Lesson Plan

51

Week 11

Y5

Activity

1

Temperature

What is temperature? (How hot or cold something is) How do we measure temperature? (thermometer) What unit of measure do we use? [degrees Celsius (metric) or degrees Fahrenheit (Imperial system)]

T shows model or diagram What can you tell me about this thermometer? e.g. using copy master:

- The unit of measure is degrees Celsius.
- It has short ticks at every 1°C, medium ticks at every 5°C and long ticks at every 10°C.
- Its range is –40°C to +60°C.
- It shows a temperature of 0°C.

Let’s show other temperatures on the thermometer.

T elicits or states temperatures and Ps come to BB to mark them on model or diagram. (If they are outside the range, Ps say in which direction they would be.)

a) At what temperature does water boil? (100°C)

b) The highest temperature in the shade ever recorded is plus 58°C (in Lybia, Africa).

c) At what temperature does water freeze? (0°C)

d) The temperature at home is about plus 20°C.

e) At the top of high mountains, or near the North and South Poles, the temperature often falls below minus 40°C.

f) The lowest temperature ever recorded was minus 89°C, near the South Pole in Antarctic.

Discuss other temperatures. (e.g. body temperature, holiday temperatures, today’s outside temperature, yesterday’s temperatures around the country and/or the world – from newspapers)

T reminds Ps that a temperature without a positive or negative sign in front of it is always regarded as being positive, e.g. +20°C = 20°C.

8 min

2

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Q.1 Read: What temperature does each thermometer show?

Set a time limit. Review with whole class. T points to each thermometer in turn and Ps show temperature on scrap paper or slates in unison. Mistakes discussed and corrected.

Solution:

- a)
- b)
- c)
- d)
- e)
- f)

Extension

Individual work, monitored, helped, then whole-class extension

Drawn on BB or use enlarged copy master or OHP

Agreement, self-correction, praising

Who can think of questions to ask about the temperatures? e.g. Which is lowest (highest)? How many °C are between a) and c), d) and e), b and e)? etc. Ps show on diagrams.

Notes

Whole class activity

T has model and if possible some real thermometers with different scale ranges, e.g. sugar, medical, bath, etc.)

Draw diagram on BB or use enlarged copy master or OHP

If possible, Ps have copies on desks too.

Involve several Ps.

Agreement, praising

[T might mention the special metal in thermometers – mercury, which is a liquid at normal temperatures but expands when hot and contracts when cold.]

At a good pace.

Ps say relevant temperatures on write on BB

(Ps mark on own diagrams too if they have them)

Agreement, praising

Elicit that when water boils it turns into steam and when it freezes it turns into ice.

T could have a map of the world to show the places mentioned.

Ps tell class temperatures they have experienced and make comparisons. (e.g. summer is hotter than winter, autumn is cooler than summer, etc.)

14 min

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### Lesson Plan 51

#### Activity

**3 Numbers**

What is the common feature of all these numbers?

- $+5$, $0$, $-3$, $-152$, $+7$, $+402$, $-1008$, $-20$, $+72$

Elicit that they are all whole numbers. What other name do we give to whole numbers? (integers) T reminds Ps if nobody can remember.

T points to certain numbers and class reads them in unison. Ps come to BB in pairs to point to two numbers. Which is more? How many more? Class agrees/disagrees.

Write the numbers in increasing order in your Ex. Bks. When most Ps have finished, Ps come to BB or dictate to T. Class agrees/disagrees.

Class agrees/disagrees.

BB: $-1008 < -152 < -20 < -3 < 0 < +5 < +7 < +72 < +402$

Which sign is not needed? (the positive sign, +, as numbers without a sign are regarded as being positive) T writes list again without '+' signs.

BB: $-1008 < -152 < -20 < -3 < 0 < 5 < 7 < 72 < 402$

---

**4 Integers**

Think about the set of whole numbers. What is their other name? (integers) T draws outer part of Venn diagram on BB and labels it.

Which numbers would be in this set? Allow Ps to suggest several numbers, then T asks which is the smallest and which is the greatest possible integer. Agree that there is no smallest or greatest integer as there is an infinite (endless) number of positive and negative numbers. Elicit or show how to indicate all the numbers in the set using ellipses.

BB: Set of Integers = \{ $\ldots$, $-3$, $-2$, $-1$, $0$, $1$, $2$, $3$, $\ldots$ \}

Could we put the integers into groups? (Yes, e.g. negative integers, or positive integers.) What other name do you know for positive integers? (Natural numbers) How could we show this subset in the diagram? Ps come to BB or dictate what T should draw and write. Class agrees/disagrees.

Let’s see if you can put these numbers in the correct places in the Venn diagram. T says one number at a time, e.g. $-20$, $30$, $0$, $-11$, $223$, and Ps come to BB to write them in the diagram. Class agrees/disagrees. Ps could suggest other numbers too.

What kind of integers are not natural numbers? (negative numbers, zero)

---

**5 Sequences**

T says the first few terms of a sequence and writes them on BB too.

Think what the rule is and continue the sequence in your Ex. Bks.

Deal with one at a time. Set a time limit of 1 minute. Review with whole class. Ps dictate terms and state the rule. Class agrees/disagrees.

a) $310$, $250$, $190$, $130$, $70$, $10$, $-50$, $-110$, $-170$, $-230$, $\ldots$

Rule: Decreasing by 60.

b) $0$, $1$, $-2$, $3$, $-4$, $(5, -6, 7, -8, 9, -10, 11, -12, 13, \ldots)$

Rule: Digits are increasing by 1 but the '+' and '-' signs alternate.

T: We could also say that the absolute value is increasing by 1. Elicit/tell that the absolute value of a number is its distance from zero.

BB: Absolute value

e.g. $-4$ is 4 units from 0, so its absolute value is 4.

We write: $| -4 | = 4$
Q.2  a) Read: Mark and label these numbers on the number line.

Set a time limit. Review with whole class. Ps come to BB to mark and label. Class agrees/disagrees. Mistakes corrected. Let’s read them in increasing (decreasing) order.

Solution:

\[ -7, +1, 0, 6, -5, -3, +10, 11 \]

T points to certain numbers and Ps give their absolute values.

b) Read: Compare the numbers. Write the missing signs in the circles.

Again set a time limit. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected by referring to number line.

Solution:

\[ -7 < +1 \quad 0 > -5 \quad -5 > -7 \quad -7 < -5 \]
\[ -5 < 0 \quad 11 > 0 \quad 6 > -3 \quad 6 < +10 \]
\[ -7 < -3 \quad 11 > 6 \quad 0 > -5 \quad -3 < +10 \]

What do you notice? T prompts where necessary. Elicit that:

- Any positive number is greater than any negative number.
- Among the positive numbers, the greatest is furthest from zero (and has the greatest absolute value).
- Among the negative numbers, the greatest is nearest zero (and has the smallest absolute value).

Also agree that as you move to the right along the number line both positive and negative numbers are increasing (and vice versa).

Feedback for T

Q.3  Read: What is the difference between the two temperatures? Answer with an operation.

Set a time limit. Ps write operations, do calculations (in Ex. Bks if they cannot calculate mentally) and write the result.

Review with whole class. T chooses a P to read each sentence, then Ps show difference on scrap paper or slates on command. P answering correctly explains to Ps who were wrong. Mistakes discussed and corrected.

Solution:

a) On a January day at dawn the temperature was \(-3 \, ^\circ\text{C}\).
   At mid-day it was \(11 \, ^\circ\text{C}\).
   BB: \(11 \, ^\circ\text{C} - (-3 \, ^\circ\text{C}) = 14 \, ^\circ\text{C}\) (11 to zero, then 3 below)

b) In the Sahara Desert, the temperature was \(43 \, ^\circ\text{C}\) at noon and \(-4 \, ^\circ\text{C}\) at night.
   BB: \(43 \, ^\circ\text{C} - (-4 \, ^\circ\text{C}) = 47 \, ^\circ\text{C}\) (43 to zero, then 4 below)

c) In Eastern Siberia the summer temperature is sometimes \(30 \, ^\circ\text{C}\) and the winter temperature is sometimes \(-70 \, ^\circ\text{C}\).
   BB: \(30 \, ^\circ\text{C} - (-70 \, ^\circ\text{C}) = 100 \, ^\circ\text{C}\) (30 to zero, then 70 below)
### Y5

#### Activity 7

(Continued)

<p>| | | |</p>
<table>
<thead>
<tr>
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</table>
| d) | On Earth, the highest air temperature ever measured is 58°C and the lowest is –89°C.  
BB: \[58°C - (-89°C) = 147°C\] (58 to zero, then 89 below)  
e) | On the Moon, the temperature can be –130°C in the day and –160°C at night.  
BB: \[-130°C - (-160°C) = 30°C\] (e.g. move 30 to the left along a horizontal number line) |

#### Activity 8

**Cash and debt**

T has cash and debt cards already prepared and stuck to side of BB. Elicit that a circle means +£1, or £1 in cash and a rectangle means –£1 or £1 in debt. The balance is the amount that you have left when you have paid all your debts.

<p>| | | |</p>
<table>
<thead>
<tr>
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</thead>
</table>
| a) | A, your balance is £3. How much cash and debt could you have?  
Come and choose appropriate cards to stick on the BB. Class checks that the balance is correct. Agree that a '+' and a '-' cancel each other out.  
Who could have shown a balance of £3 in a different way?  
B comes to BB to show it. Class agrees/disagrees. (e.g. £6 cash and £3 in debt)  
Who could tell us other ways? T asks several Ps to describe orally. e.g. £10 in cash and £7 in debt, £3 in cash and no debts, etc. For some, T could ask more able P to write an addition.  
Repeat for:  
b) | Balance: –£2  
c) | Balance: £0  
d) | Balance: £7 |

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### Lesson Plan 51

**Notes**

BB: e.g.  
\[58 + 89 = 60 + 90 - 3\]  
\[= 150 - 3 = 147\]  
or \[-89 < 58\]  
\[147\]  
or \[-160 < -130\]  
\[30\]

Whole class activity  
(Or Ps draw symbols on BB)  
BB: e.g.  
a)  
\[
\begin{array}{c}
\text{Cash:} \\
\text{Debt:} \\
\text{Balance:}
\end{array}
\]

[ £4 + (−£1) = £3 ]  

At a good pace  
Praising, encouragement only  
Extra praise if Ps can do this without T’s help.
### Lesson Plan

**Y5**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td><strong>Integers</strong></td>
<td><strong>Whole class activity</strong></td>
</tr>
<tr>
<td>Study these numbers. What do you notice? (Two numbers have vertical parallel lines on either side of them.) Elicit/tell that they mean the absolute value of the number, i.e. its distance from zero. Ps come to BB to point to the 2 numbers and write their absolute values below. Class agrees/disagrees.</td>
<td>Written on BB or SB or OHT</td>
</tr>
<tr>
<td>BB: [−7], 0, [3/4], +5, [6], [−2/3], −11, 10, opposite of −8, −20, 9</td>
<td>Discussion, agreement that:</td>
</tr>
<tr>
<td>Which of these numbers are not integers? Ps come to BB to circle them and explain reasoning. (Elicit that 3 quarters is a fraction and −1 and 2 thirds is a mixed number)</td>
<td>BB: [−7] = 7, [6] = 6</td>
</tr>
<tr>
<td>Write all the numbers in increasing order in your Ex. Bks. When majority of Ps have finished, Ps dictate order to T. Class points out errors. Ps who made a mistake write list again correctly.</td>
<td>and opposite of −8 is 8.</td>
</tr>
<tr>
<td>BB:</td>
<td>Reasoning, agreement, praising</td>
</tr>
<tr>
<td>−20 &lt; −11 &lt; −5 &lt; −2 &lt; 0 &lt; 3/4 &lt; 5 &lt; 6 &lt; 7 &lt; 8 &lt; 9 &lt; 10</td>
<td>Individual work, monitored</td>
</tr>
<tr>
<td>Let’s mark them on a number line. Which of these number lines would you choose? Who would choose the first? Who would choose the second? Why? Agree that the 2nd number line would be better as many numbers could not be shown on the first number line – the distance between integers is too great.</td>
<td>Agreement, self-correction, praising</td>
</tr>
<tr>
<td>Ps come to BB to mark and label the numbers. Class points out errors.</td>
<td>Whole class activity</td>
</tr>
<tr>
<td>BB:</td>
<td>Number lines drawn on BB</td>
</tr>
<tr>
<td>(Only 0 and 2 should already be marked on 2nd number line.)</td>
<td>or use enlarged copy master or OHP</td>
</tr>
<tr>
<td><strong>2</strong></td>
<td></td>
</tr>
<tr>
<td><strong>PhY5a, page 52</strong></td>
<td>Agreement, praising</td>
</tr>
<tr>
<td>Q.1 Read: Work out the rule and complete the table. Write the rule in different ways.</td>
<td></td>
</tr>
<tr>
<td>Set a time limit. Deal with one at a time if class is not very able. Review with whole class. Ps come to BB to complete the tables and write the rule that they used. Who agrees? Who used a different rule? etc. Mistaken discussed and corrected.</td>
<td>Extra praise if Ps suggest</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td>x + y = 0</td>
</tr>
<tr>
<td>a)</td>
<td>Agree that if v = 10, u can be either +10 or −10.</td>
</tr>
<tr>
<td>[ \begin{array}{l</td>
<td>l} x &amp; 5 \ \hline y &amp; \frac{3}{4} \ \end{array} ]</td>
</tr>
<tr>
<td>[-5, -3, 2, 14, -8, 0, -140, 479, -40.5, 12.3, \frac{5}{8}, -0.72 ]</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td></td>
</tr>
<tr>
<td>[ \begin{array}{l</td>
<td>l} u &amp; 6 \ \hline v &amp; \frac{1}{2} \ \end{array} ]</td>
</tr>
<tr>
<td>[ \frac{6}{10}, -11, 5, -93, 41, 164, -2.3, 0, 0, -10, 10, -0.15 ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>[ \begin{array}{l</td>
<td>l} v &amp; \frac{1}{2} \ \hline y &amp; \frac{1}{2} \ \end{array} ]</td>
</tr>
<tr>
<td>[ \frac{6}{10}, -11, 5, -93, 41, 164, 2.3, 0, 0, 10, 10, 0.15 ]</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>v = absolute value of u or v = [u]</td>
<td></td>
</tr>
<tr>
<td>Which numbers in the table are not integers? (fractions/decimals)</td>
<td><strong>12 min</strong></td>
</tr>
</tbody>
</table>

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**Lesson Plan 52**

### Activity 3

**Number line**

Follow my instructions and take steps along the number line. Start at zero each time. Ps come to BB to demonstrate the moves. Class points out errors. T confirms movement by drawing arrows. Who can write an operation about it? Ps come to BB or dictate to T. Class agrees/disagrees. Could it be written another way? T shows it if no P can think of another way and asks class if it is correct. e.g.

a) **Step 3 to the right, then 5 more to the right.**
   
   BB: 0 + 3 + 5 = 8
   
   or (+3) + (+5) = +8
   
   ![Number line diagram](image)

b) **Step 7 to the right, then 5 to the left.**
   
   BB: 0 + 7 – 5 = 2
   
   or +7 + (–5) = +2
   
   ![Number line diagram](image)

c) **Step 5 to the right, then 7 to the left.**
   
   BB: 0 + 5 – 7 = -2
   
   or +5 + (–7) = -2
   
   ![Number line diagram](image)

d) **Step 6 to the left, then 5 more to the left.**
   
   BB: 0 – 6 – 5 = -11
   
   or –6 + (–5) = -11
   
   ![Number line diagram](image)

e) **Step 6 to the left, then 5 to the right.**
   
   BB: 0 – 6 + 5 = -1
   
   or –6 + (+5) = -1
   
   ![Number line diagram](image)

f) **Step 6 to the left, then 9 to the right.**
   
   BB: 0 – 6 + 9 = 3
   
   or –6 + (+9) = +3
   
   ![Number line diagram](image)

---

**Addition of integers 1**

Let’s do the additions using the thermometer model (diagram) to help us. Ps set own models or use own diagrams to work out the answer. T chooses Ps to come to BB to show moves on large model or diagram and complete the operation. Class agrees/disagrees and Ps write completed addition in Ex. Bks.

After each operation has been completed, T asks pupils to explain it in a context using words. Class decides whether it is valid.

**BB:**

- **a)** 5 + (+2) = +7
  - Possible contexts: The temperature was 5°C, then it rose by 2°C and is now 7°C.
- **b)** +2 + (+5) = +7
- **c)** +4 + (–7) = -3
- **d)** –5 + (+3) = -2
- **e)** –6 + (–2) = -8
- **f)** –4 + (+8) = +4
- **g)** –6 + (+6) = 0
- **h)** +6 + (–6) = 0

---

**Notes**

Whole class activity

Number lines drawn on BB or use enlarged copy master or OHP, or class number line (Ps could have copies of copy master on desks too.)

At a good pace

Involve as many Ps as possible.

Demonstrating, reasoning, agreement, praising Ps could write the operations in Pbs too.

Accept any correct form, e.g.

- **a)** +3 + (+5) = 8
- **b)** +7 – (+5) = +2
- **c)** +5 – (+7) = -2

etc.

Agree that the 1st move from 0 does not need to be noted.

Ps could give instructions for some of the moves.

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**Y5**

**Activity 5**

**Addition of integers 2**

Let's add up the numbers and show the additions on the number line.

Ps come to BB to write operation, draw arrows on number line and then do the calculation, with T's help if necessary. Class agrees/disagrees or suggests an easier way.

BB:

- **a)** 2 and (–5) and (–1) and (+7)
  
  \[ 2 + (–5) + (–1) + (+7) = 3 \]
  
  T: What do you think about these sums? Are they correct?

  BB: 2 + 7 + (–5 + (–1)) = 9 + (–6) = 3

  \[ –5 + (–1) + 2 + 7 = –6 + 9 = 3 \]

  (Both are correct.)

- **b)** 15, –5, –4, –3, –2 and (–1)
  
  \[ 15 + (–5) + (–4) + (–3) + (–2) + (–1) = 0 \]

  What do you think about this way of calculating? Is it correct?

  \[ 15 – (5 + 4 + 3 + 2 + 1) = 15 – 15 = 0 \] (correct)

  Agree that when adding positive and negative numbers, it is sometimes easier to add all the positive numbers, then all the negative numbers, then add the two sums together.

---

**Activity 6**

**Addition of integers 3**

Listen carefully to what I say and follow it on your number line. Write an addition about it in your Ex. Bks. and do the calculation.

Show me the result when I say.

Deal with one at a time. Ps answering correctly come to BB to write addition and show on number line or thermometer diagram or model. Who agrees? Who wrote something else? etc. Mistakes discussed and corrected.

- **a)** The temperature was 0°C, then it rose by 9°C, then fell by 3°C.
  
  BB: 0 + 9 + (–3) = 6 (°C)

- **b)** The temperature fell by 7°C from +3°C.
  
  BB: 3 + (–7) = –4 (°C)

- **c)** Start at 0. Step 5 units to the left, then 2 units to the left again.
  
  BB: 0 + (–5) + (–2) = –7

- **d)** Start at 0. Step 8 units to the left, then 7 units to the right.
  
  BB: 0 + (–8) + (+7) = –1

- **e)** Start at –3. and step 10 units to the right.
  
  BB: –3 + (+10) = 7

- **f)** The temperature was 0°C. It fell by 5°C, then rose by 5°C.
  
  BB: 0 + (–5) + (+5) = 0 (°C)

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**Lesson Plan 52**

**Notes**

Whole class activity

Number lines drawn on BB or use enlarged copy master or OHP.

Numbers written on BB.

Discussion, reasoning with T's help, agreement, praising

T also writes the operation as:

\[ 2 – 5 – 1 + 7 = \frac{3}{2} \]

and shows it as steps along the number line (as opposite).

Ps could copy the additions in Ex. Bks. and make a note that they are the same.

---

Individual work, monitored, helped

Ps have number lines or thermometer models on desks.

T repeats slowly to give Ps time to think and write.

Results written on scrap paper or slates and shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Accept subtraction if it is correct, e.g.

- **a)** 9 – 3 = 6
- **b)** 3 – 7 = –4
- **c)** –5 – 2 = –7, etc., but also show as an addition.

Also accept other correct forms of addition, e.g.

- **d)** –8 + 7 = –1
- **e)** –3 + 10 = 7, etc.
Q.2 Read: Write an addition about each diagram.

Deal with one at a time.
Review at BB with whole class. Ps come to BB or dictate to T.
Class agrees/disagrees. Mistakes discussed and corrected.
Agree that the ‘+’ sign showing that a number is positive (as opposed to the ‘+’ sign for the operation of addition) is not really needed as numbers without a sign in front of them are always thought of as being positive, so the additions can be written more simply (as below).

Solution:

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Description</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>1st -5 0 5 10, 2nd 0 5 10</td>
<td>(+4) + (+6) = 10 or 4 + 6 = 10</td>
</tr>
<tr>
<td>b)</td>
<td>-10 -5 0 5</td>
<td>(+3) + (-10) = -7 or 3 + (-10) = -7</td>
</tr>
<tr>
<td>c)</td>
<td>-10 -5 0 5</td>
<td>(-5) + (+2) = -3 or (-5) + 2 = -3</td>
</tr>
<tr>
<td>d)</td>
<td>-10 -5 0 5</td>
<td>(-4) + (+9) = 5 or (-4) + 9 = 5</td>
</tr>
<tr>
<td>e)</td>
<td>-5 0 5 10</td>
<td>(+2) + (-6) + (+10) = 6 or 2 + (-6) + 10 = 6</td>
</tr>
</tbody>
</table>


Lesson Plan 52

Notes

Individual work, monitored, helped
Drawn on BB or use enlarged copy master or OHP
Discussion, reasoning, agreement, self-correcting, praising
Accept other correct forms too.
Feedback for T

Whole class activity
(or individual trial first if Ps wish)
Drawn on BB or use enlarged copy master or OHP
Discussion, reasoning, agreement, (self-correction), praising
[Practice in mental addition of 2 and 3 integers]

Review movements along number line:
- Adding a positive number: move to the right
- Adding a negative number: move to the left

Feedback for T
Opposite values
T says a quantity. Ps say the opposite quantity.
e.g. T: – 35 m, P 1 : + 35 m (or 35 m); T: £105, P 2 : – £105;
T: – 89 °C, P 3 : + 89 °C; T: Opposite of – 1000, P 4 : + 1000;
T: 6 × 5, P 5 : – (6 × 5) or – 30, etc.

Sequences
T says first few terms of a sequence. Ps say the following terms. T
decides when to stop. T asks P who gave the last term to say the rule.
Who agrees? Who can say it another way? etc.
a) 1600, 1200, 800, (400, 0, –400, –800, –1200, –1600, . . .)
   Rule: Decreasing by 400, or –400 [or + (– 400)]
b) –81, –71, –61, (–51, –41, –31, –21, –11, –1, 9, 19, . . .)
   Rule: Increasing by 10, or + 10 [or – (–10)]
c) 3, 13, –7, 3, –17, –7, –27, (–17, –37, –27, –47, –37, . . .)
   Rule: Alternating an increase by 10 with a decrease by 20,
or (+10, –20; +10, –20; etc.)

Integers
T has numbers written on BB. What kind of numbers are these?
(whole numbers or integers)
BB: –7, +5, |–10|, opposite of +9, 0, –2, 7, |+6|
What else can you tell me about any of them? Elicit that:
• |–10| = absolute value of –10 = 10
• |+6| = absolute value of +6 = 6
• opposite of +9 = –9
Write them in increasing order in your Ex. Bks. Use the number line
to help you. When task is completed, Ps dictate list to T. Mistakes
discussed and corrected
BB: –9 < –7 < –2 < 0 < 5 < 6 < 7 < 10
Let’s write them in the correct place in this Venn diagram. Ps come to
BB to write numbers and explaining reasoning. Class agrees/disagrees.

Elicit that natural numbers are positive whole numbers, and that zero
is neither positive nor negative.
### Activity 4

#### Cash and debt model

T has cash and debt cards stuck to BB. What do these symbols mean? (Circle means £1 in cash, or + £1, and rectangle means £1 owed or £1 in debt or –£1.)

T reads a problem, then calls Ps to front of class to choose and stick on BB appropriate cards or draw appropriate number of circles and rectangles. Rest of class draw circles and rectangles in Ex. Bks.

Who can write an addition about it? Who agrees? etc. Ps write the addition and the amounts of cash, debt and balance in Ex. Bks.

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- **a)** Joe’s balance was £3, as he had £7 in his piggy bank but owed a friend £4. If he earns another £5, what is his balance now?
  - BB:
    - 7 + (–4) + 5 = 3 + 5 = 8
    - Cash: £12, Debt: £4, Balance: £8

- **b)** Steve had £4 in cash and was £7 in debt. Then he bought some football stickers from a friend for £2 and promised to pay him when he could. What was Steve’s balance then?
  - BB:
    - 4 + (–7) + (–2) = −3 + (–2) = −5
    - Cash: £4, Debt: £9, Balance: −£5

- **c)** Mike has £8 in cash and has debts of £3. What is his balance?
  - BB:
    - 8 + (–3) = 5
    - Cash: £8, Debt: £3, Balance: £5

- **d)** Anna has £6 in cash and owes £10. What is her balance?
  - BB:
    - 6 + (–10) = −4
    - Cash: £6, Debt: £10, Balance: −£4

- **e)** Rosy has £7 in cash but is £7 in debt. What is her balance?
  - BB:
    - 7 + (–7) = 0
    - Cash: £7, Debt: £7, Balance: £0

---

**Notes**

Whole class activity, with individual work in Ex. Bks at same time.

Or T has symbols drawn on BB and Ps draw circles and rectangles on BB.

At a good pace

Discussion, reasoning, agreement, praising

Agree that 1 + (–1) = 0 so cancel each other out.

or 12 + (–4) = 8 (adding positive numbers first)

or 4 + (–9) = −5 (adding negative numbers first)

Show on number line too if problems or disagreement.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 53</th>
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<tbody>
<tr>
<td><strong>5</strong></td>
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<tr>
<td><strong>PbY5a, page 53</strong></td>
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<tr>
<td><strong>Q.1</strong> Read: <strong>What is the balance? Write it as an addition.</strong></td>
<td>Notes</td>
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<tr>
<td>Set a time limit. Ps count the circles and rectangles, cancel out pairs of (+ 1) and (– 1) and write an addition. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Show each operation on number line too. T ask Ps to tell a word problem for each addition. e.g. a) I owed £16, then I was given £7, so I am now only £8 in debt.</td>
<td>Individual work, monitored, helped</td>
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<td>Solution:</td>
<td>Drawn (stuck) on BB or use enlarged copy master or OHP</td>
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| a) \[
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\] \quad - 16 + (+ 7) = - 9 | Differentiation by time limit |
| b) \[
\begin{array}{cccccccccccc}
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\end{array}
\] \quad 15 + (– 5) = 10 | Reasoning, agreement, self-correction, praising, encouragement only |
| c) \[
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\end{array}
\] \quad 7 + (– 7) = 0 | Extra praise for good contexts |
| 25 min | or 7 + (– 16) = −9 |
| **6** |                |
| **PbY5a, page 53** |                |
| **Q.2** Read: **Change the diagrams so that the balance remains the same. Write an addition about it.** |                |
| How can you change the diagrams? (Draw extra circles and rectangles, or cross out some of those given.) Set a time limit. Review with whole class. T chooses Ps to come to BB to show what they drew and wrote. Is it correct? Who did they same? Who did something different? Deal with all cases. Mistakes discussed and corrected. T asks Ps to tell word problems about their diagrams. In how many ways could we show a balance of, e.g. − £4? Agree that the number of possibilities is endless or infinite. Solution: e.g. | Individual work, monitored, helped |
| | Drawn (stuck) on BB or use enlarged copy master or OHP |
| a) \[
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\end{array}
\] \quad 10 + (– 5) = 5 | Differentiation by time limit |
| b) \[
\begin{array}{cccccccccccc}
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\end{array}
\] \quad 5 + (– 9) = 4 | Discussion, reasoning, agreement, self-correction, praising |
| c) \[
\begin{array}{cccccccccccc}
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\end{array}
\] \quad 8 + (– 8) = 0 | Ask several Ps what they think. |
| 30 min | |
**Y5**

**Activity**

7  

**PbY5a, page 53**

Q.3  Read: **Draw a diagram using + 1 and – 1 for each operation. Fill in the missing number.**

Less able Ps could manipulate cash and debt cards on desks first before writing the operations. Set a time limit.

Review with whole class. Ps come to BB to draw (stick on) cash and debt symbols and write the operation. Class agrees/disagrees. Mistakes discussed and corrected.

Ps tell word problems about each operation. Class decides whether they match the given operations.

**Solution:**

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</table>

a)  5 + (– 4) = 1

b)  – 3 + (+ 3) = 0

c)  3 + (– 8) = –5

35 min

8  

**PbY5a, page 53**

Q.4  Read: **Fill in the missing numbers.**

Deal with 1 row at a time. Set a time limit. Ps can draw cash and debt symbols in Ex. Bks or manipulate cards on desks, or use number line to help them.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning using cash and debt. Class agrees/disagrees. Mistakes discussed and corrected. Show with models or on number line only if problems or disagreement.

**Solution:**

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</tbody>
</table>

a)  7 + (– 6) = 1  
b)  – 10 + (+ 10) = 0  
c)  15 + (– 13) = 2

d)  8 + (– 8) = 0  
e)  – 8 + (– 3) = – 11  
f)  12 + (– 7) = 5

g)  y + (– 12) = – 20  
h)  x + x + (– 5) = 15

i)  – 100 + 10 = – 90

40 min

9  

**PbY5a, page 53, Q.5**

a)  Read: **Solve the problem by writing an addition.**

   Sue has won £200 but she owes £100. What is her balance?

