

Example 3

Write each of the following fractions in their simplest form:

(a)
$$\frac{8}{18}$$
 (b) $\frac{5}{40}$ (c) $\frac{12}{32}$

3

Solution

(a)	$\frac{8}{18} = \frac{4}{9}$	(dividing top and bottom by 2)
(b)	$\frac{5}{40} = \frac{1}{8}$	(dividing top and bottom by 5)
(c)	$\frac{12}{32} = \frac{3}{8}$	(dividing top and bottom by 4)

Exercises

1. Write, in two different ways the fraction of each shape which has been shaded:







2. Fill in the missing number in each of the following statements:





(a) $\frac{2}{7}$ (b) $\frac{3}{8}$ (c) $\frac{5}{9}$

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Is each of the following statements *true* or *false*: 6.

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(a)
$$\frac{4}{7} = \frac{16}{21}$$
 (b) $\frac{3}{8} = \frac{12}{32}$
(c) $\frac{4}{5} = \frac{16}{20}$ (d) $\frac{5}{9} = \frac{25}{45}$

Fill in the missing number in each of the following statements: 7. (a)

Which of the fractions $\frac{4}{5}$ and $\frac{5}{8}$ is the *larger*? (b)

Fill in the missing number in each of the following statements: 8. (a)

Which of the fractions $\frac{5}{7}$ and $\frac{2}{3}$ is the *smaller*? (b)

Which of these fractions is the *largest*? 9.

$$\frac{1}{2}$$
 $\frac{3}{5}$ $\frac{4}{7}$

Write the following fractions in order of size, with the *smallest* first: 10.

$$\frac{1}{5}$$
 $\frac{1}{4}$ $\frac{2}{9}$ $\frac{1}{2}$ $\frac{5}{9}$

4.2 Fractions of Quantities

In this section we review how to find fractions of quantities; for example, $\frac{3}{4}$ of 60.

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Example 1

Calculate:

(a)
$$\frac{1}{3}$$
 of £60, (b) $\frac{1}{5}$ of £40.

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Solution

(a)
$$60 \div 3 = 20$$

So $\frac{1}{3}$ of $\pounds 60 = \pounds 20$.

(b)
$$40 \div 5 = 8$$

So $\frac{1}{5}$ of £40 = £8.

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Example 2

Calculate:

(a) $\frac{3}{4}$ of 700, (b) $\frac{5}{7}$ of 21.

Solution

(a)
$$700 \div 4 = 175$$

 $175 \times 3 = 525$
So $\frac{3}{4}$ of $700 = 525$

(b)
$$21 \div 7 = 3$$

 $5 \times 3 = 15$
So $\frac{5}{7}$ of $21 = 15$



(b) How much further does he have to drive to complete the journey?

- 7. There are 300 passengers on a train. At a station, $\frac{3}{5}$ of the passengers get off.
 - (a) How many people get off the train?
 - (b) How many passengers are left on the train?

8. Alison has £30. She decides to save $\frac{2}{5}$ of this and to spend $\frac{1}{6}$ on books.

- (a) How much money does she save?
- (b) How much does she spend on books?
- (c) How much does she have left?

9. A farmer owns 360 hectares of land. He plants potatoes on $\frac{3}{10}$ of his land. How many hectares are planted with potatoes?

- 10. An engineer tests a box of 120 floppy disks. He finds that $\frac{1}{20}$ of the disks are damaged. How many of the disks are damaged?
- 11. Sue and Ben each have 12 biscuits.
 - (a) Sue eats a quarter of her biscuits. How many biscuits does Sue eat?
 - (b) Ben eats 6 of his biscuits. What fraction of his biscuits does Ben eat?
 - (c) How many biscuits are left altogether?

(KS3/97/Ma/Tier 3-5/P1)

4.3 Operations with Fractions

Here we review how to *add*, *subtract*, *multiply* and *divide* fractions.

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Example 1

Calculate:

(a)	3 1	(b)	5	2
(a)	$\frac{-}{5} + \frac{-}{4}$	(0)	7	3



Solution

Before fractions can be added or subtracted, they must each have the same denominator (known as a *common denominator*).

