Mathematics Enhancement Programme

TEACHING SUPPORT: Year 3

SOLUTIONS TO EXERCISES

1. *Question and Solution*

Write the operations **without** brackets if possible so that the result is the same.

Do the calculations as a check.

a)	$(2 + 8) \times 7$	$= 2 \times 7 + 8 \times 7$	$= 70$
b)	$(11 - 3) \times 9$	$= 11 \times 9 - 3 \times 9$	$= 72$
c)	$(21 + 14) \div 7$	$= 21 \div 7 + 14 \div 7$	$= 5$
d)	$(24 - 8) \div 4$	$= 24 \div 4 - 8 \div 4$	$= 4$
e)	$80 \div (12 - 4)$	= Not possible	$= 10$
f)	$72 \div (3 + 6)$	$= 72 \div 3 + 72 \div 6$	$= 8$

(p18, Q3)

Notes

The first four questions, a) to d), are straightforward; for example,

d) $(24 - 8) \div 4 = \boxed{24 \div 4 - 8 \div 4} = 6 - 2 = 4$

This is correct as, if we first calculate $24 - 8$, we get

$$(24 - 8) \div 4 = 16 \div 4 = 4$$

Parts e) and f) cannot be written with brackets in this way.

This is easy to see in f), where

$$72 \div (3 + 6) = 72 \div 9 = 8$$

but

$$72 \div (3 + 6) \text{ is not equal to } 72 \div 3 + 72 \div 6 \ (= 24 + 12 = 36)$$

Your students will learn later about the BODMAS (or BIDMAS) convention which states that in calculations, the order used to complete operations is

- **Brackets**
- **Order** (powers or roots, e.g. $2^2 = 2 \times 2$) *or* **Indices**
- **Divide** or **Multiply**
- **Add** or **Subtract**

2. *Question and Solution*

Fill in the missing numbers so that the equations are true, both horizontally and vertically.

3	×	12	÷	9	= 4
×		÷		×	
3	×	3	×	2	= 18
×		×		÷	
3	×	4	÷	2	= 6
= 27		= 16		= 9	

(p19, Q4)

Notes

At this stage, we expect students to use some known factor facts; for example,

$$27 = 1 \times 1 \times 27 = 1 \times 3 \times 9 = 3 \times 3 \times 3$$

$$18 = 1 \times 1 \times 18 = 1 \times 2 \times 9 = 3 \times 2 \times 3 = 1 \times 3 \times 6$$

As the column for 27 (first column) and the row for 18 (second row) have one number in common, it could be 1, 3 or 9.

There are in fact a number of possible answers, but you could add in the constraint that,

"1 is not allowed as a factor."

This would mean that the first column would be $3 \times 3 \times 3$ and the third row would be $3 \times 2 \times 3$ or $3 \times 3 \times 2$.

Using trial and error now gives the two possible answers:

3	X	8	÷	6	= 4
X		÷		X	
3	X	2	X	3	= 18
X		X		÷	
3	X	4	÷	2	= 6
= 27		= 16		= 9	

3	X	12	÷	9	= 4
X		÷		X	
3	X	3	X	2	= 18
X		X		÷	
3	X	4	÷	2	= 6
= 27		= 16		= 9	

3. *Question and Solution*

I thought of a number. I divided it by 7 and the result was 8, remainder 6.
What is the number I was thinking of?

Calculation: $7 \times 8 + 6 = 62$

Check: $62 \div 7 = 8$, remainder 6 *Answer:* 62

(p20, Q4)

Notes

The straightforward way here is to call the missing number x.
Then the calculation is

$$x \div 7 = 8 \text{ remainder } 6$$

So

$$\begin{aligned} x &= 7 \times 8 + 6 \\ &= 56 + 6 \\ &= 62 \end{aligned}$$

4. *Question and Solution*

Which different 1-digit numbers could a, b and c be if $a + b + c = 14$ and $a \times b \times c = 84$?

$a = \boxed{3}$ $b = \boxed{4}$

$c = \boxed{7}$

(p35, Q5)

Notes

One approach would be to consider values of a, then values of b and c for each value of a; for example,

			$a \times b \times c$	
$a = 1$	$\Rightarrow b + c = 13$	$\Rightarrow b = 4, c = 9$	$1 \times 4 \times 9 = 36$	X
		$b = 5, c = 8$	$1 \times 5 \times 8 = 40$	X
		$b = 6, c = 7$	$1 \times 6 \times 7 = 42$	X
		$b = 7, c = 6$	$1 \times 7 \times 6 = 42$	X
		$b = 8, c = 5$	$1 \times 8 \times 5 = 40$	X
		$b = 9, c = 4$	$1 \times 9 \times 4 = 36$	X

So there are no solutions with $a = 1$.

