6 Probability

6.1 The Probability Scale

Probabilities are given on a scale of 0 to 1, as decimals or as fractions; sometimes probabilities are expressed as percentages using a scale of 0% to 100%, particularly on weather forecasts.

0	This is the probability of something that is *impossible.*
1	This is the probability of something that is *certain.*
$\frac{1}{2}$	This is the probability of something that is as *likely to happen* as it is *not to happen*.

Example 1

Decide whether or not each of the statements below is reasonable.

(a) The probability that it will snow on Christmas Day in London is 0.9.

(b) The probability that you will win a raffle prize is 0.5.

(c) The probability that you will go to bed before midnight tonight is 0.99.

(d) The probability that your pocket money is doubled tomorrow is 0.01.

Solution

(a) This is *not reasonable* as the probability given is much too high. It very rarely snows in London in late December, so the probability should be close to 0.

(b) This probability is far too high. You would need to have bought half of all the tickets sold to obtain this probability, so this statement is *not reasonable*.

(c) This is a *reasonable* statement as it is very likely that you will go to bed before midnight, but not certain that you will.

(d) This is a *reasonable* statement, as it is very unlikely that your pocket money will be doubled tomorrow, but not totally impossible.

Example 2

On a probability scale, mark and estimate the probability that:

(a) it will rain tomorrow,

(b) England will win their next football match,

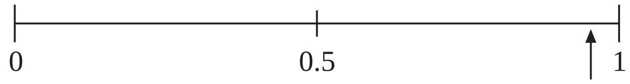
(c) someone in your class has a birthday tomorrow.

Solution

(a) This will depend on the time of year and the prevailing weather conditions.

During a dry spell in summer,

During a wet spell in winter,



(b) Based on their recent form, it is reasonable to say that England are slightly more likely to draw or lose their next match than to win it, so an estimate would be a little less than 0.5.

(c) The probability of this will be fairly small, as you can expect there to be about 2 or 3 birthdays per month for pupils in a class of about 30 pupils.

Exercises

1. Describe something that is:

   (a) *very unlikely,* (b) *unlikely,*

   (c) *likely,* (d) *very likely.*

2. State whether or not each of the statements below is reasonable.

   (a) The probability that there will be a General Election next year is 0.2.

   (b) The probability that England will win the next football World Cup is 0.8.

   (c) The probability that it will not rain tomorrow is 0.9.

   (d) The probability that your school will be hit by lightning in the next week is 0.1.

3. (a) List the things described, in order, with the most likely first.

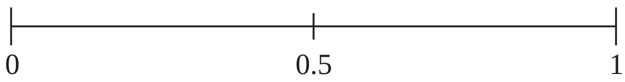
A You travel on a bus that breaks down on the way home from school.

B Your pocket money is increased during the next two weeks.

C You enjoy your school lunch tomorrow.

D You have already had a birthday this year.

(b) Mark estimates of the probabilities of each of these on a copy of the probability scale similar to the one below:



4. Explain why the probability that you will be the first person to walk on the moon is zero.

5. Describe something that has a probability of zero.

6. (a) Do you agree that the probability that you will not be abducted by aliens in the next 24 hours is 1 ?

(b) Explain why.

7. Describe something that has a probability of 1.

8. When you toss a fair coin, the probability of obtaining a head is $\frac{1}{2}$ and the probability of obtaining a tail is $\frac{1}{2}$.

Describe something else that has a probability that is equal to or close to $\frac{1}{2}$.

9. A packet of sweets contains mostly *red* sweets, a few *green* sweets and only one *yellow* sweet. You take a sweet at random from the packet.

The events A, B, C and D are listed below.

A You take a *yellow* sweet.

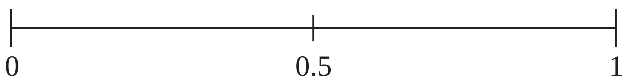
B You take a *green* sweet.

C You take a *red* sweet.

D You take a *blue* sweet.

(a) Write these outcomes in order of probability, with the most likely first.

(b) Mark the probability of each outcome on a scale similar to the one below.



10. The probability that a train is late is 0.1. Which of the following statements is the most reasonable:

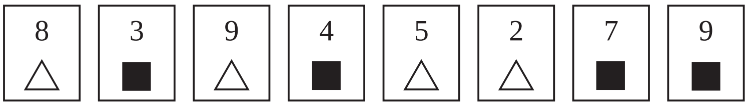
    A The train is *unlikely* to be late.

    B The train is *very unlikely* to be late.

    C The train is *likely* to be late.

    Explain why you have chosen your answer.

11. (a) Joe has these cards:



Sara takes a card without looking.

Joe says: "*On Sara's card,* ■ *is more likely than* △ ."
Explain why Joe is wrong.

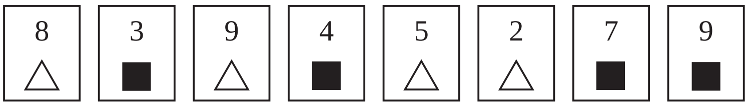
Choose one of the following words and phrases to fill in the gaps in the sentences below:

Impossible *Not Likely* *Certain* *Likely*

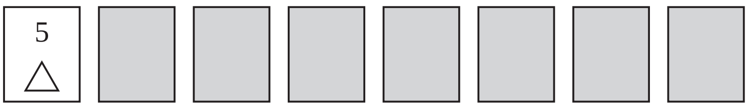
It is .... that the number on Sara's card will be *smaller than 10.*

It is .... that the number on Sara's card will be an *odd number.*

(b) Joe still has these cards:



He mixes them up and puts them face down on the table. Then he turns the first card over, like this:



Joe is going to turn the next card over.

Copy and complete this sentence:

On the next card, ... is *less likely* than ....

