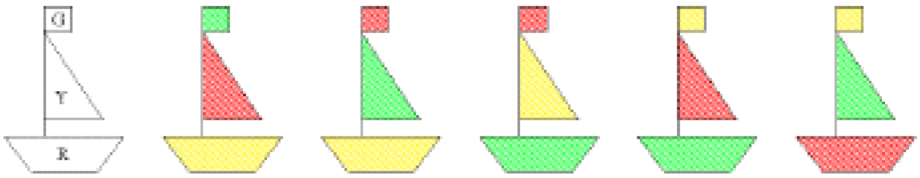
Mathematics Enhancement Programme

TEACHING SUPPORT: Year 1

SOLUTIONS TO EXERCISES

1. *Question and Solution*

Colour the boats in different ways. Use green, yellow and red.



(p4, Q5)

Notes

This is an ideal question to encourage systematic thinking rather than just random colouring in. After completing the colouring-in of the first boat, ask,

"Is there another solution with G on the flag?"

This will give the 'GRY' solution. Continue with:

"Now colour the flag R; what solutions are there?"

This gives 'RGY' and 'RYG'. Continue:

"What other colour can the flag be?"

This gives R and eventually the solutions 'YRG' and 'YGR'.

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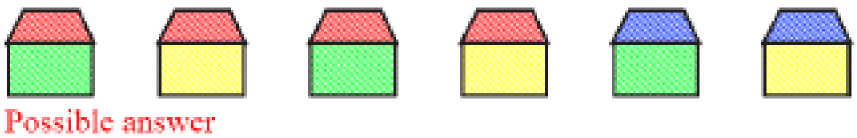
2. *Question and Solution*

Colour in the houses as shown.



(p11, Q4)

Notes

This question needs clarification! The question means that,

'There are more RED roofs than BLUE roofs;

there are the same number or less GREEN walls than YELLOW walls.'

Note the use of the symbol '≤' which means 'less than or the same as'.

(For example, the solution of $\square \leq 3$ is

$\square = 0, 1, 2$ or 3

as they all satisfy the statement.)

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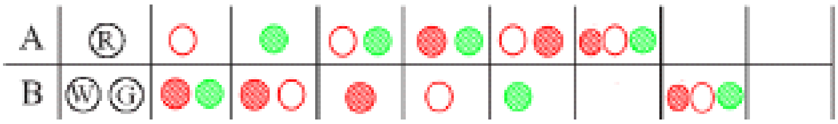
3. *Question and Solution*

Show different ways to share a Red ball, a White ball and a Green ball between Ann and Rob.



(p12, Q4)

Notes

This is another problem where a systematic approach is better than just randomly trying different answers.

You could ask,

"With just ONE ball for A, are there other answers?"

which should give the answers W/RG and G/RW.

Now ask,

"What happens if A has TWO balls?"

This is actually the same problem with B now having only ONE ball - hence there will be three answers, WG/R, RG/W and WR/G.

Note that this leaves one column with *no* entry.

Sometimes extra spaces or insufficient spaces for answers are given, to make students think!

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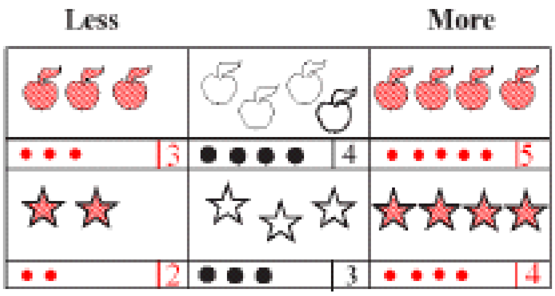
4. *Question and Solution*

Draw **less** objects and **more** objects than the number in the middle.



(p13, Q4)

Notes

The solution shown is just one possible answer.

The 'Less' column is, in fact, 'One less than' and the 'More' column is 'One more than'.

