# Mathematics Enhancement Programme

# **TEACHING SUPPORT: Year 4**

### **SOLUTIONS TO EXERCISES**

1. Question and Solution

Write the numbers which have:

- a) an even digit as their hundreds digit and 500 as their nearest ten.
   495, 496, 497, 498, 499
- an odd digit as their hundreds digit and 500 as their nearest ten. 500, 501, 502, 503, 504
- the smallest even digit as their tens digit and 1010 as their nearest ten. 1005, 1006, 1007, 1008, 1009

(p2, Q5)

Notes

Note that for a) and b), numbers that round to 500 as their nearest 10 are 495, 496, 497, 498, 499, 500, 501, 502, 503 and 504

Hence for a), we need those numbers that have an even digit as their hundreds digit, namely 495, 496, 497, 498, 499

whilst for b), we need those numbers where the hundreds digit is an odd number, 500, 501, 502, 503, 504.

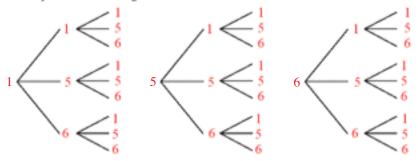
For part c), numbers that round to 1010 as their nearest 10 are 1005, 1006, 1007, 1008, 1009, 1010, 1011, 1012, 1013, 1014 and the smallest even digit is 0, so the answer needed is 1005, 1006, 1007, 1008, 1009.

**2.** *Question and Solution* 

How many 3-digit numbers can you make from these digits?

5 6 1

a) Complete the tree diagrams.



b) List the numbers. .111, 115, 116, 151, 155, 156, 161, 165, 166, 511, 515 .
516, 551, 555, 556, 561, 565, 566, 611, 615, 616, 651, 655, 656, 661, 665 .
666

(p5, Q2)

The systematic method is developed by using the tree diagram approach. In the solution shown, note that the numbers are taken in the order 1, 5, 6 throughout. This ensures that the numbers developed, that is,

are listed in increasing order (but any order is acceptable).

# **3.** *Question and Solution*

I thought of a number, then added 900.

The result was a number less than 1000.

Write  $\checkmark$  if you think the statement is true and **X** if you think it is false.

- a) The number I first thought of must be less than 100.
- The number I first thought of must be less than 99.
- c) The number I first thought of could be equal to 99.
- d) The number I first thought of cannot be more than 99.
- e) The number I first thought of could be equal to 10.
- f) The number I first thought of cannot be 100.

(p7, Q4)

Notes

Note that, if we call the number thought of x, then

$$x + 900 < 1000$$

Now take 900 from each side of the equation, to give

$$x + 900 - 900 < 1000 - 900$$
$$x + 0 < 100$$
$$x < 100$$

Hence a) is true but, as x could be equal to 99, b) is false and c) is true. Similarly, d) is true and x could be

Then e) is true, and similarly f), because  $x \ne 100$  (we know that x is less than 100).

# **4.** *Question and Solution*

Estimate the product first, then do the multiplication.

a) E: 4 2 0 E: 4 5 0 E: 7 5 0 E: 1 0 5 0

7 3 × 6 1 4 6 × 3 2 4 6 × 3 3 4 6 × 3 4 3 8 7 3 8

#### Notes

Your students should be familiar with the use of the word *product* meaning 'the result of a multiplication'.

Note that *E* is an estimation; there are no 'right' answers to this part of the question.

In the first part of a), an estimate is reached by calculating  $70 \times 6$  (= 420), or the calculation could have been  $75 \times 6$  (= 150 × 3 = 450) or even  $80 \times 5$  (= 400). Any of these answers are correct as estimations. You are just looking for a number that is reasonably close to the actual answer.

In the next calculation, E is found from  $150 \times 3$  (= 450) and then  $250 \times 3$  (= 750) and  $350 \times 3$  (= 1050).