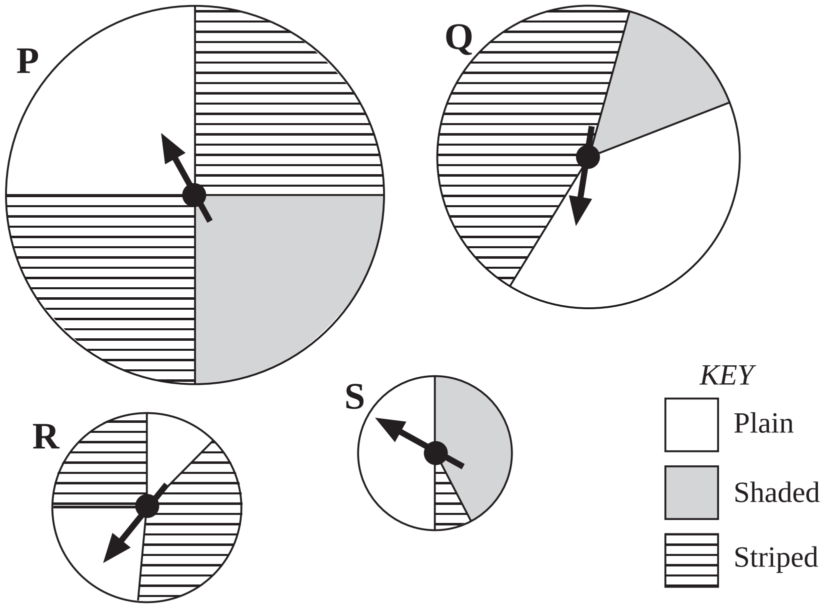
The number on the next card could be higher than 5 or lower than 5. Which of the following possibilities is *more likely*?

Higher than 5 *Lower than 5* *Cannot tell*

Explain your answer.

(KS3/97/Ma/Tier 3-5/P2)

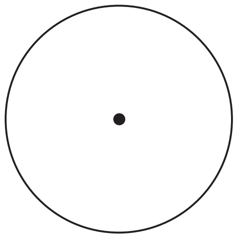
12. Here are four spinners, labelled P, Q, R and S.



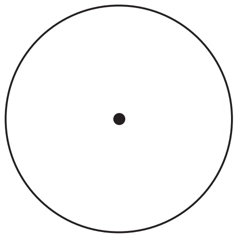
(a) Which spinner gives the *greatest* chance that the arrow will land on *plain*?

(b) Which spinner gives the *smallest* chance that the arrow will land on *shaded*?

(c) Shade a copy of the spinner shown so that it is *certain* that the arrow will land on *shaded*.



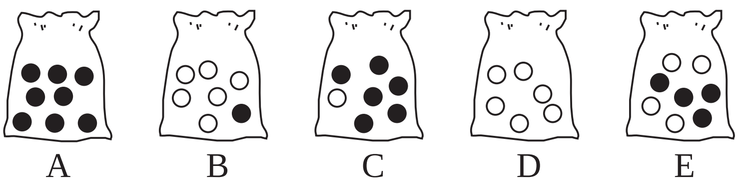
(d) Shade a copy of this spinner so that there is a 50% chance that the arrow will land on *shaded*.



(KS2/98/Ma/Tier 4-6/P2)

13. Bryn has some bags with some black beads and some white beads. He is going to take a bead from each bag without looking.

(a) Match the pictures to the statements. The first is done for you.



(i) It is *impossible* that Bryn will take a black bead from bag D.

(ii) It is *unlikely* that Bryn will take a black bead from bag ..... .

(iii) It is *equally likely* that Bryn will take a black bead or a white bead from bag ..... .

(iv) It is *likely* that Bryn will take a black bead from bag ..... .

(v) It is *certain* that Bryn will take a black bead from bag ..... .

(b) Bryn has 5 white beads in a bag.

He wants to make it *more likely* that he will take a *black* bead than a *white* bead out of the bag.

How many *black* beads should Bryn put into the bag?

(c) There are 20 beads altogether in another bag. All the beads are either black or white.

It is *equally likely* that Bryn will take a black bead or a white bead from the bag.

How many black beads and how many white beads are there in the bag?

(KS3/99/Ma/Tier 3-5/P2)

6.2 The Probability of a Single Event

In this section we consider the probabilities of equally likely events. When you roll a fair dice, each of the numbers 1 to 6 is equally likely to be on the uppermost face of the dice.

> For equally likely events:
>
> $$p(\text{a particular outcome}) = \frac{\text{number of ways of obtaining outcome}}{\text{total number of outcomes}}$$

Example 1

A card is taken at random from a full pack of 52 playing cards. What is the probability that it is:

(a) a red card,

(b) a 'Queen',

(c) a red 'Ace',

(d) the 'Seven of Hearts',

(e) an even number?

Solution

As each card is equally likely to be drawn from the pack there are 52 equally likely outcomes.

(a) There are 26 red cards in the pack, so:

$$p(\text{red}) = \frac{26}{52}$$

$$= \frac{1}{2}$$

(b) There are 4 Queens in the pack, so:

$$p(\text{Queen}) = \frac{4}{52}$$

$$= \frac{1}{13}$$

(c) There are 2 red Aces in the pack, so:

$$p(\text{red Ace}) = \frac{2}{52}$$

$$= \frac{1}{26}$$

(d) There is only one 7 of Hearts in the pack, so:

$$p(\text{7 of Hearts}) = \frac{1}{52}$$

(e) There are 20 cards that have even numbers in the pack, so:

$$p(\text{even number}) = \frac{20}{52}$$

$$= \frac{5}{13}$$

Example 2

A packet of sweets contains 18 *red* sweets, 12 *green* sweets and 10 *yellow* sweets. A sweet is taken at random from the packet. What is the probability that the sweet is:

(a) *red*,

(b) *not green*,

(c) *green* or *yellow* ?

