

# 14 Estimation and Approximation

## 14.1 Rounding

There are three main ways to round numbers:

- (i) to the *nearest* 10, 100, 1000, etc;
- (ii) to a certain number of *significant figures*;
- (iii) to a certain number of *decimal places*.

Note that a measured length such as '12 cm to the nearest cm' means that the actual length lies between 11.5 cm and 12.5 cm.

By convention, we normally round 0.5 up to the next whole number, so in fact,

$$11.5 \text{ cm} \leq \text{actual length} < 12.5 \text{ cm}$$

We call 11.5 the *lower bound* and 12.5 the *upper bound*. We can also write the upper bound as

$$12.4999 \dots \text{ or } 12.4\dot{9}$$

where the dot above the 9 means that it is repeated indefinitely, or recurs.

It is important to see that 12.49 is *not* the upper bound, as, for example, the length could have been 12.498.



### Example 1

A football match is watched by 56 742 people. Write this number correct to the nearest,

- (a) 10 000,                      (b) 1000,                      (c) 10.



### Solution

- (a)  $56\,742 = 60\,000$  to the nearest 10 000.  
(b)  $56\,742 = 57\,000$  to the nearest 1000.  
(c)  $56\,742 = 56\,740$  to the nearest 10.



### Example 2

Write each of the following numbers correct to 3 significant figures:

- (a) 47 316                      (b) 303 971  
(c) 20.453                      (d) 0.004368



### Solution

- (a)  $47\,316 = 47\,300$  to 3 significant figures.  
(b)  $303\,971 = 304\,000$  to 3 significant figures.  
(c)  $20.453 = 20.5$  to 3 significant figures.  
(d)  $0.004368 = 0.00437$  to 3 significant figures.

**Example 3**

Write each of the following numbers correct to the number of decimal places stated:

- (a) 0.3741 to 2 d.p.  
 (b) 3.8451 to 2 d.p.  
 (c) 142.8315 to 1 d.p.  
 (d) 0.000851 to 4 d.p.

**Solution**

- (a) 0.3741 = 0.37 to 2 decimal places.  
 (b) 3.8451 = 3.85 to 2 decimal places  
 (c) 142.8315 = 142.8 to 1 decimal place.  
 (d) 0.000851 = 0.0009 to 4 decimal places.

**Example 4**

State the upper and lower bounds for each of the following quantities and write an inequality for the actual value in each case.

- (a) 4 mm to the nearest mm.  
 (b) 15 kg to the nearest kg.  
 (c) 4.56 m to the nearest cm.

**Solution**

- (a)  $\left. \begin{array}{l} \text{Upper bound} = 4.5 \text{ mm} \\ \text{Lower bound} = 3.5 \text{ mm} \end{array} \right\} 3.5 \text{ mm} \leq \text{actual value} < 4.5 \text{ mm}$
- (b)  $\left. \begin{array}{l} \text{Upper bound} = 15.5 \text{ kg} \\ \text{Lower bound} = 14.5 \text{ kg} \end{array} \right\} 14.5 \text{ kg} \leq \text{actual value} < 15.5 \text{ kg}$
- (c)  $\left. \begin{array}{l} \text{Upper bound} = 4.565 \text{ m} \\ \text{Lower bound} = 4.555 \text{ m} \end{array} \right\} 4.555 \text{ m} \leq \text{actual value} < 4.565 \text{ m}$



## Exercises

- Round each of the following numbers to the nearest 100:
  - 108
  - 199
  - 3471
  - 59
  - 33
  - 451
- Round the number 4 765 173 to:
  - the nearest million,
  - the nearest 10,
  - the nearest 1000,
  - the nearest 100.
- Write each of the following numbers correct to 3 significant figures:
  - 37 412
  - 84 563
  - 261.42
  - 0.3684
  - 0.002615
  - 0.0025713
  - 3.6213
  - 4.0071
  - 18.3071
- Write each of the following numbers correct to 2 decimal places:
  - 7.431
  - 8.269
  - 4.7135
  - 11.925
  - 24.8603
  - 44.0019
- Write each of the following numbers correct to the number of significant figures stated:
  - 6.475 to 2 s.f.
  - 1473 to 1 s.f.
  - 3681 to 2 s.f.
  - 571.32 to 4 s.f.
  - 16 001 to 3 s.f.
  - 148.25 to 3 s.f.
  - 16.999 to 3 s.f.
  - 38.9712 to 2 s.f.
  - 160.37 to 4 s.f.
- Write the number 183.9591 correct to:
  - 3 decimal places
  - 3 significant figures
  - 2 significant figures
  - 5 significant figures
  - 1 decimal place
  - 1 significant figure
- Barry rounds the number 374.49 to 375.
  - Explain what mistake he has made.
  - Describe how Barry tried to round the number and then give the correct answer.