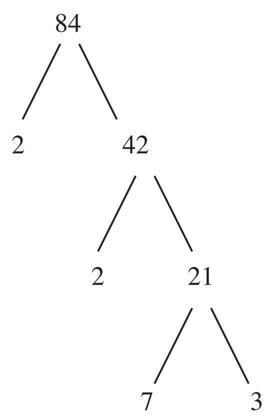
In a similar way, $a = 2$ gives no solutions, but when

$a = 3$	$\Rightarrow b + c = 11$	$\Rightarrow b = 2, c = 9$	$3 \times 2 \times 9 = 54$	X
		$b = 3, c = 8$	$3 \times 3 \times 8 = 72$	X
		$b = 4, c = 7$	$3 \times 4 \times 7 = 84$	✓

So one solution is $a = 3$, $b = 4$ and $c = 7$.

To find any more answers, we would have to continue in this way.

A more efficient approach would be to find the factors of 84.



So $84 = 2 \times 2 \times 7 \times 3$ and this gives the solutions

$$4 \times 7 \times 3 \quad \text{or} \quad 2 \times 7 \times 6$$

Hence there are six solutions:

a	b	c	
4	7	3	
4	3	7	
7	3	4	based on $4 \times 7 \times 3$
7	4	3	
3	7	4	
4	7	3	
—	—	—	
2	7	6	
2	6	7	
7	2	6	based on $2 \times 7 \times 6$
7	6	2	
6	2	7	
6	7	2	

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5. *Question and Solution*

How many different results can you find? Use +, –, or × signs.

70	+	10	+	3 =	83		70	×	10	–	3 =	697
70	+	10	–	3 =	77		70	+	10	×	3 =	100
70	–	10	+	3 =	63		70	–	10	×	3 =	40
70	–	10	–	3 =	57		70	×	10	×	3 =	2100
70	×	10	+	3 =	703		70		10		3 =	

(p45, Q3)

Notes

Note that, with three signs there will be only 9 possibilities (so one of the options remains blank).

The conventional method is to use BODMAS:

$$70 + (10 \times 3) = 70 + 30 = 100$$

but if students haven't yet met this, calculating from the left hand side will give

$$70 + 10 = 80 \text{ and } 80 \times 3 = 240$$

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6. *Question and Solution*

Two different numbers can be **rounded** to 70 as the nearest whole ten.

a) Is it possible that both numbers are less than 70?

yes (e.g. 65, 66)

b) Is it possible that one of the numbers is 10 less than the other?

no

c) Is it possible that one of them has 5 and the other has 0 as the units digits?

yes (65 and 70)

d) Is it possible that both numbers are whole tens?

no

(p49, Q4)

Notes

a) As these numbers all round to 70 to the nearest 10, any two of them would be a correct answer.

b) The full list of numbers is

65, 66, 67, 68, 69, 70, 71, 72, 73 and 74

so the greatest difference is $74 - 65 = 9$.

c) 65 and 70 is the only solution.

d) Only 70 is a whole ten.

Note that it is a convention that 65 rounds up to 70, to the nearest 10. 65 is in fact exactly half way between 60 and 70. This 'round up the middle number' rule applies for all rounding of numbers.

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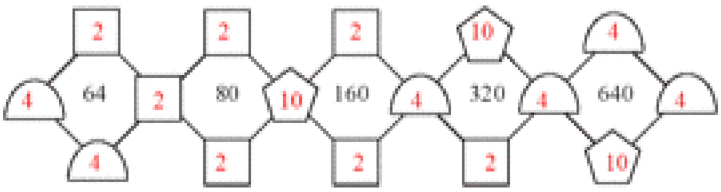
7. *Question and Solution*

The middle number is the **product** of the 4 numbers around it.



(p64, Q4)

Notes

It is tempting to guess a few numbers and see if they work, but the easiest *logical* method is to consider the last set of four numbers. Here

$$640 = a \times a \times a \times b \quad \text{where} \quad a = \text{◠} \quad \text{and} \quad b = \text{⬠}$$

Now, factorising,

$$\begin{aligned} 640 &= 64 \times 10 \\ &= 8 \times 8 \times 10 \\ &= (2 \times 2 \times 2) \times (2 \times 2 \times 2) \times (2 \times 10) \end{aligned}$$

and to put it in the form above, we could have

$$640 = 4 \times 4 \times 4 \times 10 \qquad \text{(A)}$$

or

$$640 = 2 \times 2 \times 2 \times 80 \qquad \text{(B)}$$

or

$$640 = 1 \times 1 \times 1 \times 640 \qquad \text{(C)}$$

It is clear that (C) does not work as having ⬠ $= 640$ would contradict the four numbers around 80 or 160.