Solution

The total number of sweets in the packet is 40, so there are 40 equally likely outcomes when one is taken at random.

(a) There are 18 *red* sweets in the packet, so:

$$p(\text{red}) = \frac{18}{40}$$

$$= \frac{9}{20}$$

(b) There are 28 sweets that are *not green* in the packet, so:

$$p(\text{not green}) = \frac{28}{40}$$

$$= \frac{7}{10}$$

(c) There are 22 sweets that are *green* or *yellow* in the packet, so:

$$p(\text{green or yellow}) = \frac{22}{40}$$

$$= \frac{11}{20}$$

Example 3

You roll a fair dice 120 times. How many times would you expect to obtain:

(a) a 6, (b) an *even* score, (c) a score of *less than* 5 ?

Solution

(a) $p(6) = \frac{1}{6}$

Expected number of 6s $= \frac{1}{6} \times 120$

$= 20$

(b) $p(\text{even score}) = \frac{3}{6}$

$= \frac{1}{2}$

Expected number of even scores $= \frac{1}{2} \times 120$

$= 60$

(c) $$p(\text{score less than } 5) = \frac{4}{6}$$

$$= \frac{2}{3}$$

Expected number of scores less than 5 $= \frac{2}{3} \times 120$

$= 80$

Exercises

1. You roll a fair dice. What is the probability that you obtain:

   (a) a *five*, (b) a *three*, (c) an *even* number,

   (d) a multiple of 3, (e) a number *less than* 6 ?

2. A jar contains 9 *red* counters and 21 *blue* counters. A counter is taken at random from the jar. What is the probability that it is:

   (a) *red*, (b) *blue*, (c) *green* ?

3. You take a card at random from a pack of 52 playing cards. What is the probability that the card is:

   (a) a red King, (b) a Queen or a King, (c) a 5, 6 or 7,

   (d) a Diamond, (e) *not* a Club ?

4. A jar contains 4 *red* balls, 3 *green* balls and 5 *yellow* balls. One ball is taken at random from the jar. What is the probability that it is:

   (a) *green*, (b) *red*, (c) *yellow*,

   (d) *not red*, (e) *yellow* or *red* ?

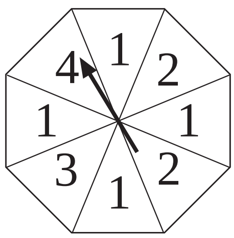
5. The faces of a regular tetrahedron are numbered 1 to 4. When it is rolled it lands face down on one of these numbers. What is the probability that this number is:

   (a) 2, (b) 3,

   (c) 1, 2 or 3, (d) an *even* number ?

6. A spinner is numbered as shown in the diagram. Each score is equally likely to occur.

   What is the probability of scoring:



   (a) 1, (b) 2,

   (c) 3, (d) 4,

   (e) 5, (f) a number less than 6 ?

7. You toss a fair coin 360 times.

   (a) How many times would you expect to obtain a head?

   (b) If you obtained 170 heads, would you think that the coin was biased? Explain why.

8. A spinner has numbers 1 to 5, so that each number is equally likely to be scored. How many times would you expect to get a score of 5, if the spinner is spun:

   (a) 10 times, (b) 250 times, (c) 400 times ?

9. A card is drawn at random from a pack of 52 playing cards, and then replaced. The process is repeated a total of 260 times. How many times would you expect the card drawn to be:

   (a) a 7, (b) a *red Queen,* (c) a *red* card,

   (d) a *Heart,* (e) a card with an *even* number ?

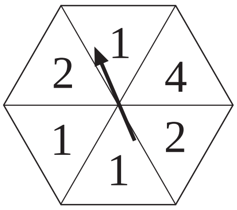
10. A six-sided spinner is shown in the diagram. It is spun 180 times.

    How many times would you expect to obtain:

    (a) a score of 1,

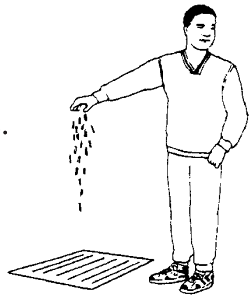
    (b) a score less than 4,

    (c) a score that is a *prime* number, (d) a score of 4 ?



11. Barry is doing an experiment. He drops 20 matchsticks at random onto a grid of parallel lines.

    Barry does the experiment 10 times and records his results. He wants to work out an estimate of probability.



Number of the 20 matchsticks that have fallen across a line									
5	7	6	4	6	8	5	3	5	7

    (a) Use Barry's data to work out the probability that a *single matchstick* when dropped will fall across one of the lines. Show your working.

    (b) Barry continues the experiment until he has dropped the 20 matchsticks 60 times.

    About how many matchsticks *in total* would you expect to fall across one of the lines? Show your working.

(KS3/96/Ma/Tier 5-7/P2)

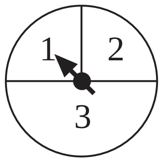
12. Les, Tom, Nia and Ann are in a singing competition. To decide the order in which they will sing all four names are put into a bag. Each name is taken out of the bag, one at a time, without looking.

(a) Write down *all* the possible orders with *Tom* singing *second.*

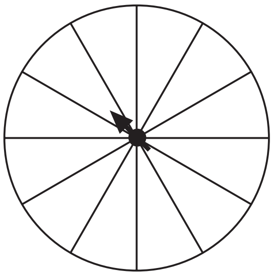
(b) In a different competition there are 8 singers. The probability that Tom sings second is $\frac{1}{8}$.
Work out the probability that Tom does *not* sing second.

(KS3/96/Ma/Tier 4-6/P1)

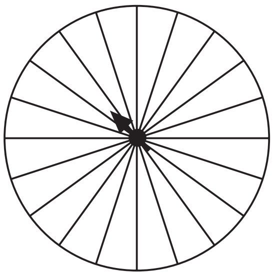
13. (a) What is the probability of getting a 3 on this spinner?