8. Use a calculator to carry out the following calculations. In each case, give the answer correct to 2 significant figures.
- (a)  $33 \div 4$                       (b)  $22 \div 0.7$                       (c)  $142 \div 0.8$   
 (d)  $66 \times 1.27$                       (e)  $3.25 \times 1001$                       (f)  $62 \times 47$   
 (g)  $3.41 \times 0.0092$                       (h)  $88 \div 0.007$                       (i)  $42 \times 1.0952$
9. The mass of a ship is stated as 47 384 tonnes to the nearest tonne. Are the following statements correct?
- (a) The mass is *less* than 47 384.5 tonnes.  
 (b) The mass is *greater* than 47 384.4 tonnes.
10. Annie says that her rabbit has a mass of 1473 grams. Give this mass correct to:
- (a) the nearest 100 grams,  
 (b) the nearest 10 grams,  
 (c) the nearest kg.
11. At an athletics meeting, the discus throws are measured to the nearest centimetre.
- (a) Viv's best throw was measured as 35.42 m.  
 Could Viv's throw actually have been more than 35.42 m ?  
 Explain your answer.
- (b) Chris won the hurdles race in a time of 14.6 seconds measured to the nearest tenth of a second.  
 Between what two values does Chris's time actually lie?

(KS3/95/Ma/Levels 5-7/P1)

12. The table shows the lengths of some rivers to the nearest km.
- (a) (i) Write the length of each river rounded to the nearest 100 km.

<i>River</i>	<i>Length in km to the nearest km</i>	<i>Length in km to the nearest 100 km</i>
Severn	354	
Thames	346	
Trent	297	
Wye	215	
Dee	113	

- (ii) Which two rivers have the *same length* to the nearest 100 km?

- (b) (i) Write the length of each river rounded to the nearest 10 km.

<i>River</i>	<i>Length in km to the nearest km</i>	<i>Length in km to the nearest 10 km</i>
Severn	354	
Thames	346	
Trent	297	
Wye	215	
Dee	113	

- (ii) Which two rivers have the *same length* to the nearest 10 km?

- (c) There is another river which is not on the list.

It has a length of 200 km to the nearest 100 km, and a length of 150 km to the nearest 10 km.

Copy and complete this sentence to give one possible length of the river to the nearest km.

*The length of the river could be ..... km.*

- (d) Two more rivers have different lengths to the nearest km.

They both have a length of 250 km to the nearest 10 km, but their lengths to the *nearest 100 km* are *different*.

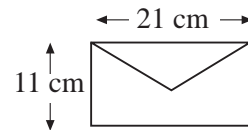
Copy and complete this sentence to give a possible length of each river to the nearest km.

*The lengths of the rivers could be ..... km and ..... km.*

(KS3/98/Ma/Tier 4-6/P2)

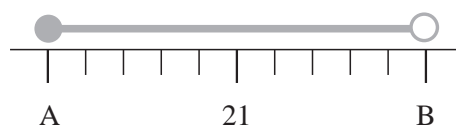
13. (a) The length of an envelope is 21 cm to the nearest cm.

What is the smallest possible real length of the envelope?

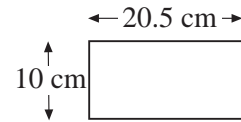


- (b) The interval on the number line below shows all the possible real lengths of the envelope.

Write down the correct numbers for the lower and upper limits of the interval, represented on the diagram by A and B.

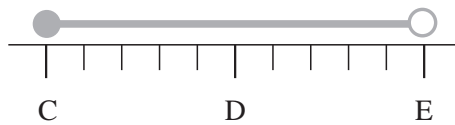


- (c) The length of a card is 20.5 cm to the nearest tenth of a cm.



