### Activity

#### Factorisation

1. **Let's factorise 140 and 141. Ps come to BB or dictate to T, drawing factor tree if needed and checking possible prime divisors.**

   - **BB:** $140 = 2 \times 2 \times 5 \times 7$; $141 = 3 \times 47$

2. **Who can tell me all the factors of 140 and 141? Ps dictate to T, using the prime factors to help them. Class agrees/disagrees.**

   - **BB:** 140: 1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, 140; 141: 1, 3, 47, 141

#### Calculation relay

- T says a multiplication or division. Ps say result (in steps if necessary). Start with practice in multiplication and division facts, e.g. $6 \times 9$, $49 \div 7$, then gradually move on to more difficult operations. e.g.
  - $14 \times 13 = 182$
  - $28 \times 7 = 196$
  - $30 \times 11 = 330$, $305 \times 11 = 3385$
  - $\frac{1}{5} \times 5, 2.7 \times 10, 3400 \div 5 = 680, \frac{100}{10} \div 10,$ etc.

#### Written exercises

- T dictates the operations and Ps write them down vertically in Ex. Bks.

  - **a)** $13 \, 0870 - 1308$
  - **b)** $428.3 - 60.2$
  - **c)** $475 \times 4$
  - **d)** $444 \times 21$
  - **e)** $651.28 + 207.43 + 1040.05 + 99.99$
  - **f)** $17 \, 253 \div 8$
  - **g)** $19 \, 605 \div 17$

Let's see how many you can do in 5 minutes! You can use any method you wish. Remember to check your results. Start . . . now! . . . Stop! Review at BB with whole class. Ps come to BB or dictate results to T, explaining reasoning loudly and with place values. Class points out errors. Mistakes discussed and corrected.

### Notes

- **Whole class activity**
- Reasoning, agreement, praising
- At a good pace
- Ps may use a calculator.
- Ps could join up the factor pairs.

- **Whole class activity**
- T chooses Ps at random.
- Allow Ps to write interim steps on scrap paper or slates or, in difficult cases, to write the whole calculation.
- Emphasis should be on quick calculation, done mentally if possible.
- At a good pace
- Class points out errors.
- Praising, encouragement only

- **Individual work, monitored, d) to g) helped**
- (Write vertically on BB or use enlarged copy master for review and for less able Ps.)
- Differentiation by time limit
- (Or deal with one at a time if class is not very able.)
- Reasoning, agreement, self-correcting, praising
- Check with reverse operations (or with a calculator).
- Feedback for T
## Y4

### Activity

<table>
<thead>
<tr>
<th><strong>4</strong></th>
<th><strong>PbY4b, page 141</strong></th>
</tr>
</thead>
</table>
| **Q.1** | **Read:**  
  i) Colour the shapes which are symmetrical and draw the lines of symmetry.  
  ii) Write the perimeter length (in grid units) below each shape.  

Elicit what 'symmetrical' and 'line of symmetry' mean.  
(If you folded the shape in half, one half would cover the other exactly. The line of symmetry is the fold line or is sometimes called the mirror line, as one half of the shape is a mirror image of the other half.)  

Set a time limit. Review at BB with whole class. Ps come to point to each shape in turn, say whether it is symmetrical, draw the lines of symmetry where appropriate and write its perimeter length. Class agrees/disagrees. Mistakes discussed and corrected.  

**Solution:**

![Perimeter lengths of shapes](image)

### Extension

What do you notice about all the shapes? (They all have an area of 6 grid squares.)  
What is the name of each shape?  
  a), b): rectangles  
  c): hexagon (plane shape with 6 straight sides)  
  d), e), g), h), j), k): octagons (plane shape with 8 straight sides)  
  f) and i) dodecagon (plane shape with 12 straight sides)  

**26 min**

### Lesson Plan 141

<table>
<thead>
<tr>
<th><strong>Notes</strong></th>
</tr>
</thead>
</table>
| Individual work, monitored, helped  

Drawn on BB or use enlarged copy master or OHP  

Revision of symmetry.  
Allow Ps to explain if they can. Ps might point out symmetrical shapes in the classroom.  
Elicit that the unit of measure for the perimeters is the side of a grid square.  
Differentiation by time limit.  
Reasoning, agreement, self-correction, praising  

Feedback for T

<table>
<thead>
<tr>
<th><strong>5</strong></th>
<th><strong>PbY4b, page 141</strong></th>
</tr>
</thead>
</table>
| **Q.2** | **Read:**  
  These shapes are congruent. What has been done to Shape 1 to make Shape 2, Shape 2 to make Shape 3, and so on? Write it in your exercise book.  

Elicit what 'congruent' means. (exactly the same shape and size)  
Revise the vocabulary for transforming shapes first. T manipulates a cardboard shape on BB and elicits the name for the movement. rotation: turning around a central point, reflection: forming a mirror image, i.e. flipping the shape over, Discuss the form of what Ps should write in Ex. Bks.  
Ps should draw the mirror line in Pbs if it is a reflection.  
Review at BB with whole class. Elicit by how much the shape has been turned if it is a rotation. T should have cut-out shapes to demonstrate on BB. Mistakes discussed and corrected.  

Individual trial first, monitored, after introductory whole class discussion (or continue as a whole class activity)  
Drawn on BB or use enlarged copy master or OHP  
Discussion and demonstration  
BB: reflection rotation  
e.g. S1 → S2: reflection  
Encourage Ps to use a ruler.  
Discussion, reasoning, agreement, self-correction, praising  
Feedback for T  

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### Y4

#### Activity

5  
(Continued)

**Solution:**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 1 → 2: reflection in mirror line
- 2 → 3: rotation of 1 by half a turn, then reflection in mirror line
- 3 → 4: rotation by a right angle
- 4 → 5: rotation by half a turn
- 5 → 6: rotation by half a turn

#### Extension

What can you tell me about the shape?
(e.g. octagon,  \( P = 12 \) units,  \( A = 6 \) square units)

31 min

---

### Lesson Plan 141

#### Notes

- **Feedback for T**
  - Elicit that
    - half a turn = 2 right angles
    - quarter of a turn = 1 right angle

- Individual work, monitored, helped

- Drawn on BB or use enlarged copy master or OHP

- Discussion, reasoning, agreement, self-correcting, praising

- Extra praise if Ps remember the vocabulary.

---

#### Extension

6  
*PbY4b, page 141*

**Q.3** Read: *What has been done to Shape A to make Shape B. Shape B to make Shape C, and so on? Write it in your exercise book.*

First discuss the form of what Ps should write in Ex. Bks.
  - e.g.  A → B: rotation

Set a time limit. Review at BB with whole class. Ps dictate findings. T helps with vocabulary. Elicit that to make bigger in all directions is to *enlarge* (and to make smaller is to *reduce*); to make bigger in only one direction is to *stretch*. Class agrees/disagrees with solutions. Mistakes discussed and corrected.

**Solution:**

- A → B: rotation (by quarter of a turn, or a right angle, or 90°)
- B → C: stretch horizontally (to twice its length)
- C → D: rotation (by a right angle or quarter of a turn, or 90°)
- D → E: stretch horizontally (to twice its length)

Which shapes are **congruent**? Which shapes are **similar**?

**Read:** *Write the area inside each shape and the perimeter below.*

Ps count the grid squares and parts of grid squares for the area. (This is not too difficult, as Ps can find parts which combine to make a complete square, but accept approximations.)

It is difficult to find the perimeter in grid units, so Ps should measure a slanting side with rulers (to the nearest mm) and as the side of each grid square is 5 mm, the total perimeter can be calculated. e.g.

- A:  \( P = 6 \times 5 \text{ mm} + 4 \times 7 \text{ mm} = 30 \text{ mm} + 28 \text{ mm} = 58 \text{ mm} \)

**Solution:**

\[ P \approx 5.8 \text{ cm} \]

Area given in grid squares.

Perimeter given in mm, but Ps could then convert to cm

36 min
### Activity 7

**Shapes**

T has shapes drawn (or stuck) on BB.

What has been done to the previous shape to make the next one?

**BB:**

Ps come to BB or dictate to T. Class agrees/disagrees. T should have cut-out versions of the small and large shape so that Ps can demonstrate rotation or measure and compare the length of the sides.

**Solution:**

A → B: rotation (by 1 sixth of a turn)
B → C: enlargement (to twice its size)
C → D: rotation (by 1 sixth of a turn)
D → E: reduction (to half its size)
E → F: rotation (by 1 sixth of a turn)
F → G: enlargement (to twice its size)

---

**Extension**

Which shapes are congruent (similar)? What is their area (perimeter)?

---

### Notes

Whole class activity

Use copy master or OHP; shapes enlarged and cut out (or draw on triangular grid)

Ps could have copy of copy master on desks too.

Discussion, reasoning, agreement, praising

Elicit that the shape is a hexagon with one of the 6 segments cut out, so rotating it by one segment will be 1 sixth of a turn.

BB: A ≅ B ≅ E ≅ F; C ≅ D ≅ G

e.g. A ~ C, etc.

---

**Lesson Plan 141**

**8 PbY4b, page 141, Q.4**

Read: *Barry Bear is planning his route to visit Piggy, then Rabbit, then Goat. He draws the possible paths he could take.*

Who can explain the diagram? (The letters stand for each of the animals and the dots are their houses. The lines are the possible paths.)

a) Read: *How many routes are possible?*

T asks several Ps what they think and why (or Ps could write number on scrap paper or slates and show on command.)

Agree that for each of the 4 different paths from B to P, there are 5 different paths from P to R and for each of these, there are 3 different paths from R to G, i.e. there are:

BB: \[4 \times 5 \times 3 = 20 \times 3 = 60\] possible routes.

b) Read: *What chance has Goat of guessing Barry’s route?*

T asks several Ps what they think and why (or Ps could show on scrap paper or slates on command).

BB: \[\frac{1}{60}\]

(Unless *Barry Bear* had a favourite route and *Goat* knew it.)

---

Whole class activity but individual calculating

Diagram drawn on BB or use enlarged copy master or OHP

BB:

Discussion, reasoning, agreement, praising

Reasoning, agreement, praising

Extra praise if a P points this out without hint from T.
Y4

Y4

R: Calculations
C: Revision and practice: Geometry
E: Geometric game. Problems

Activity

1

Factorisation
a) Let’s factorise 142. Ps come to BB or dictate to T, drawing factor tree if needed and checking possible prime divisors.

BB: 142 = 2 \times 71;

b) Who can tell me all the factors of 142? Ps dictate to T, using the prime factors to help them. Class agrees/disagrees.

BB: 142: 1, 2, 71, 142

2

Numbers
a) Let’s call a natural number a perfect number if it equals the sum of all its natural factors except for itself.

Who can tell me a perfect number? Who agrees? Let’s check it.

Who can think of another perfect number? Class checks it.

To Ts only:

Perfect numbers are rare and only 39 such numbers are known but we do not know if there are others. The first 3 perfect numbers: 6, 28 and 496 were known to the ancient mathematicians since the time of Pythagoras (C 500 BC)

All even perfect numbers are of the form \(2^n \times (2^{n+1} – 1):\)

\(6 = 2^1 \times (2^2 – 1); \quad 28 = 2^2 \times (2^3 – 1); \quad 496 = 2^4 \times (2^5 – 1);\)

\(8128 = 2^6 \times (2^7 – 1), \quad \text{then} \quad 2^{10} \times (2^{11} – 1), \quad 2^{12} \times (2^{13} – 1), \ldots\)

but note that the formula works only if \(2^n\) is a prime number.

We do not know if odd perfect numbers exist, as none have been found yet. (See http://home1.pacific.net.sg/~novelway/MEW2/)

b) Let’s call a natural number a nice number if it equals the product of all its factors except for 1 and itself.

Who can tell me a nice number? Who agrees? Let’s check it.

Who can think of another nice number? Ps try out other numbers and tell class when they have found one. Class checks it.

Problem 1

I made this solid from four 5-unit long rods, 1 unit wide and 1 unit thick. (Or T could have model constructed from strips of 5 multi-link cubes.)

Here is a diagram of it on the BB. I will ask you questions about it and you must show me the answer when I say.

BB: Ps answering correctly explain reasoning to class.

a) How many unit cubes did I use to make it?

\(4 \times 5 = 20\) unit cubes

b) How many unit squares covers its surface?

\(82\) By counting, or by calculation:

\[4 \times (4 \times 5 + 2 \times 1) – 3 \times 2 = 4 \times 2 – 6 \]

\[= 88 – 6 \]

\[= 82\] (unit squares)

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**Y4**

**Activity**

4  
*PbY4b, page 142*

Q.1 Read: How many unit cubes are needed to build each cuboid?  
*Colour the cubes which are similar.*

Elicit that the number of unit cubes is the **volume** of the cuboid and that it is calculated by: length \( \times \) width \( \times \) height.

Ps can do calculations in *Ex. Bks.* or on slates if they need to but encourage mental calculation if possible. Set a time limit.

Review at BB with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

```
\[\begin{array}{ccc}
\text{a) } & \text{b) } & \text{c) } \\
\text{a} & \text{b} & \text{c} \\
3 \text{ units} & 8 \text{ units} & 6 \text{ units} \\
2 \text{ units} & 2 \text{ units} & 4 \text{ units} \\
4 \text{ units} & 8 \text{ units} & 8 \text{ units}
\end{array}\]
```

**Extension**

What is the surface area of the cuboids? Ps come to BB or dictate to T. Class points out errors.

a)  
\[A = 2 \times (3 \times 2 + 3 \times 4 + 2 \times 4) = 26 \times 2 = 52 \text{ (unit squares)}\]

b)  
\[A = 2 \times (8 \times 2 + 8 \times 8 + 2 \times 8) = 2 \times (16 + 96 + 16) = 2 \times 138 = 256 \text{ (unit squares)}\]

c)  
\[A = 2 \times (6 \times 4 + 6 \times 8 + 4 \times 8) = 2 \times (24 + 48 + 32) = 2 \times 104 = 208 \text{ (unit squares)}\]

**Lesson Plan 142**

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

(or T has large models already made up)

Reasoning, agreement, self-correction, praising

BB:

a)  
\[V = 3 \times 2 \times 4 = 24 \text{ (unit cubes)}\]

b)  
\[V = 8 \times 2 \times 8 = 128 \text{ (unit cubes)}\]

c)  
\[V = 6 \times 4 \times 8 = 192 \text{ (unit cubes)}\]

Note: each edge of a) has been enlarged by 2 times to make c).

Reasoning, agreement, praising

Extra praise if Ps notice that the diagrams have not been drawn to scale!

---

**Y4**

5  
*PbY4b, page 142*

Q.2 Read: Find the points and join them up. Colour the shapes you make.

Who can explain to us what the numbers in the brackets mean? Ps come to BB to point and explain (The first number is the \(x\) coordinate and refers to the horizontal numbers on the \(x\)-axis in the diagram. The 2nd number is the \(y\) coordinate and refers to the vertical numbers on the \(y\)-axis.) If Ps are still unsure, T (or P) P could demonstrate how to find the first point by moving two fingers along the grid lines until they meet and drawing a dot.

Set a time limit. Review with whole class. T has solution already prepared but keeps covered until Ps have said what they have drawn. (Ps in unison: *Mickey Mouse*) T confirms it.

**Solution:**

T gives coordinates for a point on LHS (e.g. 3, 6) and Ps say the coordinates of the corresponding point on the RHS (e.g. 11, 6).

---

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### Lesson Plan 142

#### Activity

**Y4**

<table>
<thead>
<tr>
<th><strong>Activity</strong></th>
<th><strong>Notes</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6</strong></td>
<td><strong>PbY4b, page 142</strong></td>
</tr>
</tbody>
</table>
| **Q.3** Read: **A group of children are standing in a circle to play a game. Each child has been given a number in order round the circle.** If the child numbered 6 stands opposite the child numbered 15, how many children are playing the game? Ps to try to solve it using their own methods. Set a time limit. Review with whole class. Ps who have found a solution could show on scrap paper or slates in unison on command. Ps responding correctly explain at BB to rest of class. Who did the same? Who did it a different way? etc. Deal with all methods. **Possible methods of solution:**

1) Draw a diagram:  

![Diagram](image)

2) From 6 to 1 we could move 5 steps back.

5 steps back from 15 is 10, so the number opposite 1 is 10.

Therefore the greatest number must be opposite 9.

As the difference between opposite numbers is 9, (15 – 6 = 9) the greatest number must be 9 + 9 = 18

or

4) 15 – 6 = 9 is half of the children, so there are 2 × 9 = 18 children altogether.

**41 min**

<table>
<thead>
<tr>
<th><strong>7</strong></th>
<th><strong>PbY4b, page 142</strong></th>
</tr>
</thead>
</table>
| **Q.4** Read: **The Rabbit family grow their yearly supply of carrots in a rectangular garden. Its area is 180 m². How long is the garden if it is 15 m wide?** Ps solve the problem in Ex. Bks under a time limit. Review with whole class. Ps could show result on scrap paper or slates on command. Ps answering correctly come to BB to explain their reasoning. Mistakes discussed and corrected. **Solution:**

\[
A = a \times b = 180 \text{ m}^2, \quad b = 15 \text{ m}
\]

So \[a = 180 \div b = 180 \div 15 = 60 \div 5 = 12 \text{ (m)}\]

**Answer:** The garden is 12 m long.

**Extension**

- What is the perimeter of the garden? Ps come to BB or dictate to T. Class agrees/disagrees.

**BB:** \[P = 2 \times (15 + 12) = 30 + 24 = 54 \text{ (m)}\]

- What is wrong with the question? (15 m > 12 m, so 15 m should be the length and 12 m should be the width.)

**45 min**

<table>
<thead>
<tr>
<th><strong>Lesson Plan 142</strong></th>
<th><strong>Notes</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Individual work, monitored, (helped)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Discussion, reasoning, agreement, self-correction, praising</strong></td>
<td></td>
</tr>
</tbody>
</table>
| **BB:** \[b = 15 \text{ m} \quad A = 180 \text{ m}^2 \]
| **Whole class activity** |
| **Reasoning, agreement, praising** |

Agree that usually the longer side of a rectangle is its length and the shorter side is its width.
Y4

**Activity**

1. **Factorisation**
   a) Let's factorise 143. Ps come to BB or dictate to T, checking possible prime divisors. (1, 2, 3, 5, 7, 9, 11)
      BB: \(143 = 11 \times 13\);
   b) Ps dictate all its factors and T writes on BB. 143: 1, 11, 13, 143
      What is special about this number? (It is a nice number, i.e. it is equal to the the product of all its factors, apart from 1 and itself.)

2. **Problem 1**
   Listen carefully and think how you would solve the problem.
   *Donald Duck* says:
   
   *If the sum of the digits of a natural number is divisible by 3, then that number is also divisible by 3.*
   
   *Is he right or wrong?*
   
   Allow Ps to think about it and discuss with their neighbours for a couple of minutes. Who thinks that *Donald Duck* is right? T asks several Ps why they think so. Ps who think the statement is wrong try to give counter examples.
   
   Reasoning, e.g.
   
   P1: The multiples of 3 are: 0, 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, . . . and we could not find one which shows that the statement is wrong.
   
   P2: I chose numbers out of my head, e.g. 6318.
      \(6318 \rightarrow 6 + 3 + 1 + 8 = 18\), and 18 is a multiple of 3
      So I divided 6318 by 3 and found that it was also a multiple of 3.
      All the numbers I tried matched up with what *Donald Duck* said.
   
   P3: I chose a number, e.g. 7153
      \(7153 \rightarrow 7 + 1 + 5 + 3 = 16\), and \(16 \div 3 = 5, r 1\)
      So I divided 7153 by 3 and found that it also has a remainder of 1.
      Then I tried 7154.
      \(7154 \rightarrow 7 + 1 + 5 + 4 = 17\), and \(17 \div 3 = 5, r 2\)
      So I divided 7154 by 3 and found that it also has a remainder of 2.
      Then I tried 7155. 7155 \(\rightarrow 7 + 1 + 5 + 5 = 18\), and 18 is a multiple of 3.
      So I divided 7155 by 3 and found that it is a multiple of 3 too.
      No other remainders are possible, so what *Donald Duck* said seems to be true.
   
   T tells class that the statement is true but you will learn how to prove it another year. It is an easy method of telling if a number is a multiple of 3.
### Activity 3 Problem 2

Listen carefully and think how you would solve the problem.

**a)** *Mickey Mouse says:*

*If the last two digits of a natural number show a 2-digit number which is divisible by 4, then whole number is also divisible by 4.*

**Is he right or wrong?**

Who thinks that he is right? Why? Ps might suggest using some of the strategies used in the previous question. T helps with wording of reasoning. Class agrees/disagrees.

Reasoning, e.g.

1. P: The multiples of 4 are: 0, 4, 8, 12, 16, 20, 24, 28, 32, . . . and the statement is true for all these.
2. P: I chose numbers out of my head, e.g. 3728

   \[3728 \rightarrow 28,\] and 28 is a multiple of 4.

   So I divided 3728 by 4 and found that it was a multiple of 4.

   Then I tried 7328 and got the same result, so the thousands and hundreds digits do not seem to matter.

   P: Any whole hundred and any whole thousand is a multiple of 4, which is why we can decide from the tens and units digits only.

   This shows that what Mickey Mouse said is true.

**Extension

**b)** *Barry Bear says:*

*If the sum of the digits of a natural number is divisible by 6, then that number is also divisible by 6.*

**Is he right or wrong?**

Who thinks he is right? Why? Why not? Ps explain their reasoning or give counter examples. Class agrees/disagrees.

Reasoning, e.g.

1. P: The multiples of 6 are: 0, 6, 12, 18, 24, 30, . . . 12 → 3, which is not a multiple of 6, so the statement is wrong.
2. P: I chose a number whose digits add up to a multiple of 6, e.g. 2523

   \[2 + 5 + 2 + 3 = 12,\] which is a multiple of 6.

   So I divided 2523 by 6 and found that it had a remainder of 3.

   So 2523 is not a multiple of 6 and the statement is wrong.

   P: I chose a number whose digits do not add up to a multiple of 6.

   e.g. 96 → 9 + 6 = 15, which is not a multiple of 6.

   So I divided 96 by 6 and found that it is a multiple of 6.

   So I think that Barry Bear is wrong.

   Does this really show that Barry Bear is wrong? (It is not a counter-example because the reasoning is the wrong way round. He did not say that all numbers divisible by 6 have the sum of their digits as a multiple of 6.)

   Agree that only one counter example, e.g. 2523 above, is needed to prove that Barry Bear is wrong.

---

### Notes

- **Whole class activity**
- T repeats slowly to give Ps time to think.

- Discussion reasoning, agreement, praising

- Extra praise if P suggests this clever reasoning. (T gives it if no P does.)

- T repeats slowly to give Ps time to think.

- Discussion reasoning, agreement, praising

- Accept and praise P’s reasoning for the moment unless a P disagrees with it. (See P, below)

- If Ps do not suggest this or P, T might give it to make Ps think about what is a counter example and what is not.

- The same goes for P above.

- Praise all Ps who contributed to the discussion
<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY4b, page 143</strong></td>
</tr>
</tbody>
</table>

**Q.1** Read: *Snow White is painting a picture of the seven dwarfs. The area of the rectangular canvas is 4500 cm². How long is the canvas if its width is 500 mm?*

What should you do first? (Change the width to cm.)

Set a time limit. Ps write a plan, do the calculation and write the answer as a sentence in *Pbs*.

_Review with whole class. Ps could show result on scrap paper or slates on command. Ps who answered correctly explain at BB._

**Solution:**

<table>
<thead>
<tr>
<th>BB:</th>
<th><strong>A = 4500 cm²</strong></th>
<th><strong>W = 500 mm = 50 cm</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Plan:</strong></td>
<td><strong>L = A ÷ W = 4500 ÷ 50 = 450 ÷ 5 = 90 (cm)</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong></td>
<td>The canvas is 90 cm long.</td>
<td></td>
</tr>
</tbody>
</table>

| 25 min |

**Q.2** Read: *Measure the sides of each polygon. Calculate the perimeter and the area.*

Deal with the measurements first to ensure that Ps have the correct values before they do the calculations. Calculations can be done in *Ex. Bks* and only the results written in *Pbs*.

_Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected._

**Solution:**

<table>
<thead>
<tr>
<th>a) 5 cm</th>
<th>b) 5 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>3.5 cm</td>
<td>2 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>P = 2 × (5 + 3) = 2 × 8</strong></th>
<th><strong>P = (3.5 + 2 + 1.5 + 1 + 5 + 3) cm</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>= 16 (cm)</td>
<td>= 16 cm</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>A = (5 × 3) cm² = 15 cm²</strong></th>
<th><strong>A = 3.5 × 3 + 1.5 × 1</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>= 10.5 + 1.5 = 12 (cm²)</td>
<td></td>
</tr>
</tbody>
</table>

Why are the perimeters equal? Ps come to BB to explain.

| 30 min |

**Q.3** Read: *How many right angles are the angles shown by the arrows?*

What is a right angle? (1 quarter of a turn or 90°)

Set a time limit. Review at BB with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes corrected.

**Solution:**

| a) 3 right angles | b) 1 right angle | c) 1/2 right angles | d) 3/2 right angles |

| 34 min |

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### Activity 7

**PbY4b, page 143**

**Q.4** Read: *A cuboid is built from 72 unit cubes. How many units long can the edges be? First factorise 72, then show the possibilities in the table.*

- What do the circles and rectangles mean in the diagram? (The rectangles are factors which are not prime numbers and the circles are prime factors.)
- What do the letters in the table represent? (the length, width and height of the cuboid) Set a time limit.
- Review at BB with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

![Diagram with factors](image)

Agree that there are 12 different combinations.

If you think of the values of a, b and c as being interchangeable how many possibilities will there be then?

**BB:** \(3 + 6 + 6 + 6 + 6 + 3 + 6 + 3 + 6 = 4 \times 3 + 8 \times 6 = 12 + 48 = 60\) cases

---

### Extension

**PbY4b, page 143**

**Q.5** Read: *Try to divide a square into 6 smaller squares.*

Ps measure the given square and use their rulers to draw smaller squares. T might give a hint before Ps start, or if Ps are having difficulties, e.g.

Into how many congruent squares could we divide the square? (4, 9, 16, etc., i.e. square numbers)

But 6 is not a square number, so what can you say about the 6 squares? (They are not all an equal size.)

If Ps finds a solution, they come to BB to show it. If not, T shows it. Or T could leave the question open and Ps try to solve it at home or in Lesson 145.

**Solution:**

![Diagram with solution](image)

Individual work, monitored, helped
(or whole class activity if time is short)
Drawn on BB or use enlarged copy master or OHP
Initial discussion on meaning of diagram and table.
Differentiation by time limit
Reasoning, agreement, self-correction, praising
At a good pace

---

**Notes**

Individual work, monitored, helped
(or whole class activity if time is short)
Drawn on BB or use enlarged copy master or OHP
Initial discussion on meaning of diagram and table.
Differentiation by time limit
Reasoning, agreement, self-correction, praising
At a good pace

Whole class discussion, reasoning, agreement
Extra praise if a P points this out without help from T.
## Activity 1

### Factorising

a) Find the prime factors of 144 in your Ex. Bks. and write it as the product of its prime factors.

b) List all its factors using the prime factors to help you.

Set a time limit. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

<table>
<thead>
<tr>
<th>144</th>
<th>2 × 2 × 2 × 2 × 3 × 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>72</td>
<td>2 × 2 × 2 × 3 × 3</td>
</tr>
<tr>
<td>36</td>
<td>2 × 2 × 3 × 3</td>
</tr>
<tr>
<td>18</td>
<td>2 × 3 × 3</td>
</tr>
<tr>
<td>12</td>
<td>2 × 3</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>2 × 3</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

T could show this quick way too.

b) 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144  (15 factors)

What kind of number is 144? (It is a square number as 144 = 12 × 12)

5 min

### Missing numbers

Study these equations. Which numbers are missing?

When you come to the BB, first say the equation as a word problem, then explain how you will solve it. Then do the calculation at the side of the BB, explaining your reasoning.

**BB:**

a) \(645 + \boxed{3857} = 9002\)  
   **C:** \(9\ 0\ 0\ 2\)  
   \(-6\ 4\ 5\)  
   **8\ 3\ 5\ 7**
   
   e.g. How much is added to 654 to get 9002?  
   We get the unknown term of a sum if we take away the known term from the sum.

b) \(7318 – \boxed{4772} = 2546\)  
   **C:** \(7\ 3\ 1\ 8\)  
   \(-2\ 5\ 4\ 6\)  
   **4\ 7\ 7\ 2**
   
   e.g. How much is subtracted from 7318 to get 2546?  
   We get the unknown subtrahend if we subtract the difference from the reductant.

c) \(11\ 879 – 7608 = 4271\)  
   **C:** \(7\ 6\ 0\ 8\)  
   \(+4\ 2\ 7\ 1\)  
   **1\ 1\ 8\ 7\ 9**
   
   e.g. How much is 7608 taken away from to get 4271?  
   We get the reductant of a subtraction if we add the subtrahend and the difference.

d) \(4 \times \boxed{3827} = 15\ 308\)  
   **C:** \(3\ 8\ 2\ 7\)  
   \(1\ 5\ 3\ 0\ 8\)  
   **3\ 1\ 2**
   
   e.g. What is multiplied by 4 to get 15 308?  
   We get the unknown factor of a product if we divide the product by the known factor.

---

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### Y4

#### Activity

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 2 | (Continued)  
|   | e) $630 \div 7 = 90$  
|   | C:  
|   | e.g. What do we divide 630 by to get 90?  
|   | $630 \div 90 = 63 \div 9 = 7$  
|   | We get the unknown divisor if we divide the dividend by the quotient.  
|   | f) $19287 \div 9 = 2143$  
|   | C:  
|   | e.g. What is divided by 9 to get 2143?  
|   | $2143 \times 9 = 19287$  
|   | We get the unknown dividend if we multiply the quotient by the divisor.  

---

#### Notes

**15 min**

### Problem

In how many different ways can you draw 3 dots on this $3 \times 3$ grid so that there is only 1 dot in each row, column, and diagonal?  
Ps come to BB to draw dots in different configurations.  Agree that there are 6 possible ways.  Who can explain why there are 6 ways?  
BB:  

- The first row can have a dot in any of 3 squares.  
- For each of the cases in the 1st row, the 2nd row can have a dot in either of 2 squares.  
- For each of the cases in the 2nd row, there is only one possible place for the dot in the 3rd row.  
- $\text{ie. there are } 3 \times 2 \times 1 = 6 \text{ possible ways.}$

---

### PbY4b, page 144

**Q.1** Read:  
The diagram shows the plan of a house in the middle of its garden.  
Divide up the garden into 4 congruent parts in different ways.  
What does congruent mean? (the same size and shape)  
Set a time limit. T chooses Ps to BB to show their patterns (or T has some already prepared)  
Class checks that the parts are congruent. If disagreement, Ps could trace their patterns, cut out the parts and lay one on top of the other. If congruent, they should cover each other exactly.  
Elicit that each part is 1 quarter of the garden (but not 1 quarter of the diagram).
### Lesson Plan 144

**Activity**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y4</strong></td>
<td><strong>Notes</strong></td>
<td><strong>Lesson Plan 144</strong></td>
</tr>
<tr>
<td><strong>5</strong></td>
<td><strong>PbY4b, page 144</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td><strong>Q.2</strong></td>
<td><strong>Read:</strong> The perimeter of a triangle is 10 units. It has two equal sides. The length of each side is whole units. What is the length of each side?</td>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td></td>
<td>Ps can draw triangles and try out different lengths in Ex. Bks, then write the answer in Pbs. Set a time limit.</td>
<td>Discussion, reasoning, agreement, praising</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. X, what lengths did you answer? Who agrees? Who got a different answer? etc. Ps come to BB to explain their reasoning. Class agrees/disagrees.</td>
<td>T could check lengths by drawing on BB with BB compasses and ruler.</td>
</tr>
<tr>
<td></td>
<td><strong>Solution:</strong> e.g.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>List 2 numbers the same + another number to make 10: BB: 1, 1, 8; 2, 2, 6; 3, 3, 4; 4, 4, 2; 5, 5, 0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>then cross out those which cannot make a triangle, i.e. the sum of the two smallest sides must be more than the 3rd side.</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>Answer:</strong> The lengths of the sides can be 3 units, 3 units and 4 units; or 2 units, 4 units and 4 units.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>30 min</td>
</tr>
<tr>
<td><strong>6</strong></td>
<td><strong>PbY4b, page 144</strong></td>
<td>Individual work, monitored</td>
</tr>
<tr>
<td><strong>Q.3</strong></td>
<td><strong>Read:</strong> The diagram shows a 5-unit shape made from 12 equal sticks. Make another shape from 12 equal sticks which has an area of 5 units.</td>
<td>Original diagram drawn on BB (or straws or sticks stuck on BB):</td>
</tr>
<tr>
<td></td>
<td>Ps have sticks or straws on desks to manipulate. They first make the shape in Pbs and agree that it has an area of 5 squares.</td>
<td>If no P has found it after a set time, T gives hints or shows it on BB. Ps make it on desks with sticks.</td>
</tr>
<tr>
<td></td>
<td>Now make another shape which uses all the sticks and also has an area of 5 squares. When you have found it, draw it in your Pbs.</td>
<td>Discussion, demonstration, agreement, praising</td>
</tr>
<tr>
<td></td>
<td>Review at BB with whole class. P comes to BB to draw his/her shape and point to the 5 units. Class agrees/disagrees.</td>
<td>and for RHS: e.g.</td>
</tr>
<tr>
<td></td>
<td><strong>Solution:</strong></td>
<td>1 line of symmetry</td>
</tr>
<tr>
<td></td>
<td>What can you say about the shapes? e.g. for LHS shape:</td>
<td>octagon (8-sided polygon)</td>
</tr>
<tr>
<td></td>
<td>• It is symmetrical. Ps come to BB to draw lines of symmetry.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>• It is a dodecagon (12-sided polygon, if corners are joined up)</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>35 min</td>
</tr>
<tr>
<td><strong>7</strong></td>
<td><strong>PbY4b, page 144</strong></td>
<td>Individual work, monitored, (helped)</td>
</tr>
<tr>
<td><strong>Q.4</strong></td>
<td><strong>Read:</strong> Draw 12 dots on a 6 by 6 grid so that there are exactly 2 dots in each row, column and diagonal.</td>
<td>(Or whole class activity if time is short)</td>
</tr>
<tr>
<td></td>
<td>Less able Ps could have enlarged grids and counters on desks. Set a time limit. As soon as a P finds a configuration he or she comes to BB to show it. Class checks that it meets the conditions. Deal with all cases. Agree that many arrangements are possible.</td>
<td>Grids drawn on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td></td>
<td><strong>Solution:</strong> e.g.</td>
<td>Reasoning, agreement, (self-correction), praising</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Which of the patterns are symmetrical?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40 min</td>
</tr>
</tbody>
</table>
**Y4**

### Activity

**PbY4b, page 144**

Q.5 Read: *Six goblins live in 6 rooms, one goblin in each room. Make another plan of six congruent rooms but using 1 less stick.*

Ps have sticks or straws on desks to manipulate. They first make the shape in *Pbs* and agree that the rooms are 6 congruent rectangles and are made with 13 sticks.

