5 Linear Graphs and Equations

5.1 Coordinates

Firstly, we recap the concept of (x, y) coordinates, illustrated in the following examples.

Example 1

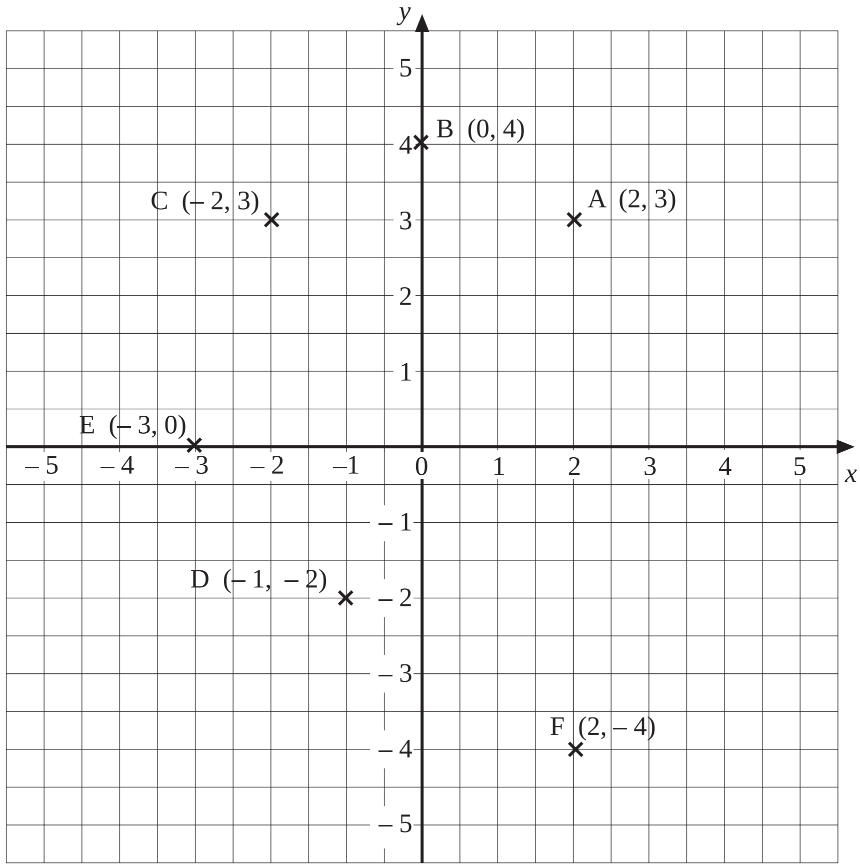
On a set of coordinate axes, plot the points

A (2, 3), B (0, 4), C (– 2, 3), D (– 1, – 2), E (– 3, 0), F (2, – 4)

Solution

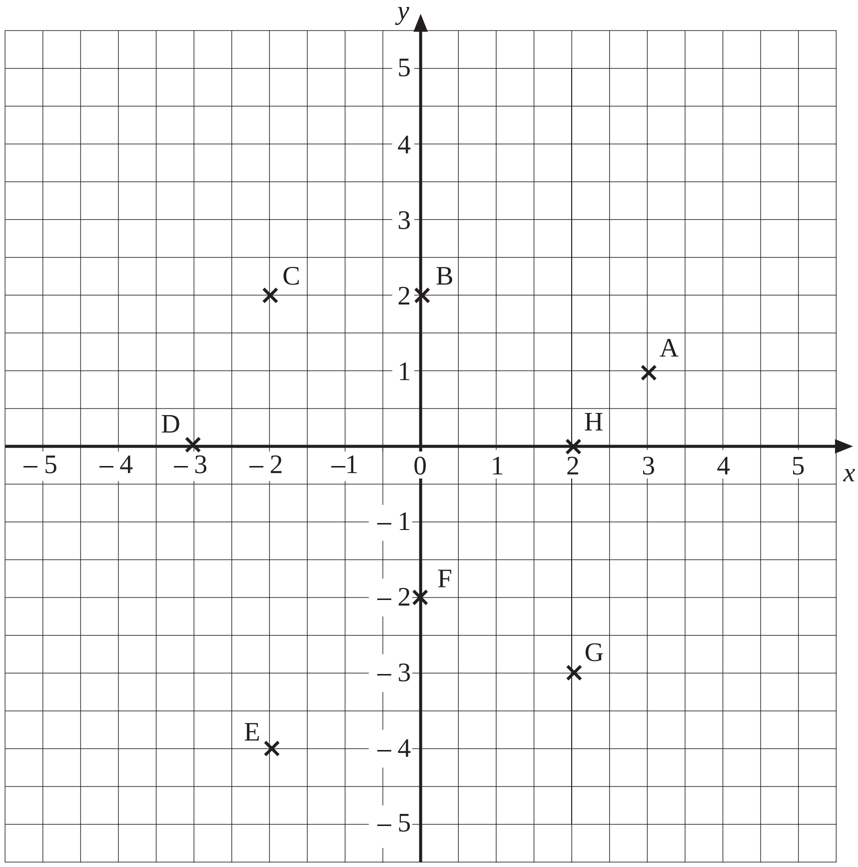
The x-axis and the y-axis cross at the origin, (0, 0).

To locate the point A (2, 3), go 2 units horizontally from the origin in the positive x-direction and then 3 units vertically in the positive y-direction, as shown in the diagram.



Example 2

Identify the coordinates of the points A, B, C, D, E, F, G and H shown on the following grid:



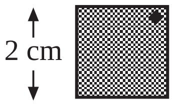
Solution

A (3, 1), B (0, 2), C (– 2, 2), D (– 3, 0),

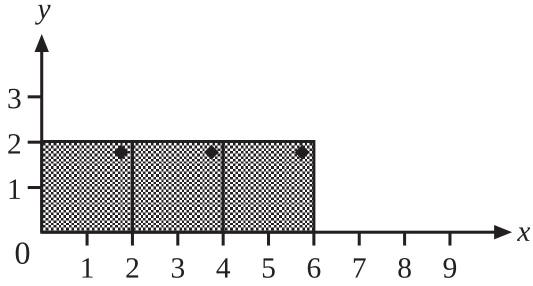
E (– 2, – 4), F (0, – 2), G (2, – 3), H (2, 0)

Example 3

Marc has ten square tiles like this:



Marc places all the square tiles in a row.
He starts his row like this:



For each square tile he writes down the coordinates of the corner which has a ◆.

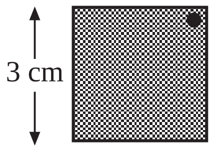
The coordinates of the first corner are (2, 2).

(a) Write down the coordinates of the next five corners which have a ◆.

(b) Look at the *numbers* in the coordinates. Describe *two* things you notice.

(c) Marc thinks that (17, 2) are the coordinates of one of the corners which have a ◆. Explain why he is *wrong*.

(d) Sam has some bigger square tiles, like this:



She places them next to each other in a row, like Marc's tiles.

Write down the coordinates of the first two corners which have a ●.

(KS3/95/Ma/Levels 4-6/P2)

Solution

(a) (4, 2), (6, 2), (8, 2), (10, 2), (12, 2)

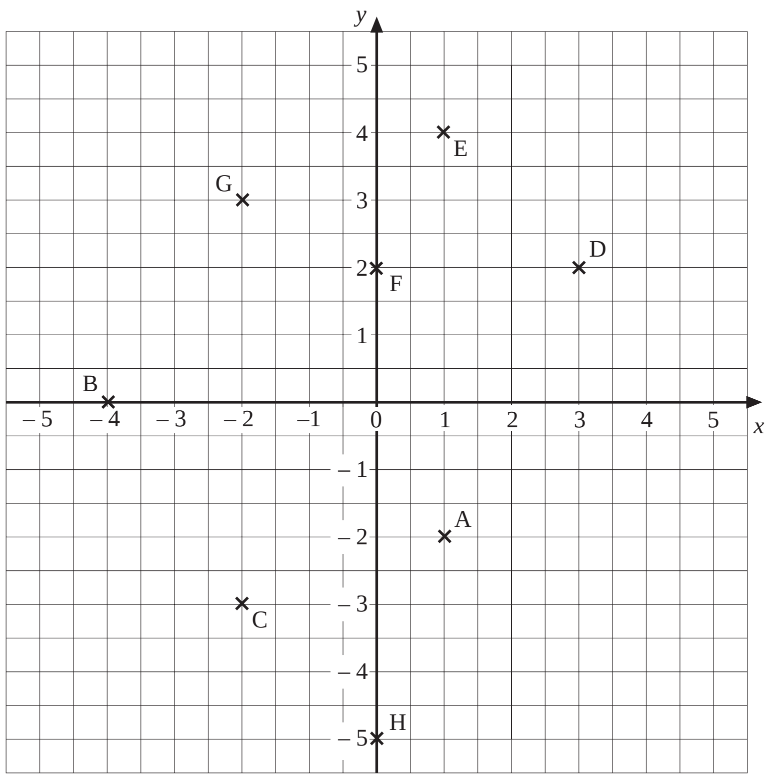
(b) The x-coordinate increases by 2 each time; the y-coordinate remains constant at 2.

(c) (17, 2) cannot be the coordinates of a corner as 17 is an *odd* number and the corners which have a ◆ all have even coordinates.

(d) (3, 3), (6, 3)

Exercises

1. Write down the coordinates of the points marked on the following grid:



2. On a set of coordinate axes, with x values from -5 to 5, y values from -5 to 5, plot the following points:

A (2, 4), B (1, 2), C (– 2, 5), D (– 3, – 3),

E (– 2, – 4), F (0, – 3), G (– 4, 0), H (2, – 3)

What can you say about A, B and E?

3. On a suitable set of coordinate axes, join the points (3, 0), (0, 4) and (– 3, 0).

What shape have you made?

4. Three corners of a square have coordinates (4, 2), (– 2, 2) and (4, – 4). Plot these points on a grid, and state the coordinates of the other corner.

5. Three corners of a rectangle have coordinates (4, 1), (– 2, 1) and (– 2, – 3). Plot these points on a grid and state the coordinates of the other corner.

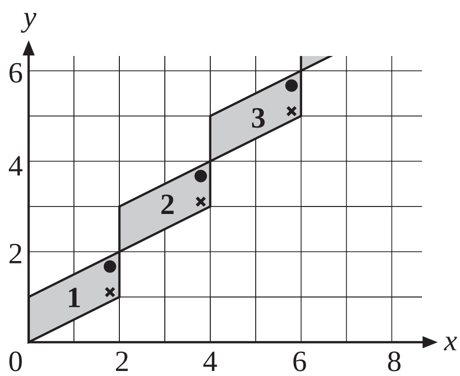
6. Two adjacent corners of a square have coordinates (– 1, 1) and (2, 1).

(a) What is the length of a side of the square?

(b) What are the possible coordinates of the other two points?

7. Daniel has some parallelogram tiles. He puts them on a grid, in a continuing pattern. He numbers each tile.

The diagram shows part of the pattern of tiles on the grid.



Daniel marks the *top right corner* of each tile with a ●.

The coordinates of the corner with a ● on tile number 3 are (6, 6).

(a) What are the coordinates of the corner with a ● on tile number 4 ?

(b) What are the coordinates of the corner with a ● on tile number 20 ?
Explain how you worked out your answer.

(c) Daniel says:

"*One tile in the pattern has a ● in the corner at* (25, 25)."

Explain why Daniel is *wrong.*

(d) Daniel marks the *bottom right corner* of each tile with a ✖ . Copy and complete the table to show the coordinates of each corner with a ✖ .

Tile Number	*Coordinates of the Corner with a* ✖
1	(2, 1)
2	
3	
4	

(e) Copy and complete the statement:

'Tile number 7 has a ✖ in the corner at (....... , ...).'

(f) Copy and complete the statement:

'Tile number .... has a ✖ in the corner at (20, 19).'

(KS3/99/Ma/Tier 4-6/P1)

8. A robot can move about on a grid. It can move North, South, East or West. It must move one step at a time.

The robot starts from the point marked ● on the grid below.

It takes 2 steps.

