Mathematics Enhancement Programme

TEACHING SUPPORT: Year 2

SOLUTIONS TO EXERCISES

1. *Question and Solution*

Share the marbles **equally** between 2 children. Complete the table.

Marbles	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Per child	2	2	3	3	4	4	5	5	6	6	7	7	8	8	9	9	10
Left over	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0

(p10, Q5)

Notes

This is an introduction to division by 2. It is important to discuss with students

- the patterns in the second and third rows
- the patterns for even and odd numbers (even numbers have nothing left over; odd numbers have 1 marble left over).

For students who find this difficult, it might be helpful for them to work in pairs using actual marbles, completing the table from their practical experience.

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2. *Question and Solution*

Complete the table. Write down the rule in different ways.

a	4	5	2	1	3	4	2	1	8	8	1	2	0	0*
b	3	5	1	6	5	2	2	3	1	1	9	2	10	5
c	3	0	7	3	2	4	6	6	1	1	0	6	0	5

(p16, Q2)

Notes

The first four columns give the clue to the 'rule' that is being used. You could make suggestions to the class if they do not quickly see the rule; for example, they could add up each column. Hopefully they will deduce the rule and complete the table.

The crucial part of this question, though, is writing the rule; in its simplest form this is

$$a + b + c = 10$$

but you could also have

$$a = 10 - (b + c) \quad \text{or} \quad b = 10 - (a + c), \text{ etc.}$$

$$a + b = 10 - c, \quad \text{etc.}$$

* Note that the final column has 11 possible answers:

0	0	0	0	0	0	0	0	0	0	0
0	1	2	3	4	5	6	7	8	9	10
10	9	8	7	6	5	4	3	2	1	0

This could be a discussion point with the class review.

3. *Question and Solution*

Colour the flags in different ways, using red, white and green.

On each flag, a colour may be used once and only once.

(p17, Q3)

Notes

It is important to encourage your students to be systematic rather than to guess.

For example, if you choose the first colour as RED, then there is a choice of WHITE or GREEN, giving two possibilities.

Similarly, starting with WHITE, and starting with GREEN, giving SIX possibilities, namely

R	R	W	W	G	G
W	G	R	G	R	W
G	W	G	R	W	R

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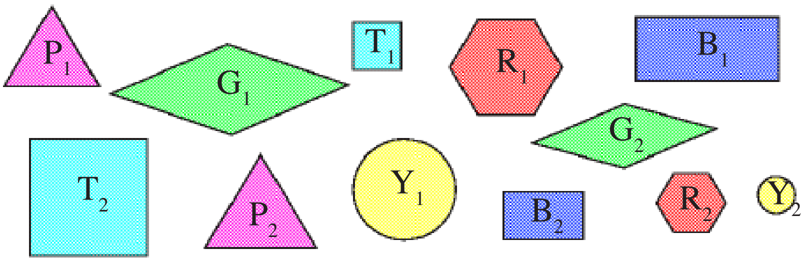
4. *Question and Solution*

Colour similar pairs of shapes in the same colour.



(p19, Q2)

Notes

This is a basic question aimed at ensuring that students understand similarity. As can be seen, the similar shapes are not identical but have the same characteristics; for example, T_1 and T_2 are both squares; Y_1 and Y_2 are both circles, etc.

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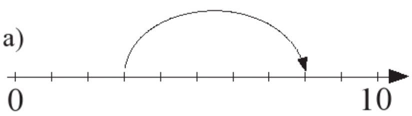
5. *Question and Solution*

Write equations and inequalities about each jump along the number line.

a)



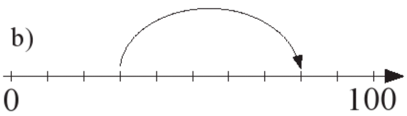
$3 <_5 8$

$3 + 5 = 8$

$8 \;{}_5> 3$

$8 - 5 = 3$

b)



$30 <_{50} 80$

$30 + 50 = 80$

$80 \;{}_{50}> 30$

$80 - 50 = 30$

(p34, Q2)

Notes

Students should write in the numbers at the points marked on each line, i.e.

a) 1, 2, 3, 4, 5, 6, 7, 8, 9

b) 20, 30, 40, 50, 60, 70, 80, 90

This question reinforces the different ways of showing an equation/inequality to illustrate this jump.