Now make another plan which uses 12 sticks. Make sure that your rooms are congruent! When you have found it, draw it in your *Pbs*. (Ps use rulers.)

Review with whole class. If a P has found a plan, he or she shows it on BB. Class check that there are 6 congruent rooms. If nobody has solved it, T gives a hint (e.g. think of triangles).

**Solution:**

What can you say about the shape? e.g.

- It is symmetrical. (It has line and rotational symmetry.)
- It is a regular hexagon.
- It contains 6 equilateral (equal-sided) triangles.

45 min

### Notes

Individual work, monitored

Original diagram drawn on BB (or straws or sticks stuck on BB):

![Diagram]

Discussion, demonstration, agreement, praising

Or T could leave question open for Ps to try at home or in *Lesson 145*.

Whole class activity

Agreement, praising
Y4

Activity

Tables and calculation practice, revision, activities, consolidation

PbY4b, page 145

Solutions:

Q.1  
   a) Symmetrical shapes: 1, 3, 5, 6
   b) 1) \( P = 16 \) units, 2) \( P = 12 \) units, 3) \( P = 16 \) units,
       4) \( P = 14 \) units, 5) \( P = 16 \) units, 6) \( P = 12 \) units,
       7) \( P = 14 \) units, 8) \( P = 16 \) units
   c) All have an area of 7 grid squares.
   d) Many shapes are possible.

Q.2  
   a) and b)
   c) 16 vertices  
   d) concave  
   e) 8-pointed star
Y4

R: Calculations
C: Collecting data. Tally charts and grouping
E: Different ways to display data

Lesson Plan

146

Activity

1

Factorisation

Let's factorise 145 and 146 and then list all their factors.
Pleaders come to BB or dictate to T, trying the prime numbers in turn as divisors. Class agrees/disagrees.

BB:

145 = 5 × 29; Factors: 1, 5, 29, 45
146 = 2 × 73; Factors: 1, 2, 73, 146

What kind of numbers are they? (Both are nice numbers.)

Notes

Whole class activity
Reasoning, agreement, praising
Ps may use a calculator to check the divisors.
Feedback for T

2

Missing numbers

Let's fill in the missing numbers.
Pleaders come to the BB to say the equation as a word problem and explain how they will solve it. Then they do the calculation at the side of the BB, explaining reasoning in detail. Class points out errors.

BB:

a) 7.32 + 2.96 = 10.28
   e.g. How much is added to 7.32 to get 10.28?
   We get the unknown term of a sum if we take away the known term from the sum.
   C: 10.28
   C: 7.32
   C: 2.96

b) 54.63 – 45.26 = 9.37
   e.g. How much is subtracted from 54.63 to get 9.37?
   We get the unknown subtrahend if we subtract the difference from the reductant.
   C: 54.63
   C: 9.37
   C: 45.26

c) 1266.3 – 452.6 = 813.7
   e.g. How much is 452.6 taken away from to get 813.7?
   We get the reductant of a subtraction if we add the subtrahend and the difference.
   C: 813.7
   C: 452.6
   C: 1266.3

D) 5 × £25.74 = £128.70
   e.g. What amount is multiplied by 5 to get £128.70?
   We get the unknown factor of a product if we divide the product by the known factor.
   C: £128.70
   C: 5
   C: £25.74

E) £17.60 ÷ 2 = £8.80
   e.g. What do we divide £17.60 by to get £8.80?
   We get the unknown divisor if we divide the dividend by the quotient.
   (Ps might notice that £8.80 is half of £17.60)
   C: £17.60
   C: 2
   C: £8.80

F) £123.20 ÷ 8 = £15.40
   e.g. What amount is divided by 8 to get £15.40?
   We get the unknown dividend if we multiply the quotient by the divisor.
   C: £123.20
   C: 8
   C: £15.40

15 min
### Activity

#### Problem 1

Listen carefully and think about how you would solve this problem.

We have several coins.

When we arrange them in rows of 2, 3 or 4, 1 coin is always left over. How many coins could we have?

Allow Ps a minute to think and discuss with neighbours if they wish.

Ps explain their ideas and findings to class. Who agrees? Who thinks something else? etc.

Elicit that the number of coins must be 1 more than a multiple of 2, 3 and 4 (or multiples of 12, as 12 is the first multiple of 2, 3, and 4).

BB:

- Multiples of 2, 3 and 4: 12, 24, 36, 48, 60, 72, 84, 96, 108, . . .
- Possible number of coins: 13, 25, 37, 49, 61, 73, 85, 97, 109, . . .

#### Problem 2

If a square means 100 units, what is the area of each of these rectangles?

Ps come to BB to count the squares and write the area below each column. Class agrees/disagrees.

BB:

- Let's list the numbers in increasing order. Ps dictate to T.
- BB: 50, 100, 200, 300, 500, 650, 800, 900, 1000
- Which is the middle number? P comes to BB to underline it.
- Who remembers the name for the middle number in a set of data? (median) T tells it and writes on BB if nobody remembers.

---

### Notes

Whole class activity

T repeats slowly to give Ps time to think and discuss.

Less able Ps could have counters on desks.

Discussion, reasoning, agreement, praising

Ps check the numbers by saying how many rows of 2, 3 and 4 coins there would be for each quantity of coins.

Whole class activity

Drawn on BB or use enlarged copy master or OHP

At a good pace

Agreement, praising

T chooses Ps at random or class shouts out in unison.

Agreement, praising

BB: median

middle value in a set of data
PbY4b, page 146

Q.1 Read: This graph shows how many people lived in Bananaville on the 1st of January in the years given.

Who can explain the graph? (the horizontal axis shows the years; the vertical axis shows the number of people and there is a grid line at every 20 people; the height of each shaded rectangle above a year shows how many people lived in Bananaville on the 1st of January that year.)

a) Read: Collect the data from the graph and write it in this table.
   Set a time limit. Review at BB with whole class. Ps come to BB to show the relevant rectangle in the graph and fill in the table. Class agrees/disagrees. Mistakes discussed/corrected.
   **Solution:**
   
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>450</td>
<td>420</td>
<td>400</td>
<td>380</td>
<td>400</td>
<td>425</td>
<td>460</td>
<td>500</td>
<td>520</td>
<td>500</td>
<td>480</td>
</tr>
</tbody>
</table>

b) Ps read questions themselves and answer them in Pbs.
   Review with whole class. Ps dictate results or show on scrap paper or slates on command. Ps answering correctly explain at BB to those who were wrong. Mistakes discussed/corrected.
   **Solution:**
   i) When was the population highest? (2000)
   ii) When was the population 500? (1999 and 2001)
   iii) When was the population increasing? (1995 to 2000)

   c) i) Read: Write the population numbers in increasing order.
       Ps dictate to T who writes on BB. Ps also write in Pbs.
       BB: 380, 400, 400, 420, 425, 450, 460, 480, 500, 500, 520
   ii) Read: Which number is the median (in the middle)?
       Ps shout out in unison. (450)

   Which number is the most frequent? Elicit that 400 and 500 both occur twice. Who remembers what the most frequent value in a set of data is called? (mode) T tells it if Ps have forgotten.
Q.2 Read: The table shows the number of pupils in the different years in a school.
   a) Show the data in the graph.
   b) Write the pupil numbers in increasing order.
   c) What is the median?

First make sure that Ps understand the table and graph. Set a time limit. Deal with one step at a time if necessary.

Review at BB with whole class. Ps come to BB to draw rectangles (bars) on the graph, then to write the data in order and underline the median. Class points out errors. Mistakes discussed and corrected.

Ps might have a problem with the median and point out that there is no single middle number. What should we do? (The median is half-way between the two numbers.) Ps suggest how to calculate it.

Solution:

   a) 38, 40, 41, 42, 42, 46

   b) Median: \((41 + 42) \div 2 = 83 \div 2 = 41.5\)

Which number is the mode? (42)

38 min

Cash and debt

T calls 5 Ps to front of class and gives them cards showing cash and debt. Each P in turn writes an operation on the BB to show their balance, explaining reasoning loudly and showing on class number line.

Let's show the balances in a table. Each of the 5 Ps complete their column. Class agrees/disagrees.

BB: e.g.

<table>
<thead>
<tr>
<th>Children</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Balance (£)</td>
<td>+3</td>
<td>-3</td>
<td>0</td>
<td>+4</td>
<td>-2</td>
</tr>
</tbody>
</table>

Let's show it as a bar graph. T has axes already prepared and Ps come to BB or OHP to draw and colour the bars. Class agrees/disagrees.

Let's put the balances in increasing order. Ps come to BB or dictate to T.

BB: \(-3, -2, 0, +3, +4\)

What is the median? (0) What is the mode? (There isn't one.)

What is the difference between the smallest and greatest balances? (£7) This is called the range of the data. Show it on the graph.

45 min

Extra praise if Ps remembered what to do without help.

(Rectangles can be wider and touch each other.)

Or \(42 - 41 = 1, 1 \div 2 = \frac{1}{2}\)

Median: \(41 + \frac{1}{2}\), or \(42 - \frac{1}{2}\)

Ps shout out in unison.

Whole class activity

T calls 5 Ps to front of class and gives them cards showing cash and debt. Each P in turn writes an operation on the BB to show their balance, explaining reasoning loudly and showing on class number line.

Let's show the balances in a table. Each of the 5 Ps complete their column. Class agrees/disagrees.

BB: e.g.

<table>
<thead>
<tr>
<th>Balance (£)</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Children</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
</tbody>
</table>

Let's put the balances in increasing order. Ps come to BB or dictate to T.

BB: \(-3, -2, 0, +3, +4\)

What is the median? (0) What is the mode? (There isn't one.)

What is the difference between the smallest and greatest balances? (£7) This is called the range of the data. Show it on the graph.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y4</strong></td>
<td><strong>Lesson Plan</strong></td>
</tr>
<tr>
<td><strong>R:</strong> Calculations</td>
<td><strong>147</strong></td>
</tr>
<tr>
<td><strong>C:</strong> Collecting and displaying data</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> Different graphs. Problems</td>
<td></td>
</tr>
</tbody>
</table>

| 1 | Factorising  
Factorise 147 in your Ex. Bks and then list all its factors.  
Review with whole class. Ps come to BB or dictate to T, explaining reasoning (e.g. 3 is a factor of 147 because \(1 + 4 + 7 = 12\), which is a multiple of 3). Class agrees/disagrees. Mistakes corrected.  
**BB:**  
\[
\begin{align*}
147 &= 3 \times 7 \times 7 \\
3 &\quad 49 \\
7 &\quad 7 \\
\end{align*}
\]  
Factors: 1, 3, 7, 21, 49, 147 | Individual work, monitored  
Discussion, reasoning, agreement, self-correction, praising  
(Ps may use a calculator.)  
Feedback for T |
| 2 | Problem  
Listen carefully and think about how you would solve this problem.  
*We have between 100 and 200 marbles. When we arrange them in rows of 2, 3, 4 or 5, one marble is always left over. How many marbles could we have?*  
Allow Ps a minute to think and discuss with neighbours if they wish. Ps explain their ideas and findings to the class. Who agrees? Who thinks something else? etc.  
Elicit that the number of marbles must be 1 more than a multiple of 2, 3, 4 and 5 (or a multiple of 60, as 60 is the first multiple of 2, 3, 4 and 5).  
**BB:**  
Multiples of 2, 3, 4 and 5: 60, 120, 180, 240, 300, . . .  
So possible numbers of marbles are: 61, 121, 181, 241, 301, . . .  
but the only numbers between 100 and 200 are **121** and **181**.  
**Answer:** We could have 121 or 181 marbles | Whole class activity  
T repeats slowly to give Ps time to note down the data.  
Discussion, reasoning, agreement, praising  
Ps check the two numbers by saying how many rows of 2, 3, 4, 5 marbles there would be. |
| 3 | Missing numbers  
Let’s complete the operations. Ps come to BB to say the operation in words, explain what they have to do to solve it and then do the calculation. Class checks mentally that they are correct.  
**BB:**  
\[
\begin{align*}
a) \quad \frac{2}{5} + \frac{4}{5} &= \frac{6}{5} \\
b) \quad 3\frac{1}{4} - \frac{3}{4} &= 2\frac{1}{2} \\
c) \quad 4 - 1\frac{1}{6} &= 2\frac{5}{6} \\
d) \quad \frac{3}{8} \times 3 &= \frac{1}{8} \\
e) \quad \frac{5}{7} \div 5 &= \frac{1}{7} \\
f) \quad \frac{2\frac{2}{3}}{4} &= \frac{2}{3} \\
\end{align*}
\]  
\(\text{How many \(\frac{1}{7}\)s are in \(\frac{5}{7}\)?}\)  
\(\text{(How many \(\frac{1}{7}\)s are in \(\frac{5}{7}\)?)}\) | Whole class activity  
Written on BB or SB or OHT  
e.g. Ps:  
a) ‘What must be added to \(2\) fifths to get \(6\) fifths?’  
‘To find the missing term in an addition, subtract the known term from the sum.’  
T helps with wording when necessary.  
At a good pace  
Reasoning, agreement, praising
Displaying data.
T has various graphs copied from books, magazines or newspapers and
enlarged (e.g. pie charts, tally charts, histograms, bar charts,
pictograms, line graphs, etc.).
Ps come to BB to explain the meaning of each graph and to ask and
answer questions about the data shown. Class agrees/disagrees.
T could have some questions already prepared, in case Ps cannot think of
any.

24 min

Median
T has sets of numbers written on BB or SB or OHT. Which number
is in the middle of the set? What is it called? (the median)
Ps come to BB circle the median, explaining reasoning. Class agrees/
disagrees. T helps in sets with even numbers if necessary.
What do you notice about any of the sets? e.g.

BB
a) 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 (increasing by 1)
b) 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24 (positive even nos.)
   ↓
   13 [ (12 + 14) ÷ 2 = 26 ÷ 2 = 13 ]
c) 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 13, 37 (prime numbers)
d) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100 (square numbers)
   ↓
   30.5 [ (25 + 36) ÷ 2 = 61 ÷ 2 = 30.5 ]
e) – 11, – 9, – 7, – 5, – 3, 1, 3, 5, 7, 9 (increasing by 2)
   ↓
   30 min

PbY4b, page 147
Q.1 Read This graph shows the highest point of some mountain
ranges and the deepest point of some seas.
T has a large map of the world beside the BB. Which of the
mountains or seas have you heard of? Where is it on this map?
Which country is it in (near)? Who has been there? etc.
Ps tell of those they know and show them on the map. T points
out any that Ps have not heard of.
Read: Read the graph and fill in the approximate missing values.
Who can explain what the graph means? (The horizontal axis
shows the mountains or seas, represented by triangles; the
vertical axis shows the height in metres, with a grid line at
every 1000 m)
Ps come to BB to read the data from the graph and fill in the
missing numbers. T helps with closer approximation if Ps’
reading is too rough. Ps fill in agreed heights in Pbss too.
Ps read questions themselves and answer in Pbss. Set a time limit.
Review with whole class. Ps dictate answers to T and confirm on
the graph. Mistakes discussed and corrected.
6. Activity (Continued)

**Solution:**

<table>
<thead>
<tr>
<th>Height (m)</th>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-5000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-10 000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. Alps = 4800 m  
2. Carpathians = 2600 m  
3. Himalayas = 8900 m  
4. Adriatic Sea = 1600 m  
5. Mediterranean Sea = -4600 m  
6. Atlantic Ocean = -9300 m  
7. Indian Ocean = -8100 m  
8. Pacific Ocean = -11 600 m

a) **Which is higher, the Alps or the Carpathian Mountains?**  (Alps)

How much higher? BB: 4800 m > 2600 m  
2200 m

b) **Which sea is deeper, the Mediterranean or the Adriatic?**  (Med.)

How much deeper? BB: -4600 m < -1600  
3000 m

c) **What is the difference between the highest mountain and the deepest sea?**

BB: 8900 m - (-11 600 m) = 8900 m + 11 600 m = 20 500 m  
Elicit that this is the range of the data. Show on the vertical axis.

38 min

7. **PbY4b, page 147**

Q.2 Read: **How many acorns did the Squirrel family collect each day? Complete the diagram.**

Tell (elicit) that data shown in the form of pictures is called a pictogram. What does half an acorn represent? (75 acorns)

Set a time limit. Ps write operations horizontally in Pbs.

Review at BB with whole class. Ps come to write on BB or dictate to T, explaining reasoning. Mistakes discussed/corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Day</th>
<th>Acorns</th>
<th>Calculation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td></td>
<td>5 x 150 = 750 + 250</td>
<td>1000</td>
</tr>
<tr>
<td>Tuesday</td>
<td></td>
<td>4 x 150 = 400 + 200</td>
<td>600</td>
</tr>
<tr>
<td>Wednesday</td>
<td></td>
<td>5 x 150 + 75 = 750 + 75</td>
<td>825</td>
</tr>
<tr>
<td>Thursday</td>
<td></td>
<td>3 x 150 + 300 = 150</td>
<td>450</td>
</tr>
<tr>
<td>Friday</td>
<td></td>
<td>4 x 150 + 75 = 600 + 75</td>
<td>675</td>
</tr>
<tr>
<td>Saturday</td>
<td></td>
<td>3 x 150 + 75 = 450 + 75</td>
<td>525</td>
</tr>
<tr>
<td>Sunday</td>
<td></td>
<td></td>
<td>0</td>
</tr>
</tbody>
</table>

Ps add up the columns and show result on command. (3825)

38 min

**Extension**

What is the mode? (none)

What is the median? (600)

What is the range? (825)

45 min

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**Y4**

**Activity**

1. **Factorising**

   Factorise 148 in your Ex. Bks and then list all its factors.

   Review with whole class. Ps come to BB to draw a factor tree and explain their reasoning. Class agrees/disagrees. Mistakes corrected.

   BB: 148 = 2 × 2 × 37

   Factors: 1, 2, 4, 37, 74, 148

   **4 min**

2. **Counting in different bases**

   Ps have 17 counters (or coins or cubes, etc.) on desks and T has 17 circles stuck on BB at random.

   a) Let's put the counters in groups of 2.

   Elicit that there are 8 groups of 2, and 1 single unit left over.

   Now let's put the groups of 2 in twos. (i.e. 2 × 2 = 4 per group)

   Elicit that there are 4 groups of 4, no groups of 2, and 1 left over.

   Now let's put the groups of 4 in twos. (i.e. 2 × 2 × 2 = 8 per group)

   Elicit that there are now 2 groups of 8, no groups of 4, no groups of 2, and 1 left over.

   If we continue in this way, what will the next grouping be?

   (Put the 8-element groups in twos.) i.e. 2 × 2 × 2 × 2 = 16 per group

   Elicit that there is now 1 group of 16, no groups of 8, no groups of 4, no groups of 2, and 1 left over.

   Let's show in a table how we can make 17 using 2 as a base for counting. T draws table and Ps dictate the headings and place-values.

   BB:

<table>
<thead>
<tr>
<th>Base 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>32</td>
</tr>
<tr>
<td>16</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

   **17 = 1 × 16 + 1 × 1**

   b) Let's group the 17 counters using 3 as the base. Ps manipulate counters on desks then dictate to T or come to BB.

   BB:

<table>
<thead>
<tr>
<th>Base 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

   **17 = 1 × 9 + 2 × 3 + 1 × 1**

   c) Let's group them using 4 as the base.

   BB:

<table>
<thead>
<tr>
<th>Base 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

   **17 = 1 × 16 + 1 × 1**

   d) Let's group them using 5 as the base.

   BB:

<table>
<thead>
<tr>
<th>Base 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>-</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

   **17 = 3 × 5 + 2 × 1**

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# Y4 Lesson Plan 148

## Activity

2  
(Continued)

e) Let's group them using 10 as the base.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base 10</strong></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>10</td>
</tr>
<tr>
<td>-</td>
<td>1</td>
</tr>
<tr>
<td>17 = 1 × 10 + 7 × 1</td>
<td></td>
</tr>
</tbody>
</table>

Of course, this is the base that we usually use in counting!

---

### Week 30

#### 3 Missing numbers

Let's complete the operations. Ps come to BB to explain the operation in words and fill in the missing number either by counting on the class number line or by calculation. Class agrees/disagrees.

**BB:**

<table>
<thead>
<tr>
<th>5 + [ ] = -4</th>
<th>4 - [ ] = -3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(by counting 9 to the left from 5)</td>
<td>(by counting 7 to the left from 4)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 [ ] = 5</th>
<th>-5 [ ] = -20</th>
</tr>
</thead>
<tbody>
<tr>
<td>[5 + (-3) = 5 - 3 = 2]</td>
<td>[(-5)+(-5)+(-5)+(-5) = -20]</td>
</tr>
</tbody>
</table>

e) \[-14 \div 7 = -2\]  
(by guessing, or \((-2)+(-2)+(-2)+(-2)+(-2)+(-2)+(-2) = -14\)
f) \[-12 \div 3 = -4\]  
(by guessing, or \((-4) \times 3 = (-4)+(-4)+(-4) = -12\)

---

#### 4 Problem

Listen carefully and think about how you would solve this problem.

*We have several coins.*

- When we arrange them in rows of 2, 1 coin is left over.
- When we arrange them in rows of 3, 2 coins are left over.
- When we arrange them in rows of 4, 3 coins are left over.

*How many coins could we have?*

Allow Ps a minute to think and discuss with neighbours if they wish.

Ps explain their ideas and findings to the class. Who agrees? Who thinks something else? etc.

Elicit that the number must be odd (1 more than a multiple of 2), 2 more than an odd multiple of 3, and 3 more than a multiple of 4.

**BB:** e.g.

Possible multiples of 3: 3, 9, 15, 21, 27, 33, 39, 45, 51, . . .

Add on 2 more: 5, 11, 17, 23, 29, 35, 41, 47, 53, . . .

Subtract 3 and underline the multiples of 4: 2, 8, 14, 20, 26, 32, 38, 44, 50, . . .

What do you notice? (The possible numbers form a sequence, starting at 11 and increasing by 12.)

**Answer:** We could have 11, 23, 35, 47, 59, . . . coins.

**Whole class activity**

Ps note data in *Ex. Bks.*

Ps could have counters on desks to help them.

Discussion, reasoning, agreement, checking

Praise all positive contributions.

If Ps have no good ideas, T gives hints and leads Ps through the solution shown opposite.
Y4

**Activity**

**Lesson Plan 148**

Q.1  
**PbY4b, page 148**

a) **Read:**  Group the elements by 3. Make groups of 3 by circling in red.  
Then circle every 3 red groups in green.  
Then circle every 3 green groups in blue.  
Write the number of different groups and the remainder in the table.

Liken the task to the earlier class activity about counting in different bases. Make sure that Ps realise that it is the final grouping which should be described in the table.

Set a time limit. Ps finished first come to BB to draw the groups (or T has solution already prepared).

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

How many stars are there altogether? (44)

**Solution:**

<table>
<thead>
<tr>
<th>Number in each group</th>
<th>27</th>
<th>9</th>
<th>3</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of groups</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

b) **Read:**  Group the elements by 4 in a similar way.  
*Fill in the table.*

As in a). If class is very able, review a) and b) together.

How many triangles are there? (44)

**Solution:**

<table>
<thead>
<tr>
<th>Number in each group</th>
<th>16</th>
<th>4</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of groups</td>
<td>2</td>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

**Notes**

- Individual work, monitored, helped
- Drawn (stuck) on BB or use enlarged copy master or OHP
- Allow Ps to explain task if possible.
- Reasoning, agreement, self-correction, praising

BB:  
1 × 27 + 1 × 9 + 2 × 3 + 2 × 1  
= 27 + 9 + 6 + 2 = 44

(Preparation for studying the number system)

Q.2  
**PbY4b, page 148**

**Read:**  The tally chart shows the months in which 37 pupils in a class were born.

Let’s see if you can do questions a) to d) on your own.

Set a time limit. (Or deal with one part at a time if class is unsure and review before next step.)

Review at BB with whole class. Ps come to BB or dictate to T (or T could have the graph already prepared). Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) **Write the number of pupils in the bottom row of the table.**

<table>
<thead>
<tr>
<th>Number born in each month</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan</td>
</tr>
<tr>
<td>1</td>
</tr>
</tbody>
</table>

**Notes**

- Individual work, monitored, helped for parts a) to d)
- Table drawn on BB or use enlarged copy master or OHP
- Discussion, reasoning, agreement, self-correction, praising

(or activity done with real data collected from Ps in class and Ps’ form own class tally chart and graph)
**Activity 6**

(Continued)

b) Draw a graph about the table.

BB:

![Graph Diagram]

Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec

Pupils

0 1 2 3 4 5 6

Months

Accept thick lines as here or shaded columns centred on the ticks for each month.

c) Put the data in order.

BB: 0, 1, 2, 3, 3, 3, 3, 4, 4, 5, 6

d) Which are the middle data? (3 and 3)

What is the median? \((3 + 3) ÷ 2 = 3\)

e) Read: Think of another 37 people. Would this statement about them be certain, possible or impossible?

At least 4 people were born in the same month.

T asks several Ps what they think and why. Class decides on correct answer. [Certain]

Reasoning: e.g.

Among 36 people, at least 3 must be born in the same month, as there are only 12 different months. So the 37th person must make 4 people born in one of the months.

37 min

**PbY4b, page 148**

Q.3 Read: 60 pupils were given a choice of 4 activities. How many pupils chose each one and what fraction of them chose it? Use the pie chart to complete the table.

How many equal parts has the pie chart been divided into? (12)

Work out the fraction first, then calculate the number of Ps for each activity. Set a time limit.

Review with whole class. Ps come to BB to complete table, explaining their reasoning by referring to the pie chart. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

![Pie Chart Diagram]

BB: 5 + 30 + 20 + 15 = 60

\(\frac{1}{12} + \frac{6}{12} + \frac{2}{12} + \frac{3}{12} = \frac{12}{12}\)

41 min

**Real data**

T chooses 12 (or 18) Ps to say, e.g. which fruit they prefer (or Ps could choose the subject). Ps come to BB to make a tally above each column in the table, fill in the numbers and fractions and complete the pie chart. Class (T) helps and corrects when necessary.

45 min

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, praising In good humour!
## Y4

### Activity

#### 1

**Factorising**

Let's factorise 149 and list all its factors. Ps try out the prime numbers 2, 3, 5, 7, 11 as divisors and dictate their findings. e.g.

- 2 is not a factor because 149 is odd;
- 3 is not a factor because $1 + 4 + 9 = 14$, which is not a factor of 3;
- 5 is not a factor because the units digit is not 0 or 5;
- 7 is not a factor because $149 \div 7 = 21$, r 2;
- 11 is not a factor because $149 \div 11 = 13$, r 6

What is the next prime number? (13) Should we try 13? (No, as $13 \times 13 = 169$, which is more than 149.)

So 149 is a prime number and has factors 1 and 149.

---

#### 2

**Numberlands**


How would they each count 149 marbles? Let's help them complete their tables. Ps come to BB to choose a base number, work out the table headings and divide up 149 into the appropriate groups. Class agrees/disagrees. T directs Ps in using a systematic method.

**BB:** e.g. Starting at LH side of table:

<table>
<thead>
<tr>
<th>Alfie</th>
<th>Jacky</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 10</td>
<td>Base 2</td>
</tr>
<tr>
<td>149 \div 100 = r 49</td>
<td>149 \div 128 = r 21</td>
</tr>
<tr>
<td>49 \div 10 = r 9</td>
<td>21 \div 64 = r 21</td>
</tr>
<tr>
<td>9 \div 1 = r 0</td>
<td>21 \div 32 = r 5</td>
</tr>
<tr>
<td><strong>Check:</strong> $1 \times 100 + 4 \times 10 + 9 \times 1 = 149 ✔$</td>
<td><strong>Check:</strong> $1 \times 128 + 1 \times 16 + 1 \times 4 + 1 \times 1 = 149 ✔$</td>
</tr>
</tbody>
</table>

**Check:** $1 \times 100 + 4 \times 10 + 9 \times 1 = 149 ✔$

<table>
<thead>
<tr>
<th>Cindy</th>
<th>Sammy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base 3</td>
<td>Base 7</td>
</tr>
<tr>
<td>149 \div 81 = r 68</td>
<td>149 \div 49 = r 2</td>
</tr>
<tr>
<td>68 \div 27 = r 14</td>
<td>2 \div 7 = r 2</td>
</tr>
<tr>
<td>14 \div 9 = r 5</td>
<td>2 \div 1 = r 2</td>
</tr>
</tbody>
</table>
| **Check:** $3 \times 49 + 0 \times 7 + 2 \times 1 = 149 ✔$ | **Check:** $1 \times 81 + 2 \times 27 + 1 \times 9 + 1 \times 3 + 2 \times 1 = 149 ✔$

---

### Notes

**Whole class activity**

At a good pace
Reasoning, agreement, checking, praising
Ps do not really need a calculator but allow them to use one if they wish.
### Activity 3

**Inequalities**

What whole numbers could the rectangles represent in these inequalities? Ps come to BB to fill in the possible numbers, explaining reasoning. Calculations can be done at side of BB. Class agrees/disagrees and checks that the numbers make the inequalities true.