(a)
$$\frac{3}{5} + \frac{1}{4} = \frac{12}{20} + \frac{5}{20} = \frac{17}{20}$$



OR

(b)
$$\frac{5}{7} - \frac{2}{3} = \frac{15}{21} - \frac{14}{21}$$

= $\frac{1}{21}$

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Calculate:

(a)

Example 2

culate.		
$\frac{4}{5} \times \frac{3}{7}$	(b)	$\frac{5}{8} \times \frac{2}{7}$

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Solution
(a)
$$\frac{4}{5} \times \frac{3}{7} = \frac{4 \times 3}{5 \times 7}$$

 $= \frac{12}{35}$
(b) $\frac{5}{8} \times \frac{2}{7} = \frac{5 \times 2}{8 \times 7}$
 $= \frac{10}{56}$
 $= \frac{5}{28}$

$$\frac{5}{4} \times \frac{5}{7} \times \frac{5}{7} = \frac{5 \times 1}{4 \times 7} = \frac{5}{28}$$

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Example 3

Calculate:
(a)
$$\frac{3}{5} \div \frac{2}{3}$$
 (b) $\frac{5}{7} \div \frac{3}{4}$

Solution

(a)
$$\frac{3}{5} \div \frac{2}{3} = \frac{3}{5} \times \frac{3}{2}$$

 $= \frac{9}{10}$
(b) $\frac{5}{7} \div \frac{3}{4} = \frac{5}{7} \times \frac{4}{3}$
 $= \frac{20}{21}$





	(g)	$\frac{3}{4} + \frac{2}{3}$	(h)	$\frac{3}{5} + \frac{2}{3}$	(i)	$\frac{4}{7} + \frac{2}{5}$	
	(j)	$\frac{5}{6} + \frac{2}{3}$	(k)	$\frac{1}{8} + \frac{2}{3}$	(1)	$\frac{4}{5} + \frac{5}{6}$	
3.	Calc	culate:					
	(a)	$\frac{1}{2} \times \frac{1}{3}$	(b)	$\frac{4}{5} \times \frac{2}{3}$	(c)	$\frac{1}{8} \times \frac{2}{3}$	
	(d)	$\frac{5}{6} \times \frac{3}{4}$	(e)	$\frac{4}{5} \times \frac{5}{7}$	(f)	$\frac{3}{8} \times \frac{1}{4}$	
	(g)	$\frac{4}{5} \times \frac{1}{2}$	(h)	$\frac{2}{3} \times \frac{3}{4}$	(i)	$\frac{5}{8} \times \frac{2}{3}$	
	(j)	$\frac{3}{7} \times \frac{2}{3}$	(k)	$\frac{4}{8} \times \frac{3}{4}$	(1)	$\frac{7}{8} \times \frac{2}{3}$	
4.	Calc	culate:					
	(a)	$\frac{1}{2} \div \frac{1}{3}$	(b)	$\frac{3}{4} \div \frac{2}{3}$	(c)	$\frac{4}{5} \div \frac{2}{3}$	
	(d)	$\frac{2}{3} \div \frac{2}{5}$	(e)	$\frac{3}{7} \div \frac{3}{4}$	(f)	$\frac{5}{8} \div \frac{3}{4}$	
	(g)	$\frac{4}{15} \div \frac{2}{3}$	(h)	$\frac{2}{3} \div \frac{5}{7}$	(i)	$\frac{3}{7} \div \frac{3}{5}$	
	(j)	$\frac{4}{9} \div \frac{2}{3}$	(k)	$\frac{3}{8} \div \frac{6}{7}$	(1)	$\frac{7}{9} \div \frac{2}{3}$	
F	Cal						
5.	Calc	1 1		1 1		1 2	
	(a)	$1\frac{1}{2} \times 2\frac{1}{4}$	(b)	$2\frac{1}{2} \times 1\frac{1}{3}$	(c)	$2\frac{1}{3} \times 1\frac{3}{4}$	
	(d)	$3\frac{1}{4} \times 1\frac{1}{3}$	(e)	$2\frac{1}{2} \times 1\frac{1}{2}$	(f)	$1\frac{1}{5} \times 1\frac{1}{2}$	
6.	Calc recta	culate the area and pe angle shown:	erimete	er of the			$\frac{2}{5}$ m
						$\frac{3}{4}$ m	
						+	

7. Julie has a vegetable plot that has an area of $\frac{2}{3}$ of an acre. She plants potatoes on $\frac{1}{4}$ of the plot.

What fraction of an acre does she plant with potatoes?

8. Which is the *larger*

$$\frac{3}{4} \times \frac{1}{2}$$
 or $\frac{3}{4} \div \frac{1}{2}$?