In the first row you could have answers such as

2 | 4 | 5 or 1 | 4 | 5

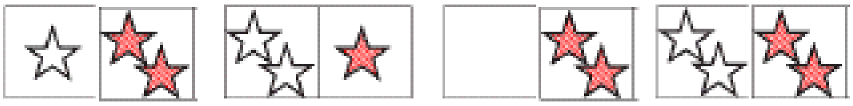
or even 0 | 4 | 5 , etc.

Encourage your students to think of all the possible answers.

(You might like to check that there is no confusion about 'rows' and 'columns'.)

5. *Question and Solution*

Complete the right-hand side of each picture to match the numbers and signs.



1	<1	2

2	1>	1

0	<2	2

2	=	2

(p22, Q1)

Notes

The new notation <1 means '1 more than',

so that | 1 | <1 | |

means that ☐ is 1 more than 1, i.e., 2.

Similarly, | 2 | 1> | | means that ☐ is 1 less than 2, i.e. 1

and | | <2 | 2 | means 2 is 2 greater than ☐ , i.e. ☐ = 0

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6. *Question and Solution*

Make the statements true by changing the place of one stick.

|| + | = |	| + | = ||
|| – | = ||	| + | = ||
| + | = |	|| - | = |

(p25, Q3)

Notes

The first question is easy but the next two do require lateral thinking. You need to recognise that both + and = are made up of two sticks. So for the second question, you move one of the first vertical sticks to change the '-' sign to '+'. (Another possibility would be | — | ≠ || , where ≠ means 'not equal to'.) The third question needs the same strategy.

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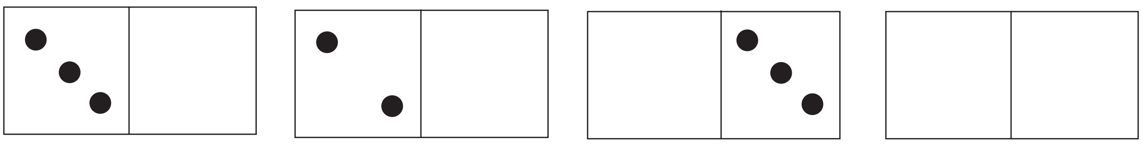
7. *Question and Solution*

Complete the drawings to match the signs.



3	2>	1

2	1>	1

3	=	3

3	3>	0

Write in the missing numbers.

(p35, Q1)

Notes

The first three parts just follow the rule for the notation. The last part,

$$\square \text{ is 3 more than } \square$$

can lead to many solutions, 3, 0 or 4, 1 or 5, 2, etc.

You could encourage the class to provide as many answers as possible.

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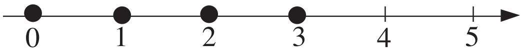
8. *Question and Solution*

Which numbers could be hidden under the cards? (0, 1, 2, 3, 4, 5)

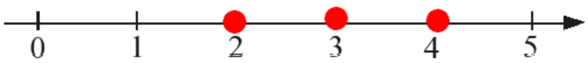
For example: $\boxtimes \leq 3$ gives $\boxtimes$: 0, 1, 2, 3



(a) $\boxtimes < 5$ $\boxtimes$: $\boxed{4, 3, 2, 1, 0}$

(b) $\boxtimes \geq 2$ $\boxtimes$: $\boxed{2, 3, 4, 5}$

(c) $2 \leq \boxtimes < 5$ $\boxtimes$: $\boxed{2, 3, 4}$



(p46, Q1)

Notes

This question involves the beginnings of algebra, where you are asked to solve inequalities.

In a), we are asked for any number less than 5, so the answer is 0, 1, 2, 3, 4. (Of course, there are many more answers such as -1, -2, $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{2}$, etc. but the question restricts answers to 0, 1, 2, 3, 4, 5.)

In b), we are asked for any number greater than or equal to 2, so the answer is 2, 3, 4, and 5.

For c) we need numbers that are both greater than or equal to 2 *and* less than 5, giving 2, 3, 4.