**BB:**

- a) $687 + \square < 1425$
- b) $7200 - \square \geq 1500$
- c) $\square - 400 > 630$
- d) $7 \times \square < 3500$
- e) $250 \div \square \geq 25$
- f) $\square \div 3 > 22$

<table>
<thead>
<tr>
<th>a) $687 + \square &lt; 1425$</th>
<th>b) $7200 - \square \geq 1500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\square &lt; 1425 - 687$</td>
<td>$\square \leq 7200 - 1500$</td>
</tr>
<tr>
<td>$\square &lt; 738$</td>
<td>$\square \leq 5700$</td>
</tr>
<tr>
<td>$\square : 737, 736, 735, \ldots$</td>
<td>$\square : 5700, 5699, 5698, \ldots$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c) $\square - 400 &gt; 630$</th>
<th>d) $7 \times \square &lt; 3500$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\square &gt; 400 + 630$</td>
<td>$\square &lt; 3500 \div 7$</td>
</tr>
<tr>
<td>$\square &gt; 1030$</td>
<td>$\square &lt; 500$</td>
</tr>
<tr>
<td>$\square : 1031, 1032, \ldots$</td>
<td>$\square : 499, 498, 497, \ldots$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>e) $250 \div \square \geq 25$</th>
<th>f) $\square \div 3 &gt; 22$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\square \leq 250 \div 25$</td>
<td>$\square &gt; 22 \times 3$</td>
</tr>
<tr>
<td>$\square \leq 10$</td>
<td>$\square &gt; 66$</td>
</tr>
<tr>
<td>$\square : 10, 9, 8, \ldots$</td>
<td>$\square : 67, 68, 69, \ldots$</td>
</tr>
</tbody>
</table>

**4 PbY4b, page 149**

**Q.1** Read: *Four children, 1 eighth of the class, have a green school bag and 3 eighths of the class have a blue bag. Eight children have a red bag and the rest have yellow bags.*

**Colour the pie chart to show the data. Complete the table.**

Set a time limit. Review with whole class. Ps come to BB to colour pie chart and fill in numbers and fractions, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Colour of bag</th>
<th>Green</th>
<th>Blue</th>
<th>Red</th>
<th>Yellow</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of pupils</td>
<td>4</td>
<td>12</td>
<td>8</td>
<td>8</td>
<td>32</td>
</tr>
<tr>
<td>Fraction</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{3}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
<td>$\frac{1}{8}$</td>
</tr>
</tbody>
</table>

Extra praise for Ps who notice that $\frac{2}{8} = \frac{1}{4}$
### Activity

**PbY4b, page 149**

#### Q.2 Read: A chain of supermarkets made a pictogram of how many pies they had sold in a year.

Each pie on the diagram means 1000 real pies.

Who can explain the pictogram? (The number of whole pies tells you how many thousands of real pies that they sold; the pies are divided into 8 equal segments, so each part means 1 eighth of 1000; half a pie means 500 real pies, 1 quarter of a pie means 250 real pies and 3 quarters of a pie means 750 real pies.)

a) Read: Fill in the missing numbers and draw pies to show the numbers given.

Set a time limit. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected before doing other questions.

**Solution:**

<table>
<thead>
<tr>
<th>Month</th>
<th>January</th>
<th>February</th>
<th>March</th>
<th>April</th>
<th>May</th>
<th>June</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>February</td>
<td></td>
<td>4000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>March</td>
<td></td>
<td>3500</td>
<td></td>
<td></td>
<td></td>
<td>3250</td>
</tr>
<tr>
<td>April</td>
<td>4250</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>May</td>
<td></td>
<td>3500</td>
<td></td>
<td></td>
<td></td>
<td>4125</td>
</tr>
<tr>
<td>June</td>
<td>3000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Ps read questions b) to d) themselves and write answers in *Pbs*. Review with whole class. Ps come to BB or dictate to T, explaining reasoning, or Ps could show results on scrap paper or slates for c) and d). Mistakes discussed and corrected.

**Solution:**

b) Write the data in increasing order:

BB: 2000, 2500, 2750, 3000, 3000, 3250, 3500, 3500, 4000, 4125, 4250, 4750

c) What is the difference between the first and last numbers?

BB: 4750 – 2000 = 2750

Elicit that this is called the range of the data.

d) Underline the two middle numbers. Which number is half-way between them? This is the median.

BB: Median: 

\[
\frac{(3250 + 3500)}{2} = 6750 \div 2 = 3375
\]

or 

\[
\frac{(3500 - 3250)}{2} = 250 \div 2 = 125,
\]

or 

\[
3250 + 125 = 3375 \text{ or } 3500 - 125 = 3375
\]

36 min

---

### Notes

**Lesson Plan 149**

Whole class discussion to start

Drawn (stuck) on BB or use enlarged copy master or OHP

Clarification of meaning of pictogram to start

Involves several Ps.

Agreement, praising

Individual work, monitored, helped

Drawings need only be rough.

Necessary calculations can be done on scrap paper or slates or in *Ex. Bks.*

BB: e.g.

\[
1000 \div 8 = 800 \div 8 + 160 \div 8 + 40 \div 8 = 100 + 20 + 5 = 125
\]

or 

\[
250 \div 2 = 125
\]

Individual work, monitored, helped

Reasoning, agreement, self-correction, praising

(If no Ps used the 2nd method, T could show it and explain by drawing on BB the relevant segment of the number line.)

Which way do you think is easiest? Why?
**Y4**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 149</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>6</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td><em>PbY4b, page 149, Q.3</em></td>
<td>Whole class activity (or individual or paired trial first if Ps wish)</td>
</tr>
<tr>
<td>Read: <strong>67 scientists are at a conference. 47 speak French, 35 speak German and 23 speak both languages. How many of them speak neither French nor German? Complete the Venn diagram.</strong></td>
<td>Drawn on BB or use enlarged copy master or OHP.</td>
</tr>
<tr>
<td>Who has an idea of what we should do? Who agrees? Who thinks something else? etc. Ps come to BB to explain their reasoning and write calculations. T gives hints only if Ps are stuck.</td>
<td>Discussion, reasoning, agreement, praising</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td><strong>BB:</strong></td>
</tr>
<tr>
<td>French: 47; German: 35; French + German: 23</td>
<td><img src="image" alt="Venn diagram" /></td>
</tr>
<tr>
<td>so number speaking: French but not German: 47 – 23 = 24</td>
<td>Number speaking German or French: 23 + 24 + 12 = 59,</td>
</tr>
<tr>
<td>German but not French: 35 – 23 = 12</td>
<td>so number not speaking German or French: 67 – 59 = 8</td>
</tr>
<tr>
<td>Number speaking German or French: 23 + 24 + 12 = 59,</td>
<td><strong>Answer:</strong> 12 dictionaries would be needed.</td>
</tr>
<tr>
<td>so number not speaking German or French: 67 – 59 = 8</td>
<td>45 min</td>
</tr>
</tbody>
</table>

| **7**    | **Notes**       |
| _PbY4b, page 149_ | Individual work, monitored, helped |
| Q.4 **Read:** How many dictionaries would be needed to translate among these languages: English, German, French, Spanish? | (T could have some such dictionaries to show to class.) |
| Allow Ps to try it in Ex. Bks. for a couple of minutes. If Ps are struggling, T could give a hint about drawing tree diagrams. | In unison on scrap paper or slates |
| If you found an answer, show me . . .now! (12) | Discussion, reasoning, agreement, self-correction, praising |
| P answering correctly explains reasoning. Some Ps might answer 6, forgetting that, e.g. E → G is not the same as G → E | Or by reasoning: |
| **Solution:** | Each of the 4 languages would need a dictionary for each of the other 3 languages. |

| **41 min** | **45 min** |
**Y4**

**Lesson Plan**

150

**Activity**

Tables and calculation practice, revision, activities, consolidation

*PhY4b, page 150*

**Notes**

**Solutions:**

Q.1  

<table>
<thead>
<tr>
<th>Children</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash (£)</td>
<td>4</td>
<td>0</td>
<td>4</td>
<td>10</td>
<td>6</td>
</tr>
<tr>
<td>Debt (£)</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>Balance (£)</td>
<td>3</td>
<td>–6</td>
<td>0</td>
<td>5</td>
<td>–2</td>
</tr>
</tbody>
</table>

b) Balance (£)

\[\begin{array}{c}
A \\
B \\
C \\
D \\
E \\
\end{array}\]

C) \(-6, -2, 0, 2, 5\)

d) Range: \(5 - (-6) = 5 + 6 = 11\)

e) Median: 0

Q.2  

a) Height (m)

\[\begin{array}{c}
0 \\
\frac{1}{2}
\end{array}\]

b) 5, 5.2, 5.4, 5.8, 6, 6.5, 13.6, 14, 14, 14.5, 15

c) Range: \(15 - 5 = 10\) (m)

d) Median: 6.5 m

(\text{Mode: 14})

Q.3  

Total children: \(10 + 20 + 20 + 30 = 80\)

Pie chart:

Q.4  

\[60 = 2 \times 2 \times 3 \times 5\]

Only 10 different possible ways
### Y4 Lesson Plan 151

#### Activity

**1**  
**Factorising**

In your Ex. Bks, factorise 150 and 151 and then list all their factors. Review at BB with whole class. Ps come to BB to draw tree diagram, show the numbers as the product of their prime factors, and list all their factors. Class agrees/disagrees. Mistakes discussed and corrected.

BB: e.g.  
150 = 2 x 3 x 5 x 5  
151 is a prime number  
(not divisible by 2, 3, 5, 7 or 11,  
and 13 x 13 = 169 > 151)

Factors: 150: 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150  
151: 1, 151

---

**2**  
**Different bases**

Imagine that we have 151 counters. Let’s group them using different numbers as the base. Ps first dictate headings for the place value table for each base number. Then Ps show the groupings in two ways: starting from the LHS and then the RHS of the table. Ps come to BB or dictate operations to T, with T's help if necessary. Class points out errors.

BB: e.g.

a) Grouping by 3

<table>
<thead>
<tr>
<th>Starting at LHS of table:</th>
<th>Or starting at RHS of table:</th>
</tr>
</thead>
<tbody>
<tr>
<td>151 ÷ 81 = 1 r 70</td>
<td>151 ÷ 3 = 50, r 1</td>
</tr>
<tr>
<td>70 ÷ 27 = 2 r 16</td>
<td>50 ÷ 3 = 16, r 2</td>
</tr>
<tr>
<td>16 ÷ 9 = 1 r 7</td>
<td>16 ÷ 3 = 5, r 1</td>
</tr>
<tr>
<td>7 ÷ 3 = 2 r 1</td>
<td>5 ÷ 3 = 1, r 2</td>
</tr>
<tr>
<td>1 ÷ 1 = 0</td>
<td>1 ÷ 3 = 0, r 1</td>
</tr>
</tbody>
</table>

Check: 1 x 81 + 2 x 27 + 1 x 9 + 2 x 3 + 1 x 1 = 151 ✓

b) Grouping by 4

<table>
<thead>
<tr>
<th>Starting at LHS of table:</th>
<th>Or starting at RHS of table:</th>
</tr>
</thead>
<tbody>
<tr>
<td>151 ÷ 64 = 2 r 23</td>
<td>151 ÷ 4 = 37, r 3</td>
</tr>
<tr>
<td>23 ÷ 16 = 1 r 7</td>
<td>37 ÷ 4 = 9, r 1</td>
</tr>
<tr>
<td>7 ÷ 4 = 1 r 3</td>
<td>9 ÷ 4 = 2, r 1</td>
</tr>
<tr>
<td>3 ÷ 1 = 0</td>
<td>2 ÷ 4 = 0, r 0</td>
</tr>
</tbody>
</table>

Check: 2 x 64 + 1 x 16 + 1 x 4 + 3 x 1 = 151 ✓

c) Grouping by 5

<table>
<thead>
<tr>
<th>Starting at LHS of table:</th>
<th>Or starting at RHS of table:</th>
</tr>
</thead>
<tbody>
<tr>
<td>151 ÷ 125 = 1 r 26</td>
<td>151 ÷ 5 = 30, r 1</td>
</tr>
<tr>
<td>26 ÷ 25 = 1 r 1</td>
<td>30 ÷ 5 = 6, r 0</td>
</tr>
<tr>
<td>1 ÷ 5 = 1</td>
<td>6 ÷ 5 = 1, r 1</td>
</tr>
<tr>
<td>1 ÷ 1 = 0</td>
<td>1 ÷ 5 = 0, r 0</td>
</tr>
</tbody>
</table>

Check: 1 x 125 + 1 x 25 + 0 x 5 + 1 x 1 = 151 ✓

---

**Notes**

Individual work, monitored, helped

Discussion, reasoning, agreement, self-correction, praising

Ps may use a calculator.

Ps could join up the factor pairs for 150.

Whole class activity

Deal with one at a time and gradually build up the tables and the two methods of divisions.

At a good pace

Reasoning, agreement, praising

(Ps may use a calculator.)

Feedback for T

---

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## Activity

**Problem**

Listen carefully and think about how you would solve this problem.

We have between 200 and 250 coins.

When we arrange them in rows of 2, 1 coin is left over.
When we arrange them in rows of 3, 2 coins are left over.
When we arrange them in rows of 4, 3 coins are left over.
When we arrange them in rows of 5, 4 coins are left over.

**How many coins could we have?**

Allow Ps a minute to think and discuss with neighbours if they wish. Ps explain their ideas and findings to the class. Who agrees? Who thinks something else? etc.

Elicit that the number of coins must be odd (1 more than a multiple of 2), so the only multiples of 5 possible have units digit 5 (as \(0 + 4 = 4\) which is even).

**BB:** e.g.

Possible multiples of 5: \(205, 215, 225, 235, 245\)
Add on 4: \(209, 219, 229, 239, 249\)

Subtract 2 and underline the multiples of 3: \(207, 217, 227, 237, 247\)

Subtract 1 more and underline the multiples of 4: \(206, 236\)

**Answer:** We have 239 coins.

---

### Notes

**Whole class activity**

T repeats slowly to allow Ps time to note the data.

Discussion, reasoning, checking, praising
Praise all positive contributions.
T gives hints if necessary.

(as the sum of all the digits is a multiple of 3)
(as 36 is divisible by 4)

---

**PhY4b, page 151**

Q.1 Read:

- **a)** Continue the list of 3-digit natural numbers with decreasing digits (to 500).
- **b)** Calculate the difference between the smallest and the greatest.
- **c)** Which are the two middle numbers?

Makes sure that Ps understand the condition for choosing the numbers. Set a time limit. Ps can list the numbers in Ex. Bks. if they need more space.

Review with whole class. Ps dictate numbers or come to BB. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- **a)** \(210; 310, 320, 321; 410, 420, 421, 430, 431, 432;\)
- **b)** \(432 - 210 = 222\) (Elicit that this is the range of the sample of 10 numbers.)
- **c)** Two middle numbers: \(410\) and \(420\)

---
PbY4b, page 151

Q.2 Read: Jack is in training for a marathon. These were the distances he ran every day last week.
What do you notice about the distances? What should you do first? (Change all the distances to metres to match the unit of measure on the graph.)
BB: 2.9 km = 2900 m, 10 km = 10 000 m

a) Read: Show the data in a graph.
   Elicit that the distances are shown on the vertical axis, with a grid line at every 200 m, and the days are on the horizontal axis. Set a time limit. Ps can draw the rectangle for each day of the week in a different colour.
   Review at BB with whole class. Ps come to BB or T has solution already prepared. Mistakes corrected.
   **Solution:**

   ![Graph showing distances run (m)]

   Mon: 2800 m
   Tue: 4300 m
   Wed: 3500 m
   Thu: 2.9 km
   Fri: 3200 m
   Sat: 10 km
   Sun: 6800 m

Ps read questions b) to d) themselves and answer them in Pbs.
Review at BB with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solutions:**

b) List the distances in increasing order.
   BB: 2800 m, 2900 m, 3200 m, 3500 m, 4300 m, 6800 m, 10 000 m

c) What is the difference between the smallest and greatest distance?
   BB: 10 000 m – 2800 m = 7200 m
   Elicit that this is the range of the data.

d) Read: What is the median (the middle number)?
   Median: 3500 m

---

**Notes**

Individual work, monitored, helped
Drawn on BB or use enlarged copy master or OHP
Initial discussion to clarify the meaning of the graph.

Discussion, reasoning, agreement, self-correction, praising

(or bars can be the width of the column and touch either)

Individual work, monitored under a time limit
Reasoning, agreement, self-correction, praising

(Or Ps could show responses for c) and d) on scrap paper or slates in unison on command.)
### Activity

**PbY4b, page 151, Q.3**

Read: Among 67 scientists at a conference:
- 47 speak French,
- 35 speak German,
- 20 speak Spanish,
- 12 speak French and Spanish,
- 11 speak German and Spanish,
- 5 speak all three languages.

Who can show us where each piece of information is in the diagram? Ps come to BB to say the information and to point to the matching area. Class agrees/disagrees.

a) Read: Complete the Venn diagram.

Who knows how to work out one of the missing numbers? Ps come to BB to fill in a number and explain their reasoning. Class agrees/disagrees. T gives hints only if Ps are stuck. Ps fill in diagram in PbS too.

**Solution:** e.g.

- number speaking only Spanish and German: 11 – 5 = 6
- number speaking only Spanish: 20 – (7 + 5 + 6) = 20 – 18 = 2
- number speaking only German: 35 – (18 + 5 + 6) = 35 – 29 = 6
- number not speaking a language:

\[
67 – (17 + 18 + 7 + 5 + 6 + 2 + 6) = 67 – 61 = 6
\]

T reads one question at a time and Ps could show results on scrap paper or slates on command. Ps responding correctly explain at BB to those who were wrong.

b) How many scientists speak:
   i) only French (17)
   ii) only German (6)
   iii) only Spanish? (2)

c) How many scientists speak Spanish and German but not French? (6)

d) How many scientists speak neither Spanish, nor German, nor French? (6)

Who can think of another question to ask about the Venn diagram?

---

### Extension

**Diagonals**

What is a pentagon? (plane shape with 5 straight sides)

Draw a pentagon in your Ex. Bks and label the vertices A, B, C, D, and E. It does not need to be a **regular** hexagon (i.e. its sides need not be equal).

What is a diagonal? (It is a straight line which joins up one vertex in a polygon to another vertex which is not adjacent to it.) Ps draw them in Ex. Bks.

How many diagonals does a pentagon have? (5)

Ps name them. (e.g. AC, AD, BD, BE, CE) and T draws them on diagram on BB.

What if I draw the pentagon like this? BB: 

Does it still have 5 diagonals? (Yes)

---

### Notes

Whole class activity

**Diagram drawn on BB or use enlarged copy master or OHP**

At a good pace

Agreement, praising

Discussion, reasoning, agreement, praising

**BB:**

(Or individual work if Ps wish, monitored, helped and reviewed with whole class)

Responses shown in unison.

Ps point to appropriate area on diagram on BB.

Agreement, (self-correcting), praising

P who asks the question chooses another P to answer it.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td><strong>Factorising</strong></td>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td>In your <em>Ex. Bk</em>, factorise 152 and then list all its factors.</td>
<td>Discussion, reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>Review at BB with whole class. Ps come to BB to draw tree diagram, show the numbers as the product of their prime factors, and list all their factors. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td>Ps may use a calculator.</td>
</tr>
<tr>
<td>BB: e.g. 152 = $2 \times 2 \times 2 \times 19$</td>
<td>Ps could join up the factor pairs.</td>
</tr>
<tr>
<td>Factors: 152: 1, 2, 4, 8, 19, 38, 76, 152</td>
<td>Whole class activity</td>
</tr>
<tr>
<td><strong>4 min</strong></td>
<td>Deal with one at a time and gradually build up the tables and the two methods of division.</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td>At a good pace</td>
</tr>
<tr>
<td><strong>Different bases</strong></td>
<td>Reasoning, agreement, praising</td>
</tr>
<tr>
<td>Imagine that we have 152 coins. How can we group them using different numbers as the base number?</td>
<td>Ps may use a calculator or do the calculations in <em>Ex. Bks</em> or at side of BB.</td>
</tr>
<tr>
<td>Ps first dictate headings for the place value table for each base number. Then Ps show the groupings in two ways: starting from the LHS and then the RHS of the table. Ps come to BB or dictate operations to T, with T's help if necessary. Class points out errors.</td>
<td>Feedback for T</td>
</tr>
<tr>
<td>BB: e.g.</td>
<td></td>
</tr>
<tr>
<td>a) <strong>Grouping by 6</strong></td>
<td></td>
</tr>
<tr>
<td>Starting at LHS of table:</td>
<td></td>
</tr>
<tr>
<td>Or starting at RHS of table:</td>
<td></td>
</tr>
<tr>
<td>152 ÷ 36 = $1 \frac{3}{8}$</td>
<td>152 ÷ 6 = 25, r 0</td>
</tr>
<tr>
<td>8 ÷ 6 = $1 \frac{2}{3}$</td>
<td>25 ÷ 6 = 4, r 1</td>
</tr>
<tr>
<td>2 ÷ 1 = 2</td>
<td>4 ÷ 6 = 0, r 4</td>
</tr>
<tr>
<td>Check: 4 × 36 + 1 × 6 + 2 × 1 = 152 ✔</td>
<td></td>
</tr>
<tr>
<td>b) <strong>Grouping by 7</strong></td>
<td></td>
</tr>
<tr>
<td>Starting at LHS of table:</td>
<td></td>
</tr>
<tr>
<td>Or starting at RHS of table:</td>
<td></td>
</tr>
<tr>
<td>152 ÷ 49 = $1 \frac{4}{7}$</td>
<td>152 ÷ 7 = 21, r 5</td>
</tr>
<tr>
<td>5 ÷ 7 = $0 \frac{5}{7}$</td>
<td>21 ÷ 7 = 3, r 0</td>
</tr>
<tr>
<td>5 ÷ 1 = 5</td>
<td>3 ÷ 7 = 0, r 3</td>
</tr>
<tr>
<td>Check: 3 × 49 + 0 × 7 + 5 × 1 = 152 ✔</td>
<td></td>
</tr>
<tr>
<td>c) <strong>Grouping by 8</strong></td>
<td></td>
</tr>
<tr>
<td>Starting at LHS of table:</td>
<td></td>
</tr>
<tr>
<td>Or starting at RHS of table:</td>
<td></td>
</tr>
<tr>
<td>152 ÷ 64 = $2 \frac{7}{8}$</td>
<td>152 ÷ 8 = 19, r 0</td>
</tr>
<tr>
<td>24 ÷ 8 = $3 \frac{0}{2}$</td>
<td>19 ÷ 8 = 2, r 3</td>
</tr>
<tr>
<td>0 ÷ 1 = 0</td>
<td>2 ÷ 8 = 0, r 2</td>
</tr>
<tr>
<td>Check: 2 × 64 + 3 × 8 + 0 × 1 = 152 ✔</td>
<td></td>
</tr>
<tr>
<td>d) <strong>Grouping by 9</strong></td>
<td></td>
</tr>
<tr>
<td>Starting at LHS of table:</td>
<td></td>
</tr>
<tr>
<td>Or starting at RHS of table:</td>
<td></td>
</tr>
<tr>
<td>152 ÷ 81 = $1 \frac{71}{9}$</td>
<td>152 ÷ 9 = 16, r 8</td>
</tr>
<tr>
<td>71 ÷ 9 = $7 \frac{8}{9}$</td>
<td>16 ÷ 9 = 1, r 7</td>
</tr>
<tr>
<td>8 ÷ 1 = 8</td>
<td>1 ÷ 9 = 0, r 1</td>
</tr>
<tr>
<td>Check: 1 × 81 + 7 × 9 + 8 × 1 = 152 ✔</td>
<td></td>
</tr>
</tbody>
</table>
Solving inequalities

Which numbers could the circles represent in these inequalities?
Ps come to BB or dictate each line of the solution to T, explaining reasoning. Calculations can be done at side of BB. Class agrees or disagrees and checks that the solution makes the inequality true.

BB:

a) \(3 \times (120 + \bigcirc) < 450\)
\[120 + \bigcirc < 450 \div 3 = 150\]
\[\bigcirc < 150 - 120\]
\[\bigcirc < 30\]
or \[\bigcirc : 29, 28, 27, \ldots\]

b) \((\bigcirc - 48) \div 7 \leq 83\)
\[\bigcirc - 48 \leq 83 \times 7 = 581\]
\[\bigcirc \leq 581 + 48\]
\[\bigcirc \leq 629\]
or \[\bigcirc : 629, 628, 627, \ldots\]

or \(\bigcirc : 29, 28, 27, \ldots\)

or \(\bigcirc : 358\) or \(359\) (if only whole numbers)

\[< 30\]

Feedback for T

Lesson Plan 152

Notes

Whole class activity
Written on BB or SB or OHT
At a good pace
Reasoning agreement, checking, praising

Discuss and agree that if solution is restricted to whole numbers, the numbers can be listed (using ellipses to save time and space). If the solution can be any number, i.e. whole numbers, fractions or decimals, then the solution should be left as a simple inequality.

e.g. \(\bigcirc < 30\)

Feedback for T

Extension

Who can think of questions to ask about the data? e.g.
What is the mode? (4000)
What is the median? (4500)
What is the range? (1400)

But also, e.g.
In which month was the play most (least) popular?
Why do you think more (fewer) people attended in these months? etc.
Q.2 Read: We heated a pan of water and noted its temperature every minute.

The temperature of the water rose steadily to 100\(^\circ\)C but did not go above it.

Who has seen a pan of water heat up? What happens to it? Why do you think that the temperature never rises above 100\(^\circ\)C?

(Because the water boils at 100\(^\circ\)C and turns into steam.)

Deal with one question at a time if class is not very able, otherwise set a time limit. Ps read questions themselves and write solutions in PbS.

Review with whole class. Ps come to BB or dictate to T. Mistakes discussed and corrected.

Solution:

a) Complete the table

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature ((^\circ)C)</td>
<td>30</td>
<td>40</td>
<td>50</td>
<td>60</td>
<td>70</td>
<td>80</td>
<td>90</td>
<td>100</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>

b) Continue drawing dots in the graph to show the data.

BB:

![Graph](image)

c) By how many \(^\circ\)C does the temperature rise each minute before it reaches 100\(^\circ\)C? (10\(^\circ\)C)

d) When does the temperature reach 100\(^\circ\)C? (after 7 minutes)

e) Is it correct to join up the dots?

(Yes, because the temperature is increasing at the same rate, so the values could also include any fraction of a minute or any fraction of 10\(^\circ\)C.) T join up dots on BB and Ps in PbS.

What does the slanting part of the graph line show? (temperature and time are increasing)

What does the horizontal part of the graph line show? (temperature staying at 100\(^\circ\)C, and time increasing)
### Activity

**6. What is the rule?**

Study these tables and think about what the rule could be. Which of the equations belongs to which table?

Ps come to BB to choose a table and colour its number and the letters of the matching equations in the same colour (or list them on BB), explaining reasoning. Class checks that they are correct.

**BB:**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>12</td>
<td>12</td>
<td>16</td>
</tr>
<tr>
<td>y</td>
<td>4</td>
<td>12</td>
<td>16</td>
</tr>
</tbody>
</table>

\[ \text{a) } x + y = 16 \]
\[ \text{b) } x \times y = 48 \]
\[ \text{c) } y = x - 8 \]
\[ \text{d) } y = x + 4 \]
\[ \text{e) } x + 2 = y - 2 \]
\[ \text{f) } y = x + 4 \]
\[ \text{g) } y = x + 2 \]
\[ \text{h) } y = x - 8 \]

**Solution:**

Each of the 5 people will clink glasses with each of the other 4 people, i.e. \(5 \times 4 = 20\), but each clinking of glasses involves 2 people, so there will actually be \(20 \div 2 = 10\) different clinkings.

Or 1st person links with 4 other people, 
2nd person clinks with 3 other people 
3rd person clinks with 2 other people 
4th person clinks with 5th person 

\[ 4 + 3 + 2 + 1 = 10 \]

**Answer:** There will be 10 clinking of glasses.

---

**Lesson Plan 152**

### Notes

Whole class activity

Written on BB or use enlarged copy master or OHP

At a good pace

Reasoning, agreement, praising

**Extension**

Ps draw a table with appropriate values to match the equations c) and f)

---

**7. PbY4b, page 152**

Q.3 Read: *There are 5 people at a party. Each person clinks glasses with each of the others. How many clinking of glasses will there be? Work it out in your exercise book and write the answer.*

I will give you 2 minutes to work out the answer!

Show me your answer... now! (10)

A, explain your reasoning to us. Who thought the same? Who worked it out in a different way? etc.

**Solution:**

Each line is a 'clinking'.

\[ 4 + 3 + 2 + 1 = 10 \]

**Answer:** There will be 10 clinking of glasses.
R: Calculations  
C: Functions, tables, graphs  
E: Problems. Numbers up to 100 000 (or above)

**Activity 1**

**Factorising**
In your Ex. Bk, factorise 153 and then list all its factors.  
Review at BB with whole class. Ps come to BB to draw tree diagram, show the number as the product of its prime factors and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.  
BB:  
\[
\begin{array}{c}
3 \\
51 \\
3 \\
17 \\
\end{array}
\]

Factors: 1, 3, 9, 17, 51, 153

**Notes**
Individual work, monitored, helped  
Discussion, reasoning, agreement, self-correction, praising  
Ps may use a calculator.  
Feedback for T

**Activity 2**

**Different bases**
Imagine that we have 153 shells. Let’s group them using 2 and then 9 as the base number.  
Ps first dictate the headings for each place value table. Then Ps show the groupings in two ways: starting from the LHS and then the RHS of the table. Ps come to BB or dictate operations to T, with T’s help if necessary. Class points out errors.

BB: e.g.

a) Grouping by 2
Starting at LHS of table:  
\[
\begin{array}{c|c|c|c|c|c}
& 1 & 2 & 8 & 6 & 4 \\
\hline
153 & 25 & 32 & 16 & 8 & 2 \\
25 & 24 & 16 & 8 & 2 & 1 \\
25 & 24 & 16 & 8 & 2 & 1 \\
9 & 8 & 6 & 4 & 2 & 1 \\
1 & 4 & 2 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 \\
\hline
\end{array}
\]

Or starting at RHS of table:  
\[
\begin{array}{c|c|c|c|c|c}
& 128 & 64 & 32 & 16 & 8 \\
\hline
76 & 38 & 19 & 9 & 4 & 2 \\
38 & 19 & 9 & 4 & 2 & 1 \\
19 & 9 & 4 & 2 & 1 & 0 \\
9 & 4 & 2 & 1 & 0 & 0 \\
4 & 2 & 1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\]

Check: 128 + 16 + 8 + 1 = 153

_4 min_

b) Grouping by 9
Starting at LHS of table:  
\[
\begin{array}{c|c|c|c|c|c}
& 8 & 1 & 9 & 1 \\
\hline
51 & 42 & 26 & 18 & 10 \\
42 & 36 & 27 & 18 & 10 \\
26 & 24 & 18 & 10 & 0 & 0 \\
18 & 16 & 9 & 1 & 0 & 0 \\
16 & 14 & 9 & 1 & 0 & 0 \\
9 & 1 & 0 & 0 & 0 & 0 \\
\hline
\end{array}
\]

Or starting at RHS of table:  
\[
\begin{array}{c|c|c|c|c|c}
& 1 & 0 & 0 & 0 & 1 \\
\hline
153 & 27 & 9 & 1 & 1 & 1 \\
27 & 18 & 9 & 1 & 0 & 0 \\
18 & 16 & 9 & 1 & 0 & 0 \\
16 & 14 & 9 & 1 & 0 & 0 \\
14 & 12 & 7 & 1 & 0 & 0 \\
7 & 6 & 3 & 1 & 0 & 0 \\
\hline
\end{array}
\]

Check: 1 × 81 + 8 × 9 + 0 × 1 = 153

_11 min_

**Activity 3**

**Place values**
T has BB already prepared. First elicit what the headings in the table mean and their relationship to one another. (Grouping by 10, i.e. base 10) Ps come to BB to read the number and write it as digits in the table, explaining place-value detail. Class points out errors.

BB:

<table>
<thead>
<tr>
<th>TTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>6</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>7</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>9</td>
<td>0</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Whole class activity  
Written on BB or use enlarged copy master or OHP  
At a good pace  
Reasoning, agreement, praising  
T covers up the words, points to a digit in the table and chooses Ps to say its real value. Which is the smallest (greatest) number?  
(26 073, 99 748)
### Activity 4

**Sequences**

T says the first 3 terms of a sequence and Ps note them in *Ex. Bks.*

I will give you 1 minute to work out the rule and continue the sequence for as many terms as you can. Start . . . now! . . . Stop!

Review at BB with whole class. Everyone stand up! T chooses a P to give a term in order round class. Ps sit down if they made a mistake or have reached their last term. Last P(s) standing gives the rule and if correct receives a round of applause for writing the most terms.

- a) 7843, 17 843, 27 843, (37 843, 47 843, 57 843, 67 843, . . .)
  
  **Rule:** The terms are increasing by 10 000. [+ 10 000]

- b) 9000, 18 000, 27 000, (36 000, 45 000, 54 000, 63 000, . . .)
  
  **Rule:** The terms are increasing by 9000. [+ 9000]

- c) 100, 300, 900, (2700, 8100, 24 300, 72 900, 218 700, . . .)
  
  **Rule:** The terms are increasing by 3 times. [× 3]

**Y4**

<table>
<thead>
<tr>
<th><strong>Lesson Plan 153</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td>Individual work, monitored</td>
</tr>
<tr>
<td>Deal with one sequence at a time.</td>
</tr>
</tbody>
</table>

If a P says an initial unexpected term, ask them to say what rule they are using.

T could write terms on BB as Ps dictate them.

Agreement, self-correction, praising (T might allow Ps to use a calculator for c).

### Activity 5

**Calculation practice**

T has additions and subtractions written on BB. Ps copy them in *Ex. Bks.* and do the calculations under a time limit. Remember to check your work!