1st step:	*West*
2nd step:	*North*

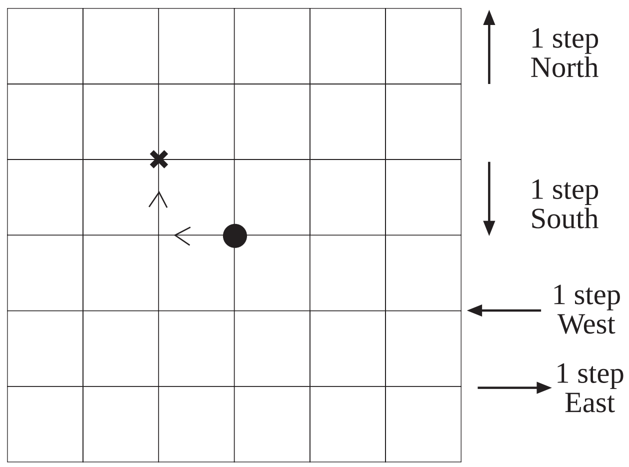
It gets to the point marked ✖.

(a) The robot *starts again* from the point marked ●.

It takes 2 steps.

1st step:	*South*
2nd step:	*South*

Copy the grid below and mark the point it gets to with a ✖.

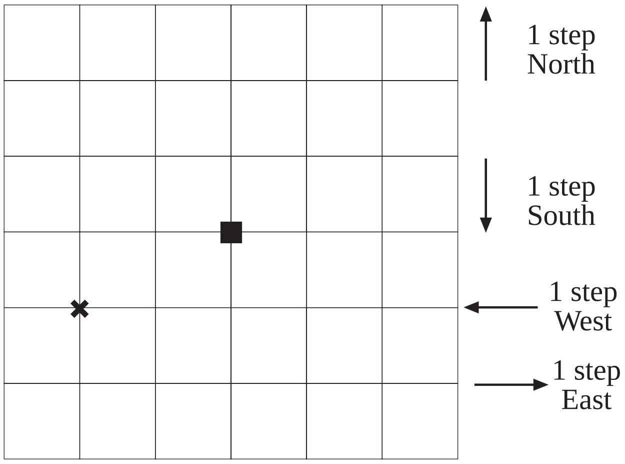


(b) The robot always starts from the point marked ●.

Find *all* the points the robot can reach in *2 steps.*

Mark each point with a ✖ on the grid you have drawn.

(c) Another robot always starts from the point marked ■ on this grid.



It takes *3 steps.*

1st step:	*South*
2nd step:	*West*
3rd step:	*West*

It gets to the point marked ✖.

The robot starts again from the point marked ■.

Copy and complete the table to show *two more ways* for the robot to get to the point marked ✖ *in 3 steps.*

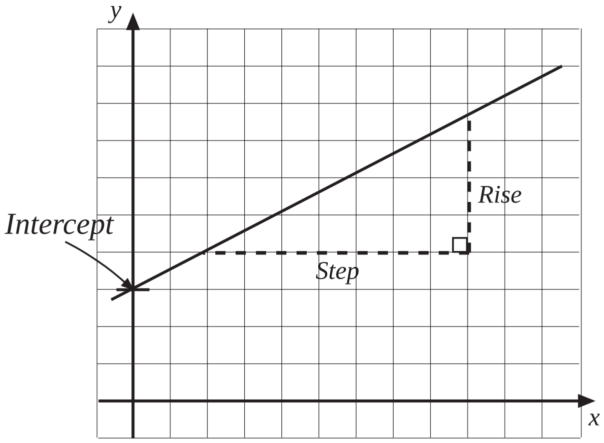
1st step	South	West	
2nd step	West		
3 rd step	West		

(KS3/96/Ma/Tier 3-5/P1)

5.2 Straight Line Graphs

We look in this section at how to calculate coordinates and plot straight line graphs. We also look at the *gradient* and *intercept* of a straight line and the *equation* of a straight line.

The gradient of a line is a measure of its steepness. The intercept of a line is the value where the line crosses the y-axis.



$$Gradient = \frac{Rise}{Step}$$

The equation of a straight line is $y = mx + c$, where

m = gradient and c = intercept (where the line crosses the y-axis).

Example 1

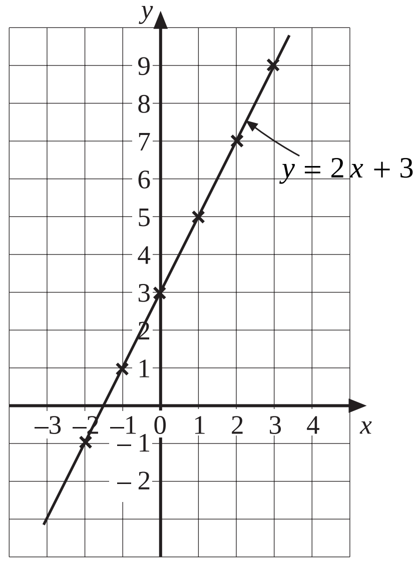
Draw the graph with equation $y = 2x + 3$.

Solution

First, find the coordinates of some points on the graph. This can be done by calculating y for a range of x values as shown in the table.

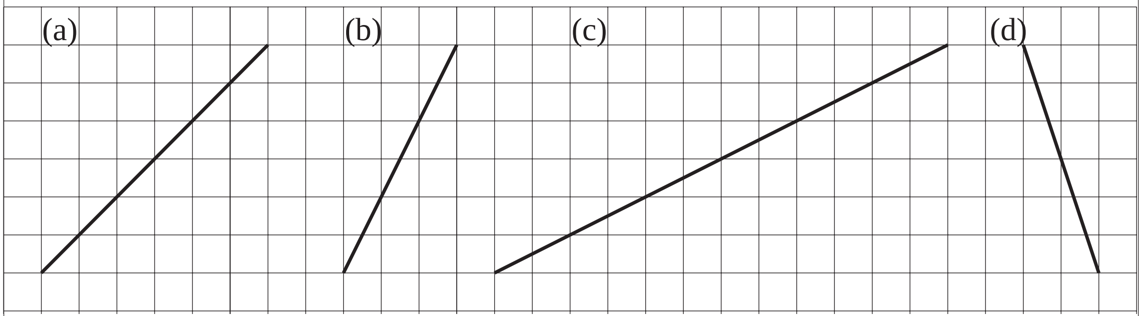
x	-2	-1	0	1	2	3
y	-1	1	3	5	7	9

The points can then be plotted on a set of axes and a straight line drawn through them.



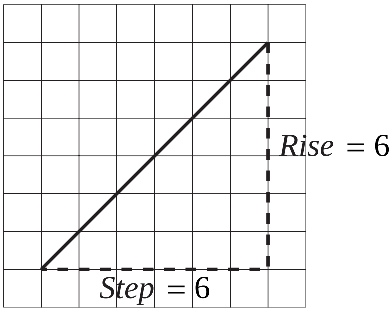
Example 2

Calculate the gradient of each of the following lines:



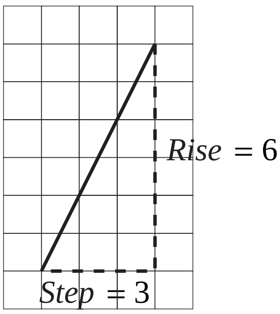
Solution

(a)



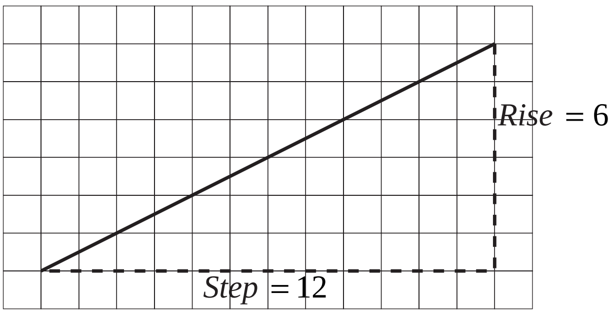
$$Gradient = \frac{6}{6}$$
$$= 1$$

(b)



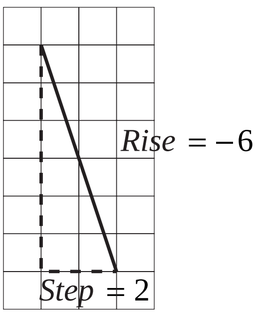
$$Gradient = \frac{6}{3}$$
$$= 2$$

(c)



$$Gradient = \frac{6}{12}$$
$$= \frac{1}{2}$$

(d)

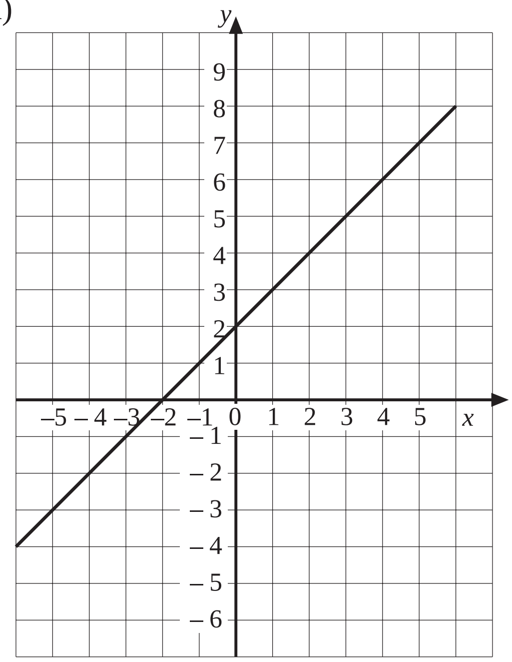


$$Gradient = \frac{-6}{2}$$
$$= -3$$

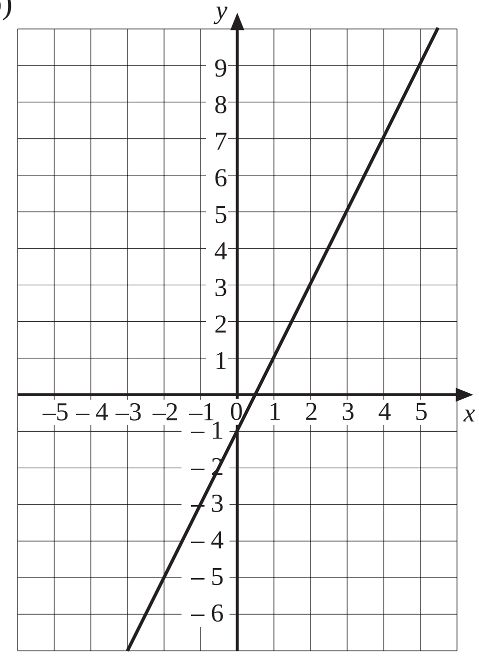
Example 3

Determine the equation of each of the following lines:

(a)

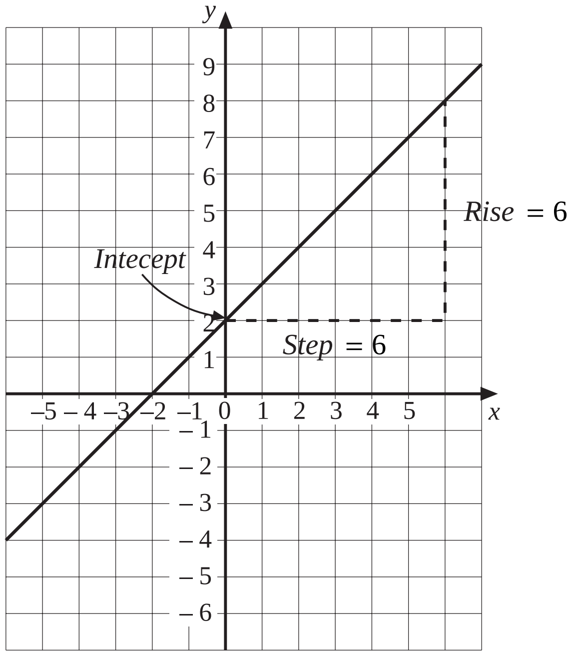


(b)



Solution

(a)



$$Gradient = \frac{6}{6}$$

$$= 1$$

$$Intercept = 2$$

So $m = 1$ and $c = 2$.