In b), the estimates can be calculated from

$$50 \times 8 \ (= 400)$$
  $150 \times 3 \ (= 450)$   $150 \times 6 \ (= 900)$   $250 \times 3 \ (= 750)$ 

# **5.** *Question and Solution*

Write the whole numbers up to 1000 which have 4 as the sum of their digits.

Notes

You need to consider what additions of four digits equals 4; for example,

 $1, 1 \text{ and } 2 \implies 112, 121, 211$ 

 $2, 2 \text{ and } 0 \implies 22, 202, 220$ 

 $1, 0 \text{ and } 3 \Rightarrow 103, 130, 310, 301, 13, 31$ 

 $4, 0 \text{ and } 0 \implies 400, 40, 4$ 

#### **6.** *Question and Solution*

a)

Are the statements true or false? Write T for true and F for false in each box.

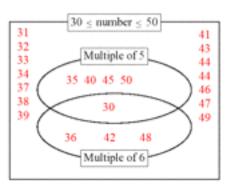
- Every number which is a whole hundred is divisible by 2.
- There is an even number which has 5 as its units digit.
- Every number which is divisible by 5 is a whole ten.
- d) 217 is divisible by neither 5 nor 2.
- e) Every number which is a whole ten is divisible by 2 and by 5.

(p29, Q1)

- a) The whole hundreds are 100, 200, 300, 400, 500, 600, 700, 800, 900 and all are divisible by 2 (in fact, any number ending in the digit zero is divisible by 2).
- b) Any number ending in 5 is not divisible by 2 and so is ODD, not EVEN; hence the statement is false.
- c) This is false as, for example, 25 is divisible by 5 but is not a whole ten.
- d) 217 is not divisible by 5 (as it does not end in 5 or 0) and not divisible by 2 (as it ends in an odd digit). Hence the statement is true (in fact,  $217 = 7 \times 31$  so it is divisible by 7 and 31 as well as by 1 and 217).
- e) Every number ending in 0 is divisible by 2 and 5 as  $2 \times 5 = 10$ .

### 7. Question and Solution

Write the whole numbers from 30 to 50 in the correct set.



(p30, Q4)

Notes

Only the number 30 is divisible by both 5 and 6 and so is positioned in the intersection of the two sets.

Other multiples of 5, that is 35, 40, 45 and 50, are written in the 'Multiple of 5' set.

The multiples of 6, that is 36, 42 and 48, belong in the 'Multiple of 6' set.

The remaining numbers are written in the diagram, but outside the two sets.

# **8.** *Question and Solution*

Write in the boxes the numbers described.

- a) The smallest 4-digit: i) number 1000 ii) odd number 1001
- b) The greatest 4-digit: i) number 9999 ii) odd number 9999
- c) The greatest 4-digit number divisible by: i) 5 9995 ii) 10 9990
- The greatest 4-digit number divisible by 100 which has the same digit in its hundreds and thousands columns.

(p33, Q4)

Notes

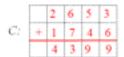
a), b), c) should be straightforward; for d), the possible answers, in numerical order, are 1100, 2200, 3300, 4400, 5500, 6600, 7700, 8800, 9900 and clearly the greatest is 9900.

**9.** Question and Solution

Estimate quickly, then calculate the sum.

a) 2653 + 1746





b) 1256 + 7902





c) 5343 + 2145





(p38, Q1)

Notes

Again, it should be stressed that estimates can vary. For example, in a), we could use

This is not a very accurate estimate but working to the nearest 1000, it is acceptable.

**10.** *Question and Solution* 



a) How many rectangles are in this diagram?

9

- b) How many rectangles would be in 874 such diagrams? .7866.
- c) What is the area of the diagram?

A = 4 square units

d) What is the perimeter of the diagram?

P = 8 units

(p46, Q1)

Notes

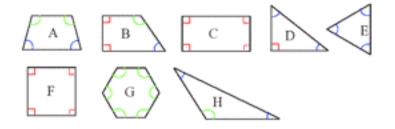
a) You can see 4 small squares, \_\_\_, and one large one, \_\_\_\_\_, (squares are also rectangles) plus two rectangles with longer horizontal sides, \_\_\_\_\_, and two with longer vertical sides, \_\_\_\_\_.