Review at BB with wholeclass. Ps come to BB to do the calculations, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

- a) 1 2806 19510 1 6044 + 17362 + 65722
- b) 3 8019 17650 9407 + 22222 87298 8 21 1
- c) 6 4715 2 4389 4 0326

Ps check a) and b) by adding in opposite direction, c) by addition.

**Notes**

Individual work, monitored, helped

Or T reads the numbers aloud and Ps write in column form in *Ex. Bks.*

Written on BB or SB or OHT

Reasoning, agreement, self-correction, praising

**Extension**

- What is the difference between the greatest and smallest answer? (46 972)
- What is the total of the three answers? (193 346)

### Activity 6

**PbY4b, page 153**

Q.1 Read: *Sammy Snail climbed up the wall at a steady speed.*

You can read from the table where he got to after the first 4 minutes.

At the end of the 5th minute, Sammy Snail turned and went back down the wall, again at a steady speed.

This time you can read from the graph where he got to in the last 5 minutes.

Who can explain the graph? What is the relationship between the table and the graph? Ps come to BB to demonstrate.

Elicit that the missing values in the table can be found from the dots on the graph and the missing dots on the graph relate to the given values in the table.

- a) Read: *Complete the table and the graph.*
  
  Set a time limit. Review with whole class. Ps come to BB to complete table and graph, explaining reasoning. Class points out errors. Mistakes discussed and corrected.

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Whole class discussion to start, with T’s where necessary

Differentiation by time limit

Reasoning, agreement, self-correction, praising
Y4

Activity

6 (Continued)

b) Read: Is it correct to join up the dots?

T asks several Ps what they think and why. (Yes, because Sammy Snail moved at a steady speed without a break and any time on his journey could be shown on the graph.)

Let’s join up the dots. T draws lines on BB and Ps in Pbs.

Solution:

<table>
<thead>
<tr>
<th>Time (minutes)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height (cm)</td>
<td>0</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
<td>60</td>
<td>45</td>
<td>30</td>
<td>15</td>
<td>0</td>
</tr>
</tbody>
</table>

---

Notes

Discussion, agreement, praising
T helps with wording of reasoning.
Where would Sammy Snail be after, e.g.
• 2 and a half minutes
• 6 and a half minutes? etc.
P.s come to BB to point and give approximate height.

Extension

Did Sammy Snail go up the wall and down the wall at the same speed? (No)
Up: 12 cm every minute
Down: 15 cm every minute
So Sammy Snail came down the wall faster than he went up! (Why?)

7 PbY4b, page 153, Q.2

Read: We ran water from a tap into a jug shaped like a cylinder and noted the water level at certain times.

We found that the relationship between the time and the water level is \( w = 2 \times t \) (where \( w \) is the water level in cm and \( t \) is the time in seconds).

If possible, T has a cylindrical jug or container to show to class. Stress that it is the same width through all its length, so will fill at a steady rate. What would happen if the container was narrower (wider) at the bottom? (The water level would increase more quickly (slowly) at first and then more slowly (quickly) later on, so we could not make a rule from the data.)

Demonstrate the experiment if there is a tap in the classroom, otherwise ask Ps to imagine it. Elicit that the water flowing from the tap must be a steady trickle or there would not be time to mark the water levels!

a) Read: Fill in the table using this rule.

Ps come to BB to complete the table, explaining reasoning. Class agrees/disagrees. Ps fill in table in Pbs too.

Solution:

<table>
<thead>
<tr>
<th>( r )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w )</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td>14</td>
<td>16</td>
<td>18</td>
<td>20</td>
</tr>
</tbody>
</table>

b) Read: Draw a graph by drawing dots on this grid and then joining them up.

Ps come to BB to choose a column, point to the values for \( r \) and \( w \) on the axes, move fingers along the grid lines until they join up, then draw (stick on) a dot. Ps draw dots in Pbs too.
7 (Continued)

Is it correct to join up the dots? (Yes, because the water is flowing continuously without a break, so there are values between the dots which are not shown in the table.) T joins up dots on BB and Ps do the same in Pbs. (See final diagram)

c) Read: *We did the same experiment another day but this time the jug already had 5 cm of water in it when we started.*

*Draw a table in your exercise books to show the new set of data. Write the rule.*

T draws/shows table on BB and Ps copy it in Ex. Bks. Why is there a 5 for the first value of the bottom row? (Because there is 5 cm of water in the jug before the start of the experiment.)

Ps complete the table in Ex. Bks, then dictate results to T, who writes in table on BB. Mistakes corrected. What is the rule? (BB)

*Solution:*

<table>
<thead>
<tr>
<th>t (sec)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>w (cm)</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>11</td>
<td>13</td>
<td>15</td>
<td>17</td>
<td>19</td>
<td>21</td>
<td>23</td>
<td>25</td>
</tr>
</tbody>
</table>

Read: *Draw its graph line on this grid in red.*

Ps come to BB as before to find the values and draw (stick on) red dots. Agree that the odd numbers are half-way between the horizontal grid lines. Should we join up the dots? (Yes, for same reason as before.) Ps draw dots and joining line in red in Pbs too.

*Solution:*

Drawn on BB or use enlarged copy master or OHP

Individual work, monitored, helped

BB: *Rule: \( w = 2 \times t + 5 \)*

Reasoning, agreement, self-correction, praising

Whole class activity
At a good pace
Agreement, praising

Extra praise if Ps notice this without hints from T.

42 min

8

**Problem**

Listen carefully, solve the problem in your *Ex. Bks* and show me the answer when I say. Discuss it with your neighbour if you wish.

*How many dictionaries would be needed to translate among these languages: English, French, German, Spanish and Italian?*

If you found an answer, show me . . . now! (20)

P answering correctly explains reasoning. Who agrees? Who did it another way? etc. (T helps Ps to solve it if no P answered correctly.)

e.g. Each of the 5 languages would need 4 different dictionaries:

\( E \rightarrow F, \ E \rightarrow G, \ E \rightarrow S, \ E \rightarrow I, \) i.e. \( 5 \times 4 = 20 \)

or \( 2 \times (4 + 3 + 2 + 1) = 2 \times 10 = 20, \) or draw tree diagrams (*LP 149,*

45 min

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### Lesson Plan 154

**Activity 1**  
**Factorising**  
In your Ex. Bk, factorise 154 and then list all its factors.  
Review at BB with whole class. Ps come to BB to draw tree diagram, show the number as the product of its prime factors and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.  
BB:  
\[
154 = 2 \times 7 \times 11
\]  
Factors: 1, 2, 7, 11, 14, 22, 77, 154

**Notes**  
Individual work, monitored, helped  
Discussion, reasoning, agreement, self-correction, praising  
Ps may use a calculator.  
Feedback for T

**Activity 2**  
**Problem**  
Listen carefully and think about how you would solve this problem.  
_Dizzy Domble has just come back from a visit to Threeland where they count in base 3. He bought a box of biscuits while he was there and on the lid it stated that there were 12201 biscuits._  
He wants to send the box of biscuits to a friend who lives in Sevenland where they count in base 7 but thinks that he had better cross out the 12201 and write the number of biscuits in base 7, otherwise his friend will expect there to be more biscuits in the box than there really are.  
What should he change it to?  
Ps suggest what to do first and how to continue. T gives hints if Ps are stuck or leads Ps through the solution if they have no ideas.  
1. **Change the number in base 3 to base 10**  
   T draws the table on BB and Ps decide on the place-value headings and digits.  
   So the number of biscuits in base 10 is:  
   BB:  
   \[
   \begin{array}{c|c|c|c|c|c|c}
   \text{Place Value} & \text{Base 3} \\
   \hline
   \text{Units} & 1 & 2 & 3 & 1 & 1 \\
   \text{Tens} & 2 & 7 & 9 & 3 & 1 \\
   \text{Hundreds} & 1 & 0 & 0 \\
   \end{array}
   \]  
   \[1 \times 81 + 2 \times 27 + 2 \times 9 + 0 \times 3 + 1 \times 1 = 135 + 19 = 154\]  
2. **Change the number in base 10 to base 7**  
   T draws the table on BB and Ps dictate the place value headings.  
   Then Ps use one of the methods of division to determine the groupings. e.g.  
   BB:  
   \[
   \begin{array}{c|c|c|c|c|c|c}
   \text{Place Value} & \text{Base 7} & \text{Or starting at RHS of table:} & \text{Base 7} \\
   \hline
   \text{Units} & 49 & 7 & \text{154 \div 7 = 22, r 0} \\
   \text{Tens} & 3 & 1 & 0 & \text{22 \div 7 = 3, r 1} \\
   \text{Hundreds} & 0 & 1 & 0 & \text{3 \div 7 = 0, r 3} \\
   \end{array}
   \]  
   \[
   \text{Check: } 3 \times 49 + 1 \times 7 + 0 \times 1 = 154 \checkmark
   \]
   \[154 \div 49 = 3, r 1\]
   \[7 \div 7 = 1, r 0\]
   \[0 \div 1 = 0\]
   \[3 \times 49 + 1 \times 7 + 0 \times 1 = 154 \checkmark\]
   **Answer:** He should change the 12201 in base 3 to 310 in base 7.

**Notes**  
Whole class activity  
T repeats slowly to give Ps time to think and discuss.  
T reads ‘12201’ as ‘one, two, two, zero, one’ and writes on BB:  
\[12201 \text{ (base 3)}\]  
Discussion, reasoning, agreement, praising  
Ps come to BB or dictate to T. Class agrees/disagrees.

Ps decide on which division method to use.  
Ps come to BB to do the calculations, explaining reasoning, and fill in the table.  
Class points out errors.  
(Ps may use a calculator.)  
Agreement, praising
Lesson Plan 154

**Y4**

**Activity**

3 Calculation practice

T has operations already written on BB. Ps copy into Ex. Bks. and do the calculations. Set a time limit.

Review at BB with whole class. Ps come to BB to explain their reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

BB:

\[
\begin{array}{ccc}
\text{a)} & 47506 & \text{b)} 47506 \\
+ & 21835 & -21835 \\
\hline
& 69341 & 25671 \\
\end{array}
\]

\[
\frac{\text{c}) \quad 8516 \times 6}{3 \quad 3}
\]

\[
\begin{array}{ccc}
\text{d)} & 27210 & \text{e)} 7836, r 4 \\
\times 3 & & \frac{5}{4133} \\
\hline
& 81630 & \frac{39184}{33333}
\end{array}
\]

18 min

4 Rounding

Let's round these numbers to the nearest 10, 100, 1000 and 10 000.

Ps come to BB to fill in the table, explaining reasoning. Class agrees/disagrees. If disagreement, draw relevant segment of the number line on the BB.

BB:

<table>
<thead>
<tr>
<th>Number</th>
<th>Rounded to nearest:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>10</td>
</tr>
<tr>
<td>21 875</td>
<td>21 880</td>
</tr>
<tr>
<td>85 000</td>
<td>85 000</td>
</tr>
<tr>
<td>36 243</td>
<td>36 240</td>
</tr>
<tr>
<td>54 999</td>
<td>55 000</td>
</tr>
</tbody>
</table>

Whole class activity

Involve several Ps.

Drawn on BB or use enlarged copy master or OHP

At a good pace

Reasoning, agreement, praising

Review the rules of rounding.

Feedback for T

23 min

5 Problem

How much is 3 quarters of £68 252?

Who can write a plan for the solution? Ps come to BB or dictate to T. Class agrees/disagrees. Now let's do the calculations.

Ps come to BB to write the calculations in column form at side of BB, explaining reasoning in detail. Class points out errors.

**Solution:**

\[
\begin{align*}
\text{Plan:} & \quad \frac{3}{4} \text{ of £68 252} = \frac{68 252}{4} \times 3 = 17 063 \\n\text{Answer:} & \quad \text{Three quarters of £68 252 is £51 189.}
\end{align*}
\]

Whole class activity

Reasoning, agreement, checking, praising

BB:

\[
\begin{array}{ccc}
\text{C:} & 17063 \times 3 \\
\hline
\frac{4}{2} \frac{68252}{2} \times 3
\end{array}
\]

Ps may check with a calculator.

26 min
Y4

Activity

6  PbY4b, page 154

Q.1  Read:  Find different rules to complete the table.
       Write each rule in different ways.

What do you notice about the tables? (The numbers are the same but the letters are different.)
Set a time limit. Ps can do necessary calculations on scrap paper or slates or in Ex. Bks.
Review with whole class. One P at a time comes to BB to complete a table and chooses another P to give the rule that they used. Class checks it with values from the table.
Who found a different rule? Come and show us. Deal with all cases.
Solution:  e.g.

<table>
<thead>
<tr>
<th>a</th>
<th>20</th>
<th>200</th>
<th>2000</th>
<th>1260</th>
<th>1400</th>
<th>70</th>
<th>2470</th>
<th>8970</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>50</td>
<td>230</td>
<td>2030</td>
<td>1290</td>
<td>1430</td>
<td>100</td>
<td>2500</td>
<td>9000</td>
</tr>
</tbody>
</table>

Rule:  $b = a + 30$,  $a = b - 30$,  $b - a = 30$

<table>
<thead>
<tr>
<th>x</th>
<th>20</th>
<th>200</th>
<th>2000</th>
<th>1260</th>
<th>1400</th>
<th>40</th>
<th>1000</th>
<th>3600</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>50</td>
<td>500</td>
<td>5000</td>
<td>3150</td>
<td>3500</td>
<td>100</td>
<td>2500</td>
<td>9000</td>
</tr>
</tbody>
</table>

Rule:  $y = x + 2 \times 5$,  $x = y + 5 \times 2$, or  $x = \frac{2}{5}$ of $y$

<table>
<thead>
<tr>
<th>u</th>
<th>20</th>
<th>200</th>
<th>2000</th>
<th>1260</th>
<th>1400</th>
<th>120</th>
<th>4920</th>
<th>17920</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>50</td>
<td>140</td>
<td>1040</td>
<td>670</td>
<td>740</td>
<td>100</td>
<td>2500</td>
<td>9000</td>
</tr>
</tbody>
</table>

Rule:  $v = u + 2 + 40$,  $u = (v - 40) \times 2$

32 min

7  PbY4b, page 154, Q.2

Read:  Tammy Tortoise went for a walk from her house to the field and back again. The graph shows how far she was from home during that time.

Talk about the meaning of the graph first. Who can explain the graph? Where is the $x$-axis? What does it show? (the time Tammy spent walking, with a grid line at every minute) Where is the $y$-axis? What does it show? (How far Tammy was from home, with a grid line at every 10 m) Where does Tammy start (finish) her walk? What part of the graph shows where she was walking away from home (coming back home, stopping for a rest)? etc.

BB:

Initial discussion to clarify the graph and what kind of motion the graph line shows.
Allow Ps to explain first without prompting if they can.
Involve several Ps.

Notes

Individual trial first, monitored, helped
Tables drawn on BB or use enlarged copy master or OHP
Discussion, reasoning, agreement, checking, self-correction, praising
Extra praise for clever or unexpected but valid rules
If problems, Ps write and say calculations with place-value detail at side of BB.  e.g.

\[
\begin{align*}
630 \times 5 &= 3150 \\
\frac{9000}{4} &= 2250 \\
2460 \times 2 &= 4920 \\
17920 &= 4920 \times 4 \\
11 &= \frac{17920}{4920}
\end{align*}
\]

Other rules are possible.

Whole class activity
Drawn on BB or use enlarged copy master or OHP
### Y4

#### Activity 7

(Continued)

Now let's see if you are clever enough to answer the questions!

T (P) reads each question aloud and Ps show answers on scrap paper or slates on command. Ps answering correctly come to BB to explain to those who were wrong. Class agrees/disagrees.

**Solution:**

a) *How far away from home did Tammy go?* (120 m)

b) *For how long was she away from home?* (21 minutes)

c) *When did she start her return journey?* (after 15 minutes)

d) *How many times did Tammy stop to rest?* (twice)

#### Extension

Who can think of other questions to ask about the graph? e.g.

- For how long did she rest?
- How far away from home was she after 2 min (10 min, etc.)?
- How far had she walked before her first rest?
- When did Tammy walk more slowly? etc.

---

#### Problem 2

Listen carefully, note the data and try to solve the problem in your *Ex.* Bks. Show me the answer when I say.

*There are 200 litres of water in my bath. When I take out the plug, the water gurgles down the plughole at a rate of 25 litres every minute. After how many minutes will my bath be empty?*

Set a time limit. Ps can discuss with their neighbours if they wish.

If you found an answer, show me . . . now! (8 minutes)

P who answered correctly explains at BB. Who did the same? Who did it a different way? etc. If no P found the answer, T gives hints and class solves it together.

**Solution:** e.g.

\[
200 - 25 - 25 - 25 - 25 - 25 - 25 = 0 \text{ (litres)} \rightarrow 8 \text{ min.}
\]

or

\[
200 \text{ litres} \div 25 \text{ litres} = 40 \text{ litres} \div 5 \text{ litres} = 8 \text{ (times)}.
\]

**Answer:** My bath will be empty after 8 minutes.

---

#### Problem 9

*PbY4b, page 154*

Read: *How many diagonals does a hexagon have?*

*Show it by drawing a hexagon and its diagonals.*

How many sides does a hexagon have? (6) I will give you 2 minutes to find the answer! You do not need to draw a regular hexagon.

Start . . . now! . . . Stop! Show me the answer . . . now! (9)

P answering correctly shows solution on BB. Class agrees/disagrees.

**Solution:** irregular regular

\[
\begin{array}{c}
\text{or} \\
9 \text{ diagonals} \\
6 \times 3 \div 2 = 2
\end{array}
\]
Y4

Lesson Plan

Notes

Tables and calculation practice, revision, activities, consolidation

PbY4b, page 155

Solutions:

Q.1
Table (1): equations (a) and (g)
Table (2): equations (b) and (i)
Table (3): equations (d) and (f)

Q.2
Harvey's age (years) | 0 | 1 | 2 | 4 | 7 | 15 | 18 | 27 | 8 | 19 | 28
Dad's age (years)    | 28| 29| 30| 32| 35| 43| 46| 55| 36| 47| 56

a) Harvey’s Dad will be 46 years old
b) Harvey: 36 years old, Harvey’s Dad: 64 years old
c) \[ D = H + 28, \quad H = D - 28, \quad 28 = D - H \]

Q.3

a) Time (minutes) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9
Outflow (litres)  | 0 | 35| 70| 105|140|175|210|245|280|315
Water left (litres) | 320|285|250|215|180|145|110|75 |40 | 5

b) Half-full: 320 litres \( \div 2 = 160 \) litres
   After 5 minutes the tank was less than half full.
c) After 10 minutes the tank was empty.
d) 5 litres flowed out in the last minute.

d) 5 litres flowed out in the last minute.

Q.4

7.5 cm = 75 mm, \[ 75 \text{ mm} \div 5 = 15 \text{ mm} = 1.5 \text{ cm} \]
**Y4**

**R:** Calculations

**C:** Probability games. Fair and unfair games

**E:** Problems

### Activity 1

**Factorising**

In your Ex. Bk, factorise 155 and 156 and then list all their factors.

Review at BB with whole class. Ps come to BB to draw tree diagrams, show the numbers as the product of their prime factors and list all their factors. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

\[
\begin{align*}
155 &= 5 \times 31 \\
156 &= 2 \times 2 \times 3 \times 13 \\
\end{align*}
\]

Factors:

155: 1, 5, 31, 155 (a nice number!)

156: 1, 2, 3, 4, 6, 12, 13, 26, 39, 52, 78, 156

---

**Notes**

Individual work, monitored, helped

Discussion, reasoning, agreement, self-correction, praising

Ps may use a calculator.

Ps may join up the factor pairs for 156.

Feedback for T

---

### Activity 2

**Calculation practice**

Ps come to BB to do the calculations, explaining reasoning in detail.

Class checks mentally and points out errors.

**BB:**

\[
\begin{align*}
a) & \quad 64 \times 43 &= 2732 \\
b) & \quad 18 \times 43 &= 774 \\
c) & \quad 7347 \div 12 &= 612.25 \\
d) & \quad 80 \div 8 &= 10 \\
\end{align*}
\]

---

**Notes**

Whole class activity

Written on BB or use enlarged copy master or OHP

At a good pace

Reasoning, checking, agreement, praising

(If disagreement, check with a calculator.)

Extra praise if Ps notice without help from T.

---

### Activity 3

**Rounding**

Who can explain to us what these statements really mean?

\[
\begin{align*}
a) & \quad 64000 \text{ is the value of a number which has been rounded to the nearest thousand.} \\
P: & \quad \text{e.g. The number is at least 63 500 and it is less than 64 500.} \\
\end{align*}
\]

Who can write it as an inequality? BB: 63 500 ≤ n < 64 500

If the number is a natural number, what could the number be?

\[
\begin{align*}
n: & \quad 63 500, 63 501, \ldots, 64 499 \\
\end{align*}
\]

If the number can be a whole number or a fraction or a decimal, who can show us the possible values on this number line? T draws on BB and Ps come to BB to draw circles and join them up, explaining reasoning. Class agrees/disagrees. Revise the notation if necessary.

**BB:**

\[
\begin{align*}
63 000 & \quad 64 000 & \quad 65 000 \\
\end{align*}
\]

---

**Notes**

Whole class activity

BB: 64 000 (to nearest 1000)

Agreement, praising

Elicit that a natural number is a positive whole number.

Ps dictate possible numbers.

Extra praise if Ps remember without help from T how to show the complete solution.
### Activity 3

(Continued)

b) 64000 is the value of a number which has been rounded to the nearest hundred.

P: e.g. The number is at least 63 950 and it is less than 64 050.

Who can write it as an inequality? BB: 63 950 ≤ n < 64 050

If the number is a natural number, what could the number be?

(n: 63 950, 63 951, . . ., 64 049)

If the number can be a whole number or a fraction or a decimal, who can show us the possible values on this number line? T draws on BB and Ps come to BB to draw circles and join them up, explaining reasoning. Class agrees/disagrees.

BB:

<table>
<thead>
<tr>
<th>63 900</th>
<th>63 950</th>
<th>64 000</th>
<th>64 050</th>
<th>64 100</th>
</tr>
</thead>
</table>

#### Notes

BB: 64 000 (to nearest 100)
Agreement, praising

Ps dictate possible numbers.
Reasoning, agreement, praising

### Activity 4

#### Perimeter

This is an equilateral triangle. What does equilateral mean? (Its sides are equal in length.)

The two smaller triangles are also equilateral, and their perimeters are 15 units and 24 units long.

What is the perimeter of the largest triangle?

Ps suggest what to do first and how to continue. T gives hints only if necessary.

e.g. Each side of the smallest triangle is: 15 units ÷ 3 = 5 units
Each side of the middle-sized triangle is: 24 units ÷ 3 = 8 units
So each side of the large triangle is: 5 units + 8 units = 13 units
and its perimeter is: 3 × 13 units = 39 units.

Or: The perimeter of the largest triangle is equal to the sum of the perimeters of the two smaller triangles.

BB:

<table>
<thead>
<tr>
<th>A</th>
<th>F</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>8</td>
<td></td>
</tr>
</tbody>
</table>

Whole class activity
Drawn on BB or use enlarged copy master or OHP
BB:

\[
P = 15 \text{ units} \]
\[
P = 24 \text{ units} \]

T gradually adds lengths to diagram as each value is worked out.

BB:

Reasoning, agreement, praising

If no P suggests the 2nd method, T shows it.

### Activity 5

**PbY4b, page 156**

Q.1 Read: *If we put a 3-volume encyclopedia back on the shelf without looking at the volume numbers, in what order might they end up? Show all the possibilities.*

Set a time limit. Review with whole class. Ps come to BB or dictate to T. Mistakes corrected and omissions added.

Agree that there are only 6 different possible arrangements.

BB:

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Individual work, monitored, helped
Drawn on BB or use enlarged copy master or OHP
Agreement, self-correcting, praising

Or by calculation:

\[
3 \times 2 \times 1 = 6
\]
<table>
<thead>
<tr>
<th>Y4</th>
<th>Lesson Plan 156</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Activity</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td>(Continued)</td>
<td>Individual work, monitored, helped, then reviewed with whole class, or whole class activity, with Ps showing answers on scrap paper or slates in unison on command.</td>
</tr>
</tbody>
</table>
| a) Read: **What chance is there of them being in the order 2 3 1?**  
(1 out of 6, or 1 sixth, as each of the 6 possibilities has an equal chance of occurring.) | Reasoning, agreement, praising |
| b) Read: **What chance is there of these events happening?**  
i) *the book on the left-hand side is Volume 1*  
(2 out of 6, or 2 sixths, or 1 third)  
\[ \frac{2}{6} = \frac{1}{3} \]  
ii) *the volume numbers are decreasing from the left.*  
(1 out of 6, or 1 sixth) | Whole class discussion  
Ps think of other events too!  
e.g. What is the probability of volume 2 being in the middle?  
\[ \frac{1}{3} \] |
| If necessary T revises the vocabulary of probability:  
- the chance (probability) of something (an event) happening (occurring) is usually given as a fraction between 0 and 1.  
- the less chance there is of an event occurring, the nearer the fraction is to 0.  
- the greater the chance, the nearer the fraction is to 1. | |
| **6** | Whole class activity  
BB: \[ \begin{array}{ccccccc} 0 & 1 & 2 & 3 & 4 & 5 \ \end{array} \]  
Demonstration of card game |
| **PbY4b, page 156, Q.2** | Individual work, monitored, helped |
| Read: **Four children are playing a game with these cards.**  
T has a large set stuck to BB for demonstration. Let’s play the game!  
T calls 4 Ps to front of class to be A, B, C and D. | Agreement, self-correction, praising |
| T or P reads out one rule at a time and the group carries it out. Repeat until all 4 Ps in the group have drawn a 2-digit number and written it on the BB. Repeat if some Ps still do not understand the game. | Extra praise if Ps reason without prompting that 01, 02, etc. contain only units, so are not 2-digit numbers. |
| a) Read: **List in your exercise book all the 2-digit numbers that could be chosen.**  
Set a time limit. Review with whole class. Ps dictate the numbers and T writes on BB in a logical order. Class points out any missed.  
BB: \( \begin{array}{ccccccc} 01, 02, 03, 04, 05; 10, 12, 13, 14, 15; 20, 21, 23, 24, 25; 30, 31, 32, 34, 35; 40, 41, 42, 43, 45; 50, 51, 52, 53, 54 \end{array} \)  
Agree that there are 30 possible combinations. Are they all 2-digit numbers? (No, 01, 02, 03, 04 and 05 are 1-digit numbers, so the extra rules do not apply to them, only to the 25 2-digit numbers.) | Whole class activity  
Or Ps could show results on scrap paper or slates in unison on command.  
Reasoning, agreement, praising |
| b) Read: **Who might complain because the extra rules are unfair?**  
Let’s work out the probability of each person missing a turn.  
T reads out the extra rules one at a time and Ps count how many of of that type of number there are (not counting the 1-digit numbers), then give the probability. Class agrees/disagrees.  
\[ \begin{align*}  
\text{Alan misses a turn if the 2-digit number is even.} & \quad \frac{13}{25} \\
\text{Becky misses a turn if the 2-digit number is odd.} & \quad \frac{12}{25} \\
\text{Callum misses a turn if the 2-digit number is a whole 10.} & \quad \frac{5}{25} \\
\text{Diana misses a turn if the 2-digit number is divisible by 5.} & \quad \frac{9}{25} \\
\end{align*} \]  
All but Callum might complain as he has least chance of missing a turn. | T asks several Ps what they think and why. |
Q.3 Read: A marble is dropped into this maze and has an equal chance of falling to the left or to the right.  
   a) In how many ways can the marble come out at A, B, C, D or E?  

Set a time limit. Review at BB with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. If disagreement, Ps come to BB to show the different routes. Mistakes corrected.

How many different routes are there altogether? (16)

BB: A: 1, B: 4, C: 6, D: 4, E: 1

Total routes: $1 + 4 + 6 + 4 + 1 = 16$

b) Read: Where is the marble most likely to come out?  
Show me . . . now! (C) (as it has most possible routes)

c) Read: Write the ratio of the chance of where it comes out.  
What kind of number should we write in the boxes? (fraction)  
What will the denominator be? (16) What will the numerator be? (the number of possible routes leading to that letter).

Ps write fractions in Pb, then come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

BB: A B C D E

\[
\frac{1}{16} : \frac{4}{16} : \frac{6}{16} : \frac{4}{16} : \frac{1}{16}
\]

[Note: For ratio, it is also acceptable to use $1 : 4 : 6 : 4 : 1$ as the denominator is the same for each fraction.]

---

40 min

---

8 Problem

Listen carefully and think about how to solve this problem  
There are 5 locked doors and 5 keys but the keys are not marked.  
What is the most number of tries that have to be made to be certain of finding the correct key for each door?  
Set a time limit. and allow Ps to discuss with neighbours if they wish.

If you have an answer, show me . . . now! (14)

Ps who responded correctly explain their reasoning. Who agrees?  
Who did it a different way? etc. If no P found the solution, class solves it with hints from T. (Praise Ps who answer with 15, but see below.)

Solution: e.g.

\[
\begin{array}{cccc}
\text{Doors:} & A & B & C & D & E \\
\text{No of keys that must be tried:} & 5 & 4 & 3 & 2 & 0 \\
\end{array}
\]

The worst possible case is that always the last key tried fits the lock, and after D there is only one key left for E, so there is no need to try it.

So the most number of tries to be certain is: $5 + 4 + 3 + 2 + 0 = 14$

(Or the 1st key can be tried in 5 doors, the 2nd key in 4 doors, etc.)

Answer: The most number of tries that have to be made is 14.

---

45 min
## Lesson Plan

### Y4

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **1** Factorising | Whole class activity  
At a good pace  
Ps explain reasoning or do divisions at side of BB.  
Class agrees/disagrees  
Praising, encouragement only |
| **2** Problem | Whole class activity  
T repeats slowly to give Ps time to think and discuss.  
Discussion, reasoning, agreement, praising |
| **3** Calculation practice | Individual calculation, then whole class review  
(Or whole class activity, with Ps coming to BB to write the operations and do the calculations, explaining reasoning. Class points out errors.) |

### Activity 1

#### Factorising

Let's factorise 157 and then list all its factors.

Ps dictate or come to BB to try each of the prime numbers, 2, 3, 5, 7 and 11 as divisors, using 'quick' methods where possible. Should we try dividing by 13? (No, as $13 \times 13 = 169 > 157$)

Elicit that 157 is a **prime number**, and its factors are 1 and 157.

**Whole class activity**
- At a good pace
- Ps explain reasoning or do divisions at side of BB.
- Class agrees/disagrees
- Praising, encouragement only

### Activity 2

#### Problem

Listen carefully, picture the story in your head and think about how you would work out the answer.

**A knight fell in love with a young princess and promised her that every Sunday he would bring her as many roses as it is the day of the month.**

What is the most number of roses that he might take to the princess in a month?

**A**, tell us what you would do. Who agrees? Who would do it another way? etc. T gives hints if nobody is on the right track.

**Solution:**

The most number of days in a month is 31.

In a 31-day month, the most roses that the knight could take to the princess would be if the 31st was a Sunday.

So the roses he took to the princess during that month would be:

$BB: \quad 31 + 24 + 17 + 10 + 3 = 85$

**Answer:** The most roses that the knight might take to the princess in a month is 85.

**Whole class activity**
- T repeats slowly to give Ps time to think and discuss.
- Discussion, reasoning, agreement, praising
- T helps with wording of reasoning.
- Ps do addition on slates or scrap paper or in Ex. Bks and dictate result to T.
- T chooses a P to say the answer in a sentence.

### Activity 3

#### Calculation practice

Listen carefully, write down the numbers and do the calculation in your Ex. Bks. Show me the result when I say.

Deal with one question at a time.

Ps responding correctly explain at BB to those who were wrong.

Class agrees/disagrees. Mistakes discussed and corrected.

1. **Which number is 4 times as much as 9350?**  (37 400)
   
   $BB: \quad 9350 \times 4 = \underline{37400}$

2. **Five times a number is 43810. What is the number?**  (8762)
   
   $BB: \quad 43810 \div 5 = \underline{8762}$

3. **Which number is 2 fifths of 45 600?**
   
   $(45 600 \div 5 \times 2 = 9120 \times 2 = \underline{18240})$

4. **Three quarters of a number is 45 600. What is the number?**
   
   $(45 600 \div 3 \times 4 = 15 200 \times 4 = \underline{60800})$

**Whole class activity**
- T gives hints if nobody is on the right track.
- Discussion, reasoning, agreement, praising
- T helps with wording of reasoning.
- Ps do addition on slates or scrap paper or in Ex. Bks and dictate result to T.
- T chooses a P to say the answer in a sentence.

---

157

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### Activity 4

**Converting units of measure**

Let's convert these quantities. Revise units of measure first if necessary. Ps come to BB or dictate what T should write. Class agrees/disagrees.