Now consider (B); here ⬠ $= 80$ and ◠ $= 2$. Working on the right hand set of numbers, we would have

$$2 \times 2 \times \square \times \square = 64$$

or

$$\square \times \square = 16$$

So

$$\square = 4$$

In the next set of 4 numbers,

$$4 \times 4 \times 4 = 80 \text{ should equal } 80$$

but this is not true. Hence (B) does not work.

Thus (C) is the correct solution and we can see that

$$◠ = 4, \quad ⬠ = 10, \quad \square = 2$$

satisfies each set of numbers in the diagram.

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8. *Question and Solution*

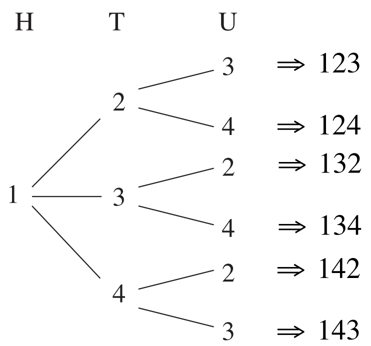
Create as many different 3-digit numbers as you can from the digits 1, 2, 3 and 4.
Do not use a digit more than once in any number.

123	234	124	134
213	243	142	143
312	324	214	314
132	342	241	341
231	432	412	413
321	423	421	431

(p67, Q2)

Notes

In this question, we need to be logical and systematic. Start with the first digit:



So there are 6 3-digit numbers starting with 1.

Exactly the same holds for numbers starting with 2, 3 and 4. Hence there are

$$6 + 6 + 6 + 6 = 4 \times 6 = 24$$

different 3-digit numbers using each of the digits 1, 2, 3 and 4 not more than once.

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9. *Question and Solution*

Continue the sequences.

a) 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 . . .

b) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 . . .

c) 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 . .

d) 1, 3, 6, 10, 15, 21, 28, 36, 45, 55 . . . .

(p74, Q4)

Notes

For sequences, we need to find the 'rule' on which they are based. So here we have

a) $1, \quad 2 = 2 \times 1, \quad 4 = 2 \times 2, \quad 8 = 2 \times 4, \quad 16 = 2 \times 8$

and the rule is

$$\boxed{\text{next number} = 2 \times \text{previous number}}$$

Using this rule, we obtain

$$2 \times 16 = 32, \quad 2 \times 32 = 64, \quad 2 \times 64 = 128, \text{ etc.}$$

b) $1 = 1 \times 1, \quad 4 = 2 \times 2, \quad 9 = 3 \times 3, \quad 16 = 4 \times 4, \quad 25 = 5 \times 5$

The rule is

$$\boxed{n \times n}$$

when n is 1, 2, 3, 4, 5, 6, 7, ... (The product of a number which has been multiplied by itself is called a *square number*.)

Thus the next terms are

$$6 \times 6 = 36, \quad 7 \times 7 = 49, \quad 8 \times 8 = 64, \quad 9 \times 9 = 81, \quad 10 \times 10 = 100, \quad \ldots$$

c) This is not so straightforward. If we look carefully, though, we can see that

$$\begin{aligned} 0 + 1 &= 1 \\ 1 + 1 &= 2 \\ 1 + 2 &= 3 \\ 2 + 3 &= 5 \\ 3 + 5 &= 8 \end{aligned}$$

that is,

$$\boxed{\text{next number} = \text{sum of previous two numbers in the sequence}}$$

This is known as a *Fibonacci sequence*. There are numerous examples of Fibonacci sequences in nature including the breeding pattern for rabbits (and also mice) in ideal conditions.

The next terms are

$$5 + 8 = 13, \quad 8 + 13 = 21, \quad 13 + 21 = 43, \quad \text{etc.}$$

Teaching point

The number of petals in a flower often follows the Fibonacci sequence (there are very few flowers with 4 petals; 4-leaf clovers are considered lucky as they are very rare!)

d) Again we need to look carefully and consider the difference in each term:

Sequence:	1		3		6		10		15
Difference:		2		3		4		5	

Now we can see that the rule is

$$\boxed{\text{differences increase by one}}$$

and hence the next terms are

$$15 + 6 = 21, \quad 21 + 7 = 28, \quad 28 + 8 = 36, \quad \text{etc.}$$

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10. *Question and Solution*

Fill in the missing digits.

i)

		3	2	5
+	1	2	4	3
	1	5	6	8

ii)

	1	3	5
+	9	1	3
1	0	4	8

iii)

	5	3	9
+	8	0	1
1	3	4	0

iv)

		5	0	7	
+	1	9	8	8	
	2	6	9	5	*

v)

	9	7	8	
+	3	6	1	
1	3	3	9	*

* Possible answer

(p94, Q3)

Notes

a) Each one of these can be deduced from the logic of calculation; for example,

iii)

		5	3	___
	+	___	___	1
	___	3	4	0

First note that in the sum the THOUSAND digit can only be 1.