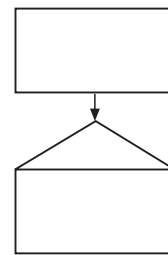
The interval on the number line below shows all the possible real lengths of the card.

Write down the numbers represented on the number line by C, D and E.



- (d) Can you be sure the card will fit in the open envelope like this?

Explain why.



(KS3/94/Ma/5-7/P2)

## 14.2 Estimation

Estimation is important because it allows you to check that the answers you obtain on a calculator are reasonable. It is easy to make a simple mistake when using a calculator or working out a problem by hand, so an estimation of the expected answer is a useful check.

One of the simplest approaches to estimation is to round all the numbers involved to 1 significant figure.



### Example 1

- Estimate the value of  $4.73 \times 18.4$ .
- Use a calculator to find  $4.73 \times 18.4$ .
- Compare the estimate with the exact value.



### Solution

- $4.73 \times 18.4 \approx 5 \times 20 = 100$
- $4.73 \times 18.4 = 87.032$
- Because in (a) we rounded both numbers up, the estimate is slightly bigger than the actual value, but it does give us an idea of the size of the answer.



### Example 2

The correct answer to  $14.1 \times 18.3$  is listed below, along with 3 incorrect answers.

25.803

258.03

2580.3

25803

Use estimation to decide which is the correct answer.



### Solution

$$14.1 \times 18.3 \approx 10 \times 20 = 200$$

So the correct answer must be 258.03

Although we often round to 1 significant figure, we can sometimes produce better estimates by using other values that are still easy to work with. For example, in Example 2 above, we could have said that

$$14.1 \times 18.3 \approx 14 \times 20 = 280$$

which is closer to the correct answer 258.03

The important thing to remember is that you must be able to do the estimation calculation *in your head*.



### Example 3

Estimate the value of:

$$(a) \sqrt{\frac{3.87 \times 14.6}{9.32}}$$

$$(b) \sqrt{\frac{72 \times 33.4}{41}}$$



### Solution

$$(a) \sqrt{\frac{3.87 \times 14.6}{9.32}} \approx \sqrt{\frac{4 \times 10}{10}} = \sqrt{4} = 2$$

or

$$\sqrt{\frac{3.87 \times 14.6}{9.32}} \approx \sqrt{\frac{4 \times 16}{9}} = \frac{2 \times 4}{3} = \frac{8}{3} = 2\frac{2}{3} = 2.\dot{6}$$

Note that the actual value is 2.462 to 3 d.p.

$$(b) \sqrt{\frac{72 \times 33.4}{41}} \approx \sqrt{\frac{70 \times 30}{40}} = \sqrt{\frac{2100}{40}} = \sqrt{\frac{2000}{40}} = \sqrt{50}$$

Note that  $\sqrt{50}$  is between 7 and 8, but will be closer to 7 since  $7 = \sqrt{49}$ . We can therefore take 7 as our estimate.

The actual value is 7.659 to 3 d.p.

You might also need to approximate when converting between different units. The following list reminds you of some of the important approximations between metric and imperial units.

8 km	≈	5 miles
1 m	≈	40 inches
30 cm	≈	1 foot
2.5 cm	≈	1 inch
1 kg	≈	2.2 lbs
1 litre	≈	$1\frac{3}{4}$ pints
1 gallon	≈	$4\frac{1}{2}$ litres
1 acre	≈	$\frac{2}{5}$ hectare
450 g	≈	1 lb



## Exercises

1. Estimate the answer to each of the following calculations:

- |                         |                         |                         |
|-------------------------|-------------------------|-------------------------|
| (a) $3.6 \times 14.3$   | (b) $47 \times 192$     | (c) $33.6 \times 403$   |
| (d) $11.25 \times 76.3$ | (e) $12.84 \times 3.94$ | (f) $103.6 \div 21.72$  |
| (g) $44.32 \div 1.987$  | (h) $68.39 \div 7.48$   | (i) $12.021 \div 5.917$ |

2. Estimate the value of each of the following:

- |  |   |
|--|---|
| (a) $\sqrt{\frac{3.9 \times 17.45}{4.23}}$ | (b) $\sqrt{8.1^2 + 7.32^2}$                   |
| (c) $\sqrt{\frac{62.4 \times 18.3}{4.7}}$  | (d) $\sqrt{\frac{11.42 \times 19.99}{3.851}}$ |