<table>
<thead>
<tr>
<th>BB:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 32 m 35 cm = (3235 cm = 32350 mm)</td>
</tr>
<tr>
<td>b) 15 684 mm = (1568 cm 4 mm = 15 m 68 cm 4 mm)</td>
</tr>
<tr>
<td>c) 57 litres 24 cl = (5724 cl = 57 240 ml)</td>
</tr>
<tr>
<td>d) 28 315 ml = (2831 cl 5 ml = 28 litres 31 cl 5 ml)</td>
</tr>
<tr>
<td>e) 46 kg 380 g = (46 380 g)</td>
</tr>
<tr>
<td>f) 65 904 g = (65 kg 904 g)</td>
</tr>
<tr>
<td>g) 98 km 540 m = (98 540 m)</td>
</tr>
<tr>
<td>h) 21 480 m = (21 km 480 m)</td>
</tr>
</tbody>
</table>

#### Notes
Whole class activity
Written on BB or SB or OHT
At a good pace
Reasoning, agreement, praising
Feedback for T

### Activity 5

**PbY4b, page 157, Q.1**

Read: *Three boys, A, B and C, decided to have a race. We know that there was a tie but not for which place.*

What possibilities could there be? (there is a winner and 2 boys tie for 2nd place, 2 boys tie for 1st place and there is a 3rd place, 3 boys tie for 1st place)

a) Read: *What could the finishing order be? Show all the possibilities.*

Ps come to BB or dictate to T. Class agrees/disagrees. Ps complete the tables in *Pb*s too.

<table>
<thead>
<tr>
<th>BB:</th>
<th>1st</th>
<th>2nd/3rd</th>
<th>1st/2nd</th>
<th>3rd</th>
<th>1st/2nd/3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B, C</td>
<td>A, B</td>
<td>C</td>
<td>A, B, C</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>A, C</td>
<td>A, C</td>
<td>B</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>A, B</td>
<td>B, C</td>
<td>A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) Read: *If each possible result has an equal chance of happening, what is the chance that there was a tie for 1st place?*

How many different possibilities are there? (7) How many of them have ties for 1st place? (4)

What is the probability of a 1st place tie? Show me . . now! ($\frac{4}{7}$)

#### Notes
Whole class activity
(or individual work if Ps wish)
Discussion, agreement
If Ps do not suggest the last possibility, T asks them to keep thinking!

Drawn on BB or SB or OHT
At a good pace
Agreement, praising

### Activity 6

**PbY4b, page 157, Q.2**

Read: *Predict the results for each outcome first, then do the experiment.*

T puts 2 red, 2 white and 2 green counters in a bag and chooses A to take out 2 counters with his/her eyes shut. Before he/she does so, T asks Ps to predict the outcome. Will they both be the same (s) or different (d); will there be 1 red + 1 white (R + W) or 2 green? (2G) Ps write their prediction on slates and show in unison on command. A takes out the counters. Ps who predicted correctly stand up and class gives them a round of applause!

This is only one outcome from one experiment! If we do the experiment 15 times, how many times do you think the different outcomes will occur? Write your prediction in this column in the table. (T points on BB.)

Now let's do the experiment properly!

#### Notes
Whole class introduction
Table drawn on BB or use enlarged copy master or OHP
Ps work in pairs and each pair has appropriately coloured counters and bag on desks.

Demonstration of experiment to show Ps what to do.

Ps write predictions in table in *Pb*s and T could write on table on BB.
Y4

Activity

(Continued)

Keep class together on each experiment. Ps work in pairs, taking turns to shake the bag and take out the counters but both Ps record result in table in Pbh with a tally mark. Make sure that Ps realise that they must mark either of the top two rows each time, and also one of the bottom two rows where appropriate.

After 15th experiment, Ps add up the totals in each row of their table. Write a tick beside your prediction if it is equal to the result of your experiment.

e.g. | Outcome | Prediction | Experiment | Totals |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Both the same</td>
<td>7</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Both different</td>
<td>8</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1 red + 1 white</td>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 green</td>
<td>1 ✔</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Who has 4 ticks (3 ticks)? Let’s give them a clap!

Do not worry if you have no ticks – it is very difficult to make predictions!

Ps read the questions themselves and use their own data to estimate the chances. T chooses one or two Ps to show their data and explain their answers to class.

e.g. Solution using the data in the table above:

What chance is there of you taking out of the bag:

a) 2 counters of the same colour

\[
\frac{4}{15}
\]

b) 2 counters of different colours

\[
\frac{11}{15}
\]

c) a red and a white counter

\[
\frac{1}{15}
\]

d) 2 green counters?

\[
\frac{1}{15}
\]

Extension

Elicit that Ps have different results because the experiment has not been done enough times. How could we get a better estimate? (Collect the data for the class.)

T adds another column to table above, or uses a class table. Ps dictate their results to T or come to BB to fill in a column in class table. Ps keep running totals or T uses a calculator.

BB: e.g.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>P1</th>
<th>P2</th>
<th>P3</th>
<th>P4</th>
<th>P5</th>
<th>P6</th>
<th>P7</th>
<th>P8</th>
<th>P9</th>
<th>P10</th>
<th>P11</th>
<th>P12</th>
<th>P13</th>
<th>P14</th>
<th>P15</th>
<th>P16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Both the same</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Both different</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 red + 1 white</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 green</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The chances for a), b), c) and d) are estimated using class totals divided by the total number of trials (e.g. 15 \times 16 = 240 if there are 16 pairs of Ps). Ps use calculators to work out the chances.

If possible, use a computer program to generate more data, then the closer the probabilities will get to the theoretical results.

Lesson Plan 157

Notes

Paired work, monitored
If class has an odd number of Ps, T could work with a P.
In any case, T has table drawn on BB, or uses enlarged copy master or OHP for reference.
Ps should check that their totals in the top 2 rows of the table add up to 15.

P working with T could fill in table on BB.

Individual work, monitored, helped

Reasoning, agreement, (self-correcting), praising

Whole class activity
Class table drawn on BB or use copy master or OHP

[Note to Ts]
Theoretically, there are 30 possible outcomes using \( R_1, \ R_2 \), for the 2 red, etc. and all are equally likely:

\[
\begin{array}{cccc}
R_1 & R_2 & W_1 & W_2 \\
\checkmark & \checkmark & \checkmark & \checkmark \\
W_1 & W_2 & R_1 & R_2 \\
\checkmark & \checkmark & \checkmark & \checkmark \\
G_1 & G_1 & G_2 & G_2 \\
\checkmark & \checkmark & \checkmark & \checkmark \\
G_2 & G_2 & G_1 & G_1 \\
\checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

So the probabilities are:

a) \( \frac{6}{30} = \frac{1}{5} \)

b) \( \frac{24}{30} = \frac{4}{5} \)

c) \( \frac{8}{30} = \frac{4}{15} \)

d) \( \frac{2}{30} = \frac{1}{15} \)

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### Lesson Plan 157

**Notes**

Individual work, monitored, helped

Grids drawn on BB or use enlarged copy master or OHT

(or Ps have copies of copy master on desks)

Discussion, demonstration, agreement, self-correction, praising

Extra praise for Ps who found all 20 squares without help

---

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 157</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>7</strong></td>
<td><strong>PbY4b, page 157</strong></td>
</tr>
<tr>
<td>Q.3 Read: <em>How many squares which have vertices on the grid dots can you draw on this diagram?</em></td>
<td></td>
</tr>
</tbody>
</table>
| T could draw a square on grid on BB if Ps do not understand what they have to do. Set a time limit. Ps draw copies of the grid in Ex. Bks for their trials. Review at BB with whole class. A, how many squares did you find? Come and show them to us. Who agrees? Who found more? etc. (T could have solution already prepared on 6 grids as below and uncover those that Ps did not find.) Agree that it is possible to draw 20 squares on the grid.
| **Solution:** |
| ![Grids](image) |
| (9 of this) (4 of this) (4 of this) |
| (1 of this) (1 of this) (1 of this) |
| **39 min** |

| **8**    | **PbY4b, page 157** |
| Q.4 Read: *Which digits can be the last digits of the square numbers? Continue the list in your exercise book.* Let's see how many more you can find in 2 minutes! Review at BB with whole class. Ps dictate the list to the T who writes on BB. Continue the list as far as any P has reached.
| BB: $1 \times 1 \rightarrow 1$, $2 \times 2 \rightarrow 4$, $3 \times 3 \rightarrow 9$, $4 \times 4 \rightarrow 6$
| $5 \times 5 \rightarrow 5$, $6 \times 6 \rightarrow 6$, $7 \times 7 \rightarrow 9$, $8 \times 8 \rightarrow 4$
| $9 \times 9 \rightarrow 1$, $10 \times 10 \rightarrow 0$, $11 \times 11 \rightarrow 1$, $12 \times 12 \rightarrow 4$
| etc. |
| Agree that the last digit can be 0, 1, 4, 5, 6 or 9. |
| T reads the statements and Ps write T or F in Pbs, then show their answer (on scrap paper or slates or by pre-agreed actions) in unison on command.
| a) *Is it true or false that in 7 different square numbers there are at least 2 in which the units digits are the same?* (T)
| X, why do you think so? (The first 6 numbers could all have different units digits, but the 7th number must have a units digit the same as one of the previous 6 numbers.)
| b) *Is it true or false that in 7 different square numbers there are at least 2 in which their difference is divisible by 10?* (T)
| Y, why do you think so? (The first 6 numbers could all have different units digits, but the 7th number must have a units digit the same as one of the previous 6 numbers, so their difference must have 0 as the units digit and is therefore divisible by 10.) |
| **45 min** |
Lesson Plan

158

Activity

1

Factorising

In your Ex. Bk, factorise 158 and list all its factors.

Review at BB with whole class. Ps come to BB to draw tree diagram, show the number as the product of its prime factors and list all its factors. Class agrees/agrees. Mistakes discussed and corrected.

BB: \[158 = 2 \times 79\]

Factors: 1, 2, 79, 158 (It is a nice number.)

2

Problem 1

Draw a 4 by 4 grid in your Ex. Bk. like this.

a) In how many ways can you place 4 dots on the grid so that there is 1 dot in each row and column?

Set a time limit of 3 minutes. Review at BB with the whole class. A, how many ways have you find? How did you work it out? Who found more than A? Who worked it out without needing to draw the grids? If nobody did, allow Ps the chance to explain if they can, otherwise T gives hints

For each of the 4 possible rows in the first column, there are 3 possible rows in the 2nd column, and for each of these there are 2 possible rows in the 3rd column, and for each of these there is only 1 possible row left in the 4th column. i.e. there are \[4 \times 3 \times 2 \times 1 = 24\] possible ways.

If we wanted to show all the different patterns, we could do it without needing to draw 24 grids!

Let's label the rows 1 to 4 and the columns A to D and show one pattern of the dots.

The first dot is in column A, row 1, so we can call it A1. What will the 2nd dot be? (B2) etc. We can write each pattern in brackets:

BB: \[(A1, B2, C3, D4)\]

To list all the possibilities, we could write:

\[(A1, B2, C3, D4), (A1, B2, C4, D3), (A1, B3, C2, D4), (A1, B3, C4, D2), \ldots\]

Or, keeping the same order: A, B, C, D, we could leave out the letters and just write:

\[(1234), (1243), (1324), (1342), (1423), (1432), \ldots\]

When Ps understand the pattern, allow them to dictate all 24 cases.

b) If we did not label the rows or columns and could rotate the grid like this (T demonstrates), would there be more or less different patterns? (Less, as e.g.1234 would be the same as 4321 and 1243 would be the same as 3421 etc.)

T sticks the different grids one at a time on BB and Ps dictate the patterns that they show. T helps by rotating the grids where necessary. Ps underline or tick each pattern as it is dealt with.

BB:

Accept and praise any number found, but also elicit/tell the method of calculation opposite, explaining by referring to the diagram.

Whole class discussion.

Whole class activity

Grids drawn on card, cut out and stuck to BB (or use enlarged copy master)

Agreement, praising

Elicit that there are only 9 different ways if the grid can be rotated.

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Q.1 Read: Predict the results for each outcome first, then do the experiment.

Toss 2 coins one after the other 20 times and note how they land in this table.

Demonstrate the experiment first. Ps decide which outcome they think will happen and write on scrap paper or slates. T chooses a P to come to front to toss two coins one after the other and T writes the result on BB.

Show me what you predicted . . . now! If you guessed correctly, give yourself a pat on the head.

Now predict how many times out of 20 you would get the outcomes in the table. Ps fill in 2nd column in table.

Ps work in pairs, taking turns to toss the coins, but both write the result in table in Pbs. After the 20th toss, Ps count their totals in each row.

Write a tick beside your prediction if it is equal to your real data.

e.g.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Prediction</th>
<th>Tosses</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Heads</td>
<td>6</td>
<td>1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20</td>
</tr>
<tr>
<td>1 Head + 1 Tail</td>
<td>3</td>
<td>✔</td>
</tr>
<tr>
<td>1 Tail + 1 Head</td>
<td>4 ✔</td>
<td>✔</td>
</tr>
<tr>
<td>2 Tails</td>
<td>7</td>
<td>✔</td>
</tr>
</tbody>
</table>

Who has 4 (3) ticks? Let’s give them a clap!

Now answer the questions using your own data. Set a time limit.

T chooses one or two Ps to give their outcome totals and their fractions. Class points out errors.

Solution e.g. using the data in the table above:

What fraction of the tosses resulted in:

a) 2 heads  b) 2 tails  c) a head and a tail  d) at least 1 head?

\[
\text{a) } \frac{5}{20}, \quad \text{b) } \frac{6}{20}, \quad \text{c) } \frac{9}{20}, \quad \text{d) } \frac{14}{20}
\]

How many different outcomes are there? (4) HH, HT, TH, TT

If the coins were evenly balanced so that one side had an equal chance of landing heads up as tails up, what fractions would you expect to get?

(4 possible outcomes with an equal chance of happening, so each outcome would have a probability of \(\frac{1}{4}\) quarter.)

But you have to be careful which coins you use, as some coins are biased. Who knows what that means? (They are not made evenly and one side tends to land face up more often than the other side.)

T might tell class:

Have you heard about an experiment done by some Polish mathematicians? They found that Euro coins are biased, because when they are tossed, they land tails up more often than heads up!

T could collect class data and/or do the experiment using a computer program to show that the more times the experiment is done, the closer the results get to the expected values.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Prediction</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Heads</td>
<td>6</td>
</tr>
<tr>
<td>1 Head + 1 Tail</td>
<td>3</td>
</tr>
<tr>
<td>1 Tail + 1 Head</td>
<td>4 ✔</td>
</tr>
<tr>
<td>2 Tails</td>
<td>7</td>
</tr>
</tbody>
</table>

**Extension**

Whole class discussion about expected outcomes.

(It might be a surprise to some Ps that the outcome 1H + 1T occurred more frequently than HH or TT.)

Individual work, monitored, helped

Whole class discussion about expected outcomes.

i.e. 5 times out of 20

BB: biased

Elicit that it unfair to those who do not know!

In good humour!

(Could be done in Lesson 160)


**Activity 4**  
*PbY4b, page 158*

Q.2 Read: *At the entrance to a wood there are 5 paths leading to the first clearing. From the first clearing there are 6 paths leading to the 2nd clearing. From the 2nd clearing there are 3 paths leading to the 3rd clearing.*

a) Draw a diagram to show it in your exercise book.

b) How many routes could you take from the 1st clearing to the 3rd clearing?

c) What chance would you have of guessing correctly a person’s route from the entrance of the wood to the 3rd clearing?

Deal with part a) first. Ps come to BB to draw the diagram (with T's help) and rest of Ps draw it in Ex. Bks.

Ps read questions b) and c) themselves and write the answers.

Review with whole class. Ps could show responses on scrap paper or slates on command. Ps answering correctly explain at BB to those who were wrong. Class agrees/disagrees.

Mistakes discussed and corrected.

**Solution:**

a) BB: e.g.

b) For each of the 6 paths to the 2nd clearing, there are 3 paths to the 3rd clearing, so there are: \(6 \times 3 = 18\) routes.

c) Altogether, there are \(5 \times 6 \times 3 = 90\) possible routes from the entrance to the 3rd clearing, so the chance of guessing correctly is:

\[
\frac{1}{90}, \quad \text{or} \quad \frac{1}{90}
\]

**Lesson Plan 158**

**Notes**

Individual work monitored (but diagram helped or done with whole class first)

Agreement, praising

Reasoning, agreement, self-correction, praising

(Unless you know that the person has a usual or favourite route.)

**Activity 5**  
*PbY4b, page 158*

Q.3 Read: *Predict the results for each outcome first, then do the experiment.*

*Throw a dice 20 times and keep a tally of how it lands in this table.*

T or P demonstrates experiment first if necessary. Set a time limit. Ps have dice on desks and work in pairs (or individually if they prefer).

Review table totals and compare with predictions.

Ps read questions themselves and write answers in Pb's using their own data.

<table>
<thead>
<tr>
<th>Prediction</th>
<th>Tally of 20 throws</th>
<th>Totals</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>#</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Individual (or paired) work, monitored

Table drawn on BB or use enlarged copy master or OHP for reference

Remind Ps that their final totals should add up to 20.

Class applauds Ps who made accurate predictions by reasoning rather than by guessing.
T chooses one or two Ps to give their totals and answers and class checks whether they are correct.

Solutions: e.g. using data from table above

How many times did you get:

- a) a 2 or a 3 b) less than 5 c) not less than 5
  - (6) (13) (7)
- d) not more than 6 e) more than 6?
  - (20) (0)

What fraction of the 20 throws were these numbers?

- a) \( \frac{6}{20} = \frac{3}{10} \)
- b) \( \frac{13}{20} \)
- c) \( \frac{7}{20} \)
- d) \( \frac{20}{20} = 1 \)
- e) \( \frac{0}{20} = 0 \)

If you throw a dice once, how many possible outcomes are there? (6) If each number has an equal chance of being thrown, what is the probability of you throwing a 6?

(1 out of 6 times, or \( \frac{1}{6} \))

Extension

T could collect class data or use a computer program to show that the more times a dice is thrown, the closer the real data gets to what is expected.

Problem 2

Listen carefully and tell me whether you think that the statement is true or false. (Ps show T or F on slates or use pre-agreed actions.)

There are 2 red, 2 white and 2 green counters in a bag. I take out two counters with my eyes shut. Are these statements true or false?

- a) It is possible that both counters are green. (T)
- b) It is certain that both counters are green. (F)
- c) It is impossible that both counters are green. (F)
- d) It is certain that one of the 2 counters is green. (F)
- e) It is possible that one of the 2 counters is green. (T)

Whole class activity
Ps can choose the actions.

Responses shown in unison on command.
Ps responding correctly explain to those who were wrong
In good humour!
Praising, encouragement only
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factorising</strong></td>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td>In your <em>Ex. Bk</em>, factorise 159 and list all its factors. Review at BB with whole class. Ps come to BB to draw tree diagram, show the number as the product of its prime factors and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td>Discussion, reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>BB: [159] = 3 × 53</td>
<td>Ps may use a calculator.</td>
</tr>
<tr>
<td>Factors: 1, 3, 53, 159 (It is a nice number.)</td>
<td></td>
</tr>
<tr>
<td><strong>Problem 1</strong></td>
<td>Individual trial first, then whole class solution</td>
</tr>
<tr>
<td>Listen carefully and think about how you would solve this problem. The sum of 5 adjacent natural numbers is 5 times 25. What are the numbers? Allow Ps 2 or 3 minutes to think about it and try to work it out. Ps may discuss with their neighbours if they wish. Who thinks that they know what to do. Come and explain it to us. Who agrees? Who would do it another way? etc. e.g. ( 5 \times 25 = 25 + 25 + 25 + 25 + 25 = 125 ) so the 5 adjacent numbers must each be close to 25: ( 23 + 24 + 25 + 26 + 27 = 125 ) <strong>Answer:</strong> The 5 adjacent numbers are 23, 24, 25, 26 and 27.</td>
<td>Ps tell their ideas and findings to class.</td>
</tr>
<tr>
<td><strong>Problem 2</strong></td>
<td>Reasoning, checking, agreement, praising</td>
</tr>
<tr>
<td>If we throw a red and a white dice at the same time, what are the possible outcomes? Let’s write the red number first, then the white number. Ps dictate what T should write on BB. BB: (1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6); (2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6); (3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6); (4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6); (5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6); (6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6) [\text{Agree that there are 36 possible outcomes. Could we have worked it out without writing them all down? (Yes—for each of the 6 possible outcomes on the red dice, there are 6 possible outcomes on the white dice, i.e. } 6 \times 6 = 36)]</td>
<td>Elicit that: 24 + 26 = 23 + 27 = 50 T chooses a P to answer in a sentence.</td>
</tr>
<tr>
<td><strong>Extension</strong></td>
<td>Whole class activity</td>
</tr>
<tr>
<td>If the dice are not biased, what is the probability of you throwing 2 sixes? (( \frac{1}{36} ), as each of the 36 outcomes has an equal chance of happening)</td>
<td>Ps dictate in order round class.</td>
</tr>
<tr>
<td></td>
<td>Class points out errors.</td>
</tr>
<tr>
<td></td>
<td>Agreement, praising</td>
</tr>
<tr>
<td></td>
<td>Discussion, agreement, praising</td>
</tr>
<tr>
<td></td>
<td>Ps could show fraction on scrap paper or slates in unison on command.</td>
</tr>
</tbody>
</table>
**Y4**

### Activity 4

**Problem 3**

Listen carefully and think about how you would solve this problem.

*In how many ways could you draw 5 dots on this \(5 \times 5\) grid so that there is 1 dot in each row and column?*

**Reasoning:** e.g.

The rows and columns are labelled, so we cannot turn the grid.

For each of the possible 5 rows in column A, there are 4 possible rows in column B, 3 possible rows in column C, 2 possible rows in column D and only 1 possible row in column E, so there are

BB: \(5 \times 4 \times 3 \times 2 \times 1 = 120\) ways

---

### Lesson Plan 159

**Notes**

- **Whole class activity**
  - Drawn on BB or use enlarged copy master or OHP
  - BB: e.g.
  - Discussion, reasoning, agreement, praising
  - T helps with wording if necessary.

- **Individual (or paired) work, monitored, helped**
- **Tables drawn on BB or use enlarged copy master or OHP for reference**
  - (or for collecting class data)
- Individual work, monitored closely, praising
- Reasoning, checking, agreement, praising

---

**PhY4b, page 159**

Q.1 Read: *Throw 2 dice at the same time 36 times. Keep a tally of the outcomes here.*

If possible, Ps should have 2 different coloured dice each, but otherwise Ps work in pairs with one dice each.

- Set a time limit or keep class together on the throws.
- e.g. 1 and 1, 2 and 2, 3 and 3, 4 and 4, 2 and 3, 3 and 4, 4 and 5, 2 and 4, 3 and 5, 4 and 6, 2 and 5, 3 and 6, 5 and 5, 6 and 6

[T could collect the class data if there is time.]

After the 36 throws, Ps read the questions themselves and answer the using their own data (or if class is not very able, deal with one question at a time). T monitors thoroughly, correcting mistakes.

Choose some Ps to show and explain their results to class.

Class agrees/disagrees with their reasoning and answers.

**Solution:** (e.g. using the data in the tables above)

- **a)** *How many times were these numbers the product of the two numbers?*

  \[
  \begin{array}{cccccccccccc}
  1 & 2 & 3 & 4 & 5 & 6 & 8 & 9 & 10 & 12 & 15 & 18 & 20 & 24 & 25 & 30 & 36 \\
  1 & 2 & 1 & 4 & 3 & 4 & 2 & 1 & 2 & 5 & 2 & 0 & 1 & 3 & 1 & 2 & 1 & 1 \\
  \end{array}
  \]

- **b)** *How many times was the product of the two numbers even? (26)*

  What fraction is it of the 36 throws? \(\frac{26}{36} = \frac{13}{18}\)

- **c)** *How many times were these numbers the sum of the two numbers thrown?*

  \[
  \begin{array}{cccccccccccc}
  \text{Y} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 12 & 14 & 16 & 18 & 20 & 24 & 25 & 30 & 36 \\
  \text{B} & 1 & 2 & 3 & 4 & 6 & 7 & 4 & 3 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
  \end{array}
  \]

Elicit that 0 and 13 are impossible!
Y4

Activity

5

(Continued)

d) How many times was the sum of the two numbers even? (18)

What fraction is it of the 36 throws? \( \frac{18}{36} = \frac{1}{2} \)

6

PbY4b, page 159

Q.2 Read: Leslie threw a pyramid-shaped dice 100 times. It has 5 written on its square base and 1, 2, 3 and 4 written on its triangular sides.

T has a large model to show to class. Ps could demonstrate the experiment a few times so that T can explain the table. Elicit that there are 5 possible outcomes: landing on 1, 2, 3, 4 or 5.

Read: Leslie made this table to show how many times (T points to frequency in table) the dice landed on each number (T points to outcome in the table). We say that it shows the frequency of each outcome. (i.e. how many times it landed on each number)

a) Read: Write in the bottom row of the table what fraction of the 100 times each number was landed on.

This is called the probability of an outcome happening.

Ps complete table. Review at BB with whole class.

What would the fractions be as decimals? Ps dictate to T.

b) Read: How many times did Leslie throw:

i) at most a 3 \( (15 + 18 + 19 = \frac{52}{50} ) \)

ii) at least a 3? \( (19 + 16 + 32 = \frac{67}{50} ) \)

7

Problem 3

Listen carefully, picture the problem in your head and think about how you would solve it. Discuss it with your neighbour if you wish.

In a game, they chose only players whose birthdays are on the 13th of any month

How many players could they have chosen if it is certain that among them were 3 players who were born in the same month?

Who thinks that they know what to do? Who agrees? Who would do it another way? etc. Ps tell their ideas to class. If no P is on the right track, T gives hints and class solve it together.

Solution: e.g.

If the first 12 players all have birthdays on the 13th day of different months, and so do the next 12 players, the 25th player must have a birthday in the same month as two other players.

(However, it is possible, but not certain, that the first 3 players chosen could all have the same birthday!)

Answer: The least number of players chosen to be certain of there being 3 players who were born in the same month is 25.

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Y4

**Activity**

8

*PbY4b, page159*

Q.3  Read:  *If we toss a 10 p, a 20 p and a 50 p coin at the same time just once, which sides could face up?*

*Write T or H in the table.*

Less able Ps could have model coins on desks to help them.

Set a time limit.  Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>H</th>
<th>H</th>
<th>H</th>
<th>H</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 p</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 p</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>50 p</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Who could have worked out the answer without a table?

(For each of the two possible sides of the 50 p coin, there are two possible sides of the 20 p coin, and for each of these there are 2 possible sides of the 10 p coin, i.e. there are:

\[2 \times 2 \times 2 = 8\] possible outcomes

so the last 2 columns in the table are not needed.)

Ps think of probability questions to ask about the data.

(e.g. What is the chance of the 3 coins landing Heads up?) \(\frac{1}{8}\)

**Notes**

- Individual work, monitored, helped
- Drawn on BB or use enlarged copy master or OHP
- Agreement, self-correction, praising
- With T's help with wording if P has the right idea
- Extra praising
- Whole class activity
- In good humour!

---

Lesson Plan 159

45 min
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collect and analyse class data in LP 157/6, LP 158/3, LP 158/5 and LP 159/5, or use computer programs to simulate tossing coins and throwing dice, and compare with expected probabilities.</td>
<td></td>
</tr>
</tbody>
</table>

**PbY4b, page 160**

**Solutions:**

Q.1 Answers depend on Ps' data.

Q.2 No. of possibilities: $4 \times 3 \times 2 \times 1 = 24$

- ABCD, ABDC, ACBD, ACDB, ADBC, ADCB
- BACD, BADC, BCAD, BCDA, BDAC, BDCA
- CABD, CADB, CBAD, CBDA, CDAB, CDBA
- DABC, DACB, DBAC, DBCA, DCAB, DCBA

a) $\frac{1}{24}$  
b) $\frac{6}{24} = \frac{1}{4}$

Q.3 Impossible event, as there are only 4 chocolate wafers in the tin, so the probability will be 0!
Lesson Plan

Y4

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>R: Mental calculation</td>
<td></td>
</tr>
<tr>
<td>C: Revision and practice</td>
<td></td>
</tr>
<tr>
<td>E: Problems</td>
<td></td>
</tr>
</tbody>
</table>

**Y4 Lesson Plan 161**

**Week 33**

**Activity 1**

**Factorising**

In your *Ex. Bk*, factorise 160 and 161 and then list all their factors.

Review at BB with whole class. Ps come to BB to draw tree diagrams, show the numbers as the product of their prime factors and list all their factors. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

\[
\begin{array}{c}
160 = 2 \times 2 \times 2 \times 2 \times 5 \\
4 \quad 40 \\
2 \quad 2 \quad 2 \quad 10 \\
2 \quad 2 \\
4 \\

161 = 7 \times 23 \\
7 \quad 23
\end{array}
\]

Factors:

160: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160
161: 1, 7, 23, 161  (It is a nice number!)

---

**Problem 1**

Listen carefully and think about how you would solve this problem.

*In how many ways could you draw 6 dots on this 6 \times 6 grid so that there is 1 dot in each row and column?*

X, what do you thank that we should do? Who agrees? Who thinks something else? etc. Ps tell class their ideas.

**Reasoning:** e.g.

The rows and columns are labelled, so we cannot turn the grid.

For each of the possible 6 rows in column A, there are 5 possible rows in column B, for each of these there are 4 possible rows in column C, for each of these there are 3 possible rows in column D, for each of these there are 2 possible rows in column E and for each of these there is only 1 possible row in column F, so there are

**BB:**  \(6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720\) ways

---

**Problem 2**

Three friends, Alan, Ben and Charlie, live in the same street. Alan lives at no. 2, Ben lives at no. 4 and Charlie lives at no. 12. The houses in their street are the same distance apart.

Here is a diagram of their street showing the house numbers. They want to meet at a house on the street where the total distance they have to walk is as short as possible. Let's work out where they should meet.

What should we do first? (Write the number of houses they have to pass in total above each house, then see which is the smallest.)

Ps come to BB or dictate to T, explaining reasoning. Class checks that they are correct.

**BB:**

Ps: 15 12 11 10 11 12 13 14 15 16 17 18 21 24 27

Agree that the meeting point should be at no. 4 (Ben's house).

---

**Notes**

Individual work, monitored, helped

Discussion, reasoning, agreement, self-correction, praising

Ps may use a calculator.

Ps join up the factor pairs for 160.

Feedback for T

---

Whole class activity

Drawn on BB or use enlarged copy master or OHP

BB:

Discussion, reasoning, agreement, praising

T helps with wording if necessary.

---

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Discussion on strategy for solution.

Accept and praise any positive contribution. If no P thinks of the idea opposite, T gives hints or suggests it.

At a good pace

Ps can work out the totals in *Ex. Bks* first before coming to BB.

Reasoning, agreement, praising
Lesson Plan 161

Y4

Activity

4

PbY4b, page 161
Q.1  Read:  Calculate the product of the 7 smallest
       a) positive, even whole numbers
       b) 1-digit numbers.

Ps write plans in Pbs, do calculations in Ex. Bks, then write
answers in Pbs.  Set a time limit.

Review with whole class.  Ps could show results on scrap paper
or slates on command.  Ps answering correctly explain
reasoning at BB.  Who did the same?  Who did it another way? etc.  Mistakes discussed and corrected.

Solution:  e.g.
   a) 2 \times 4 \times 6 \times 8 \times 10 \times 12 \times 14
            = 8 \times 6 \times 8 \times 10 \times 168 = 8 \times 6 \times 1680
            = 8 \times 6 \times 13440 = 8 \times 80640 = 645120
   b) 0 \times 1 \times 2 \times 3 \times 4 \times 5 \times 6 = 0

What is another name for positive, even whole numbers?
(natural numbers)

What is another name for a whole number?  (integer)

Remind Ps that an integer can be positive or negative or zero.
Elicit that zero is neither positive nor negative.

21 min

5

PbY4b, page 161
Q.2  Read:  Circle the natural numbers up to 100 which have only two
       factors.  (e.g. the only factors of 7 are 7 and 1)

We call these numbers prime numbers.
List them in increasing order.

Ps try out divisors 2, 3, 5, 7 and 9 in Pbs if necessary, although
Ps might use other strategies (e.g. after circling 2, we know that
any other even number is not a prime number, so can be crossed
out; after circling 3, we know that any other multiple of 3 is not a
prime number, so can be crossed out, etc.)

Review with whole class.  Ps come to BB or dictate to T.  Class
agrees/disagrees.  Mistakes discussed and corrected.

Elicit that 1 is not a prime number as it has only 1 factor – itself!