   Show me . . . now! (£100)  

   BB:  £200 + (– £100) = £100

b)  Read: **Work out the answer to the addition.**

   (Think of it as ‘cash and ‘debt’.)

   Show me . . . now! (– 100)  

   BB: (+ 150) + (– 250) = – 100

Ps think of contexts for the operation. Class decides whether they match the addition.

45 min

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Lesson Plan

54

Investigation

Whole class activity
Written on BB or SB or OHT
T has large number line for demonstration and Ps have individual number lines on desks.

In good humour!
T repeats vague explanations more clearly.
Praising, encouraging only
Draw or stick cash and debt symbols on BB if necessary.

Uncle Jim had £150, then he spent £180. Is it possible?
Show me what you think ... now!
T asks Ps with opposite opinions to explain their reasoning. e.g.
P: It is impossible, as 150 < 180.
P: It is possible, as he could have borrowed the extra £30.
P: It is possible, as he could have had £150 in his bank account, then paid £180 by credit card and now has a balance of £30.

BB: Had: £150
Spent: £180
Borrowed: £30
So has debt: £ -30

Agree that Uncle Jim is left with a debt of £30.

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Reading information from a diagram

Look at this vertical number line. What do you notice? (There are three dots marked and labelled with letters.) What could the letters represent? (e.g. temperatures, money, heights above sea level, capacity, etc.) Ps suggest values for each letter in each of the suggested contexts. e.g. B = 3°C, C = 11°C, K = 14°C, or B = £3, C = £11, etc.

What can you tell me about their relationship with one another? e.g. K is 3 units more than C; C is 8 units more than B; B is 11 units less than K, etc.

Who can think of an operation about them? e.g. 3 + 8 = 11, etc.

A, come and mark the point S on the number line if its value is –2. Is A correct? What can you tell me about S? (e.g. it is 2 units less than 0; it is 4 units less than B; it is 13 units less than K, etc.)

Does the presence of this negative value affect the possible contexts? (e.g. the values cannot be capacity, as you cannot have a negative capacity)

Who can think of a real-life context for all these values? Ps come to BB and tell class their contexts, referring to the diagram where relevant. Class decides whether or not it is a good context and if not, why not.

### Missing numbers

Which numbers are missing from the equations? Draw a diagram to help you in your Ex. Bks. (cash and debt symbols or number line)

Ps come to BB to fill in the missing number, explaining reasoning in context and drawing a suitable diagram. Class agrees/disagrees.

**BB:**

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**a)** $-5 + 1 + (+3) = -3$  e.g. [1 1 1 1 1]

**b)** $8 + (-5) = +3$  e.g. [2 2 2 2 2 2 2 2 2]

**c)** $-4 + (-2) = -6$  e.g. [3 3 3 3 3 3 3 3 3]

### Car model

T has BB already prepared and a car model cut from cardboard or wood.

**BB:**

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<th>8</th>
<th>9</th>
<th>10</th>
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<th>12</th>
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*a)* The car is at 0 and faces the house. It moves 3 units ahead, then another 5 units ahead. Where does it end up?

Let's write an addition about it. Ps dictate what T should write.

**BB:** $0 + 3 + (+5) = +8$  or  $0 + 3 + 5 = 8$ (omitting positive signs)

Or I could have said:

*The car is at +3 and faces the house. It moves 5 units to the right. Where does it end up?*

Ps dictate the addition and agree that result is the same.

**BB:** $+3 + (+5) = +8$  or  $3 + 5 = 8$ (omitting positive signs)
b) BB: The car is at 0 and faces the tree. It moves 3 units ahead, then another 5 units ahead. Where does it end up?
Let's write an addition about it. Ps dictate what T should write.
BB: \(0 + (-3) + (-5) = -8\) or \(-3 + (-5) = -8\) (as the first move from zero does not need to be written down)

c) BB: The car is at +3 and faces the tree. It moves 5 units ahead. Where does it end up?
Let's write an addition about it. Ps dictate what T should write.
BB: \(+3 + (-5) = -2\) or \(3 + (-5) = -2\)

Discuss what the rules are when using the little cars. Elicit that:
• When we add a positive number to any number, the car faces the house (right) and goes forward.
• When we add a negative number to any number, the car faces the tree (left) and goes forward.

[If there is time, allow Ps to demonstrate other movements at BB, e.g. \(P_1\) gives the instructions, \(P_2\) moves the car and \(P_3\) writes the addition.]

T gives the instructions. P moves car on BB and rest of Ps move own cars if they have them.
Discussion, agreement on the matching operations.

Ps need not write this down but encourage them to try to memorise it.

Or if Ps have own models, paired practice, monitored, helped

Individual work, monitored, helped
(or whole class activity if Ps are still unsure)
Drawn on BB or use enlarged copy masters or OHP from Activity 6.
Demonstration, agreement, self-correction, praising
BB:
Q.2 Read: Use the idea of the car moving along the number line to help you calculate these sums.

Ps use own car models if they have them, or use the number line in Q.1. Encourage mental calculation if Ps are able.

Set a time limit. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees.

Mistakes discussed and corrected. Demonstrate movements with car model (or use money model too if problems).

Solution:

a) \((-5) + (+7) = 2\)  

b) \((+6) + (−8) = −2\)

c) \((-3) + (+3) = 0\)  
d) \((-5) + (−2) = −7\)

e) \((+6) + (+3) = 9\)  
f) \((+4) + (−4) = 0\)

36 min

Q.3 Read: Use the number lines to help you calculate the sums.

Deal with one part at a time.

a) What must I do to my number lines on the BB to make them the same as the number lines in part a) in your Ps? (Move the bottom number line 5 units to the right, i.e. + 5 units)

How can it help us do the calculations? T gives hints or explains if Ps cannot see it. (One term in each addition in part a) is + 5, which we have just done by moving the bottom number line 5 units to the right. So if we find the other term on the bottom number line, the sum is the number directly above it on the top number line.)

T demonstrates with the given sum and Ps find numbers in diagram in Ps too. Who does not understand how to use the diagram? If necessary, ask a P who does understand to demonstrate another addition.

Continue as individual work under a time limit.

Review with whole class. Ps dictate results to T, saying the whole addition. Class agrees/disagrees. If problems, Ps demonstrate on number lines on BB. Mistakes corrected.

Extra praise if Ps noticed that the 2nd terms and sums are decreasing by 1 if worked through down the columns instead of across the rows.

b) T moves the two number lines on BB so that they match again.

What must I do the number lines to make them the same as those in part b)? (Move bottom number line 3 units to the left, i.e. − 3 units.) T does so and Ps explain and demonstrate how to use them. (One term in each addition is − 3, which we have already done, so find 2nd term on bottom number line and the number directly above it is the sum.) Continue as in a).
(Continued)

Solution:

(a) \[(+ 5) + (+ 6) = \quad (+ 5) + (+ 1) = \quad (+ 5) + (– 4) = \quad 11\]  
\[(+ 5) + (+ 5) = \quad (+ 5) + 0 = \quad (+ 5) + (+ 5) = \quad 0 \]
\[(+ 5) + (+ 4) = \quad (+ 5) + (– 1) = \quad (+ 5) + (– 6) = \quad – 1\]
\[(+ 5) + (+ 3) = \quad (+ 5) + (– 2) = \quad (+ 5) + (– 7) = \quad – 2\]
\[(+ 5) + (+ 2) = \quad (+ 5) + (– 3) = \quad (+ 5) + (– 8) = \quad – 3\]

(b) \[\quad (+ 3) + (+ 5) = \quad (+ 3) + (+ 4) = \quad (+ 3) + (+ 3) = \quad (+ 3) + (+ 2) = \quad (+ 3) + (+ 1) = \quad 1\]  
\[\quad (+ 3) + (+ 5) = \quad (+ 3) + (+ 4) = \quad (+ 3) + (+ 3) = \quad (+ 3) + (+ 2) = \quad (+ 3) + (+ 1) = \quad 0\]
\[\quad (+ 3) + 0 = \quad (+ 3) + (– 1) = \quad (+ 3) + (– 2) = \quad (+ 3) + (– 3) = \quad (+ 3) + (– 4) = \quad – 4\]
\[\quad (– 3) + (– 3) = \quad (– 3) + (– 2) = \quad (– 3) + (– 1) = \quad (– 3) + 0 = \quad (– 3) + (+ 1) = \quad 0\]
\[\quad (– 3) + (– 3) = \quad (– 3) + (– 2) = \quad (– 3) + (– 1) = \quad (– 3) + 0 = \quad (– 3) + (+ 1) = \quad 0\]

Whole class activity
Discussion, reasoning, checking with values in Pbs, agreement, praising

If we let \(x\) be any number on the bottom number line (i.e. the 2nd term) and \(y\) be any number on the top number line (i.e. the sum), who can write an equation about each pair?

Ps come to BB or dictate to T. Who agrees? Who could write it a different way? etc.

BB:

(a) \(y = (+ 5) + x\), or \(y = 5 + x\) \[or \ y = x + 5\]

(b) \(y = (– 3) + x\), or \(y = – 3 + x\) \[or \ y = x – 3\]

Ps could make graphs for a) and b) in Lesson 55, either as a whole class activity, with T drawing axes on BB and marking some points before Ps come to BB, or as individual work in Ex. Bks. or on worksheets.

(a) \(y = 5 + x\)

(b) \(y = – 3 + x\)
Tables and calculation practice, activities, consolidation

[Extension to Lesson 54, Activity 8]

PbY5a, page 55

Solutions:

Q.1

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a) $+3 + (-1) = \boxed{2}$
b) $-1 + (-2) = \boxed{-3}$
c) $-3 + (-2) = \boxed{-5}$
d) $+3 + (-3) = \boxed{0}$
e) $0 + (-3) = \boxed{-3}$
f) $+2 + (-3) = \boxed{-1}$
g) $-1 + 0 = \boxed{-1}$

Q.2

a) $256 + 137 + 44 + 64 = (256 + 44) + (137 + 63 + 1)$
   $= 300 + 201 = \boxed{501}$
b) $125 + 49 + 151 + 50 = (125 + 50) + (49 + 151)$
   $= 175 + 200 = \boxed{375}$
c) $43 + 291 + 69 + 17 = (43 + 17) + (291 + 9 + 60)$
   $= 60 + 360 = \boxed{220}$
d) $1324 + 9999 + 1001 = 1324 + (9999 + 1 + 1000)$
   $= 1324 + 11000 = \boxed{12324}$

Q.3

a) $\begin{array}{c}
+3 \\
+2 \\
+3 \\
\hline
15 \\
\end{array}$
b) $\begin{array}{c}
+3 \\
+5 \\
+3 \\
\hline
15 \\
\end{array}$
c) $\begin{array}{c}
+2 \\
+3 \\
+4 \\
\hline
10 \\
\end{array}$
d) $\begin{array}{c}
+3 \\
+4 \\
+1 \\
\hline
18 \\
\end{array}$

Accept vertical calculations too.

Q.4

a) $472 + 123 - 172 = (472 - 172) + 123 = 300 + 123 = \boxed{423}$
b) $89 + 111 - 27 + 30 = (89 + 111) + (30 - 27)$
   $= 200 + 3 = \boxed{203}$
c) $216 - 90 - 66 + 39 = (216 + 39) - (90 + 66)$
   $= 255 - 156 = 100 - 1 = \boxed{99}$
d) $426 + 117 - 125 - 67 = (426 - 125) + (117 - 67)$
   $= 301 + 50 = \boxed{351}$
e) $1725 + 310 - 525 + 90 = (1725 - 525) + (310 + 90)$
   $= 1200 + 400 = \boxed{1600}$

Q.5

a) $\begin{array}{c}
-4 \\
-5 \\
-2 \\
\hline
-11 \\
\end{array}$
b) $\begin{array}{c}
-3 \\
-1 \\
-1 \\
\hline
-5 \\
\end{array}$
c) $\begin{array}{c}
-3 \\
-6 \\
-3 \\
\hline
-12 \\
\end{array}$
d) $\begin{array}{c}
-3 \\
-6 \\
-3 \\
\hline
-12 \\
\end{array}$

Accept vertical calculations too.
Y5

Activity

1

Addition of integers 1

R: Mental calculation. Models for addition of integers
C: Constructional models. Comparison, subtraction as difference
E: Preparation for subtraction of integers

Lesson Plan

56

Week 12

Notes

Whole class activity
Use copy masters for car model in LP 55/6 or draw diagram on BB ro SB or OHT
(If not using car model, draw number lines with arrows to show the additions.)
Discussion, reasoning, agreement, praising
Agree that the first move, 0 + (+ 5), can be ignored – the car could be thought of as starting at + 5 facing the tree.
[Use cash and debt cards stuck to BB or draw symbols on BB if Ps have difficulty.]

T has sentences written on BB or SB or OHT with underlined words missing or covered up.
Ps read the sentences in unison, adding the missing words
Or Ps demonstrate movements of car and choose Ps to write additions and also explain in cash and debt context.

Whole class activity
Written on BB or SB or OHT
Involve several Ps.
Discussion, reasoning, agreement, praising
T shows orders not suggested by Ps.
Which do you think is the easiest order? Why?
[It is probably easiest to match up positive and negative terms which cancel each other out (i.e. sum to zero) but T should stress that all the methods are correct and acceptable.]

10 min
Activity

3

**Missing terms**

Which numbers are missing from the additions? P's come to BB to explain reasoning and write numbers. Class agrees/disagrees. P's show on number line or use car or cash and debt models.

BB:

a) \(-7 + (−3) = −10\)  
e) \(-6 + 4 = −2\)  

b) \(+5 + 7 = 12\)  
f) \(-3 + (+10) = +7\)  

c) \(+8 + (−5) = +3\)  
g) \(−8 + 5 = −3\)  

d) \(+2 + (−9) = −7\)  
h) \(+8 + (−8) = 0\)  

**Comparison**

a) Compare the two temperatures. Which is higher and by how many degrees?

P's find temperatures on thermometer models or diagrams, then come to BB to write missing signs and differences. Class agrees/disagrees. How would we normally check a horizontal subtraction? (with addition, starting at the result) T writes operations with P's help.

BB:  

\[\begin{align*}
5 ^\circ C &\quad 2 ^\circ C &\quad 3 ^\circ C\\
3 ^\circ C &\quad 5 ^\circ C &\quad 7 ^\circ C\\
\text{5 is 3 more than 2, so } +5 - (+2) = +3 &\quad \text{Check: } +3 + (+2) = +5\\
\text{5 is 7 more than -2, so } +5 - (-2) = +7 &\quad \text{Check: } +7 + (-2) = +5\\
\text{5 is 3 more than -4, so } -1 - (-4) = +3 &\quad \text{Check: } +3 + (-4) = -1\\
\text{-3 equals -3} &\quad \text{Check: } 0 + (-3) = -3
\end{align*}\]
(Continued)
b) We could also do the calculations the other way round. T explains.

i) \(2^\circ C\) is less than \(5^\circ C\) by \(3^\circ C\), so \(+2 - (+5) = -3\)
   Check: \(-3 + (+5) = +2\)

ii) \(-2^\circ C\) is less than \(5^\circ C\) by \(7^\circ C\), so \(-2 - (+5) = -7\)
   Check: \(-7 + (+5) = -2\)

iii) \(-4^\circ C\) is less than \(-1^\circ C\) by \(3^\circ C\), so \(-4 - (-1) = -3\)
   Check: \(-3 + (-1) = -4\)

Agree that iv) can only be done one way.
What do you notice about the differences done this way?
Elicit that if you subtract the larger number from the smaller number, the difference is the opposite of subtracting the smaller number from the larger.

---

5 Difference of integers

Where are +4 and +1 on this number line? Ps come to BB to mark them. Which is more? (+4) How much more? (+3)
Let's write a subtraction about it. Ps come to BB or dictate to T.
Which number is the subtrahend (amount being subtracted)? Which number is the reductant (amount being subtracted from)? Ps come to BB to stick name arrows below appropriate numbers, draw an arrow between them and write the difference.

\[
\begin{align*}
\text{BB:} & \\
\text{T:} & \text{We can read the difference by drawing an arrow from the subtrahend to the reductant.} \\
\text{Repeat for some other subtractions, with T helping only if necessary.} \\
-4 - (+2) & = -6, \quad -5 - (-2) = -7, \quad +2 - (-4) = +6, \text{ etc.}
\end{align*}
\]

---

6 PbY5a, page 56

Q.1 a) Read: Find the reductant and subtrahend on the number line. Read the difference.

Set a time limit or deal with one part at a time.
Review with whole class. Ps come to BB to demonstrate and complete subtractions. Class agrees/disagrees. T shows how to check with reverse addition, also referring to number line. Mistakes discussed and corrected.

\[
\begin{align*}
\text{Solution:} & \\
i) & 8 - (+3) = +5, \quad 8 - 0 = +8, \quad 4 - (-2) = +6, \quad 0 - (-5) = +5 \\
ii) & +3 - (+8) = -5, \quad 0 - (+8) = -8, \quad -2 - (+4) = -6, \quad -5 - 0 = -5.
\end{align*}
\]

---

Notes
T demonstrates the reverse comparisons on large model or diagram, e.g.

BB: \(9^\circ C\) T explains reasoning clearly, and elicits Ps' help when they understand.
Ps copy subtractions and inverse additions in Ex.Bks.
e.g. \(+5 - (+2) = +3\)
\(+2 - (+5) = -3\)

Whole class activity
Number line drawn on BB and name arrows cut from card, or use enlarged copy master or OHP
Ps could have smaller version on desks too.

BB: \(+4 - (+1) = +3\)
Check: \(+3 + (+1) = +4\)
Discussion, reasoning, checking, agreement, praising

Ps repeat the rule in unison.

Ask Ps to say what they are doing in a loud voice.

Individual work, monitored, helped
(or whole class activity if Ps are unsure)
Number line drawn on BB or use enlarged copy master or OHP from LP 56/5 or class number line.
Reasoning, agreement, self-correction, praising
Ps repeat T's checking addition in unison.
Lesson Plan 56

### Activity 6 (Continued)

b) Read: *Compare the two numbers. Which is more? How many more?*

Set a time limit. Ps find the two numbers on number line, note which is more and fill in the missing signs and differences. Review with whole class. Ps come to BB to explain reasoning, drawing arrows from subtrahend to redundant on number line. Class agrees/disagrees. Mistakes discussed and corrected.

How did you find the difference? (Subtracted the smaller number from the greater number) Let’s see what the differences would be if we start at the greater number and draw an arrow to the smaller number, so we make the greater number the subtrahend. P comes to BB to demonstrate, with T’s help. Elicit that the result is the opposite of the previous difference.

e.g. BB: +8 – (+3) = +5 but +3 – (+8) = –5

**Solution:**

<table>
<thead>
<tr>
<th>i)</th>
<th>ii)</th>
<th>iii)</th>
<th>iv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>+8</td>
<td>&gt;3</td>
<td>&gt;2</td>
<td>&gt;8</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

34 min

### Notes

Individual work, monitored, helped (or whole class activity if Ps had difficulty with part a)
Written on BB or SB or OHT
Reasoning, agreement, self-correcting, praising

---

**Lesson Plan 56**

Whole class discussion
BB: ![Number Line Diagram]

Agree that:
If we subtract a greater number from a smaller number, the difference is the opposite of the result of subtracting the smaller number from the greater number.

---

7 **PbY5a, page 56**

Q.2 Read: *Write a subtraction to work out the difference, then check it with an addition.*

Go through part a) with the whole class first. Ps come to BB to point to 3 and – 6 on the number line and count the units between them. Then they fill in the difference and explain the subtraction and the checking addition.

Who does not understand what to do? (If there is a substantial number, continue as a whole class activity until they do.) Follow the pattern and see how many you can do in 3 minutes!

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Show on number line if problems.

**Solution:**

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
<th>f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3°C</td>
<td>–6°C</td>
<td>So</td>
<td>3 – (–6) = 9</td>
<td>Check:</td>
<td>9 + (–6) = 3</td>
</tr>
<tr>
<td>9</td>
<td>8</td>
<td>7</td>
<td>6</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>9°C</td>
<td>8°C</td>
<td>So</td>
<td>–6 – (+3) = –9</td>
<td>Check:</td>
<td>–9 + (+3) = –6</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>–1</td>
</tr>
<tr>
<td>So</td>
<td>4 – (+7) = –3</td>
<td>Check:</td>
<td>–3 + (+7) = 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>–1</td>
<td>–2</td>
</tr>
<tr>
<td>3°C</td>
<td>4°C</td>
<td>So</td>
<td>+7 – (+4) = +3</td>
<td>Check:</td>
<td>+3 + (+4) = +7</td>
</tr>
<tr>
<td>–9</td>
<td>–8</td>
<td>–7</td>
<td>–6</td>
<td>–5</td>
<td>–4</td>
</tr>
<tr>
<td>So</td>
<td>–8 – (–2) = –6</td>
<td>Check:</td>
<td>–6 + (–2) = –8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>So</td>
<td>–2 – (–8) = +6</td>
<td>Check:</td>
<td>+6 + (–8) = –2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

38 min

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<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y5</strong></td>
</tr>
</tbody>
</table>

**Lesson Plan 56**

PbY5a, page 56

Q.3 Read: *Do the subtractions, then check with an addition.*

T advises Ps to compare the two numbers. Which is more? How many more? Is it the bigger number the reductant or the subtrahend?

Deal with part a), then part b). Set a time limit.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. If problems, show on number line.

**Solution:**

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
<th>Check:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3 - (+4))</td>
<td>(-1)</td>
<td>((-1) + (+4) = +3)</td>
</tr>
<tr>
<td>(3 - (+3))</td>
<td>(0)</td>
<td>((0) + (+3) = +3)</td>
</tr>
<tr>
<td>(3 - (+2))</td>
<td>(+1)</td>
<td>((+1) + (+2) = +3)</td>
</tr>
<tr>
<td>(3 - (+1))</td>
<td>(+2)</td>
<td>((+2) + (+1) = +3)</td>
</tr>
<tr>
<td>(3 - 0)</td>
<td>(3)</td>
<td>(+3 + 0 = +3)</td>
</tr>
<tr>
<td>(3 - (-1))</td>
<td>(+4)</td>
<td>((+4) + (-1) = +3)</td>
</tr>
</tbody>
</table>

**b)** \((-3) - (+1)\) = \(-4\)  
Check: \(-4 + (+1)\) = \(-3\)

<table>
<thead>
<tr>
<th>Expression</th>
<th>Result</th>
<th>Check:</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-3) - 0)</td>
<td>(-3)</td>
<td>((-3) + 0 = -3)</td>
</tr>
<tr>
<td>((-3) - (-1))</td>
<td>(-2)</td>
<td>((-2) + (-1) = -3)</td>
</tr>
<tr>
<td>((-3) - (-2))</td>
<td>(-1)</td>
<td>((-1) + (-2) = -3)</td>
</tr>
<tr>
<td>((-3) - (-3))</td>
<td>(0)</td>
<td>((0) + (-3) = (-3))</td>
</tr>
<tr>
<td>((-3) - (-4))</td>
<td>(+1)</td>
<td>((+1) + (-4) = -3)</td>
</tr>
</tbody>
</table>

What do you notice? (The subtrahends are decreasing by 1, so the sums are increasing by 1.)

**Notes**

- Individual work, monitored, helped
- Written on BB or SB or OHT
- Differentiation by time limit
- Ps use number line to help them.
- Reasoning, agreement, self-correction, praising
- T could ask Ps to write and say some operations without the positive signs. e.g.
  - a) \(3 - 4 = -1\)
    \(3 - 3 = 0,\) etc
  - Agree that the positive sign in front of a number is not really needed, as numbers without signs are positive, but the + sign as an operation sign **cannot** be omitted.

Feedback for T

**[Note to Ts only]**

Do not think that Ps have learned the subtraction of integers just because they have obtained the correct results – they could just have followed the pattern!"
### Y5
**Activity**

#### 1 Find the mistakes

I did these calculations in a hurry and have not had time to check them yet. Help me to find any mistakes and correct them.

BB:

- a) \(-7 + (-12) = \not{19}\)
- b) \(+6 + (-5) + (-1) = 0\)
- c) \(+8 + (-3) + (-5) = 0\)
- d) \([-5] + (-7) + (+6) = -6\)
- e) \(+1 + (-5) + (+5) = 1\)
- f) \(-4 + [-12] + (+4) = +16\)

Ps come to BB to do calculations mentally and either tick the operation or cross out and correct the mistake. Class agrees/disagrees or suggests an easier grouping.

*E.g.* c) \(+8 + [(-3) + (-5)] = +8 + (-8) = 0\)

Discuss opposite numbers, absolute values and how the order of terms in an addition does not matter as long as the sign in front of a number stays the same.

**Notes**

- **Lesson Plan**
  - Week 12
  - **Notes**
    - Whole class activity
    - Written on BB or SB or OHT
    - At a good pace

- **Activity**
  - **Find the mistakes**
    - I did these calculations in a hurry and have not had time to check them yet. Help me to find any mistakes and correct them.
    - BB:
      - a) \(-7 + (-12) = \not{19}\)
      - b) \(+6 + (-5) + (-1) = 0\)
      - c) \(+8 + (-3) + (-5) = 0\)
      - d) \([-5] + (-7) + (+6) = -6\)
      - e) \(+1 + (-5) + (+5) = 1\)
      - f) \(-4 + [-12] + (+4) = +16\)
    - Ps come to BB to do calculations mentally and either tick the operation or cross out and correct the mistake. Class agrees/disagrees or suggests an easier grouping.
    - *E.g.* c) \(+8 + [(-3) + (-5)] = +8 + (-8) = 0\)
    - Discuss opposite numbers, absolute values and how the order of terms in an addition does not matter as long as the sign in front of a number stays the same.
  - **Notes**
    - **Activity**
      - **Find the mistakes**
        - I did these calculations in a hurry and have not had time to check them yet. Help me to find any mistakes and correct them.
        - BB:
          - a) \(-7 + (-12) = \not{19}\)
          - b) \(+6 + (-5) + (-1) = 0\)
          - c) \(+8 + (-3) + (-5) = 0\)
          - d) \([-5] + (-7) + (+6) = -6\)
          - e) \(+1 + (-5) + (+5) = 1\)
          - f) \(-4 + [-12] + (+4) = +16\)
        - Ps come to BB to do calculations mentally and either tick the operation or cross out and correct the mistake. Class agrees/disagrees or suggests an easier grouping.
        - *E.g.* c) \(+8 + [(-3) + (-5)] = +8 + (-8) = 0\)
        - Discuss opposite numbers, absolute values and how the order of terms in an addition does not matter as long as the sign in front of a number stays the same.

#### 2 Addition of integers: car model

a) P comes to BB and T gives P instructions on how to move the car. The little car is standing on –3 and faces the tree. It moves forward 4 units then turns to face the house and moves forward 11 units. Where does it end up? Show me . . . now! (4 or +4)

BB:

- Write an addition about it in your Ex. Bks.

BB: \(-3 + (-4) + (+11) = +4\)

b) Ps describe own movements of the car (with car always moving forward) along the number line. Class writes additions in Ex. Bks (or calculate mentally) and show result on command

Ps answering correctly dictate operations to T (or come to BB).

**Notes**

- **Activity**
  - **Addition of integers: car model**
    - a) P comes to BB and T gives P instructions on how to move the car. The little car is standing on –3 and faces the tree. It moves forward 4 units then turns to face the house and moves forward 11 units. Where does it end up? Show me . . . now! (4 or +4)
    - BB:
      - Write an addition about it in your Ex. Bks.
    - BB: \(-3 + (-4) + (+11) = +4\)
    - b) Ps describe own movements of the car (with car always moving forward) along the number line. Class writes additions in Ex. Bks (or calculate mentally) and show result on command
    - Ps answering correctly dictate operations to T (or come to BB).
  - **Notes**
    - **Activity**
      - **Addition of integers: car model**
        - a) P comes to BB and T gives P instructions on how to move the car. The little car is standing on –3 and faces the tree. It moves forward 4 units then turns to face the house and moves forward 11 units. Where does it end up? Show me . . . now! (4 or +4)
        - BB:
          - Write an addition about it in your Ex. Bks.
        - BB: \(-3 + (-4) + (+11) = +4\)
        - b) Ps describe own movements of the car (with car always moving forward) along the number line. Class writes additions in Ex. Bks (or calculate mentally) and show result on command
        - Ps answering correctly dictate operations to T (or come to BB).

#### 3 Addition of integers: cash and debt model

a) Think of these additions as cash and debt, with the answer as the balance. Ps come to BB to describe the addition in context and to fill in the missing result. Class agrees/disagrees. (Use cash and debt cards if problems or draw circles and rectangles on BB).

BB:

- i) \(+4 + (-11) = \not{-7}\)
- ii) \(-8 + (+8) = 0\)
- iii) \(-4 + (-3) + (+4) + (+9) = 6\)
  - (e.g. £7 in debt, £13 in cash, balance: £6)

b) Ps write own additions on BB, describe them as debt and cash and choose Ps to calculate their balances. Class agrees/disagrees.