This gives a total of 9 rectangles.

Parts b), c) and d) are straightforward.

- **11.** *Question and Solution* 
  - a) In each diagram, mark
    - right angles in red like this, R
    - angles **smaller** than a right angle in *blue* like this,  $\sqrt{B}$
    - angles **larger** than a right angle in *green* like this.





- List the letters of the shapes for which each statement is true.
  - It is a square. . . . F . . .
- ii) It is a rectangle. . S.F. . . . .
- iii) It is a quadraliteral. A, B, C, F iv) It is a triangle. . D, E, H. . .
- v)
- vi) Every angle is a right angle.
- vii) It has at least one angle smaller than a right angle. A, B, D, E, H
- viii) All its angles are smaller than a right angle. E . . . . . . . . .
- A, B, G, H It has at least one angle larger than a right angle. **G**....
- All its angles are larger than a right angle. x)

(p49, Q2)

#### Notes

Part a) is straightforward but in part b), note that both C and F are rectangles, whilst A, B, C and F are all quadrilaterals (that is, they are closed shapes formed by 4 straight lines).

#### 12. *Question and Solution*

Complete these non-convex shapes so that they become **convex** shapes.













(p52, Q4)

#### Notes

The question would have been better worded as

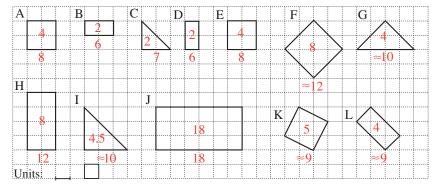
"Change these shapes into convex shapes."

There are many possible correct answers!

#### 13. Question and Solution

List the **similar** shapes.

Write the **area** inside each shape and the length of the **perimeter** below.

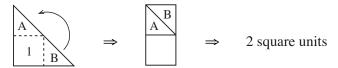


Similar shapes: A E F K; B D H J L; C G I

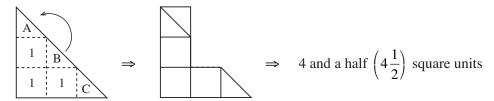
There are 3 distinct sets of similar shapes:

- A, E, F and K these are all squares
- B, D, H, J and L these are all rectangles with side lengths in the ratio 2 to 1, i.e. 2:1 (note that in J the ratio is 6:3 this is the same as 2:1)
- C, G and I these are all triangles with a right angle and two equal sides enclosing the right angle.

The areas are straightforward, but note that for C, we can work out the area by moving part of the shape:



For I, we can redraw in this way:



K is more complicated:

$$A + B = 1$$

$$B \quad I \quad F$$

$$\Rightarrow \quad C + D = 1$$

$$\Rightarrow \quad E + F = 1$$

$$G + H = 1$$

$$middle = 1$$

$$5 \text{ square units}$$

For the perimeters, some need to be estimated, that is, C, J, K and L.

### **14.** *Question and Solution*

Complete the fractions.

a) 
$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{3}{6} = \frac{5}{10} = \frac{10}{20} = \frac{50}{100} = \frac{100}{200}$$

b) 
$$\frac{1}{4} = \frac{4}{16} = \frac{2}{8} = \frac{5}{20} = \frac{8}{32} = \frac{25}{100} = \frac{50}{200} = \frac{3}{12}$$

c) 
$$\frac{1}{3} = \frac{2}{6} = \frac{4}{12} = \frac{3}{9} = \frac{5}{15} = \frac{8}{24} = \frac{10}{30} = \frac{100}{300} = \frac{6}{18}$$

(p80, Q2)

Note that in a), to get from the first fraction,  $\frac{1}{2}$ , to any of the other 'equivalent' fractions, you multiply 'top' (numerator) and 'bottom' (denominator) by the same number. For example,

$$\frac{1}{2} = \frac{3}{6}$$
 or  $\frac{1}{2} = \frac{100}{200}$ 

At this level, we say that all the fractions are equal but the technical term is 'equivalent fractions', and  $\frac{1}{2}$  is the *simplest form* of all the fractions in a).