Solution:

\begin{align*}
1 & 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8 \ 9 \ 10 \ 11 \ 12 \ 13 \ 14 \ 15 \ 16 \ 17 \ 18 \ 19 \ 20 \\
21 \ 22 \ 23 \ 24 \ 25 \ 26 \ 27 \ 28 \ 29 \ 30 \ 31 \ 32 \ 33 \ 34 \ 35 \ 36 \ 37 \ 38 \ 39 \ 40 \\
41 \ 42 \ 43 \ 44 \ 45 \ 46 \ 47 \ 48 \ 49 \ 50 \ 51 \ 52 \ 53 \ 54 \ 55 \ 56 \ 57 \ 58 \ 59 \ 60 \\
61 \ 62 \ 63 \ 64 \ 65 \ 66 \ 67 \ 68 \ 69 \ 70 \ 71 \ 72 \ 73 \ 74 \ 75 \ 76 \ 77 \ 78 \ 79 \ 80 \\
81 \ 82 \ 83 \ 84 \ 85 \ 86 \ 87 \ 88 \ 89 \ 90 \ 91 \ 92 \ 93 \ 94 \ 95 \ 96 \ 97 \ 98 \ 99 \ 100 \\
\end{align*}


29 min

Notes

Individual work, monitored (less able helped)

Ps can use any combination of multiplications.

In unison

Reasoning, agreement, self-correction, praising

Revision of types of numbers.
Agreement, praising

Erratum

In Pbs, 
'31, 31' should be 
'31, 32'

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Discussion, agreement, self-correction, praising

Feedback for T
Y4

Activity

6  PbY4b, page 161

Q.3  Read: Practise calculation.

Let’s see how many you can do in 5 minutes! Remember to check your answers! Start . . . now! . . . Stop!

Review with whole class. Ps come to BB to write results, explaining with place-value detail. Class agrees/disagrees.

Mistakes discussed and corrected.

Show e) and f) as long multiplication, and g) and h) as long division, if problems.

Who had all 8 correct? Let’s give them a round of applause!

Who had 1 mistake (2, 3, 4, more than 4 mistakes)? T notes Ps having difficulty and sets them extra similar calculations for homework.

Solution:

\[
\begin{align*}
\text{a)} & \quad 8 \times 104 \div 7 = 118 \quad 1 \text{ min} \\
\text{b)} & \quad 4 \times 1031 \div 7 = 144 \quad 1 \text{ min}
\end{align*}
\]

Week 33

Lesson Plan 161

Notes

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Evaluation, praising

Feedback for T

Individual work, monitored, helped

Less able Ps could have 20 unit cubes on desks.

Or ask Ps to think of 20 as the product of 3 numbers.

Discussion, reasoning, checking, agreement, self-correcting, praising

Model, or diagram on BB:

\[\begin{align*}
\text{a)} & \quad 20 = 2 \times 2 \times 5, \text{ and there is no other 3-term multiplication possible, the three edges are:} \\
\text{b)} & \quad A = 2 \times (2 \times 2) + 4 \times (2 \times 5) \\
\quad & \quad = 2 \times 4 + 4 \times 10 = 8 + 40 = 48 \text{ (unit squares)}
\end{align*}\]
**Y4**

### Lesson Plan 161

**Activity**

8

*PbY4b, page 161*

Q.5  Read: *Tom has ducks and pigs on his farm, 8 in total. They have 22 legs altogether. How many ducks and how many pigs does Tom have? Work out the answer in your exercise book.*

Set a time limit. Ps can use any method, but encourage Ps to use mathematical reasoning.

Review with whole class. If you have an answer, show me... now! (*D = 5, P = 3*) P answering correctly explains at BB. Who did the same? Who worked it out in a different way? etc.

**Solution:** e.g.

Let number of pigs be *P* and number of ducks be *D*.

\[
P + D = 8, \text{ so } D = 8 - P
\]

\[
2 \times D + 4 \times P = 22
\]

Putting \(8 - P\) instead of \(D\) in equation:

\[
2 \times (8 - P) + 4 \times P = 22
\]

\[
16 - 2 \times P + 4 \times P = 22
\]

\[
2 \times P = 22 - 16 = 6
\]

\[
P = 6 \div 2 = 3 \text{ and so } D = 8 - P = 8 - 3 = 5
\]

**Check:** \(5 \times 2 + 3 \times 4 = 10 + 12 = 22\) ✅

**Answer:** Tom has 3 pigs and 5 ducks.

---

**Notes**

Individual work, monitored, helped

(or whole class activity if time is short)

Discussion, reasoning, agreement, self-correcting, praising

Accept any valid method which gives the correct result, including trial and error, but also show the solution opposite.

(If no P has correct answer, class solves it with T’s help.)
Y4

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>R: Mental calculation</td>
<td>Lesson Plan 162</td>
</tr>
<tr>
<td>C: Revision and practice</td>
<td>Notes</td>
</tr>
<tr>
<td>E: Problems</td>
<td>Individual work, monitored, helped</td>
</tr>
</tbody>
</table>

**Activity 1**

**Factorising**

In your Ex. Bk, factorise 162 and then list all its factors.

Review at BB with whole class. Ps come to BB to draw tree diagram, show the numbers as the product of its prime factors and list all its factors. Class agrees or disagrees. Mistakes discussed and corrected.

BB:

\[ 162 = 2 \times 3 \times 3 \times 3 \]

Factors: 1, 2, 3, 6, 9, 18, 27, 54, 81, 162

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Discussion on strategy for solution.

At a good pace
Reasoning, agreement, self-correction, praising

Feedback for T

**Problem 1**

Which point on the line is the shortest total distance from the 4 points, A, B, C and D?

How could we work it out? (Ps might notice the similarity to Activity 3 in Lesson 161, but if not, T reminds Ps about it.)

T points to each number marked on the number line in turn and Ps come to BB or dictate its total distance from the 4 points. Encourage mental calculation if possible. Class points out errors.

BB:

\[
\begin{array}{cccccccccccc}
B & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
C & & & & & & & & & & & & & & & & \\
D & & & & & & & & & & & & & & & & \\
\end{array}
\]

Agree that there are 3 points with the shortest total distance (15) from A, B, C and D: at 2 (B), 3 and 4 (C).

**Problem 2**

Q.1 Read: Practise calculation. Do the operations in the correct order.

Revise order of operations first if necessary. Set a time limit.

Ps do necessary calculations in Ex. Bks, write the interim results above each operation sign and write the answers in Pbs.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. If problems or disagreement, Ps do calculations at side of BB, reasoning with place-value detail. Mistakes discussed and corrected.

What did you notice about the two operations in each part?

**Solution:**

| a) \(2756 - 1348 + 220\) | \(2756 - (1348 \div 220)\) |
| b) \(2756 \times 4 + 1348 \div 4\) | \((2756 + 1348) \div 4\) |
| c) \((6315 \div 1726) \times 3\) | \((6315 \div 3) \times 1726\) |
| d) \((10256 \times 4 - 2372 \div 4)\) | \((10256 - 2372) \div 4\) |
| e) \(2187 \div (9 \div 3)\) | \(2187 \div 9 \div 3\) |
| f) \(2187 \times 9 \div 3\) | \(2187 \times (9 \div 3)\) |

Extra praise if Ps noticed that [apart from e)] the calculations on RHS have the same result as on LHS, so they only had to do half of the calculations.

In e), elicit that dividing by 9 and then by 3 is the same as dividing by 27.

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit (or if class is not very able, T chooses only one or two)
Reasoning, agreement, self-correction, praising

Extra praise if Ps noticed that [apart from e)] the calculations on RHS have the same result as on LHS, so they only had to do half of the calculations.

In e), elicit that dividing by 9 and then by 3 is the same as dividing by 27.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y4</td>
<td></td>
</tr>
<tr>
<td><strong>PbY4b, page 162</strong></td>
<td>Individual work, monitored helped</td>
</tr>
<tr>
<td>Q.2 Read: <em>Plan, estimate, calculate and check in your exercise book. Write the answers here.</em></td>
<td>Discussion, reasoning, agreement, checking, self-correction, praising</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Lesson Plan 162</strong></td>
<td></td>
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<td></td>
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<tr>
<td><strong>Notes</strong></td>
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<tr>
<td><strong>Activity 4</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Y4</strong></td>
<td></td>
</tr>
<tr>
<td><strong>PbY4b, page 162</strong></td>
<td></td>
</tr>
<tr>
<td>Q.2 Read: <em>Plan, estimate, calculate and check in your exercise book. Write the answers here.</em></td>
<td></td>
</tr>
<tr>
<td>Deal with one question at a time. Ps read problems themselves, solve in <em>Ex. Bks.</em> and write only the answers in <em>Pbs.</em></td>
<td></td>
</tr>
<tr>
<td>Review with whole class. Ps could write answers on slates or scrap paper and show on command. Ps answering correctly explain at BB to those who were wrong. Who agrees? Who did it a different way? etc. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td><strong>Solutions:</strong></td>
<td></td>
</tr>
<tr>
<td>a) <em>In a large container there are 18 649 litres of water.</em></td>
<td></td>
</tr>
<tr>
<td><em>In a smaller container there are 12 450 litres less.</em></td>
<td></td>
</tr>
<tr>
<td><strong>How much water is in the smaller container?</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Plan:</strong> 18 649 – 12 450 (litres)</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> 19 000 – 12 000 = 7 000 (litres)</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong> There are 6199 litres in the smaller container.</td>
<td></td>
</tr>
<tr>
<td>b) <em>Andrew has £6278 and James has £2327 more.</em></td>
<td></td>
</tr>
<tr>
<td><strong>How much money will James have left after spending £1796?</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Plan:</strong> £6278 + £2327 – £1796</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> £6000 + £2000 – £2000 = £6 000</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong> James will have £6809 left.</td>
<td></td>
</tr>
<tr>
<td>c) <em>A cruise to a certain holiday destination costs £875 per person.</em></td>
<td></td>
</tr>
<tr>
<td>i) <strong>How much would it cost for a group of 4 people?</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Plan:</strong> £875 × 4</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> £900 × 4 = £3600</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong> It would cost £3500 for 4 people.</td>
<td></td>
</tr>
<tr>
<td>ii) <strong>How much would it cost for a group of 8 people?</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Plan:</strong> £875 × 8 (or £3500 × 2)</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> £900 × 8 = £7200</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong> It would cost £7000 for 8 people.</td>
<td></td>
</tr>
<tr>
<td>iii) <strong>How much would it cost for each group if they travelled by plane for £400 less each?</strong></td>
<td></td>
</tr>
<tr>
<td><strong>4-group:</strong> <strong>Plan:</strong> (£875 – £400) × 4 = £475 × 4</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> £500 × 4 = £2000</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong> It would cost £1900 for 4 people.</td>
<td></td>
</tr>
<tr>
<td><strong>8-group:</strong> <strong>Plan:</strong> (£875 – £400) × 8 = £475 × 8</td>
<td></td>
</tr>
<tr>
<td><strong>E:</strong> £500 × 8 = £4000</td>
<td></td>
</tr>
<tr>
<td><strong>Answer:</strong> It would cost £3800 for 8 people.</td>
<td></td>
</tr>
<tr>
<td><strong>36 min</strong></td>
<td></td>
</tr>
</tbody>
</table>
Y4

**Activity**

5  
*PbY4b, page 162*

Q.3  Read: *Where could you put ‘+’ signs among the digits 1 to 7 so that the sum is 100? You must keep the digits in increasing order!*

T writes the digits 1 to 7 on BB. Allow Ps a couple of mintues to try it in *Ex. Bks.* Who has solved it? Come and show us. Who agrees? Who has found a different solution? etc.

If no P has found an answer, T gives hint about 2-digit numbers and class solves it together. Ps write a solution in *Pbs.*

_Solution:_

BB: \[1 + 2 + 34 + 56 + 7 = 100\]

or \[1 + 23 + 4 + 5 + 67 = 100\]

59 min

6  
*PbY4b, page 162*

Q.4  Read: *Point A stands for 1 fifth and Point B stands for 7 tenths. Mark the positions of 0 and 1.*

How can we do it? T asks several Ps what they think. Elicit that the distance between A and B is 5 tenths (7 tenths – 2 tenths) of a unit, so if we measure it, we can work out what 1 tenth of a unit is and then where 0 and 1 should be.

Ps measure with rulers and mark the tenths and 0 and 1 in *Pbs.* Review with whole class. Ps come to BB to explain and mark the tenths with a BB ruler. Class agrees/disagrees.

_Solution:_

A to B: 5 tenths of a unit → 5 cm

1 tenth of a unit → 1 cm

0 is 2 tenths of a unit, i.e. 2 cm, to the left of A.

1 is 3 tenths of a unit, i.e. 3 cm, to the right of B.

BB:  

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1/10</td>
<td>1</td>
</tr>
<tr>
<td>1/10</td>
<td>2/10</td>
<td>2</td>
</tr>
<tr>
<td>3/10</td>
<td>3/10</td>
<td>3</td>
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<tr>
<td>4/10</td>
<td>4/10</td>
<td>4</td>
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<tr>
<td>5/10</td>
<td>5/10</td>
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<td>6/10</td>
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<td>6</td>
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<tr>
<td>7/10</td>
<td>7/10</td>
<td>7</td>
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<tr>
<td>8/10</td>
<td>8/10</td>
<td>8</td>
</tr>
<tr>
<td>9/10</td>
<td>9/10</td>
<td>9</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td></td>
</tr>
</tbody>
</table>

39 min

7  
*PbY4b, page 162, Q.5*

Read: *Check the results and correct the answer if it is wrong.*

What should we do first? (Change the Roman numerals into Arabic numbers.) Revise Roman numerals if necessary.

Ps come to BB or dictate what T should write. Class agrees/disagrees.

In the case of b), which is wrong, Ps suggest how to correct it.

_Solution:_

a) \[C = 100,\]

\[LXXX = 50 + 30 = 80\]

\[VI = 5 + 1 = 6\]

\[100 + 80 + 6 = 186\]

b) \[MMII = 2000 + 2 = 2002\]

\[M = 1000\]

\[CM = 1000 – 100 = 900\]

\[X = 100 – 10 = 90\]

\[IX = 10 – 1 = 9\]

\[1000 + 900 + 90 + 9 = 1999\]

45 min

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<table>
<thead>
<tr>
<th>Y4</th>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>1</strong></td>
<td><strong>2</strong></td>
</tr>
<tr>
<td></td>
<td>Factorising</td>
<td><strong>Lesson Plan</strong></td>
</tr>
<tr>
<td></td>
<td>Let's factorise 163 and then list all its factors.</td>
<td>163</td>
</tr>
<tr>
<td></td>
<td>Ps dictate or come to BB to try each of the prime numbers, 2, 3, 5, 7 and 11 as divisors, using 'quick' methods where possible. Should we try dividing by 13? (No, as $13 \times 13 = 169 &gt; 163$)</td>
<td>Whole class activity</td>
</tr>
<tr>
<td></td>
<td>Elicit that 163 is a prime number, and its factors are 1 and 163.</td>
<td>At a good pace</td>
</tr>
<tr>
<td></td>
<td><strong>4 min</strong></td>
<td>Ps explain reasoning or do divisions at side of BB.</td>
</tr>
<tr>
<td></td>
<td><strong>2</strong></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>PbY4b, page 163</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Read: <em>In your exercise book, write 2-term additions using the numbers in Set A.</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB: $A = { -3, 2, 1, 0, -5, 6 }$</td>
<td></td>
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<tr>
<td></td>
<td>What could we do first to make the task easier? (Put them in increasing order.) Ps dictate to T who writes on BB and Ps write in Pbs. Let's see how many you can write and solve in 5 minutes!</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Start . . . now! . . . Stop!</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps dictate additions and T writes on BB in a logical order. Class points out errors in solution or additions missed. Accept what Ps dictate for the moment.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Read: <em>How many additions are possible?</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Elicit that for each of the 6 possible 1st terms, there are 5 possible 2nd terms, i.e. $6 \times 5 = 30$ possible additions, but as the terms of an addition are inter-changeable, e.g. $2 + 1 = 1 + 2$, we must divide 30 by 2, so 15 different additions are possible.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ps check that there are 15 additions on BB and dictate any missing.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB: $-5 + (-3) = -8$, $-5 + 0 = -5$, $-5 + 1 = -4$, $-5 + 2 = -3$, $-5 + 6 = 1$; $-3 + 0 = -3$, $-3 + 1 = -2$, $-3 + 2 = -1$, $-3 + 6 = 3$; $0 + 1 = 1$, $0 + 2 = 2$, $0 + 6 = 6$; $1 + 2 = 3$, $1 + 6 = 7$; $2 + 6 = 8$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Read: <em>How many results are</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>i) positive</em> (8, but only 6 different results)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><em>ii) negative?</em> (7, but only 6 different results)</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>14 min</strong></td>
<td>Ps shout out numbers in unison.</td>
</tr>
<tr>
<td></td>
<td><strong>R:</strong> Mental calculation</td>
<td>Elicit how many different results there are.</td>
</tr>
<tr>
<td></td>
<td><strong>C:</strong> Revision and practice</td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>E:</strong> Problems</td>
<td></td>
</tr>
</tbody>
</table>

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Y4

Activity

Lesson Plan 163

Notes

Y4 Lesson Plan 163

PbY4b, page 163

Q.2 Read: Solve this problem in your exercise book. Write only the answer here.

Ps read problem themselves and solve it in Ex. Bks. Set a time limit. Remind Ps to check their answers!

Review with whole class. Ps could show answer on scrap paper or slates on command. P answering correctly explains at BB to those who were wrong. Mistakes discussed and corrected.

Solution:
If my father takes 20 paces forward, he covers a distance of 16 m.
If I take 10 paces forward, I cover a distance of 7 m.
How much longer is one of my father's paces than one of mine?
BB: e.g.
F: 20 paces → 16 m
10 paces → 8 m
Me: 10 paces → 7 m
Difference in 10 paces: 8 m – 7 m = 1 m = 100 cm
Difference in 1 pace: 100 cm ÷ 10 = 10 cm
Answer: The father's pace is 10 cm longer than his child's pace.

Q.3 Read: The price of 0.7 litres of syrup is £5.60. How much would 1 litre of syrup cost?
Ps decide what to do first, then how to continue. Ps come to BB to explain reasoning. Class points out errors in calculations or reasoning or suggests another way to solve it. T intervenes and helps only when necessary.
BB: e.g.
0.7 litres → £5.60
0.1 litres → £5.60 ÷ 7 = £0.80
1 litre → £0.80 × 10 = £8.00
or 0.7 litres = 70 cl → £5.60 = 560 p
10 cl → 560 p ÷ 7 = 80 p
100 cl → 80 p × 10 = 800 p = £8
Answer: 1 litre of syrup would cost £8.

Q.4 Read: 8 = 2 × 4 and 8 + 4 = 12 is exactly divisible by 3, as 3 × 4 = 12
14 = 2 × 7 and 14 + 7 = 21 is exactly divisible by 3, as 3 × 7 = 21
Is this statement true or false? Give a reason for your answer.
If we add a natural number and its double, then the sum is exactly divisible by 3.
T gives Ps time to think about it, discuss with neighbours and try to find a counter example. Show me what you think . . now! (T)
A, why do you think so? Who agrees? Who can give another reason?
BB: e.g.
2 × 4 = 8 = 4 + 4,
8 + 4 = (4 + 4) + 4 = 3 × 4
2 × 7 = 14 = 7 + 7,
14 + 7 = (7 + 7) + 7 = 3 × 4
Adding a natural number to its double means that you have 3 times the number, so the sum must be a multiple of 3.

Whole class activity
(Individual work if Ps wish, monitored, helped)
Ps try out other examples in Ex. Bks. then show responses on slates or by agreed actions.
T shows the general solution:
Let any natural number be n:
n + 2 × n = n + n + n
= 3 × n
which is divisible by 3.
Ps write agreed answer in own words in Pbs.

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**Activity 6**

**PbY4b, page 163**

Q.5 Read: *Factorise these numbers.*

What will you have to do in b) and c)? (Do the calculation first, then factorise the result.)

Ps try out prime numbers as divisors and draw tree diagrams in *Ex. Bks*, then write the number as the product of its prime factors in *Pbs*. Set a time limit or deal with one part at a time.

Review with whole class. Ps come to BB to draw tree diagrams and write the multiplications. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) \(720 = 2 \times 2 \times 2 \times 3 \times 3 \times 5\)

b) \(8 \times 8 - 7 \times 7 = 64 - 49 = 15 = 3 \times 5\)

c) \(10 \times 10 - 1 = 100 - 1 = 99 = 3 \times 3 \times 11\)

**34 min**

---

**Activity 7**

**PbY4b, page 163**

Q.6 a) Read: *Factorise 1250 and 175 in your exercise books.*

Set a time limit. Review with whole class. Ps come to BB to draw tree diagrams and write the multiplications. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) \(1250 = 2 \times 5 \times 5 \times 5 \times 5\)

b) \(175 = 5 \times 5 \times 7\)

i) Read: *What is the greatest natural number which is a factor of both numbers?*

Show me . . . now! (25) Ps come to BB to point to 5 × 5.

ii) Read: *What is the smallest natural number which is a factor of both numbers?*

Show me . . . now! (1)

b) As in a) above.

**Solution:**

a) \(68 = 2 \times 2 \times 17\)

b) \(170 = 2 \times 5 \times 17\)

i) Greatest factor of both 68 and 170 is 34 (2 × 17)

ii) Smallest factor of both 68 and 170 is 1.

Tell class that factors of more than one number are **common** factors.

**42 min**

---

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<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8</strong></td>
<td><strong>Lesson Plan 163</strong></td>
</tr>
<tr>
<td><strong>Problem</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td>T sticks these number cards on BB: 1 7 7 1</td>
<td>Whole class activity (or individual work in Ex. Bks. if Ps prefer)</td>
</tr>
<tr>
<td>How many different 4-digit numbers can be made with these number cards? Ps come to BB or dictate numbers to T.</td>
<td>Agreement, (self-correcting) praising</td>
</tr>
<tr>
<td>Agree that there are 6 possible different 4-digit numbers</td>
<td></td>
</tr>
<tr>
<td>BB: 1177, 1717, 1771, 7117, 7171, 7711</td>
<td></td>
</tr>
</tbody>
</table>

*45 min*
Factorising

In your Ex. Bk, factorise 164 and list all its factors.

Review at BB with whole class. Ps come to BB to draw tree diagram, show the number as the product of its prime factors and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.

BB:

\[
164 = 2 \times 2 \times 41
\]

Factors: 1, 2, 4, 41, 82, 164

---

Plane shapes

Ps work in pairs and each pair has the same set of three different sizes of squares on desk. T has larger set for demonstration only.

a) Take one of the small squares BB:

- Measure its sides.
- Calculate its perimeter and area.
- Measure its diagonals.

Agree that the 2 diagonals of a square are equal.

Ps calculate in Ex. Bks. then dictate measurements to T.

b) Take one of the middle-sized squares BB:

- Measure its sides.
- Calculate its perimeter and area.
- Measure its diagonals.

Ps dictate measurements to T.

- Can you tile over (tessellate) this square with the small squares? (Yes, 4 small squares cover it exactly)

Think of a number line which is endless in 2 directions. Who remembers the mathematical name for endless or never ending? (infinite)

Now think of the plane (flat surface) that the square is on and imagine it being infinite and spreading out wider and wider in all 4 directions.

Could you use the small or middle-sized squares to tessellate the whole plane so that there are no gaps and no overlaps?

Ask several Ps what they think and why. There are two arguments, e.g.

P₁: No, the plane is endless, so we would never be able to finish tiling.

P₂: Yes, we could tile the plane using equal (congruent) squares in any of the 4 possible directions but we would need an infinite number of squares.

Praise both arguments but T supports P₁, as in practice nobody could ever tessellate an infinite plane!
Y4

Activity

2

(Continued)

c) Take one of the large squares BB:

- Measure its sides.
- Calculate its perimeter and area. (For the area, T advises changing the lengths to mm, then Ps can do long multiplication, or $56 \times 7 \times 8$)
- Measure its diagonals.

Ps dictate measurements to T.

- Can you tessellate this square with the small or middle-sized squares? (No)

If we cut the squares in half along a diagonal, could we tessellate the large square with the small or the middle-sized half squares? (i.e. right-angled triangles)

Ps try it out and confirm that it can be done. T shows it on diagram or model on BB (as in diagram above).

Lesson Plan 164

Notes

Measurements of sides need only be approximate.

BB: $a = 5.6$ cm

$P \approx 4 \times 5.6 = 22.4$ cm

$A = 5.6 \times 5.6 = 31.36$ cm$^2$

(as 100 mm$^2$ = 1 cm$^2$)

$d = 8$ cm

Discussion, demonstration, agreement, praising

3  
PbY4b, page 164

Q.1 Read: The rectangle is the plan of a garden.

1 mm on the diagram means 1 m in real life.

Measure the sides and complete the table.

Agree on values of $a$ and $b$ first before Ps continue with table.

Review with whole class. Ps come to BB to complete the table, explaining reasoning and showing calculations on BB. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

<table>
<thead>
<tr>
<th>Side</th>
<th>On diagram</th>
<th>In real life</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>40 mm</td>
<td>40 m</td>
</tr>
<tr>
<td>$b$</td>
<td>30 mm</td>
<td>30 m</td>
</tr>
<tr>
<td>Perimeter</td>
<td>140 mm</td>
<td>140 m</td>
</tr>
<tr>
<td>Area</td>
<td>1200 mm$^2$</td>
<td>1200 m$^2$</td>
</tr>
</tbody>
</table>

Discussion, demonstration, agreement, praising

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

BB: $P = 2 \times (40 + 30) = 2 \times 70 = 140$ (mm)

$A = 30 \times 40 = 1200$ (mm$^2$)

4  
PbY4b, page 164

Q.2 Read: The square is the plan of a table.

1 mm on the diagram means 3 cm in real life.

Measure the sides and complete the table.

Agree on values of $a$ and $b$ first before Ps continue with table.

Review with whole class. Ps come to BB to complete the table, explaining reasoning and showing calculations on BB. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

<table>
<thead>
<tr>
<th>Side</th>
<th>On diagram</th>
<th>In real life</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>30 mm</td>
<td>90 cm</td>
</tr>
<tr>
<td>Perimeter</td>
<td>120 mm</td>
<td>360 cm</td>
</tr>
<tr>
<td>Area</td>
<td>900 mm$^2$</td>
<td>8100 cm$^2$</td>
</tr>
</tbody>
</table>

Elicit that if the lengths of the sides of a square are increased by 3 times, the area increases by $3 \times 3 = 9$ times.

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

BB: $P = 4 \times 30 = 120$ mm

$A = 30 \times 30 = 900$ (mm$^2$)

BUT in real life:

$A = 90 \times 90 = 8100$ cm$^2$
5 \textit{PbY4h, page 164, Q.3}

Read: \textit{On the outside of a measuring cylinder, there are marks at every 10 cl. Join up the quantities to the corresponding marks.}

Who can explain the diagram? (The cylinder has been divided up into 10 equal parts, with a mark at every 1 tenth of a litre, so if you pour in 1 tenth of a litre of water, it will be level with the first mark.)

How many cl (ml) are in 1 tenth of a litre? (10 cl, 100 ml)

Ps come to BB to choose a quantity, convert to litres if necessary and join up to appropriate mark on diagram. Class agrees/disagrees. Ps draw joining lines in \textit{Pbs} too.

\textbf{Solution:}

\begin{center}
\begin{tikzpicture}
\begin{scope}
\clip (-0.5,0) rectangle (10,10);
\draw (3,0) circle [radius=3];
\draw (5,0) circle [radius=5];
\draw (7,0) circle [radius=7];
\draw (9,0) circle [radius=9];
\draw (3,3) circle [radius=3];
\draw (5,5) circle [radius=5];
\draw (7,7) circle [radius=7];
\draw (9,9) circle [radius=9];
\draw (1,0) -- (2,2) -- (3,3) -- (4,4) -- (5,5) -- (6,6) -- (7,7) -- (8,8);
\draw (1,0) -- (2,2) -- (3,3) -- (4,4) -- (5,5) -- (6,6) -- (7,7) -- (8,8);
\end{scope}
\end{tikzpicture}
\end{center}

6 \textit{PbY4b, page 164}

\textbf{Q.4 Read: Change the units of measure, then round them to the nearest whole unit required.}

Do a) i) with the whole class first if necessary as an example for the class to follow. Set a time limit. T writes an extra question iv) for each part on BB for Ps who finish early.*

Review with whole class. Ps dictate to T or come to write on BB, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected

\textbf{Solution:}

\begin{itemize}
\item[a)] i) \(678 \text{ m} = 0 \text{ km}\) \(678 \text{ m} = 1 \text{ km}\)
\item[ii)] \(15 240 \text{ m} = 15 \text{ km}\) \(240 \text{ m} = 15 \text{ km}\)
\item[iii)] \(5648 \text{ mm} = \frac{5}{6} \text{ m}\) \(648 \text{ mm} = \frac{6}{6} \text{ km}\)
\item[iv)] \(5648 \text{ cm} = \frac{56}{66} \text{ m}\) \(48 \text{ cm} = \frac{66}{66} \text{ m}\)
\item[b)] i) \(3518 \text{ ml} = 3 \text{ litres}\) \(518 \text{ ml} = 4 \text{ litres}\)
\item[ii)] \(3518 \text{ cl} = \frac{35}{35} \text{ litres}\) \(18 \text{ cl} = \frac{35}{35} \text{ litres}\)
\item[iii)] \(18 450 \text{ ml} = 18 \text{ litres}\) \(450 \text{ ml} = 18 \text{ litres}\)
\item[iv)] \(18 450 \text{ cl} = \frac{184}{35} \text{ litres}\) \(50 \text{ cl} = \frac{185}{35} \text{ litres}\)
\end{itemize}

Mental practice

\begin{itemize}
\item[a)] T says an amount in kg and Ps change it to grams. e.g.
\begin{align*}
1 \text{ tenth of a kg} &= 100 \text{ g}, & 1 \text{ fifth of a kg} &= 200 \text{ g}, \\
0.1 \text{ of a kg} &= 100 \text{ g}, & 3 \text{ tenths of a kg} &= 300 \text{ g}, \\
3 \text{ fifths of a kg} &= 600 \text{ g}, & 0.3 \text{ of a kg} &= 300 \text{ g}, \text{ etc.}
\end{align*}
\item[b)] T says an amount in g and Ps give it in kg (fraction or decimal).
\begin{align*}
\text{(Ps can say the amounts in g too and choose Ps to convert it to kg.)}
\end{align*}
\end{itemize}
Y4

Activity

Calculation and tables practice, revision, activities, consolidation

PbY4b, page 165

Notes

Lesson Plan

165

Q.1 a)

<p>| | | | | |</p>
<table>
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<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>30</td>
<td>15</td>
<td>10</td>
<td>6</td>
<td>5</td>
</tr>
</tbody>
</table>

b) Its edges must be 2 units, 3 units and 5 units long.

c) There are 4 faces (2 of each type) which are 5 units long:
\[ A_1 = 5 \text{ units} \times 3 \text{ units} = 15 \text{ unit squares} \]
\[ A_2 = 5 \text{ units} \times 2 \text{ units} = 10 \text{ unit squares} \]

Q.2 a) \[ 360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5 \]
b) \[ 768 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \]
c) Greatest common factor of 360 and 768:
\[ 2 \times 2 \times 2 \times 3 = 24 \]

Q.3

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>8</td>
<td>7</td>
</tr>
</tbody>
</table>

(A to B is 30 mm, so eighths are 6 mm apart)

Q.4 a) CDLX \times VII = MMMCCXX
\[ 460 \times 7 = 3210 \text{ (Should be 3220)} \]
Correction: e.g. CDLX \times VII = MMMCCXX

b) MMCXII – MCMXV = XCVII
\[ 2112 - 1915 = 97 \text{ (Should be 197)} \]
Correction: e.g. MMCXII – MCMXV = XCVII

c) MMMLXIX \div IX = CCCXL
\[ 3069 \div 9 = 340 \text{ (Should be 341)} \]
Correction: e.g. MMMLXIX \div IX = CCCXL

d) CCCLXXXVII + MCCXIII = MCD
\[ 387 + 1213 = 1400 \text{ (Should be 1600)} \]
Correction: e.g. CCCLXXXVII + MCCXIII = MCD

Q.5 Quantity of punch made:
\[ \frac{3}{4} + \frac{1}{2} + 2 \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = 11 \text{ (litres)} \]
(or \[ 1.75 + 0.50 + 2.50 + 1.50 + 4.75 = 11.00 \])
So 6 2-litre jugs will be needed, but the last jug will contain only 1 litre of punch.

Ps could build a model with unit cubes as a check.
## Lesson Plan 166

### Activity

#### 1. Factorising

In your Ex. Bk, factorise 165 and 166 and then list all their factors. Review at BB with whole class. Ps come to BB to draw tree diagrams, show the numbers as the product of their prime factors and list all their factors. Class agrees/disagrees. Mistakes discussed and corrected.