**Notes**

- **Activity**
  - **Addition of integers: cash and debt model**
    - a) Think of these additions as cash and debt, with the answer as the balance. Ps come to BB to describe the addition in context and to fill in the missing result. Class agrees/disagrees. (Use cash and debt cards if problems or draw circles and rectangles on BB).
    - BB:
      - i) \(+4 + (-11) = \not{-7}\)
      - ii) \(-8 + (+8) = 0\)
      - iii) \(-4 + (-3) + (+4) + (+9) = 6\)
        - (e.g. £7 in debt, £13 in cash, balance: £6)
    - b) Ps write own additions on BB, describe them as debt and cash and choose Ps to calculate their balances. Class agrees/disagrees.
  - **Notes**
    - **Activity**
      - **Addition of integers: cash and debt model**
        - a) Think of these additions as cash and debt, with the answer as the balance. Ps come to BB to describe the addition in context and to fill in the missing result. Class agrees/disagrees. (Use cash and debt cards if problems or draw circles and rectangles on BB).
        - BB:
          - i) \(+4 + (-11) = \not{-7}\)
          - ii) \(-8 + (+8) = 0\)
          - iii) \(-4 + (-3) + (+4) + (+9) = 6\)
            - (e.g. £7 in debt, £13 in cash, balance: £6)
        - b) Ps write own additions on BB, describe them as debt and cash and choose Ps to calculate their balances. Class agrees/disagrees.
### Activity 4  

**Subtraction of integers: temperature**

T describes a temperature change and Ps write a subtraction about it in Ex. Bks. A, come and show us what you wrote. Who agrees? Who could write it another way? (i.e. with and without the positive signs).

Show on model thermometer or vertical number line drawn on BB.

a) The temperature was 11°C at 2.00 pm but by 8.00 pm it had fallen by 7°C. What was the temperature at 8.00 pm?

BB: + 11°C – (+ 7°C) = + 4°C or 11°C – 7°C = 4°C

b) The temperature was 11°C and fell by 14°C. What was the temperature then?

BB: + 11°C – (+ 14°C) = −3°C or 11°C – 14°C = −3°C

c) The temperature was −2°C and fell by 3°C. What was the temperature then?

BB: −2°C – (+ 3°C) = −5°C or −2°C − 3°C = −5°C

---

### Activity 5  

**Cash and debt: addition and subtraction**

a) Sue’s balance is £7. How much cash and how much debt could she have?

In your Ex. Bks, show at least 3 different possible solutions by drawing cash and debt symbols and writing matching additions. Review quickly with whole class. Ps come to BB to stick (draw) symbols and write possible additions. Class points out errors.

BB: e.g.  

\[
\begin{align*}
\text{£1} & \quad \text{or} \quad \text{£1} + (\text{−} \text{£1}) \quad \text{or} \quad \text{£1} + (\text{−} \text{£1}) + (\text{−} \text{£1}) \\
8 + (\text{−} 1) &= 7 \\
9 + (\text{−} 2) &= 7 \\
10 + (\text{−} 3) &= 7
\end{align*}
\]

b) Charlie had £12, then he spent £7. How much did he have left?

Ps come to BB to draw symbols and write subtraction, explaining reasoning. Class agrees/disagrees. Ps write subtractions in Ex. Bks.

BB:  

\[
\begin{align*}
\text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} \quad \text{or} \quad \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} \quad \text{or} \quad \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} \\
12 – 7 &= \text{£5} \quad \text{or} \quad + 12 – (+ 7) = + \text{£5}
\end{align*}
\]

c) Bob’s balance was −£8. Then the bank cancelled a debt of £3. What is Bob’s balance now?

As b).

BB:  

\[
\begin{align*}
\text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} + \text{£1} \\
\text{−} 8 – (\text{−} 3) &= \text{−} 5
\end{align*}
\]

---

**Notes**

Individual trial in Ex. Bks, monitored  
If possible, Ps have own thermometer models or diagrams on desks, otherwise Ps use vertical number line on Pb Y5a, page 56.  
(Or Ps could show final temperature on scrap paper or slates in unison and Ps answering correctly come to BB to explain.)

Reasoning, agreement, self-correction, praising

Agree that + signs in front of positive numbers are not really needed.

Individual work, monitored  
Quick discussion, reasoning, agreement, self-correction, praising

Agree that the possibilities are infinite.

Whole class activity  
(or individual trial in Ex. Bks first if Ps wish)

Reasoning, agreement, praising

Agree that Charlie had less money than before, so his balance has worsened.

Agree that Bob has more money than before, so his balance has improved.
### Activity 6

**PbY5a, page 57**

**Q.1** Read: *Draw diagrams using + 1 and – 1 to model each problem, then write the operation.*

Deal with one at a time. Set a time limit. Ps read problems themselves, amend/draw diagram and write operation.

Review with whole class. Ps come to BB to explain solution. Class agrees/disagrees. Mistakes discussed and corrected.

Let’s check it with an addition. Ps dicate to T.

**Solution:**

a) **Paula had £7, then she spent £6. How much did she have left?**

BB: \[\begin{array}{c}
+7 \\
\hline
\end{array}\]

\[+ 7 – (+ 6) = + 1 \checkmark\]

Check: \[+ 1 + (+ 6) = + 7 \checkmark\]

b) **Roy owed £7, then £4 of his debt was cancelled. What is his balance now?**

BB: \[\begin{array}{c}
-7 \\
\hline
\end{array}\]

\[-7 – (– 4) = – 3 \checkmark\]

Check: \[– 3 + (– 4) = – 7 \checkmark\]

c) **Lee had £3, then he spent £3. What is his balance now?**

BB: \[\begin{array}{c}
0 \\
\hline
\end{array}\]

\[+ 3 – (+ 3) = 0 \checkmark\]

Check: \[0 + (+ 3) = + 3 \checkmark\]

d) **Tina was £4 in debt, then £4 of her debt was cancelled. What is her balance now?**

BB: \[\begin{array}{c}
0 \\
\hline
\end{array}\]

\[-4 – (– 4) = 0 \checkmark\]

Check: \[0 + (– 4) = – 4 \checkmark\]

**30 min**

### Notes

Individual work, monitored, helped

Reasoning, checking, agreement, self-correction, praising

(T could have cash and debt cards stuck to BB for Ps to manipulate.)

or \[7 – 6 = \frac{1}{\checkmark}\]

\[\frac{1}{\checkmark} + 6 = 7\]

Show on number line too.

T stresses the idea of cash and debt being ‘taken away’.

Elicit that:

- taking away cash means that the balance is less.
- taking away debt means that the balance is more.

Discuss how debt can be cancelled in real life, e.g. buying an item in a shop, taking it back later on and getting your money back.

---

### Activity 7

**PbY5a, page 57**

**Q.2** Read: *Draw diagrams using + 1 and – 1 to help you work out the differences.*

Set a time limit. Review with whole class. Ps come to BB to draw (stick on) symbols, write differences and check with an addition. Class agrees/disagrees. Mistakes discussed/corrected.

T asks Ps to give a word problem for each subtraction.

**Solution:**

a) \[(+ 6) – (+ 4) = + 2\]

Check: \[+ 2 + (+ 4) = + 6\]

b) \[– 6 – (– 4) = – 2\]

Check: \[– 2 + (– 4) = – 6\]

c) \[(+ 5) – (+ 5) = 0\]

Check: \[0 + (+ 5) = + 5\]

d) \[– 6 – (– 6) = 0\]

Check: \[0 + (– 6) = – 6\]

**34 min**
Q.3 Read: Fill in the missing amounts in the questions. Solve them in your exercise book.

Less able Ps can use cash and debt cards on desk. Deal with one at a time (or set a time limit if class is very able). Review with whole class. Ps come to BB to explain solution. Class agrees/disagrees. Mistakes discussed and corrected. Show long form of the operation (i.e. with positive signs) if Ps do not.

Let’s check it with the inverse operation. Ps come to BB or dictate what T should write.

Solution:

a) Sue’s starting balance was £2, as she had £5 in cash and was £3 in debt. Then she spent £5.

How much is her balance now?

BB: + £2 – (+ £5) = – £3

Check: – £3 + (+ £5) = + £2

b) Rob’s starting balance was – £3, as he had £2 in cash and was £5 in debt. Then he spent £2.

How much is his new balance?

BB: – £3 – (+ £2) = – £5

Check: – £5 + (+ £2) = – £3

c) Billy’s starting balance was – £3, as he had £1 in cash and was £4 in debt. Then £4 of his debts were cancelled.

How much is his balance now?

BB: – £3 – (– £4) = + £1

Check: £1 + (– £4) = – £3

d) Mary’s starting balance was £2, as she had £5 in cash and was £3 in debt. Then £3 of her debts were repaid.

What is her balance now?

BB: + £2 – (– £3) = + £5

Check: + £5 + (– £3) = + £2

Individual work, monitored, helped (or whole class activity if Ps are still unsure, or completing sentences with whole class them individual calculation)

Written on BB or use enlarged copy master or OHP (Ps can draw circles and rectangles in Ex. Bks. if they have no cards to manipulate.)

Discussion, reasoning, checking, agreement, self-correction, praising

Ps decide whether each new balance is better or worse than the old one.

Feedback for T

Note to Ts: Cash and debt is not the usual preferred model for problems such as c) and d) but it can be used if the initial balance is shown as the sum of appropriate amounts of cash and debt, i.e. the balance consists of at least as much debt as is being taken away]
**Activity 9**

*PbY5a, page 57. Q.4*

Read: *Show the subtractions using the cash and debt model. Complete the calculations.*

T has cash and debt cards stuck to BB. Ps come to BB to model each subtraction in turn (showing the starting balance as an addition of cash and debt as in *Activity 8*) and to write the differences. Rest of class helps with modelling or points out errors. P at BB chooses another P to check the difference with an addition.

**Solution:** (with starting balances shown as the same amount of debt as is being taken away and the corresponding amount of cash)

<table>
<thead>
<tr>
<th>Starting Balance</th>
<th>Check</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (+3) – (–4) = [+7 + (–4)] – (–4) = +7</td>
<td>+7 + (–4) = +3</td>
</tr>
<tr>
<td>b) (+3) – (+8) = [+8 + (–5)] – (+8) = –5</td>
<td>–5 + (+8) = +3</td>
</tr>
<tr>
<td>c) (–2) – (–5) = [+3 + (–5)] – (–5) = +3</td>
<td>–3 + (–5) = –2</td>
</tr>
<tr>
<td>d) (–2) – (+3) = [+3 + (–5)] – (+3) = –5</td>
<td>–5 + (+3) = –2</td>
</tr>
<tr>
<td>e) 0 – (+4) = [+4 + (–4)] – (+4) = –4</td>
<td>–4 + (+4) = 0</td>
</tr>
<tr>
<td>f) 0 – (–4) = [+4 + (–4)] – (–4) = +4</td>
<td>+4 + (–4) = 0</td>
</tr>
</tbody>
</table>

When will the balance be better? (If we take away a negative number.)

When will the balance be worse? (If we take away a positive number.)

**Notes**

Whole class activity

Written on BB or SB or OHT

Discussion on how to model the starting balance first, e.g.

a) Reasons checking, agreement, praising

Ps say for each whether the new balance is better or worse.

Check with other models too.

Ps say word problems about the operations.

Point out that for e) and f), the operations can be written without zero, with the same result.

e) – (+4) = –4

f) – (–4) = +4
Are they correct?
What do you think about these operations? Are they correct? Could they be written in a clearer way? T asks several Ps what they think. Class agrees/disagrees. Ps write modified operations in Ex Bks.

a) \(-4 + (+6) + (-5) = -3\) (Correct, but clearer if brackets were drawn around ‘\(-5\)’)

b) \(7 + (-5) - 4 = -2\) (Correct, but clearer if written as:
\[
7 + (-5) + (-4) = -2
\]
or
\[
(+7) + (-5) + (+4) = -2
\]

Addition of integers
Let’s write these operations as additions. Ps come to BB or dictate what T should write. Class agrees/disagrees

a) \(-7 + 8 - 6 = [\ -7 + 8 + (-6) = -13 + 8 = -5\] \)
or \[
[\ (-7) + (+8) + (-6) = (-13) + (+8) = -5]
\]
b) \(6 - 5 - 6 + 5 = [\ (+6) + (-5) + (-6) + (+5) = 0\] \)
c) Write this operation in a shorter way.
\(-5 + (-7) + (+3) + (+5) = [-5 - 7 + 3 + 5 = -4\] \)

Find the mistake
I did these subtractions too quickly and have not had time to check them. Can you check them for me?
Psin BB to work through each calculation in order, write a tick if it is correct and if wrong, cross out the mistake and correct it. Class checks with addition. If problems or disagreement, show on number line or with cash and debt model or ask Ps to explain in context.

BB:

a) \(-6 - (-5) = -1\)
Check: \(-1 + (-5) = -6\)
(e.g. If you were £6 in debt, then a £5 debt was cancelled, you would now be only £1 in debt.)

b) \(-5 - (-6) = \#\)
\(-5 - (-6) = +1\)
Check: \(+1 + (-6) = -5\)

c) \(+7 - (+8) = -1\)
Check: \(-1 + (+8) = +7\)

d) \(0 - (-8) = +8\)
Check: \(+8 + (-8) = 0\)

e) \(-8 - (+6) = \#\)
\(-8 - (+6) = -8\)
Check: \(-8 + 0 \neq -8\)

Elicit that:
• if debts are reduced (taken away), then the balance increases. or (i.e. if a negative number is subtracted, the result increases)
• if cash is reduced (taken away), the balance decreases. (i.e. if a positive number is subtracted, the result decreases)
### Two number line model

**a)** T has two number lines stuck to BB with zeros matching at the start. Then T moves top number line 5 units to the left as below.

BB:

<table>
<thead>
<tr>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>-7</td>
<td>-6</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

What can you say about the matching pairs of numbers? (Each top number is 5 units more than bottom number, or each bottom number is 5 units less than top number, or difference is 5 units)

Let’s write subtractions from the number lines. Ps come to BB or dictate to T. Class checks mentally with an addition and points out errors.

BB: e.g.

7 – 2 = 5 [or +7 – (+2) = +5], 8 – 3 = 5, 9 – 4 = 5, etc.
5 – 0 = 5, 4 – (–1) = 5, 3 – (–2) = 5, 2 – (–3) = 5, etc.
1 – (–6) = 5, –2 – (–7) = 5, etc.

Elicit the rule of subtraction: reductant – subtrahend = difference and that its check is: difference + subtrahend = reductant.

Which number line is the reductant for these subtractions? (top)
Which is the subtrahend? (bottom) What is the difference? (5)

**b)** T moves the number lines back to their original position (i.e. zeros matching), then moves the top number line 3 units to the right.

BB:

<table>
<thead>
<tr>
<th>-11</th>
<th>-10</th>
<th>-9</th>
<th>-8</th>
<th>-7</th>
<th>-6</th>
<th>-5</th>
<th>-4</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8</td>
<td>-7</td>
<td>-6</td>
<td>-5</td>
<td>-4</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
</tr>
</tbody>
</table>

If the top number line is still the reductant and the bottom number line is still the subtrahend, what is their difference now? (–3)

Let's write subtractions about them. Ps come to BB or dictate to T. Class checks mentally and points out errors.

BB: e.g.

5 – 8 = –3, 4 – 7 = –3, 3 – 6 = –3, 2 – 5 = –3, etc.
0 – 3 = –3, –1 – 2 = –3, –2 – 1 = –3, –3 – 0 = –3
–4 – (–1) = –3, –5 – (–2) = –3, etc.

Elicit what the differences be? (5)

**Extension**

If the top number was subtracted from the bottom number what would the differences be? [a) – 5, b) + 3, i.e. opposite of previous difference]

---

### PbY5a, page 58

**Q.1** Read: *Do the subtractions. Use the number line to help you.*

First elicit that if subtracting a positive number, move to the left and if subtracting a negative number, move to the right.

Remember to check your result with a mental addition! Set a time limit. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>a) + 9 – (+ 2) = + 7</th>
<th>d) + 2 – (– 5) = + 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) + 3 – (+ 6) = – 3</td>
<td>e) – 1 – (+ 2) = – 3</td>
</tr>
<tr>
<td>c) – 5 – (– 2) = – 3</td>
<td>f) – 1 – (– 8) = + 7</td>
</tr>
</tbody>
</table>
Subtraction of integers: car model

T has large car model on BB and Ps have smaller versions on desks if possible. Ps move the car along the number line according to T’s instructions.

a) The car is at + 6 and faces the house. Move it 4 units backwards. Where does it end up? (+ 2)

BB: + 6 – (+ 4) = + 2

We can write it as a subtraction like this.

b) The car is now at + 2 and faces the house. Move it another 5 units backwards. Where does it end up this time? (– 3)

Who can write it as a subtraction? P comes to BB or class dictates to T. Who agrees? Who thinks something else? etc.

BB: + 2 – (+ 5) = – 3

c) The car is at + 2 and faces the tree. Move it 5 units backwards. Where does it end up? (+ 7) Who can write the subtraction?

BB: + 2 – (– 5) = + 7

d) The car is at – 3 and faces the tree. Move it 4 units backwards. Where does it end up? (+ 1) Ps come to BB to write subtraction.

BB: – 3 – (– 4) = + 1

Discuss and agree on the rules when using the car model for subtraction.

• When we subtract a positive number from any number, then the car faces the house and moves backwards. (decreasing)
• When we subtract a negative number from any number, the car faces the tree (left) and moves backwards. (increasing)

Notes

Whole class activity
Use copy masters from LP54/6

P moves car on BB and rest of class use own car models if they have them.

(T might pretend to be the car and step backwards for the relevant number of paces.)

Reasoning, agreement, praising

Check the subtractions in different ways: with addition and using other models (e.g. cash and debt, temperature)

Ps could write the agreed subtraction in Ex. Bks.

If time, Ps come to BB in threes, one to give own instructions, one to move the car and the other to write the subtraction.

Class points out errors.

Do not make Ps write the rules but encourage them to memorise them and visualise the movements in their heads.

Solution:

a) The car is at (+ 4) and faces the house. Move it 3 units backwards.

+ 4 – (+ 3) = + 1 Check: + 1 + (+ 3) = + 4

b) The car is at (+ 4) and faces the house. Move it 7 units backwards.

+ 4 – (+ 7) = – 3 Check: – 3 + (+ 7) = + 4

[Note to Ts]

Each model has advantages and disadvantages. Use the models your Ps find easiest to understand.

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c) The car is at (– 5) and faces the tree. Move it 3 units backwards.

\[-5 - (-3) = -2\]  
Check: \[2 + (-3) = -5\]

d) The car is at (+3) and faces the tree. Move it 4 units backwards.

\[+3 - (+4) = -1\]  
Check: \[-1 + (+4) = +3\]

1. The car is at (– 5) and faces the tree. It moves 7 units backwards.

BB: \[-5 - (-7) = +2\]  
Check: \[+2 + (-7) = -5\]

2. The car is at (+ 3) and faces the tree. It moves 4 units backwards.

BB: \[+3 - (-4) = +7\]  
Check: \[+7 + (-4) = +3\]

---

**Extension**

1. The car is at (– 5) and faces the tree. It moves 7 units backwards.

BB: \[-5 - (-7) = +2\]  
Check: \[+2 + (-7) = -5\]

2. The car is at (+ 3) and faces the tree. It moves 4 units backwards.

BB: \[+3 - (-4) = +7\]  
Check: \[+7 + (-4) = +3\]

---

**Lesson Plan 58**

**Notes**

Elicit that:

- when the car faces the house (right) and moves backwards, a positive number is subtracted.
- when the car faces the tree and moves backwards, a negative number is subtracted

Extra practice for quicker Ps. T gives instructions orally.

---

8  **PbY5a, page 58**

Q.3 Read: Do the subtractions and join them to the matching car.

Again, less able Ps should have car models to help them.

Set a time limit or deal with one at a time. Encourage Ps to check results mentally with addition.

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) \[(+ 8) - (+ 2) = +6\]  
\[\text{decreasing}\]

b) \[(-8) - (-2) = -6\]

c) \[(+2) - (+8) = -6\]  
\[\text{decreasing}\]

\[\text{increasing}\]

d) \[(-2) - (-8) = +6\]

e) \[(+4) - (+3) = +1\]  
\[\text{increasing}\]

f) \[(-4) - (-3) = -1\]

---

9  **PbY5a, page 58, Q.4**

Read: Do each calculation, then join it to the matching car.

Ps come to BB to calculate mentally if they can, or to use the car model to help them, then write result. (Extra praise for Ps who do not need the model.) Class checks mentally with inverse operation or by visualising other models. Then Ps join operation to matching car. Class points out errors. If problems or disagreement, demonstrate with other models too.

**Solution:**

a) \[ (+ 3) + (- 1) = +2\]  
\[\text{increasing}\]

b) \[ (+ 3) + (- 5) = -2\]  
\[\text{decreasing}\]

c) \[ (+ 3) + (+ 2) = +5\]  
\[\text{increasing}\]

d) \[ (+ 3) + 0 = +3\]  
\[\text{increasing}\]

e) \[ (-4) + (+ 1) = -3\]  
\[\text{decreasing}\]

f) \[ (-4) + (+ 6) = +2\]  
\[\text{increasing}\]

g) \[ (-4) + (- 3) = -7\]  
\[\text{decreasing}\]

h) \[ (-4) + 0 = -4\]  
\[\text{decreasing}\]

i) \[ 0 + (+ 2) = +2\]  
\[\text{increasing}\]

j) \[ 0 + (-3) = -3\]  
\[\text{decreasing}\]

k) \[ (+ 3) - (+ 1) = +2\]  
\[\text{increasing}\]

l) \[ (+ 3) - (+ 5) = -2\]  
\[\text{decreasing}\]

m) \[ (+ 3) - (- 2) = +5\]  
\[\text{increasing}\]

n) \[ (+ 3) - 0 = +3\]  
\[\text{increasing}\]

o) \[ (-4) - (- 1) = -3\]  
\[\text{decreasing}\]

p) \[ (-4) - (- 6) = +2\]  
\[\text{increasing}\]

q) \[ (-4) - (- 3) = -7\]  
\[\text{decreasing}\]

r) \[ (-4) - 0 = -4\]  
\[\text{decreasing}\]

s) \[ 0 - (-2) = +2\]  
\[\text{increasing}\]

f) \[ 0 - (+ 3) = -3\]  
\[\text{decreasing}\]

---

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### Lesson Plan

#### Week 12

**Y5**

**Activity**

1. **Sequences**
   
   T says first few terms of a sequence. Ps continue it and give the rule. e.g.
   
   a) \(-57, -42, -27, (-12, +3, 18, 33, 48, 63, 78, 93, 108, \ldots)\)
   
   **Rule:** \(+15\), or increasing by 15
   
   b) \(555, 444, 333, (222, 111, 0, -111, -222, -333, -444, \ldots)\)
   
   **Rule:** \(-111\), or decreasing by 111
   
   c) \(-1, +2, -4, +8, (-16, +32, -64, +128, -256, +512, -1024, +2048, -4096, +8192, -16384 \ldots)\)
   
   **Rule:** Each following term is twice the opposite of the previous term, or absolute value is increasing by 2 times and negative and positive signs are alternating
   
   d) \(150000, -75000, +37500, (-18750, +9375, -4687.5, \ldots)\)
   
   **Rule:** Each following term is half the opposite of the previous term, or absolute value is decreasing by 2 times and positive and negative signs are alternating

   - **5 min**

2. **What is the rule?**
   
   Ps come to BB to fill in missing numbers or dictate what T should write. Class agrees/disagrees. Who can write the rule? Who can write it another way? Class checks each version with values from table.

<table>
<thead>
<tr>
<th>(a)</th>
<th>8</th>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>-1</th>
<th>-2</th>
<th>-3</th>
<th>-4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(b)</td>
<td>5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>-1</td>
<td>-2</td>
<td>-3</td>
<td>-4</td>
<td>-5</td>
<td>-6</td>
<td>-7</td>
</tr>
</tbody>
</table>

   **Rule:** \(a - 3 = b\)  \(a - (+3) = b\)  \(a + (-3) = b\)  \(a = b + 3\)
   
   \(a - b = 3\)  \(b - a = -3\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>5</th>
<th>-1</th>
<th>2</th>
<th>0</th>
<th>6</th>
<th>7</th>
<th>-97</th>
<th>3</th>
<th>-40</th>
<th>-8</th>
<th>8</th>
<th>-3</th>
<th>-5.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>-2</td>
<td>4</td>
<td>1</td>
<td>3</td>
<td>-3</td>
<td>-4</td>
<td>100</td>
<td>0</td>
<td>43</td>
<td>11</td>
<td>-5</td>
<td>6</td>
<td>8.5</td>
</tr>
</tbody>
</table>

   **Rule:** \(x + y = 3\)  \(x = 3 - y\)  \(y = 3 - x\)

   - **10 min**

3. **Two number line model**
   
   T has number lines already prepared.

   - **15 min**

---

**Notes**

Whole class activity

- Ps stand up.
- At speed, in order round class
- If a Ps makes a mistake, the next Ps must correct it.
- Ps sit down if they make or miss a mistake.
- T decides when to stop and final P gives the rule.
- Ps can calculate in Ex. Bks or on slates for c) and d) if necessary. T helps with explaining the rules.
- (or individual competition in Ex. Bks. under a time limit)

Whole class activity

- Tables drawn on BB or use enlarged copy master or OHP
- At good pace
- Reasoning, agreement, praising
- T points to columns in table and Ps say the operation it shows.
- Ps give pairs of values not shown in tables.

Whole class activity

- Actual prepared number lines stuck on BB, or use enlarged copy master or OHP
- Discussion, reasoning, agreement, praising
- At a good pace
- Elicit that:
  - top number + bottom number = -4
  - (-4) - bottom number = top number
  - (-4) - top number = bottom number
Calculation practice

Let's do these calculations. Ps come to BB to explain reasoning and fill in results. Class points out errors or suggests an easier order of calculation, or helps with reasoning.

BB: Reasoning, e.g.

a) \((+ 12) + (– 12) – (– 7) = 7\) [as \(– (– 7)\) is the opposite of the opposite of 7]

b) \((- 15) + (+ 10) – (+ 5) = – 10\) [as \(- 10 + (+ 5) = (– 5)\)]

c) \((+ 5) – (– 3) + (+ 7) = 15\) [as 15 is 7 more than 8]

d) \(- 7 + (+ 5) – (– 8) = 6\) [as \(6 + (– 8) = – 2\)]

e) If \(x = – 8\), \(y = – 7\) and \(z = + 5\),
\[x – y + z = [– 8 – (– 7) + (+ 5) = – 1 + 5 = + 4]\]

[I was £8 in debt, then £7 of my debts were cancelled, so I was only £1 in debt. Then I earned £5 and paid off the £1 debt, so my balance is now £4.]

20 min

PbY5a, page 59

Q.1 Read: Fill in the missing numbers. Continue drawing the graphs.

Deal with one part at a time. Ps fill in missing numbers under a time limit. Review with whole class. Ps dictate results, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Complete the graph with the whole class. Ps work at BB and rest of class works in Pbs.

What have the operations to do with the graph?

Using one of the dots and operations already given, elicit that, e.g. \((+ 4) – (– 3) = + 1\) is shown by the dot with coordinates \((3, 1)\), so \(x = \frac{3}{2}\) (or \(+ 3\)), which is the number being subtracted from 4, and \(y = \frac{1}{2}\) which is the difference between 4 and \(x\).

T tells class that all the equations in a) can be shown by the general rule (or formula) above the graph: \(4 – x = y\)

Let's draw a dot on the graph for each equation.

Ps come to BB, point to an operation, identify the relevant \(x\) and \(y\) coordinates, point to their positions on the \(x\) and \(y\) axes with both hands, move their fingers along grid lines until they meet and mark the point by drawing a dot. Class agrees/disagrees. Rest of Ps draw dots in Pbs too.

Deal with part b) in a similar way but if Ps wish, allow them to continue drawing the dots as individual work under a time limit.

What do you notice about the graphs? (e.g. Dots are in a straight line. If we drew lines through the dots, the two lines would be parallel. The reductant in each formula is where its graph line crosses the \(x\) and \(y\) axes. etc.)
### Activity 5 (Continued)

**Solution:**

a) \((+4) - (+6) = -2\)
\((+4) - (+5) = -1\)
\((+4) - (+4) = 0\)
\((+4) - (+3) = +1\)
\((+4) - (+2) = +2\)
\((+4) - (+1) = +3\)
\((+4) - 0 = +4\)
\((+4) - (-1) = +5\)
\((+4) - (-2) = +6\)

b) \((-4) - (+2) = -6\)
\((-4) - (+1) = -5\)
\((-4) - 0 = -4\)
\((-4) - (-1) = -3\)
\((-4) - (-2) = -2\)
\((-4) - (-3) = -1\)
\((-4) - (-4) = 0\)
\((-4) - (-5) = +1\)
\((-4) - (-6) = +2\)

### Lesson Plan 59

**Notes**

Given dots are circled.

---

### Activity 6

**PbY5a, page 59**

Q.2 Read: *Calculate the sums and differences.*

Ps may use number lines to help them. Set a time limit.

Deal with part a) then review before dealing with part b).

Ps dictate results to T, saying the whole operation, and check with reverse operation. Class agrees/disagrees. Mistakes discussed and corrected. Show on number line, or reason with cash and debt model for additions and appropriate subtractions (highlighted), if problems or disagreement.

**Solution:**

a) \((+3) + (-5) = -2\)
\((+3) + (-1) = -4\)
\((+3) + 0 = 3\)
\((+3) + (+1) = 4\)
\((+3) + (+2) = 5\)

b) \((-3) - (+2) = -5\)
\((-3) - (+1) = -4\)
\((-3) + 0 = -3\)
\((-3) + (+1) = -2\)
\((-3) + (+2) = -1\)

\(\text{Individual work, monitored, (helped)}\)
\(\text{Written on BB or use enlarged copy master or OHP}\)

Reasoning, agreement, checking, self-correction, praising

N.B. In subtractions which are not appropriate for using cash and debt, use comparison.

e.g. \((+3) - (-2) = +5\)

\(3 \text{ is 5 more than } -2\)

\(-3 - (+2) = -5\)

\(\text{‘} -3 \text{ is 5 less than } +2\)’

T points out that:

- Subtracting a **positive** number can be replaced by adding the opposite negative number.
- Subtracting a **negative** number can be replaced by adding the opposite positive number.
### Lesson Plan 59

**Activity**

(Continued)

T: We can think of the addition of integers in a different way:

a) To add two positive numbers, add their absolute values and the sum is positive.

b) To add two negative numbers, add their absolute values and the sum is negative.

c) To add a positive and a negative number, calculate the difference of their absolute values and take the sign of the greater absolute value.