Note also that the statement in part b) is essentially the same as in part a) except that 3 is replaced by 30, 5 by 50 and 8 by 80, as the jump is now from 30 to 80 rather than 3 to 8.

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6. *Question and Solution*

List the numbers which make the statement true.

a) $40 < \square < 47$ $\square$: 41, 42, 43, 44, 45, 46

b) $30 + 20 < \bigcirc < 10 + 50$ $\bigcirc$: 51, 52, 53, 54, 55, 56, 57, 58, 59

(p36, Q5)

Notes

Here a) is straightforward but in b), students must first calculate each side of the inequality (see solution below).

a) $\square$ can be 41, 42, 43, 44, 45 or 46

b) $30 + 20 < \bigcirc < 10 + 50$

$50 < \bigcirc < 60$

So $\bigcirc$ can be 51, 52, 53, 54, 55, 56, 57, 58 or 59.

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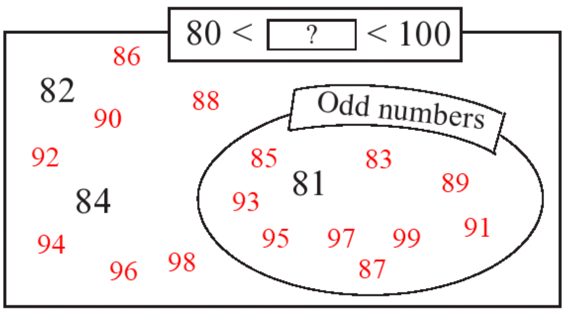
7. *Question and Solution*

Which numbers make the inequality true?

Write them in the correct places on the diagram.



(p37, Q1)

Notes

This diagram is known as a Venn diagram; these are very useful ways of illustrating different sets of numbers. (The Venn diagram is named after the Rev John Venn who introduced them in a paper published in 1880.) The interior of the oval part of the diagram symbolically represents the elements of the set, while the exterior represents elements that are not members of the set.

The numbers that must be put in the Venn diagram are given by

$$80 < \ ? \ < 100$$

which means 81, 82, 83, . . . , 98, 99 (? is a whole number greater than 80 and less than 100.)

All the odd numbers, that is,

81, 83, 85, 87, 89, 91, 93, 95, 97, 99

are placed *inside* the oval shape as they are odd numbers between 80 and 100.

The remaining (even) numbers,

82, 84, 86, 88, 90, 92, 94, 96, 98

are placed *outside* the oval.

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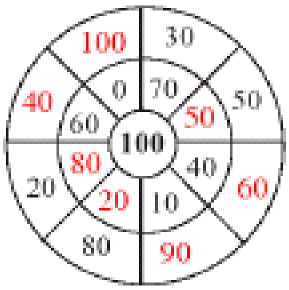
8. *Question and Solution*

Fill in the missing numbers.



(p43, Q1)

Notes

The RULE for the first diagram is

'outer number' + 'inner number' = 100

This leads to the answers above.

For the second diagram, we could use the RULE

'outer number' + 'inner number' = 50

which gives many possible answers, for example,

25 + 25 = 50, 10 + 40 = 50, 0 + 50 = 50, 20 + 30 = 50, etc.

Enterprising students might, though, choose a different RULE; for example

'outer number' - 'inner number' = 50

giving

60 – 10 = 50, 100 – 50 = 50, etc.

