13$$

[Subtract 7 from both sides]

$$2x = 6$$

[Divide both sides by 2] $x = 3$

Example 4

Solve the equation,

$$6x - 2 = 4x + 8$$

Solution

[Subtract $4x$ from both sides] $2x - 2 = 8$

[Add 2 to both sides] $2x = 10$

[Divide both sides by 2] $x = 5$

Example 5

Solve the equation,

$$\frac{p}{2} + 3 = 7$$

Solution

EITHER

[Subtract 3 from both sides] $\frac{p}{2} = 4$

[Multiply both sides by 2] $p = 8$

OR

[Multiply both sides by 2] $p + 6 = 14$

[Subtract 6 from both sides] $p = 8$

Example 6

Solve the equation

$$\frac{p + 3}{2} = 7$$

Solution

[Multiply both sides by 2] $p + 3 = 14$

[Subtract 3 from both sides] $p = 11$

Exercises

1. Solve the following equations:

(a) $3x + 2 = 17$ (b) $5x - 6 = 9$ (c) $6x - 4 = 8$

(d) $3(x + 4) = 30$ (e) $5(2x - 3) = 15$ (f) $7 - 2x = 3$

(g) $6x - 4 = 32$ (h) $6x + 7 = 1$ (i) $7x + 6 = 34$

(j) $6x - 7 = 11$ (k) $2x + 15 = 16$ (l) $8 - 2x = 5$

(m) $38 = 3y + 2$ (n) $35 = 5(7 + 2p)$ (o) $56 = 7(2 - 3q)$

2. Solve the following equations:

(a) $\frac{x}{2} + 5 = 9$ (b) $14 = \frac{x}{3} - 8$ (c) $\frac{y}{5} - 9 = -2$

(d) $\frac{z}{4} + 8 = 3$ (e) $7 = \frac{p}{4} - 6$ (f) $\frac{x + 5}{2} = 9$

(g) $14 = \frac{x - 8}{3}$ (h) $\frac{y - 9}{5} = -2$ (i) $\frac{z + 8}{4} = 3$

(j) $7 = \frac{p - 6}{4}$ (k) $\frac{2x}{3} + 1 = 9$ (l) $\frac{5x}{4} - 7 = 3$

3. Solve the following equations:

(a) $2x + 3 = x + 10$ (b) $6x - 2 = 4x + 7$

(c) $16x - 7 = 8x + 17$ (d) $11x + 2 = 8x + 7$

(e) $x + 1 = 2(x - 1)$ (f) $3(x + 4) = 5(x - 2)$

(g) $9(x + 7) = 2(5x - 7)$ (h) $3(2x - 1) = 4(3x - 4)$

4. The formula $F = 32 + \frac{9C}{5}$ can be used to convert temperatures from degrees Celsius (C) to degrees Fahrenheit (F).

(a) Copy and complete the following solution to calculate the value of C when F is $86°$:

$$F = 32 + \frac{9C}{5}$$

[Substitute 86 for *F*] $86 = 32 + \frac{9C}{5}$

[Subtract 32 from both sides] $=$

[Multiply both sides by 5] $=$

[Divide both sides by 9] $=$

$C =$

(b) Calculate the value of C when F is 41, using the process as in part (a).

(c) Calculate the value of C when F is 23.

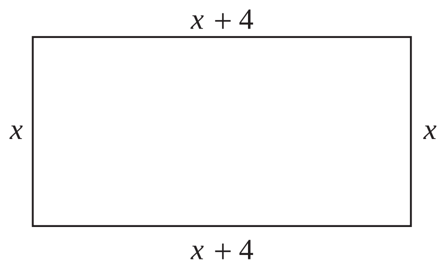
5. The formula $p = 2(x + y)$ can be used to work out the perimeter of a rectangle with sides x and y. Use the same approach as in question 4, to set up and solve an equation to calculate the value of x, if $p = 50$ and $y = 8$.

6. A formula states that $v = u + at$. Set up and solve an equation to determine the value of a, if,

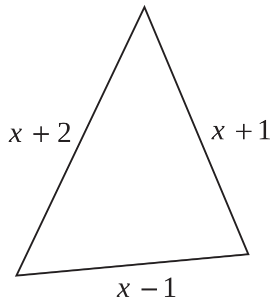
(a) $v = 10$, $u = 3$ and $t = 5$,

(b) $v = 2$, $u = 5$ and $t = 3$.

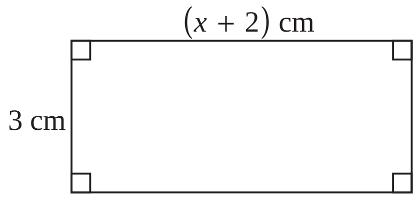
7. The perimeter of the rectangle shown is 16 cm. Calculate the value of x.