3. James writes down this statement:

$$14.62 \times 401 = 586.262$$

- (a) Use estimation to describe why James must be wrong.
- (b) Use a calculator to determine the correct answer, and then describe the mistake that James made.
4. Use estimation to decide which of the following calculations are definitely incorrect:
- (a)  $15.2 \times 6120 = 930240$       (b)  $65.224 \div 12.4 = 5.26$
- (c)  $192 \times 4587 = 880704$       (d)  $346.92 \div 2.36 = 14.7$
5. For each of the following calculations three possible answers are given, but only one is correct. Use estimation to select the correct answer.

(a)  $3.712 \times 805 = \begin{cases} 298.816 \\ 2988.16 \\ 298881.6 \end{cases}$

(b)  $2955.82 \div 9.82 = \begin{cases} 301 \\ 3010 \\ 30100 \end{cases}$

(c)  $\sqrt{\frac{7.2^2 + 8.1^2}{4}} = \begin{cases} 541.87 \\ 54.187 \\ 5.4187 \end{cases}$

(d)  $\frac{84.2 + 73.6}{2.2} = \begin{cases} 71.73 \\ 7.173 \\ 0.7173 \end{cases}$

6. The month of December has 31 days. Estimate the number of seconds in December. Compare your estimate with the correct value.
7. Estimate the circumference of a circle with a radius of 20.5 cm.
8. A book has 326 pages with an average of 268.4 words on each page. Estimate the number of words in the book.
9. At a school the average daily amount spent by each child in the canteen is £1.42. There are 1264 pupils in the school. Estimate the total amount spent in the canteen each day.

10. Estimate the area of a circle of radius 6.27 cm.
11. Carl has an old recipe for egg custard with raisins.
- (a) The custard must be cooked at 320 degrees Fahrenheit.  
Carl has a rule to change the temperature to degrees Celsius.

*Rule:*

To change the temperature to degrees Celsius,  
subtract 32 from the temperature in degrees Fahrenheit,  
then multiply the answer by 5,  
then divide by 9.

Use Carl's rule to change 320 degrees Fahrenheit to degrees Celsius.  
You must show each step in your calculation.

- (b) Carl is using this recipe:

*Egg Custard with Raisins*

$\frac{1}{4}$  pound of raisins

1 pint of milk

3 eggs

Put the raisins in an 8 inch bowl.

Mix the eggs and milk, and pour over the raisins.

Bake in the oven at 320 ° Fahrenheit for about an hour.

He starts to change the amounts into *metric* measures.

Write down the words or numbers missing from the sentences below.

*Egg Custard with Raisins*

About 100 grams of raisins

About 0.5 ..... of milk

3 eggs

Put the raisins in a ..... centimetre bowl.

(KS3/96/Ma/Tier 4-6/P1)

## 14.3 Calculator Use

In this section we consider how to make the best use of calculators. We focus on the use of *brackets*. We will then look at the use of the *memory* functions.

Calculators vary a great deal both in the number of memories available and in how they are used. It is essential for you to learn how *your* calculator memory works.



### Example 1

Calculate  $\frac{4.7 + 3.2}{8.6 - 3.1}$ , giving your answer to 3 decimal places.



### Solution

Use the following keys to obtain the correct answer:

( 4 . 7 + 3 . 2 ) ÷

( 8 . 6 - 3 . 1 ) =

You should obtain 1.436363636, which rounds to 1.436 to 3 decimal places.

Note how brackets have been used around both the numerator and the denominator of the fraction.



### Example 2

Zarah uses her calculator to obtain an answer to,

$$\frac{4.7}{102 - 3.8}$$

She obtains the answer  $-3.75$  to 2 decimal places.

- Use estimation to show that her answer is incorrect.
- Describe the error she made when using her calculator, and write down the calculation that she actually carried out.
- Calculate the correct answer, correct to 3 significant figures.



### Solution

$$(a) \quad \frac{4.7}{102 - 3.8} \approx \frac{5}{100 - 4} \approx \frac{5}{100} = 0.05$$

The estimate indicates that Zarah's negative answer must be incorrect.