(b) Shade a copy of the following spinner so that the chance of getting a shaded section is double the chance of getting a white section.



(c) Shade a copy of the following spinner so that there is a 40% chance of getting a shaded section.



(KS3/95/Ma/Levels 4-6/P1)

14. Pat has 5 white beads and 1 black bead in her bag. She asks two friends about the probability of picking a black bead without looking in the bag.

Owen says: "*It is* $\frac{1}{5}$ *because there are 5 white beads and 1 black bead.*"

Maria says: "*It is* $\frac{1}{6}$ *because there are 6 beads and 1 is black.*"

(a) Which of Pat's friends is *correct*? Explain why the other friend is *wrong.*

(b) Tracy has a different bag of black beads and white beads.

The probability of picking a black bead from Tracy's bag is $\frac{7}{13}$.

What is the probability of picking a white bead from Tracy's bag?

(c) How many black beads and how many white beads could be in Tracy's bag?

(d) Peter has a different bag of black beads and white beads.

Peter has more beads in total than Tracy.

The probability of picking a black bead from Peter's bag is also $\frac{7}{13}$.

How many black beads and how many white beads could be in Peter's bag?

(KS3/94/Ma/4-6/P1)

15. Brightlite company makes light bulbs. The state of the company's machines can be:

available for use and being used
or available for use but not needed
or broken down.

(a) The table shows the probabilities of the state of the machines in July 1994. What is the missing probability?

State of machines: July 1994	*Probability*
Available for use, being used	
Available for use, not needed	0.09
Broken down	0.03

(b) During another month the probability of a machine being available for use was 0.92. What was the probability of a machine being broken down?

(c) Brightlite calculated the probabilities of a bulb failing within 1000 hours and within 2000 hours.

Copy and complete the table below to show the probabilities of a bulb still working at 1000 hours and at 2000 hours.

Time	*Failed*	*Still working*
At 1000 hours	0.07	
At 2000 hours	0.57	

(KS3/95/Ma/Levels 5-7/P1)

16. A machine sells sweets in *five* different colours:

red, green, orange, yellow, purple.

You *cannot choose* which colour you get.

There are the *same number* of each colour in the machine.

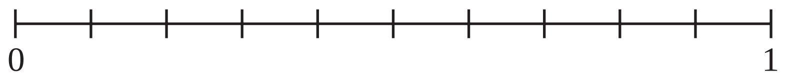
Two boys want to buy a sweet each.

Ken does not like orange sweets or yellow sweets. Colin likes them all.

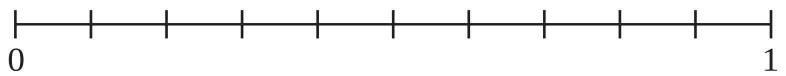
(a) What is the probability that Ken will get a sweet that he likes?

(b) What is the probability that Colin will get a sweet that he likes?

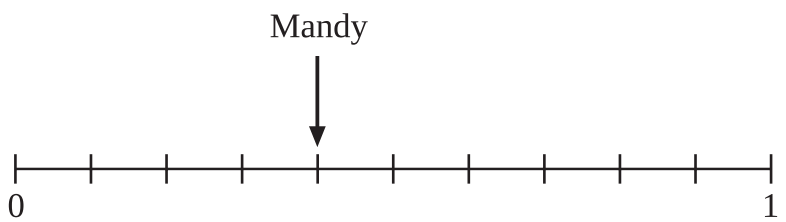
(c) Copy the following scale and draw an arrow to show the probability that Ken will get a sweet that he likes. Label the arrow 'Ken'.



(d) On your scale from (c), draw an arrow to show the probability that Colin will get a sweet that he likes. Label this arrow 'Colin'.



(e) Mandy buys one sweet. The arrow on the following scale shows the probability that Mandy gets a sweet that she likes.



Write a sentence that *could* describe which sweets Mandy likes.

(KS3/96/Ma/Tier 3-5/P2)

6.3 The Probability of Two Events

In this section we review the use of *listings*, *tables* and *tree diagrams* to calculate the probabilities of two events.

Example 1

An unbiased coin is tossed twice.

(a) List *all* the possible outcomes.

(b) What is the probability of obtaining *two heads*?

(c) What is the probability of obtaining a *head* and a *tail* in any order?

Solution

(a) The possible outcomes are:

H H
H T
T H
T T

So there are 4 possible outcomes that are all equally likely to occur as the coin is not biased.

(b) There is only one way of obtaining 2 heads, so:

$$p(2 \text{ heads}) = \frac{1}{4}$$

(c) There are two ways of obtaining a head and a tail, H T and T H, so:

$$p(\text{a head and a tail}) = \frac{2}{4} = \frac{1}{2}$$

Example 2

A red dice and a blue dice, both unbiased, are rolled at the same time. The scores on the two dice are then added together.

(a) Use a table to show all the possible outcomes.

(b) What is the probability of obtaining:

(i) a score of 5,

(ii) a score which is *greater than* 3,

(iii) a score which is an *even number*?

Solution

(a) The following table shows all of the 36 possible outcomes:

Red Dice

Blue Dice	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

(b) (i) There are 4 ways of scoring 5, so:

$$p(5) = \frac{4}{36}$$

$$= \frac{1}{9}$$

(ii) There are 33 ways of obtaining a score greater than 3, so:

$$p(\text{greater than } 3) = \frac{33}{36}$$

$$= \frac{11}{12}$$

(iii) There are 18 ways of obtaining a score which is an even number, so:

$$p(\text{even score}) = \frac{18}{36}$$

$$= \frac{1}{2}$$

Example 3

A card is taken at random from a pack of 52 playing cards, and then replaced. A second card is then drawn at random from the pack.