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9. *Question and Solution*

The same shape means the same number. Write the numbers in each shape.

a) $\boxed{20} + \boxed{20} + \boxed{20} = 90 - 30$

b) $\boxed{50} + \boxed{50} = 60 + 20 + 20$

c) $\boxed{30} + \boxed{30} + 10 = 100 - 30$

d) $30 + \boxed{30} = 90 - \boxed{30}$

(p49, Q3)

Notes

This question involves the beginnings of algebra, using shapes for the unknowns instead of x, which will be introduced later.

a) $\underbrace{\boxed{} + \boxed{} + \boxed{}}_{\text{all equal}} = 60 \quad \Rightarrow 3 \text{ lots of } \boxed{} = 60$

$\Rightarrow \boxed{} = 60 \div 3 = 20$

(*Check*: $20 + 20 + 20 = 90 - 30$ i.e. $60 = 60$)

b) $\boxed{} + \boxed{} = 60 + 20 + 20 = 100 \quad \Rightarrow 2 \text{ lots of } \boxed{} = 100$

$\boxed{} = 100 \div 2 = 50$

(*Check*: $50 + 50 = 60 + 20 + 20$ i.e. $100 = 100$)

c) $\boxed{} + \boxed{} + 10 = 100 - 30$

$\boxed{} + \boxed{} + 10 = 70$

$\boxed{} + \boxed{} + 10 = 60 + 10$

$\boxed{} + \boxed{} = 60$

2 lots of $\boxed{} = 60 \quad \Rightarrow \boxed{} \div 2 = 30$

(*Check*: $30 + 30 + 10 = 100 - 30$ i.e. $70 = 70$)

d) $30 + \boxed{} = 90 - \boxed{}$

$= 30 + 60 - \boxed{}$

So $\boxed{} = 60 - \boxed{}$

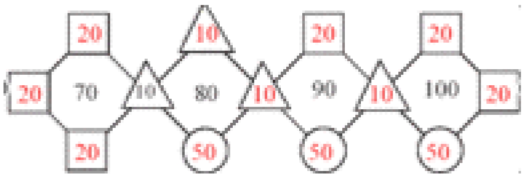
$= 30 + 30 - \boxed{}$

and $\boxed{} = 30$ satisfies this.

10. *Question and Solution*

The same shape means the same number.

The sum of the 4 numbers at the corners equals the middle number.



(p55, Q5)

Notes

Again, this is really an algebraic problem but we can use logic to determine the solution.

Considering the '70' on the LHS,

$$\square + \square + \square + 10 = 70$$

$$\square + \square + \square = 60$$

$$3 \text{ lots of } \square = 60 \Rightarrow \square = 60 \div 3 \Rightarrow \square = 20$$

For the '80', we now have,

$$10 + \triangle + \triangle + \bigcirc = 80$$

$$10 + 10 + 10 + \bigcirc = 80$$

$$30 + \bigcirc = 80 \Rightarrow \bigcirc = 50$$

Now check the '90',

$$\triangle + \square + \triangle + \bigcirc = 90$$

$$10 + 20 + 10 + 50 = 90$$

$$90 = 90$$

Finally we check the '100',

$$\triangle + \square + \square + \bigcirc = 100$$

$$10 + 20 + 20 + 50 = 100$$

$$100 = 100$$

So we have the solution,

$$\triangle = 10, \quad \square = 20, \quad \bigcirc = 50$$

11. *Question and Solution*

I thought of a number. I multiplied it by 3, then divided by 6 and got 2.

What was the number I first thought of?

$\boxed{4}$

(p102, Q4)

Notes

We can deduce the answer by working backwards from the final answer, 2, and using the 'reverse' operations; that is,

$$2 \times 6 = 12$$

$$12 \div 3 = 4$$

So the answer is 4.

(*Check*: $4 \times 3 = 12$, $12 \div 6 = 2$)

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12. *Question and Solution*

Find a rule.
Complete the table.

Write the rule in different ways.

■	2	7	12	8	9	9	3	11	8	5	10	1
▲	4	5	2	3	10	2	6	3	5	6	7	9
●	8	35	24	24	90	18	18	33	40	30	70	9

● = ■ × ▲ ▲ = ● ÷ ■ ■ = ● ÷ ▲

(p104, Q4)

Notes

There are two stages to this problem; firstly completing the table and secondly, writing the RULE in different ways.