9. Solve these equations:

(a)
$$\frac{2}{3}x = \frac{4}{9}$$
 (b) $\frac{3}{5}x = \frac{9}{4}$

10. If the area of the rectangle shown is $1\frac{1}{2}$ m², what is the length of the rectangle?

 $\frac{2}{3}$ m

11.

(a)

4.3

Advert 1 uses $\frac{1}{4}$ of the pageAdvert 2 uses $\frac{1}{8}$ of the pageAdvert 3 uses $\frac{1}{16}$ of the page

In a magazine there are three adverts on the same page.

In total, what fraction of the page do the three adverts use? Show your working.

(b) The cost of an advert is £10 for each
$$\frac{1}{32}$$
 of a page.

An advert uses $\frac{3}{16}$ of a page. How much does the advert cost?

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(KS3/99/Ma/Tier 4-6/P1)



4.4 Fraction, Decimal and Percentage Equivalents

In this section we revisit the equivalence of fractions, decimals and percentages; for example, we could write $\frac{1}{2}$ as 0.5 or as 50%.

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Example 1

Write each of the following percentages as decimals and fractions in their simplest form:

(a) 75% (b) 32%

Solution

(a)
$$75\% = \frac{75}{100}$$

= 0.75 as a decimal
 $75\% = \frac{75}{100}$
= $\frac{3}{4}$ as a fraction in its simplest form
(b) $32\% = \frac{32}{100}$
= 0.32 as a decimal
 $32\% = \frac{32}{100}$

$$\frac{3}{25}$$
 as a fraction in its simplest form

(i)

Example 2

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Write each of the following decimals as a percentage and as a fraction in its simplest form:

(a) 0.72 (b) 0.08

Solution

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(a)
$$0.72 = \frac{72}{100}$$

 $= 72\%$ as a percentage
 $0.72 = \frac{72}{100}$
 $= \frac{18}{25}$ as a fraction in its simplest form
(b) $0.08 = \frac{8}{100}$
 $= 8\%$ as a percentage
 $0.08 = \frac{8}{100}$
 $= \frac{2}{25}$ as a fraction in its simplest form

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Example 3

Write each of the following fractions as a decimal and as a percentage:

(a)
$$\frac{3}{10}$$
 (b) $\frac{4}{25}$ (c) $\frac{3}{8}$

Solution

(a)
$$\frac{3}{10} = \frac{30}{100}$$
 (multiply top and bottom by 10)
= 0.3 as a decimal
= 30% as a percentage
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$$\frac{4}{25} = \frac{10}{100}$$
 (multiply top and bottom by 4)
= 0.16 as a decimal

= 16% as a percentage

(b)
$$\frac{3}{8} = \frac{37.5}{100}$$
 (multiply top and bottom by 12.5)
= 0.375 as a decimal
= 37.5% as a percentage

4.4

Exercises 1. Write each of the following percentages as a decimal: 60% 70% (a) (b) (c) 20% (d) 45% (e) 31% (f) 82% (g) 14% (h) 4% (i) 63% 2% (k) 1% 19% (j) (1)Write each of the following percentages as a fraction in its simplest form: 2. 80% 25% 40% (a) (b) (c) 35% (e) 65% 4% (d) (f) 64% (h) 82% 28% (g) (i) 6% (k) 7% (1) 92% (j) 3. Write each of the following decimals as a percentage: 0.74 0.99 (a) (b) 0.5 (c) (d) 0.06 (e) 0.26 (f) 0.02 0.3 (h) 0.002 (i) 0.042 (g) 4. Write each of the following decimals as a fraction in its simplest form: (a) 0.5 (b) 0.25 (c) 0.4 (d) 0.7 (e) 0.62 (f) 0.44 (g) 0.37 (h) 0.04 (i) 0.05 0.24 (k) 0.1 (1) 0.74 (j) Write each of the following fractions as a decimal: 5. $\frac{4}{5}$ $\frac{1}{2}$ $\frac{3}{4}$ (a) (b) (c) $\frac{9}{20}$ $\frac{7}{10}$ $\frac{3}{100}$ (f) (d) (e) $\frac{19}{100}$ $\frac{23}{50}$ $\frac{7}{25}$ (g) (h) (i) $\frac{5}{8}$ $\frac{8}{25}$ $\frac{1}{8}$ (j) (k) (l)