Use a tree diagram to determine the probability that:

(a) *both* cards are Diamonds,

(b) *at least one* card is a Diamond,

(c) *exactly one* card is a Diamond,

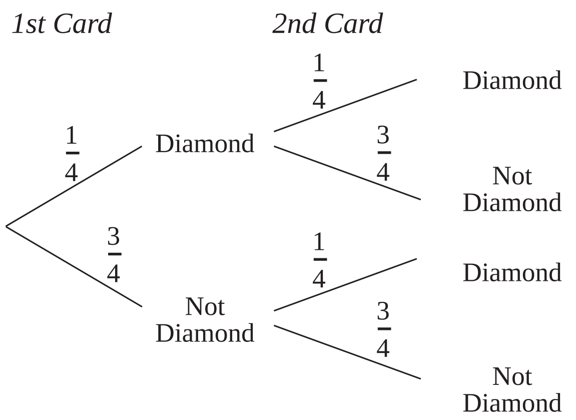
(d) *neither* card is a Diamond.

Solution

We first note that, for a single card drawn from the pack,

$$p(\text{Diamond}) = \frac{13}{52} = \frac{1}{4} \text{ and } p(\text{not Diamond}) = \frac{39}{52} = \frac{3}{4}.$$

We put these probabilities on the branches of the tree diagram below:



PROBABILITIES

$$\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$$

$$\frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

$$\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

$$\text{Total} = \frac{16}{16} = 1$$

Note also that the probability for each combination, for example, two Diamonds, is determined by *multiplying* the probabilities along the branches.

(a) $p(\text{both Diamonds}) = \frac{1}{16}$

(b) $p(\text{at least one Diamond}) = \frac{1}{16} + \frac{3}{16} + \frac{3}{16}$

$= \frac{7}{16}$

(c) $p(\text{exactly one Diamond}) = \frac{3}{16} + \frac{3}{16}$

$= \frac{6}{16}$

$= \frac{3}{8}$

(d) $p(\text{neither card a Diamond}) = \frac{9}{16}$

Exercises

1. The faces of an unbiased dice are painted so that 2 are *red*, 2 are *blue* and 2 are *yellow*. The dice is rolled twice. Three of the possible outcomes are listed below:

   R R   R B   R Y

   (a) List all 9 possible outcomes.

   (b) What is the probability that:

   (i) both faces are *red*,

   (ii) both faces are the *same colour*,

   (iii) the faces are of *different colours*?

2. A spinner is marked with the letters A, B, C and D, so that each letter is equally likely to be obtained. The spinner is spun twice.

   (a) List the 16 possible outcomes.

   (b) What is the probability that:

   (i) A is obtained *twice*,

   (ii) A is obtained *at least once*,

   (iii) *both* letters are the *same*,

   (iv) the letter B is *not* obtained at all?

3. Two fair dice are renumbered so that they have the following numbers on their faces:

   1, 1, 2, 3, 4, 6

   The dice are rolled at the same time, and their scores added together.

   (a) Draw a table to show the 36 possible outcomes.

   (b) What is the probability that the total score is:

   (i) 6,   (ii) 3,

   (iii) greater than 10,   (iv) less than 5 ?

4. A red spinner is marked with the numbers 1 to 4 and a blue spinner is marked with the numbers 1 to 5. On each spinner all the numbers are equally likely to be obtained. The two spinners are spun at the same time and the two scores are added together.

   (a) Draw a table to show the 20 possible outcomes.

   (b) What is the probability that the total score on the two spinners is:

   (i) an *even* number,   (ii) the number 7,

   (iii) a number greater than 4,   (iv) a number less than 7 ?

5. An unbiased dice is rolled and a fair coin is tossed at the same time.

   (a) *Either* list all the possible outcomes *or* show them in a table.

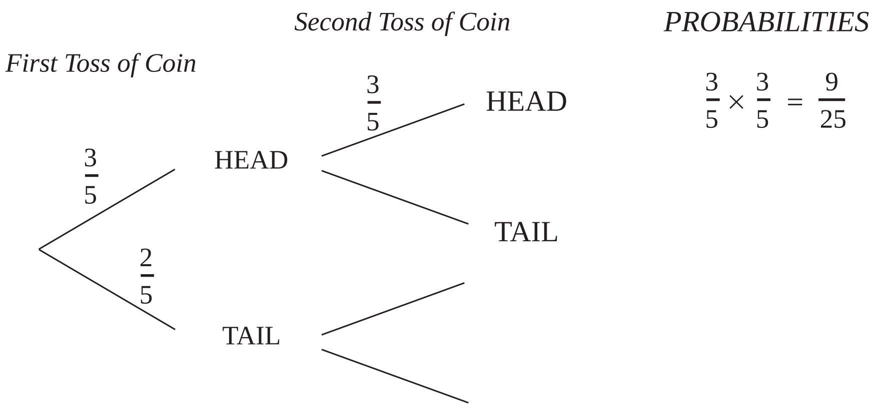
   (b) What is the probability of obtaining:

   (i) a head and a 6, (ii) a tail and an odd number,

   (iii) a tail and a number less than 5 ?

6. A coin is biased so that the probability of obtaining a head is $\frac{3}{5}$ and the probability of obtaining a tail is $\frac{2}{5}$.

   (a) Copy and complete the following tree diagram to show the possible outcomes and probabilities if the coin is tossed twice.