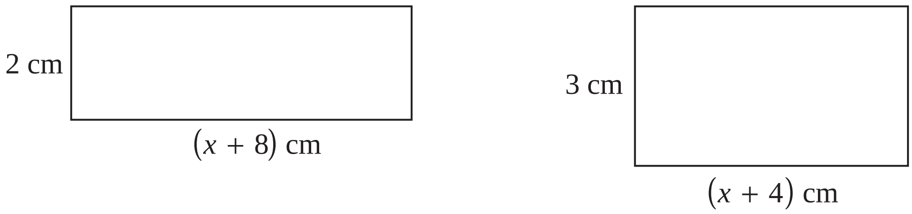
8. The perimeter of the triangle shown is 23 cm. Calculate the value of x.



9. The area of the rectangle shown is $19.5\ \text{cm}^2$. Determine the value of x.



10. The following two rectangles have the same areas:



(a) Determine the value of x.

(b) Write down the lengths of the two rectangles.

12.5 Non-Linear Equations

Solutions to non-linear equations are not always possible with the methods we have been using for linear equations: sometimes it is necessary to use a method called *trial and improvement*, where you make a sensible first guess at the solution, and then you try to improve your estimate. The method is illustrated in the examples which follow.

Example 1

Solve the equation,

$$x^3 = 100$$

giving your answer correct to 1 decimal place.

Solution

If we substitute $x = 4$ into the expression x^3, we get $4^3 = 64$, which is *less* than 100.

If we substitute $x = 5$, we get $5^3 = 125$, which is *more* than 100.

This tells us that there is a solution between 4 and 5.

We now improve our solution. A good way to record the values that you calculate is to use a table, and to make comments as you go:

x	x^3	*Comments*
4	64	4 is *too low*
5	125	5 is *too high*: the solution is between 4 and 5
4.5	91.125	4.5 is *too low*
4.6	97.336	4.6 is *too low*
4.7	103.823	4.7 is *too high*: the solution is *between 4.6 and 4.7*
4.65	100.544625	4.65 is *too high*

From the table we can see that 4.6 is *too low*, and 4.65 is *too high*, so the solution is *between* 4.6 *and* 4.65.

So, we may write

$$4.6 < x < 4.65$$

The statement means that, when it is rounded, x will be 4.6 correct to one decimal place:

$$x = 4.6 \quad \text{(to 1 d.p.)}$$

Note If we need the solution to a greater degree of accuracy, then we continue the process and extend the table.

Example 2

Use trial and improvement to solve the equation $x^3 + x = 20$, giving your answer correct to 2 decimal places.

Solution

x	$x^3 + x$	*Comments*
2	10	2 is *too low*
3	30	3 is *too high*: solution is *between 2 and 3*
2.5	18.125	2.5 is *too low*
2.6	20.176	2.6 is *too high*: solution is *between 2.5 and 2.6*
2.55	19.131	2.55 is *too low*
2.58	19.754	2.58 is *too low*
2.59	19.964	2.59 is *too low*: solution is *between 2.59 and 2.6*
2.595	20.070	2.595 is *too high*

From the table we can see that the solution of x is between 2.59 and 2.595, so

$$2.59 < x < 2.595$$

When we round off, $x = 2.59$ correct to 2 decimal places.

Exercises

1. Use the trial and improvement method to solve the following equations for positive x:

   (a) $x^2 = 15$ to 1 decimal place,

   (b) $x^2 + x = 28$ to 1 decimal place,

   (c) $x^4 + 5 = 80$ to 1 decimal place,

   (d) $\frac{6}{x^2} = 0.1$ to 1 decimal place.

2. (a) Show that the equation $x^3 - x^2 = 2$ has a solution between 1 and 2.

   (b) Use trial and improvement to solve the equation $x^3 - x^2 = 2$, giving your solution correct to 2 decimal places.

3. (a) Show that the equation $x^2 + 2x + 3 = 15$ has a solution between 2 and 3.

   (b) Use trial and improvement to solve the equation $x^2 + 2x + 3 = 15$, giving your answer correct to 2 decimal places.

4. Use a trial and improvement method to solve $x^3 - 5x - 1 = 0$ for positive x, giving your answer correct to 2 decimal places.

12.6 Changing the Subject of a Formula

We say that v is the *subject* of the formula $v = u + at$.

The formula can be rearranged so that $a = \frac{v - u}{t}$, and we now say that a is the subject of the formula. When rearranging a formula you must use the same approach as when you solve equations.

Note When giving your solution at the end, remember to write the *subject* of the formula on the *left hand side* of the equation.