6. Write each of the following fractions as a percentage:

- (a) $\frac{9}{10}$ (b) $\frac{17}{100}$ (c) $\frac{14}{25}$
- (d) $\frac{3}{20}$ (e) $\frac{2}{5}$ (f) $\frac{3}{5}$ (g) $\frac{9}{20}$ (h) $\frac{9}{100}$ (i) $\frac{1}{100}$
- (j) $\frac{3}{50}$ (k) $\frac{7}{8}$
- 7. Copy and complete this table of equivalent fractions, decimals and percentages:

Fraction	Decimal	Percentage
$\frac{4}{5}$		
	0.68	
		85%
	0.76	
$\frac{8}{25}$		
		3%
	0.005	

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8. In a survey, 400 people were asked how they would vote at the next election. The results are listed below:

Labour	220
Conservative	160
Other	20

Express these results as percentages.

- 9. In a school there are 50 Manchester City supporters out of a total of 2000 pupils.
 - (a) What percentage of the pupils support Manchester City?
 - (b) What percentage of the pupils do *not* support Manchester City?

- 10. In a group of 40 pupils there are 7 who cannot swim.What percentage of the pupils *can* swim?
- 11. Simon is growing vegetables in three vegetable patches.
 - (a) About 50% of this vegetable patch is for *carrots*.



Write down the missing *percentages*:

- (i) about ...% of the patch is for *cabbages*,
- (ii) about ...% of the patch is for *lettuces*.

(b) About $\frac{1}{8}$ of this vegetable patch is for *beetroot*.



Write down the missing *fractions*:

- (i) about ... of the patch is for *broad beans*.
- (ii) about ... of the patch is for *peas*.
- (c) About $\frac{4}{5}$ of this vegetable patch is for *potatoes*.

Copy the diagram below and draw a straight line to show how much of the patch is for *potatoes*. Shade in the area for potatoes.



The rest of the patch is for *turnips*.

About what fraction of the patch is for *turnips*?

(KS3/96/Ma/Tier 4-6/P1)

12. $\frac{1}{2}$ of the diagram below is shaded.



(a) Look at this diagram:



What *fraction* is shaded?

What percentage is shaded?

(b) Copy the diagram below and shade $\frac{2}{5}$ of it.



What *percentage* of the diagram have you shaded?

(KS3/97/Ma/Tier 3-5/P1)

4.5 Percentage Increases and Decreases

Often prices are increased or decreased by a percentage. In this section we consider how to increase or decrease quantities by using percentages.

Example 1

Katie earns £40 per week for her part-time job. She is to be given a 5% pay rise. How much will she earn per week after the pay rise? 4.5

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Solution

5% of £40 = $\frac{5}{100} \times £40$ OR 100% + 5% = 105% = £2 which is 1.05 as a decimal New pay = £40 + £2 New pay = £40 × 1.05 = £42 = £42

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Example 2

The prices of all the televisions in a shop are to be increased by 8%. Calculate the new price of a television that originally cost ± 150 .

Solution

8% of £150 = $\frac{8}{100} \times $ £150	OR	100% + 8% = 108%
= £12		which is 1.08 as a decimal
New price = $\pounds 150 + \pounds 12$		New price = $\pounds 150 \times 1.08$
= £162		$= \pounds 162$

Example 3

In a sale the cost of a computer is reduced by 30%. The normal price of the computer was £900. Calculate the sale price of the computer.

Solution

30% of £900 =
$$\frac{30}{100} \times £900$$
OR100% - 30% = 70%= £270which is 0.7 as a decimalSale price = £900 - £270New price = £900 × 0.7= £630= £630

Exercises

- 1. (a) Increase $\pounds 100$ by 20%.
 - Increase £80 by 25%. (c)
 - Increase 40 kg by 3%. (e)
- 2. (a) Decrease $\pounds 60$ by 30%.
 - (c) Decrease 80 kg by 10%.
 - Decrease 90 m by 2%. (f) (e)
- 3. A company increases the cost of all its products by 5%. Calculate the new price of each of the items listed below:
 - (a) a tent that previously cost £60.
 - a rucksack that previously cost £15, (b)
 - (c) a sleeping bag that previously cost $\pounds 24$.
- Joe was paid £30 per week for delivering papers. He was given a 3% pay 4. rise. How much will he now earn each week?
- A small firm employs 4 staff. They are all given a 4% pay rise. The 5. original salaries are as follows:

John Smith	£24 000
Alice Holland	£22 500
Graham Hall	£14 000
Emma Graham	£8500

Calculate the new salary for each member of staff.