	5	3	___
+	___	___	1
1	3	4	0

Working from the right hand side, the missing UNIT digit has to be 9.

$$\begin{array}{cccc} & 5 & {}^{1}3 & \mathbf{9} \\ + & \_\_ & \_\_ & 1 \\ \hline 1 & 3 & 4 & 0 \\ \hline \end{array}$$

Now we can see that the missing TENS digit has to be 0.

$$\begin{array}{cccc} & 5 & {}^{1}3 & \mathbf{9} \\ + & \_\_ & \mathbf{0} & 1 \\ \hline \mathbf{1} & 3 & 4 & 0 \\ \hline \end{array}$$

Finally, the HUNDREDS digit has to be 8 (as we need to carry 1 into the thousands column).

$$\begin{array}{cccc} {}^{1}0 & 5 & {}^{1}3 & 9 \\ + & \mathbf{8} & \mathbf{0} & 1 \\ \hline \mathbf{1} & 3 & 4 & 0 \\ \hline \end{array}$$

b) This is more of a challenge but able students will not only enjoy this but will be able to find many solutions. For example, we can see that,

1. The missing THOUSAND digit in the sum has to be 1.
2. Check whether you can solve the puzzle with only the HUNDREDS digit 'crossing ten'; for example, $6 + 4 = 10$.
3. We could look now for combinations from the remaining digits, 2, 3, 5, 7, 8, 9, that do *not* cross 10; for example, $2 + 7 = 9$, $5 + 3 = 8$.
4. We now have many solutions; for example,

$$\begin{array}{cccc} & 6 & 2 & 5 \\ + & 4 & 7 & 3 \\ \hline 1 & 0 & 9 & 8 \\ \hline \end{array} \quad \text{or} \quad \begin{array}{cccc} & 6 & 2 & 3 \\ + & 4 & 7 & 5 \\ \hline 1 & 0 & 9 & 8 \\ \hline \end{array}$$

(How many solutions can you find that take this form?)

11. *Question and Solution*

Which is more? How many more? Write subtractions and inequalities.

a) The smallest 4-digit number compared with the greatest 3-digit number.
 1000 > 999 1000 – 999 = 1

b) The smallest 4-digit number compared with the smallest 3-digit number.
 1000 > 100 1000 – 100 = 900

c) The smallest 4-digit number compared with the smallest 2-digit number.
 1000 > 10 1000 – 10 = 990

d) The greatest 3-digit whole ten compared with the greatest 3-digit hundred.
 990 > 900 990 – 900 = 90

e) The smallest 4-digit hundred compared with the smallest 4-digit whole ten.
 1000 = 1000 1000 – 1000 = 0

f) The smallest whole hundred compared with the smallest whole ten.
 100 > 10 100 – 10 = 90

(p97, Q3)

Notes

This tests a number of concepts, especially place value and understanding of greatest, least, etc.

a)	1000 – 999 = 1	$1000 \overset{1}{>} 999$
b)	1000 – 100 = 900	$1000 \overset{900}{>} 100$
c)	1000 – 10 = 990	$1000 \overset{990}{>} 10$
d)	990 – 900 = 90	$990 \overset{90}{>} 900$
e)	1000 – 1000 = 0	(careful with this one!)
f)	100 – 10 = 90	$100 \overset{90}{>} 10$

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12. *Question and Solution*

Use every number on a dice only once in each subtraction, so that the subtraction makes sense and the difference is:

a) at least 300 — e.g.
$$\begin{array}{r} 6\;5\;4 \\ -\;2\;3\;1 \\ \hline 4\;2\;3 \end{array}$$

b) the smallest possible
$$\begin{array}{r} 4\;1\;2 \\ -\;3\;6\;5 \\ \hline 4\;7 \end{array}$$

c) between 200 and 300 — e.g.
$$\begin{array}{r} 6\;5\;3 \\ -\;4\;1\;2 \\ \hline 2\;4\;1 \end{array}$$

d) even — e.g.
$$\begin{array}{r} 4\;6\;5 \\ -\;3\;2\;1 \\ \hline 1\;4\;4 \end{array}$$

e) the greatest possible
$$\begin{array}{r} 6\;5\;4 \\ -\;1\;2\;3 \\ \hline 5\;3\;1 \end{array}$$

f) divisible by 10
$$\begin{array}{r} \square\;\square\;\square \\ -\;\square\;\square\;\square \\ \hline \square\;\square\;\square \end{array}$$
Impossible