#### 15. Question and Solution

a) 
$$\frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \boxed{\frac{3}{5}}$$
 b)  $\frac{3}{8} + \frac{2}{8} = \boxed{\frac{5}{8}}$  c)  $\frac{7}{12} - \frac{2}{12} = \boxed{\frac{5}{12}}$  d)  $\frac{11}{20} - \frac{9}{20} = \boxed{\frac{2}{20}}$  e)  $\frac{7}{10} + \frac{3}{5} = \boxed{\frac{13}{10}}$  f)  $\frac{3}{4} - \frac{3}{8} = \boxed{\frac{3}{8}}$ 

b) 
$$\frac{3}{8} + \frac{2}{8} = \boxed{\frac{5}{8}}$$

c) 
$$\frac{7}{12} - \frac{2}{12} = \boxed{\frac{5}{12}}$$

d) 
$$\frac{11}{20} - \frac{9}{20} = \boxed{\frac{2}{20}}$$

e) 
$$\frac{7}{10} + \frac{3}{5} = \begin{vmatrix} \frac{13}{10} \end{vmatrix}$$

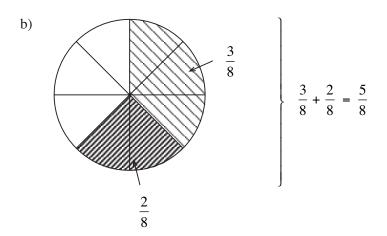
f) 
$$\frac{3}{4} - \frac{3}{8} = \frac{3}{8}$$

(p83, Q4)

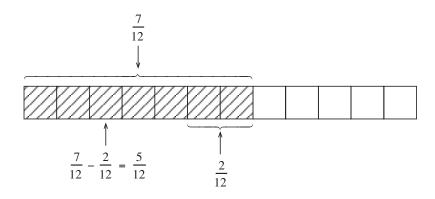
### Notes

This exercise introduces the formal addition of fractions. You can easily illustrate the answers with diagrams. For example,

a) 
$$\frac{\frac{1}{5}}{\frac{1}{5}} \longrightarrow \left\{ \begin{array}{c} \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = \frac{3}{5} \end{array} \right\}$$



Subtraction follows in a similar way:



Parts e) and f) require the use of equivalent fractions to ensure that the denominators are the same, that is,

e) 
$$\frac{7}{10} + \frac{3}{5} = \frac{7}{10} + \frac{6}{10} = \frac{13}{10}$$
 (as  $\frac{3}{5} = \frac{6}{10}$ )

$$(as \frac{3}{5} = \frac{6}{10})$$

f) 
$$\frac{3}{4} - \frac{3}{8} = \frac{6}{8} - \frac{3}{8} = \frac{5}{8}$$
 (as  $\frac{3}{4} = \frac{6}{8}$ )

(as 
$$\frac{3}{4} = \frac{6}{8}$$
)

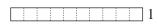
#### **16.** Question and Solution

Compare the pairs of numbers and fill in the missing signs. (<,>,=)Use the diagrams to help you.

a) 
$$\frac{2}{10}$$
  $< \frac{7}{10}$   $\frac{8}{10}$   $< 0.9$  0.6  $> 0.3$ 

$$\frac{8}{10}$$
 < 0.9

0.6 
$$\geq$$
 0.3



b) 
$$\frac{15}{100}$$
  $\leq$   $\frac{72}{100}$ 

$$\frac{15}{100}$$
  $<$   $\frac{72}{100}$   $\frac{43}{100}$   $<$  0.70 0.52  $>$  0.49