The equation is:

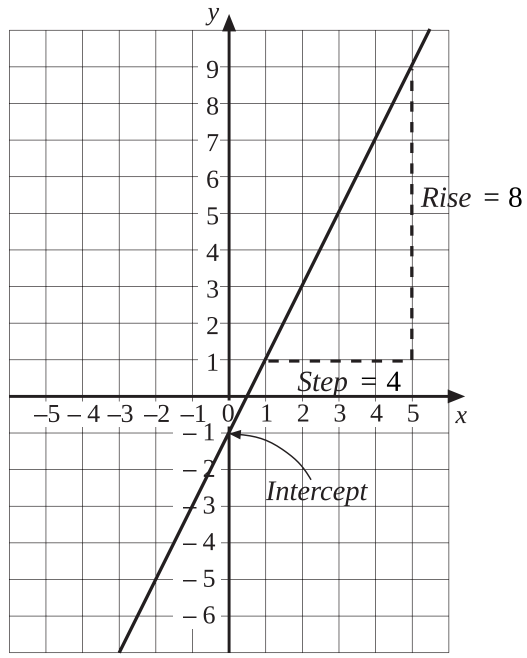
$$y = mx + c$$

$$y = 1x + 2$$

or

$$y = x + 2$$

(b)



$$Gradient = \frac{8}{4}$$

$$= 2$$

$$Intercept = -1$$

So $m = 2$ and $c = -1$.

The equation is:

$$y = mx + c$$

$$y = 2x + (-1)$$

or

$$y = 2x - 1$$

Exercises

1. (a) Copy and complete the following table for $y = 2x - 2$.

x	-2	-1	0	1	3	5
y						

(b) Draw the graph of $y = 2x - 2$.

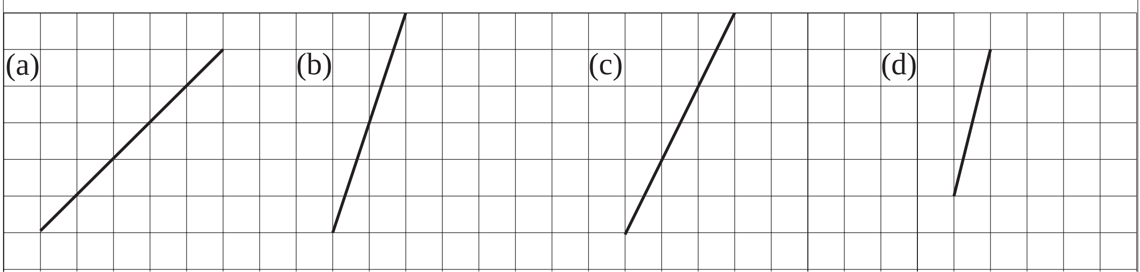
2. Draw the graphs with the equations given below, using a new set of axes for each graph.

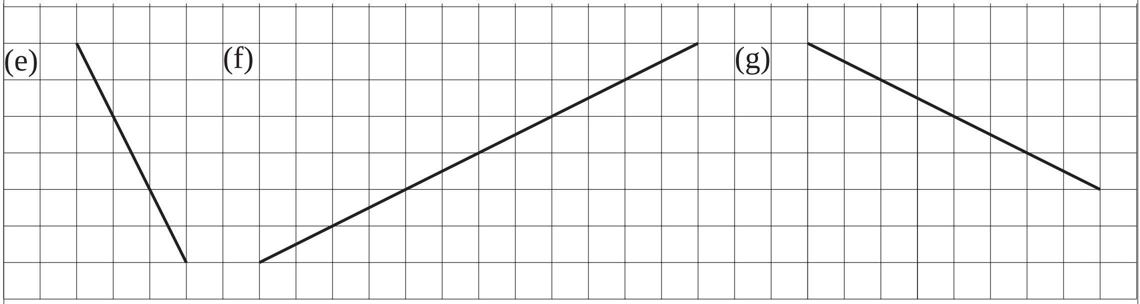
(a) $y = x + 3$ (b) $y = x - 4$

(c) $y = 4x - 1$ (d) $y = 3x + 1$

(e) $y = 4 - x$ (f) $y = 8 - 2x$

3. Calculate the gradient of each of the following lines, (a) - (g):





4. Write down the equations of the lines with gradients and intercepts listed below:

(a) Gradient $= 4$ and intercept $= 2$.

(b) Gradient $= 2$ and intercept $= -5$.

(c) Gradient $= \frac{1}{2}$ and intercept $= 1$.

(d) Gradient $= -1$ and intercept $= -5$.

5. Copy and complete the following table, which gives the equation, gradient and intercept for a number of straight lines.

Equation	*Gradient*	*Intercept*
$y = 5x + 7$		
	3	-2
$y = -3x + 2$		
$y = -4x - 2$		
	-2	3
	$\frac{1}{2}$	1
$y = 4 - x$		
$y = 10 - 3x$		

6. (a) Plot the points A, B and C with coordinates:

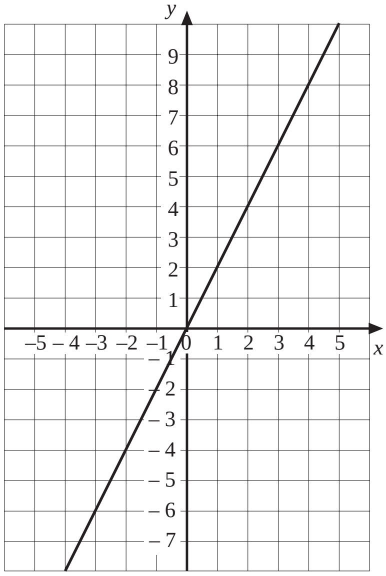
A (2, 4)
B (7, 5)
C (0, 10)

and join them to form a triangle.

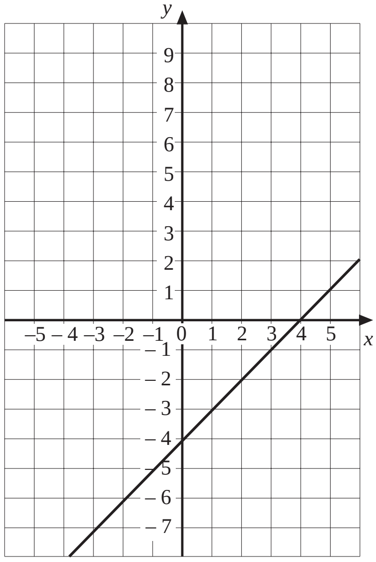
(b) Calculate the gradient of each side of the triangle.

7. Determine the equation of each of the following lines:

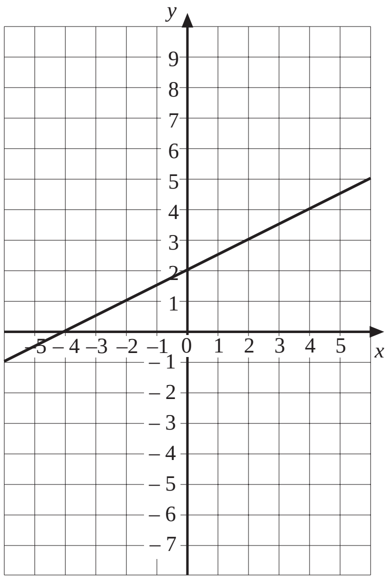
(a)



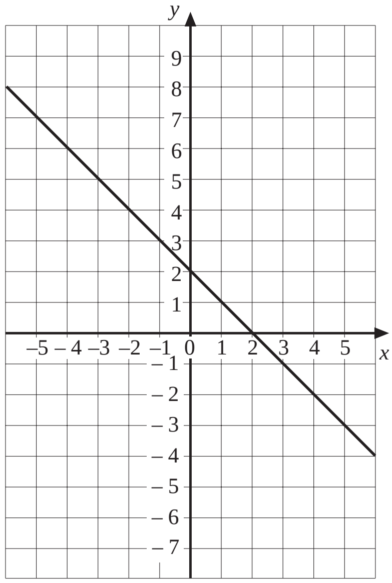
(b)



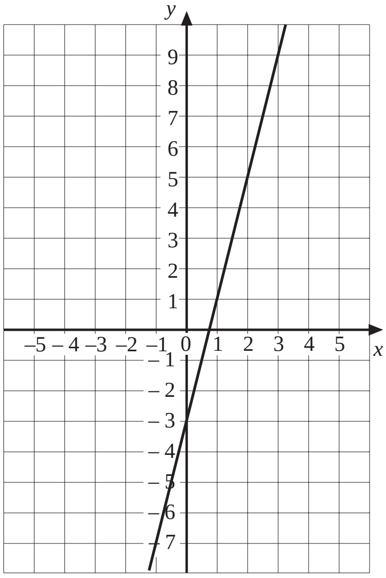
(c)



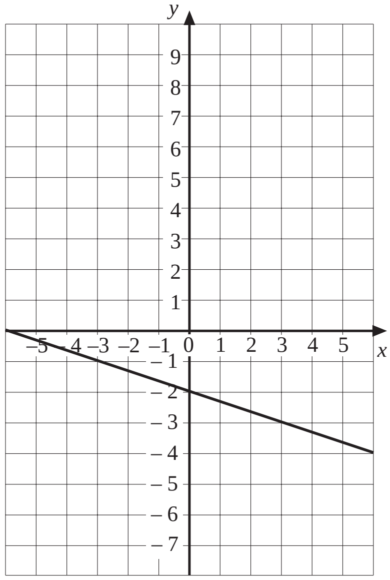
(d)



(e)



(f)



8. (a) On a set of axes, plot the points with coordinates

$$(-2, -2), \quad (2, 0), \quad (4, 1) \text{ and } (6, 2)$$

and then draw a straight line through these points.

(b) Determine the equation of the line.

9. (a) On the same axes, draw the lines with equations $y = 2x + 3$ and $y = 8 - \frac{1}{2}x$.

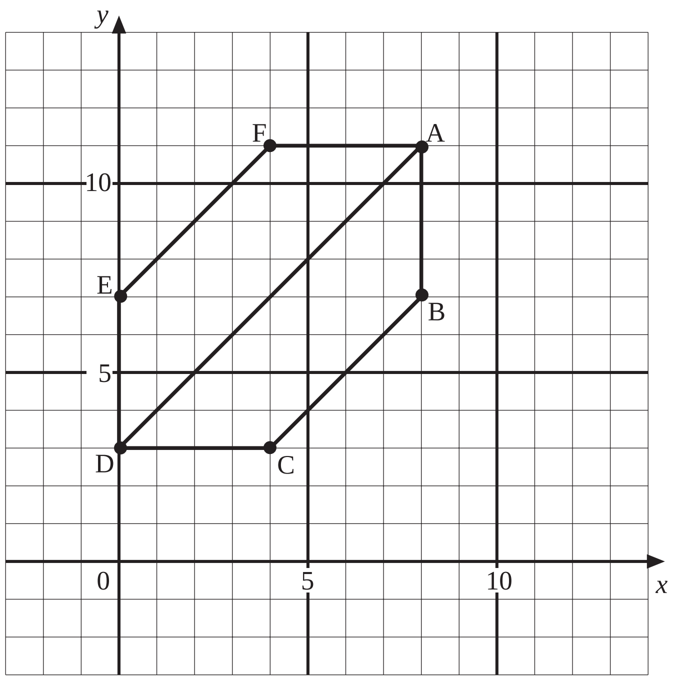
(b) Write down the coordinates of the point where the lines cross.

10. The point A has coordinates (4, 2), the point B has coordinates (8, 6) and the point C has coordinates (5, 9).

(a) Plot these points on a set of axes and draw straight lines through each point to form a triangle.

(b) Determine the equation of each of the lines you have drawn.