- Rachel puts £50 into a bank account. After one year 5% interest is added to 6. her money. How much does she have then?
- Add $17\frac{1}{2}$ % VAT to each of the following prices: 7. (c) £42 £200 (a) £70 (b)
- 8. A rope is 8 m long but it shrinks when it gets wet. What would be the new length of the rope if its length is reduced by:

(a) 2% (b) 7% (c) 12% ?

- (b) Increase $\pounds 400$ by 30%.
- (d) Increase £50 by 6%.
- (f) Increase 250 m by 7%.
- Decrease 8 m by 5%. (b)
- (d) Decrease $\pounds 44$ by 20%.
- Decrease 420 kg by 25%.

9. In a sale the prices of each of the items listed below is to be reduced by 35%.

Coat	£28	Jeans	£42
Trainers	£36	Shirt	£14

Calculate the sale price of each item.

- 10. A mountain bike was priced at £180. Its price was increased by 8%. Later, this increased price was reduced by 20% in a sale.Calculate the sale price of the bike.
- 11. This is how Caryl works out 15% of 120 in her head.
 - 10% of 120 is 12,
 - 5% of 120 is 6,
 - so 15% of 120 is 18.
 - (a) Copy and complete the following calculations to show how Caryl can work out $17\frac{1}{2}$ % of 240 in her head.
 - % of 240 is % of 240 is % of 240 is so $17\frac{1}{2}$ % of 240 is
 - (b) Work out 35% of 250. Show your working.

(KS3/98/Ma/Tier 3-5/P1)

12. Look at this table:

	1961	1994
England	17.6	
Wales	17.0	12.2

- (a) In England, from 1961 to 1994, the birth rate fell by 26.1%What was the birth rate in England in 1994 ? Show your working.
- (b) In Wales, the birth rate also fell.
 Calculate the percentage fall from 1961 to 1994. Show your working. (KS3/98/Ma/Tier 5-7/P2)

Birth rate per 1000 population

4.5

Continent	Land Area (in 1000 km^2)
Africa	30 264
Antarctica	13 209
Asia	44 250
Europe	9 907
North America	24 398
Oceania	8 534
South America	17 793
World	148 355

13. The table shows the land area of each of the World's continents.

- (a) Which continent is approximately 12% of the World's land area?
- (b) What percentage of the World's land area is Antarctica? Show your working.
- (c) About 30% of the World's area is land. The rest is water. The amount of land in the World is about 150 million km^2 .

Work out the approximate total area (land and water) of the World. Show your working.

(KS3/98/Ma/Tier 6-8/P2)

14. In 1995, the Alpha Company employed 4000 people. For each of the next 2 years, the number of people employed increased by 10%.

1995	employed 4000 people
1996	employed 10% more people
1997	employed 10% more people

(a) Tony said: *"Each year, the Alpha Company employed another 400 people."*

Tony was wrong. Explain why.

(b) Which of the calculations below shows how many people worked for the company in 1997:

(i) $4000 \times 0.1 \times 2$ (ii) 4000×0.1^2 (iii) $(4000 \times 0.1)^2$ (iv) $4000 \times 1.1 \times 2$ (v) 4000×1.1^2 (vi) $(4000 \times 1.1)^2$



(c) Look at these figures for the Beta Company:

1995	employed <i>n</i> people
1996	employed 20% fewer people
1997	employed 10% more people

Write an expression using n to show how many people the company employed in 1997. Show your working and write your expression as simply as possible.

(KS3/99/Ma/Tier 6-8/P1)

- 15. A clothes shop had a closing down sale. The sale started on Tuesday and finished on Saturday. For each day of the sale, prices were reduced by 15% of the prices on the day before.
 - (a) A shirt had a price of £19.95 on Monday. Kevin bought it on Wednesday. How much did he pay? Show your working.
 - (b) Ghita bought a dress on Tuesday for £41.48. What was its price on Monday? Show your working.
 - (c) A jacket had a price of £49.95 on Monday. What was its price on Friday? Show your working.
 - (d) Another shop is reducing its prices each day by 12% of the prices on the day before. How many days would it take for its original prices to be reduced by more than 50%? Show your working.

(KS3/96/Ma/Tier 6-8/P2)