c) 0.04 
$$<$$
 0.1  $\frac{2}{10}$   $>$   $\frac{18}{100}$  0.27  $<$  0.3

$$\frac{2}{10} > \frac{18}{100}$$

d) 
$$\frac{1}{5} = 0.2$$

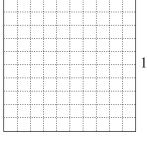
$$\frac{2}{5}$$
 < 0.3

$$\frac{1}{5} = 0.2$$
  $\frac{2}{5} < 0.3$   $\frac{3}{10} < 0.6$ 

e) 
$$\frac{1}{5} > \frac{17}{100}$$

$$\frac{3}{10}$$
 < 0.5

$$\frac{1}{5} > \frac{17}{100} \qquad \frac{3}{10} < 0.51 \qquad \frac{78}{100} > 0.53$$



(p93, Q3)

Notes

This exercise involves both fractions and decimals. Any decimal can be written as a fraction. For example,

$$0.9 = \frac{9}{10}$$

$$0.04 = \frac{4}{100}$$

and any fraction can be written as a decimal by forming an equivalent fraction with one of 10, 100, 1000, etc. as the denominator.

For example,

$$\frac{2}{5} = \frac{4}{10} = 0.4$$

$$\frac{1}{4} = \frac{5}{100} = 0.05$$

Hence in a) 
$$\frac{8}{10} < 0.9 \ \left( = \frac{9}{10} \right)$$
 and in b)  $\frac{43}{100} < 0.70 \ \left( = \frac{70}{100} \right)$ ,

whilst in d) 
$$\frac{1}{5} = 0.2 \left( as \frac{1}{5} = \frac{2}{10} = 0.2 \right)$$

#### Question and Solution 17.

Which quantity is greater? Fill in the missing signs.

a) 
$$\frac{3}{10}$$
 m < 54 cm

- a)  $\frac{3}{10}$  m < 54 cm b) 0.9 kg > 90 g c)  $\frac{1}{6}$  hour < 30 min
- d) £150 20 p = £150.2 e)  $5\frac{7}{100}$  litres < 5 litres 700 ml

f) 
$$4\frac{1}{2}$$
 weeks  $\leq$  29 days

f)  $4\frac{1}{2}$  weeks 29 days g) 84.3 cm = 843 mm < 8.43 m

(p105, Q3)

Notes

Now we have not only fractions and decimals but also units to consider.

a) 
$$\frac{3}{10}$$
 m =  $\frac{30}{100}$  m = 30 cm < 54 cm

b) 
$$0.9 \text{ kg} = 0.9 \times 1000 \text{ g} = 900 \text{ g} > 90 \text{ g}$$

c) 
$$\frac{1}{6}$$
 hours = 60 min ÷ 6 = 10 min < 30 min

£150 20 p can be written as £150.20 which is £150.2 as a decimal number (although we do not normally write it like this).

e) 
$$5\frac{7}{1000}$$
 litres = 5 litres 7 ml < 5 litres 700 ml

f) 
$$4\frac{1}{2}$$
 weeks =  $\left(4 \times 7 + \frac{1}{2}\right)$  days =  $28\frac{1}{2}$  days < 29 days

g) 
$$84.3 \text{ cm} = 843 \text{ mm} = 0.843 \text{ m} < 8.43 \text{ m}$$

# **18.** *Question and Solution*

Imagine these cubes built from unit cubes.

Fill in the missing numbers.









Length of 1 edge ⊢	1	2	3	4	5	6
Area of cube	6	24	54	96	150	216
Volume of cube	1	8	27	64	125	216

(p109, Q1)

Notes

This is a straightforward question but note the pattern emerging.

For the area, we have

$$6 \times 1$$
,  $6 \times 4$ ,  $6 \times 9$ ,  $6 \times 16$ ,  $6 \times 25$ ,  $6 \times 36$ , ...,

The numbers 1, 4, 9, 16, ..., are called 'square numbers' as

$$1 = 1 \times 1$$
,  $4 = 2 \times 2$ ,  $9 = 3 \times 3$ , etc.