BB:

\[
\begin{align*}
165 & = 3 \times 5 \times 11 \\
166 & = 2 \times 83
\end{align*}
\]

Factors:

- 165: 1, 3, 5, 11, 15, 33, 55, 165
- 166: 1, 2, 83, 166 (It is a nice number!)

---

#### 2. Plane shapes

Pairs of Ps each have the same type and number of plane shapes on desks and T has larger version of the same set for demonstration.

BB: e.g.

- What name could you give to all these shapes? (polygons, i.e. plane shapes with many straight sides) T chooses Ps to hold up a shape and say what they know about it. (e.g. name, number of sides, types of angles, etc.). Elicit that acute angle < right angle (90°) < obtuse angle
- Which shapes are similar (congruent)? (e.g. A ~ C, L ~ M ~ N, etc. but none are congruent)

a) Let’s measure the sides of some of these shapes (T writes letters on BB) and calculate their perimeters and areas. Ps dictate to T.

b) Let’s tile (tessellate) some of the larger shapes with the smaller shapes. Ps come to BB to show class when they have found shapes which can be tessellated.

- G with A
- K with I
- K with P

---

### Notes

- Individual work, monitored, helped
- Discussion, reasoning, agreement, self-correction, praising
- Ps may use a calculator.
- Feedback for T

---

Whole class activity to start

Use copy master copied on to card and either kept as a sheet for Ps, or cut out.

(T’s version enlarged and stuck on BB.)

(If class is not very able, T can choose which shapes to deal with.)

[Pts and T should have extra copies of some of the smaller shapes cut out for tessellation in b.)]

Initial discussion to revise plane shapes.

(Extra praise for clever facts, e.g. 2-dimensional)

Praising, encouragement only

Paired work in measuring, whole class calculation

Paired work, monitored, helped, then demonstration and discussion with large models with whole class

Agreement, praising
**Activity**

### 3 PbY4b, page 166

**Q.1** Read: A gang of workmen repaired 5 km 300 m of road in the 1st week of March, 8 km 60 m in the 2nd week and 4 km 700 m in the 3rd week. What length of road did the gang repair in the 3 weeks?

Write a plan in your *Pbs*, do the calculation in your *Ex. Bks*, then write the answer in the box in your *Pbs*. Remember to estimate before doing the calculation and then to check it.

Set a time limit. Review with whole class. Ps could show responses on scrap paper or slates in unison on command.

Ps responding correctly explain at BB to those who were wrong. Who agrees? Who did it another way, etc. Mistakes discussed and corrected.

**Solution:**

**Plan:** 5 km 300 m + 8 km 60 m + 4 km 700 m

**E:** 5 km + 8 km + 5 km = 18 km

**Answer:** The gang repaired 18 km 60 m of road in 3 weeks.

---

### 4 PbY4b, page 166

**Q.2** Read: There were 5 litres 400 ml of syrup in a container. Another 680 ml were poured in. How much syrup is in the container now?

T elicits or tells what syrup is (liquid sugar or a mixture of sugar and water as in tins of fruit or a fruit salad).

Set a time limit. Review with whole class. Ps could show responses on scrap paper or slates in unison on command.

Ps responding correctly explain at BB to those who were wrong. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

**Solution:**

**Plan:** 5 litres 400 ml + 680 ml

**E:** 5 litres + 1 litre = 6 litres

**Answer:** The container now holds 6 litres 80 ml of syrup.

---

### 5 PbY4b, page 166

**Q.3** Read: In a granary, there are 14 650 kg of grain. 8750 kg is wheat, 230 kg is rye and the rest is oats. How many kg of oats are in the granary?

Elicit or tell what a granary is. T could have samples or pictures of the different grains or plants to show to class.

Set a time limit. Review with whole class. A, come and show us what you did. Who agrees? Who did it another way, etc. Mistakes discussed and corrected.

**Solution:**

**Plan:** 14 650 kg – (8750 kg + 230 kg) (or 14650 – 8750 – 230)

**E:** 1500 kg – (9000 kg + 0 kg) = 6000 kg

**Answer:** There are 5670 kg of oats in the granary.
<table>
<thead>
<tr>
<th>Activity</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY4b, page 166</strong></td>
<td></td>
</tr>
<tr>
<td>Q.4 Read: <strong>Draw around the whole rectangle if the shaded area is this fraction of the whole rectangle.</strong></td>
<td></td>
</tr>
<tr>
<td>T explains task, or does part a) with the whole class first if Ps are unsure what to do. Set a time limit.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong> e.g.</td>
<td></td>
</tr>
<tr>
<td>a)</td>
<td>b)</td>
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<tr>
<td><img src="image1" alt="Fraction Grid" /></td>
<td><img src="image2" alt="Fraction Grid" /></td>
</tr>
<tr>
<td>39 min</td>
<td></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY4b, page 166</strong></td>
<td></td>
</tr>
<tr>
<td>Q.5 Read: <strong>Fill in the missing numbers.</strong></td>
<td></td>
</tr>
<tr>
<td>Let's see how many of these you can do in 3 minutes! Start . . . now! . . . Stop!</td>
<td></td>
</tr>
<tr>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. If Ps did not finish the questions, complete with the whole class. Ps show each answer on diagram on BB or on model or real clock.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td><img src="image5" alt="Fraction Grid" /></td>
<td><img src="image6" alt="Fraction Grid" /></td>
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<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>45 min</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

BB: \(\frac{4}{5} = \frac{8}{10}\)

(Accept any solution with the correct area in grid squares.)

Feedback for T
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **1** Factorising | Whole class activity  
| Let’s factorise 167 and then list all its factors. | At a good pace  
| Ps dictate or come to BB to try each of the prime numbers, 2, 3, 5, 7 and 11 as divisors, using ‘quick’ methods where possible. Should we try dividing by 13? (No, as \(13 \times 13 = 169 > 167\)) | Ps explain reasoning or do divisions at side of BB.  
| Elicit that 167 is a prime number, and its factors are 1 and 167. | Class agrees/disagrees  
| **6 min** | Praising |
| **2** Loci | Ps have rulers and if possible, compasses, on desks.  
| a) Draw a dot in your Ex. Bks. (in the middle of the page) | Individual work, but Ps kept together and follow T's instructions.  
| Draw dots which are 2 cm away from it. Try to draw dots in several different places. How could we show all the dots possible? (Join up all the dots to show the complete set.) Elicit that the shape is a circle. | Ps should use well-sharpened pencils!  
| (If Ps have compasses, T demonstrates how to use them with BB compasses to make it easier to draw the circles, otherwise once Ps know the shape is a circle, Ps draw it freehand as best they can.) BB:  
| b) In your Ex. Bks. draw a straight line 3 cm long. | Discussion on shape formed by dots  
| Draw dots which are 2 cm from the line. Try to draw dots in several different places. Draw the whole set when you are sure of the shape that the line will form. | Once Ps have agreed on shape, T confirms on BB.  
| (Ps use compasses if they have them to draw the semi-circles at each end of the line, otherwise Ps draw them freehand as best they can.) BB:  
| c) In your Ex. Bks. mark two dots A and B, in similar positions to these two dots. (T draws them on BB.) | (Extra praise if a P thinks of a point being outside the plane, i.e 3-dimensional. In such a case, the set of points would form the surface of a sphere.)  
| Draw dots which are an equal distance from A and B. Try to draw dots in several different places. Draw the whole set when you are sure of the shape of the line. BB:  
| | If Ps find only the points parallel to the 3 cm line, i.e. above and below it, T gives hint about the sides too. |  
| **16 min** | Agree that the shape is a straight line but then discuss how long it should be.  
| | Extra praise if Ps suggest a never-ending (infinite) line. |
### Activity 3

**PbY4b, page 167**

**Q.1 Read:** Do the calculations in your exercise book. Write the answers here.

- **Set a time limit. Review at BB with whole class.**
- **Ps could show responses on scrap paper or slates on command.**
- **Ps answering correctly explain with place-value detail to Ps who were wrong.**
- **Mistakes discussed and corrected.**

**Solutions:**

**a) 1 m of material costs £6.70. How much do 8 m cost?**

- **Plan:** £6.70 \( \times \) 8 or \( 670 \text{ p} \times 8 \)
- **E:** \( £7 \times 8 = £56 \) or \( 700 \text{ p} \times 8 = 5600 \text{ p} \)
- **Answer:** The cost of 8 m of material is £53.60.

**b) 7 kg of apples cost £13.30. How much does 1 kg cost?**

- **Plan:** £13.30 \( \div \) 7 or \( 1330 \text{ p} \div 7 \)
- **E:** < £14 \( \div \) 7 = £2 or \( E < 1400 \text{ p} \div 7 = 200 \text{ p} \)
- **C:**

\[
\begin{array}{c}
\text{7} \\
\text{3} \text{.} \text{3} \text{.} \text{0} (\text{£}) \\
\text{6}
\end{array}
\]

- **Answer:** The cost of 1 kg is £1.90.

**c) 5 litres of oil cost £16.50. How much do 7 litres cost?**

- **Plan:** £16.50 \( \div \) 5 \( \times \) 7 or \( 1650 \text{ p} \div 5 \times 7 \)
- **E:** > £15 \( \div \) 5 \( \times \) 7 = £21 or \( E > 1500 \text{ p} \div 5 \times 7 = 2100 \text{ p} \)
- **C:**

\[
\begin{array}{c}
\text{5} \\
\text{3} \text{.} \text{3} \text{.} \text{0} (\text{£}) \\
\text{1}
\end{array}
\]

- **Answer:** The cost of 7 litres of oil is £23.10.

---

### Activity 4

**PbY4b, page 167**

**Q.2 Read:** Kate had 360 pennies. On Friday, she spent 7 ninths of them on stamps.

- **a) How much did the stamps cost?**
- **b) What part of her money was left?**

- **Review with whole class. Ps come to BB to show their solution, explaining reasoning and referring to the diagram. Class agrees or disagrees. Mistakes discussed and corrected.**

**Solution:** e.g.

**a) Plan:** 360 \( \div \) 9 \( \times \) 7 = 40 \( \times \) 7 = \( 280 \text{ p} \)

- **Answer:** The stamps cost 280 p.

**b) Plan:** \( 1 - \frac{7}{9} = \frac{9}{9} \times \frac{7}{9} = \frac{2}{9} \)

- **Answer:** Kate had 2 ninths of her money left.

---

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### Activity

#### 5

**PhY4b, page 167**

Q.3 Read: Danny has already run 900 m, which is 3 fifths of the distance he has to run.

a) What distance is he running?

b) i) What part of the distance does he still have to run?

ii) How many metres does he still have to run?

Deal with one part at a time or set at a time limit.

Review whole class. Ps come to BB to show their solution, explaining reasoning and referring to the diagram. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:** e.g.

a) **Plan:** $900 \div 3 \times 5 = 300 \times 5 = 1500$ m

   **Answer:** Danny is running 1500 m.

b) i) **Plan:** $1 - \frac{3}{5} = \frac{5}{5} - \frac{3}{5} = \frac{2}{5}$

   **Answer:** He still has 2 fifths of the distance to run.

ii) **Plan:** $1500 - 900 = 600$ m

   **Answer:** He still has 600 m to run.

**30 min**

#### 6

**PhY4b, page 167**

Q.4 Deal with one at a time or set a time limit.

Review at BB with whole class. Ps could show answers on scrap paper or slates on command.

P answering correctly explains at BB to Ps who were wrong. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

a) Read: How much does Peter have if half of his money is 50 p more than 1 quarter of it?

**Solution:**

$$\frac{1}{2} - \frac{1}{4} = \frac{2}{4} - \frac{1}{4} = \frac{1}{4} \rightarrow 50 \text{ p}, \text{ or } \frac{1}{2} = \frac{2}{4} > \frac{1}{4}$$

So $\frac{4}{4} \rightarrow 50 \times 4 = 200 \text{ p} = \£2$

**Answer:** Peter has £2.

b) Read: 2 fifths of Veronica’s money is 120 p less than 3 fifths of it. How much money does Veronica have?

**Solution:** e.g.

$$\frac{3}{5} - \frac{2}{5} = \frac{1}{5} \rightarrow 120 \text{ p}, \text{ so } \frac{5}{5} \rightarrow 120 \times 5 = 600 \text{ p} = \£6$$

**Answer:** Veronica has £6.

---

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### Activity

#### 6

(Continued)

**c)** Read: Wendy spent half of her money on Monday, half of what was left on Tuesday and she had 40 p left.

*How much money did Wendy have at first?*

**Solution:** e.g.

- **Monday:** part spent: $\frac{1}{2}$, part left: $\frac{1}{2}$
- **Tuesday:** part spent: $\frac{1}{2}$ of $\frac{1}{2} = \frac{1}{4}$
  
  Part spent altogether: $\frac{1}{2} + \frac{1}{4} = \frac{2}{4} + \frac{1}{4} = \frac{3}{4}$

  Part left: $1 - \frac{3}{4} = \frac{1}{4} \rightarrow 40$ p.

  so $\frac{4}{4} \rightarrow 40 \times 4 = 160 = \£1.60$

  *Answer:* Wendy had £1.60 at first.

---

#### 7

**PbY4b, page 167, Q.5**

Read: *Solve these equations in your exercise book.*

Deal with one at a time. Ps decide what to do first and how to continue. Ps work on BB and rest of class in *Ex. Bks*. Calculations done at side of BB if necessary. Class checks that the solution is correct by inserting values in original statements. T helps with d) and shows on number line.

**BB:**

- a) \[3 \times a - 410 = 4690\]
  
  \[3 \times a = 4690 + 410 = 5100\]
  
  \[a = \frac{5100}{3} = 1700\]

- b) \[4 \times b + 40 = 3 \times b + 110\]
  
  \[4 \times b = 3 \times b + 70\]
  
  \[b = 70\]

- c) \[5 \times c + 2000 < 7400\]
  
  \[5 \times c < 5400\]
  
  \[c < 1080\]

- d) \[87 < 6 \times d - 320 < 113\]
  
  \[407 < 6 \times d < 433\]
  
  \[d < 68, 69, 70, 71, 72\]

- e) \[d: 68, 69, 70, 71, 72\]

---

Whole class activity

(or individual work if Ps wish)

Written on BB or SB or OHT Discussion, reasoning, checking, agreement, (self-correction), praising

Accept trial and error too.

**BB:**

- $d: \frac{67}{45} \times 5 \quad \frac{72}{43} \times 1$

  (If $d$ is a natural number)
**Lesson Plan**

### Activity 1: Factorising

In your *Ex. Bk*, factorise 168 and list all its factors.

Review at BB with whole class. Ps come to BB to draw tree diagram, show the number as the product of its prime factors and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

\[
168 = 2 \times 2 \times 2 \times 3 \times 7
\]

Factors: 1, 2, 3, 4, 6, 7, 8, 12, 14, 21, 24, 28, 42, 56, 84, 168

**Notes**

Individual work, monitored, helped

Discussion, reasoning, agreement, self-correction, praising

Ps may use a calculator.

Ps join up the factor pairs.

Feedback for T

---

### Activity 2: Points on a plane

#### a) Draw a dot A in your *Ex. Bks.* (in the middle of the page)

Draw dots on the same plane which are not more than 2 cm away from it. How could we show all the dots possible? (Colour over them.) Let's colour the area covering the complete set of dots green. What shape have you made? (a circular plane shape)

Now let's find dots which are more than 2 cm away from point A. Let's colour this part red. What colour should the points which are exactly 2 cm away from A be? (green, as not more than 2 cm)

T tells class that the points in the plane less than 2 cm from A are called the inside points, the points exactly 2 cm away from A are the border points and the points more than 2 cm away from A are the outside points. Elicit that the red area covers the whole plane to infinity in all directions, except for the green circular plane shape.

#### b) In your *Ex. Bks.* draw a straight line 3 cm long.

Find dots in the same plane which are less than 2 cm from the line. Try to draw dots in several different places. How can we show all the dots possible? (Colour over them.) Let's colour the area covering all these points red. Should the border points be red? (No)

Let's colour the points on the plane which are exactly 2 cm from the line in yellow. (border points)

Colour the dots on the plane which are more than 2 cm from the line segment in green. Elicit that these are the outside points.

#### c) In your *Ex. Bks.* mark three dots A, B and C in similar positions to these dots. (T draws them on BB.)

Find dots on the same plane which are an equal distance from A and B. Colour them red. (Elicit that they form a straight line.)

How long is the line? (it is never-ending or infinite.)

Find dots on the plane which are an equal distance from A and C. Colour the set of dots blue. (Elicit that they form a straight line.)

Find dots on the plane which are an equal distance from A, B and C. How many dots did you find? (one) Where is it? (At the point where the red and blue lines cross over each other)
**Activity 3**  
*PbY4b, page 168*

**Q.1** Read: What rule has been used to group the natural numbers? Write these numbers in the correct set.

10, 72, 38, 13, 54, 96, 61, 87, 75, 49, 172, 359, 648, 975, 831, 570, 903, 184, 657

Set a time limit. Ps decide on a rule to use and fill in the numbers in the sets. Advise Ps to write lightly at first in case they want to change their minds.

Review with whole class. A, tell us the rules you used. Who agrees? Who thought of them in a different way? e.g.

P₁: In the 1st set, I put numbers with units digit 5 or 0; in the 2nd set, I put numbers with units digit 1 or 6; in the 3rd set, I put numbers with units digit 2 or 7; in the 4th set, I put numbers with units digit 3 or 8; in the 5th set, I put numbers with units digit 4 or 9.

P₂: I put numbers exactly divisible by 5 in the 1st set, then in the next sets, numbers with remainders 1, 2, 3 and 4 after dividing by 5.

**Solution:**

| 5, 25, | 1, 6, 21, | 2, 7, 42, | 3, 8, 63, | 4, 9, 99, |
| 100, 1201, 66, | 5317, | 172, 657, | 4218, | 1644 |
| 10, 75, | 96, 61, | 72, 87, | 38, 13, | 54, 49, |
| 831 | 172, 657 | 648, 903 | 359, 184 |

**Bold** numbers are added.

---

**Activity 4**  
*PbY4b, page 168*

**Q.2** Read: Write the numbers 1, 2, 3, 6, 9 and 18 in the suitable circles if the arrows point towards the multiples. Complete the missing arrows.

Let's see if you can do it without help first. (If T sees that majority of Ps are struggling, T gives hint, e.g. Which is the biggest number? Where are most of the arrows pointing? If Ps are still struggling, stop the individual work and continue as a whole class activity.)

Review with whole class. Ps come to BB to write numbers and draw the missing arrows, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

Individual work, monitored, helped.

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

Elicit that each number is also a multiple of itself.

**Broken** arrows are missing.

---

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### Activity 5

**PbY4b, page 168**

Q.3 Read: It takes 45 minutes for 7200 litres of water to flow out of the dam.

How much water would flow out after these times? Fill in the missing numbers.

What is a dam? Why do we build them? Who has seen one? (T has picture to show if possible.)

Set a time limit, Ps can do necessary calculations in Ex. Bks. and write only answers in Pbs.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**
- a) 15 minutes: \(7200 \div 3 = 2400\) litres
- b) 5 minutes: \(2400 \div 3 = 800\) litres
- c) 3 minutes: \(2400 \div 5 = 480\) litres
- d) 1 minute: \(480 \div 3 = 160\) litres
- e) 30 minutes: \(2400 \times 2 = 4800\) litres
- f) 1 hour: \(4800 \times 2 = 9600\) litres

**Notes**

Individual work, monitored, helped

Written on BB or SB or OHT

Initial discussion to clarify the context. Involve several Ps.

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Show calculations in detail on BB if problems.

Ps might show other ways to calculate, e.g.

5 min: \(7200 \div 9 = 800\) litres

Feedback for T

---

### Activity 6

**Pb Y4b, page 168**

Q.4 Deal with one question at a time. Ps read question themselves, work out the answer in Ex. Bks, check it and and write only the numerical solution in Pbs.

Ps write answer on scrap paper or slates and show to T in unison on command. P responding correctly explains at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solutions:**
- a) **Lennie Lion eats about 16 kg of meat every day.**

  About how much meat does Lennie Lion eat in a year?

  **Plan:** \(365 \times 16 = 730 \times 8\) kg

  \(E: 700 \times 8 = 5600\) kg

  **Answer:** Lennie Lion eats about 5840 kg of meat in a year.

- b) **In one year, Ellie Elephant drinks about 150 times.**

  Each time, she drinks about 200 litres of water.

  How much water does Ellie Elephant drink in a year?

  **Plan:** \(150 \times 200 = 300 \times 10\) litres

  \(= 3000 \times 10 = 30000\) litre

  **Answer:** Ellie Elephant drinks about 30000 litres in 1 year.

- c) **Daisy Dragonfly flies around for 2 and a half hours.**

  How far does she fly if she covers 625 m per min?

  **Plan:** \(2 + 30 = 150\) (minutes)

  \(625 \times 15 = 6250 \times 15 = 6200 + 625 \times 5 = 6250 + 3125 = 93750\) (m)

  **Answer:** Daisy Dragonfly flies 93 km 750 m in 2.5 hours.

**Notes**

Individual work, monitored, helped

(or whole class activity if time is short)

Reasoning, agreement, self-correction, praising

C: \(730 \times 8 = 5840\)

\(\frac{5840}{1}\)

or T might show: \(5840\)

T chooses Ps to say answers in sentences.

[Familiarisation with large numbers.]

C: \(6250 \times 5 = 31250\)

\(\frac{31250}{12}\)

or T might show: \(93750\)
### Lesson Plan 169

**Activity 1**

**Factorising**

Let’s factorise 169 and then list all its factors.

Ps try the prime numbers 2, 3, 5, 7 and 11 in turn, using quick methods where they can. Should we try 13? (Yes)

BB: \(13 \times 13 = 130 + 39 = 169,\)

So \(169 = 13 \times 13\) (It is a **square number**). Its factors are 1, 13, 169

---

**Notes**

Whole class activity

Extra praise if Ps remember 169 = 13 × 3 from the trials of previous numbers.

Reasoning, agreement, praising

BB: \[\begin{array}{c} 13 \hspace{1cm} 169 \end{array}\]

---

**Activity 2**

**Missing numbers**

Where should these numbers go in the diagram if the arrows are pointing towards the greater number?

BB: –5, 40.93, 0, \(-\dfrac{2}{7}\), 562, –72.3

Who knows where one of the numbers should go? Why? Ps come to BB to write a number and explain their thinking. Class agrees/disagrees.

BB:

![Diagram with missing numbers](image)

Reasoning: e.g.

- The circle with no arrows pointing towards it must contain the smallest number, which is –72.3.
- The circle with 5 arrows pointing towards it and none away from it must be the biggest number, which is 562.
- The circle with 4 arrows pointing towards it and only one away from it must be the 2nd greatest number, which is 40.93, etc.

---

**Notes**

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

At a good pace

---

**Activity 3**

**Shortest distance**

Where should we measure how far apart one element is from the other?

Ps come to BB to say what the two elements are, then to show where they would measure their distance apart. Class agrees/disagrees.

Ps draw the measuring line using BB ruler (with T’s help). Let’s label the distance between them \(d\).

BB:

- a)
- b)
- c)
- d)
- e)
- f)
- g)
- h)
- i)

---

**Notes**

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Ps could have rulers and copies of copy master too.

Discussion, reasoning, agreement, praising

Elicit that the shortest distance between:

- 2 points is a straight line;
- 2 lines is the perpendicular distance between them;
- 2 shapes is the distance between the 2 closest points.

Point out that lines, e.g. \(e\) and \(f\) in h) can be extended to infinity and if not parallel will cross one other eventually, but that line segments, e.g. AB, have a finite length.
Lesson Plan 169

Notes

Individual work, monitored, helped

Discussion, reasoning, agreement, self-correction, praising

T could have a globe to show the position of the Equator and to clarify the context.

C: \[465 \times 6 = \frac{27 900}{3}\]

(= 27 km 900 m per minute
= 1674 km per hour
≈ 40 000 km per day)

Ps might tell class of own experiences of thunderstorms and what happened to their gardens.

BB: \[\begin{array}{c}
30 \\
50
\end{array}\]

Revise notation for metre square (m²) if necessary. T could show similar notation for ‘cm cube’ in c). (cm³)

Whole class activity

Ps suggest what to do first and how to continue. Class agrees/disagrees.

T helps with the calculation if necessary.

C: \[\frac{3}{7}\]

or T shows: 34 740

---

Activity 4  PbY4b, page 169

Q.1 Read: Solve the problems in your exercise book.

Deal with one part at a time. Ps read problem themselves and write a plan and do the calculation in Ex. Bks. Set a time limit. Review with whole class. Ps who have found an answer could show results on scrap paper or slates on command. P answering correctly explains at BB to Ps who were wrong. Mistakes discussed and corrected.

Solutions:

a) A point on the Equator turns on the Earth’s axis at a speed of 465 metres per second.

How many metres does it turn every minute?

Plan: 1 minute = 60 seconds

1 second → 465 m

60 seconds → 465 m × 60 = 4650 m × 6

= 27 900 m

Answer: It turns 27 900 m every minute.

b) During a thunderstorm, 30 mm of rain fell. It means that 30 litres of rain fell on an area of 1 square metre.

After the same thunderstorm, how many litres of rain fell on a rectangular garden which is 30 m wide and 50 m long?

Plan: Area of garden: 30 m × 50 m = 300 m × 5 m

= 1500 m²

1 m² → 30 litres

1500 m² → 1500 × 30 litres = 15 000 × 3 litres

= 45 000 litres

Answer: 45 000 litres of rain fell on the garden.

c) One centimetre cube of gold has mass 19.3 g. What would be the mass of a cuboid made of gold if it is 20 cm long, 10 cm wide and 9 cm high?

Plan: Volume of cuboid = 20 cm × 10 cm × 9 cm

= 1800 cm³ (or cm³)

1 cm³ → 19.3 g

1800 cm³ → 19.3 g × 1800 = 193 g × 180

= 1930 g × 18 = 3860 g × 9

= 34 740 g = 34 kg 740 g

Answer: The mass of the cuboid would be 34 kg 740 g.
Y4

Activity

5  PbY4b, page 169
Q.2 Read: Practise calculation.
Let's see how many of these you can do in 4 minutes!
Remember to check your calculations.  Start . . . now! . . . Stop!
Review with whole class.  Ps come to BB or dictate to T,
explaining reasoning (with place-value details if problems).
Class agrees/disagrees.  Who made a mistake?  What was your
mistake?  Deal with all types of errors made.
Stand up if you had all 8 correct.  Let's give them a clap!
Solution:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>x</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>+</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>

31 min

6  PbY4b, page 169
Q.3 Read: List the natural numbers about which this statement is true.
It is a multiple of 8, the sum of its digits is 7 and the
product of its digits is 6.
Allow Ps a couple of minutes to try it out in Ex. Bks.
X, what number have you found?  Let's check it.  e.g.
BB: 16: 1 + 6 = 7, 1 × 6 = 6  
Who has found another such number?  (Class checks it if Ps have
found one but it is unlikely that they will find 1312 and 3112
without help from T.)
T gives hints or tells class about 1312 and asks Ps to check it.
BB: 1 6 4
8 1 3 1 2
   5 3
1 + 3 + 1 + 2 = 7, 1 × 3 × 1 × 2 = 6  
How could you rearrange these digits to form another multiple
of 8?  (3112)

35 min

Lesson Plan 169

Notes
Individual work, monitored, helped
Written on BB or use enlarged
copy master or OHP
Differentiation by time limit.
Reasoning, agreement, self-
correction, praising
In unison

Individual trial first, monitored
Reasoning, checking,
agreement, praising
Extra praise if they have
found another number.
Or ask Ps to think of factors
of 6, then form numbers and
check to see if they are
divisible by 8.
Ps may use a calculator.

Check: 3 8 9
8 3 1 1 2
7 7


### Activity

#### 7

**PbY4b, page 169, Q.4**

Read: *Three travellers met on a road. One of them had 3 loaves of bread, another had 5 loaves of bread and the third had no food at all. They shared the bread equally.*

*The third person then offered 8 coins to the others to pay for his food.*

*How can the other two travellers share the money fairly?*

Allow Ps time to think about it and discuss with their neighbours.

Ps come to BB to explain their reasoning. Class points out errors or suggests improvements to the method of solution. T helps where necessary.

**Solution:** e.g. Let the travellers be A, B and C

Each person ate: \(8 \div 3 = 6 \div 3 + 2 \div 3 = 2 + \frac{2}{3} = \frac{2}{3}\) (loaves)

**BB:**

\[
\begin{array}{c|c}
\text{Bread} & \text{Coins} \\
\hline
\text{A} & \text{A} \\
\text{B} & \text{B} \\
\text{C} & \text{C} \\
\end{array}
\]

A gave \(2\frac{1}{3}\) to C, B gave \(\frac{1}{3}\) to C, so \(A : B = \frac{7}{3} : \frac{1}{3} = 7 : 1\)

So the coins should also be in the ratio 7 : 1

**Answer:** Traveller C should give 7 coins to Traveller A and 1 coin to Traveller B.

___40 min___

#### 8

**PbY4b, page 169, Q.5**

Read: *27 players took part in a knockout singles tennis competition. The winner from each pair went through to the next round and the person without an opponent qualified automatically.*

*How many matches were played before the winner was decided?*

T gives Ps a couple of minutes to think about it and discuss with their neighbours if they wish. Who has an idea what to do? Who agrees? Who would do it another way? etc.

Ps suggest what to do first and how to continue. T gives hints or helps only if Ps are stuck.

**Solution:** e.g.

1st round: \(13\) matches \(\rightarrow\) \(13\) winners + 1 = \(14\) players

2nd round: \(7\) matches \(\rightarrow\) \(7\) winners

3rd round: \(3\) matches \(\rightarrow\) \(3\) winners + 1 = \(4\) players

4th round: \(2\) matches \(\rightarrow\) \(2\) winners

5th round: \(1\) match \(\rightarrow\) \(1\) winner

**Total:** \(26\) matches

Or the simplest solution:

If there were 27 players and only 1 winner, there were 26 losers, so 26 matches must have been played.

**Answer:** After 26 matches the winner was decided.

___45 min___
### Activity

Calculation and tables practice. Revision, activities, consolidation

*PbY4b, page 170*

### Solutions:

**Q.1**

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>387</td>
<td>80432</td>
<td>1352</td>
<td>91032</td>
</tr>
<tr>
<td>5342</td>
<td>5679</td>
<td>5</td>
<td>81288</td>
</tr>
<tr>
<td>7793</td>
<td>74753</td>
<td>76410</td>
<td>14012</td>
</tr>
<tr>
<td>13722</td>
<td></td>
<td></td>
<td>25216</td>
</tr>
</tbody>
</table>

**Q.2** Several solutions are possible. e.g.

**Q.3**

- a) i) 360 min = 6 hours  
  ii) 25 min = \( \frac{5}{12} \) hour
- b) i) 36 hours = 1 \( \frac{1}{2} \) days  
  ii) 2 days = \( \frac{2}{7} \) week
- c) i) 700 g = \( \frac{7}{10} \) kg  
  ii) \( \frac{1}{5} \) kg = 200 g
- d) i) 40 cm = \( \frac{2}{5} \) m  
  ii) \( \frac{3}{20} \) m = 15 cm
- e) i) 250 m = \( \frac{1}{4} \) km  
  ii) \( 2\frac{1}{2} \) km = 2500 m
- f) i) 200 cl = 2 litres  
  ii) 200 ml = \( \frac{1}{5} \) litre

### Notes

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### Activity

#### 1. Factorising

In your *Ex. Bk*, factorise 170 and 171 and then list all their factors. Review at BB with whole class. Ps come to BB to draw tree diagrams, show the numbers as the product of their prime factors and list all their factors. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

- **170:** \[ 2 \times 5 \times 17 \]
- **171:** \[ 3 \times 3 \times 19 \]

Factors:

- **170:** 1, 2, 5, 10, 17, 34, 85, 170
- **171:** 1, 3, 9, 19, 57, 171

**6 min**

#### 2. Missing numbers

Where should these numbers go in the diagram if the arrows are pointing towards the smaller number? Review at BB with whole class. Ps dictate to T. Class agrees/disagrees. Omissions added and mistakes corrected.

**BB:**

- \[ 3 \frac{2}{3}, -2, 0, -\frac{1}{3}, 0.7, \frac{4}{3} \]

Who knows where one of the numbers should go? Why? Ps come to BB to write a number and explain their thinking. Class agrees/disagrees.

**BB:**

- \(-2\)
- \(0\)
- \(0.7\)
- \(\frac{4}{3}\)

**11 min**

#### 3. *PbY4b, page 171*

**Q1** Read: *How many routes lead from A to K, L, M, N and O if you can only move down to the left or to the right?*

Let's see how many ways you can find in 4 minutes!