38 min

**Notes**

Whole class activity

T explains the idea and Ps check that it is correct by pointing to relevant additions in the list.

[Preparation for addition and subtraction of integers.]

**True or False?**

I will read a statement. If you think it is true write T on your slates and if you think it is false write F. Show me what you have written when I say.

T asks Ps with opposing views to give an example or counter example to support their response. Agree that if false, only one counter example is needed and if true, no counter example is possible.

<table>
<thead>
<tr>
<th>Statement</th>
<th>True/False</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) The sum of any two positive integers is always positive.</td>
<td>(T)</td>
</tr>
<tr>
<td>b) The absolute value of the sum of two negative integers is positive.</td>
<td>(T)</td>
</tr>
<tr>
<td>c) Instead of subtracting a negative number, we can add its opposite.</td>
<td>(T)</td>
</tr>
<tr>
<td>d) The difference between two integers is always less than the subtrahend.</td>
<td>(F)</td>
</tr>
<tr>
<td>e) The difference between two integers is always less than the reductant.</td>
<td>(F)</td>
</tr>
<tr>
<td>f) Subtracting a whole number from a greater whole number gives a positive difference.</td>
<td>(T)</td>
</tr>
</tbody>
</table>

42 min

**Notes**

Whole class activity

(Or Ps agree on certain actions for T and F)

Responses shown in unison.

Examples/counter examples:

- a) $3 + 4 = 7$
- b) $-2 + (-5) = -7$, $|7| = 7$
- c) $5 - (-7) = 5 + (+7) = 12$
- d) $8 - (-3) = 11$, and $11 > -3$
- e) $-5 - (-3) = -2$, and $-2 > -5$
- f) $(-5) - (+2) = +3$
  $(-3) - (-5) = +2$

**PbY5a, page 59**

Q.3 Read: *Tick the solution to the equation if it is correct.*

I will give you 1 minute to work out whether they are correct.

Review one at a time. Ps show whether correct or not by writing a tick or a cross on slates or scrap paper and showing to T on command. P who thinks it is wrong, comes to BB to point out mistake and correct it. Class agrees/disagrees. Ps who have changed their minds annotate Pbs appropriately.

**Solution:**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Solution</th>
<th>Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) $x - (-12) = 20$</td>
<td>$20 + (-12) = 8$</td>
<td>$x = 8$ ✓</td>
</tr>
<tr>
<td>b) $-12 - y = -15$</td>
<td>$-12 - (-15) = 3$</td>
<td>$y = 3$ ✓</td>
</tr>
<tr>
<td>c) $z - (+3) = -2$</td>
<td>$-2 - (+3) = -5$</td>
<td>$z = -5$ ✗</td>
</tr>
</tbody>
</table>

**Checks:**

- $8 - (-12) = 20$
- $-12 - (+3) = -15$
- $+1 - (+3) = -2$

Review the general rules:

- a) and c):
  We calculate the **reductant** by adding the difference to the subtrahend.

- b) We calculate the **subtrahend** by subtracting the difference from the reductant.

45 min

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Calculation practice, revision, consolidation

PhY5a, page 60

(Could be set as a timed test, reviewed with whole class.)

Solutions:

Q.1  a) \(+ 6 + (– 3) = + 3\)  
b) \(0 – (– 10) = + 10\)  
c) \(– 8 + (– 2) = – 10\)  
d) \(– 6 + (– 6) = – 12\)  
e) \(– 15 + (– 8) = – 23\)  
f) \(– 15 – (– 8) = – 7\)

Q.2  a) \((- 2) + (– 2) + (– 2) + (– 2) = 4 \times (– 2) = – 8\)  
b) \((- 3) + (– 3) + (– 3) + (– 3) + (– 3) = 5 \times (– 3) = – 15\)  
c) \((- 3) + (– 3) + (– 3) + (– 3) + (– 3) + (– 3) + (– 3) = 7 \times (– 3) = – 21\)

Q.3  a) \(12 \times 12 = 144\)  
b) \(20 \times 20 = 400\)  
c) \(13 \times 13 = 169\)  
d) \(12 \times 21 = 252\)  
e) \(19 \times 20 = 380\)  
f) \(49 \times 8 = 392\)  
g) \(30 \times 31 = 930\)  
h) \(29 \times 12 = 348\)

Q.4  a) \(\frac{13 \times 13}{11 \times 12} = 1\)  
b) \(\frac{14 \times 14}{12 \times 12} = 1\)  
c) \(\frac{16 \times 16}{14 \times 14} = 1\)  
d) \(\frac{17 \times 17}{15 \times 15} = 1\)

Q.5  a) \(45 \div 9 = 5\)  
b) \(24 \div 8 = 3\)  
c) \(63 \div 7 = 9\)  
d) \(40 \div 10 = 4\)  
e) \(15 \div 3 = 5\)  
f) \(28 \div 7 = 4\)  
g) \(81 \div 9 = 9\)  
h) \(42 \div 7 = 6\)  
i) \(48 \div 8 = 6\)  
j) \(26 \div 3 = 8\frac{2}{3}\)  
k) \(52 \div 6 = 8\frac{2}{3}\)  
l) \(60 \div 8 = 7\frac{1}{2}\)  

Q.6  a) \(217 \div 3 = 72, r 1\)  
b) \(2170 \div 30 = 72, r 10\)  

Q.7  a) \(\frac{13}{10} - \frac{6}{10} = \frac{7}{10}\)  
b) \(\frac{17}{10} - \frac{9}{10} = \frac{8}{10}\)  
c) \(\frac{19}{10} - \frac{12}{10} = \frac{7}{10}\)  
d) \(\frac{21}{10} - \frac{18}{10} = \frac{3}{10}\)  

Notes
Lesson Plan

61

Y5

R: Calculation. Addition, subtraction of integers with models
C: Meaning of multiplying and dividing integers by natural numbers
E: Preparation for multiplying and dividing integers by natural numbers

Activity

1

Revision of addition

Write these numbers below each other by place value and add them up. Estimate first and check your result.

Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

BB:

\[
\begin{align*}
\text{a) } & 7080 + 70 + 150 + 56200 = 70800 + 150 + 56200 = 63600 \\
& \quad + 56200 \\
& \quad = 63600 + 56200 = 698200 \\
\text{b) } & 6010 + 4009 + 261 + 4020 \\
& \quad = 6000 + 4000 + 261 + 4000 = 14300
\end{align*}
\]

Notes

Individual work, monitored

T dictates numbers and writes them on BB.

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Ask Ps to explain with place value detail.

T points to a column and Ps say its place value.

(TH, TH, H, T, U)

Feedback for T

2

Revision of subtraction

Do these subtractions in your Ex. Bks. Estimate first, then check your result with an addition. Set a time limit.

Review with whole class. Ps come to BB to explain reasoning with place value detail. Class agrees/disagrees. Who had all five correct? Let's give them a clap! Who made a mistake? What was your mistake? Who did the same? etc. Mistakes discussed and corrected.

BB:

\[
\begin{align*}
\text{a) } & 567 - 456 \\
& \quad = 111 \\
\text{b) } & 4453 - 7009 \\
& \quad = 3444 \\
\text{c) } & 75038 - 2890 \\
& \quad = 72148 \\
\text{d) } & 56709 - 30567 \\
& \quad = 26142 \\
\text{e) } & 13067 - 6094 \\
& \quad = 6973
\end{align*}
\]

Notes

Individual work, monitored

Written on BB or SB or OHT

Reasoning, agreement, self-correction, evaluation, praising

T points to a column and Ps say its place value.

Feedback for T

3

Revision of multiplication

Let's multiply 316 by 524 using the place value table.

How could we do it? T asks several Ps what they think. If no P suggests the idea, T starts the explanation and chooses Ps to continue it. Calculations can be written at side of BB. Class points out errors.

BB:

\[
\begin{align*}
316 \times 4 &= 3160 \\
316 \times 20 &= 6320 \\
316 \times 500 &= 31600 \\
316 \times 524 &= 158000
\end{align*}
\]

Notes

Whole class activity

Place-value table drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

Or start with multiplication by 500, then by 20, then by 4 if T prefers.

Feedback for T
## Revision of long multiplication

Do these multiplications in column form in your Ex. Bks using the same idea of breaking up the multiplier into units, tens, hundreds, etc. Set a time limit. Ask Ps to estimate mentally first and to check their results against their estimate. Review with whole class. Ps come to BB to show calculations, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed or corrected.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revision of long multiplication</td>
<td>Individual work, monitored, (helped) Written on BB or SB or OHT Differentiation by time limit Reasoning, agreement, self-correction, praising T shows short forms of b) and c) on BB. T: To save us writing lots of zeros we can combine some rows. T shows on BB. b) The first 2 rows of working can be combined as $516 \times 30$ (not shown) c) the first 3 rows of working can be combined as $516 \times 300$ (as shown)</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
</tbody>
</table>

### BB:

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$516 \times 23$</td>
<td>$516 \times 230$</td>
<td>$516 \times 2300$</td>
</tr>
</tbody>
</table>
| $\begin{array}{l}
516 \\
\times 23
\end{array}$ | $\begin{array}{l}
516 \\
\times 230
\end{array}$ | $\begin{array}{l}
516 \\
\times 2300
\end{array}$ |
| $15480$ | $154800$ | $1548000$ |
| $+ 10320$ | $+ 103200$ | $+ 1032000$ |
| $118680$ | $1186800$ | $1186800$ |
| shorter form: | | |
| $\begin{array}{l}
516 \\
\times 30
\end{array}$ | $\begin{array}{l}
516 \\
\times 300
\end{array}$ |
| $15480$ | $154800$ |
| $+ 10320$ | $+ 103200$ |
| $118680$ | $1186800$ |
| $154800$ | $1548000$ |
| $1186800$ | $1186800$ |
| $20\text{ min}$ | | |

### Revision of division

Do these divisions in column form your Ex. Bks. I will give you 3 minutes! Review with whole class. Ps come to BB to show calculations, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed or corrected.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Revision of division</td>
<td>Individual work, monitored, (helped) Written on BB or SB or OHT Differentiation by time limit Reasoning, agreement, self-correction, praising If problems, show as long division. Feedback for T</td>
</tr>
<tr>
<td>5</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
</tr>
</tbody>
</table>

### BB:

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9 \div 9$</td>
<td>$108 \div 9$</td>
<td>$1107 \div 9$</td>
<td>$11106 \div 9$</td>
<td>$11105 \div 9$</td>
</tr>
</tbody>
</table>
| $\begin{array}{l}
9 \\
\div 9
\end{array}$ | $\begin{array}{l}
12 \\
\div 9
\end{array}$ | $\begin{array}{l}
123 \\
\div 9
\end{array}$ | $\begin{array}{l}
1234 \\
\div 9
\end{array}$ | $\begin{array}{l}
12345 \\
\div 9
\end{array}$ |
| $1$ | $110$ | $1107$ | $11106$ | $11105$ |
| $1$ | $12$ | $123$ | $1234$ | $12345$ |
| $22$ | $23$ | $233$ | $234$ | $2344$ |
| $20\text{ min}$ | | | | |

## PbY5a, page 61

Q.1 Read: Write two possible plans for solving each question. Calculate one of the plans and write the answer in a sentence.

Ps read problems themselves and answer in Pbs. Set a time limit or deal with one at a time if class is not very able. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

### Solution:

a) Adrian had no money, neither cash nor debt, so we can say that he had £0. Then he ran up debts of £3 each day for a week. What is his balance now?

**Plans:**

$$(-£3) + (-£3) + (-£3) + (-£3) + (-£3) + (-£3) + (-£3) = -£21$$

or $$7 \times (-£3) = -£21$$

**Answer:** Adrian’s balance is now –£21.
b) Five boys were £20 in debt. If they shared the debt equally, how much was each boy in debt?

Plans: e.g.

\[-£4 \times 5 = -£20, \text{ or} \]
\[-£4 + -£4 + -£4 + -£4 + -£4 = -£20\]

or \[-£20 \div 5 = -£4\]

Answer: Each boy was £4 in debt.

---

Q.2 Read:

b) Write the additions as multiplications.

Set a time limit or deal with one at a time.

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes corrected.

T asks Ps to give word problems about the operations. Class decides whether or not the context is valid.

Solution:

a) i) \((-2) \times 5 = (-2) + (-2) + \left[ (-2) + (-2) + (-2) \right] = -10\)
   
   ii) \(4 \times (-3) = (-3) + (-3) + \left[ (-3) + (-3) \right] = -12\)
   
   iii) \((-10) \times 6 = (-10) + (-10) + \left[ (-10) + (-10) + (-10) \right] + (-10) = -60\)

b) i) \((-5) + (-5) + (-5) + (-5) + (-5) + (-5) + (-5) + (-5) + (-5) + (-5)\)
   
   \[= \left[ (-5) \times 7 = -35\right]\]
   
   ii) \((-6) + (-6) + (-6) + (-6) + (-6) + (-6)\)
   
   \[= \left[ (-6) \times 6 = -36\right]\]
   
   iii) \((-100) + (-100) + (-100) + (-100)\)
   
   \[= \left[ (-100) \times 4 = -400\right]\]

---

Q.3 Read:

The car starts at 0 each time and faces the house. Write its moves as a multiplication or a division.

Ps come to front of class in threes, one to read the instructions, one to move the car and one to write the operation. Class agrees/disagrees. Once operation is agreed, Ps write it in Pbs.

Solution:

a) It moves 4 units per second for 3 seconds towards the house.

\[3 \times 4 = 12 \text{ or } 3 \times (+4) = (+12)\]
### Activity

8

(Continued)

b) It moves 4 units per second for 4 seconds towards the tree.

BB: \(4 \times (-4) = -16\) It moves (-16) units, or 16 units to the left.

c) It moves 15 units towards the tree in 3 seconds. How many units does it move each second on average?

BB: \(-15 \div 3 = -5\)

Each second it moves (-5) units, or 5 units to the left.

---

9

**PbY5a, page 61, Q.4**

Read: Write the 7th, 10th and 20th terms of each of these sequences in your exercise book.

Deal with one at a time. First elicit the rule. If Ps suggests addition or subtraction, praise them but then draw Ps' attention to multiplication by asking, e.g. What is the 1st term? What is the 2nd term (3rd term)? How did we get it from the 1st term? Do we need to write out all the first 6 terms to get the 7th term? etc.

T shows the quick way of writing 1st term, 2nd term, etc.

BB: 1st term: \(a_1\) 2nd term: \(a_2\) 3rd term: \(a_3\) etc.

Once class has agreed on the rule, Ps come to BB or dictate the required terms. Class agrees/disagrees. T asks for other terms too.

**Solution:**

BB:

<table>
<thead>
<tr>
<th>Sequence</th>
<th>Rule</th>
<th>7th Term (a_7)</th>
<th>10th Term (a_{10})</th>
<th>20th Term (a_{20})</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (-9, -18, -27, \ldots)</td>
<td>(+(-9)) or (-(+9))</td>
<td>(-9 \times 7 = -63)</td>
<td>(-9 \times 10 = -90)</td>
<td>(-9 \times 20 = -180)</td>
</tr>
<tr>
<td>b) (-12, -24, -36, \ldots)</td>
<td>(-12)</td>
<td>(-12 \times 7 = -108)</td>
<td>(-12 \times 10 = -120)</td>
<td>(-12 \times 20 = -240)</td>
</tr>
<tr>
<td>c) (-40, -80, -120, \ldots)</td>
<td>(-40)</td>
<td>(-40 \times 7 = -280)</td>
<td>(-40 \times 10 = -400)</td>
<td>(-40 \times 20 = -800)</td>
</tr>
</tbody>
</table>

45 min

---

**Notes**

Whole class activity
(or individual or paired trial first if Ps wish)
Discussion, reasoning, agreement, praising
T directs Ps' thinking when required.
\[e.g. \text{in a):}\]
BB: \(a_1 = -9 = (-9) \times 1\)
\(a_2 = -18 = (-9) \times 2\)
\(a_3 = -27 = (-9) \times 3\)

Extra praise for Ps who realise that any term can be found easily by multiplication
T could have the first 20 terms of each sequence already prepared and uncover the required terms as confirmation.
(Or Ps could be asked to list them for homework.)
### Activity

#### Problems 1

Listen carefully, note the data and write a plan in your Ex. Bks. Do the calculation and show me the answer when I say.

Ps answering correctly come to BB to explain solution. Who agrees? Who did it another way? etc. Mistakes discussed and corrected. Ps give answer in a sentence.

**a)** Bonny has to swim 2500 m in training. How far has she swum if she still has 1750 m to go?

BB: **Plan:** 2500 m – 1750 m

Answer: Bonny has already swum 750 metres.

**b)** In a subtraction, the subtrahend is 6579 and the reductant is 17,532. What is the difference?

BB: **Plan:** 17,532 – 6579 =

Answer: The difference is 10,953.

**c)** In a subtraction, the subtrahend is 567 and the difference is 465. What is the reductant?

BB: **Plan:** – 567 = 465

Answer: The reductant is 1032.

**d)** What is the difference between the sum and the difference of 4508 and 3972?

BB: **Plan:** (4508 + 3972) – (4508 – 3972)

Answer: The difference is 7944.

**e)** Which number is more than 382 by as many as it is less than 734?

Let unknown number be \( x \). \( x \) must be halfway between 382 and 734.

BB: **Plan:** \( x \) = 382 + (734 – 382)

Answer: The number is 558.

### Multiplication practice

Calculate these products in your Ex. Bks. Set a time limit.

BB: a) \( 3367 \times 3 \)  b) \( 3367 \times 11 \)  c) \( 3367 \times 33 \)

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Who did the same? Who did it another way? Discuss the relationships which make the calculations easier. Mistakes discussed and corrected. T shows the forms below and asks what Ps think of them.

BB: a) \[
\begin{align*}
3367 & \times 3 \\
10101 & \\
\hline
\end{align*}
\]

b) \[
\begin{align*}
3367 & \times 11 \\
37037 & +101010 \\
\hline
407407 &
\end{align*}
\]

c) \[
\begin{align*}
3367 & \times 33 \\
10101 & \\
\hline
\end{align*}
\]

Individual work, monitored

Written on BB or SB or OHT

Discussion, reasoning, agreement, self-correcting, praising

Extra praise if Ps notice that:

- when multiplying by 10, each digit is moved to the next greater place value and 0 is written in units column;
- result of a) can be used in c).
Problems 2

a) Listen carefully, note the data and write a plan in your Ex. Bks. Do the calculation and show me the answer when I say.

Ps answering correctly come to BB to explain solution. Who agrees? Who did it another way? etc. Mistakes discussed and corrected. Ps give answer in a sentence.

Five sacks of rice weigh 490 kg. What would two of the sacks of rice weigh?

BB:  \[ \begin{array}{c}
\frac{490 \text{ kg}}{5 \times 2} = 98 \text{ kg} \times 2 = 196 \text{ kg} \\
\end{array} \]

Answer: Two of the sacks of rice would weigh 196 kg. (We are assuming that each sack is the same size and contains the same amount of rice, although the question does not state it.)

b) Listen carefully, note the data in your Ex. Bks and think how you would work out the answer. You may discuss it with your neighbour if you wish.

Two trains were carrying oil. There were 57 oil drums on the first train and 65 oil drums on the second train.

If the first train carried 128 tonnes more oil than the second train, how many tonnes of oil did they each carry?

Ps tell their ideas and come to BB to show solution, explaining reasoning. Class agrees/disagrees or suggests an easier method. T gives hints or intervenes only if necessary. T chooses a P to give answer in a sentence.

BB: Difference in amount of oil carried: \[ 65 - 57 = 8 \text{ drums} \]

\[ \begin{array}{c}
8 \text{ drums} \rightarrow 128 \text{ t} \\
1 \text{ drum} \rightarrow 128 \text{ t} \div 8 = 16 \text{ t} \\
\end{array} \]

1st train: \[ 57 \text{ drums} \rightarrow 16 \text{ t} \times 57 = 912 \text{ t} \]

2nd train: \[ 65 \text{ drums} \rightarrow 16 \text{ t} \times 65 = 1040 \text{ t} \]

Answer: The first train carried 912 tonnes of oil and the 2nd train carried 1040 tonnes of oil.

Problems 3

Listen carefully and think about the plan you would use to solve the problem.

Ps suggest plans and class decides which to use. Ps dictate the calculations and results. Class agrees/disagrees. T chooses Ps to give answers in sentences.

a) A submarine dived 15 metres every minute. To what depth did the submarine dive in 6 minutes?

BB:  \[ \begin{array}{c}
(-15 \text{ m}) \times 6 = -90 \text{ m} \\
\end{array} \]

Answer: The submarine dived to a depth of –90 m in 6 minutes.

b) Joe has house bills of £600 per year. What are his house bills every month?

BB:  \[ \begin{array}{c}
-600 \div 12 = -\£50 \\
\end{array} \]

Answer: Joe has house bills of £50 each month.
### Activity 5  
**PbY5a, page 62**

**Q.1** Read: *Fill in the products and notice how they change.*

Set a time limit. Review quickly with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes corrected. What did you notice? (Multipliers are decreasing by 1 unit and products are decreasing by 5 units.)

Read: *Complete the graph.*

Discuss the relationship between the multiplications, the given formula (general rule) and the graph. Elicit that the \(x\) values are the multipliers and the \(y\) values are the products and that the coordinates of the two given dots are \((3, 15)\) and \((2, 10)\).

Ps come to BB to point to the next operation, say the formula with the appropriate values substituted and write coordinates of the matching point. If correct, they find the point on the graph and mark with a dot. Rest of Ps draw dots in \(Pbs\) too.

**Solution:**

\[
\begin{align*}
5 \times 3 &= 15 \\
5 \times 2 &= 10 \\
5 \times 1 &= 5 \\
5 \times 0 &= 0 \\
5 \times (-1) &= -5 \\
5 \times (-2) &= -10 \\
5 \times (-3) &= -15
\end{align*}
\]

Individual work, monitored

Drawn/written on BB or use enlarged copy master or OHP

Agreement, self-correction, praising

**Extension**

What do you notice about the graph? (T could prompt with appropriate questions.) e.g.

[A straight line can be drawn through the dots, as multipliers could be fractions between the integers.]

The graph line crosses the \(x\) and \(y\) axes at zero but does not cut the quadrants in half, as the \(x\) and \(y\) values are not equal.

The \(y\) coordinates are greater, so the graph line is closer to the \(y\)-axis (vertical) than to the \(x\)-axis.


### Activity 6  
**PbY5a, page 62**

**Q.2** Read: *Fill in the quotients and notice how they change.*

*Complete the graph.*

Deal with this activity in a similar way to Activity 5, reviewing the quotients and discussing relationships between the divisions, formula and given dots before Ps come to BB to explain reasoning and draw dots on the graph. Class points out errors. Rest of Ps complete own graphs in \(Pbs\) at the same time

**Solution:**

\[
\begin{align*}
9 \div 3 &= 3 \\
6 \div 3 &= 2 \\
3 \div 3 &= 1 \\
0 \div 3 &= 0 \\
-3 \div 3 &= -1 \\
-6 \div 3 &= -2 \\
-9 \div 3 &= -3
\end{align*}
\]

Individual work, monitored, to start, then whole class activity in completing the graph

Drawn/written on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

Elicit that the \(x\) values are the dividends and the \(y\) values are the quotients.

**Extension**

What can you say about the graph? [As in Q.5 – but in this case the \(y\) values are less than the \(x\) values, so the graph line is closer to the \(x\)-axis than to the \(y\)-axis.]
Activity 7

Q3  a) **Read**: In your exercise book or on a grid, draw a house according to these coordinates.

Wall: (–3, 1), (–2, 1), (–2, –2), (2, –2), (2, 1), (3, 1)

Roof: (–3, 1), (0, 4), (3, 1)

Window: (–1, 0), (1, 0), (1, 1), (–1, 1), (–1, 0)

Which coordinate is written first? (x-coordinate)

T chooses Ps to work at BB while rest of class work in Pbs.

Ps compare own diagram with the one on the BB and correct any mistakes.

**Solution**:  

![House on grid]

b) **Read**: Form new coordinates from those in part a) and draw the new images.

Deal with one part at a time. P reads instructions aloud, then Ps write new coordinates in Ex. Bks.

New coordinates reviewed and corrected before Ps draw the diagram. Review the new diagram (already prepared by T) and compare it with the diagram in a) before moving on.

Elicit that:

i) has been stretched horizontally by 2 times.

ii) has been stretched vertically by 3 times

iii) has been enlarged by 2 times, i.e. it is twice the size of a)

iv) has been reduced by 4 times from iii), so by 2 times from a): i.e. it is 1 quarter of iii) and 1 half of a).

**Solution**: 

![Enlarged house on grid]

**Notes**

Individual work, monitored, helped

Grids drawn on BB or use enlarged copy master or OHP

Ps have copies of copy master on desks too (extra grid provided in case of errors)

Ps should mark points and draw lines in pencil so that they can be rubbed out easily.

Agreement, self-correction, praising

Ps kept together on exercises, with quicker Ps helping the less able.

Discussion, agreement, self-correction, praising

T helps with coordinates of iv).

T reminds Ps of mathematical terms and writes them on BB if necessary.

Elicit that houses iii) and iv) are similar to the house in a).

**Homework** (optional)

**PhY5a, page 62, Q.4**

Less able Ps just write the additions and results.

More able Ps could use a) to form a graph, as in Activity 6, by:

1. letting x be the multiplicand and y be the product;
2. writing a formula;
3. forming equations with the x-value increasing by 1 unit each time to +5;
4. plotting points on a graph.

Review before Lesson 63.

(Ps could use grids on copy master in LP 64/12.)
# Lesson Plan

**Week 13**

**Y5**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| Multiples 1 | **R:** Calculations. Multiples, divisors, factors  
**C:** Operations with integer numbers  
**E:** Equations and inequalities with integers |
| **Lesson Plan** | **Notes** |
| 1 | **Notes** |
| | Whole class activity  
Written on BB or SB or OHT  
At a good pace  
Reasoning, agreement, praising  
Extra praise if Ps remember that each number is also a multiple and factor of itself.  
$4 \times 1 = 4$, $4 \div 4 = 1$  
| | **Notes** |
| | Whole class activity but individual listing in Ex. Bks. at same time.  
Drawn on BB or SB or OHT  
Discussion, agreement, self-correction, praising  
[Agree that 0 is a multiple of 4 and also of 6, as $4 \times 0 = 0$ and $6 \times 0 = 0$  
We say that 0 is a common multiple of 4 and 6.]  
| | **Notes** |
| | T repeats Ps' description of each area in a clearer way if necessary.  
Practice using 'set' vocabulary:  
NOT, AND, OR  
| | **Notes** |
| | BB: $A \cap B$  
intersection of sets A and B  
BB: $A \cup B$  
union of sets A and B  
| | **Notes** |

T has the numbers, 1, 2, 3, 4, 6 and 12 written on BB as in diagram below. Let’s draw arrows from each number towards its multiples. What is a multiple of a number? (e.g. The result of multiplying that number by another number.)

Ps come to BB to draw arrows, explaining reasoning e.g. 'I draw an arrow from 2 to 6, as 6 is 3 times two.’ Class points out errors or missed arrows.

BB:

If the arrows pointed in the opposite direction, what would it mean? (The arrows would be pointing towards the factors of each number.)

Elicit that a factor of a number divides into that number exactly.)

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiples 2</td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Notes</strong></td>
</tr>
</tbody>
</table>
| | 5 min  
**Notes** |
| | **Notes** |
| | 10 min  
**Notes** |

Draw this Venn diagram in your Ex. Bks. and write the integers from 0 to 30 in the correct sets if:

BB: $A = \{\text{multiples of 4}\}$ and $B = \{\text{multiples of 6}\}$

T chooses Ps to work at BB while rest of Ps draw diagram and write numbers in Ex. Bks. Class points out errors or omissions from diagram on BB. T introduces the term common multiple (as opposite).

T numbers the various areas of the diagram and asks Ps to describe their features. Who agrees? Who would describe it in a different way? etc.

BB:

Agree that:

(i) This set contains numbers which are not multiples of 4 and not multiples of 6.

(ii) This set contains numbers which are multiples of 4 but are not multiples of 6.

(iii) This set contains numbers which are multiples of 4 and multiples of 6 also. (Or this set contains common multiples of 4 and 6.)

Elicit/tell that iii) is the intersection of sets A and B. T shows how to write it mathematically.

iv) This set contains numbers which are multiples of 6 but are not multiples of 4.

If we combined all the numbers in (ii), (iii) and (iv), how could we describe them?

Set (ii)–(iii)–(iv) contains numbers which are either multiples of 4 or multiples of 6. Elicit/tell that this is called the union of sets A and B.

BB: $A \cap B$  
intersection of sets A and B  
BB: $A \cup B$  
union of sets A and B  

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### Activity

#### Factors 1

Let's factorise these numbers. Who remembers what it means? (Break down the number into its prime factors, i.e. factors which can only be divided exactly by themselves and 1.)

Who remembers how to do it? Come and show us! If no P remembers, T starts and Ps come to BB to continue, or dictate what T should write. Show both methods below and ask Ps which they prefer and why. Rest of Ps work in Ex. Bks at the same time.