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9. *Question and Solution*

How many different results can be found? Use + or – signs.

a) 2 $\boxed{+}$ 2 $\boxed{+}$ 1 = $\boxed{5}$

2 $\boxed{+}$ 2 $\boxed{-}$ 1 = $\boxed{3}$

2 $\boxed{-}$ 2 $\boxed{+}$ 1 = $\boxed{1}$

b) 3 $\boxed{+}$ 2 $\boxed{+}$ 1 = $\boxed{6}$

3 $\boxed{+}$ 2 $\boxed{-}$ 1 = $\boxed{4}$

3 $\boxed{-}$ 2 $\boxed{+}$ 1 = $\boxed{2}$

3 $\boxed{-}$ 2 $\boxed{-}$ 1 = $\boxed{0}$

(p46, Q3)

Notes

Again, encourage your students to work systematically to find the solution. Note that there is one possible missing combination,

$$2 \boxed{-} 2 \boxed{-} 1$$

but this leads to the answer $\boxed{-1}$. Discuss with your students if you think that they will understand the concept of negative numbers (you could use temperature as an example, or land/sea level).

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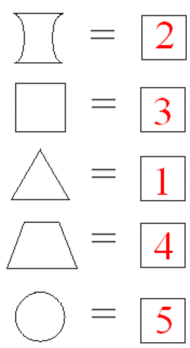
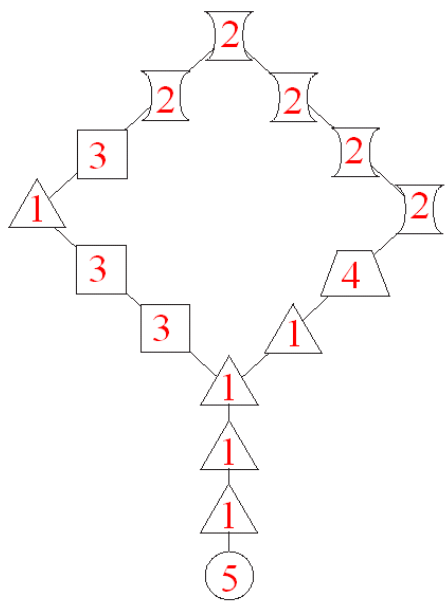
10. *Question and Solution*

Each shape represents a number.

The sum of the four numbers along each line must equal 8.

Do not use 0.

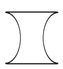




(p66, Q2)

Notes

The key to this question is to consider the possibilities when four numbers sum to 8. Note also that in the top RHS combination, all numbers are equal (same shape),

so  = 2.

Next, the bottom vertical set of numbers must be

$$1 + 1 + 1 + 5 = 8$$

(as no value apart from 1 will work for the △). Hence △ = 1 and ○ = 5.

The values of the other shapes are now easy to calculate; □ = 3 and ⏢ = 4.

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11. *Question and Solution*

What is the machine doing? Complete the table and write down the rule.

□	4	5	8	3	9	6	7	9	10
△	1	2	5	0	6	3	4	6	7

△ = $\boxed{□ - 3}$ □ = $\boxed{△ + 3}$

(p83, Q2)

Notes

This is another signpost for algebra. You first need to consider how to get from the □ row to the △ row.

You can see that 3 is subtracted in each column. So the machine takes away 3 from each of the numbers going into it. That is the straightforward part of the question.

We now need to write down this relationship, namely

△ = □ − 3

How do we get the □ values from the △ values? We add 3, so the relationship is

□ = △ + 3

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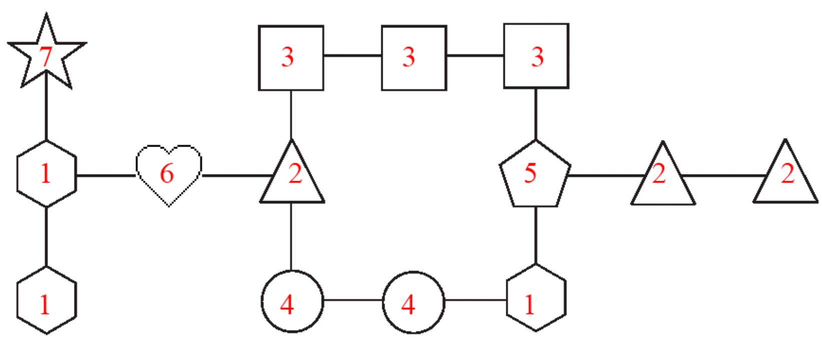
12. *Question and Solution*

Divide the number 9 into three parts. **Do not use 0.**

The same shape stands for the same number.