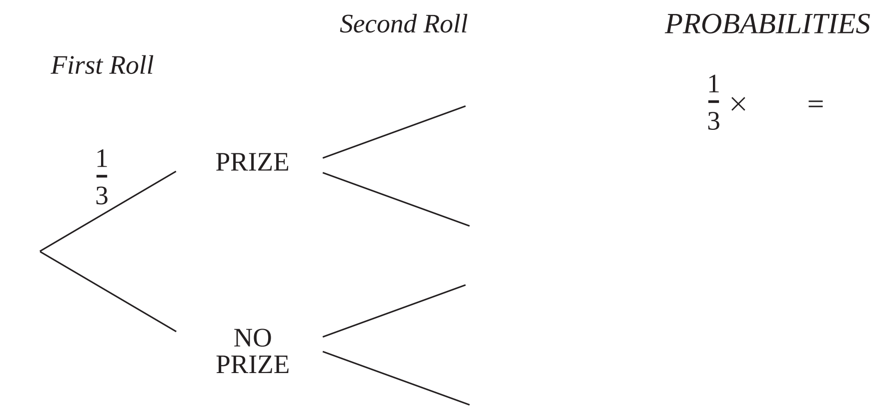
   (b) What is the probability of obtaining:

   (i) 2 heads, (ii) at least one head,

   (iii) 2 tails, (iv) exactly 1 tail ?

7. An unbiased dice is rolled twice in a game. If a 1 or a 6 is obtained, you win a prize.

   (a) Copy and complete the following tree diagram:



   (b) What is the probability that a player wins:

   (i) 2 prizes, (ii) 1 prize, (iii) at least 1 prize ?

8. A card is taken at random from a pack of 52 playing cards. It is replaced and a second card is then taken at random from the pack.
A card is said to be a 'Royal' card if it is a *King, Queen* or *Jack*.

Use a tree diagram to calculate the probability that:

(a) *both* cards are Royals,
(b) *one* card is a Royal,
(c) *at least one* card is a Royal,
(d) *neither* card is a Royal.

9. The probability that a school bus is late on any day is $\frac{1}{10}$. Use a tree diagram to calculate the probability that on two consecutive days, the bus is:

(a) late *twice*,
(b) late *once*,
(c) *never* late.

10. The probability that a piece of bread burns in a toaster is $\frac{1}{9}$. Two slices of bread are toasted, one after the other.

(a) Use a tree diagram to calculate the probability that at least one of these slices of bread burns in the toaster.

(b) Extend your tree diagram to include toasting 3 slices, one at a time. Calculate the probability of at least one slice burning in the toaster.

11. A coin has two sides, heads and tails.

(a) Chris is going to toss a coin. What is the probability that Chris will get heads? Write your answer as a fraction.

(b) Sion is going to toss 2 coins. Copy and complete the following table to show the different results he could get.

First coin	*Second coin*
heads	heads

(c) Sion is going to toss 2 coins. What is the probability that he will get tails with both his coins? Write your answer as a fraction.

(d) Dianne tossed one coin. She got tails.
Dianne is going to toss another coin.
What is the probability that she will get tails again with her next coin? Write your answer as a fraction.

(KS3/99/Ma/Tier 3-5/P1)

12. I have two fair dice. Each of the dice is numbered 1 to 6.

(a) The probability that I will throw *double 6 (*both dice showing number 6) is

$$\frac{1}{36}$$

What is the probability that I will *not* throw double 6 ?

(b) I throw both dice and get double 6. Then I throw both dice again.

Which one answer from the list below describes the probability that I will throw *double 6* this time?

less than $\frac{1}{36}$

$\frac{1}{36}$

more than $\frac{1}{36}$

Explain your answer.

I start again and throw both dice.

(c) What is the probability that I will throw *double 3* (both dice showing number 3) ?

(d) What is the probability that I will throw a double? (It could be double 1 or double 2 or any other double.)

(KS3/98/Ma/Tier 4-6/P2)

13. On a road there are two sets of traffic lights. The traffic lights work independently.

For each set of traffic lights, the probability that a driver will have to *stop* is 0.7.

(a) A woman is going to drive along the road.

(i) What is the probability that she will have to *stop* at *both* sets of traffic lights?

(ii) What is the probability that she will have to *stop* at *only one of the two sets* of traffic lights?

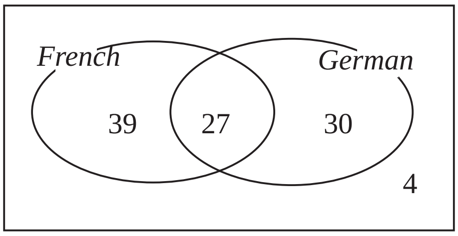
Show your working.

(b) In one year, a man drives 200 times along the road. Calculate an estimate of the number of times he drives through *both sets* of traffic lights *without stopping.* Show your working.

(KS3/99/Ma/Tier 6-8/P2)

14. 100 students were asked whether they studied French or German.

Results:



27 students studied both French *and* German.

(a) What is the probability that a student chosen at random will study only *one* of the languages?

(b) What is the probability that a student who is studying German is also studying French?

(c) Two of the 100 students are chosen at random.

From the following calculations, write down one which shows the probability that *both* students study French and German.

$$\frac{27}{100} \times \frac{26}{100} \qquad \frac{27}{100} + \frac{26}{99} \qquad \frac{27}{100} + \frac{27}{100}$$

$$\frac{27}{100} \times \frac{26}{99} \qquad \frac{27}{100} \times \frac{27}{100}$$

(KS3/98/Ma/Tier 6-8/P1)

15. A company makes computer disks. It tested a random sample of the disks from a large batch. The company calculated the probability of any disk being defective as 0.025.

Glenda buys 2 disks.

(a) Calculate the probability that *both* disks are defective.

(b) Calculate the probability that *only one* of the disks is defective.