**BB:** e.g.

<table>
<thead>
<tr>
<th>Number</th>
<th>Prime Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>150</td>
<td>2 × 3 × 5 × 5</td>
</tr>
<tr>
<td>672</td>
<td>2 × 2 × 2 × 2 × 3 × 7</td>
</tr>
<tr>
<td>375</td>
<td>3 × 5 × 5 × 5</td>
</tr>
</tbody>
</table>

How can we check that we are correct? (Multiply all the factors together and the result should be the original number.)

15 min

#### Factors 2

In your Ex. Bks. use either of these methods to find the prime factors, then all the factors of 48 and 144. List them in increasing order and join up the factor pairs. Set a time limit.

Review with whole class. How many factors did you find for 48 (144)? Show me . . . now! (10, 15) Ps with correct responses come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

**BB:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Prime Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>48</td>
<td>2 × 2 × 2 × 2 × 3</td>
</tr>
<tr>
<td>144</td>
<td>2 × 2 × 2 × 2 × 3 × 3</td>
</tr>
</tbody>
</table>

Individual work, monitored, helped
(or whole class activity if Ps are unsure what to do)
Differentiation by time limit
Discussion, reasoning, agreement, self-correction, praising

(Or T could show prepared solutions on SB or OHT to save time.)

#### Problem

Listen carefully and think how you would solve this problem.

*At a concert in a school hall, the 5 rows of seats which had been put out were filled and 98 people had to stand at the back and sides of the hall. During the interval, an extra row of seats was put out so that everyone could sit down. In fact, there were two empty seats.*

a) How many seats were in each row?
b) How many people were in the audience?

Allow Ps a minute to think about it and discuss with their neighbours. Who has an idea what to do? Come and show us! Who agrees? etc. T gives hints or directs Ps’ thinking where necessary. Check the answer in the context.

Whole class activity
T repeats slowly and also writes data on BB. T asks a P to repeat in own words.
Encourage logical reasoning rather than trial and error.
Discussion on strategy for solution, agreeing that it can only be solved if we assume that there are the same number of seats in each row.
Reasoning, agreement, checking, praising
### Lesson Plan 63

**Activity**

<table>
<thead>
<tr>
<th>5</th>
</tr>
</thead>
</table>

(Continued)

**Solution:** e.g.

a) Number of seats in 6th row: \( 98 + 2 = 100 \)
   
   **Answer:** There were 100 seats in each row.

b) Number of people:
   
   e.g. 1st half: \( 100 \times 5 + 98 = 500 + 98 = 598 \)
   
   or 2nd half: \( 100 \times 6 - 2 = 600 - 2 = 598 \)
   
   **Answer:** There were 598 people in the audience.

**Notes**

or let number in each row be \( x \),
then \( x \times 5 = x \times 6 - 100 \)
so \( x = 100 \)

Ps say answers in sentences.

(Agree that we are assuming that everyone came back after the interval!)

**Notes**

Individual work, monitored, helped

Deal with one at a time if Ps are not very able.

Reasoning, agreement, self-correcting, praising

Show on number line too.

| 6 |

**Erratum**

In **Ps** in a): 2nd sentence should start, 'How much should Joe give to Tina so that . . .'

**PbYsA, page 63**

Q.1 Read: Write an operation for each question and underline the result.

Set a time limit. Ps read problems themselves and solve them.

Review at BB with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to those who were wrong. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

a) Tina has £2 and Joe has £17. How much should Joe give to Tina so that they both have the same amount?
   
   **Plan:** \( (\£17 - \£2) \div 2 = \£15 \div 2 = \£7.50 \)
   
   **Check:** T: \( \£2 + \£7.50 = \£9.50 \)
   
   J: \( \£17 - \£7.50 = \£9.50 \ ✔ \)
   
   **Answer:** Joe should give £7.50 to Tina.

b) Colin has £23. If we add his money to Kate's money, the total amount is £11. How much does Kate have?
   
   **Plan:** \( \£23 + K = \£11 \), so \( K = \£11 - \£23 = -\£12 \)
   
   **Answer:** Kate has debts of £12, (or has a balance of – £12).

c) Arnie has a bank balance of – £43. If he adds it to Christine's bank account, the balance is £17 altogether. How much does Christine have in her bank account?
   
   **Plan:** \( \£17 - (\£43) = \£17 + \£43 = \£60 \)
   
   or \( -\£43 + C = \£17 \), so \( C = \£17 - (\£43) = \£60 \)
   
   **Answer:** Christine has £60 in her bank account.

---

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**Y5**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 63</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>PbY5a, page 63</td>
</tr>
<tr>
<td>Q.2 Read: Do the numbers in the set make the statements true or false? Complete the table.</td>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td>Set a time limit or deal with one row at a time. Less able Ps could have number lines to help them.</td>
<td>Draw on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Show on number line or with a model or reason with words if problems or disagreement.</td>
<td>Discussion, reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td>Feedback for T</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Statement</th>
<th>Numbers which make it true</th>
<th>Numbers which make it false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-5 \leq x = 8$</td>
<td>$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$</td>
<td>$-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$</td>
</tr>
<tr>
<td>$6 + \square = 1$</td>
<td>$-5$</td>
<td>$-4, -3, -2, -1, 0, 1, 2, 3, 4, 5$</td>
</tr>
<tr>
<td>$\square &lt; 3$</td>
<td>$-5, -4, -3, -2, -1, 0, 1, 2$</td>
<td>$3, 4, 5$</td>
</tr>
<tr>
<td>$5 - \square &gt; 6$</td>
<td>$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$</td>
<td>$-6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$</td>
</tr>
<tr>
<td>$4 + \square \leq 8$</td>
<td>$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$</td>
<td>$5$</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>8</th>
<th>PbY5a, page 63</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.3 Read: List the integers represented by the shapes. Show the solutions on a number line.</td>
<td>Individual work, monitored, helped</td>
<td></td>
</tr>
<tr>
<td>What kind of numbers should not be listed? (Fractions or decimals, as the solutions are integers, i.e. whole numbers)</td>
<td>Written on BB or use enlarged copy master or OHP</td>
<td></td>
</tr>
<tr>
<td>Elicit/ remind Ps how to list lots of numbers (using <em>ellipses</em>).</td>
<td>BB: <em>ellipses</em> . . . (stands for numbers or words which are not shown)</td>
<td></td>
</tr>
<tr>
<td>Deal with one at a time (or set a time limit if class is very able). P’s draw the relevant segment of the number line in <em>Ex. Bks</em> and mark the solution with dots. (Accept rough drawings – Ps should not spend too much time drawing exact number lines!)</td>
<td>For difficult inequalities, advise Ps to think of it an an equality first, then decide in which direction to move along the number line.</td>
<td></td>
</tr>
<tr>
<td>Review with whole class. Ps could show solutions on scrap paper or slates on command. Ps answering correctly come to BB to explain to Ps who were wrong. Class checks mentally that each number in solution makes the statement true. Mistakes discussed and corrected.</td>
<td>Discussion, reasoning, checking, agreement, self-correction, praising</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td>Accept trial and error but ask for logical reasoning too; e.g. using reverse operation or showing on number line or using a model or explaining in words.</td>
<td></td>
</tr>
</tbody>
</table>

| a) $-4 - \square = -18$ | b) $\square \times \triangle < 1$ | c) $13 + \square = -10$ |
| [Diagram] (numbers shaded) | [Diagram] (numbers shaded) | [Diagram] (numbers shaded) |
| $\square = 14$ | $\triangle = -23$ | $\square = -5$ |
| $-4, -5, 5, 10, 15$ | $-5, 0, 1, 2, 3, \ldots$ | $-20, -15, -10, -5, 0, 5, 10, 15$ |
| d) $-3 \quad \square \leq -15$ | e) $10 - \square = 15$ | f) $\square - (-2) > 5$ |

| [Diagram] (numbers shaded) | [Diagram] (numbers shaded) | [Diagram] (numbers shaded) |
| $\square = 14, 13, 12, \ldots$ | $\triangle = -5$ | $\square = 4, 5, 6, \ldots$ |
| $-8, -5, 5, 10, 15$ | $-5, 0, 5, 10, 15$ | $0, 5, 10, 15$ |
| g) $\square + (-3) = -5$ | h) $-8 \quad \square \geq -10$ | i) $-10 + (-x) = -11$ |

| [Diagram] (numbers shaded) | [Diagram] (numbers shaded) | [Diagram] (numbers shaded) |
| $\square = -2$ | $\triangle = -10, -9, -10, \ldots$ | $\triangle = 1$ |
| $-5, -2, 0, 3, 5$ | $-10, -9, -8, -7, -6, \ldots$ | $-5, 0, 5, 10$ |
**Activity 9**

*PbY5a, page 63, Q.4*

Read: *List the integers represented by the shapes. Show the solutions on a number line.*

Ps come to BB to explain reasoning, writing interim steps in working where appropriate (as below) and showing solution on class number line or on relevant segment of number line drawn on BB. Class checks solutions by mentally substituting values in the equations or inequalities and points out errors.

**Solution:**

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>□ × 6 ≥ −18</td>
<td>b)</td>
<td>+8 × □ ≤ 0</td>
<td>c)</td>
<td>△ × 4 = △ + (−12)</td>
<td>d)</td>
</tr>
<tr>
<td></td>
<td>□ ≥ −3</td>
<td></td>
<td>□ ≤ 0</td>
<td></td>
<td>△ × 3 = −12</td>
<td></td>
</tr>
<tr>
<td></td>
<td>−3, −2, −1, …</td>
<td></td>
<td>0, −1, −2, …</td>
<td></td>
<td>△ = −4</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>e)</td>
<td>○ + 5 = −3</td>
<td>f)</td>
<td>(−5) + △ ≤ +6</td>
<td></td>
<td>△ &lt; 11</td>
<td></td>
</tr>
<tr>
<td></td>
<td>□ = −15</td>
<td></td>
<td>□ = −15</td>
<td></td>
<td>△: 10, 9, 8, …</td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

Whole class activity (or individual trial first if Ps wish and there is time)

Written on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, (self-correction), praising

Ps work in Pbs too.

Praising, encouragement only

Feedback for T
## Y5

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>Lesson Plan</strong></td>
</tr>
<tr>
<td><strong>Addition: order of terms</strong></td>
<td><strong>64</strong></td>
</tr>
<tr>
<td>Let's do this calculation in two ways.</td>
<td></td>
</tr>
<tr>
<td>BB: $-5 + (+7) + (-6) + (-7) + (+5) + (+6) =$</td>
<td></td>
</tr>
<tr>
<td>A, come and show us one way. Who agrees? Who can write it another way? Which way do you think is best? Why?</td>
<td></td>
</tr>
<tr>
<td>a) Adding all the negative numbers, and then all the positive numbers:</td>
<td></td>
</tr>
<tr>
<td>BB: $-5 + (-6) + (-7) + (+5) + (+6) + (+7) = -18 + (+18) = 0$</td>
<td></td>
</tr>
<tr>
<td>b) Cancelling out opposite numbers:</td>
<td></td>
</tr>
<tr>
<td>BB: $-5 + (-5) + (-6) + (+6) + (-7) + (+7) = 0 + 0 + 0 = 0$</td>
<td></td>
</tr>
<tr>
<td><strong>2</strong> Problem 1</td>
<td>Whole class activity</td>
</tr>
<tr>
<td>Listen carefully, follow my instructions, and show me the result when I say. Nod your heads when you have done each step.</td>
<td>Written on BB or SB or OHT</td>
</tr>
<tr>
<td>Think of the opposite of $+5$. Take its absolute value. Add $-10$. Multiply by 4. Divide by 5. Add a secret number to the result so that the sum is $+10$. What is the secret number?</td>
<td>Reasoning, agreement, praising</td>
</tr>
<tr>
<td>Show me . . . now! (14)</td>
<td>At a good pace</td>
</tr>
<tr>
<td>Ps with correct answer explain each step at BB.</td>
<td>Of course, also accept normal order of calculating (from left to right).</td>
</tr>
<tr>
<td>$- (+5) = -5$ $</td>
<td>{-5}</td>
</tr>
<tr>
<td><strong>3</strong> Number sets</td>
<td>Whole class activity</td>
</tr>
<tr>
<td>Study this set diagram. What do you think the labels mean?</td>
<td>Encourage mental calculation but less able Ps may write the interim steps in Ex. Bks. or on slates or scrap paper.</td>
</tr>
<tr>
<td>BB:</td>
<td>In unison.</td>
</tr>
<tr>
<td>Ask several Ps what they think. Elicit that:</td>
<td>Reasoning, agreement, praising</td>
</tr>
<tr>
<td>Z = { whole numbers or integers }</td>
<td>Ask Ps with incorrect answers what kind of mistakes they made.</td>
</tr>
<tr>
<td>N = { positive whole numbers or natural numbers }</td>
<td></td>
</tr>
<tr>
<td>Who can tell me other numbers which belong in the sets but are not shown? (i.e. represented by the ellipsis in each set) Ps suggest a few.</td>
<td></td>
</tr>
<tr>
<td>Who can think of a true statement to say about the diagram? T gives an example first if necessary, then Ps make own statements and class checks that it is true.</td>
<td></td>
</tr>
<tr>
<td>e.g. 'There is an integer which is not negative and not positive.' (0)</td>
<td></td>
</tr>
<tr>
<td>'All natural numbers are integers.'</td>
<td></td>
</tr>
<tr>
<td>'Not all integers are natural numbers.'</td>
<td></td>
</tr>
<tr>
<td>'Every natural number has an opposite integer.'</td>
<td></td>
</tr>
<tr>
<td><strong>Extension</strong></td>
<td>Extra praise for creativity!</td>
</tr>
<tr>
<td>T: We can say that set N is part of of set Z, or N is a subset of Z and write it mathematically like this.</td>
<td>BB: $N \subset Z$</td>
</tr>
<tr>
<td>N is a subset of Z</td>
<td></td>
</tr>
</tbody>
</table>
**Y5**

**Activity**

4 **Missing signs**

Let’s fill in the missing signs. Ps come to BB to write signs and explain reasoning. Class agrees/disagrees. Show on number line if problems.

**BB:**

- a) \(-5 \underline{x} |5|\)
- b) \(-3 + (+2) \underline{\times} (-3) \times 2\)
- c) \(-6 \underline{=} |+6|\)
- d) \((-4) + (-5) \underline{=} |-10|\)
- e) \((-4) \times 3 \underline{=} (-4) + (-4) + (-5)\)
- f) \(3 \times 0 + (-2) \underline{=} (-3) \times 2\)
- g) \((-18) \div 2 \underline{=} 3 \times [-3]\)
- h) \((-15) \div 5 \underline{=} |-3|\)

4 Missing signs

**Notes**

**Lesson Plan 64**

6 **PbY5a, page 64**

Q.1 Read: Find a rule and complete the table.

Set a time limit. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees.

(Who can write the rule? Who agrees? Who can write it another way? etc. Check with values from the table.

**Solution:**

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c}
& x & 4 & -1 & 0 & 17 & -29 & -165 & +40 & -1024 & +12 & \text{LHS}\ 
\hline
y & -4 & 1 & 0 & -17 & +29 & 165 & -40 & +1024 & -12 & +309 & \text{RHS}\ 
\end{array}
\]

\[y = \text{opposite of } x, \text{ or } y = -x, \quad x = \text{opposite of } y, \text{ or } x = -y\]

Individual work, monitored

Drawn on BB or use enlarged copy master or OHT

Differentiation by time limit.

Reasoning, agreement, self-correction, praising

Feedback for T

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**Y5**

**Activity**

6  
(Continued)

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>b)</td>
<td>a</td>
<td>5</td>
<td>-4</td>
<td>+ 11</td>
<td>0</td>
<td>+105</td>
<td>-48</td>
</tr>
<tr>
<td>b</td>
<td>5</td>
<td>4</td>
<td>+11</td>
<td>0</td>
<td>105</td>
<td>48</td>
<td>+382</td>
</tr>
</tbody>
</table>

\[ b = \text{absolute value of } a, \, \text{or } b = |a| \quad \text{or } a = \pm b \]

---

7  
**PbY5a, page 64**

Q.2 Read: *Which is more? How many more? Fill in the missing signs and write the differences.*

Set a time limit. Ps can write results of each side above operation signs first if necessary before writing signs and differences. (Less able Ps could have number line or thermometer model to help them.) Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Mistakes discussed and corrected. Show on class number line if problems. T chooses Ps to read the operations in different ways (with and without positive signs, and inequalities from left to right and right to left.)

**Solution:**

\[
\begin{align*}
\text{a) } & -3 + 2 = -3 + ( + 2) \\
\text{b) } & 4 - 3 = 4 + (-3) \\
\text{c) } & -4 - 3 < 4 + (-3) \\
\text{d) } & -4 - 5 = -4 + (-5) \\
\text{e) } & 3 + (-4) \geq 3 - 4 \\
\text{f) } & 5 - 2 \leq -5 + (-2) \\
\end{align*}
\]

---

8  
**Erratum**

In PbS:

h) ’+ 10’ should be ’– 10’

---

**Notes**

Agree that \( a \) can be either + \( b \) or – \( b \), as the absolute values are the same. T shows how to write it mathematically.

---

Individual work, monitored  
Written on BB or SB or OHT  
Differentiation by time limit  
Discussion, reasoning, agreement, self-correction, praising  
Show on number line or with a model if problems.

Feedback for T

---

Individual work, monitored, (helped)  
Written on BB or SB or OHT  
Differentiation by time limit  
Reasoning, agreement, self-correction, praising

Reasoning:

\( a \):

’I had debts of £7, then I earned £12 and paid back the £7 that I owed, so I now have £5 left. or ’+ 5 is 12 more than – 7’ or Ps show on number line.
**Activity 9**  

**Rules for addition and subtraction of integers**

Let's think of rules about adding and subtracting integers. T poses the question and asks one or two Ps what they think. Then T states the rule given below and asks Ps whether it is correct. Ps test the rule with examples on BB. T says the rule again (and/or uncovers rule written on BB or SB or OHT) and Ps repeat it in unison.

**a) How do we add whole numbers which have the same sign?**

**Rule:** We can add whole numbers which have the same sign by adding their absolute values and keeping that sign.

**b) How do we add a positive and a negative whole number?**

**Rule:** We can add a positive and a negative whole number by calculating the difference of their absolute values and taking the sign of the greater absolute value.

**c) How do we subtract an integer?**

**Rule:** We can subtract an integer by adding its opposite number.

---

**Homework (optional)**

**PbY5a, page 64, Q.6**

Find a rule. Complete the table. Draw a graph to show the data.

(Use copy master grid.)

**Solution:**

<table>
<thead>
<tr>
<th>x</th>
<th>+3</th>
<th>+5</th>
<th>+1</th>
<th>-1</th>
<th>0</th>
<th>+8</th>
<th>-5</th>
<th>-6</th>
<th>e.g.</th>
<th>+7</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>+1</td>
<td>-1</td>
<td>+5</td>
<td>+4</td>
<td>-4</td>
<td>+9</td>
<td>+9</td>
<td>+10</td>
<td>-3</td>
<td>-3</td>
</tr>
</tbody>
</table>

**Rule:** \(x + y = 4,\) \(x = 4 - y,\) \(y = 4 - x\)
Week 13

Lesson Plan

65

Review of homework in Lesson 64:

**PbY5a, page 64**

Q.6  **Graph:**

**PbY5a, page 65**

**Solutions:**

Q.1  

a) $3.5 \times 4 = \underline{14} \text{ units}$  

b) $2.5 \times 4 \text{ units} = \underline{10} \text{ units}$  

c) $10 \text{ units} \div 3 = 3 \frac{1}{3} \text{ units}$  

d) $25 \text{ units} \div 5 \text{ units} = \underline{5} \text{ (seconds)}$

Q.2  

a) $(-7) + (-7) + (-7) + (-7) + (-7) \equiv (-7) \times 5 = -35$

b) $(-11) + (-11) + (-11) + (-11) + (-11) + (-11) = (-11) \times 6 = -66$

c) $(-30) + (-30) + (-30) + (-30) = (-30) \times 4 = -120$

Q.3

<table>
<thead>
<tr>
<th>Statement</th>
<th>Numbers which make it true</th>
<th>Numbers which make it false</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x = 9$</td>
<td>$-3$</td>
<td>$-6, -5, -4, -2, -1, 0, 1, 2, 3, 4$</td>
</tr>
<tr>
<td>$x = 0$</td>
<td>$-6$</td>
<td>$-5, -4, -3, -2, -1, 0, 1, 2, 3, 4$</td>
</tr>
<tr>
<td>$x &gt; 2$</td>
<td>$-6, -5, -4, -3, -2, 1, 0, 1$</td>
<td>$2, 3, 4$</td>
</tr>
<tr>
<td>$x &lt; 4$</td>
<td>$-6, -5, -4, -3, -2$</td>
<td>$-1, 0, 1, 2, 3, 4$</td>
</tr>
<tr>
<td>$x \leq 2$</td>
<td>$-6, -5, -4, -3, -2$</td>
<td>$-1, 0, 1, 2, 3, 4$</td>
</tr>
</tbody>
</table>

Q.4

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3$</th>
<th>$+4$</th>
<th>$+1$</th>
<th>$-5$</th>
<th>$-2$</th>
<th>$0$</th>
<th>$-1$</th>
<th>$+2$</th>
<th>$+3$</th>
<th>$-4$</th>
<th>$+5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>$+2$</td>
<td>$-5$</td>
<td>$-2$</td>
<td>$+4$</td>
<td>$+1$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$-3$</td>
<td>$-4$</td>
<td>$+3$</td>
<td>$-6$</td>
</tr>
</tbody>
</table>

**Rule:** $x + y = -1$, or $x = -1 - y$, or $y = -1 - x$

**Graph:**

Use grid on copy master for LP64/12
### Y5 Lesson Plan

**Week 14**

| R: Calculations                  | Lesson Plan 66 |
| C: Operations with natural numbers (mental and written) |
| E: Word problems with integers, fractions and decimals |

#### Activity

**Reading numbers**

Let’s read out these numbers together. T points to each number in turn and class reads it in unison. If T notes Ps having difficulties, ask them to repeat it alone. Ps can write some numbers too and class reads them.

BB: e.g., Ps:
- 65 960 (sixty-five thousand, nine hundred and sixty)
- 100 083 (one hundred thousand and eighty-three)
- 3 212 345 (three million, two hundred and twelve thousand, three hundred and forty-five)

**Writing numbers**

- a) T dictates the numbers below and Ps write them as digits in Ex. Bks. Review quickly with whole class. Ps come to BB or dictate digits to T. Mistakes discussed and corrected.

- b) Underline the odd numbers. A, what did you underline? (404 827) Who agrees? How do you know it is odd? (If the units digit is odd, the whole number is odd.)

- c) In your Ex. Bks, round each number to the nearest ten, then the nearest hundred, then the nearest thousand, one below the other in your Ex. Bks. Set a time limit. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected. Agree that the whole number should be rounded according to the required place-value and not 1 digit at a time.

<table>
<thead>
<tr>
<th>BB:</th>
<th>Rounded:</th>
</tr>
</thead>
<tbody>
<tr>
<td>27 048 (\approx) 27 050</td>
<td>to nearest 10</td>
</tr>
<tr>
<td>27 000</td>
<td>to nearest 100</td>
</tr>
<tr>
<td>27 000</td>
<td>to nearest 1000</td>
</tr>
<tr>
<td>30 000</td>
<td>to nearest 10 000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BB:</th>
<th>Rounded:</th>
</tr>
</thead>
<tbody>
<tr>
<td>404 827 (\approx) 404 830</td>
<td>to nearest 10</td>
</tr>
<tr>
<td>404 800</td>
<td>to nearest 100</td>
</tr>
<tr>
<td>405 000</td>
<td>to nearest 1000</td>
</tr>
<tr>
<td>400 000</td>
<td>to nearest 10 000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>BB:</th>
<th>Rounded:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 413 652 (\approx) 1 413 650</td>
<td>to nearest 10</td>
</tr>
<tr>
<td>1 413 700</td>
<td>to nearest 100</td>
</tr>
<tr>
<td>1 414 000</td>
<td>to nearest 1000</td>
</tr>
<tr>
<td>1 410 000</td>
<td>to nearest 10 000</td>
</tr>
</tbody>
</table>

**Notes**

Whole class activity

Written on BB or SB or OHT (with spaces or commas between Th and H, M and HTm, to make numbers easier to read)

In unison

Praising, encouragement only

Individual work, monitored

T reads out numbers while walking round class.

Discussion, reasoning, agreement, self-correction, praising

T could demonstrate layout on BB (as below left).

**Extension**

Who can think of other questions to ask about the numbers? e.g.

- [Which is smallest/greatest? What is their sum? What is the difference between the greatest and smallest? Which are multiples of 4? (only tens and units digits need to be taken into account as 100 is exactly divisible by 4) etc.] Extra praise for clever questions
**Activity 3**

**PbY5a, page 66**

Q.1 Read: *Practise mental calculation.*

How many operations are there? \((4 \times 4 = 16)\)

Let's see how many you can do in 3 minutes. Do the calculations in your head and write only the results. Start... now! ... Stop!

Review with whole class. Ps dictate results to T, saying the whole operation. Who made a mistake? What was your mistake? Ask for details of calculation only if problems.

Who had all 16 correct? Let's give them a clap! Who made 1 (2, 3, more than 3) mistakes? Let's see if you can do better next time!

**Solution:**

a) \(6 + 8 = 14\)  
b) \(24 + 5 = 29\)  
c) \(32 + 19 = 51\)  
d) \(250 + 190 = 440\)  
e) \(13 - 8 = 5\)  
f) \(26 - 12 = 14\)  
g) \(54 - 18 = 36\)  
h) \(350 - 140 = 210\)  
i) \(6 \times 7 = 42\)  
j) \(14 \times 5 = 70\)  
k) \(6 \times 90 = 540\)  
l) \(18 \times 100 = 1800\)  
m) \(30 \div 5 = 6\)  
n) \(42 \div 7 = 6\)  
o) \(150 \div 10 = 15\)  
p) \(250 \div 10 = 25\)  

\[12\text{ min}\]

**Notes**

Individual work, monitored
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
At speed
Agreement, self-correction, evaluation, praising
Ps mark and correct own work and give themselves a score out of 16.
Feedback for T

---

**Activity 4**

Erratum

In Pbs, final 'd)' should be 'f)'

**PbY5a, page 66**

Q.2 Read: *Do these calculations in your exercise book.*

Set a time limit. Ps can use any method of calculation. Ps finished first come to BB to write their calculations (but keep them hidden from rest of class).

Review with whole class. Ps dictate results to T, saying the whole operation and T uncovers calculation on BB. Who made a mistake? What was your mistake? Who did it this way? Who did it another another way? etc.

Who had all 6 correct? Let's give them a clap! Who made fewer mistakes than in Q.1? Give them a pat on the back!

**Solution:**

a) \(4335\)  
b) \(613\)  
c) \(63\)  
d) \(98\)  
e) \(7015\)  
f) \(74412\)

\[18\text{ min}\]

**Notes**

Individual work, monitored
Written on BB or SB or OHT
Reasoning, agreement, self-correcting, praising
Only go through details of working if several Ps made a mistake.
Feedback for T

\[f)\]

\[\frac{7152623}{-497}\]

\[\frac{-292}{-284}\]

\[83\]

\[-71\]

\[72\]

---

**Activity 5**

**PbY5a, page 66**

Q.3 Read: *Solve these problems in your exercise book.*

Deal with one at a time or set a time limit. (If Ps are not very able, agree on the plan first.) Ps read problems themselves and solve in Ex. Bks.

Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly come to BB to explain to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

T chooses a P to give the answer in a sentence.

\[\text{In unison}\]

Reasoning, agreement, self-correction, praising

---

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5 (Continued)

**Solution:**

a) How much money has Philip saved if he still needs £217 before he has enough money to buy the £1520 boat that he wants?

**Plan:** £1520 – £217

**C:**

\[
\begin{array}{c}
1520 \\
217 \\
1303 \\
\end{array}
\]

**Answer:** Philip has saved £1303.

b) Andrew has saved £385, which is £127 less than the amount that Ben has saved. Ben’s sister, Kate has saved £82.

How much money have the two boys saved?

**Plan:**

\[
A: \text{£385, } B: \text{£385 + £127},
\]

**C:**

\[
\begin{array}{c}
385 \\
127 \\
512 \\
\end{array}
\]

**Answer:** The two boys have saved £897.

c) Charlie has gathered 258 kg of pears. How much money will he make if he sells the pears for 91 p per kg?

**Plan:**

\[
258 \times 91 \text{ p}
\]

**C:**

\[
\begin{array}{c}
258 \\
91 \\
23478 \\
\end{array}
\]

**Answer:** Charlie will make £234.78.

6 **Number line**

Let’s mark the numbers from – 5 to + 5 on the number line. Which number line would you choose? Ask several Ps what they think.

Elicit that a) is impossible, as the distances between the integers are not equal, so it is impossible to decide where to mark the other numbers.

Agree that in b), the distance between – 1 and 2 can be measured and then divided into 3 equal lengths (as there are 3 units between – 1 and 2).

Ps come to BB to measure with BB ruler, work out the length between each integer, draw the ‘ticks’ and mark and label the other 9 numbers.

Class points out errors.

**BB:**

a)

b)

7 *PbY5a, page 66, Q.4*

Read: Write an operation for each problem and calculate the result in your exercise book.

Deal with one part at a time. T chooses a P to read each sentence and Ps come to BB to write operations and work out the results, explaining reasoning. Class points out errors. Show on number line if problems.

**Solution:**

a) How much is Linda’s balance if she owes £24 and has only £11 in her account?

**BB:** \[-24 + 11 = -13\]

Linda’s balance is –£13.