(p87, Q3)

Notes

There are many ways to solve this problem ranging from using logic to trial and improvement. The first stage is to note that the top horizontal lines give

$$3 + 3 + 3 \quad (= 9)$$

so □ $= 3$. The next stage is the crucial one. As there are three triples with 2 shapes the same, these must be $1 + 1 + 7$, $2 + 2 + 5$ and $4 + 4 + 1$ as there are no others. If you look at these digits you will see that the number 1 occurs three times. Now look at the three rows/columns that must represent these combinations of numbers. You can see that only the hexagon occurs three times, so

⬡ $= 1$ and the rest of the solution now follows.

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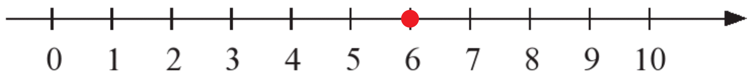
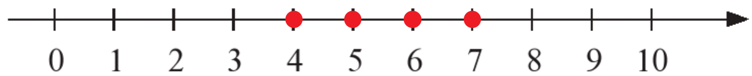
13. *Question and Solution*

Write down the answers. Mark them with dots on the number line.

a) 10 − 3 > ○ > 2 + 3 (7, 5) ○ : 6



b) 1 + 2 < △ < 9 − 1 (3, 8) △ : 4, 5, 6, 7



(p90, Q1)

Notes

The crucial first stage is to calculate both the RHS and LHS of each statement.

So, in a), $10 - 3 >$ ○ $> 2 + 3$ becomes $7 >$ ○ > 5 and hence ○ $= 6$ as it is the only number that satisfies this statement.

Similarly in b), $1 + 2 <$ △ $< 9 - 1$ becomes $3 <$ △ < 8 and hence △ $= 4, 5, 6, 7$ as they all satisfy the statement.

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14. *Question and Solution*

Make the statements correct by changing the place of one stick.

$\boxed{\text{V + IV = IX}}$ $\boxed{\text{X − VI = IV}}$ $\boxed{\text{VI + IV = X}}$

(p100, Q3)

Notes

As in a question 6, part a) can only be made correct by moving one vertical stick to change the 'minus' sign into a 'plus' sign, giving $\boxed{\text{V + IV = IX}}$.

Parts b) and c) are more straightforward.

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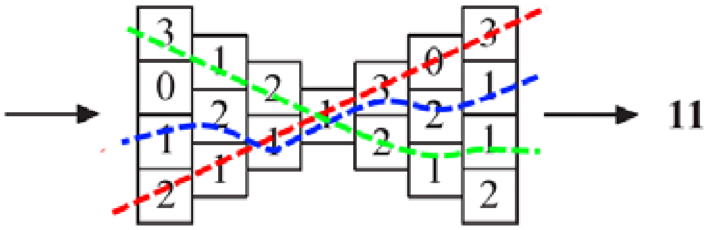
15. *Question and Solution*

Find ways through the maze so that the sum of the numbers used is 11.



(p110, q5)

Notes

Whilst you can use trial and improvement, a more systematic method is to consider the LHS and RHS. This gives the possible combinations,

LHS		RHS	
3 + 1 = 2	(= 6)	3 + 0 + 3	(= 6)
0 + 1 + 2	(= 3)	3 + 0 + 1	(= 4)
0 + 2 + 2	(= 4)	3 + 2 + 1	(= 6)
0 + 2 + 1	(= 3)	3 + 2 + 1	(= 6)
1 + 2 + 2	(= 5)	2 + 2 + 1	(= 5)
1 + 2 + 1	(= 4)	2 + 2 + 1	(= 5)
1 + 1 + 1	(= 3)	2 + 1 + 1	(= 4)
2 + 1 + 1	(= 4)	2 + 1 + 2	(= 5)