To complete the table, we must find the RULE. Most students will recognise this as

first row × second row = final row

For example, $2 \times 4 = 8$; $7 \times 5 = 35$.

Once completed, we can write the rule as

● = ■ × ▲

and

▲ = ● ÷ ■ (*Check*: $4 = 8 \div 2$)

■ = ● ÷ ▲ (*Check*: $2 = 8 \div 4$)

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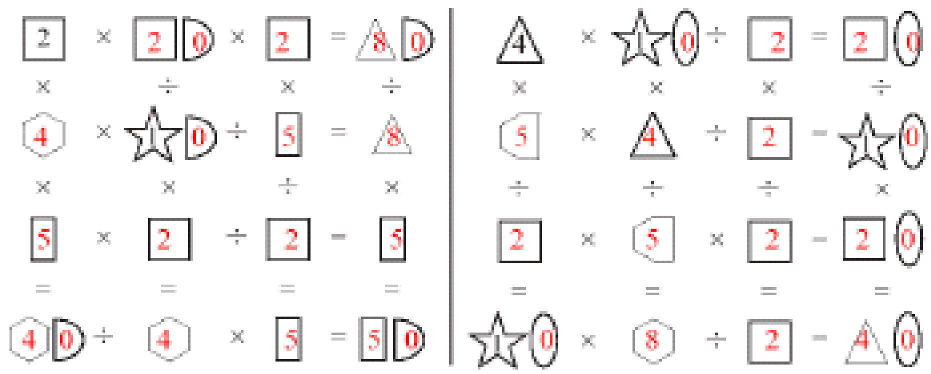
13. *Question and Solution*

In each part, the same shape stands for the same digit. Fill in the digits.

a) b)



(p105, Q1)

Notes

To start part a), write 2 in each □.

Looking at the first row and taking D = 0 as the simplest possible answer, gives

$$2 \times 20 \times 2 = 80 \quad \Rightarrow \quad \triangle = 8$$

The second row gives,

$$⬡ \times 10 \div ▯ = 8$$

The simplest answer is ⬡ = 4 and ▯ = 5 (*Check*: $4 \times 10 \div 5 = 8$)

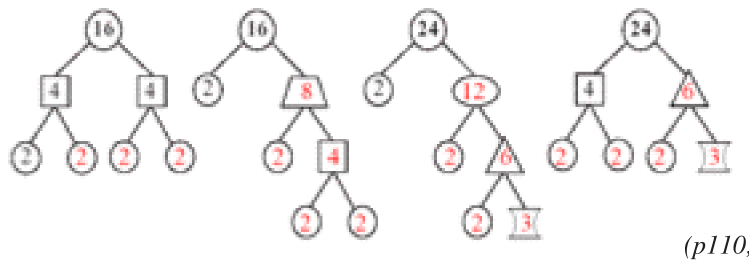
Now write in all the numbers and check all the other rows and columns.

A similar technique can be used for part b).

14. *Question and Solution*

Break down the numbers into their factors.

Write each as a multiplication.



(p110, Q2)

Notes

For the (16) tree diagrams, ○ = 2, □ = 4, ⏢ = 8

and $16 = 2 \times 2 \times 2 \times 2$ (for both equations).

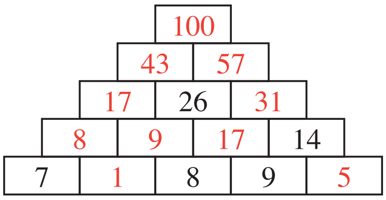
For the (24) tree diagrams, ○ = 2, ⬭ = 12, △ = 6,)(= 3

and $24 = 2 \times 2 \times 2 \times 3$ (for both equations).

15. *Question and Solution*

Each number is the **sum** of the 2 number directly below it.

Fill in the missing numbers.



(p112, Q2)

Notes

Start with the bottom right hand space where the missing number can be easily calculated as 5 $(14 = 9 + 5)$. Also, $8 + 9 = 17$, so 17 can be inserted.