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### Activity

#### Y5

**Erratum**  
In Pbs in c), 'then' should be 'than'

(Continued)

b) **How much is Kate's balance if she is £100 in debt and has £170 in her account?**  
   BB: \(-100 + 170 = 70\)  
   Kate's balance is £70.

c) **How much more or less is £110 in cash than £80 in debt?**  
   BB: \(110 - (-80) = 190\)  
   Having £110 in cash is £190 more than being £80 in debt.

d) **How much higher or lower is \(-170\) m than \(-4900\) m?**  
   BB: \(-170 - (-4900) = -170 + 4900 = 4730\)  
   \(-170\) m is 4730 m higher than \(-4900\) m.  
   (or \(-4900\) m is 4730 m lower than \(-170\) m.)

e) **How much more or less is £800 outgoings than £700 income?**  
   BB: \(-800 - (+700) = -1500\)  
   or \(700 - (-800) = 700 + 800 = 1500\)  
   £800 outgoings is £1500 less than £700 income.  
   (or £700 income is £1500 more than £800 outgoings)

---

### Lesson Plan 66

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discuss what the negative values could mean (e.g. depth below sea level)</td>
</tr>
<tr>
<td>Elicit that:</td>
</tr>
<tr>
<td>• outgoings are bills or debts to be paid;</td>
</tr>
<tr>
<td>• income is money earned or received as a gift, or won.</td>
</tr>
</tbody>
</table>

### 8 PbY5a, page 66

Q.5 Read:  
  a) **Write the operations in a shorter form.**  
  b) **Calculate.**

Set a time limit. Ps can use Ex. Bks if they need more space.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Use a model or diagram to explain if necessary.

Who can think of a word problem about it? Class decides whether it matches the operation and makes sense.

**Solution:**

a)  
  - i) \(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{1}{8} \times 5 = \frac{5}{8}\)
  - ii) \(\frac{2}{9} + \frac{2}{9} + \frac{2}{9} = \frac{2}{9} \times 3 = \frac{6}{9} = \frac{2}{3}\)
  - iii) \(£4.50 + £4.50 + £4.50 + £4.50 = £4.50 \times 4 = £18\)

b)  
  - i) \(\frac{7}{10} - \frac{3}{10} = \frac{4}{10} = \frac{2}{5}\)
  - ii) \(1 - \frac{4}{5} = \frac{1}{5}\)  
  - iii) \(3 - \frac{1}{6} = 2 - \frac{1}{6} = \frac{11}{6}\)

Extra work for quick Ps can be written on BB: 
  e.g.  
  - c) i) \(£4.50 - £2.20 = (£2.30)\)
  - ii) \(£10 - £3.50 = (£6.50)\)

---

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**Y5**

<table>
<thead>
<tr>
<th>Activity</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY5a, page 66. Q.6</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Read:</strong> Find a rule. Complete the table.</td>
<td></td>
</tr>
<tr>
<td>Study the completed columns and think of the rule. A, what do you think it is? Who agrees? Who thinks something else? Class decides which rule to use (it need only be in words at this stage), then Ps come to BB to choose a column and fill in the missing numbers, explaining reasoning. Class points out errors. Rest of Ps fill in table in <em>Pbs</em> at same time.</td>
<td></td>
</tr>
<tr>
<td>Let’s write the rule in different ways. Ps come to BB or dictate to T. Class checks rules mentally by inserting values from a column in table and agrees/disagrees. Ps complete rules in <em>Pbs</em> too.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>5</th>
<th>9</th>
<th>6</th>
<th>7</th>
<th>10</th>
<th>6</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>3</td>
<td>2</td>
<td>5</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>7</td>
<td>8</td>
<td>11</td>
<td>30</td>
</tr>
<tr>
<td>z</td>
<td>22</td>
<td>17</td>
<td>46</td>
<td>82</td>
<td>61</td>
<td>50</td>
<td>81</td>
<td>67</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

**Rule:**

\[
\begin{align*}
z &= x \times y + 1 \\
y &= \frac{z - 1}{x} \\
x &= \frac{z - 1}{y}
\end{align*}
\]

Who can think of other values for extra columns in the table?  

---

**Lesson Plan 66**

**Notes**

Whole class activity
(or individual work if Ps wish and there is enough time, reviewed with whole class)

Drawn on BB or use enlarged copy master or OHP

Discussion on the rule

At a good pace

Involve several Ps.

Reasoning, agreement, (self-correction), praising

Also accept:

\[
\begin{align*}
x \times y &= z - 1, \\
x \times y - z &= 1
\end{align*}
\]

Who can think of other values for extra columns in the table?
**Lesson Plan 67**

**Week 14**

### Activity 1

**Operations with negative integers**

Let’s continue the pattern. Ps come to BB or dictate to T. Class points out errors. Ask Ps to explain some using the models suggested below.

**BB:**

- a) $3 + (-5) = -2$
- b) $-3 - (+2) = -5$
- c) $-3 \times 5 = -15$

- $3 + (-4) = -1$
- $3 - (+1) = -4$
- $3 \times 4 = -12$

- $3 + (-3) = 0$
- $3 - 0 = -3$
- $3 \times 3 = 9$

- $3 + (-2) = 1$
- $3 - (-1) = 2$
- $3 \times 2 = 6$

**Models:**

- a) Cash and debt, or car moving (or P stepping) along number line
- b) Comparison (e.g. $-3$ is 2 more than $-5$), or drawing an arrow from the subtrahend to the reductant on the number line.
- c) In words, or if Ps would like to try it, using the car model:

  - If multiplicand negative, multiplier positive (e.g. $-3 \times 5 = -15$): car faces left and starts at 0, then moves 3 units forward 5 times.
  - If multiplicand negative, multiplier negative (e.g. $-3 \times (-1) = +3$): car faces left and starts at 0, then moves 3 units backwards 1 time.

**Extension**

**Mental practice**

Study these operations, do the calculations in your head and show me the result when I say. Look for easy ways to do it quickly!

**BB:**

- a) $-5 + 3 - 8 + 6 + 5 - 6 - 3 + 8 = 0$ (opposite pairs cancel out)
- b) $-6 - (-8) = -6 + (+8) = +2$ (subtracting a negative number is the same as adding the opposite positive number)
- c) $+12 - (+10) = +12 + (-10) = +2$
- d) $-20 - (-20) + (-10) + (+10) = 0$

**Fractions of a shape**

Each shape is 1 unit. What part of each shape has been shaded?

Ps come to BB to write and say fractions and explain reasoning. (e.g. in a), 'The circle has been divided into 4 equal parts, so each part is quarter of the circle. Three of the parts have been shaded, so the part shaded is 3 quarters of the circle.' Class points out errors.

**BB:**

- a) $\frac{1}{4}$
- b) $\frac{3}{8}$
- c) $\frac{2}{3}$
- d) $\frac{1}{4}$
- e) $\frac{3}{6} = \frac{1}{2}$

Revise meaning of numerator and denominator and that:

**BB:**

$1 = \frac{4}{4} = \frac{8}{8} = \frac{3}{3} = \frac{6}{6} = \ldots$ (Ps dictate others.)

Which part is not shaded? Ps come to BB to write as a subtraction. Is it sensible to add all the fractions together? (No, as units not the same.)

**Whole class activity**

Written on BB or SB or OHT

At a good pace

Agreement, praising

In c), have no expectations beyond $-3 \times 0$, but if a P continues with $-3 \times (-1) = +3$ give extra praise and ask following Ps to follow the pattern.

If P gets it wrong, T could use the car model as described opposite to show the correct answer, but do not expect Ps to learn it yet.

**Whole class activity**

Written on BB or SB or OHT, with underlined numbers missing.

Responses shown on scrap paper or slates in unison.

Reasoning, agreement, praising

**Whole class activity**

Drawn on BB or use enlarged copy master or OHP (Or Ps could show fractions on scrap paper or slates in unison on command.)

Discussion, reasoning, agreement, praising

**BB:** numerator $\rightarrow \frac{3}{4}$

denominator $\rightarrow \frac{4}{4}$

Elicit that fractions which have the same value are called equivalent fractions.

**BB:**

e.g. $1 - \frac{3}{8} = \frac{5}{8}$
**MEP: Primary Project**

**Week 14**

### Activity 4

#### Problems

Listen carefully and show me the answer as a fraction when I say. Ps responding correctly come to BB to explain to those who were wrong. Class agrees/disagrees.

What would the fraction be as a decimal? Ps come to BB to write it and class agrees/disagrees. Who can think of another question to ask?

**a)** In a class of 25 pupils, 11 pupils are girls. What part of the class are girls?

Show me . . . now! \( \frac{11}{25} \) BB: \( \frac{11}{25} = \frac{44}{100} = 0.44 \)

[What part are boys? (14 twenty-fifths) ]

**b)** There are 10 balls in a box and 7 of them are white. What part of the total number of balls are white?

Show me . . . now! \( \frac{7}{10} \) BB: \( \frac{7}{10} = 0.7 \)

[What part is not white? (3 tenths) ]

**c)** Alice has 11 dolls and Barbara has 10 dolls. What is the ratio of the number of dolls they have?

Show me . . . now! \( \frac{B}{A} = \frac{10}{11} \), or \( \frac{A}{B} = \frac{11}{10} \) Accept both!

BB: \( \frac{11}{10} = \frac{10}{10} + \frac{1}{10} = 1 + \frac{1}{10} = 1\frac{1}{10} = 1.1 \)

Ps use a calculator to show that \( \frac{10}{11} = 0.909 \) (to 3 decimal places)

---

### Lesson Plan 67

**Notes**

Whole class activity Responses shown on scrap paper or slates and shown in unison.

Reasoning, agreement, praising

Elicit, or remind Ps, that multiplying or dividing the numerator and denominator of a fraction by the same number does not change the value of the fraction.

Other questions:

**a)** Part boys: \( \frac{14}{25} = \frac{28}{50} = \frac{56}{100} = 0.56 \)

**b)** Part not white: \( \frac{3}{10} = 0.3 \)

Discuss what ratio means (the value of one amount in relation to another, or others)

Tell/elicit that it is usually written: BB: \( A : B = 11 : 10 \) or \( B : A = 10 : 11 \) but can be written as a fraction too.

---

### Activity 5

#### Perimeter and area

What is the perimeter and area of each of the rectangles? Ps come to BB to show the perimeter (distance around the outside of the shape) and the area (amount of space it covers) and count or calculate the values. Class agrees/disagrees.

BB:

- **a)**
  - Perimeter: 12 units
  - Area: 8 unit squares

- **b)**
  - Perimeter: 16 units
  - Area: 15 unit squares

- **c)**
  - Perimeter: 16 units
  - Area: 16 unit squares

- **d)**
  - Perimeter: 12 units
  - Area: 9 unit squares

If we let the length of the horizontal side be \( a \) and the vertical side be \( b \), what is the general rule (formula) for perimeter (area)?

Ps come to BB or dictate what T should write. Class agrees/disagrees.

BB: Perimeter of a rectangle: \( P = 2 \times a + 2 \times b = 2 \times (a + b) \)

Area of a rectangle: \( A = a \times b \) \( [= ab] \)

Perimeter of a square: \( P = 4 \times a \) \( [= 4a] \)

Area of a square: \( A = a \times a \) \( [= a^2] \)

T asks Ps to come to BB to shade different parts of the rectangles, e.g.

**a)** \( \frac{5}{8} \) of 8 = 5 (squares)  \( \frac{5}{8} \) of 16 = \( \frac{2}{16} \times 8 \times 5 = 10 \) (grid squares)

---

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**Lesson Plan 67**

### Notes

Whole class activity
- Drawn on BB or use enlarged copy master or OHP
- Responses shown in unison.
  - [Diagram of cube in Pbs is drawn in such a way that Ps can only determine it is a cube from the lengths of the sides.]
- Ps tell what they know: e.g. right angles; no. of sides, edges, faces, vertices; parallel, perpendicular edges /faces, etc.
- Agreement, praising Feedback for T

Individual work for a) and b), monitored, helped
- Individual measuring for c) to e), then whole class calculation
- Revise how to calculate the surface area and volume of cuboids and cubes.
- Refer to diagrams (or copy master or models) in *Activity 4.*
  - Discussion, reasoning, agreement, self-correction, praising

Extra praise if Ps point out equivalent values in cm.
- BB: 10 mm = 1 cm
  - 100 mm² = 1 cm²
  - 1000 mm³ = 1 cm³
- e.g. 46 mm = 4.6 cm
  - 112 mm² = 1.12 cm²
  - 1331 mm³ = 1.331 cm³
- etc.
  - (but T need not enforce it if Ps do not suggest it themselves)
### Lesson Plan 67

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

**Extension**

What is the perimeter of each rectangle?

Ps write below areas in *Pbs* or in *Ex. Bks*.

\[
P_1 = 2 \times (4 + 1) = 10 \text{ cm}
\]

\[P_2 = 2 \times (4 + 2) = 12 \text{ cm}
\]

\[P_3 = 2 \times (4 + 3) = 14 \text{ cm}
\]

\[P_4 = 2 \times (4 + 7) = 22 \text{ cm}
\]

\[P_5 = 2 \times (4 + 30) = 68 \text{ cm}
\]

Whole class discussion to decide on plan, then individual calculation in *Ex. Bks.*

(or whole class calculation on BB if time is short)

Drawn on BB or use enlarged copy master or OHP

(If possible, T also has real models made from cm cubes and nets in cm squares for demonstration.)

Involve as many Ps as possible.

At a good pace

Discussion, reasoning, agreement, (self-correction), praising

### Bb:

\[
V_1 = 5 \times 4 \times 3 = 60 \text{ (cm}^3)\]

\[V_2 = 4 \times 3 \times 3 = 36 \text{ (cm}^3)\]

\[V_3 = 4 \times 4 \times 4 = 64 \text{ (cm}^3)\]

Review the general rules for each type of cuboid. e.g.

\[A_{cube} = 6 \times a \times a \times (= 6a^2)\]

\[V_{cube} = a \times a \times a \times (= a^3)\]

etc.
Activity

Sequences
T writes the first 3 terms of a sequence on BB. Think of a rule and continue the terms. Ps dictate terms to T who writes them on BB. Class points out errors. T decides when to stop and then asks several to explain the rule. If Ps have difficulty, T gives hints. Elicit the actual values of the terms. If Ps have difficulty, T gives hints.

BB: 6, 24, 54, 96, 150, 216, 294, 384, ...
Elicit the actual values of the terms.

Rule: 6 times the square numbers in increasing order.

Elicit the actual values of the terms: 1, 8, 27, 64, 125, 216, ...
Rule: the number is the cube of an integer.

a) We can work out the perimeter of a rectangle if we add the lengths of its sides.

\[ P_{\text{rectangle}} = 2(a + b) \]

b) We can work out the area of a rectangle if we multiply two adjacent sides.

\[ A_{\text{rectangle}} = ab \]
\[ A_{\text{square}} = a^2 \]

If \( a \) and \( b \) are lengths in cm, what is the unit of area?

BB: Unit of area: \( 1 \text{ cm}^2 \)
What does it mean?

(a square which is 1 cm long by 1 cm wide)

If \( a \), \( b \) and \( c \) are the lengths in cm of 3 edges which meet at a vertex, what is the unit of volume?

BB: Unit of volume: \( 1 \text{ cm}^3 \)
What does it mean?

(a cube which is 1 cm long by 1 cm wide by 1 cm high)
Lesson Plan 68

Y5

Activity

3

PbY5a, page 68

Q.1  Read: The volume of a cuboid is 36 unit cubes and its edges are a whole number of units.

Fill in the table to show how long its edges could be.

What do the letters in the table mean? (The length, width and height of the cube in units) T demonstrates on a cuboid.

Encourage a logical listing. Set a time limit. Ps can discuss with their neighbours if they wish.

Review at BB with whole class. Ps come to BB or dictate to T, explaining reasoning. Class checks mentally that the 3 numbers multiply to make 36, agrees/disagrees, suggests values not shown or points out duplications.

Demonstrate that the cuboid can be turned around, so, e.g., \(a=1, b=4, c=9\), is the same cube as \(a=9, b=1\) and \(c=4\), i.e. the order of the values in each column does not matter.

Mistakes discussed and corrected.

Solution: What do you notice about any of the cuboids shown in the table? (Three are square-based cuboids: 1, 1, 36; 2, 2, 9; 3, 3, 4, but none of them are cubes.)

T points to certain columns and asks Ps for the surface area of the cuboids they show.

<table>
<thead>
<tr>
<th>a</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>8 possibilities, so 9th column is not needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>6</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>c</td>
<td>36</td>
<td>18</td>
<td>12</td>
<td>9</td>
<td>6</td>
<td>9</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

What do you notice about any of the cuboids shown in the table? (Three are square-based cuboids: 1, 1, 36; 2, 2, 9; 3, 3, 4, but none of them are cubes.)

T points to certain columns and asks Ps for the surface area of the cuboids they show.

4

Problem

Listen carefully, note the data and think how you would do the calculation.

The floor of a kitchen is 4 m 20 cm long and 3 m 60 cm wide. We want to cover the floor with 20 cm by 20 cm square tiles. How many tiles would we need? Ps suggest what to do first and how to continue. Who agrees? Who would do it another way? etc. T intervenes only if necessary.

Solution: e.g.

BB: Diagram:

\[
\begin{array}{c}
3 \text{ m} \ 60 \text{ cm} \\
(= 360 \text{ cm})
\end{array}
\quad
\begin{array}{c}
20 \text{ cm}
\end{array}
\]

\[
4 \text{ m} \ 20 \text{ cm} (= 420 \text{ cm})
\]

Plan:

\[
\begin{align*}
(420 \times 360) \text{ cm}^2 & \div (20 \times 20) \text{ cm}^2 \\
= (42 \times 36) \text{ cm}^2 & \div (2 \times 2) \text{ cm}^2 \\
= (21 \times 18) \text{ cm}^2 & \div (1 \times 1) \text{ cm}^2 \\
= 378 \text{ cm}^2 & \div 1 \text{ cm}^2 = 378 \times \frac{18}{21} \\
& = 378 \text{ (times)} + 3 \frac{6}{7} \\
& = 3 \frac{7}{8}
\end{align*}
\]

Answer: We would need 378 tiles.
Y5

**Activity**

5  
*PbY5a, page 68*

Q.2 Read: **Write a plan and calculate the result. Write the answer as a sentence.**

Set a time limit or deal with one part at a time. Ps read questions themselves, write a plan, do the calculation (in Ex. Bks if they need more space) and write the answer.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly come to BB to explain their solution. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

Solution: e.g.

The cost of hiring a 45-seater coach for a tour is £3780.

a) How much would it cost per person if 42 people go on the tour?

Plan: £3780 ÷ 42

Answer: It would cost £90 per person.

b) How much would it cost each person if 45 people go on the tour?

Plan: £3780 ÷ 45

Answer: It would cost each person £84.

31 min

6  
*PbY5a, page 68*

Q.3 Read: **Solve this problem in your exercise book.**

T chooses a P to read out the problem while the other Ps visualise the story in their heads and think how to solve it.

Last year Uncle Alex planted cabbages in a field which was 15 m wide and 40 m long.

This year he wants to plant cabbages in a new field but has not decided whether to use the 5 m wide field, the 24 m wide field or the 30 m wide field.

If he plants the same amount of cabbages as last year, what lengths will each of these fields have to be?

Who has an idea what to do? Ps make suggestions and class agrees or suggests better ways. Let’s see if you can solve it!

Set a time limit. Review with whole class. Ps come to BB to show solutions, explaining reasoning. Who did the same? Who did it a different way? etc. Mistakes discussed/corrected.

Solution: e.g.

Last year, area of cabbages: 15 m × 40 m = 600 m²

This year, area of cabbages: 600 m² (the same as last year)

5 m wide field: Plan: 600 m² ÷ 5 m = 120 m

24 m wide field: Plan: 600 m² ÷ 24 m = 25 m

30 m wide field: Plan: 600 m² ÷ 30 m = 60 m² ÷ 3 m = 20 m

Answer: The 5 m wide field would have to be 120 m long, the 24 m field 25 m long and the 30 m field 20 m long.

35 min

Lesson Plan 68

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td>Differentiation by time limit</td>
</tr>
<tr>
<td>Reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>Whole class discussion to start, then individual (or paired) work in Ex. Bks, monitored, helped</td>
</tr>
<tr>
<td>(Or Ps could show lengths on scrap paper or slates in unison on command.)</td>
</tr>
<tr>
<td>Discussion, reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>(inverse proportion)</td>
</tr>
</tbody>
</table>
Activity

7  

**PbY5a, page 68**

Q.4 Read: *Solve this problem in your exercise book. Write the answer here.*

*We have 48 cards and want to put them into envelopes so that there is the same number of cards in each envelope and none are left over.*

*How many envelopes could we use?*

Ps can choose to work individually or in pairs. Allow 3 minutes.

Who has an answer? Come and tell us what you did. Who did the same? Who did it a different way? etc.

**Solution:** e.g.

- Listing: 48 cards per envelope → 1 envelope
  - 24 → \( \frac{1}{2} \)
  - 16 → \( \frac{1}{3} \)
  - 12 → \( \frac{1}{4} \), etc.

Or drawing a table:

<table>
<thead>
<tr>
<th>No. of cards per envelope</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>12</th>
<th>16</th>
<th>24</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of envelopes</td>
<td>48</td>
<td>24</td>
<td>16</td>
<td>12</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>

Or number of envelopes possible must be the factors of 48:

1, 2, 3, 4, 6, 8, 12, 16, 24, 48

**Answer:** We could use 1, 2, 3, 4, 6, 8, 12, 16, 24 or 48 envelopes.

---

8  

**PbY5a, page 68**

Q.5 Read: *A strange clock whistles every 8 minutes, clicks every 3 minutes and chings every 12 minutes.*

*When it is turned on, after how many minutes will it whistle, click and ching at the same time?*

I will give you 3 minutes to work out the answer. Discuss it with your neighbour if you wish and try to solve it in your Ex. Bks. Start . . . now! . . . Stop!

If you have an answer, show me . . . now! Ps with correct solution come to BB to explain reasoning. Who did the same? Who did it a different way? etc. If no P responded correctly, T gives hint about **common multiples** and class solve it together.

**Solution:** e.g.

- Multiples of 8: 8, 16, \( \underline{24} \), 32, 40, \( \underline{48} \), 56, . . .

  Underline those which are also multiples of 3.
  Circle the first of these which is also a multiple of 12.

  **Answer:** The clock will whistle, click and ching at the same time after 24 minutes.

Elicit that **every** 24 minutes after that the same thing will happen.

(after 48 minutes, 72 minutes, etc., i.e. the multiples of 24)

Let's demonstrate what the clock does! T divides class into 3 groups: one to whistle, one to click their fingers and one to say 'ching'. Ps count from zero in relay round class but instead of saying the relevant multiples, they make the appropriate noises.
Y5

Lesson Plan

69

Activity

1

Sequences

T writes first few terms of a sequence on BB. Ps think of a rule and dictate the following terms. Ps can do necessary calculations in Ex. Bks. or on slates if they cannot do them mentally. Class points out errors. T decides when to stop and final P gives the rule.

BB:

a) 0, – 3, – 6, – 9, (– 12, – 15, – 18, – 21, – 24, – 27, – 30, . . .)
   Rule: Decreasing by 3, or [– 3]

   Rule: Decreasing by 3 times, or [× 3]

c) – 60, – 30, – 10, (0, 0, – 10, – 30, – 60, – 100, . . .)
   + 30, + 20, + 10, 0, – 10, – 20, – 30, – 40, . . .
   Rule: Difference between terms is decreasing by 10.

d) 2, 5, 11, 23, (47, 95, 191, 383, 767, 1535, 3071, 6143, . . .)
   Rule: Each following term is 1 more than twice the previous term, or [× 2, + 1]

     8 min

2

Find the rule

Study these tables and think what the rule could be. Ps come to BB to fill in missing numbers. Class agrees/disagrees. P who completes last column says the rule. Who agrees? Who could write it a different way? Class checks mentally that the rule is correct by inserting values from one of the columns in the table.

BB:

a) \n
   \begin{array}{|c|c|c|c|c|c|c|c|}
   \hline
   & 8 & 14 & 7 & 10 & 13 & 29 & 31 \hline
   b & 22 & 30 & 16 & 23 & 20 & 17 & 1 \hline
   \hline
   \end{array}
   \begin{array}{|c|c|}
   \hline
   2 & 28 \hline
   \end{array}

   Rule: \ a + b = 30, \ a = 30 – b, \ b = 30 – a

b) \n
   \begin{array}{|c|c|c|c|c|c|c|c|}
   \hline
   & 26 & 42 & 88 & 110 & 66 & 98 & 1410 \hline
   x & 13 & 21 & 55 & 33 & 49 & 705 & 400 \hline
   \hline
   \end{array}

   Rule: \ x \div y = 2, \ x = y \times 2 \ y = x \div 2

     15 min

3

Problem 1

Listen carefully, note the data, and think about how you would work out the answer.

A lift is limited to carrying a maximum of 500 kg. John is 76 kg and his friends weigh much the same as John but not less than John.

How many people, including John, can take the lift in safety?

A, what do you think we should do? Who agrees? Who could write it a different way? T directs Ps thinking if Ps have no ideas.

Solution: e.g.

If each person weighed 76 kg, then: \ 76 kg \times n \leq 500 kg \n
so \ n \leq 500 kg \div 76 kg \n
= 6 \text{ (times), } r 44 kg

Answer: John and 5 of his friends can take the lift together in safety.

     15 min

Notes

Whole class activity
Written on BB or SB or OHT
Tses Ps at random.
At a good pace
Agreement, correcting, praising

Accept any rule which is reasoned correctly.

T might allow Ps to use calculators for difficult numbers in b) and d).

Feedback for T
Problem 2

Listen carefully, note the data and think about how you would work out the answer.

Charlie lives 810 m from his school. If the length of Charlie’s step is 60 cm, how many steps does he take to walk from his house to his school?

Who thinks that they know what to do? Come and explain to us. Who agrees? Who would do it a different way? T draws sketch on BB to help Ps visualise the problem. Ps write solution in Ex. Bks.

Solution: e.g.

BB: 810 m  =  81 000 cm

Plan: 81 000 cm ÷ 60 cm

= 8100 cm ÷ 6 cm

= 1350 (times)

Answer: He takes 1350 steps.

Q.1 Read: Write plans and do the calculations in your exercise book. Write the answer here.

Set a time limit. Ps read problems themselves and solve them. Review with whole class. P reads out the question, then asks class to show results on scrap paper or slates on command. P answering correctly explains solution on BB to Ps who were wrong. Who agrees? Who did it another way? etc. How can we check it? Mistakes discussed and corrected.

Solution:

a) How many 60 cm lengths can be cut from a ribbon which is 8 m 90 cm long?

Plan: 8 m 90 cm ÷ 60 cm = 890 cm ÷ 60 cm

= 14 (times), r 50 cm

Answer: 14 lengths of 60 cm can be cut from the ribbon. (and 50 cm will be left over)

b) 12 litres 50 cl of milk is poured into glasses which can hold 30 cl when full. How many glasses are needed?

Plan: 12 litres 50 cl ÷ 30 cl = 1250 cl ÷ 30 cl

= 41 (times), r 20 cl

Answer: 42 glasses are needed.

(41 full glasses and one holding only 20 cl)

PbY5a, page 69

Lesson Plan 69

Notes

Whole class activity

T repeats slowly to give Ps time to think.

Discussion, reasoning, agreement, praising

Individual work, monitored, helped

Discussion, reasoning, agreement, checking, self-correction, praising

Check:

BB: 1 4 r 50

6 8 9 0

- 0 0

2 9 0

- 2 4 0

5 0

Chec:

14 × 60 cm + 50 cm = 840 cm + 50 cm = 890 cm

BB: 3 0

2 5 0

- 1 2 0

5 0

- 3 0

0

Check:

41 × 30 cl + 20 cl = 1230 cl + 20 cl = 1250 cl
**Y5**

**Activity**  

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
</table>
| 6 | *PbY5a, page 69* | Q.2 | Read: *At a birthday party, 6 friends shook hands with one another. How many handshakes were there? Complete the diagram and list all the possibilities.*  
Set a time limit. Review with whole class. Ps could show number of handshakes on scrap paper or slates on command. Ps responding incorrectly come to BB to complete diagram and list the possibilities. Class points out errors or omissions. Who can think of another way to solve it?  
**Solution:**  
<p>| | | | | | |</p>
<table>
<thead>
<tr>
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<tbody>
<tr>
<td>AB</td>
<td>BC</td>
<td>CD</td>
<td>DE</td>
<td>EF</td>
<td></td>
</tr>
<tr>
<td>AC</td>
<td>BD</td>
<td>CE</td>
<td>DF</td>
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<tr>
<td>AD</td>
<td>BE</td>
<td>CF</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AE</td>
<td>BF</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>AF</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

\[
5 + 4 + 3 + 2 + 1 = 15
\]

**Answer:** There were 15 handshakes.

---

<p>| | | | | | |</p>
<table>
<thead>
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<th></th>
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</thead>
</table>
| 7 | *PbY5a, page 69* | Q.3 | Read: *From the entrance to a park, there are 3 different paths to the fountain. From the fountain there are 4 different paths to the play area. From the play area there are 5 different paths to the bandstand. How many different ways are there to get to the bandstand from the entrance? Draw a diagram to show it.*  
Discuss how to how to draw the diagram first (write initial letters for the places and draw lines between them to show the paths)  
Set a time limit. Review with whole class. How many routes are possible? Show me . . . now! (60)  
Ps answering correctly come to BB to draw diagram and explain reasoning. Who did the same? Who drew a different diagram? Mistakes discussed and corrected.  
**Solution:**  
**Diagram:** e.g.  
\[
\begin{array}{c}
E \quad F \quad P \quad B \\
(3) \quad (4) \quad (5)
\end{array}
\]

Reasoning: e.g.  
For each of the 3 possible paths from E to F there are 4 possible paths from F to P, and for each of these there are 5 possible paths from P to B, so number of possible paths is  
BB: \[
3 \times 4 \times 5 = 12 \times 5 = 60
\]

**Answer:** There are 60 different ways to get to the bandstand from the entrance.

---

**Lesson Plan 69**

**Notes**  
Individual work, monitored, helped  
Written/drawn on BB or SB or OHT  
Discussion, reasoning, agreement, self-correction, praising  
Demonstrate with 6 Ps at front of class if necessary.  
Or  
Each of the 6 friends shakes hands with the other 5, but e.g. A–B and B–A are the same handshake, so the calculation is:  
\[
6 \times 5 \div 2 = 30 \div 2 = 15
\]
Extra praise for Ps who think of this.