11. Look at this diagram:



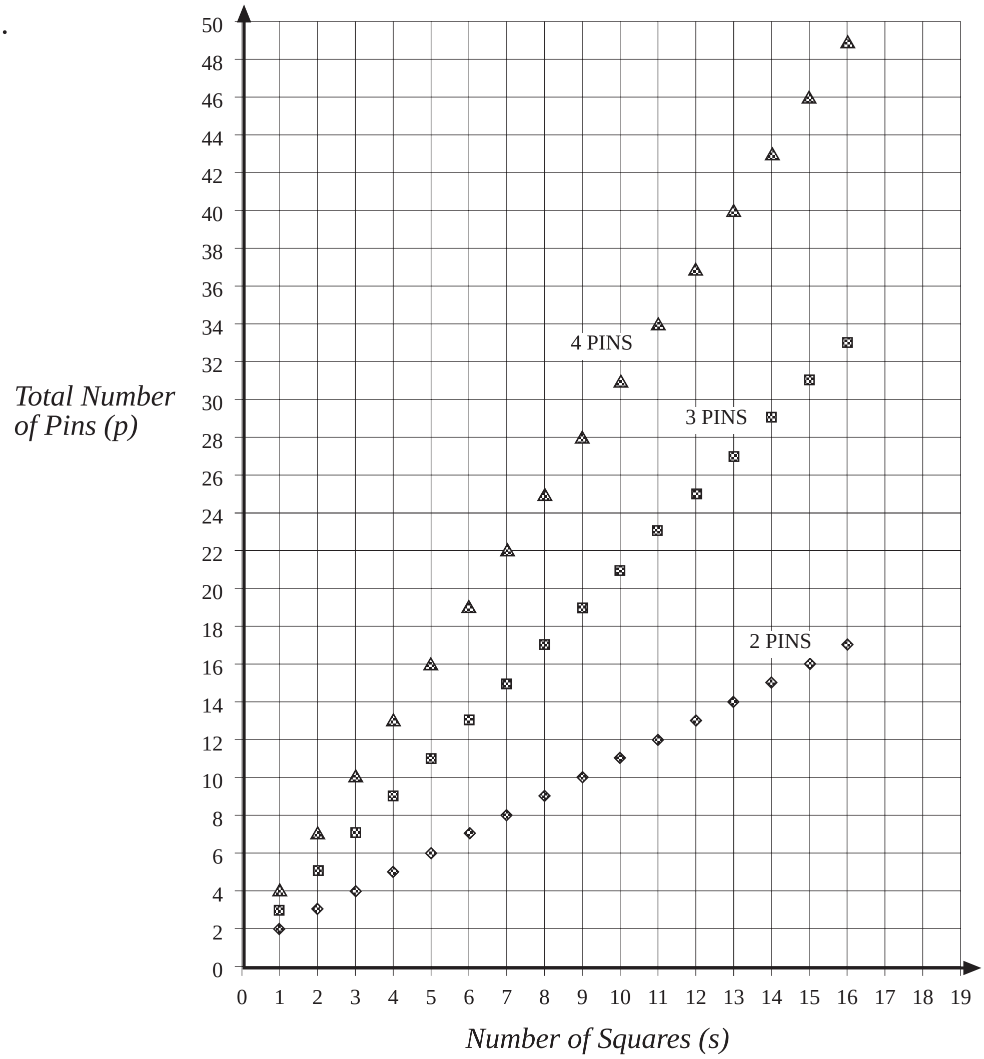
(a) The line through points A and F has the equation $y = 11$.
What is the equation of the line through points A and B ?

(b) The line through points A and D has the equation $y = x + 3$.
What is the equation of the line through points F and E ?

(c) What is the equation of the line through points B and C ?

(KS3/98/Ma/Tier 4-6/P1)

12.



The △s give the graph $p = 3s + 1$.

The ⊠s give the graph $p = 2s + 1$.

The ◇s give the graph $p = s + 1$.

Selma has 16 pins.

(a) Use the correct graph to find the number of squares she can pin up with 4 pins in each square.

How many squares can she pin up with 3 pins in each square?

(b) The line through the points for $p = 3s + 1$ climbs more steeply than the line through the points for $p = 2s + 1$ and $p = s + 1$.

Which part of the equation $p = 3s + 1$ tells you how steep the line is?

(c) On a copy of the grid at the beginning of this question, plot three points to show the graph for 8 pins in each square.

(d) What is the equation of this graph?

(KS3/95/Ma/Levels 6-8/P2)

5.3 Linear Equations

In this section we consider solving linear equations, using both algebra and graphs.

Example 1

Solve the following equations:

(a) $x + 6 = 13$ (b) $x - 7 = 11$ (c) $4x = 72$ (d) $\frac{x}{3} = 11$

Solution

(a)
$$\begin{aligned} x + 6 &= 13 \\ x &= 13 - 6 && \textit{(subtracting 6 from both sides)} \\ x &= 7 \end{aligned}$$

(b)
$$\begin{aligned} x - 7 &= 11 \\ x &= 11 + 7 && \textit{(adding 7 to both sides)} \\ x &= 18 \end{aligned}$$

(c)
$$\begin{aligned} 4x &= 72 \\ x &= \frac{72}{4} && \textit{(dividing both sides by 4)} \\ x &= 18 \end{aligned}$$

(d)
$$\begin{aligned} \frac{x}{3} &= 11 \\ x &= 11 \times 3 && \textit{(multiplying both sides by 3)} \\ x &= 33 \end{aligned}$$

Example 2

Solve the following equations:

(a) $2x + 4 = 20$ (b) $\frac{x + 4}{6} = 3$ (c) $4(x + 4) = 18$

Solution

(a)
$$\begin{aligned} 2x + 4 &= 20 \\ 2x &= 20 - 4 && \textit{(subtracting 4 from both sides)} \\ 2x &= 16 \\ x &= \frac{16}{2} && \textit{(dividing both sides by 2)} \\ x &= 8 \end{aligned}$$

(b) $\dfrac{x+4}{6} = 3$

$x + 4 = 3 \times 6$ *(multiplying both sides by 6)*

$x + 4 = 18$

$x = 18 - 4$ *(subtracting 4 from both sides)*

$x = 14$

(c) $4(x+4) = 18$

$4x + 16 = 18$ *(removing brackets)*

$4x = 18 - 16$ *(subtracting 16 from both sides)*

$4x = 2$

$x = \dfrac{2}{4}$ *(dividing both sides by 4)*

$x = \dfrac{1}{2}$

Example 3

Solve the following equations:

(a) $4x + 2 = 3x + 5$ (b) $4x - 4 = 10 - 3x$

Solution

(a) $4x + 2 = 3x + 5$

$x + 2 = 5$ *(subtracting $3x$ from both sides)*

$x = 5 - 2$ *(subtracting 2 from both sides)*

$x = 3$

(b) $4x - 4 = 10 - 3x$

$7x - 4 = 10$ *(adding $3x$ to both sides)*

$7x = 10 + 4$ *(adding 4 to both sides)*

$7x = 14$

$x = \dfrac{14}{7}$ *(dividing both sides by 7)*

$x = 2$

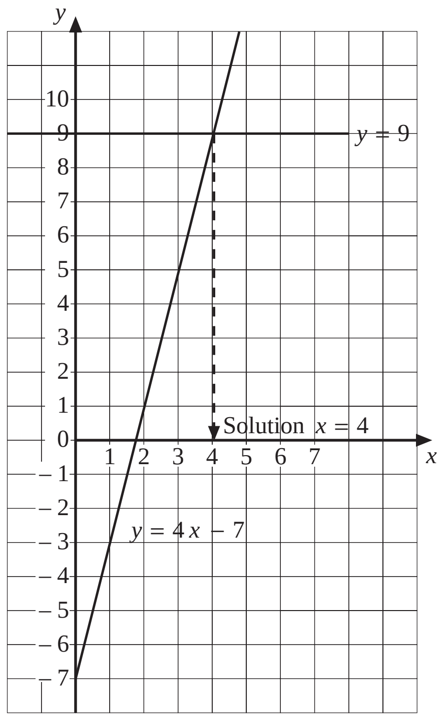
Example 4

Use graphs to solve the following equations:

(a) $4x - 7 = 9$ (b) $x + 7 = 3x - 3$

Solution

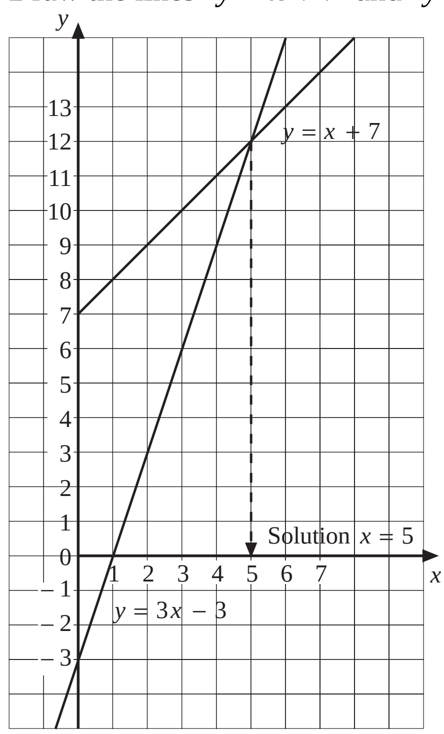
(a) Draw the lines $y = 4x - 7$ and $y = 9$.



The solution is given by the value on the x-axis immediately below the point where $y = 4x - 7$ and $y = 9$ cross.

The solution is $x = 4$.

(b) Draw the lines $y = x + 7$ and $y = 3x - 3$.



The lines cross where $x = 5$, so this is the solution of the equation.

Exercises

1. Solve the following equations:

(a) $x + 6 = 14$ (b) $x - 3 = 8$ (c) $7x = 21$

(d) $\frac{x}{3} = 10$ (e) $10x = 80$ (f) $5x = 35$

(g) $x + 9 = 22$ (h) $x - 4 = 3$ (i) $x - 22 = 18$

(j) $\frac{x}{5} = 100$ (k) $3x = 96$ (l) $x + 22 = 47$

2. Solve the following equations:

(a) $2x + 7 = 15$ (b) $5x - 3 = 32$ (c) $6x + 4 = 22$

(d) $11x - 3 = 19$ (e) $5x + 2 = 37$ (f) $\frac{x + 4}{3} = 21$

(g) $\frac{2x - 1}{3} = 5$ (h) $4(x + 2) = 28$ (i) $3(5x - 6) = 147$

(j) $2(3x - 7) = 46$ (k) $\frac{2(x + 6)}{3} = 6$ (l) $5(2x + 3) = 35$

3. (a) $x + 1 = 2x - 1$ (b) $2x + 4 = 3x - 1$

(c) $7x - 2 = 5x + 6$ (d) $4x + 7 = 10x - 11$

(e) $x + 18 = 9x - 22$ (f) $7x + 1 = 3x + 17$

(g) $6(x + 1) = 14(x - 1)$ (h) $2(5x + 3) = 12x - 3$

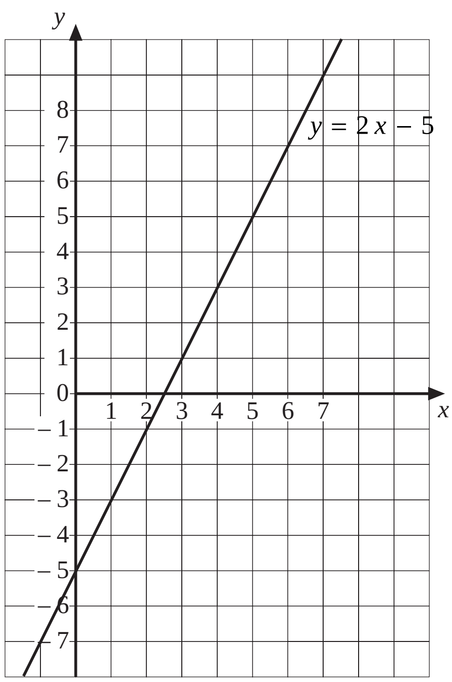
4. The graph $y = 2x - 5$ is shown:

Use the graph to solve the equations:

(a) $2x - 5 = 1$

(b) $2x - 5 = 7$

(c) $2x - 5 = -3$



5. Solve the equation $2x - 3 = 9$ by drawing the graphs $y = 2x - 3$ and $y = 9$.

6. Use a graph to solve the equation $4x - 5 = 3$.

7. (a) On the same set of axes, draw the lines with equations $y = x + 1$ and $y = 2x - 3$.