(p103, Q4)

Notes

Some of these questions have many answers, but care is needed. We use only the digits 1, 2, 3, 4, 5 and 6 in the first *two* rows, but any digits in the final row; for example,

e) Here 654 is the greatest number possible whilst 123 is the smallest, to give

$$\begin{array}{r} 6\ \ 5\ \ 4 \\ -\ \ 1\ \ 2\ \ 3 \\ \hline 5\ \ 3\ \ 1 \\ \hline \end{array}$$

f) Now we need the sum to end in a zero , but this cannot be possible! Hence no answer! (Hungarians often set impossible questions in order to make students think.)

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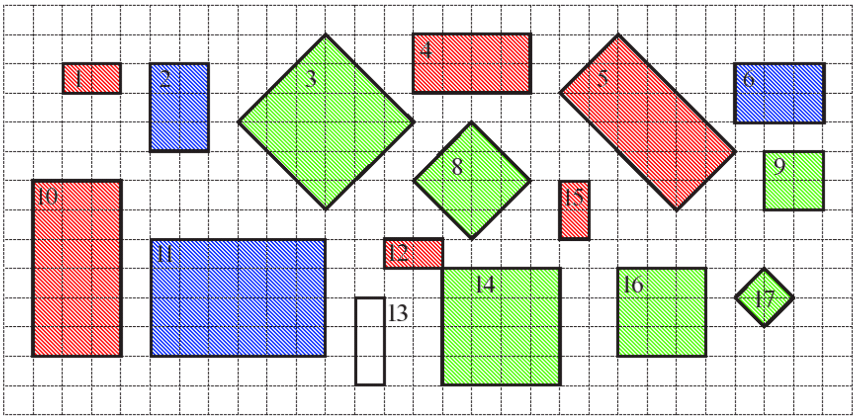
13. *Question and Solution*

Colour in the same colour shapes which are **similar** to

i) rectangle 1 ii) rectangle 2 iii) rectangle 3.

Use a different colour for each set of shapes.



(p114, Q1)

Notes

Note that, for rectangles to be SIMILAR, the adjacent sides must be in the same ratio; for example,

In RECTANGLE 1, the adjacent sides are in the ratio 1 : 2 and in RECTANGLE 4, the ratio is 2 : 4 which is equivalent to 1 : 2.

Also, in RECTANGLE 5, the ratio is 2 : 4 (but using diagonal sides) and this is also 1 : 2.

(We often write this as $2 : 4 \equiv 1 : 2$, the sign '$\equiv$' meaning 'is equivalent to', but for Year 3 students, it is sufficient to say 'equal to' or 'the same as'.)

The ratio of the sides in RECTANGLE 10 is 6 : 3 $\equiv$ 2 : 1. This is the same as 1 : 2 (but rotated by one right angle).

RECTANGLE 12 is identical to RECTANGLE 1; they are congruent and hence also similar.

Thus RECTANGLES 1, 4, 5, 10 and 12 are similar.

We can see that RECTANGLES 2 and 6 are similar (in fact, they are identical, with adjacent sides in the ratio 2 : 3) and RECTANGLE 11, with sides in the ratio $4 : 6 \equiv 2 : 3$ is also similar.

RECTANGLES 3, 8, 9, 14, 16 and 17 are similar (they are all squares).

RECTANGLE 11 (with sides in the ratio $4 : 6 \equiv 2 : 3$) and RECTANGLE 13 (with sides in the ratio 3 : 1) are not similar to any of the other rectangles.

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14. *Question and Solution*

Write these numbers in the correct place in the diagrams.

0, 4, 13, 30, 72, 95, 100, 321, 679, 1000, 1006, 1027, 2000

a)

Even	Odd
0 4 30 72	13 95
100 1000	321 679
1006 2000	1027

b)

Whole tens	Not whole tens
0 30 100	4 13 72
1000	95 321 679
2000	1006 1027

c)

3-digit	Not 3-digit
100 321	0 4 13 72
679	95 1000
	1027 2000

d)

Whole hundreds	Not whole hundreds
0 100	4 13 30 72
1000	95 321 679
2000	1006 1027

(p126, Q3)

Notes

This is a straightforward question on classification. You can revise or extend the question by asking students to decide on their own method of classification (this is the Japanese way of using 'open approach' problem solving when there are many possible answers). In fact, you could give ownership to your students by first asking each of them to produce their own set of 1, 2 or 3 digit numbers to classify!