Noting that there is $\boxed{1}$ in the middle, we can look for combinations to give a total of 10, that is

6 + 4

5 + 5

4 + 6

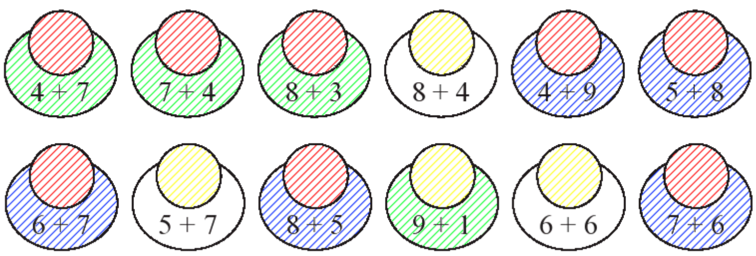
(a total of 3 on the RHS or LHS will not give a solution as there are no totals of 7). This will give a total of

$$\begin{array}{ccccccc} 2 & + & 2 & + & 9 & = & 13 \\ (6+4) & & (5+5) & & (3+6) & & \end{array}$$

different routes!

16. *Question and Solution*

Write the sums into the circles. Colour the shapes as shown.



(p117, Q2)

Notes

This is a straightforward question - but note that there are two totals of 12 $(8 + 4,\ 6 + 6)$ and these are not coloured as the instructions are for 'Greater than 12' and 'Smaller than 12', so 'exactly 12' should not be coloured.

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17. *Question and Solution*

Complete the table. Write down the rule in different ways.

$\boxed{a + b = 14}$ $\boxed{a = 14 - b}$ $\boxed{b = 14 - a}$

a	0	1	2	3	2	5	6	7	8	9	10	11	12	13	14
b	14	13	12	11	12	9	8	7	6	5	4	3	2	1	0

(p125, Q3)

Notes

Completion of the table for a and b is straightforward, being based on the number bonds for 14. The two equations are another staging point for later algebra. Try to get your students to devise the rule (e.g. you subtract b from 14) and then translate this to $a = 14 - b$. Similarly for $b = 14 - a$.

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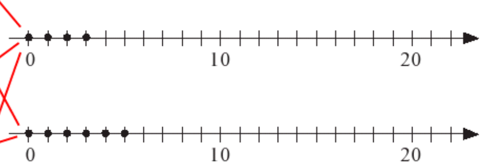
18. *Question and Solution*

Which numbers can be written instead of the letters so that the inequalities are correct? Join each solution to the matching number line.

$13 + p \leq 16$	p:	0, 1, 2, 3
$10 + a < 16$	a:	0, 1, 2, 3, 4, 5
$16 - r > 12$	r:	0, 1, 2, 3
$b + b < 11$	b:	0, 1, 2, 3, 4, 5
$s + 16 < 20$	s:	0, 1, 2, 3



(p140, Q2)

Notes

These questions look challenging, but one-by-one, there should not be problems. Here is the algebraic approach for the first inequality,

$$13 + p \leq 16$$

If you subtract 13 from both sides,

$$13 + p - 13 \leq 16 - 13$$

$$p \leq 3 \qquad (\text{as } 13 - 13 = 0,\ 16 - 13 = 3)$$

that is,

$$p = 0,\ 1,\ 2,\ 3$$

You can check these answers in the statement:

$$13 + 0 = 13 \leq 16$$
$$13 + 1 = 14 \leq 16$$
$$13 + 2 = 15 \leq 16$$
$$13 + 3 = 16 \leq 16$$

The second inequality follows in a similar way with

$$10 + a < 16$$

$$10 + a - 10 < 16 - 10$$

$$a < 6 \Rightarrow a = 0, 1, 2, 3, 4, 5$$

For the next inequality, start with

$$16 - 0 = 16 > 12$$

$r = 1 \Rightarrow 16 - 1 = 15 > 12$

$r = 2 \Rightarrow 16 - 2 = 14 > 12$

$r = 3 \Rightarrow 16 - 3 = 13 > 12$ but not satisfied by $r = 4, 5, ...,$ etc.