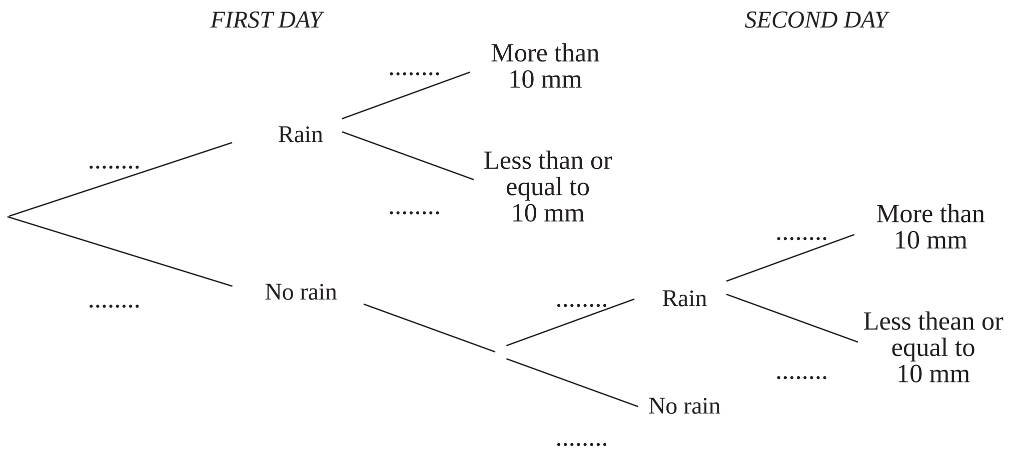
(c) The company found 3 defective disks in the sample they tested.
How many disks were likely to have been tested?

(KS3/96/Ma/Tier 6-8/P2)

16. On a tropical island the probability of it raining on the first day of the rainy season is $\frac{2}{3}$. If it does not rain on the first day, the probability of it raining on the second day is $\frac{7}{10}$. If it rains on the first day, the probability of it raining more than 10 mm on the first day is $\frac{1}{5}$. If it rains on the second day

but not on the first day, the probability of it raining more than 10 mm is $\frac{1}{4}$.

You may find it helpful to copy and complete the tree diagram before answering the questions.



(a) What is the probability that it rains more than 10 mm on the second day, and does not rain on the first? Show your working.

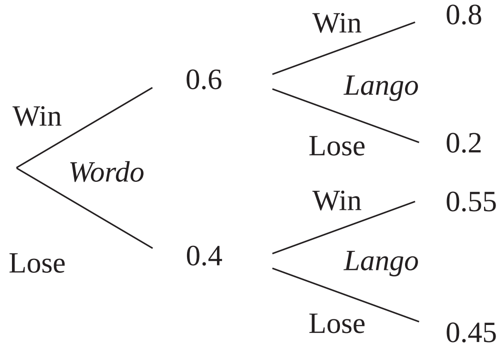
(b) What is the probability that it has rained by the end of the second day of the rainy season? Show your working.

(c) Why is it not possible to work out the probability of rain on both days from the information given?

(KS3/96/Ma/Ext)

17. Pupils at a school invented a word game called Wordo. They tried it out with a large sample of people and found that the probability of winning Wordo was 0.6.

The pupils invented another word game, Lango. The same sample who had played Wordo then played Lango. The pupils drew this tree diagram to show the probabilities of winning.



(a) What was the probability of someone from the sample winning Lango?

(b) What was the probability of someone from the sample winning *only* one of the two word games?

(c) The pupils also invented a dice game. They tried it out with the same sample of people who had already played Wordo and Lango.

The probability of winning the dice game was 0.9. This was found to be independent of the probabilities for Wordo and Lango.

Calculate the probability of someone from the sample winning two out of these three games.

(d) Calculate the probability of someone from the sample winning *only* one of these three games.

(KS3/95/Ma/Levels 9-10)

6.4 Theoretical and Experimental Probabilities

In this section we compare *theoretical* and *experimental* probabilities.

The term 'theoretical probabilities' describes those which have been calculated, for example by the methods described in sections 6.2. and 6.3.

'Experimental probabilities' are estimates for probabilities that cannot be determined logically. They can be derived from the results of experiments, but often they are obtained from the analysis of statistical data or historical records.

Here we obtain experimental probabilities from simple experiments and compare them with the theoretical probabilities.

Example 1

An unbiased dice is to be rolled 240 times.

(a) Calculate the number of times you would expect to obtain each of the possible scores.

(b) Now roll the dice 240 times and collect some experimental results, presenting them in a bar chart.

(c) Compare the theoretical and experimental results.