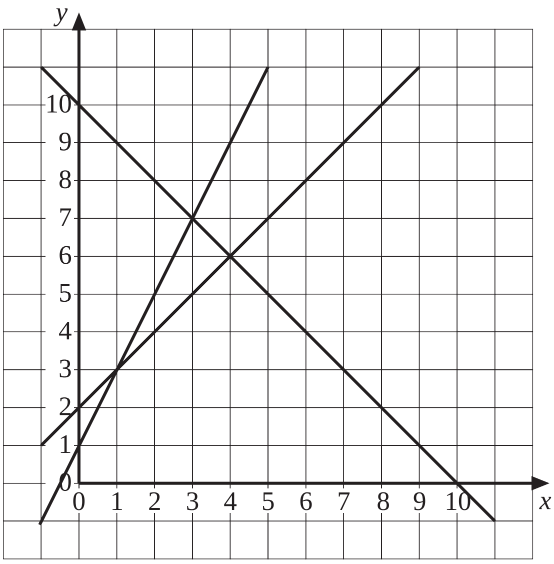
   (b) Use the graph to find the solution of the equation

$$x + 1 = 2x - 3$$

8. Use a graph to solve the following equations:

   (a) $2x = -x + 3$ (b) $4 - 2x = 2x - 8$

9. The following graph shows the lines with equations $y = 2x + 1$, $y = x + 2$ and $y = 10 - x$.



Use the graph to solve the equations:

(a) $2x + 1 = 10 - x$

(b) $x + 2 = 10 - x$

(c) $2x + 1 = x + 2$

10. On the same set of axes, draw the graphs of three straight lines and use them to solve the equations:

   (a) $2x - 2 = x + 3$

   (b) $2x - 2 = 8$

   (c) $x + 3 = 8$

11. Solve these equations. Show your working.

(a) $4 - 2y = 10 - 6y$ (b) $5y + 20 = 3(y - 4)$

(KS3/99/Ma/Tier 6-8/P1)

5.4 Parallel and Perpendicular Lines

In this section we consider the particular relationship between the equations of parallel lines and perpendicular lines. The key to this is the *gradient* of lines that are parallel or perpendicular to each other.

Example 1

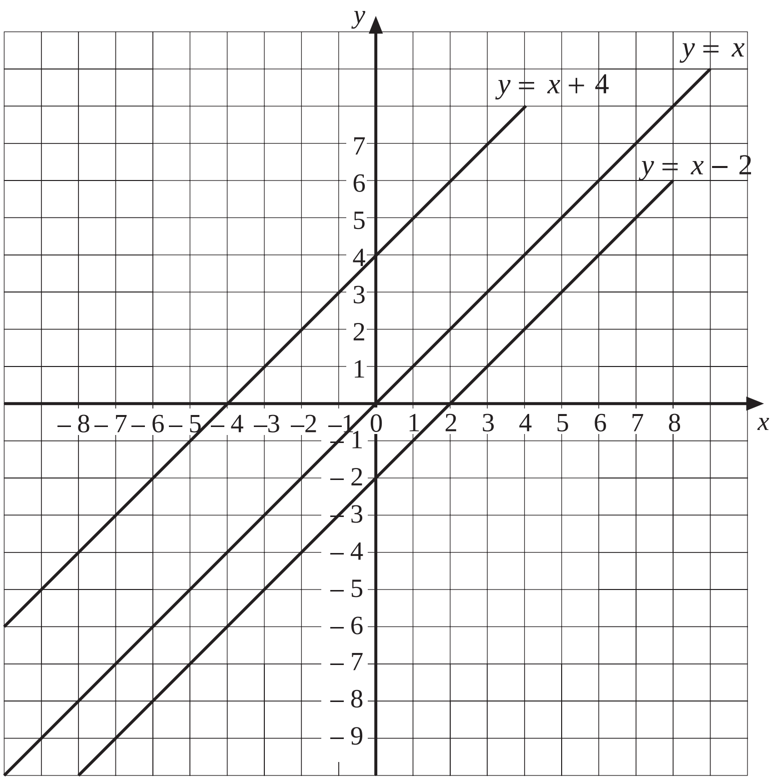
(a) Draw the lines with equations

$y = x$ $y = x + 4$ $y = x - 2$

(b) What do the three equations have in common?

Solution

(a) The following graph shows the three lines:



(b) Note that the three lines are parallel, all with gradient 1. All the equations of the lines contain 'x'. This is because the gradient of each line is 1, and so the value of m in the equation $y = mx + c$ is always 1.

Parallel lines will always have the *same gradient*, and so the equations of parallel lines will always have the same number in front of x (known as the *coefficient* of x).

For example, the lines with equations:

$$y = 4x - 2$$

$$y = 4x$$

$$y = 4x + 10$$

will all be parallel (the coefficient of x is 4 in each case).

Example 2

The equations of four lines are listed below:

A	$y = 3x + 2$	B	$y = 4x + 2$
C	$y = 3x - 8$	D	$y = 4x + 12$

(a) Which line is parallel to A ?

(b) Which line is parallel to B ?

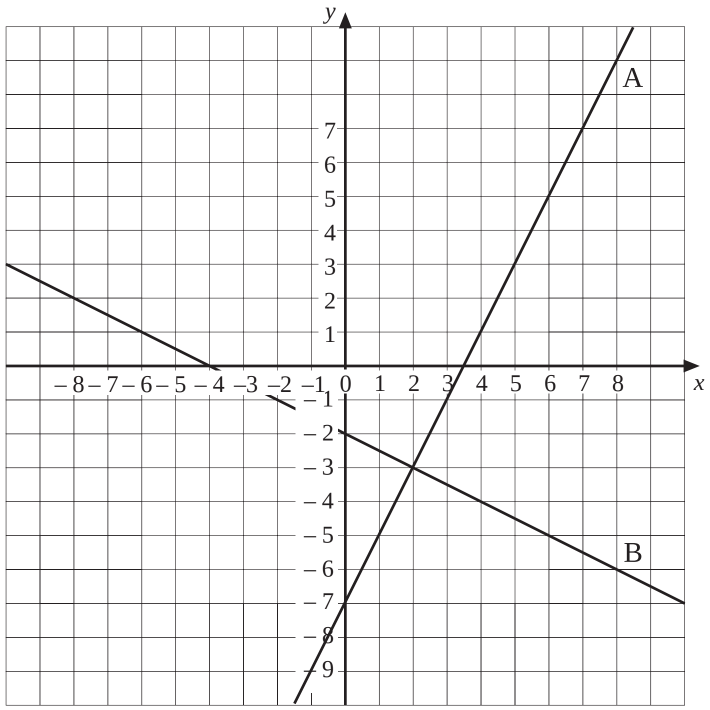
Solution

(a) C is parallel to A, because both equations contain $3x$ (the coefficient of x in both cases is 3).

(b) D is parallel to B, because both equations contain $4x$ (the coefficient of x in both cases is 4).

Example 3

The graph shows two perpendicular lines, A and B:

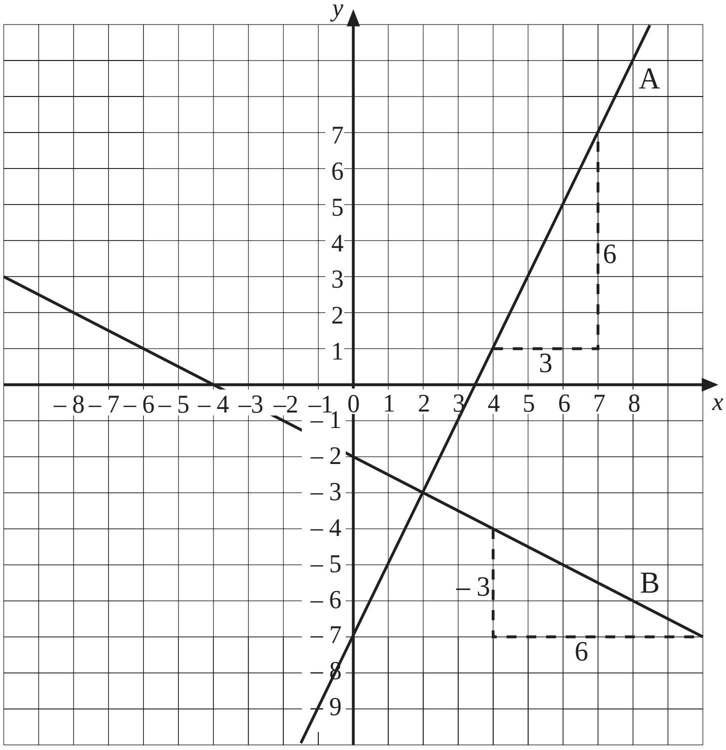


(a) Calculate the gradient of A and write down its equation.

(b) Calculate the gradient of B and write down its equation.

(c) Describe how the gradients of the lines are related.

Solution



(a) Gradient of A $= \dfrac{6}{3}$

$= 2$

Intercept of A $= -7$

Equation of A is $y = 2x - 7$

(b) Gradient of B $= \dfrac{-3}{6}$

$= -\dfrac{1}{2}$

Intercept of B $= -2$

Equation of B is $y = -\dfrac{1}{2}x - 2$

(c) The gradients of the lines are 2 and $-\dfrac{1}{2}$.

So:

$$\text{Gradient of B} = \frac{-1}{\text{Gradient of A}}$$

If two lines A and B are perpendicular,

$$\text{Gradient of B} = \frac{-1}{\text{Gradient of A}}$$

OR

$$\text{Gradient of A} \times \text{Gradient of B} = -1$$

Example 4

Line A has equation $y = 3x + 2$. Write down the gradient of line B that is perpendicular, and a possible equation for B.

Solution

(a) Gradient of A $= 3$

$$\text{Gradient of B} = \frac{-1}{\text{Gradient of B}}$$

$$= \frac{-1}{3}$$

Equation of B will be $y = -\frac{1}{3}x + c$.

This will be perpendicular to A for any value of c, so a possible equation is $y = -\frac{1}{3}x + 4$.

Exercises

1. (a) Draw the lines with the following equations on the same set of axes:

$$y = 2x + 5$$

$$y = 2x + 1$$

$$y = 2x - 3$$

(b) Draw two other lines that are parallel to these lines and write down their equations.

2. (a) Draw the line with equation $y = 3x - 2$.

   (b) Draw a line parallel to $y = 3x - 2$ that passes through the point with coordinates $(0, 3)$

   (c) Determine the equation of the second line.

3. The equations of five lines are listed below.

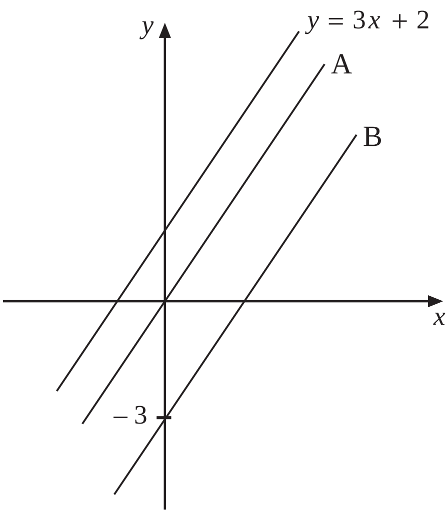
A	$y = 5x - 7$
B	$y = 2x + 8$
C	$y = 3x + 3$
D	$y = 3x - 8$
E	$y = 5x + 2$

   (a) Which line is parallel to A ?

   (b) Which line is parallel to C ?

   (c) Are there any lines parallel to B ? Explain why.

4. The diagram shows the line with equation $y = 3x + 2$ and two other lines, A and B, parallel to it.

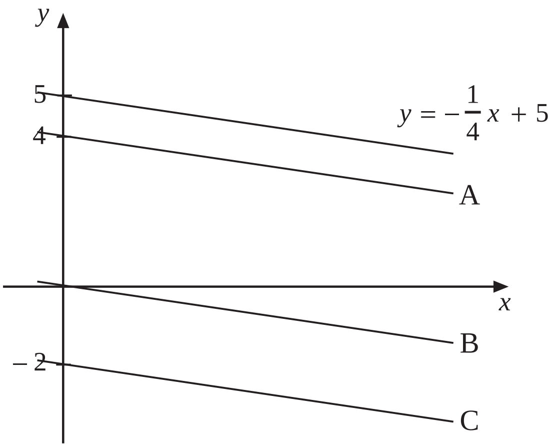


   (a) What is the *gradient* of the line A ?