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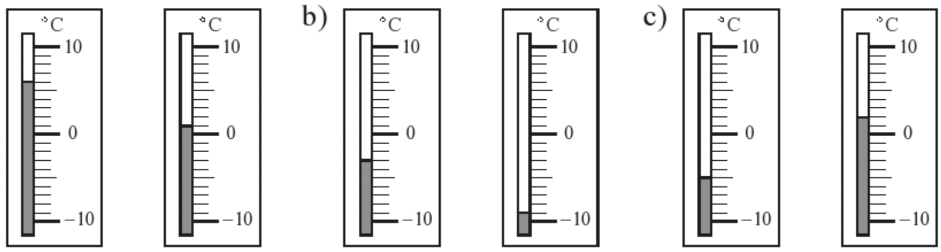
15. *Question and Solution*

Write the temperature below the thermometers. Write in the missing sign.

a) b) c)



a) 6 °C > 1 °C b) –3 °C > –9 °C c) –5 °C < 2 °C

(p128, Q1)

Notes

This question focuses on negative numbers using temperature as the context. Your students should not have difficulty identifying the values, that is,

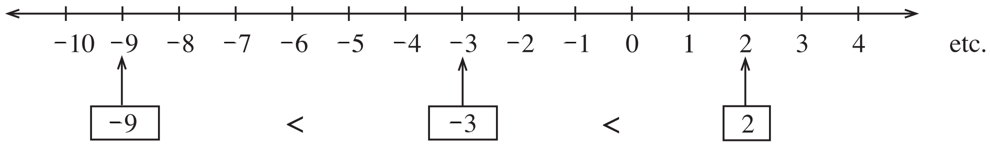
7, 1, –3, –9, –5, 2 (°C)

but might have more difficulty with the inequality signs between the values.

$$7 > 1 > -3 > -9 < -5 < 2$$

The $-3 > -9$ *sometimes looks odd as* $3 < 9$, *but keep referring to the positions of the numbers on a number line.*

You might find it helpful to have a horizontal extended number line on the board.



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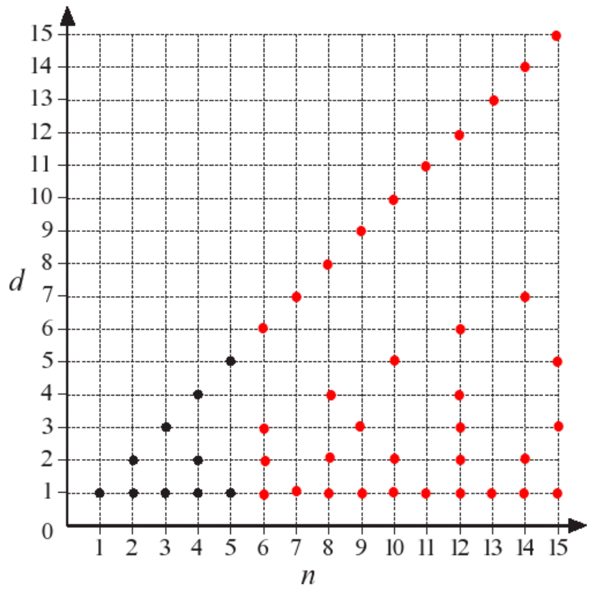
16. *Question and Solution*

What is the rule? Complete the table and the graph.

n	d
1	1
2	1, 2
3	1, 3
4	1, 2, 4
5	1, 5
6	1 , 2, 3, 6
7	1, 7
8	1, 2, 4, 8
9	1, 3, 9
10	1, 2, 5, 10
11	1, 11
12	1, 2, 3, 4, 6, 12
13	1, 13
14	1, 2, 7, 14
15	1, 3, 5, 15



23)

Notes

This is a really useful question for illustrating a number of concepts. First, though, your students must complete the table (which shows the divisors of each number) and the chart.

You could ask students to spot the patterns and deduce that

- numbers divisible by 2 are in the '2' row
- numbers divisible by 3 are in the '3' row, etc.
- only one number, 1, has just one divisor.

Or ask

- which numbers have exactly 2 divisors? (These are prime numbers.)
- which numbers have exactly 3 divisors? (These are square numbers.)

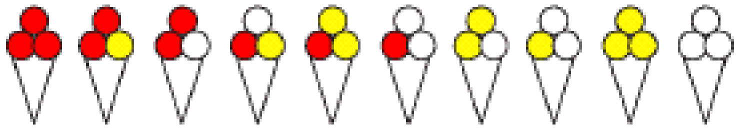
An extension is to look for any numbers whose divisors, excluding the number itself, add up to itself. We can see that there is only one, that is 6 (as $1 + 2 + 3 = 6$). This is called a PERFECT number.