- Zarah forgot to include brackets around the denominator of the fraction. She actually calculated

$$\frac{4.7}{100} - 3.8$$

- (c) The correct answer can be obtained by using the following sequence of keys:

$$\boxed{4} \boxed{\cdot} \boxed{7} \boxed{\div} \boxed{(} \boxed{1} \boxed{0} \boxed{2} \boxed{-} \boxed{3} \boxed{\cdot} \boxed{8} \boxed{)} \boxed{=}$$

This gives a value of 0.047861507, which is 0.0479 to 3 significant figures.

Note that this compares well with the estimate.



## Exercises

1. Calculate the answers to the following, giving your answers to 2 significant figures:

(a)  $\frac{3.8 \times 4.7}{6.2 + 8.9}$

(b)  $\frac{42.3}{8.6 + 12.9}$

(c)  $\frac{56.7 - 8}{3.2}$

(d)  $\frac{11.4 + 19.6}{7.31}$

(e)  $\frac{4.87 + 3.62}{5.9 - 0.62}$

(f)  $\frac{14.6}{8.2} - \frac{3.7}{4.9}$

(g)  $\frac{4.3 \times 3.07 \times 6.109}{2.9 \times 9.81 \times 5.006}$

(h)  $\frac{21 \times (3 - 2.1)}{4.7 + 12.8}$

2. Calculate the following, giving your answers correct to 3 significant figures:

(a)  $(6.72 - 3.8)^2$

(b)  $(4.62 + 3.84)^3$

(c)  $\sqrt{6.2 + 7.4}$

(d)  $(3.68 + 2.41) \times (8.21 - 3)$

(e)  $6.2^2 \times (14.2 - 8.03)$

(f)  $11.4^2 + (3.8 - 4.2)^2$

3. For each calculation below, first obtain an estimate of the answer and then calculate the actual answer, where necessary rounding the value to 3 significant figures.

(a)  $\frac{81.4}{12 - 3.2}$

(b)  $\frac{33.6}{4.52} - \frac{11.2}{21.5}$

(c)  $\frac{88.92 - 14.37}{2.69}$

(d)  $\frac{84.3 + 18.92}{3(4.6 - 0.17)}$

4. Joseph and Jenny use their calculators to try to calculate,

$$\frac{3.7 + 8.9}{4.61}$$

When they round to 3 significant figures, Joseph obtains the answer 5.63 and Jenny gets 2.73.

- (a) Use estimation to decide who has the *incorrect* answer.  
 (b) Describe the calculation that was used to obtain the *incorrect* answer.  
 (c) Check that the other answer is *correct*.
5. Dee and Daniella press the keys on their calculators in the following sequences:

Dee      (4) (÷) (7) (+) (6) (=) (÷) (3) (=)

Daniella      (4) (÷) (7) (+) (6) (÷) (3) (=)

- (a) What answers do they obtain?  
 (b) Write down the calculation that Dee has the answer to.  
 (c) Write down the calculation that Daniella has the answer to.
6. Adrian calculates the number of seconds in 2 hours and 40 minutes as 2520. His brother, Richard, says that he is wrong and that the correct answer is 9600 seconds.
- (a) Use an estimate to show that Adrian must be wrong.  
 (b) Write down the calculations that the brothers did to obtain their answers.
7. Denise says that she can obtain the answer to

$$\frac{3.6}{17.2} + \frac{8.6}{3.5}$$

using her calculator, but without using brackets. Hannah disagrees. Who is correct? Explain why.

8. Which of the following can be calculated on a calculator, without using brackets or a memory?

(a)  $3 \times 7 + 8 \times 32$

(b)  $\frac{4}{9} + \frac{6}{25}$

(c)  $\frac{11 + 6}{4}$

(d)  $\frac{3 \times 2}{4}$

(e)  $\frac{5 \times 2}{8 \times 3}$

(f)  $\frac{22 + 1}{5}$

9. For each of the following calculations, first obtain an estimate and then calculate the actual value, where necessary giving your answer correct to 3 significant figures.