---

**Individual work, monitored, helped**

Discussion, reasoning, agreement, self-correction, praising  
(Ps might suggest labelling the routes with a, b, c, etc. and then listing all the possibilities, or drawing a tree diagram. Praise them but ask whether any P used a quicker method.)  
T repeats reasoning more clearly if necessary.

T chooses a P to say the answer in a sentence.
**Y5**

**Activity 8**

**PbY5a, page 69, Q.4**

Read: *In a class of 29 pupils, 15 pupils play volleyball and 17 pupils play football. Each pupil plays at least one of the two games. How is it possible? Draw a set diagram to show it.*

Ps suggest what to do first and how to continue. Ps come to BB to draw the two sub-sets and label them, then discuss how to work out which numbers should go where. T helps only if necessary.

Accept trial and error: e.g.  
14 + 1 + 16 = 31 > 29  
13 + 2 + 15 = 30 > 29  
12 + 3 + 14 = 29 ✔

but also ask for logical reasoning. T shows it if no P thinks of it.

BB: (15 + 17) – 29 = 32 – 29 = 3  
So 3 Ps must play two games, i.e. are in the intersection of V and F.

So number playing: only volleyball: 15 – 3 = 12  
only football: 17 – 3 = 14

Ask Ps to explain in words what each area in the set diagram means.
- 12 Ps play volleyball but not football.
- 3 Ps play volleyball and football
- 14 Ps play football but not volleyball.
- There are no Ps who play neither volleyball nor football.

**Notes**

**Lesson Plan 69**

Whole class activity

Drawn on BB or SB or OHT

Discussion involving several Ps, reasoning, agreement, praising

BB:

T explains the difference between, e.g., the number 12 being an element in a set and the number 12 showing that there are 12 elements in the set (latter written in brackets). T could show correct notation for no elements in a set:

BB: Ø: empty set

**Activity 9**

**PbY5a, page 69**

Q.5 Read: *In a bag there are 3 red, 4 white and 5 green marbles. What is the least number of marbles that we must take out of the bag (with our eyes closed) so that we are certain of getting:  
  a) at least one of each colour  
  b) at least one white marble  
  c) 2 marbles of the same colour?*  

I will give you 2 minutes to think of the answers.

Then T reads out each question and Ps show number on scrap paper or slates on command. T asks Ps with different answers to explain their reasoning. Demonstrate if there is disagreement!

**Solution:**

a) 10 as the first 9 could be 5 green and 4 white marbles but the 10th marble must be red.

b) 9 as the first 8 could be 3 red and 5 green marbles, but the 9th marble must be white.

c) 4 as the first 3 could be a red, a white and a green marble but the 4th must be another green, or white or red marble.

Individual trial first, then whole class review and demonstration

T should have a real bag of marbles already prepared.

Responses shown in unison.

Reasoning, agreement, self-correction, praising

In good humour!

Stand up if you worked out the correct answer to all 3 questions. Let’s give them a clap!

[Note to Ts: This type of problem is known as a 'pigeon-hole' problem.]
Activity

Calculation practice, revision, activities, consolidation

*PbY5a, page 70*

**Solutions:**

<table>
<thead>
<tr>
<th>Q.1</th>
<th>x</th>
<th>4</th>
<th>5</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>10</th>
<th>6</th>
<th>5</th>
<th>9</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>y</td>
<td>2</td>
<td>3</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>1</td>
<td>6</td>
<td>4</td>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>z</td>
<td>7</td>
<td>14</td>
<td>17</td>
<td>27</td>
<td>24</td>
<td>9</td>
<td>35</td>
<td>19</td>
<td>-1</td>
<td>20</td>
</tr>
</tbody>
</table>

c.e.g. 

<table>
<thead>
<tr>
<th>Q.2</th>
<th>a) £1152 ÷ 16 = £576 ÷ 8 = £72</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>It would cost £72 per person for 16 people.</td>
</tr>
<tr>
<td></td>
<td>b) £1152 ÷ £64 = £576 ÷ £32 = £72 ÷ £4 = 18 (times)</td>
</tr>
<tr>
<td></td>
<td>18 people went on holiday at a cost of £64 each.</td>
</tr>
</tbody>
</table>

**Q.3**

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(25)</td>
<td>Class</td>
<td>(4)</td>
<td></td>
</tr>
</tbody>
</table>

**Q.4**

<table>
<thead>
<tr>
<th>a) £2.75 × 300 ÷ 50 = £2.75 × 6 = £16.50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Each goose cost £16.50.</td>
</tr>
<tr>
<td>b) 300 × 12 ÷ 50 = 3600 ÷ 50 = 360 ÷ 5 = 72 (min.)</td>
</tr>
<tr>
<td>or 300 ÷ 50 = 6 (min); 6 min × 12 = 72 min</td>
</tr>
<tr>
<td>It will take her 72 minutes (or 1 h 12 min) to type 12 pages.</td>
</tr>
</tbody>
</table>

**Q.5**

<table>
<thead>
<tr>
<th>a) 4 km – 1.5 km = 2.5 km = 2500 m = 25 × 100 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost for 1.5 km: £2.20</td>
</tr>
<tr>
<td>Cost for 2.5 km: 14 p × 25 = 250 p + 100 p</td>
</tr>
<tr>
<td>= 350 p = £3.50</td>
</tr>
<tr>
<td>Total cost: £3.50 + £2.20 = £5.70</td>
</tr>
<tr>
<td>b) £7.66 – £2.20 = £5.46 = 546 p</td>
</tr>
<tr>
<td>14 p → 100 m</td>
</tr>
<tr>
<td>546 p → 100 m × (546 ÷ 14) = 100 m × (273 ÷ 7)</td>
</tr>
<tr>
<td>= 100 m × 39</td>
</tr>
<tr>
<td>= 3900 m</td>
</tr>
<tr>
<td>= 3.9 km</td>
</tr>
<tr>
<td>Total distance: 3.9 km + 1.5 km = 5.4 km</td>
</tr>
</tbody>
</table>
**Lesson Plan**

### Y5

**R:** Logic expressions and operations: and/or  
**C:** Sets of points. Distance between shapes. Circle. Sphere  
**E:** Sets of points with given properties

#### Activity

1. **Distance from a point**

   a) Draw a dot in the middle of a sheet of paper and label it O.
   Use your ruler to measure and mark different points which are 3 cm away from O. Draw as many as you can in 1 minute.
   Join up all your dots with a curved line. What have you drawn?
   (a circle) Study it carefully. Do you think it is an exact circle?
   Is every point on the line exactly 3 cm from O? (Ps might agree that there are some wobbles, or that their centre dots are too thick.)

   b) How could we draw a circle around a point more accurately?
   (e.g. draw around a circular object, or Ps might mention using compasses. If no P does so, T suggests it and explains their use.
   On a new sheet, mark a new point O more accurately by drawing a cross and labelling the point where the lines cross O.
   T demonstrates on BB, using BB compasses and BB ruler, how to set the compasses to 3 cm and draw a circle around O. Now you try it.
   Encourage Ps to keep practising until they draw a perfect circle.

   c) Who remembers the name of the line around the edge of a circle? (circumference) We call point O the centre point of the circle and we say that the the circle we have drawn has centre point O and radius 3 cm. The radius of a circle is the distance from the centre point to the circumference. (T refers to circle on BB.)

   **6 min**

2. **PbY5a, page 71**

   Q.1 Read: Join up points A and B and measure the distance between them.
   I will give you 1 minute!
   Show me the distance . . . now! (32 mm) (On slates / scrap paper)
   Ps with wildly inaccurate results tell class how they got them!
   Agree that the distance between 2 points is the length of the straight line between them. Ps who did not draw a straight line, do so now using a ruler and check that the distance is 32 mm.

   **8 min**

3. **PbY5a, page 71**

   Q.2 Read: Colour the points on the straight line in:
   a) red if they are 3 cm from P  
   b) blue if they are more than 3 cm from P  
   c) yellow if they are less than 3 cm from P.
   Ps may use rulers or compasses to mark the points which are 3 cm on either side of P before colouring.
   Set a time limit. Review with whole class. Ps come to BB to show which parts of the line are which colour. Class points out errors. Mistakes corrected. Agree that the point P is yellow.
   If we label the two red points A and B, what is the length of AB? (6 cm) T writes on BB: AB = AP + PB = 6 cm
   Do you agree that the set of blue points are two rays, one from A and one from B, but without their start points? (Yes, as the start point A and the start point B are red.)
   Who can describe the set of yellow points? (e.g. the line segment AB without its start and end points)

   **12 min**

---

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### Activity 4

**PbY5a, page 71**

Q.3 Read:

- **a)** Draw the set of points in the frame which are 2 cm from point O.
- **b)** Colour red the points which are less than 2 cm from point O.
- **c)** Colour blue the points which are more than 2 cm from point O.

If necessary, T reminds Ps by demonstration how to set their compasses to 2 cm and use them to measure. Set a time limit.

Review with whole class. T has circle already drawn inside the rectangular frame. Ps come to BB to show the different sets of points and say what colour they should be. Class agrees/disagrees.

T asks Ps to describe each set of points. After Ps' descriptions, T repeats in a clear, precise way:

- a) This set of points forms the circumference of a circle, with centre O and radius 2 cm.
- b) The set of red points forms a circular plane shape with centre O and radius 2 cm, but without the points on its circumference.
- c) The set of blue points forms a rectangular plane shape but without the red circle.

If we were not limited to the rectangular frame, how could we describe the set of blue points?

(The set of blue points forms a plane without the red circle.)

Elicit that the blue points would extend in all directions to infinity (i.e. they would be never ending or infinite).

### Extension

**Sphere**

Ps each have a circular piece of card (e.g. radius 3 cm) with a straw stuck along its diameter. T has large model for demonstration.

Where is the surface of your circle? Where is its centre point? Where is its circumference? What length is its radius? Ps measure. (3 cm)

Rotate your circle around its vertical axis (i.e. on the bottom of the straw) like this. T demonstrates and Ps copy.

What solid shape does the circular card form in the air? (e.g. a ball, or a sphere) T reminds Ps of the mathematical name if necessary.

Every point in the air which the circular card touches when it is rotated is part of our imaginary sphere. What point is at its centre?

(Ø) What is its radius? (3 cm)

T: We say that it forms a sphere with centre Ø and radius 3 cm.

What part of the sphere is formed by the circumference of the circle when we rotate the card? (its surface) Elicit that all points on the surface of the sphere are 3 cm from the centre point, Ø.

T has different spherical objects to show to class. (e.g. marble, football, hamster's ball, cricket ball, etc.) Which of these is a solid sphere? (i.e. with all its inner points) (e.g. marble, cricket ball)

How would you describe, e.g. the hamster's ball? (the surface of a sphere without the inner points) We cannot see its centre point but we can imagine where it is.

### Notes

Individual work, monitored helped, corrected

Drawn on BB or SB or OHT

Discussion, reasoning, agreement, self-correction, praising

BB:

T uses correct vocabulary as an example for Ps to follow. Agreement, praising

Whole class activity but individual manipulation of prepared models.

Models cut from card or use enlarged copy master or OHP

BB: sphere

Agreement, praising

T sets an example of vocabulary for Ps to follow, indicating the relevant parts of the model.

Ps come to front of class to choose appropriate objects, or suggest other objects of their own. Class agrees/disagrees.

T helps Ps to use correct vocabulary. Praising only
## Activity

### Missing words

T has sentences written on BB or SB or OHT, with underlined words missing (or covered up).

Study these sentences. Which words do you think are missing?

Ps come to BB to say and write the missing words. Class agrees/disagrees. (Or Ps say the words, class agrees/disagrees, and Ps uncover the words to confirm.) After agreement, class reads out the sentence together. T asks Ps to show the relevant part or model.

**BB:**

a) The **circumference** of a circle is the set of points in a plane which are a given distance (not zero) from a given point in the **plane**.

b) A **circle** is the set of points in a plane which are equal to or **less than** a given distance from a point in the plane.

c) The surface of a sphere is the set of points in **space** which are a given **distance** (not zero) from a given point in space.

d) A **sphere** is the set of points in space which are equal to or less than a given distance from a **given point** in space.

---

### Circle vocabulary

Let’s join up the names to the corresponding parts of the circle.

Ps come to BB to choose a name, say it, find where it is in the diagram and draw a joining line. Tell us in your own words what it means.

T explains the meaning if Ps do not know. Ps repeat unknown name in unison and T chooses one or two Ps to try to repeat T’s explanation.

**BB:**

- **circumference** curved line around the edge of the circle
- **radius** straight line joining a point on the circumference to the centre point, (i.e. distance between centre point and circumference)
- **diameter** straight line joining 2 points on the circumference and passing through the centre point. (twice length of radius)
- **arc** part of the circumference of a circle
- **chord** straight line joining 2 points on the circumference but not passing through the centre point.
- **tangent** straight line drawn outside the circle but which touches the circumference of the circle at a point.
- **tangent point** point at which the tangent touches the circle (or common point of tangent and circle)
- **semicircle** half a circle
- **segment** part of a circle bordered by a chord and an arc
- **sector** part of a circle bordered by two radii and an arc

---

### Notes

Whole class activity

Or T has missing words written on card and stuck randomly to side of BB and Ps choose correct cards for each sentence, or use enlarged copy master or OHP, with boxes covered up.

At a good pace

Agreement, demonstrating, praising

T could also choose individual P to read each definition.

(T could tell Ps that an exact explanation of something in mathematics is called a definition, and to define something is to explain it exactly.)

---

Do not expect Ps to learn the meaning of all the words yet – this is just an introduction to the vocabulary

T tells Ps that we talk about one **radius** but two or more **radii**.
### Lesson Plan 71

#### Activity

**8**

**Measuring distance**

Show me the distance between your desk and your head. How did you choose the path to show through the air? T asks 2 or 3 Ps to explain. (Ps might use words such as closer, closest, straight line, perpendicular, etc.)

T chooses two objects (or two Ps) in the classroom and asks Ps to show how they would measure the distance between them. Elicit that when we say the distance between two things, we usually mean the shortest distance between them.

T draws two sets of points on the BB. How would you measure the distance between them? Would you measure from here to here? (T draws line (1)) or here to here? (T draws line (2)) or here to here? (T draws line (3)) Ps agree that the 3rd line is correct, as it joins the two nearest points in each set, i.e. is the shortest distance between them.

**PbY5a, page 71**

**Q.4** Read: *What is the shortest distance between the two shapes? Draw a measuring line, measure it and write its length beside it.*

Ps measure with rulers, or compasses and rulers, in mm.

Review with whole class. Ps come to BB to draw measuring lines and write lengths. Class agrees/disagrees. T asks Ps what each length is in cm. Who measured in a different place? Who drew the correct measuring line but read the wrong length? Ps correct their mistakes.

**Solution:**

<p>| | |</p>
<table>
<thead>
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<tbody>
<tr>
<td>a)</td>
<td><img src="a.png" alt="Image" /></td>
</tr>
<tr>
<td>b)</td>
<td><img src="b.png" alt="Image" /></td>
</tr>
<tr>
<td>c)</td>
<td><img src="c.png" alt="Image" /></td>
</tr>
<tr>
<td>d)</td>
<td><img src="d.png" alt="Image" /></td>
</tr>
<tr>
<td>e)</td>
<td><img src="e.png" alt="Image" /></td>
</tr>
<tr>
<td>f)</td>
<td><img src="f.png" alt="Image" /></td>
</tr>
</tbody>
</table>

#### Notes

Whole class activity to start.

T notes whether Ps show the distance between the two closest points (i.e. from chin to nearest edge of desk)

Discussion, agreement

BB: e.g.

Set A

Set B

Agreement, praising

Individual work, monitored, helped, corrected

Drawn on BB or use enlarged copy master or OHP

Discussion, agreement, self-correction, praising

Elicit that the shortest distance between a point and a line, and between two lines is the perpendicular distance between them.

In f), accept 3 to 4 mm but T points out that a line extends in both directions to infinity. If the two lines are extended, they have a common point, so their distance is really 0 mm.

**9**

**PbY5a, page 71. Q.5**

Read: *Draw all the points on the plane which are:*

* a) an equal distance from the two lines.
* b) 1 cm from line segment AB
* c) 1 cm from line e and from point A.

Deal with one at a time. Ps come to BB to mark different points and then agree on the set. After agreement, Ps draw the set of points in "Pbs." In c), T suggests drawing set of points 1 cm from line e first (parallel line), then the set of points 1 cm from point A (circle, radius 1 cm).

What do you notice? (They have two common points.) Let's label them B and C. Only points B and C are 1 cm from line e and from point A.

**Solution:**

<p>| | |</p>
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<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td><img src="a.png" alt="Image" /></td>
</tr>
<tr>
<td>b)</td>
<td><img src="b.png" alt="Image" /></td>
</tr>
<tr>
<td>c)</td>
<td><img src="c.png" alt="Image" /></td>
</tr>
</tbody>
</table>

**Extension** for c):

Which set of points are 1 cm from line e or from point A? (All points on dashed line and all points on circumference of the circle, i.e. the union of the line and the circle.)

Whole class activity (or individual trial first if Ps wish and there is time, reviewed with whole class)

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

T helps Ps at BB to use the BB ruler and compasses.

Ps use compasses and rulers to draw in "Pbs."
**Lesson Plan**

**Y5**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distance</strong></td>
<td>Whole class activity but individual or paired measuring with rulers and compasses. Use any geographical map which is clearly marked. Agreement, correcting, praising.</td>
</tr>
</tbody>
</table>

Ps have copy of a national or county or city map, rulers and compasses on desks. T has large copy for demonstration only.

Discuss and agree on the scale of the map first and write on BB.

e.g. BB: **Scale**: 1 cm → 10 km

T (Ps) suggests two places and Ps measure the distance between them.

T asks several Ps what they measured. Allow some slight variations but ask Ps with wildly inaccurate results to measure again more carefully. After Ps agree on measurement, ask what the distance would be in real life.

Measure the distance between, e.g.

- a) Exeter and London (. . . cm → . . . km)
- b) Devon and Yorkshire (. . . cm → . . . km)
- c) The River Thames and the sea (0)
- d) Exeter and Devon (0)

Review how to measure distances between two shapes or objects. (straight line joining the nearest points of each shape or object) T draws some random shapes on BB and Ps come to BB to show where they would measure the shortest distance between them.

| 5 min |

| Drawing circles | Individual construction but class kept together on tasks. T demonstrates with BB ruler and compasses. |

Ps have rulers, compasses and plain sheets of paper on desks.

- a) Mark a point O with a small cross, then draw a circle with centre O and radius 4 cm. T shows Ps how to set their compasses to 4 cm on their ruler, to stick the pointed arm on point O and rotate the compasses by a whole turn.
- b) Colour red the set of points which are not less than 3 cm from O. (Ps draw a 3 cm circle around O and colour appropriately. Elicit that the 3 cm circumference should be drawn over in red, as not less than 3 cm means more than or equal to 3 cm)
- c) Colour blue the set of points which are not more than 1 cm from O. (Ps draw a 1 cm circle around O and colour appropriately. Elicit that the 1 cm circumference should be drawn over in blue, as not more than 3 cm means less than or equal to 1 cm.)
- d) What is the common property of the points which are not coloured? T asks several Ps what they think, then T confirms that: The white points are those points on the plane which are more than 1 cm but less than 3 cm from point O.
- e) How could we write the distances of the 3 sets of points mathematically? Ps come to BB or dictate what T should write.

| 10 min | R: Circles. Ordering triangles by length of sides  
C: **Sets of points with given properties. Construction of triangles**  
E: Distance between shapes (objects)  

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<table>
<thead>
<tr>
<th>Activity Number</th>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>True or false?</td>
<td><strong>Lesson Plan 72</strong></td>
</tr>
<tr>
<td></td>
<td>Think of all the points in a plane. Are these statements true or false? Write T if you think it is true and F if you think it is false. T reads statement and Ps show appropriate letter on command. T chooses Ps with different responses to explain their reasoning.</td>
<td>Whole class activity&lt;br&gt;Written on BB or SB or OHT&lt;br&gt;Responses shown on scrap paper or slates in unison (or use pre-agreed actions)&lt;br&gt;Discussion, reasoning, agreement, praising</td>
</tr>
<tr>
<td></td>
<td>a) If point A is 3 cm away from point P, then A is on the circumference of a circle with radius 3 cm and centre P. (T)</td>
<td>BB:</td>
</tr>
<tr>
<td></td>
<td>b) If point A is on the circumference of a circle with radius 3 cm and centre P, then A is 3 cm away from P. (T)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Point A is on the circumference of a circle with radius 3 cm and centre P only if A is 3 cm away from P. (T)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Draw a diagram on BB to show that all 3 statements are true.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Sets of points</td>
<td></td>
</tr>
<tr>
<td></td>
<td>I will describe a set of points and you must tell me in a sentence what their distance is from point P.</td>
<td>Whole class activity&lt;br&gt;Written on BB or SB or OHT&lt;br&gt;Ps explain their answers on diagram drawn on BB.</td>
</tr>
<tr>
<td></td>
<td>a) The points on the circumference of a circle with radius 4 cm and centre P. (P: Their distance from P is 4 cm.) [d = 4\text{ cm}]</td>
<td>BB:</td>
</tr>
<tr>
<td></td>
<td>b) The points in a circle with radius 4 cm and centre P, including the points on the circumference. (P: Their distance from P is equal to or less than 4 cm) [d \leq 4\text{ cm}]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) The points in a circle with radius 4 cm and centre P, but without the points on the circumference. (P: Their distance from P is less than 4 cm) [d &lt; 4\text{ cm}]</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d) The points in the plane outside the circle with radius 4 cm and centre P. (P: Their distance from P is more than 4 cm.) [d &gt; 4\text{ cm}]</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Missing words</td>
<td>Whole class activity&lt;br&gt;Written on BB or SB or OHT&lt;br&gt;Written on scrap paper or slates and shown on command in unison.</td>
</tr>
<tr>
<td></td>
<td>T has sentences written on BB with underlined words covered up. Show me the word you think is missing . . . now! T confirms by uncovering the word and class reads the whole sentence in unison. BB:</td>
<td>Agreement, praising</td>
</tr>
<tr>
<td></td>
<td>a) The length of the diameter of a circle is twice the length of the radius.</td>
<td>Let (d) be the diameter and (r) be the radius. BB: [d = 2 \times r] [r = d \div 2]</td>
</tr>
<tr>
<td></td>
<td>b) The length of the radius of a circle is half the length of the diameter.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Who can write the statements in a mathematical way? Ps come to BB or dictate to T. Class agrees/disagrees. Ps write equations n Ex. Bks. Would these equations also be true for a sphere? (Yes)</td>
<td></td>
</tr>
</tbody>
</table>
PbY5a, page 72

Q.1 Read: Complete the sentences.
   Deal with one at a time or set a time limit. Ps read sentences themselves and write the missing words.
   Review with whole class. Ps read out the whole sentence. Who agrees? Who wrote something else? Deal with all cases. Mistakes discussed and corrected. Accept any valid form of words.

Solution:
   a) The circumference of a circle is the set of points in a plane which are an equal distance from the centre point of the circle.
   b) The radius of a circle is a straight line which connects the centre of the circle with a point on the circumference.
   c) A sphere is the set of points which are not more than a given distance from a point in space, as long as the given distance is not zero.

Triangles 1

T has various types of triangles stuck to BB and, if possible, Ps have copies on desks too.
How could we group these triangles? Ps suggest ways.
(e.g. according to size of angles: right-angled (90°), acute-angled (< 90°), obtuse-angled (> 90°);
or according to length of sides: 3 different sides, at least 2 equal sides, 3 equal sides)
Who remembers the names for triangles with 2 (3) equal sides? T reminds Ps if nobody remembers.
BB: at least 2 equal sides: isosceles triangle
   3 equal sides: equilateral triangle

Let's use the lengths of sides to group the triangles and BB: show it in a Venn diagram.
T draws it on BB as directed by Ps and Ps come to BB to stick the triangles in the corresponding set.
Class points out errors.

Triangles 2

Ps have 2 cm, 3 cm, 4 cm, 5 cm and 6 cm straws on desks.
Use your straws to form a triangle with these side lengths:
   a) 3 cm, 4 cm, 5 cm (right-angled triangle)
   b) 2 cm, 4 cm, 7 cm (impossible, as straws do not meet up)
   c) 3 cm, 3 cm, 5 cm (isosceles)
   d) 4 cm, 4 cm, 4 cm (equilateral)

Individual work, monitored
Ps say what they know about the triangles that they have made.
Discussion, agreement, praising
Agree that the sum of the lengths of the 2 shortest sides of a triangle must be more than the length of the longest side.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td><strong>PbY5a, page 72</strong></td>
</tr>
<tr>
<td><strong>Q.2</strong></td>
<td>Read: Points A, B and C are in the same plane but do not form a straight line. Join up the points and measure the connecting line segments.</td>
</tr>
<tr>
<td></td>
<td>I will give you 2 minutes! Ps draw lines and measure with rulers, then write lengths in mm in <em>Pbs</em>.</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. What shape have you drawn? (triangle) Ps dictate lengths of sides and T writes on BB. Class agrees/disagrees. Mistakes corrected.</td>
</tr>
<tr>
<td></td>
<td>What kind of triangle have you drawn? ( obtuse-angled) <strong>Solution:</strong></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td><strong>35 min</strong></td>
</tr>
<tr>
<td>10</td>
<td><strong>PbY5a, page 72</strong></td>
</tr>
</tbody>
</table>
| **Q.3**  | Read: In a park garden there is a monument and a well. The park gardeners were asked to plant some rose bushes that were both:  
|          | • 2 m from the monument, and also  
|          | • 2 m from the well.  
|          | Show on the diagrams where the rose bushes should be planted if the distance between the well and the monument is: a) 3 metres b) 4 metres c) 5 metres. |
|          | First ask Ps to explain what a monument and a well are. Relate to local parks or monuments if possible. |
|          | What do you notice about the diagrams? (They have been drawn in cm, not metres.) What scale has been used? (BB) T demonstrates in a) how to use a ruler and compasses to draw two arcs (parts of circumference of a circle) with radius 2 cm, one with centre M and the other with centre W. Ps copy in *Pbs*. Elicit that the points of intersection (or common points) are where rose bushes should be planted. Agree that in a), two rose bushes could be planted. |
|          | Now let's see if you can do parts b) and c) on your own. Set a time limit. Review with whole class. Ps come to BB to set compasses and draw arcs, then explain the results. Class agrees/disagrees. Mistakes discussed and corrected. **Solution:** |
|          | ![Diagram](image) |
|          | **40 min** |
### Lesson Plan 72

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y5</strong></td>
<td>Whole class activity, with individual construction, and class kept together on each step.</td>
</tr>
<tr>
<td><strong>PbY5a, page 72</strong></td>
<td>Freehand sketch drawn on BB or use enlarged copy master or OHP</td>
</tr>
</tbody>
</table>
| Q.4 Read: **Draw accurately the triangle which has these sides:**  
  \[a = 2 \text{ cm}, \quad b = 4 \text{ cm}, \quad c = 4 \text{ cm}\]  
  **Follow the order of construction.** | Discussion, demonstration, agreement, praising |
| Who can explain the diagram? (Circled numbers show the steps.) | BB: Freehand sketch |
| T works on BB and Ps in Phs. Ps dictate what class should do at each step (with T's help where necessary) e.g. |  
  (1) Draw line segment BC, using ruler or ruler and compasses. (length 2 cm) |
| (2) Set compasses to 4 cm and draw arc with centre B and radius 4 cm. | |  
  (3) Check that arms of compasses are still 4 cm apart and draw arc with centre C and radius 4 cm. |
| (4) Label the point of intersection of the 2 arcs A. | |  
  (5) Join up AB and AC. |
| T explains that when drawing triangles, it is usual to label the side opposite point A as lower case \(a\), the side opposite point B as lower case \(b\), etc. | |
| What kind of triangle have we drawn? (isosceles triangle, as two sides are of equal length; or acute-angled triangle, as all the angles are less than \(90^\circ\)) | |
| **Actual triangle:** | |
| | |

Extra praise if Ps point out that there are two possible positions for \(A\) (above and below BC), but T explains that the freehand sketch only shows the triangle with \(A\) above BC so that is what we should draw. In such a case, elicit that: \(\Delta ABC\) would be **congruent** to \(\Delta A'BC\) (i.e. exactly the same size and shape) |
| BB: \(\Delta ABC \cong \Delta A'BC\) | |
| Extra praise for Ps who remember the word and the notation. | |
### Lesson Plan 73

#### Vocabulary
T has BB or SB or OHT already. Ps come to BB to say and write the name for each diagram. Class agrees/disagrees. T tells Ps that we usually label points with capital letters and lines with lower case letters.