Solution

(a) $p(6) = \frac{1}{6}$

Expected number of 6s $= \frac{1}{6} \times 240$

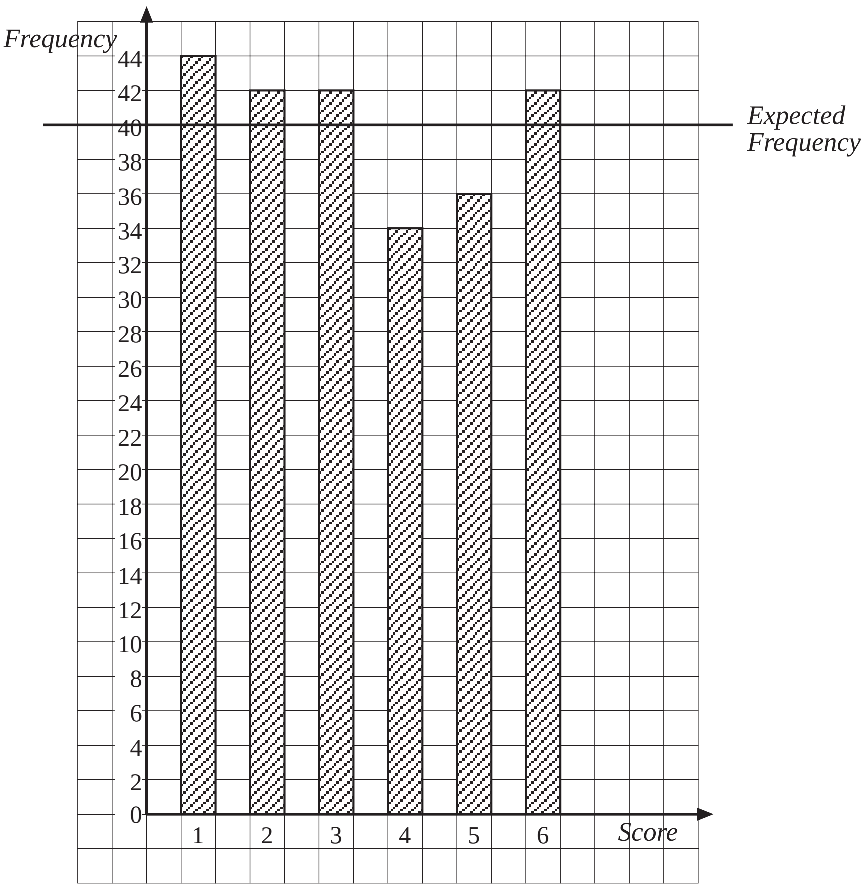
$= 40$

Similarly, you would expect to obtain each of the possible scores 40 times.

(b) The results of the experiment are recorded in the following table:

Score	*Tally*	*Frequency*
1	卌 卌 卌 卌 卌 卌 卌 卌 ||||	44
2	卌 卌 卌 卌 卌 卌 卌 卌 ||	42
3	卌 卌 卌 卌 卌 卌 卌 卌 ||	42
4	卌 卌 卌 卌 卌 卌 ||||	34
5	卌 卌 卌 卌 卌 卌 卌 |	36
6	卌 卌 卌 卌 卌 卌 卌 卌 ||	42

These results are illustrated in the following bar chart. A horizontal line has been drawn to show the expected frequencies for the scores.



Note that none of the bars is of the expected height; some are above and some are below. However, all the bars are *close* to the predicted number. We would not expect to obtain *exactly* the predicted number. The more times the experiment is carried out, the closer the experimental results will be to the theoretical predictions.

Exercises

1. (a) A fair coin is tossed 100 times. How many heads and how many tails would your expect to obtain?
   (b) Toss a fair coin 100 times and display your results using a bar chart.
   (c) Compare your theoretical predictions with your experimental results.

2. Two fair coins are to be tossed at the same time.
   (a) Calculate the probability of obtaining:
   (i) 2 heads, (ii) a head and a tail, (iii) 2 tails.
   (b) Calculate the number of times you would expect to obtain each outcome if the coins are tossed 100 times.
   (c) Toss two coins 100 times and illustrate your results using a bar chart.
   (d) Compare your theoretical predictions with your experimental results.

3. (a) List the 8 possible outcomes when 3 fair coins are tossed at the same time.
   (b) If three fair coins were tossed 32 times, how many times would you expect to obtain:
   (i) 3 heads, (ii) 2 heads,
   (iii) 1 head, (iv) 0 heads ?
   (c) Carry out an experiment and compare your theoretical predictions with your experimental results.

4. (a) What are the expected frequencies of the totals 2, 3, 4, ..., 11, 12 when two fair dice are thrown at the same time and the experiment is repeated 36 times?
   (b) Carry out the experiment in (a) and compare the predicted and experimental frequencies.
   (c) Repeat (a) and (b) for 144 throws.
   (d) Comment on how carrying out the experiment more times influences the differences between the predicted and experimental frequencies.

5. A fair coin and an unbiased dice are thrown at the same time. A score is then calculated using the following rules:
   - *if the coin shows a head, you double the score shown on the dice;*
   - *if the coin shows a tail, you subtract 1 from the score on the dice.*

   (a) Use a two-way table to show all the possible scores.
   (b) Draw up a table showing the theoretical probabilities for the various scores.

(c) If the coin and the dice are thrown 120 times, how many times would you expect to obtain each score?

(d) Conduct an experiment and compare your experimental results with your answers to part (c).

6. A dice with 4 faces has one blue, one green, one red and one yellow face.

Five pupils did an experiment to investigate whether the dice was biased or not.

The following table shows the data they collected.

Pupil's Name	*Number of Throws*	*Face Landed On*			
		Red	*Blue*	*Green*	*Yellow*
Peter	20	9	7	2	2
Caryl	60	23	20	8	9
Shana	250	85	90	36	39
Keith	40	15	15	6	4
Paul	150	47	54	23	26

(a) Which pupil's data is most likely to give the best estimate of the probability of getting each colour on the dice? Explain your answer.

The pupils collected all the data together.

Number of Throws	*Face Landed On*			
	Red	*Blue*	*Green*	*Yellow*
520	179	186	75	80

(b) Consider the data. Write down whether you think the dice is biased or unbiased, and explain your answer.

(c) From the data, work out the probability of the dice landing on the blue face.

(d) From the data work out the probability of the dice landing on the green face.

(KS3/95/Ma/Levels 5-7/P1)

7. Some pupils threw 3 fair dice. They recorded how many times the numbers on the dice were the same.

Name	*Number of throws*	*Results*		
		all different	*2 the same*	*all the same*
Morgan	40	26	12	2
Sue	140	81	56	3
Zenta	20	10	10	0
Ali	100	54	42	4

(a) Write the name of the pupil whose data are *most likely* to give the best estimate of the probability of getting each result. Explain your answer.

(b) This table shows the pupils' results collected together:

Number of throws	*Results*		
	all different	*2 the same*	*all the same*
300	171	120	9

Use these data to estimate the *probability* of throwing numbers that are *all different.*

(c) The theoretical probability of each result is shown below:

	all different	*2 the same*	*all the same*
Probability	$\frac{5}{9}$	$\frac{5}{12}$	$\frac{1}{36}$

Use these probabilities to calculate, for 300 throws, *how many times* you would theoretically expect to get each result. Copy and complete the table below.

Number of throws	*Results*		
	all different	*2 the same*	*all the same*
300	....	....	....

(d) Explain why the pupils' results are not the same as the theoretical results.

(KS3/98/Ma/Tier 5-7/P2)