Review a BB with whole class. Ps dictate to T. Class agrees/disagrees. Omissions added and mistakes corrected.

**Solution:**

- **A to K:** 1 route (ABDGK)
- **A to L:** 4 routes (ABDGL, ABDHL, ABEHL, ACEHL)
- **A to M:** 6 routes (ABDHM, ABEHM, ABEIM, ACEHM, ACEIM, ACFIM)
- **A to N:** 4 routes (ABEIN, ACEIN, ACFIN, ACFIN)
- **A to O:** 1 route (ACFJO)

**Whole class activity**

- Drawn on BB or use enlarged copy master or OHP

Ps might suggest putting the numbers in order first. Discussion, reasoning, agreement, praising

At a good pace

**T helps with wording of reasoning if necessary.**

---

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Y4

**Activity**

(Continued)

Let's write how many routes lead to each letter. Ps dictate to T and T writes on BB in same pattern as the letters.

**BB:** A to A: 1

B: 1 route

C: 1 route

D: 1 route

E: 2 routes (1 + 1)

F: 1 route

G: 1 route, etc.

What do you notice about the numbers in the pattern? (Each number is the sum of the two numbers directly above it in the previous row.)

T tells class that this formation of numbers is called **Pascal's Triangle**, named after the mathematician who first discovered it.

What would the next row in the triangle be? (1, 5, 10, 10, 5, 1)

---

**Notes**

Whole class activity

At a good pace

Agreement, praising

Extra praise if Ps notice the pattern without help.

Ps dictate in unison. Praising

---

**PbY4b, page 171**

Q.2 Read: *Colour the shapes on the grid and fill in the missing numbers if the sum of the numbers in each shape is 10 000.*

T explains task. Ps first find the relevant numbers, write them in the shape, then colour the shape and its position in the grid in the same colour. Set a time limit.

Review with whole class. Ps come to BB to show solutions, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

---

**PbY4b, page 171**

Q.3 Read: *Write the missing numbers in the puzzles if the sum of the 3 numbers along each side is 15 000.*

Choose from:

a) 4200, 4000, 5200, 5400, 5600, 5800

b) 5400, 5600, 5800, 4800, 5000, 5200, 4000, 4600

Deal with one part at a time. Set a time limit. Encourage logical thinking rather than trial and error.

Review with whole class. Ps come to BB or dictate to T. Class checks that they are correct. Ps explain how they solved it.

**Solution:**

---
**Activity 6**

**PbY4b, page 171**

Q.4 Read: *Fill in the missing numbers.*

Set a time limit. Review with whole class.

Ps come to BB to fill in numbers and explain their reasoning in detail. Who did the same? Who did it a different way? etc.

- e.g. P₁: \(900 \times 4 = 3600\), so \(3600 \times 2 = 7200\),

  then \(7200 \div 900 = 72 \div 9 = 8\)

- P₂: \((900 \times 4) \times 2 = 900 \times \frac{8}{3}\) etc.

T shows the 2nd strategy if no P used it. Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
\text{a) } & \quad 900 \times 4 = 3600, \quad 3600 \times 2 = 7200, \quad 7200 \div 900 = 72 \div 9 = 8 \\
\text{b) } & \quad 8000 \div 4 = 2000, \quad 2000 \times 2 = 4000, \quad 4000 \div 8 = 500 \\
\end{align*}
\]

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit.

Discussion, reasoning, agreement, self-correction, praising

Extra praise for Ps who noticed 'quick' ways.

(Or could be done as a competition between 2 teams of 4 Ps.

T times how long it takes each team to reason aloud to class and fill in missing numbers.

Class points out errors and Ps have to start their reasoning all over again if incorrect.

Team finished correctly in the quickest time is the winner.

Let's give them three cheers!)

---

**Activity 7**

**Combinatorics**

*In how many ways can you put 2 circles and 3 triangles in order?*

Ps draw the different orders in Ex. Bks and/or have shapes or shape cards on desks to manipulate. Set a time limit.

Show me the most number of ways that you found . . . now! (10)

Ps with the correct answer explain how they worked it out. T gives extra praise for a systematic method of solution. e.g.

BB:

- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)
- \(\bigcirc \bigcirc \bigcirc \bigcirc \bigcirc \)

10 different orders

**Notes**

Individual or paired work, monitored

(or whole class activity if time is short and Ps come to BB for dictate to T)

Responses shown on scrap paper or slates in unison.

Discussion, reasoning, agreement, praising

T might show the tree diagram if no P has thought of it.
### Activity

**1. Factorising**

In your *Ex. Bks.*, factorise 172 and then list all its factors.

Review at BB with whole class. Ps come to BB to draw a tree diagram showing the number as the product of its prime factors, and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

\[
\begin{array}{c}
172 \\
\downarrow \\
2 \quad 86 \\
\downarrow \\
2 \quad 43 \\
\end{array}
\]

Factors: 1, 2, 4, 43, 86, 172

**Notes**

Individual work, monitored

Discussion, reasoning, agreement, self-correction, praising

Ps may use a calculator.

**2. Combinatorics**

In how many ways can we get to B from A?

**BB:**

Let’s choose some interim steps first.

In how many ways can we get from A to here? (T points to a grid point.)

Ps come to BB to show the different routes.

Class agrees/disagrees. Ps write number of ways in the appropriate circle.

Repeat for one or two other grid points until Ps realise that the number of ways for any grid point is the sum of the 2 numbers directly above it in the previous row. e.g. \(15 = 10 + 5\)

Agree that the number of different ways from A to B is 70.

**Notes**

Whole class activity

Drawn on BB or use enlarged copy master or OHP

At a good pace

Involve several Ps

Reasoning, agreement, praising

Extra praise if Ps realise from the beginning that they should add the 2 numbers in the row above.

Elicit that the shape forms part of Pascal’s Triangle.

**3. PbY4b, page 172, Q.1**

Read:

a) List the natural numbers up to 100 which have an odd number of factors.

b) What are these numbers called?

Let’s do it logically. What is the first natural number? (1) How many factors does it have? (1, which is an odd number)

Ps factorise the following natural numbers in *Ex. Bks.*, then come to BB to show the next appropriate numbers and list their factors as a check.

Class agrees/disagrees. (It will be rather slow at first until Ps realise that the numbers they are looking for are square numbers.)

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>4</th>
<th>9</th>
<th>16</th>
<th>25</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1, 2)</td>
<td>(1, 3, 9)</td>
<td>(1, 2, 4, 8, 16)</td>
<td>(1, 5, 25)</td>
<td>(1, 2, 3, 4, 6, 9, 12, 18, 36)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>49</th>
<th>64</th>
<th>81</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1, 7, 49)</td>
<td>(1, 2, 4, 8, 16, 32, 64)</td>
<td>(1, 3, 9, 27, 81)</td>
<td>(1, 2, 4, 5, 10, 20, 25, 50, 100)</td>
</tr>
</tbody>
</table>

**Notes**

Whole class activity

(or individual work if Ps wish)

At a good pace

Reasoning, agreement, praising

Extra praise for the first P to realise that the numbers are square numbers.

What is a square number?

(A number which is formed by multiplying another number by itself.) Elicit that it can form a square.

**BB:** e.g.

\[
\begin{array}{c}
A = 25 \\
\downarrow \\
5 \\
\end{array}
\]
Y4

Activity

4 PbY4b, page 172

Q.2 Read: a) How many zeros are at the end of the number which is the result of: $10 \times 11 \times 12 \times 13 \times 14 \times 15$?

Set a time limit. Ps may discuss with their neighbours if they wish. Trials can be done on slates or scrap paper or in Ex. Bks.

Show me how many zeros there are at the end . . . now! (2)

Ps answering correctly explain how they worked out the answer. Who did the same? Who did it a different way? etc. If no P had the right answer, class solves it under T's direction.

Solution: e.g.

• Multiply out all the numbers. (Difficult without a calculator!)

• Factorise the numbers, and count how many pairs of $(2 \times 5)$ there are, as $2 \times 5 = 10$

$10 \times 11 \times 12 \times 13 \times 14 \times 15$

$= 2 \times 5 \times 11 \times 2 \times 2 \times 3 \times 13 \times 2 \times 7 \times 3 \times 5$

$= (10 \times 10) \times (11 \times 2 \times 3 \times 13 \times 2 \times 7 \times 3)$

$= 100 \times (a \text{ number which is not a whole ten})$

So there must be 2 zeros at the end of the number.

• $10 \times 11 \times 12 \times 13 \times 14 \times 15$

$= 10 \times 11 \times 12 \times 13 \times 7 \times 2 \times 3 \times 5$

$= 100 \times (11 \times 12 \times 13 \times 21)$

which has units digit 6, so is not a whole 10.

So the number must be a whole 100, i.e. 2 zeros at the end.

Read: b) Check your answer on a calculator.

Write the product in words.

Ps work out the product and dictate digits to T who writes on BB. T points to each digit in turn and Ps say its place-value.

BB: 3 603 600

$= (3\text{M} + 6\text{HTh} + 0\text{TTh} + 3\text{Th} + 6\text{H} + 0\text{T} + 0\text{U})$

If Ps say the 1st '3' as 3ThTh, T asks who knows what 1000 thousands is called? (1 million) T writes word on BB.

T chooses Ps to read the number aloud. Let's see if you can write the number in words in your Pbs. T writes it on BB too so that Ps can correct their spelling mistakes.

BB: 3 million, six hundred and three thousand, six hundred

T points out that it is usual to leave a gap (or write a comma) between every 3 digits to make the number easier to read.

Notes

Individual trial first, monitored, helped

Do not allow Ps to use calculators until part b)!

In unison

Discussion, reasoning, agreement, self-correcting, praising

Whole class activity or individual work if Ps have a calculator each.

Agreement, praising

In unison

BB: 1000 Th = 1 million

Individual work in Pbs, monitored, helped, corrected
Activity 5

PbY4b, page 172

Q.3 Read: The product of the ages of my children is 1664. The youngest is half the age of the oldest. I am 50 years old. How many children do I have and what are their ages?

Allow Ps time to think and discuss with their neighbours. Ps can do calculations and checks in Ex. Bks. If Ps are struggling, T could give a hint about factorising 1664.

Who thinks that they have an answer? Ps tell their ideas and findings to class. If no P knows what to do, class solves it together with T’s help.

Solution:

First factorise 1664. Then try to form numbers with the prime factors which fulfil the conditions.

BB: \[ 1664 = 2^8 \times 13 \]

The only possible solution is 8, 13 and 16.

Answer: I have 3 children aged 8 years, 13 years and 16 years.

25 min

Activity 6

PbY4b, page 172, Q.4

Read: Two positive whole numbers have these factors in common: 1, 2, 3 and 6. If we combine their factors, we get this set: \{1, 2, 3, 4, 6, 9, 12, 18\}

Write the factors in the correct set if:

- \( A = \) factors of the 1st number
- \( B = \) factors of the 2nd number

Which factors should we write in first? (the common factors) P comes to BB to write 1, 2, 3, and 6 in intersection of the two sets. Which set should the other factors be in? Ask several Ps what they think and why. Class agrees/disagrees. Ps fill in diagram in Pbs too.

What are the two numbers? (12 and 18) Let’s check their factors.

Factors of 12: \{1, 2, 3, 4, 6, 12\} ✔ Factors of 18: \{1, 2, 3, 6, 9, 18\} ✔

30 min
### Activity 7

**PbY4b, page 172**

**Q.5** Read: List all the positive integers up to 100 which are exactly divisible by 2, 3, 4 and 5.

Think carefully before you start! Set a time limit.

Review with whole class. Elicit that only one number is possible: 60. Ps explain how they worked out the answer.

**Reasoning:**

It must be an even number as it is divisible by 2, and it must be a whole 10 if it also divisible by 5. The only whole ten divisible by both 3 and 4 is 60.

---

### Activity 8

**PbY4b, page 172**

**Q.6** Read: I am thinking of a positive number.

Its half is 15 more than its third and its quarter is 15 more than its sixth. What is the number?

Ps use the diagrams to help them and do any necessary calculations in Ex. Bks. Set a time limit. Remind Ps to check their solutions in the context of the question.

Review with whole class. Who found an answer? Let’s check it. Elicit that such a number is impossible!

**Reasoning:**

Using 1st clue: \( \left( \frac{1}{2} - \frac{1}{3} \right) \) of \( n \) = \( \left( \frac{3}{6} - \frac{2}{6} \right) \) of \( n \) = \( \frac{1}{6} \) of \( n \) = 15

So \( n = 15 \times 6 = 90 \)

But if we check it with the other clue:

\( \left( \frac{1}{4} - \frac{1}{6} \right) \) of \( n \) = \( \left( \frac{3}{12} - \frac{2}{12} \right) \) of \( n \) = \( \frac{1}{12} \) of \( n \) \( \neq \) 15

So the number is impossible or the clues are wrong!

---

### Activity 9

**PbY4b, page 172, Q.7**

Read: In how many different orders can you put these shapes?

T starts, then Ps come to BB or dictate what T should write once they can see the pattern. Class points out errors or duplications.

Agree that 30 different orders are possible.

**Solution:**

\[ \begin{array}{cccc}
\bigcirc & \bigcirc & \bigtriangleup & \square \\
\bigcirc & \bigcirc & \bigtriangleup & \square \\
\bigcirc & \bigcirc & \bigtriangleup & \square \\
\bigcirc & \bigcirc & \bigtriangleup & \square \\
\bigcirc & \bigcirc & \bigtriangleup & \square \\
\end{array} \]

---

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### Lesson Plan

**Activity**

<table>
<thead>
<tr>
<th><strong>1</strong></th>
<th><strong>Factorising</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Let’s factorise 173 and then list all its factors.</td>
<td></td>
</tr>
<tr>
<td>Ps dictate or come to BB to try each of the prime numbers, 2, 3, 5, 7, 11 and 13 as divisors, using ‘quick’ methods where possible. Should we try dividing by the next prime number, 17?</td>
<td></td>
</tr>
<tr>
<td>(No, as $17 \times 17 = 289 &gt; 173$)</td>
<td></td>
</tr>
<tr>
<td>Elicit that 173 is a prime number, and its factors are 1 and 173.</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

- Whole class activity
- At a good pace
- Ps explain reasoning or do divisions at side of BB or use a calculator.
- Class agrees/disagrees
- Praising

<table>
<thead>
<tr>
<th><strong>2</strong></th>
<th><strong>Game of 20</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Let's play a game.</td>
<td></td>
</tr>
<tr>
<td>Two players, A and B, start from zero and take turns to count in steps of 1 or 2 up to 20. e.g. A says 2, B says 5, A says 6, B says 8, and so on. The first player to reach ‘20’ is the winner.</td>
<td></td>
</tr>
<tr>
<td>Play the game in pairs and keep a record of the steps on scrap paper or in your Ex. Bks.</td>
<td></td>
</tr>
<tr>
<td>e.g. A: 2 6 . . . (T shows on BB.)</td>
<td></td>
</tr>
<tr>
<td>B: 5 8 . . .</td>
<td></td>
</tr>
<tr>
<td>Ps take turns to be Player A, i.e. start the game. Ps play the game several times and note the winner each time.</td>
<td></td>
</tr>
<tr>
<td>What did you find? (Hopefully, Player B won more often.)</td>
<td></td>
</tr>
<tr>
<td>b) T plays the game in front of whole class with one or two Ps. Who thinks that they can beat me?</td>
<td></td>
</tr>
<tr>
<td>If T is Player A, T exploits B’s possible weaknesses, but if T is Player B, T always wins the game.</td>
<td></td>
</tr>
<tr>
<td>Who has noticed a strategy for playing the game so that you always win? Ask several Ps what they think.</td>
<td></td>
</tr>
<tr>
<td>Strategy:</td>
<td></td>
</tr>
<tr>
<td>e.g. To get to 20, I have to say 16, as then the other player cannot reach 20. In order to say 16, I have to say 12 the turn before. etc.</td>
<td></td>
</tr>
<tr>
<td>The best strategy is to be Player B and to say 4, 8, 12, 16, 20.</td>
<td></td>
</tr>
<tr>
<td>c) If we changed the rules and the person who reaches 20 is the loser, what would the winning strategy be?</td>
<td></td>
</tr>
<tr>
<td>(To be Player A and say: 3, 7, 11, 15, 19, so B has to reach 20.)</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

- Paired work, monitored
- T walks round listening to the games.
- Whole class activity
- In good humour!
- Discussion, reasoning, agreement, praising

<table>
<thead>
<tr>
<th><strong>3</strong></th>
<th><strong>Problem</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>T has BB already prepared.</td>
<td></td>
</tr>
<tr>
<td>BB: D I V I S</td>
<td></td>
</tr>
<tr>
<td>In how many ways could we read the word divisor on this diagram?</td>
<td></td>
</tr>
<tr>
<td>Ps come to BB to point out different routes and write them on the BB.</td>
<td></td>
</tr>
<tr>
<td>Let's think of each letter as a point on a grid and write the number of possible routes to each point. Ps come to BB or dictate to T.</td>
<td></td>
</tr>
<tr>
<td>BB: 1 1 1 1 1</td>
<td></td>
</tr>
<tr>
<td>1 2 3 4 5</td>
<td></td>
</tr>
<tr>
<td>1 3 6 10 15</td>
<td></td>
</tr>
<tr>
<td>Agree that the number of different ways is 15.</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

- Whole class activity
- Written on BB or SB or OHT
- BB: e.g. D I V I S O R S O R |
- At a good pace
- Agreement, praising
**Lesson Plan 173**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y4</strong></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td><em>PbY4b, page 173</em></td>
<td>Individual work, monitored, helped</td>
</tr>
</tbody>
</table>
| Q.1 Read: *The perimeter of a triangle is 10 cm and the length of each side is a whole cm.* Are these statements true or false? Write a tick if true and a cross if false. Ps can draw diagrams or do calculations in Ex. Bks. Set a time limit. Review with whole class. T (P) reads each question and Ps show responses by writing T or F on scrap paper or slates or by pre-agreed actions, and show on command. T asks Ps with different viewpoints to explain their reasoning and class decides on correct answer. Mistakes corrected. **Solutions:**

a) *The triangle has only one side which is 1 cm long.* (F) If \(a = 1\), then possible values for \(b\) and \(c\) are:

<table>
<thead>
<tr>
<th>(b)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>(7)</td>
<td>(6)</td>
<td>(5)</td>
</tr>
</tbody>
</table>

but \(1 + 2 < 7\), \(1 + 3 < 6\), \(1 + 4 = 5\), which are impossible. (In a triangle the sum of the 2 shorter sides must be more than the longest side.)

b) *The triangle could have only one side which is 2 cm long.* (T) If \(a = 2\), then possible values of \(b\) and \(c\) are:

<table>
<thead>
<tr>
<th>(b)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(c)</td>
<td>(5)</td>
<td>(4)</td>
</tr>
</tbody>
</table>

The first column is impossible as \(2 + 3 = 5\), but the 2nd column is possible.

The sides are 2 cm, 4 cm and 4 cm.

c) *The triangle has only one side which is 3 cm long.* (F) If \(a = 3\), the two other sides must be 3 and 4 but then there would be two sides which are 3 cm long!

d) *The triangle has only one side which is 5 cm long.* (F) The sum of the other two sides must be 5, which is not more than 5, so the triangle is impossible.

**30 min**

<table>
<thead>
<tr>
<th>5</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><em>PbY4b, page 173</em></td>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td>Q.2 Read: We want to rearrange some books on two bookshelves. At the moment, there are 156 books on the bottom shelf and on the top shelf there are 30 books more than there are on the bottom shelf. Rearrange the books so that there are: a) the same number of books on both shelves, b) one shelf has twice as many books as the other. Ps draw diagrams or do calculations in Ex. Bks. then write only the numbers of books on each shelf in Pbs. Set a time limit. Review with whole class. Ps come to BB to show their solutions and explain reasoning. Class agrees/disagrees. Mistakes corrected. <strong>Solution:</strong> a) 171 and 171 b) 228 and 114 (or vice versa)</td>
<td></td>
</tr>
</tbody>
</table>

| \(186 - 156\) | \(15\) |
| \(186\) | \(156\) |
| \(156\) | \(171\) |
| \(228\) | \(114\) |
| \(186\) | \(156\) |
| \(172\) | \(114\) |
| \(186\) | \(156\) |
| \(172\) | \(114\) |
| \(186\) | \(156\) |
| \(172\) | \(114\) |
| **35 min** |       |
Lesson Plan 173

Activity

6 PbY4b, page 173

Q.3 Ps read questions themselves, write a plan, do the calculations and write the answer as a sentence in Pbs. Set a time limit.

Review at BB with whole class. Ps could show numerical solutions on scrap paper or slates on command. Ps responding correctly explain to those who were wrong. Mistakes discussed and corrected.

Solutions:
The children are making up gift boxes for a large party.

a) If they put 4 sweets in each box, they can make 139 boxes and 2 sweets will be left over.

How many sweets did they have?

Plan: \(139 \times 4 + 2\)

Answer: They had 558 sweets.

b) How many gift boxes would they make if they put 9 sweets in each box?

Plan: \(558 \div 9\)

Answer: They would make 62 gift boxes, with no sweets over.

Notes

Individual work, monitored, (helped)

Differentiation by time limit.

Reasoning, agreement, self-correction, praising

Feedback for T

Whole class activity

BB:

\[
\begin{array}{c}
A \\
\hline
1.5 \\
6 \\
\end{array}
\]

\[
\begin{array}{c}
D \\
\hline
6 \\
G \\
\end{array}
\]

\[
A = 6 \times 12 = 72 \text{ m}^2
\]

Discussion, reasoning, agreement, praising

T helps where necessary but allow Ps to do most of the work.

How could we show the areas in the diagram?

Ps come to BB to show each child’s area of the garden on the diagram. (Need only be rough!)

45 min
**Lesson Plan 174**

**Notes**

Individual work, monitored
Discussion, reasoning, agreement, self-correction, praising
Ps may use a calculator.
Ps join up the factor pairs.
Feedback for T

**Activity**

1. **Factorising**

   In your Ex. Bks. factorise 174 and then list all its factors.

   Review at BB with whole class. Ps come to BB to draw a tree diagram, show the number as the product of its prime factors, and list all its factors. Class agrees/disagrees. Mistakes discussed and corrected.

   BB:
   \[
   174 = 2 \times 3 \times 29
   \]

   Factors: 1, 2, 3, 6, 29, 58, 87, 174

   Paired work, monitored
   T walks round listening to the games.

   Ask several pairs of Ps about their matches, e.g. how many games they played, who won most often and what position they went in when they won.

   Whole class activity
   In good humour!

   Individual work, monitored
   T calls 2 Ps to front of class to try out the strategy.

   Individual work, monitored, helped
   Discussion, reasoning, agreement, self-correction, praising
   Ps may use a calculator.
   Ps join up the factor pairs.

2. **Game of 21**

   a) Let's play a game which is similar to the one we played yesterday.

      Two players, A and B, start from zero and take turns to count in steps of 1, 2, 3 or 4 up to 21. e.g. A says 1, B says 5, A says 8, B says 11, and so on. The first player to reach '21' is the winner.

      Play the game in pairs and keep a record of the steps on scrap paper or in your Ex. Bks.

      Ps take turns to be Player A, i.e. start the game. Ps play the game several times and note the winner each time.

      What did you find? (Hopefully, Player A won more often.)

   b) T plays the game in front of the whole class with two Ps (one P as Player A and the other as Player B).

      If T is Player B, T exploits Ps possible weaknesses, but if T is Player B, T wins the game.

      Who has noticed a strategy for playing the game so that you always win? Ask several Ps what they think.

      Strategy:
      Be Player A and say 1, 6, 11, 16, 21.

   Paired work, monitored
   T walks round listening to the games.

   Ask several pairs of Ps about their matches, e.g. how many games they played, who won most often and what position they went in when they won.

   Whole class activity
   In good humour!

   Discussion, reasoning, agreement, praising
   T calls 2 Ps to front of class to try out the strategy.

   Individual work, monitored, helped
   Discussion, reasoning, agreement, self-correction, praising
   Deal with all methods used.

3. **PbY4b, page 174**

   Q.1 Read: Sue spent half of her money. Then she spent another £20 and had £80 left.

   How much money did Sue have at first?

   Ps solve problem in Ex. Bks. then write the answer as a sentence in Pbs. Set a time limit.

   Review at BB with whole class. Ps could show result on scrap paper or slates on command. P answering correctly explains at BB to those who were wrong. Who agrees? Who did it in a different way? etc. Mistakes discussed and corrected.

   Solution: e.g. Do the reverse operations in the opposite order:

   Plan: \( (\£80 + \£20) \times 2 = \£100 \times 2 = \£200 \)

   or \( \frac{1}{2} \) of \( S - \£20 = \£80 \), so \( \frac{1}{2} \) of \( S = \£80 + \£20 = \£100 \)

   So \( S = \£100 \times 2 = \£200 \) (where \( S \) = Sue's money)

   Answer: Sue had £200 at first.

   Individual work, monitored, helped
   Discussion, reasoning, agreement, checking, self-correction, praising
   Deal with all methods used.

   e.g. BB:
   \[
   \begin{array}{ccccccccc}
   \hline
   & & & & & & & & & \\
   \hline
   \hline
   \end{array}
   \]

   Spent: \( \frac{1}{2} \) \£20

   Check: \( 200 - 100 - 20 = 80 \)

   17 min

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**Y4**

### Activity

**PbY4b, page 174**

#### Q.2 Read: Which positive integer can be written instead of the letter \(x\) so that the inequality is true?

BB: \(48 + x < 52 - x\)

Let's see if you can solve it without any help. Set a time limit.

Review with whole class. Ps could write solution on scrap paper or slates and show in unison on command. Ps answering correctly explain their reasoning. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

- **Trial and error:**
  
  If \(x = 1\), then \(48 + 1 < 52 - 1\), and \(50 < 51\), so \(x = 1\) ✅
  
  If \(x = 2\), then \(48 + 2 < 52 - 2\), but \(50 \not< 50\), so \(x \neq 2\) ❌
  
  Ps might have tried 3, 4, 5, . . . to confirm that the only possible solution is \(x = 1\)

- **By calculation:** (T might show this method)
  
  \[
  48 + x < 52 - x \\
  \Rightarrow x < 4 - x \\
  \Rightarrow x < 2 \\
  \Rightarrow x = 1
  \]

- or  \(2 \times x < 52 - 48 = 4\)
  
  \(\Rightarrow x < 4 \div 2 = 2\)
  
  so \(x = 1\)

---

**Notes**

Individual work, monitored

T notes the methods that Ps are using.

Ps try it out in *Ex. Bks.* or on scrap paper then write only the solution in *Pbs.*

Discussion, reasoning, agreement, checking, self-correcting, praising

---

**Lesson Plan 174**

#### Q.3 Read: An antiques dealer bought a vase for £700, then sold it for £800. Then he bought the vase back again for £900 and sold it for £1000. Did the antiques dealer make a profit or a loss?

Ps write a plan and do calculations in *Ex. Bks.* then write the answer as a sentence in *Pbs.* Set a time limit.

Stand up if you think that the antiques dealer made a profit . . .now! A, tell us why you think so. X, tell us why you think he did not.

Class listens to their reasoning and decides who is correct.

What did X do wrong? Who made the same mistake? etc.

**Solution:** e.g.

**Plan:**

\[-700 + 800 - 900 + 1000 = 1800 - 1600 = 200 (\text{\£})\]

or  \(-£700 + £800 = £100\) and \(-£900 + £1000 = £100, £100 + £100 = £200\)

**Answer:** The antiques dealer made a profit of £200.

---

**Notes**

Individual work, monitored, helped

Initial discussion about antiques and dealers and auctions to clarify the context.

In unison

Discussion, reasoning, agreement, self-correcting, praising

Feedback for T

---

**Lesson Plan 174**

#### Q.4 Read: What is half of double the greatest 2-digit number?

Show me . . . now! (99) P responding correctly explains to class.

The greatest 2-digit number is 99, and half of its double is itself!

---

**Notes**

Whole class activity

Responses show on slates or scrap paper in unison.

Reasoning, agreement, praising

---

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## Y4

### Activity

#### 7

*PbY4b, page 174, Q.5*

Read: *On a sheet of paper there are these 4 statements. Tick the only true one.*

**BB:**
1. *On this sheet there is exactly one false statement.*
2. *On this sheet there are exactly two false statements.*
3. *On this sheet there are exactly three false statements.*
4. *On this sheet there are exactly four false statements.*

Allow Ps time to think about it and discuss with their neighbours first.

Who thinks that Statement 1 is true? Why do you think so? Who disagrees? Why? Class agrees that it is not the true one.

Deal with each statement in turn in the same way, involving as many Ps as possible in the discussions.

**Solution:**
1. If this statement is true, then 2, 3 and 4 are false, which makes 3 false statements, so it is a contradiction and cannot be true.
2. If this statement is true, then 1, 3 and 4 are false, which makes 3 false statements, so it is a contradiction again and cannot be true.
3. If this statement is true, then 1, 2 and 4 are false, which makes 3 false statements, so it is true. ✓
4. If this statement is true, than all the others including itself are false, which is a contradiction again, so it cannot be true.

**Answer:** Statement 3 is the only true one.

---

#### 8

*PbY4b, page 174*

**Q.6 Read:** *At the market in Hobbitland, they offered 4 roosters for 2 geese or 2 roosters for 4 chickens. How many roosters did Mrs Hobbit get for 1 goose and 2 chickens?*

Ps work out solution in *Ex. Bks* and write the answer as a sentence in *Pbs*. Set a time limit. Remind Ps to check their solution in the context of the question.

If you have found the answer, show me . . . now! (3)

P with correct answer explains at BB to Ps who were wrong. Who agrees? Who did it another way? etc. Mistakes discussed and corrected. If no P found the answer, T helps class to solve it.

**Solution:**

\[
\begin{align*}
4R &= 2G \\
2R &= 4C \\
1G + 2C &= 2R + 1R = 3R
\end{align*}
\]

**Answer:** Mrs Hobbit got 3 roosters for 1 goose and 2 chickens.
**Y4**

<table>
<thead>
<tr>
<th>Activity</th>
<th>9</th>
</tr>
</thead>
</table>

**PbY4b, page 174, Q.7**

Read: *We want to cut out a cross from a square piece of material which has sides of length 7 cm. The width of each arm of the cross is 1 cm.*

*How much material will be wasted?*

Who thinks that they know how to work it out? Who agrees? Who can think of another way to solve it? etc.

Ps come to BB to explain, giving their reasoning in detail, writing calculations on BB and referring to the diagram where relevant. Class points out errors.

**Solution:** e.g.

Area of the cross: \((7 \times 1) + (7 \times 1 - 1)\) [as middle square is included in both arms]

\[= 7 + 6 = 13 \text{ cm}^2\]

Area of material: \(7 \text{ cm} \times 7 \text{ cm} = 49 \text{ cm}^2\)

Area wasted: \(49 \text{ cm}^2 - 13 \text{ cm}^2 = 36 \text{ cm}^2\)

Or taking the 4 pieces wasted:

Area wasted: \(4 \times (3 \times 3) = 4 \times 9 = 36 \text{ (cm}^2)\)

Or putting the waste material together: \(6 \times 6 = 36 \text{ (cm}^2)\)

**Answer:** The amount of material wasted is 36 cm².

---

**Notes**

Whole class activity

(or individual trial first if Ps wish)

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

BB:

Ps write answer as a sentence in Pbs.
Ps finished early could make up their own cross-number or crossword puzzle and clues for their neighbours to solve.

Solutions:

**Q.1 Horizontal Clues**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>6</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>8</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

**Vertical Clues**

<table>
<thead>
<tr>
<th></th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
<th>f</th>
<th>g</th>
<th>h</th>
<th>i</th>
<th>j</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>56Th + 7H + 5T + 3U</td>
<td>18975 ÷ 5</td>
<td>1 quarter of 100</td>
<td>65 000 + 1872</td>
<td>( \frac{2}{5} ) of 15 \times (140 ÷ 20)</td>
<td>A 3-digit number with all its digits the same</td>
<td>10 000 – 9163</td>
<td>( \frac{1}{4} ) of 2000 + ( \frac{1}{4} )</td>
<td>518 \times 4</td>
<td>5Th + 7H + 5T + 3U</td>
<td>The 10th prime number</td>
</tr>
</tbody>
</table>

Q.2 Horizontal clues only

1. 6-sided plane shape
2. 3-D shape with many plane faces
3. To make bigger
4. Plane shape with no straight sides
5. Opposite of multiply
6. A triangle has 3 of them
7. A shape has this if one half is a mirror image of the other half
8. The same shape but not necessarily the same size

Q.3 Twice the half of any number is itself, so the answer is \( 2 \frac{1}{2} \).