   (b) What is the *equation* of the line A ?

   (c) What is the *equation* of the line B ?

5. The diagram shows the line with equation $y = -\frac{1}{4}x + 5$, and three other parallel lines.



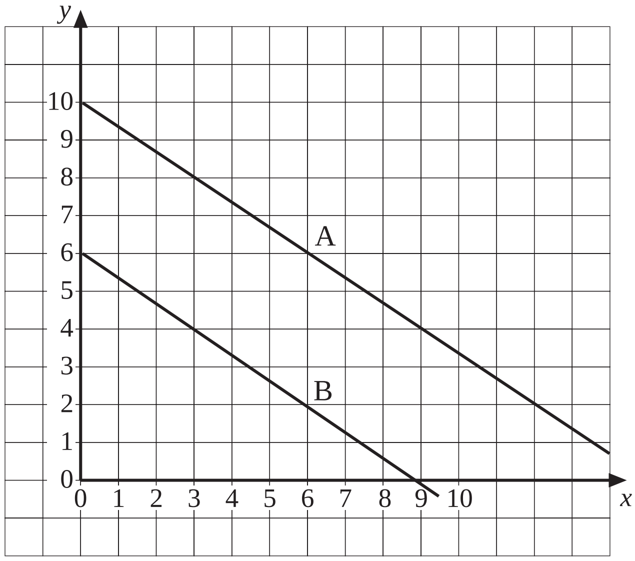
   What is the equation of:

   (a) line A,

   (b) line B,

   (c) line C ?

6. The graph shows two lines, A and B.

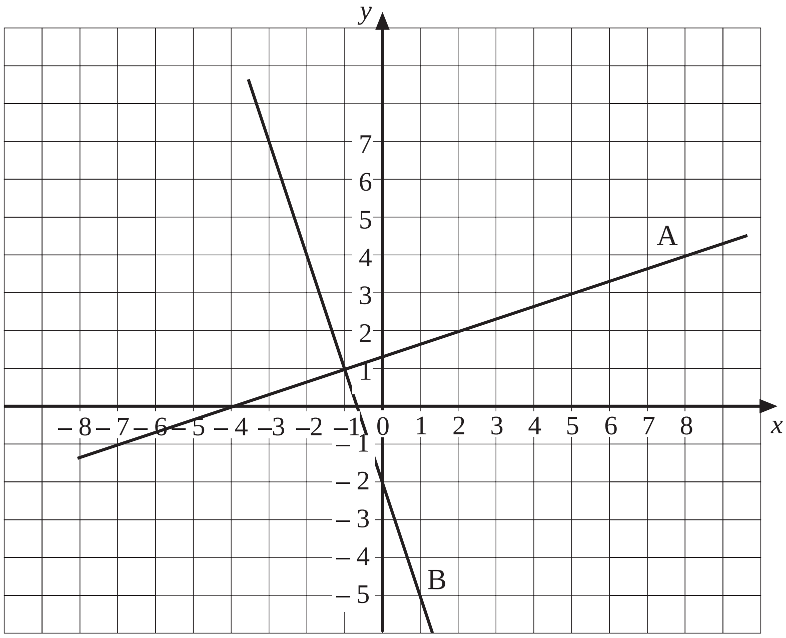


(a) Calculate the *gradient* of the line A.

(b) What is the *equation* of the line A?

(c) What is the *equation* of the line B ?

7. The graph shows two lines, A and B.



(a) Calculate the *gradient* of A.

(b) Calculate the *gradient* of *B.*

(c) Explain why the lines are *perpendicular*, using your answers to (a) and (b).

8. The equations of five lines are given below:

A $y = 5x + 2$

B $y = \frac{1}{2}x + 4$

C $y = 2x + 1$

D $y = -\frac{1}{5}x + 6$

E $y = -2x + 3$

(a) Which line is perpendicular to A ?

(b) Which line is perpendicular to B ?

(c) Which line is *not* perpendicular to any of the other lines?

9. The line A joins the points with coordinates (4, 2) and (6, 8).

The line B joins the points with coordinates (5, 5) and (11, 3).

The line C joins the points with coordinates (6, 8) and (11, 4).

(a) Calculate the gradient of each line.

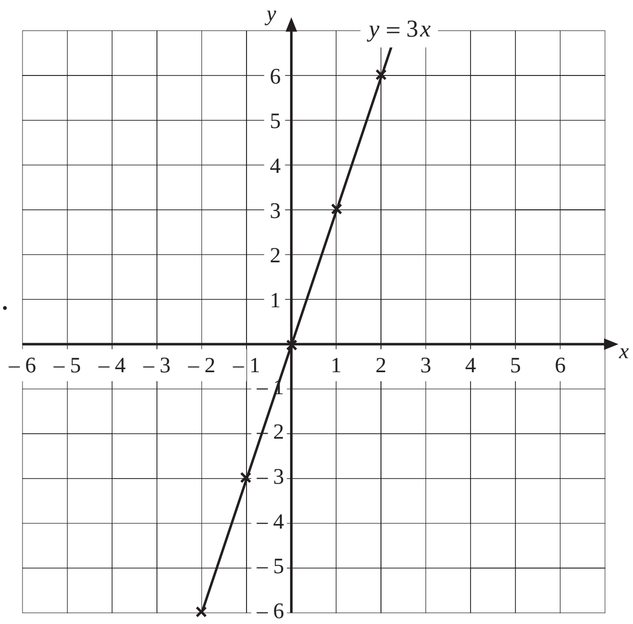
(b) Which two lines are perpendicular?

10. A line has equation $y = 4x + 3$.

(a) Write down the equation of 2 lines that are *parallel* to $y = 4x + 3$.

(b) Write down the equation of 2 lines that are *perpendicular* to $y = 4x + 3$.

11. The diagram shows the graph of the straight line $y = 3x$.



(a) On a copy of the diagram, draw the graph of the straight line $y = 2x$.

Label your line $y = 2x$.

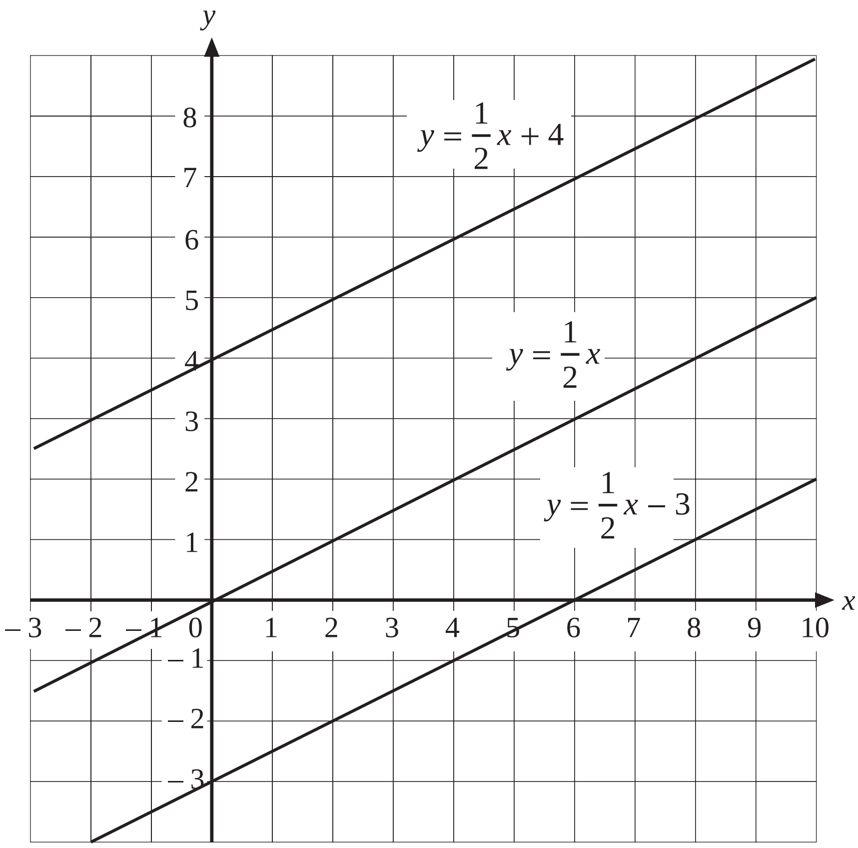
(b) Write the equation of another straight line which goes through the point (0, 0).

(c) The straight line with the equation $y = x - 1$ goes through the point (4, 3). On your diagram, draw the graph of the straight line $y = x - 1$. Label your line $y = x - 1$.

(d) Write the equation of the straight line which goes through the point (0, – 1) and is *parallel* to the straight line $y = 3x$.

(KS3/96/Ma/Tier 6-8/P2)

12. Lucy was investigating straight lines and their equations. She drew the following lines.



(a) $y = \frac{1}{2}x$ is in each equation.

Write one fact this tells you about all the lines.

(b) The lines cross the *y* axis at (0, – 3), (0, 0) and (0, 4).

Which part of each equation helps you see where the line crosses the *y* axis?

(c) Lucy decided to investigate more lines. She needed longer axes.

Where will the line $y = \frac{1}{2}x - 20$ cross the *y* axis?

(d) On a copy of the graph, draw another line which is parallel to $y = \frac{1}{2}x$.

Write the equation of your line.

(KS3/94/Ma/5-7/P2)

5.5 Simultaneous Equations

Simultaneous equations consist of two or more equations that are true at the same time. Consider the following example:

Claire and Laura are sisters; we know that

(i) *Claire is the elder sister,*

(ii) *their ages added together give 20 years,*

(iii) *the difference between their ages is 2 years.*

Let $x =$ Claire's age, in years and $y =$ Laura's age, in years.

$$x + y = 20$$
$$x - y = 2$$

This is an example of a pair of simultaneous equations.

In this section we consider two methods of solving pairs of simultaneous equations like these.

Example 1

Use a graph to solve the simultaneous equations:

$$x + y = 20$$
$$x - y = 2$$

Solution

We can rewrite the first equation to make y the subject:

$$\begin{aligned} x + y &= 20 \\ y &= 20 - x \end{aligned}$$

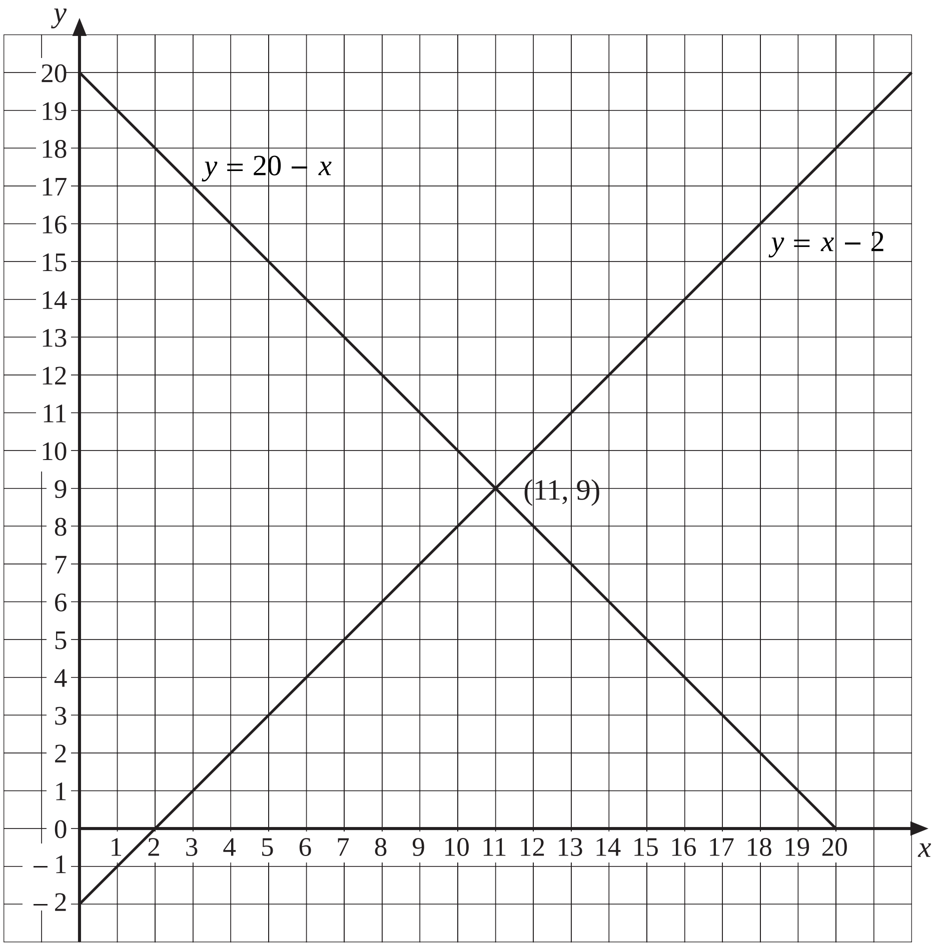
For the second equation,

$$\begin{aligned} x - y &= 2 \\ x &= y + 2 \\ x - 2 &= \text{y} \end{aligned}$$

or

$$y = x - 2$$

Now draw the graphs $y = 20 - x$ and $y = x - 2$.