Example 1

Make x the subject of each of the following formulae:

(a) $y = x + 8$ (b) $y = 2x - 4$

Solution

(a) $y = x + 8$

[Subtract 8 from both sides] $y - 8 = x$

$$x = y - 8$$

(b) $y = 2x - 4$

[Add 4 to both sides] $y + 4 = 2x$

[Divide both sides by 2] $\frac{y+4}{2} = x$

$$x = \frac{y+4}{2}$$

Example 2

Make t the subject of each of the following equations:

(a) $v = u + at$ (b) $p = k(b + t)$

Solution

(a) $v = u + at$

[Subtract u from both sides] $v - u = at$

[Divide both sides by a] $\frac{v-u}{a} = t$

$$t = \frac{v-u}{a}$$

(b) $p = k(b + t)$

EITHER

[Multiply out the brackets]

$$p = kb + kt$$

[Subtract kb from both sides]

$$p - kb = kt$$

[Divide both sides by k]

$$\frac{p-kb}{k} = t$$

$$t = \frac{p-kb}{k}$$

OR

[Divide both sides by k]

$$\frac{p}{k} = b + t$$

[Subtract b from both sides]

$$\frac{p}{k} - b = t$$

$$t = \frac{p}{k} - b$$

Note that these two formulae are equivalent, even though they look different, because

$$\frac{p - kb}{k} = \frac{p}{k} - \frac{kb}{k}$$

$$= \frac{p}{k} - b$$

Exercises

1. Make x the subject of each of the following formulae:

   (a) $y = x - 2$ (b) $y = x + 7$ (c) $y = 4x$

   (d) $y = \frac{x}{3}$ (e) $y = 2x + 1$ (f) $y = 4x - 3$

   (g) $y = 2(x + 3)$ (h) $y = 3(x - 4)$ (i) $y = mx$

   (j) $y = x + a$ (k) $y = kx - c$ (l) $y = ax + b$

2. (a) Make a the subject of the formula $y = ax + b$.

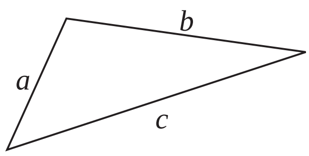
   (b) Make b the subject of the formula $y = ax + b$.

3. If $y = \frac{3x - 7}{2}$, express x in terms of y.

4. The formula $F = 32 + \frac{9C}{5}$ is used for converting temperatures for degrees Celsius (C) to degrees Fahrenheit (F). Make C the subject of this formula.

5. The perimeter of the triangle shown is

   $$p = a + b + c$$

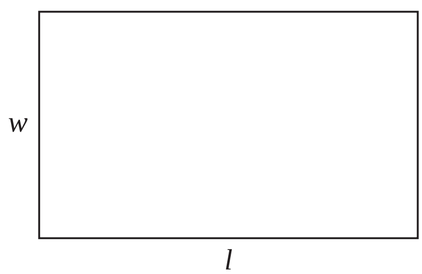


   (a) Make a the subject of this formula.

   (b) Make c the subject of this formula.

6. (a) Complete the following formula for the perimeter of the rectangle shown:

   $$p = 2w + \ldots\ldots$$



   (b) Make w the subject of your formula.

(c) Complete the following formula for the area of the rectangle:

$A = \ldots\ldots\ldots\ldots\ldots$

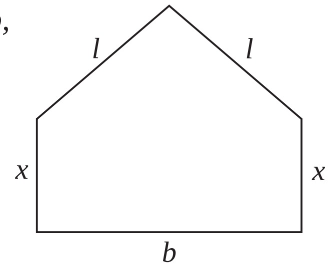
(d) Make l the subject of your formula.

7. (a) Write down a formula for the perimeter, p, of the shape shown.

(b) Make x the subject of your formula.

(c) Make l the subject of your formula

(d) Make b the subject of your formula.

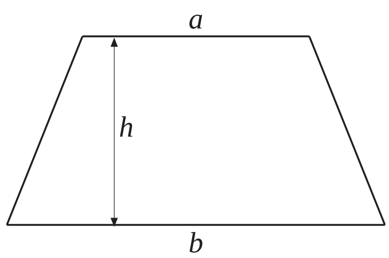


8. The area of the trapezium shown is given by the formula

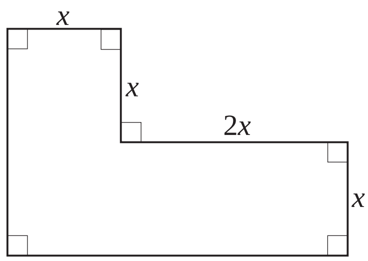
$$A = \frac{h}{2}(a + b)$$

(a) Make h the subject of the formula.

(b) Make a the subject of the formula.



9. Write down a formula for the perimeter of the shape shown, and then make x the subject of the formula.



10. (a) Write down a formula for the shaded area, A, in the diagram shown.

(b) Make x the subject of the formula.

(c) Make y the subject of the formula.