Are there any more?

(Not in $1 \rightarrow 15$, but 28 is the second perfect number.) A useful reference for perfect numbers is http://mathforum.org/dr.math/faq/faq.perfect.html

17. *Question and Solution*

How could a 3-scoop ice-cream be made from vanilla or strawberry or lemon?

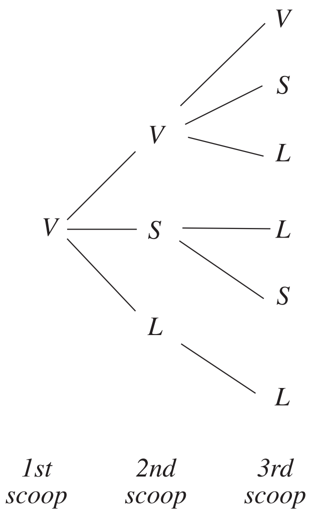


(p154, Q3)

Notes

This is an example of a 'combinatorics' question, that is, a counting question. We look for efficient ways to answer such questions or at least systematic ways of counting.

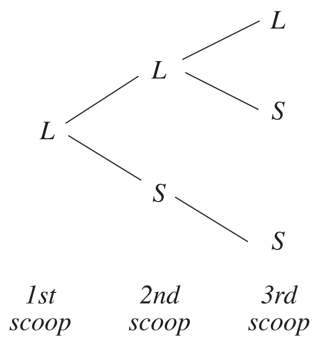
We first assume that the actual position of each scoop does not matter. Then, we could start by choosing vanilla (V) and could list the possible combinations, making sure that there are no repeats.



V V V
V V S
V V L
V S L
V S S
V L L

6 possibilities

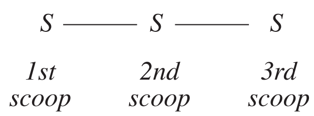
Now start with lemon (L) to give new possibilities.



L L L
L L S
L S S

3 possibilities

Finally, starting with strawberry (S),



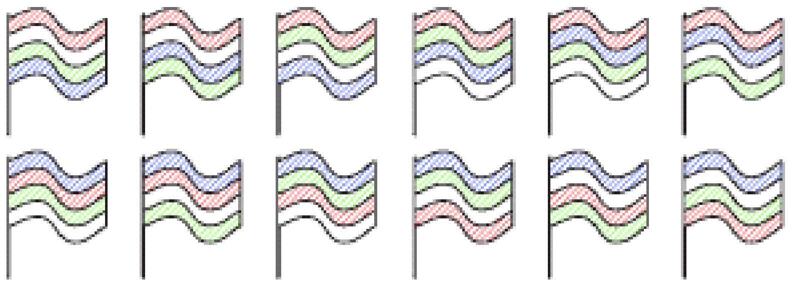
S S S

1 possibility

This gives $6 + 3 + 1 = 10$ possibilities.

18. *Question and Solution*

In how many different ways can you colour the flags *red*, *white*, *green* and *blue*?
Use every colour only once in each flag.

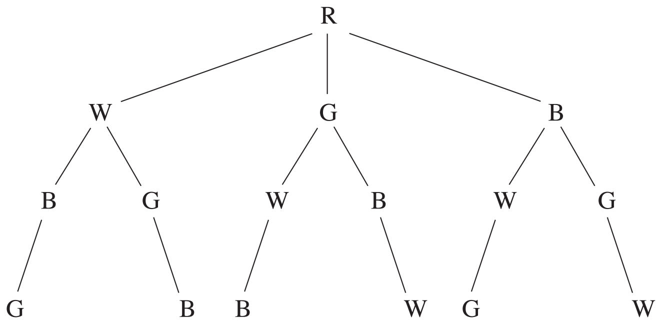


There are 24 different ways (or 12, as the flags can be flown upside-down).

(p167, Q1)

Notes

This is another combinatorics problem. Yes, we could start colouring in, but it is much more efficient to make a chart (R, W, G and B). Start with red (R).



So, starting with R, we have 6 possible flags,

R W B G	R G W B	R B W G
R W G B	R G B W	R B G W

The same arrangement can be used for starting with W (white), G (green) or B (blue), to give

$$6 + 6 + 6 + 6 = 4 \times 6 = 24 \text{ possibilities}$$

Why are there only 12 empty flags in the book?

This is partly to encourage students to think rather than rush into colouring and also as, for example,

R W B G

will also occur in the reverse colouring, G B W R.