(a)  $\sqrt{\frac{3.7}{0.84}}$

(b)  $\sqrt{\frac{8.2}{1.6}} + \sqrt{\frac{9.2}{1.3}}$

(c)  $\sqrt{8.4^2 + 4.8^2 - 2 \times 8.4 \times 4.8}$

(d)  $\frac{\sqrt{82} - \sqrt{63}}{\sqrt{47} \times \sqrt{32}}$

10. (a) Write down the sum that you would use to calculate the number of seconds in 3 days, 4 hours and  $28\frac{1}{2}$  minutes.  
 (b) Obtain the answer on your calculator.

11. David is studying blood cells through a microscope.

The diameter of a red cell is 0.000714 cm and the diameter of a white cell is 0.001243 cm.

- (a) Use a calculator to work out the difference between the diameter of a red cell and the diameter of a white cell.

Give your answer in *millimetres*.

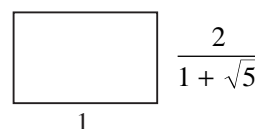
David wants to explain how small the cells are.

He calculates how many white cells would fit across a full stop which has a diameter of 0.65 mm.

- (b) How many whole white cells would fit across the full stop?

(KS3/95/Ma/Levels 5-7/P1)

12. Rectangles with length and width in the special ratio  $1 : \frac{2}{1 + \sqrt{5}}$  are called golden rectangles.



Some artists use them because the proportions look attractive.

- (a) Work out  $\frac{2}{1 + \sqrt{5}}$ .

Write this using all the decimal places your calculator shows.

- (b) Ramesh used his calculator efficiently to work out  $\frac{2}{1 + \sqrt{5}}$ .

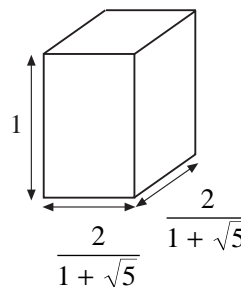
He did *not* have to write down any numbers and then key them back into his calculator.

Show the steps he might have used with his calculator.

- (c) Four faces of this cuboid are golden rectangles.

The volume of the cuboid is:

$$\left(\frac{2}{1+\sqrt{5}}\right) \times \left(\frac{2}{1+\sqrt{5}}\right) \times 1$$



Use a *short* and *accurate* method on your calculator to calculate this. Write the volume using all the decimal places your calculator shows.

- (d) Show the steps you used to make your method *short* and *accurate*.

(KS3/94/Ma/5-7/P1)

## 14.4 Error Propagation

If there is an error in a value that is used in a calculation, that error can become more significant when the calculation is made. For example, if the radius of a circle is rounded from 2.57 cm to 2.6 cm, an error of  $0.49 \text{ cm}^2$  would be made when calculating the area of the circle.

In this section we consider how errors introduced by rounding can be increased (or *propagated*) in subsequent calculations.



### Example 1

The radius of a circle is given as 31 cm, correct to the nearest cm. What are the possible errors when calculating its area?



### Solution

As the radius,  $r$  cm, is given as 31 cm to the nearest cm, we have

$$30.5 \leq r < 31.5$$

$$\begin{aligned} \text{If } r = 30.5, \quad A &= \pi \times 30.5^2 \\ &= 2922.466566 \text{ cm}^2 \\ &= 2920 \text{ cm}^2 \quad \text{to 3 s.f.} \end{aligned}$$

$$\begin{aligned} \text{If } r = 31, \quad A &= \pi \times 31^2 \\ &= 3019.07054 \text{ cm}^2 \\ &= 3020 \text{ cm}^2 \quad \text{to 3 s.f.} \end{aligned}$$

$$\begin{aligned} \text{If } r = 31.5, \quad A &= \pi \times 31.5^2 \\ &= 3117.245311 \text{ cm}^2 \\ &= 3120 \text{ cm}^2 \quad \text{to 3 s.f.} \end{aligned}$$

If  $r = 30.5$ , then the error is

$$3019.07054 - 2922.466566 = 96.603974 \text{ cm}^2$$

whilst, if  $r = 31.5$ , the error is

$$3117.245311 - 3019.07054 = 98.17477043 \text{ cm}^2$$

Hence the maximum possible error occurs when  $r = 31.5$ , and is approximately  $98.2 \text{ cm}^2$ . In other words, there is a potential error here of almost  $100 \text{ cm}^2$  in the area if we calculate it from the rounded radius.