Next, $26 - 17 = 9$ and 9 is written to the left of 17; $9 - 8 = 1$, so 1 can be inserted on the bottom row.

As $7 + 1 = 8$, 8 can be inserted on the second to bottom row. This row is now complete.

For the LHS of the middle row, $8 + 9 = 17$ so 17 is inserted.

For the RHS of the middle row, $17 + 14 = 31$ so 31 is inserted.

In a similar way, the next row is given by $17 + 26 = 43$ and $26 + 31 = 57$.

Finally, the topmost number is given by $43 + 57 = 100$.

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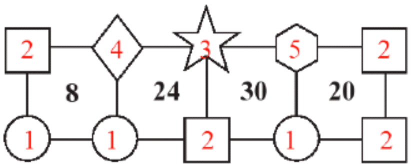
16. *Question and Solution*

The same shape means the same number. Choose from 1, 2, 3, 4 or 5.

The middle number is the **product** of the 4 numbers around it.

Fill in the missing numbers.



(p115, Q4)

Notes

The method here is to use the factors of the numbers given. (A factor is a whole number which divides exactly into a whole number, leaving no remainder.)

Starting with the '20', there are only two ways of writing the factors with two the same, that is,

$$20 = 2 \times 2 \times 5 \times 1 \quad \text{or} \quad 20 = 1 \times 1 \times 5 \times 4$$

But if □ = 1, then ⬡ and ○ = 5 and 4, but this would not work in the first '8'.

Hence we take

□ = 2

and in the '8', we use $8 = 2 \times 4 \times 1 \times 1$, giving

○ = 1 and = 4

Now consider the '24', using $24 = 4 \times 1 \times 2 \times 3$.

Clearly,

 = 3

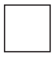
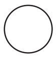
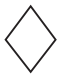
and for '30', using $30 = 3 \times 2 \times 1 \times 5$, so

 = 5

Finally we can check the '20' with

$$20 = 5 \times 1 \times 2 \times 2$$

So we have the solution,

 = 2,  = 1,  = 4,  = 3 and  = 5

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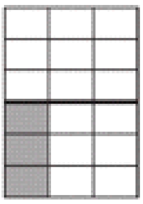
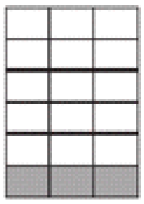
17. *Question and Solution*

Compare the shaded parts. Which is more? Write in the correct sign.

 $\boxed{=}$ 

1 third of a half **1 half** of a third

(p144, Q4)

Notes

Note that here

1 unit $\equiv$ 18 rectangles so 1 half $=$ 9 rectangles and 1 third of this $=$ 3 rectangles (the LHS diagram).

Similarly,

1 third $\equiv 18 \div 3 = 6$ rectangles and 1 half of this $=$ 3 rectangles (the RHS diagram).

So the two parts shaded are equal and the ' = ' sign should be written in the box.

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18. *Question and Solution*

Use the digits 1, 2, 3 and 4 to make pairs of 2-digit numbers.

Each digit can be used only once in every pair, but can be in any order.

An example of such a pair is: 21 and 34.

a) Which pairs have the largest **sum**? $\boxed{3\,2} + \boxed{4\,1} = \boxed{7\,3}$

and $\boxed{3\,1} + \boxed{4\,2} = \boxed{7\,3}$

b) Which pairs have the smallest difference? $\boxed{3\,1} - \boxed{2\,4} = \boxed{\,7}$

and $\boxed{\,} - \boxed{\,} = \boxed{\,}$

(p150, Q4)

Notes

a) Largest sum $41 + 32 = 73$

or $42 + 31 = 73$

b) Smallest difference $31 - 24 = 7$ (the extra line of boxes is a Hungarian-style distraction!)