So there are only 12 distinct colourings if you ignore the vertical direction of the flag!

An even quicker (and more mathematical) way of obtaining the answer is as follows:

For the top row there are 4 choices of colour

Having found the colour of the top row, there will be 3 choices of colour for row 2.

Having found the colours of the first 2 rows, there will be 2 choices of colour for row 3.

Having found the colours of the first 3 rows, there will be only 1 possible colour for row 4.

This gives

$$4 \times 3 \times 2 \times 1 = 24$$

possibilities. (This method is only for very, very able Year 3 students!)

19. *Question and Solution*

a) List in increasing order all the 3-digit numbers which have digits 1 or 2.

111, 112, 121, 122, 211, 212, 222

b) List in decreasing order all the 2-digit numbers which have digits 1, 2 or 3.

33, 32, 31, 23, 22, 21, 13, 12, 11

(p170, Q3)

Notes

Logical, systematic thinking is again required to give, for example:

a) 111, 112, 121, 122, 211, 212, 221, 222

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20. *Question and Solution*

Make two 3-digit numbers using the numbers 0, 1, 3, 4, 5 and 8 so that:

a)	their sum is the least possible,	108	and	345
b)	their sum is the greatest possible,	841	and	530
c)	their difference is the least possible,	401	and	385
d)	their difference is the greatest possible.	854	and	103

(p172, Q4)

Notes

Again, logical thinking is needed.

a) We need the two smallest numbers, so the HUNDREDS digits must be 1 and 3 (we cannot start the number with 0) and use 0 and 4 for the TENS and 5 and 8 for the UNITS. For example,

105 and 348 to give sum = 453

(there are a number of possible solutions but they have the same sum,

i.e. 145 and 308
148 and 305
108 and 345)

b) This is the reverse of the problem in a), so we put 8 and 5 as the HUNDREDS, 4 and 3 as the TENS and 0 and 1 as the UNITS, to give, for example,

840 and 531 with sum = 1371

c) We need to make the numbers as close to one another as possible so the first digits must 3 and 4 or 4 and 5.

For 3 and 4, we have 385 (largest number) and 401 (smallest number) to give a difference of 16.

For 4 and 5, we have 483 (largest number) and 501 (smallest number) to give a difference of 18.

Hence we choose 401 and 385.

d) This is the reverse of the problem in c). We cannot start the number with 0, so the two numbers will start with 1 and 8, and we choose

103 (smallest number) and 854 (largest number)

to give a difference of 751.

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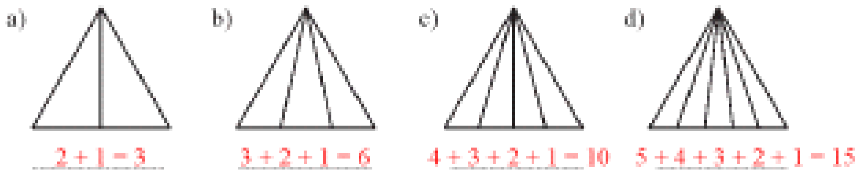
21. *Question and Solution*

How many triangles can you see in each diagram?



(p174, Q4)

Notes

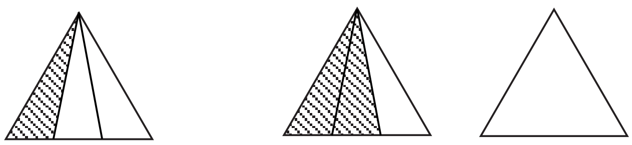
The important aspect of this problem is to be able to generalise.

For a), we clearly have

$$2\ (\text{half-size }\Delta) + 1\ (\text{large }\Delta) = 3\ \Delta \qquad (= 2 + 1)$$

For b), we have

$$3\ (\text{one third-size }\Delta) + 2\ (\text{two third-size }\Delta) + 1\ (\text{large }\Delta) = 6\ \Delta \qquad (= 3 + 3)$$



For c), we have

$$4\ (\text{quarter-size }\Delta) + 3\ (\text{half-size }\Delta) + 2\ (\text{three-quarter size }\Delta)1 + 1\ (\text{large }\Delta) = 10\ \Delta$$
$$(= 4 + 6)$$

For d), we have

$$5\ (\text{one fifth-size }\Delta) + 4\ (\text{two fifth-size }\Delta) + 3\ (\text{three fifth-size }\Delta) +$$
$$2\ (\text{four fifth-size }\Delta) + 1\ (\text{large }\Delta) = 15\ \Delta \quad (= 5 + 10)$$

So what about the next triangle? It will have $6 + 15 = 21\ \Delta$, etc.