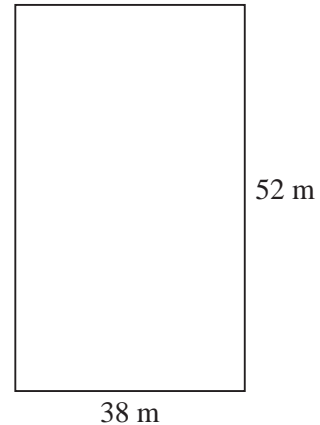


### Example 2

A rectangular plot of land has sides with lengths of 38 m and 52 m correct to the nearest m.

Calculate the maximum and minimum possible values of:

- the *perimeter* of the rectangle,
- the *area* of the rectangle.



### Solution

- The sides have been given to the nearest metre, so

$$51.5 \text{ m} \leq \text{length} < 52.5 \text{ m}$$

$$37.5 \text{ m} \leq \text{width} < 38.5 \text{ m}$$

$$\begin{aligned} \text{Minimum perimeter} &= 2(37.5 + 51.5) \\ &= 178 \text{ m} \end{aligned}$$

$$\begin{aligned} \text{Maximum perimeter} &= 2(38.5 + 52.5) \\ &= 182 \text{ m} \end{aligned}$$

- Minimum area =  $37.5 \times 51.5$   
=  $1931.25 \text{ m}^2$

$$\begin{aligned} \text{Maximum area} &= 38.5 \times 52.5 \\ &= 2021.25 \text{ m}^2 \end{aligned}$$



### Example 3

The values of  $x$  and  $y$  are given to 1 decimal place as  $x = 4.2$  and  $y = 7.3$

Determine the minimum and maximum values of:

- $x + y$
- $y - x$
- $\frac{x}{y}$



### Solution

First note that  $4.15 \leq x < 4.25$  and  $7.25 \leq y < 7.35$ .

- Minimum value of  $x + y = 4.15 + 7.25$  (minimum value of  $x$  + minimum value of  $y$ )  
= 11.4



$$\begin{aligned} \text{Maximum value of } x + y &= 4.25 + 7.35 && (\text{maximum value of } y \\ &= 11.6 && + \text{maximum value of } x) \end{aligned}$$

$$(b) \quad \begin{aligned} \text{Minimum value of } y - x &= 7.25 - 4.25 && (\text{minimum value of } y \\ &= 3 && - \text{maximum value of } x) \end{aligned}$$

$$\begin{aligned} \text{Maximum value of } y - x &= 7.35 - 4.15 && (\text{maximum value of } y \\ &= 3.2 && - \text{minimum value of } x) \end{aligned}$$

$$(c) \quad \begin{aligned} \text{Minimum value of } \frac{x}{y} &= \frac{4.15}{7.35} = 0.56462585 && (\text{minimum value of } x \\ &= 0.565 \quad \text{to 3 s.f.} && \div \text{maximum value of } y) \end{aligned}$$

$$\begin{aligned} \text{Maximum value of } \frac{x}{y} &= \frac{4.25}{7.25} = 0.586206896 && (\text{maximum value of } x \\ &= 0.586 \quad \text{to 3 s.f.} && \div \text{minimum value of } y) \end{aligned}$$

Note that, for  $x$  and  $y$  both positive,

To find *maximum* value of  $x + y$  or  $xy$ ,  
use the largest value of  $x$  and largest value of  $y$ .

To find the *minimum* value of  $x + y$  or  $xy$ ,  
use the smallest value of  $x$  and smallest value of  $y$ .

To find *maximum* value of  $x - y$  or  $\frac{x}{y}$ ,  
use the largest value of  $x$  and smallest value of  $y$ .

To find the *minimum* value of  $x - y$  or  $\frac{x}{y}$ ,  
use the smallest value of  $x$  and largest value of  $y$ .



## Exercises

- A car park is a square with sides of length 48 m correct to the nearest metre. Calculate the maximum and minimum possible values for:
  - the *perimeter* of the car park,
  - the *area* of the car park.
- The radius of a circle is given as 9 cm correct to the nearest cm. Calculate the maximum and minimum possible values for the area of the circle. Round your upper and lower bounds to 2 decimal places.