The volume pattern is

and these are 'cubic numbers', that is,

$$1 = 1 \times 1 \times 1$$
,  $8 = 2 \times 2 \times 2$ ,  $27 = 3 \times 3 \times 3$ , etc.

# **19.** *Question and Solution*

Follow the example. Complete the sentences. Use the number line to help you.

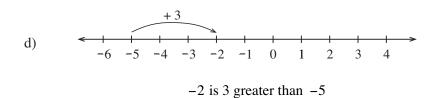
- b)  $3^{\circ}$ C is less than  $8^{\circ}$ C by  $5^{\circ}$ C. 3-8=[-5], [-5] + 8 = 3
- c)  $8^{\circ}$ C is greater than  $0^{\circ}$ C by  $8^{\circ}$ C. 8-0=8, 8+0=8
- d)  $3^{\circ}$ C is greater than  $-2^{\circ}$ C by  $5^{\circ}$ C.  $3 (-2) = 5^{\circ}$ ,  $5 + (-2) = 3^{\circ}$
- e)  $-2^{\circ}$ C is less than  $3^{\circ}$ C by  $5^{\circ}$ C . -2-3 = -5, -5 + 3 = -2
- f)  $-2^{\circ}$ C is greater than  $-5^{\circ}$ C by  $3^{\circ}$ C. -2 (-5) = 3, 3 + (-5) = -2

(p134, Q4)

Notes

Dealing with addition/subtraction of negative numbers can often be confusing so it helps if you illustrate what you are doing on a number line.

For example,



(In context, we are showing that a temperature of  $-5^{\circ}$ C is  $3^{\circ}$ C colder than  $-2^{\circ}$ C.)

# **20.** *Question and Solution*

In an opaque bag there are 10 black and 30 white marbles.

What is the smallest number of marbles you must take out of the bag (with your eyes closed) to be **certain** of getting 2 marbles which are the same colour?

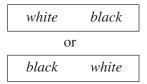
3

(p137, Q4)

Notes

This is almost a 'trick question': many people will give the answer '11'.

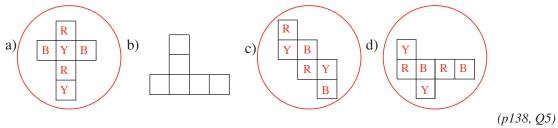
It is clearly 3, as the worst outcome is to obtain



for the first two marbles. The third one taken must be either *black* or *white*, giving 2 marbles that are the same colour.

### **21.** *Question and Solution*

Circle the nets which can make a cube. Colour their opposite faces in the same colour.



Notes

It is always best to cut out models of the nets and try them out. Some students will be able to visualise without the physical model, but going through this process is valuable for many.

### **22.** *Question and Solution*

What is the smallest natural multiple of 2, 3, 4, 5 and 8?

120

(p140, Q2)

Here you are looking for the smallest whole number that can be divided exactly by 2, 3, 4, 5 and 8 (no remainder).

Both 2 and 4 are factors of 8, so really we are finding the smallest multiple of 3, 5 and 8.

These numbers have no common factor, so the number required must be

$$3 \times 5 \times 8 = 120$$

#### 23. Question and Solution

In an opaque bag, there are 5 black, 10 red and 5 white marbles.

What is the smallest number of marbles you must take out of the bag (with your eyes closed) to be **certain** of getting:

- 3 marbles which are the same colour
- a red marble? b)

b)

(p140, Q3)

Notes

- The worst situation is to obtain 2B, 2R and 2W for the first 6 marbles taken. a) The next one has to give you 3 of the same colour, whatever colour it is.
- Think very carefully about this question. The worst scenario is to first take all the B and W marbles, that is 10 marbles. The next one *must* be R!

#### 24. Question and Solution

List in your exercise book all the numbers between 999 and 10 000 which have 4 as the sum of their digits. How many did you find?