It is easy to try out numbers in the next inequality.
For example,

$b = 0$, then $0 + 0 < 11$ so $b = 0$ is a solution

$b = 1$, then $1 + 1 = 2 < 11$ so $b = 1$ is a solution

$b = 5$ $5 + 5 = 10 < 11$

$b = 5$ $5 + 5 = 10 < 11$

$b = 5$ $5 + 5 = 10 < 11$

$b = 5$, then $5 + 5 = 10 < 11$ so $b = 5$ is a solution

but not $b = 6\ (6 + 6 = 12 > 11)$.

So the full answer is $b = 0, 1, 2, 3, 4, 5$

In a similar way, for the final inequality, we try $s = 0, 1, ...$

giving

$0 + 16 = 16 < 20$ so $s = 0$ is a solution

$1 + 16 = 17 < 20$ so $s = 1$ is a solution

$2 + 16 = 18 < 20$ so $s = 2$ is a solution

$3 + 16 = 19 < 20$ so $s = 3$ is a solution

but $s = 4$ is not a correct solution as

$4 + 16 = 20$ (not less than 20)

The algebraic method gives

$$s + 16 < 20$$

so taking 16 from each side,

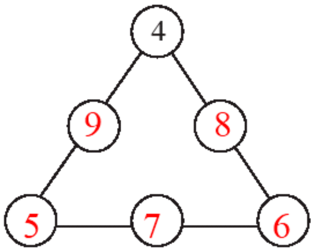
$$s + 16 - 16 < 20 - 16$$

$$s < 4 \qquad \text{(as } 16 - 16 = 0,\ 20 - 16 = 4\text{)}$$

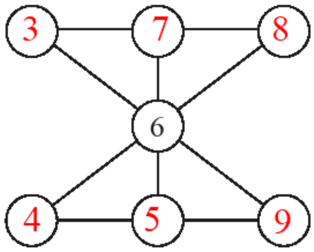
and this gives (as above) $s = 1, 2, 3.$

19. *Question and Solution*

Write the numbers in the correct places so that the sum of the 3 numbers on each line will be 18.



5
6
7
8
9



3
4
5
7
8
9

(p145, Q4)

Notes

In the LHS diagram, we first need to find pairs of numbers that sum to $18 - 4 = 14$; we have

9 and 5   and   8 and 6

This leaves 7 to be placed in the centre circle on the horizontal row, and the two other circles must sum to $18 - 7 = 11$. So 5 and 6 will go in these circles and 9 and 8 in the remaining two circles on the sides.

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20. *Question and Solution*

The same letter stands for the same number.

$$A + N + N + A = 20$$

Which number could each letter stand for? Write your answers in the table.

A	0	1	2	3	4	5	6	7	8	9	10				
N	10	9	8	7	6	5	4	3	2	1	0				

(p174, Q1)

Notes

Able students will recognise that this equation

$$A + N + N + A = 20$$

is in fact the same as

$$A + N = 10$$

(Algebraically, we can write the equation as

$$(A + A) + (N + N) = 20$$

$$2A + 2N = 20 \quad \text{as} \quad 2A = A + A, \text{ etc.}$$

Divide both sides by 2 to give

$$A + N = 10)$$

So there are just 11 distinct answers (as shown above) and the extra columns given are just a distraction!

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21. *Question and Solution*

I thought of a number. I added 8 to it then I took away 6 and got 5.

What is the number I first thought of?

5	+	6	-	8	=	3

(p175, Q4)

Notes

You can write the instruction as

$\boxed{} + 8$

$\boxed{} + 8 - 6 = \boxed{} + 2$

$\boxed{} + 2 = 5$

So clearly, $\boxed{} = 3$

Another approach is to start at 5 and do the operations in reverse:

$5 + 6 = 11$

$11 - 3 = 8$

So the starting value is 3.