3. A rectangle has sides of length 1.27 m and 2.43 m correct to 2 decimal places. Calculate the maximum and minimum possible values for:
- (a) the *perimeter*;                      (b) the *area*.
4. The formula  $V = \frac{4\pi r^3}{3}$  is used to calculate the volume of a sphere of radius  $r$ . A sphere has radius 18 cm correct to the nearest cm. Calculate the maximum and minimum possible values for the volume of the sphere. Round your upper and lower bounds to 2 decimal places.
5. If  $x$  and  $y$  are given correct to 1 decimal place as  $x = 5.2$  and  $y = 11.6$ , calculate the maximum and minimum possible values of:
- (a)  $x^2$                                       (b)  $x + y$                                       (c)  $xy$
- (d)  $y - x$                                       (e)  $x^2 + y^2$                                       (f)  $\frac{x}{y}$
6. Hannah measures with a ruler that has marks every 0.5 cm. She says that the lengths of the side of a rectangle are 8.5 cm and 12 cm correct to the nearest 0.5 cm. Calculate the minimum possible values of:
- (a) the *perimeter* of the rectangle,
- (b) the *area* of the rectangle.
7. The radius of a circle is said to be  $18 \text{ m} \pm 0.5 \text{ m}$ .
- (a) What is the *maximum* possible radius of the circle?
- (b) Write the *circumference* of the circle in the form  $a \text{ m} \pm b \text{ m}$ , giving the values of  $a$  and  $b$  correct to 2 decimal places.
8. A square has sides with lengths that are given as  $1.8 \text{ cm} \pm 0.05 \text{ cm}$ .
- (a) Write the *perimeter* of the square in the form  $a \text{ cm} \pm b \text{ cm}$ .
- (b) Write the *area* of the square in the form  $a \text{ cm}^2 \pm b \text{ cm}^2$ .
9. The area of a square is given as  $14 \text{ m}^2$  correct to the nearest  $\text{m}^2$ . Calculate the maximum and minimum possible values for the length of the side of the square, rounding your answers to 3 significant figures.
10. A circle has area  $50 \text{ cm}^2 \pm 2 \text{ cm}^2$ . What are the maximum and minimum possible values of the radius of the circle? Round your upper and lower bounds to 2 decimal places.

11. Mary, Arvind and Nesta wrote a mass to different numbers of decimal places.

Mary wrote 1.7 to 1 decimal place.

Arvind wrote 1.748 to 3 decimal places.

- (a) Can both of them be correct?  
Explain how you decided.
- (b) Nesta wrote 1.7474 to 4 decimal places.  
Can both Nesta and Arvind be correct?  
Explain how you decided.
- (c) They wrote a volume to different numbers of decimal places.  
Mary wrote 2.6 to 1 decimal place.  
Arvind wrote 2.65 to 2 decimal places.  
They are both correct.  
What are the lower and upper bounds of the volume?
- (d) The mass of a block is 0.10 kilograms to 2 decimal places.  
Its volume is 0.02 litres to 2 decimal places.

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Work out the minimum and maximum possible density of the block.

Show your working.

- (e) The maximum possible percentage error in the density of the block can be thought of in different ways and therefore calculated using different methods.  
Investigate two different methods which give a maximum percentage error greater than 30%.  
Calculate the values they give for the percentage error.  
Show your working.

(KS3/94/Ma/9-10/Ext)

12. The exact formula to convert a temperature measured in degrees Celsius (C) to degrees Fahrenheit (F) is

$$F = \frac{9C}{5} + 32$$

An approximate formula often used for this conversion is

$$F = 2C + 30$$

- (a) The temperature is  $20.0\text{ }^{\circ}\text{C}$ , measured to the nearest tenth of a degree. What would you obtain as the upper and lower bounds of the temperature in degrees Fahrenheit if you used the approximate formula for the conversion?
- (b) What are the upper and lower bounds of the temperature in degrees Fahrenheit obtained by using the exact formula to convert  $20.0\text{ }^{\circ}\text{C}$  to Fahrenheit?
- (c) Calculate the percentage error when the approximate formula, rather than the exact formula, is used to convert  $30.0\text{ }^{\circ}\text{C}$  to Fahrenheit.
- (d) For what range of values of  $C$  does the approximate formula give an answer in degrees Fahrenheit which differs by a percentage error of no more than 5% from the exact formula?  
Give your answers to one decimal place.

(KS3/95/Ma/Levels 9-10)