(p140, Q4)

20

Notes

Be systematic in finding all the possible answers. The full list is shown below.

'1, 1, 1, and 1' 1 1 1 1

'1, 1, 2 and 0' 2 1 1 0, 2 1 0 1, 2 0 1 1,  $\Rightarrow$ 1 2 1 0, 1 2 0 1, 1 1 2 0, 1 1 0 2 1 0 1 2, 1 0 2 1

'2, 2, 0 and 0' 2 2 0 0, 2 0 2 0, 2 0 0 2

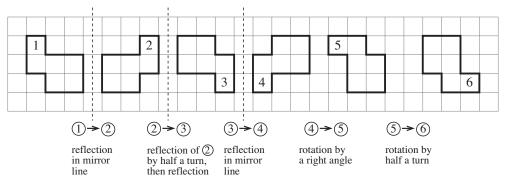
'1, 3, 0 and 0' 3 1 0 0, 3 0 1 0, 3 0 0 1 1 3 0 0, 1 0 3 0, 1 0 0 3

'4, 0, 0 and 0' 4 0 0 0

20

# **25.** *Question and Solution*

These shapes are **congruent**. What has been done to *Shape 1* to make *Shape 2*, *Shape 2* to make *Shape 3*, and so on? Write it in your exercise book.



(p141, Q2)

Notes

Note that

half a turn = 2 right angles

quarter turn = 1 right angle

It might be helpful to use tracing paper for the rotations.

# **26.** *Question and Solution*

Among 67 scientists at a conference,

- 47 speak French,
- 35 speak German,
- 20 speak Spanish,
- 12 speak French and Spanish,
- 11 speak German and Spanish,
- 5 speak all three languages.
- a) Complete the *Venn* diagram.
- b) How many scientists speak:
  - i) only French 17 ii) only German
    - man 6
- iii) only Spanish?

67 scientists

(18)

(5)

(2)

Spanish | 20

French

(17)

(6)

47

(7)

- 2
- c) How many scientists speak Spanish and German but not French?
- 6

German 35

(6)

6)

- d) How many scientists speak neither Spanish nor German nor French?
- 6

(p151, Q3)

Notes

To complete the Venn diagram, note that

11 speak G and S  $\Rightarrow$  6 speak G and S but not F so 6 can be inserted.

Looking at G, the total is 35, so the missing number is

$$35 - (18 + 5 + 6) = 35 - 29 = 6$$

Similarly, 20 is the total for S, and the missing number is

$$20 - (7 + 5 + 6) = 20 - 18 = 2$$

Finally, there are 67 scientists in total. Adding up all the numbers inserted gives

$$17 + 18 + 5 + 7 + 6 + 6 + 2 = 61$$

So there are 67 - 61 = 6 in the outside region of the Venn diagram.

### **27.** *Question and Solution*

Circle the natural numbers up to 100 which have only two factors. (e.g. the only factors of **7** are 7 and 1)

List these numbers in increasing order.

(p161, O2)

Notes

Numbers with only 2 factors are called PRIME numbers and are very important in the development of mathematics. Although it appears that they occur less frequently as the numbers get larger, this is not the case, and they keep on occurring as there is an *infinite* number of them.

Many recent applications of prime numbers have occurred, including a method of coding and decoding electronic messages using two very large prime numbers multiplied together.

### **28.** *Question and Solution*

The perimeter of a triangle is 10 cm and the length of each side is a whole cm.

Are these statements true or false? Write a  $\checkmark$  if true and a **X** if false.

a) The triangle has only one side which is 1 cm long.

b) The triangle could have only one side which is 2 cm long.

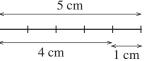
c) The triangle has only one side which is 3 cm long.

d) The triangle has only one side which is 5 cm long.

(p173, Q1)

a) If a triangle has only one side of length 1 cm, then the possible lengths could be

1 cm, 5 cm, 4 cm - this is a straight line.



b) The triangle could have sides of



c) The possible lengths are

d) Here you can have