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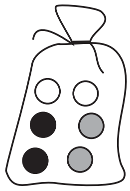
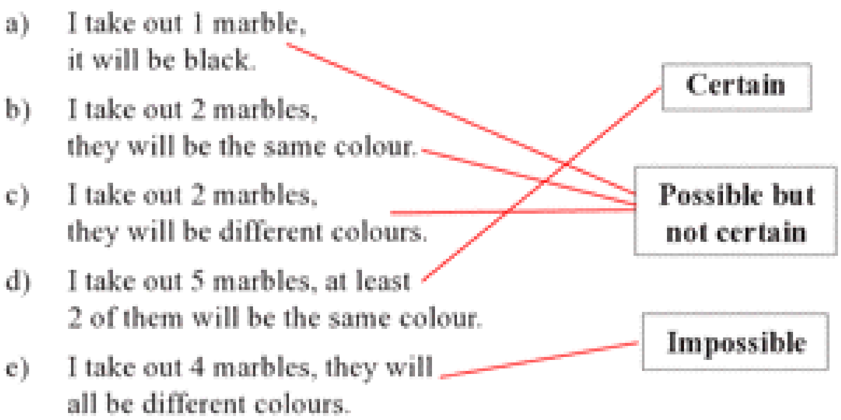
19. *Question and Solution*



There are 2 white, 2 black and 2 striped marbles in a bag.
The bag is tied with cord and you cannot see inside.

Join up the statements on the left to the labels on the right.

How certain can I be that if, with my eyes shut:



a) I take out 1 marble, it will be black.

b) I take out 2 marbles, they will be the same colour.

c) I take out 2 marbles, they will be different colours.

d) I take out 5 marbles, at least 2 of them will be the same colour.

e) I take out 4 marbles, they will all be different colours.

Certain

Possible but not certain

Impossible

(p154, Q1)

Notes

This question introduces your students to the beginnings of probability. They have to read the statements carefully and decide which of

Certain | Possible but not certain | Impossible

is most appropriate in each case.

a) This is Possible but not certain as there are also white and striped marbles.

b) Again, this is Possible but not certain as I could take out 2 WHITE or 2 BLACK or 2 STRIPED marbles but also 1 BLACK and 1 WHITE.

c) This is also Possible but not certain as explained in b).

d) As there are only 3 colours, if I take out 5 marbles I am certain to have two of the same colour (in fact I will have 2 colour repeats). Here this event is Certain.

e) The marbles taken out cannot all be of different colours as there are only 3 colours and I am taking out 4 marbles. Hence the event is Impossible.

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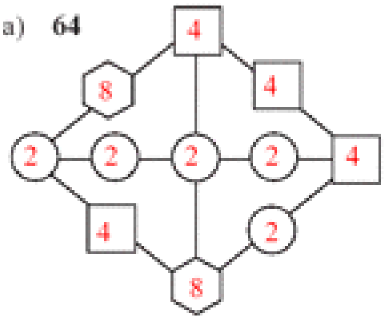
20. *Question and Solution*

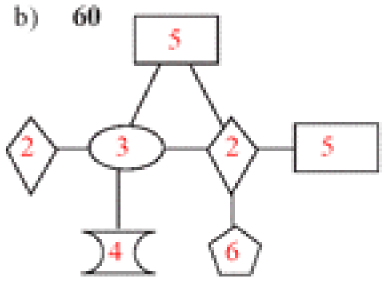
The same shape stands for the same 1-digit number greater than 1.

Fill in the numbers if the **product** of the numbers along each line equals:

a) 64



b) 60



(p170, Q3)

Notes

This question can be answered using logic and rigour.

a) The □ must be 4 as only $4 \times 4 \times 4 = 64$ (topmost diagonal on RHS)

It is also clear that ○ × ○ × ○ × ○ × 4 = 64.

○ × ○ × ○ × ○ = 64 ÷ 4 = 16

So ○ = 2 and we can see that ⬡ = 8.

b) As ◇ is repeated is must be 2, giving ⬭ and □ as 3 and 5.

In fact, we can determine the solutions that work, namely

◇ = 2, ⬭ = 3, □ = 5, ⟩⟨ = 4, ⬠ = 6

or

◇ = 2, ⬭ = 5, □ = 3, ⟩⟨ = 4 but ⬠ = 10