The lines cross at the point with coordinates (11, 9), so the solution of the pair of simultaneous equation is $x = 11$, $y = 9$.

Note: this means that the solution to the problem presented at the start of section 5.5 is that Claire is aged 11 and Laura is aged 9.

Example 2

Use a graph to solve the simultaneous equations:

$$x + 2y = 18$$
$$3x - y = 5$$

Solution

First rearrange the equations in the form $y = \ldots$

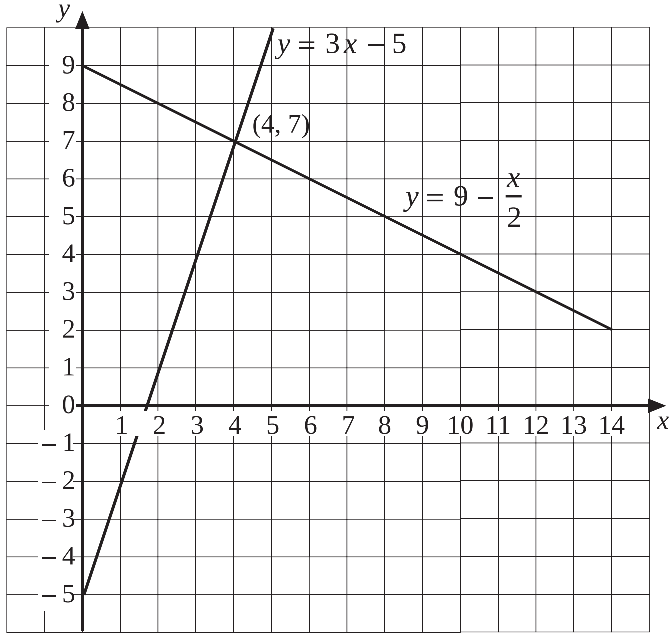
$$\begin{aligned} x + 2y &= 18 \\ 2y &= 18 - x \\ y &= \frac{18 - x}{2} \\ y &= 9 - \frac{x}{2} \end{aligned}$$

$$3x - y = 5$$
$$3x = y + 5$$
$$3x - 5 = y$$

or

$$y = 3x - 5$$

Now draw these two graphs:



The lines cross at the point with coordinates (4, 7), so the solution is $x = 4$, $y = 7$.

An alternative approach is to solve simultaneous equations *algebraically*, as shown in the following examples.

Example 3

Solve the simultaneous equations:

$$x + 2y = 29 \qquad (1)$$
$$x + y = 18 \qquad (2)$$

Solution

Note that the equations have been numbered (1) and (2).

Method 1 *Substitution*

Start with equation (2)

$$x + y = 18$$
$$y = 18 - x$$

Now replace y in equation (1)

Using $y = 18 - x$

$$x + 2y = 29$$
$$x + 2(18 - x) = 29$$
$$x + 36 - 2x = 29$$
$$36 - x = 29$$
$$36 = 29 + x$$
$$36 - 29 = x$$
$$x = 7$$

Method 2 *Elimination*

Take equation (2) away from equation (1).

$$x + 2y = 29 \qquad (1)$$
$$x + y = 18 \qquad (2)$$
$$y = 11 \qquad (1) - (2)$$

In equation (2), replace y with 11.

$$x + 11 = 18$$
$$x = 18 - 11$$
$$= 7$$

Finally, using $y = 18 - x$ gives

$$y = 18 - 7$$
$$y = 11$$

So the solution is $x = 7, \ y = 11$

Example 4

Solve the simultaneous equations:

$$2x + 3y = 28 \qquad (1)$$
$$x + y = 11 \qquad (2)$$

Solution

Method 1 *Substitution*

From equation (2)

$$\begin{aligned} x + y &= 11 \\ y &= 11 - x \end{aligned}$$

Substitute this into equation (1)

$$\begin{aligned} 2x + 3(11 - x) &= 28 \\ 2x + 33 - 3x &= 28 \\ 33 - x &= 28 \\ 33 &= 28 + x \\ 33 - 28 &= x \\ x &= 5 \end{aligned}$$

Finally use $y = 11 - x$

$$\begin{aligned} y &= 11 - 5 \\ y &= 6 \end{aligned}$$

So the solution is,

$x = 5, \ y = 6$

Method 2 *Elimination*

Subtract $2 \times$ equation (2) from equation (1).

$$\begin{array}{rcll} 2x + 3y &=& 28 & (1) \\ 2x + 2y &=& 22 & 2 \times (2) \\ \hline y &=& 6 & (1) - 2 \times (2) \end{array}$$

Now replace y in equation (2) with 6.

$$\begin{aligned} x + 6 &= 11 \\ x &= 11 - 6 \\ x &= 5 \end{aligned}$$

So the solution is,

$x = 5, \ y = 6$

Example 5

Solve the simultaneous equations:

$$x - 2y = 8 \qquad (1)$$
$$2x + y = 21 \qquad (2)$$

Solution

Method 1 *Substitution*

From equation (2)

$$\begin{aligned} 2x + y &= 21 \\ y &= 21 - 2x \end{aligned}$$

Substitute this into equation (1)

$$\begin{aligned} x - 2y &= 8 \\ x - 2(21 - 2x) &= 8 \\ x - 42 + 4x &= 8 \\ 5x - 42 &= 8 \\ 5x &= 8 + 42 \\ 5x &= 50 \\ x &= 10 \end{aligned}$$

Now substitute this into $y = 21 - 2x$

$$\begin{aligned} y &= 21 - 2 \times 10 \\ y &= 21 - 20 \\ y &= 1 \end{aligned}$$

So the solution is,

$x = 10, \quad y = 1$

Method 2 *Elimination*

Subtract $2 \times$ equation (1) from equation (2).

$$\begin{aligned} 2x + y &= 21 && (2) \\ 2x - 4y &= 16 && 2 \times (1) \\ \hline 5y &= 5 && (2) - 2 \times (1) \\ y &= 1 \end{aligned}$$

Now replace this in equation (1).

$$\begin{aligned} x - 2y &= 8 \\ x &= 8 + 2 \\ x &= 10 \end{aligned}$$

So the solution is,

$x = 10, \quad y = 1$

Exercises

1. (a) Draw the lines with equations $y = 10 - x$ and $y = x + 2$.

   (b) Write down the coordinates of the point where the two lines cross.

   (c) What is the solution of the pair of simultaneous equations,

$$\begin{aligned} y &= 10 - x \\ y &= x + 2 \end{aligned}$$

2. (a) Draw the lines with equations $y = 5 - 2x$ and $y = 4 - x$.

   (b) Determine the coordinates of the point where the two lines cross.

   (c) Determine the solution of the simultaneous equations,

$$2x + y = 5$$
$$x + y = 4$$

3. Use a graphical method to solve the simultaneous equations,

$$x - 2y = 5$$
$$x + y = 8$$

4. Use a graph to solve the simultaneous equations,

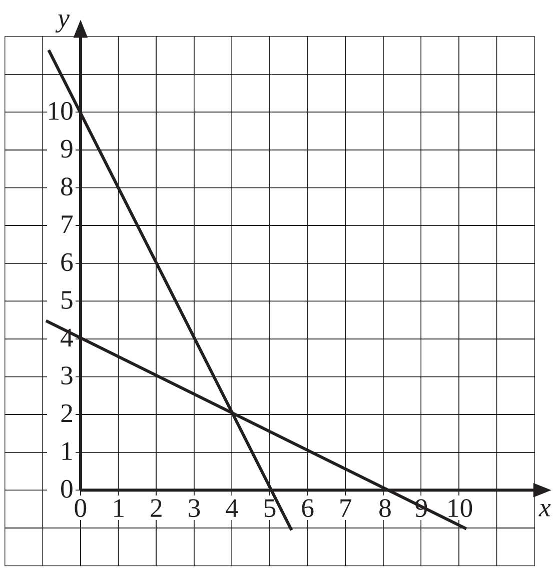
$$x + 2y = 10$$
$$2x + 3y = 18$$

5. Two numbers, x and y, are such that their sum is 24 and their difference is 6.

   (a) If the numbers are x and y, write down a pair of simultaneous equations in x and y.

   (b) Use a graph to solve the simultaneous equations and hence identify the two numbers.

6. Michelle obtains the solution $x = 4$, $y = 2$ to a pair of simultaneous equations by drawing the following graph:



What are the equations that she has solved?

7. A pair of simultaneous equations are given below:

$$2x + 4y = 14 \qquad (1)$$
$$2x + y = 8 \qquad (2)$$

(a) Explain why subtracting equation (2) from equation (1) helps to solve the equations.

(b) Solve the equations.

8. Solve the following pairs of simultaneous equations, using algebraic methods:

(a) $x + 5y = 8$
$x + 4y = 7$

(b) $2x + 3y = 16$
$8x + 3y = 46$

(c) $2x + 6y = 26$
$2x + 3y = 20$

(d) $x + 2y = 3$
$x + y = 7$

(e) $x + 3y = 18$
$x - 2y = 3$

(f) $2x + 4y = 32$
$2x - 3y = 11$

9. A pair of simultaneous equations is given below:

$$4x + 2y = 46 \qquad (1)$$
$$x + 3y = 14 \qquad (2)$$

(a) Explain why you could calculate

four times equation (2) – equation (1)

to determine one solution.

(b) Calculate the solution of this pair of equations.

10. Solve the following pairs of simultaneous equations, using an algebraic method:

(a) $x + 2y = 7$
$2x + 3y = 11$

(b) $4x + 9y = 47$
$x + 2y = 11$

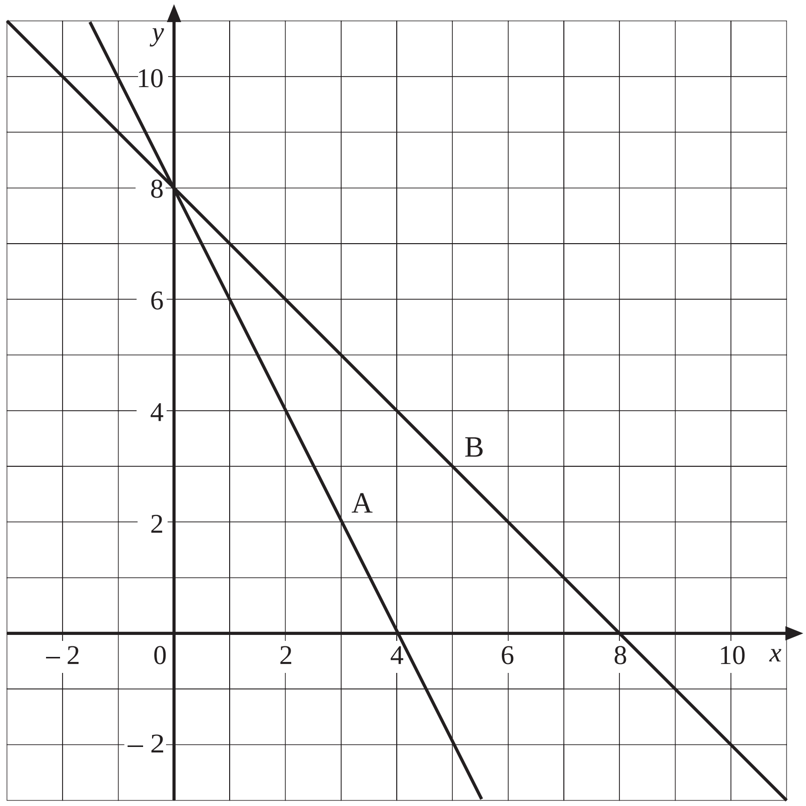
(c) $4x + 5y = 25$
$x - y = 4$

(d) $2x + 6y = 20$
$x + 2y = 9$

(e) $x - 8y = 4$
$2x + y = 42$

(f) $4x - 2y = 24$
$8x - 3y = 50$

11. Look at this graph:



(a) Show that the equation of line A is $2x + y = 8$.

(b) Write the equation of line B.

(c) On a copy of the graph, draw the line whose equation is $y = 2x + 1$. Label your line C.

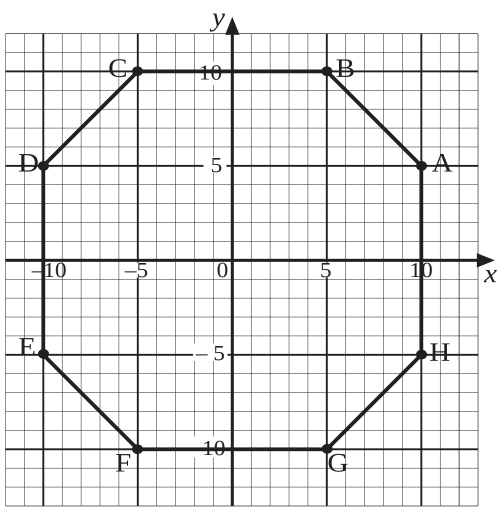
(d) Solve these simultaneous equations:

$$\begin{aligned} y &= 2x + 1 \\ 3y &= 4x + 6 \end{aligned}$$

Show your working.

(KS3/99/Ma/Tier 6-8/P1)

12. Look at this octagon:



(a) The line through A and H has the equation $x = 10$.

What is the equation of the line through F and G ?

(b) Copy the following statement, adding in the missing words to make it correct:

$x + y = 15$ is the equation of the line through .... and ....

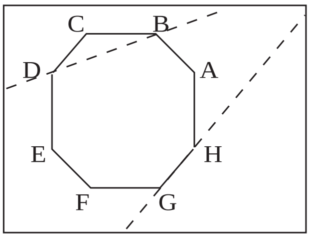
(c) The octagon has four lines of symmetry. One of the lines of symmetry has the equation $y = x$.

On a copy of the diagram, draw and label the line $y = x$.

(d) The octagon has three other lines of symmetry. Write the equation of one of these three other lines of symmetry,

(e) The line through D and B has the equation $3y = x + 25$.

The line through G and H has the equation $x = y + 15$.



Solve the simultaneous equations

$$3y = x + 25$$
$$x = y + 15$$

Show your working.

(f) Copy and complete this sentence:

The line through D and B meets the line through G and H at (......... , ...).

(KS3/97/Ma/Tier 5-7/P1)

5.6 Equations in Context

In this section we determine the solutions to a variety of problems by forming and solving suitable linear equations.

Example 1

Apples cost 55p per kg. Alan buys a bag of apples that costs £1.65.

If the bag contains x kg of apples,

(a) write down an equation involving x,

(b) solve the equation.

Solution

(a) It is easier to work in pence.

$$x \times 55 = 165$$

$$55x = 165$$

(b) $$x = \frac{165}{55}$$

$$x = 3$$

Example 2

Three consecutive whole numbers add up to 36. Determine the three numbers.

Solution

If x = first number,

then $x + 1$ = second number,

and $x + 2$ = third number.

Adding these gives:

$$x + (x + 1) + (x + 2) = 36$$

$$3x + 3 = 36$$

$$3x = 33$$

$$x = \frac{33}{3}$$

$$x = 11$$

and the three numbers are 11, 12 and 13.

Example 3

A taxi driver charges £2.00 plus £1.10 per mile for all journeys.

(a) Write down the cost, in pence, for travelling m miles.

(b) The charge for a journey is £3.65. Write down an equation and use this to determine the distance travelled.

Solution

(a) Basic cost + 110 $\times$ number of miles = $200 + 110m$ pence

(b) $$200 + 110m = 365$$

$$110m = 365 - 200$$

$$110m = 165$$

$$m = \frac{165}{110}$$

$$m = 1.5$$

So the distance travelled is 1.5 miles.

Exercises

1. The cost of a ticket for a football match is £9.

   (a) Write down an expression for the cost of n tickets.

   (b) Solve an equation to determine how many tickets could be bought with £108.

2. The cost of hiring a van is £20 per day, plus 50p for each mile travelled.

   (a) Write down an expression for the cost, c, in pounds, of travelling m miles in one day in a hired van.

   (b) Write down an expression for the cost in pounds of travelling m miles during a two-day hire period.

   (c) James hires a van for 2 days. He has to pay a total of £68.50. Write down an equation and solve it to determine how far he travelled.

3. Two consecutive odd numbers are x and $x + 2$.

   When these numbers are added together they total 100. Write down and solve an equation to obtain the value of x.

4. A removals firm charges £4 per mile plus a fixed charge of £25. Use an equation to determine the distance travelled if the bill is £39.

5. The price of petrol is given in pence per litre. To convert this to £ per gallon, use the flow chart given below.

   Price in pence per litre → $\boxed{\times\ 4.5}$ → $\boxed{\div\ 100}$ → *Price in £ per gallon*

   (a) Convert a price of 80p per litre to £ per gallon.

   (b) If the price is x pence per litre, write down the cost in £ per gallon.

   (c) Convert a price of £4.14 per gallon to pence per litre.

6. A rectangle has length 10 m and width x m.

   (a) Write down a formula for the area of the rectangle.

   (b) Use an equation to determine x if the area is 16 m^2.

   (c) Write down a formula for the perimeter of the rectangle.

   (d) Use an equation to determine x, if the perimeter is 39 m.

7. A repairman charges £40 for the first hour of his time and £15 for each hour after that.

   (a) Write down a formula for the cost of a repair that takes n hours.

   (b) Use an equation to determine the time for a repair, if the cost is £52.50.

8. At a bank a charge of £2 is made for changing British Pounds (£) into French Francs (Fr). The charge is deducted first and then 9 Fr are issued for every £1 left.

   (a) Write down a formula for the number of Fr issued in exchange for £x.

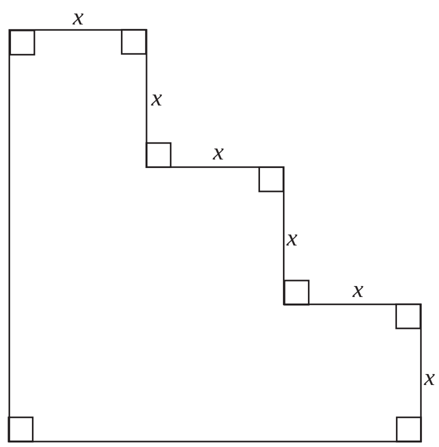
   (b) Use an equation to determine how many £ you would need to change to get 900 Fr.

9. (a) Write down a formula for the perimeter of the shape shown.

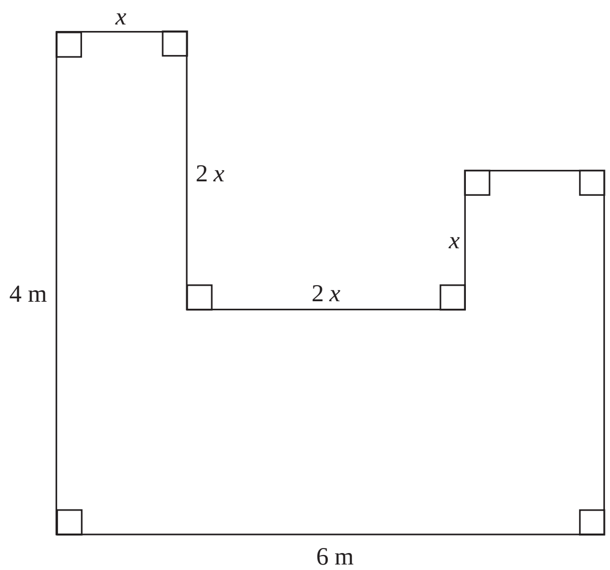
   (b) Calculate x if the perimeter is 2.76 m.

   (c) Write down a formula for the *area* of the shape.

   (d) Calculate x if the area is $8.64\ \text{m}^2$.



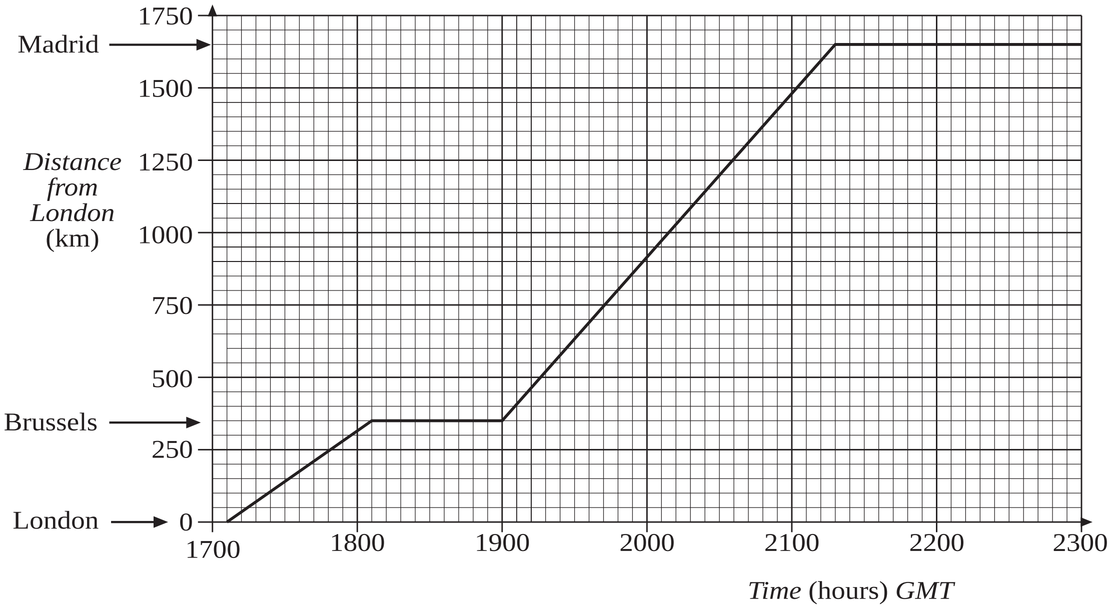
10.



   (a) Write down a formula for the perimeter of the shape shown.

   (b) If the perimeter is 23 m, determine the length x.

11. The simplified graph shows the flight details of an aeroplane travelling from London to Madrid, via Brussels.



(a) What is the aeroplane's average speed from London to Brussels?

(b) How can you tell from the graph, *without calculating*, that the aeroplane's average speed from Brussels to Madrid is *greater* than its average speed from London to Brussels?

(c) A different aeroplane flies *from* Madrid *to* London, via Brussels. The flight details are shown below.

Madrid	depart	1800
Brussels	arrive	2000
	depart	2112
London	arrive	2218

On a copy of the graph, show the aeroplane's journey from Madrid to London, via Brussels. (Do not change the labels on the graph.)

Assume constant speed for each part of the journey.

(d) At what time are the two aeroplanes the same distance from London?

(KS3/99/Ma/Tier 5-7/P2)