<table>
<thead>
<tr>
<th>BB:</th>
<th>a) ×</th>
<th>b) e</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>point</td>
<td>straight line (endless in both directions)</td>
<td>half-line or ray (endless in 1 direction)</td>
</tr>
<tr>
<td>d) A</td>
<td>e)</td>
<td>f)</td>
<td>g)</td>
</tr>
<tr>
<td></td>
<td>line segment (part of a line)</td>
<td>circumference of a circle</td>
<td>centre point of a circle</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>radius of a circle</td>
</tr>
</tbody>
</table>

#### Activity 1

**Vocabulary**

T has BB or SB or OHT already. Ps come to BB to say and write the name for each diagram. Class agrees/disagrees.

T tells Ps that we usually label points with capital letters and lines with lower case letters.

**Notes**

Whole class activity

Or use enlarged copy master or OHT

At a good pace

Agreement, praising

Elicit/tell that the radius is the distance from the centre of the circle to a point on the circumference.

**PbY5a, page 73**

**Q.1 Read:** In your exercise book, construct triangles which have these sides.

Sketch the triangle first, with Ps dictating what T should draw or coming to BB and writing the order of the steps on diagram. In a), agree that it does not matter which side is used as the base of the triangle.

In b), T advises Ps to measure the line segments using a ruler, or ruler and compasses, and to write the lengths in Pbs before drawing the triangle.

Set a time limit. Review with whole class. T has enlarged triangles drawn on BB or SB or OHT for discussion. T chooses Ps to say what they did and asks Ps to say if they had any difficulties.

**Solution:** (reduced to scale)

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Freehand sketch" /></td>
<td><img src="image" alt="Freehand sketch" /></td>
</tr>
</tbody>
</table>

What is the perimeter of each triangle?

- a) $24 + 42 + 48 = 114$ (mm)
- b) $38 + 28 + 30 = 96$ (mm)

How could we label the vertices? Pbs dictate to T.

(A opposite side a, B opposite side b, C opposite side c)

Praising
### Lesson Plan 73

#### Activity

**PbY5a, page 73**

**Q.2** Read: *In your exercise book, construct the triangle which has:*

- a) perimeter of 12 cm and sides of equal length;
- b) a perimeter of 15 cm, two sides of equal length and its third side half as long as the others.

Deal with one at a time. Set a time limit. Ps draw a freehand sketch and do calculations in Ex. Bks to find the lengths of a, b and c.

Review lengths with whole class before Ps start drawing the triangles. Ps come to BB to expain reasoning and draw a freehand sketch on BB. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

Now draw the triangle using your ruler and compasses.

**Solution:**

a) \(a = b = c\), so \(P = 3 \times a = 12\) cm

\[a = 4\text{ cm}\]

Elicit that it is an **equilateral** triangle.

b) \(b = c = 2a\), so \(P = a + 2a + 2a = 15\) cm

\[5a = 15\text{ cm}\]

\[a = 3\text{ cm}\]

So \(a = 3\) cm, \(b = 6\) cm and \(c = 6\) cm.

Elicit that it is an **isosceles** triangle.

c) A triangle has perimeter 12 cm. The difference between the longest and shortest side is 2 cm and the third side is 1 cm longer than the shortest side. *What is the length of its sides?*

Ps suggest what to do and how to continue, with hints from T if necessary. Class agrees/disagrees. e.g.

**BB:** \(a < b < c\), \(c - a = 2\) or \(c = a + 2\), \(b = a + 1\)

\[P = a + b + c = 12\text{ cm}\]

\[a + (a + 1) + (a + 2) = 12\text{ cm}\]

\[3 \times a + 3 = 12\text{ cm}\]

\[3 \times a = 9\text{ cm}\]

\[a = 3\text{ cm}\]

So \(b = 4\text{ cm}, c = 5\text{ cm}\)

Once the lengths have been determined, Ps could draw the triangle accurately in their Ex. Bks. *What do you notice about it? (It is a right-angled triangle.)*

#### Notes

Individual work, monitored, helped

(or whole class activity in determining the lengths of the sides, then individual drawing)

Discussion, reasoning, agreement, self-correction, praising

Accept trial and error but T should lead Ps through the solutions opposite if no P has used them.

T monitors, helps, corrects Ps in drawing accurate triangles.

**Check:** \(6 + 6 + 3 = 15\) cm

Whole class activity

(or individual draw first if Ps wish)

Written on BB or SB or OHT

Discussion, reasoning, agreement, checking, praising

Accept ‘trial and error’ too.

Only rough sketches are required.

Involve several Ps.

**Check:** \(3 + 4 + 5 = 12\) cm

Individual work, monitored, helped, corrected (or drawing can be set as homework)
Activity

4 Shortest route
T has diagrams drawn on BB or SB or OHT. How could we get:

a) CDAB or CB
b) BAC or BC
c) ABC or ADC

Ps come to BB to show the different routes by drawing arrows and listing the letters passed through.

Which is the shortest route? Ps come to BB to underline it. Agree that in c) both routes are the same length but elicit that the shortest route is the diagonal, AC.

Agree that the shortest distance between two points is a straight line.

Ps think of other questions to ask about the shapes. (e.g. What is their name? Which lines are parallel (perpendicular)? What kind of angles do they have? How would we label the sides? etc.)

22 min

5 Shortest distance
T has straight line e and point A drawn on BB.

a) If we think of the point as being Ann’s house and the line e as being the road, in how many ways can Ann get from her house to the road?

Ps come to BB to draw some. Agree that there are an infinite number of ways.

b) Which is the shortest way? Is it this route? (T deliberately points to longer distances to make Ps think.) Elicit that the shortest route is the perpendicular distance from A to line e, i.e. the line drawn from A to point B on line e is perpendicular to line e.

BB:

T: The distance between point A and line e is the length of AB.

25 min

6 Drawing perpendicular lines

Each P has a ruler and a set square on desk and T has BB versions too.

This is called a set square. We can use it to draw perpendicular lines.

What is special about it? (right-angled triangle, so 2 sides perpendicular)

a) Draw a straight line in your Ex. Bk. (or on a plain sheet of paper) and label it e. Mark a point P on the line e.

Let’s draw a line from point P which is perpendicular to line e.

T demonstrates on BB and Ps copy in Ex. Bks.

(Lay ruler along line e as in diagram, then place set square above ruler, with bottom LH corner lined up with point P. Draw a line down the edge of the set square to P. Mark the angle with a square to show that it is a right angle.)
b) Draw another straight line in your Ex. Bk. (or on a plain sheet of paper) and label it \(f\). Mark a point \(Q\) somewhere above the line. Let's draw a perpendicular line from point \(Q\) to line \(f\).

T demonstrates on BB and Ps copy in Ex. Bks.

(Lay ruler below line \(f\) as in diagram, then place set square above the ruler so that its LH edge is lined up with point \(Q\). Draw a line down the edge of the set square from \(Q\) to line \(f\).)

Mark the angle with a square to show that it is a right angle.

\[\text{PbY5a, page 73}\]

Q.3 Read: Draw a line which is perpendicular to the given line and passes through the given point.

Set a time limit. Ps finished first could help other Ps to complete their drawings. Ps suggest labels for the new lines. (e.g. \(s\) and \(t\))

Review with whole class. Ps come to BB to explain and demonstrate what they did. Is it correct to extend the lines \(s\) and \(t\) below the horizontal lines? (Yes, as lines extend to infinity.)

T shows Ps how to do it using ruler or ruler and set square.

Who can write a mathematical statement about the two lines?

Solution: 

\[\begin{align*}
\text{i) } & s \perp e \\
\text{ii) } & B \perp t, \quad t \perp f \\
\end{align*}\]

Elicit that lines \(s\) and \(t\) are parallel to one another. Ps come to BB to mark them with arrowheads and to mark all the right angles.

\[\text{34 min}\]

\[\text{PbY5a, page 73, Q.4}\]

Read: In a field there are two paths, \(a\) and \(b\), as shown in the diagram. What is the shortest route from path \(a\) to path \(b\)? Draw and then measure it.

What kind of line should we draw to be sure that it is the shortest possible route? (Line which is perpendicular to each of the two given lines. How could we do it? Ps make suggestions or come to BB to demonstrate and explain (with T's help if necessary).

(e.g. Lay a ruler below the bottom line, place the set square above the ruler and draw a line down the edge of the set square from line \(a\) to line \(b\).) Ps work in Pbs at the same time. Let's label the new line \(f\). Ps check that the two angles formed are right angles and then measure line \(f\). What is its length? (12 mm = 1.2 cm)

T marks points A, B and P on diagram. How could we draw the shortest route from line \(a\) to line \(b\) which passes through point A (B, P)? Ps come to BB to draw and measure them. Class points out errors. What do you notice? All the drawn lines are the same length.

What does this tell us about lines \(a\) and \(b\)? (They are parallel.)

BB:

\[\text{Whole class activity, but individual drawing in Pbs too}\]

Lines drawn on BB or SB or OHT (12 units apart when measured by BB ruler)

Discussion, reasoning, agreement, praising

T repeats steps suggested by Ps more clearly if necessary and Ps follow them in Pbs.

Ps check right angles with corner of ruler, set square, etc.

Discussion, reasoning agreement, praising

Who can write mathematical statements about the diagram?

BB: 

\[a \parallel b, \quad f \perp a, \quad f \perp b\]
### Activity

**PbY5a, pge 73**

**Q.5** Read: *Mark on the diagrams, or list by their letters, the perpendicular and parallel lines.*

Revise the notation for indicating sets of parallel lines. (1st set: 1 arrowhead; 2nd set: 2 arrowheads, etc.) Set a time limit. Review with whole class. Ps come to BB to explain, draw or write. Class agrees/disagrees. Mistakes discussed and corrected. Accept either method of notation (marking on diagram or listing).

**Solution:**

<table>
<thead>
<tr>
<th>Diagram</th>
<th>Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram a)" /></td>
<td>(a \parallel c, \quad d \perp a, \quad d \perp c)</td>
</tr>
<tr>
<td><img src="image" alt="Diagram b)" /></td>
<td>(a \parallel c, \quad b \parallel d)</td>
</tr>
<tr>
<td><img src="image" alt="Diagram c)" /></td>
<td>(a \perp b, \quad a \perp d, \quad a \perp c, \quad b \perp c, \quad b \perp d, \quad c \perp b, \quad c \perp d)</td>
</tr>
<tr>
<td><img src="image" alt="Diagram d)" /></td>
<td>(a \parallel d, \quad a \perp e, \quad d \perp e)</td>
</tr>
<tr>
<td><img src="image" alt="Diagram e)" /></td>
<td>(e \parallel f, \quad f \parallel ), (e \perp f, \quad f \perp ), (e \perp g, \quad f \perp g, \quad f \perp h, \quad f \perp i)</td>
</tr>
<tr>
<td><img src="image" alt="Diagram f)" /></td>
<td>(e \parallel f, \quad e \parallel g, \quad e \parallel h, \quad f \parallel g, \quad f \parallel h, \quad e \parallel g, \quad e \parallel h, \quad f \parallel g, \quad f \parallel h)</td>
</tr>
</tbody>
</table>

### Extension

What other information do you know about the shapes? [e.g. names of shapes: plane shapes (trapezium, rectangle, right-angled triangle, pentagon) and line shapes; types of angles; convex or concave, symmetrical or not symmetrical, etc.]

### Notes

Individual work, monitored, helped
Drawn on BB or use enlarged copy master or OHP
BB: \(\quad \quad \quad \quad \)

Agreement, self-correction, praising
Whole class activity activity
Agreement, praising
Extra praise for unexpected properties
Week 15

Y5

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong> Parallel lines 1</td>
<td>Whole class activity Discussion, agreement, praising Feedback for T</td>
</tr>
<tr>
<td>Ps each have a sheet of paper and T has large sheet for demonstration. Fold the sheet in half like this, then fold it again. Now unfold it. Let's find lines which are parallel. Ps come to front of class to show them on the large sheet of paper. Class agrees/disagrees. Elicit that there are 3 horizontal parallel lines and 3 vertical parallel lines. Let's find lines which are perpendicular. Ps come to front of class to show them on the large sheet of paper. Class agrees/disagrees. Elicit that each vertical parallel line is perpendicular to each horizontal parallel line and vice versa.</td>
<td>Elicit/stress that: • parallel lines stay the same distance apart along their lengths. • perpendicular lines form a right angle (or an angle of 90°) where they meet.</td>
</tr>
<tr>
<td><strong>2</strong> Parallel lines 2</td>
<td>Individual drawing, monitored, helped, corrected, then whole class discussion BB:</td>
</tr>
<tr>
<td>Draw a straight line on your sheet of paper and label it e. Now draw dots which are 2 cm from your line. T allows 1 minute. How many dots can be drawn? (infinite number) What shape would the dots form? (2 straight lines) Draw the 2 lines with your ruler and label them f and g. T chooses a P to work on BB at the same time. We can say that f is parallel to e, g is parallel to e and f is parallel to g. Who can write it mathematically? Who agrees?</td>
<td></td>
</tr>
<tr>
<td><strong>3</strong> PbY5a, page 74</td>
<td>Whole class discussion to start, then individual work, monitored, helped, corrected Why use a set square? (It has 2 sides which are perpendicular to one another.) T could demonstrate how to draw the first point on BB using BB ruler and compasses, while Ps work in Pbys.</td>
</tr>
<tr>
<td>Q.1 Read: Draw the set of points which are exactly 2 cm away from the straight line e. How could we do it accurately? Ps suggest what to do first and how to continue. If no P mentions using a set square, T suggests it. Agree on the following steps. 1) Lay set square so that its base is resting along the line e. 2) Draw a perpendicular line down its LH edge. 3) Measure 2 cm from line e along the perpendicular line using a ruler (mark with a dot) or compasses (mark with an arc) Ps repeat steps 1) to 3) to mark other dots which are 2 cm above line e, then draw a line through them using a ruler. Extra praise for Ps who realise that: a) only 2 points are needed to draw a straight line b) the same procedure should be done below line e (or the perpendicular lines can be extended 2 cm below line e, as shown in diagram). Solution:</td>
<td>(It is easier if the two points are as far away from each other as possible.) What can you tell me about the 3 lines? (They are parallel to one another.) Who can write it mathematically?</td>
</tr>
<tr>
<td></td>
<td>BB: e</td>
</tr>
</tbody>
</table>
Notes

Individual work, monitored, helped
Deal with one part at a time if Ps are unsure what to do.

Grid drawn on BB or use enlarged copy master or OHP
Discussion, reasoning, agreement, self-correction, praising

BB: $a \parallel x \parallel b$, $c \parallel y \parallel d$
$a \perp c$, $a \perp y$, $a \perp d$, etc.

Rules:
- line $a$: $y = 2$
- line $b$: $y = -2$
- line $c$: $x = -3$
- line $d$: $x = 3$

(T should write the rules beside corresponding lines in diagram if there is room.)

Individual work, monitored, helped, reviewed with whole class.
Agreement, praising

Whole class activity
T demonstrates each step on BB and Ps follow in Pbs.
T monitors, helps, corrects

Ps suggests names for the 3 lines (e.g. $f$, $g$ and $h$) and write a mathematical statement about them. e.g.
BB: $e \parallel f \parallel g \parallel h$
(or $e \parallel f$, $e \parallel g$, etc.)
### Y5

#### Activity

- **Lines**

Ps each have two straws on desks.

Place your straws according to my instructions and think of them as straight lines extending to infinity in both directions.

- **a)** Place one straw so that it crosses over the other straw.
  
  What is their distance apart? (0) T confirms on BB.

- **b)** Lay one straw on your desk at any angle. Now place the 2nd straw on your desk so that is a distance of 4 cm from the 1st straw.
  
  How many positions are possible? (2)

  Hold one straw in the air, and hold the 2nd straw 4 cm away from it, keeping the 2 straws parallel. How many positions of the 2nd straw are possible? (endless or infinite number of positions)

- **c)** Place the two straws on your desk something like this. (T draws 2 lines on BB.) If we think of the straws as endless lines, will they have any common points? (Yes, 1) What would be their distance apart? (0) P (or T) extends the lines on the BB to show that they will cross each other eventually.

How many common points could two straight lines in the same plane have? (0 if they are parallel, 1 if they are not parallel, infinite number of common points if the lines are touching all along their length)

Let’s draw the 3 different cases. T draws on BB and Ps draw in Ex. Bks.

#### Notes

Whole class activity, but individual manipulation of straws, monitored, corrected

BB:

or

Agreement, praising

Extra praise if a P thinks of the last idea, but if nobody does so, T asks if two lines could have an infinite number of common points.

Ps manipulate their straws and agree that it occurs when the two lines are touching along their length, i.e. they have an infinite number (∞) of common points. In effect, they are the same line.

---

#### Lines in space

- **a)** T has a diagram, similar to the one shown, drawn on BB, or preferably has a model road with toy cars for demonstration.

  What can you tell me about the two roads? BB:

  (e.g. not parallel, in different planes, traffic on one does not interfere with the traffic on the other; they will never meet each other)

  Hold your 2 straws in the air so that they will never meet each other, however far they are extended. T quickly checks each P, correcting where necessary.

- **b)** Ps have cubes on desks, and/or T has large cube for demonstration.

  Hold up the cube and show me two different edges which are not parallel and will never meet each other.

  What is the distance between the two edges? Where should we measure? P comes to front to show on large cube. Agree that the distance is the length of the vertical edge, i.e. the perpendicular distance between then. T draws diagram on BB and Ps measure their cubes (or the large cube) and dictate length to T. Class agrees/disagrees.

**35 min**
**Lesson Plan 74**

**Notes**

If possible, Ps have rods and sheets of card on desks, otherwise T demonstrates in front of class.

Discussion, agreement, praising

Individual work, monitored, helped
(or continue as a whole class activity if time is short)

Discussion, reasoning, agreement, self-correction, praising

Short individual trial in planning first, then whole class discussion

Reasoning, agreement, praising

**Parallel planes**

Stand some 3 cm Cuisennaire rods (or 3-unit strips of plastic cubes) upright on your desk and lay the sheet of card over them. What can you say about the position of the card compared with the surface of your desk?

Elicit that the surface of the desk and the sheet of paper can be thought of as two different planes which are **parallel** to one another, i.e. they will never meet, however far and in whatever direction they are extended. What is their distance apart? (3 cm)

**PbY5a, page 74**

Q.4 Read: Do the calculation in your exercise book and write the answer here.

*Imagine a block of flats which has 6 storeys, all equal in height.*

Where are the points which are an equal distance from the floor level of the 2nd storey and the floor level of the 6th storey?

Make sure that Ps understand what a storey is. Set a time limit. Ps read question again themselves and draw a diagram first to help them. Then they write answer as a sentence in Pbs.

Review with whole class. T asks several Ps to read their answers to the class. Class decides whether or not they are correct. T chooses P with correct answer to draw a diagram on BB and explain their reasoning (with T’s help if necessary).

**Solution:**

e.g. 6th – 2nd = 4 (storeys), 4 storeys ÷ 2 = 2 storeys

6th – 2 storeys = 4th storey or 2nd + 2 storeys = 4th storey

Answer: The points are on the floor of the 4th storey.

**PbY5a, page 74, Q.5**

Read: Construct the rectangle which has these adjacent sides:

\[ a = 4 \text{ cm}, \quad b = 2 \text{ cm}. \]

*Make a freehand sketch to show the order of construction.*

Allow Ps a minute to think about it and to draw a rough sketch in Ex. Bks or on scrap paper. Ps tell class what they think should be done first and the order of the following tasks. When agreement has been reached on the order of construction and correct labelling on diagram on BB, Ps draw their sketches in Pbs, revising if necessary.

Let’s draw the rectangle accurately. T dictates each step and works on BB. Ps follow instructions in Pbs.

1. Draw line segment AB, using ruler or ruler and compasses. (4 cm)
2. Lay set square on AB with LH edge at A and draw a line along its LH edge. Set compasses to 2 cm and draw arc with centre A and radius 2 cm (or measure with a ruler). Label vertex D.
3. Repeat the procedure to draw vertical line BC.
4. Join up points D and C.

What can you say about the rectangle ABCD? (AB \parallel DC, AD \parallel BC, AB \parallel DC, AB \parallel AD, AD \parallel BC, BC \parallel AB, BC \parallel CD)

**Extension**

45 min

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### Activity

Calculation and tables practice, revision, activities, consolidation  
*PbY5a, page 75*

**Solutions:**

Q.1  

a) i)  

\[a = 5 \text{ cm}, \quad b = 5 \text{ cm},\]

\[c = 18 \text{ cm} - (a + b) = 18 \text{ cm} - 10 \text{ cm} = 8 \text{ cm}\]

(isosceles triangle)

ii)  

\[a = 5 \text{ cm}, \quad b = 3 \text{ cm}, \quad c = 4 \text{ cm}\]

(right-angled triangle)

iii)  

\[c = 8 \text{ cm}, \quad a + b = 20 \text{ cm} - 8 \text{ cm} = 12 \text{ cm}\]

\[a = b = 6 \text{ cm}\]

(isosceles triangle)

b) triangle ii) should be coloured. [It is also half of triangle i)]

Q.2  

a)  

b)  

c) none  

d)  

Q.3  

b)  

![Graph](image)

![Graph](image)

A \((-2, \ -4)\)  
B \((2, \ -4)\)  
C \((2, \ 4)\)  
D \((-2, \ 4)\)

Use copy master from *LP 74/4*

Q.4  

a)  

b)  

![Graph](image)
**Y5**

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<td><strong>Lesson Plan</strong> 76</td>
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<tr>
<td>R: Parallel and perpendicular lines</td>
<td></td>
</tr>
<tr>
<td>C: Special quadrilaterals (trapezium, parallelogram, deltoid, rhombus)</td>
<td></td>
</tr>
<tr>
<td>E: Sets and properties. Definitions. Logic statements</td>
<td></td>
</tr>
</tbody>
</table>

### Constructing rectangles

Each sketch shows a different plan for constructing the same rectangle. Who can explain how each was drawn?

Ps come to BB to say each step in their own words, demonstrating (with T’s help) where necessary. Class points out errors.

**BB:**

a) ![Rectangle 1](image1.png)

b) ![Rectangle 2](image2.png)

c) ![Rectangle 3](image3.png)

d) ![Rectangle 4](image4.png)

---

**2**

**Constructing a square**

Choose one of these methods to construct a square with perimeter 16 cm in your Ex. Bks. Set a time limit.

As Ps finish, T asks them to explain the method that they used. Agree that: $P = 4 \times a$, so $a = 16 \text{ cm} / 4 = 4 \text{ cm}$

---

**3**

**Grouping shapes**

T has various plane shapes stuck on BB (different types of polygons and shapes with curved lines) (If possible, Ps have smaller version on desks.)

**BB:**

a) How could we put the shapes into 2 groups? (Ps will probably suggest polygons and not polygons). Ps come to BB to group them and Ps group their shapes on desks too.

b) Let’s put the polygons into groups. How could we do it? Who agrees? Who can think of another way to do it? (e.g. by number of sides; or convex and concave shapes; or has/have no parallel pairs of sides (perpendicular sides, equal sides), etc.

Ps choose the criteria and group the shapes (on BB and on desks).

---

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**Activity**

**PbY5a, page 76**

Q.1 Read: a) Write the number of the polygons in the correct place in the set diagram if:
- \( A \) = [It has at least 1 pair of parallel sides]
- \( B \) = [It is a quadrilateral]

b) Write ∅ in the area where there are no numbers.

c) Colour red the area where the polygons have parallel sides and are quadrilaterals.

What is a polygon? (plane shape with many straight sides)
What is a quadrilateral? (polygon with 4 sides)

T explains that in b) the symbol ∅ means an **empty set**.

Set a time limit. Review with whole class. Ps come to BB to choose a shape, say it s name, mark the parallel sides and write in the correct place in the Venn diagram. Class agrees/disagrees.

**Solution:**

a) ![Diagram](image)

b) There is no empty set!

c) Intersection of \( A \) and \( B \)  BB: \( A \cap B \)

Which part of the diagram shows the polygons which have parallel sides or are quadrilaterals?

Elicit that it is \( A + B \), i.e. the union of sets \( A \) and \( B \). BB: \( A \cup B \)

**Extension**

**Trapezium**

Draw a horizontal line across your page with a ruler. Label it e.

Now draw a parallel line below it (using ruler and set square). Label it f. Mark two points, A and B, on line f and 2 points, D and C, on line e. Join up AD and CD.

What shape have you drawn? T asks one or two Ps to describe their shapes. What name can we give to all your shapes? (quadrilaterals) What is special about all these quadrilaterals? (They have at least 1 pair of parallel sides.) What name do we give such a shape? (trapezium)

T has diagram already prepared on BB. T: Each of these quadrilaterals is called a trapezium because it has at least 1 pair of opposite sides parallel.

BB: 

Which of them is a special trapezium? Ps come to BB to point, say its name and explain the special property. [e.g. parallelogram: (1, 5, 6, 8); rhombus: (6, 8); rectangle: (5, 6); square: (6)]

Ps draw all the different types of trapeziums on their lines too.
Let's describe the different types of shapes exactly.

What is a quadrilateral?
(A quadrilateral is a plane shape which has 4 straight sides.)

When do we call a quadrilateral a trapezium?
(A quadrilateral is called a trapezium when it has at least 1 pair of parallel sides.)

When do we call a trapezium a parallelogram?
(A trapezium is called a parallelogram when it has 2 pairs of parallel sides.)

When do we call a parallelogram a rectangle?
(A parallelogram is called a rectangle when it has only right angles; or when its adjacent sides are perpendicular to one another.)

T: We could also say that a parallelogram is a rectangle if its angles are equal.

When do we call a parallelogram a rhombus?
(We call a parallelogram a rhombus when its sides are equal.)

What is a square?
(A square is a rectangle which has equal sides; or a square is a parallelogram which has equal sides and angles.)

T: We could also say that a square is a regular parallelogram.

28 min

PbY5a, page 76

Q.2 Read: List the numbers of the quadrilaterals which belong in each set.

Ask Ps to mark the parallel, perpendicular and equal sides on the diagrams. Set a time limit. Deal with one set at a time if Ps are not very able.

Review with whole class. Ps come to BB to list the shapes and mark the relevant criteria. Class agrees/disagrees. Mistakes discussed and corrected. Elicit the notation to show an empty set.

Solution:

A = {It has a pair of parallel sides}: 1, 2, 3, 4, 6, 9
B = {Its opposite sides are equal in length}: 1, 2, 6, 9
C = {Its opposite sides are parallel}: 1, 2, 6, 9
D = {All its sides are equal in length}: 2, 9
E = {It has a pair of perpendicular sides}: 1, 3, 9, 11
F = {It has a pair of parallel sides and its opposite sides are equal}: 1, 2, 6, 9
G = {It has a pair of parallel sides but not all its sides are equal}: 1, 3, 4, 6

H = {All its sides are equal but it has no pair of parallel sides}: ∅ (empty set)
I = {Its opposite sides are equal and parallel}: 1, 2, 6, 9
J = {Its opposite sides are equal but not parallel}: ∅
K = {It has a pair of parallel and a pair of perpendicular sides}: 1, 3, 9

35 min

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### Lesson Plan 76

<table>
<thead>
<tr>
<th>Activity 7</th>
<th>Notes</th>
</tr>
</thead>
</table>
| Y5 | Whole class activity

#### Deltoids

What do these quadrilaterals have in common? (e.g. They have 2 pairs of adjacent sides equal.)

**BB:**

```
 a) a 
 b b 
```

```
 b b 
``` 

```
 a a 
 b b 
```

Who remembers the mathematical name for such a shape? (deltoid)

T tells it if no P remembers.

T: A quadrilateral which has 2 pairs of adjacent equal sides is called a deltoid.

I will give you 2 minutes to draw an accurate deltoid in your Ex. Bks. using your rulers and compasses.

Is a rhombus a deltoid? (Yes, as it has 2 pairs of adjacent equal sides, but the pairs are also equal to each other.)

Is a square a deltoid? (Yes, as a square is also a rhombus.)

What else can you tell me about the deltoids in the diagram? (LH and middle deltoids are convex and RH deltoid is concave.)

---

### 8

**PbY5a, page 76, Q.3**

Read: Decide whether the statements are true or false. Write a tick or a cross.

Give Ps a couple of minutes to read the the criteria by themselves first and write ticks or crosses in Pbs.

Review with whole class. T reads statement and Ps put their hands on their heads if they think it is true and knock on the desk if they think it is false. T chooses Ps with different views to give example or counter example by drawing diagrams on BB. (Only one counter example is needed to prove that it is false.) Mistakes corrected.

**Solution:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Every rectangle is a trapezium.</td>
<td>(T)</td>
</tr>
<tr>
<td>b) Every trapezium is a rectangle.</td>
<td>(F)</td>
</tr>
<tr>
<td>c) Every rhombus is a parallelogram.</td>
<td>(T)</td>
</tr>
<tr>
<td>d) Every parallelogram is a rhombus.</td>
<td>(F)</td>
</tr>
<tr>
<td>e) A parallelogram can be a trapezium.</td>
<td>(T) [In fact, all are!]</td>
</tr>
<tr>
<td>f) All parallelograms are trapeziums.</td>
<td>(T)</td>
</tr>
<tr>
<td>g) Not all parallelograms are trapeziums.</td>
<td>(F) [negative of f)]</td>
</tr>
<tr>
<td>h) A trapezium can be concave.</td>
<td>(F) [example impossible]</td>
</tr>
<tr>
<td>i) A trapezium need not be a quadrilateral.</td>
<td>(F)</td>
</tr>
<tr>
<td>j) There is no rhombus which is concave.</td>
<td>(T)</td>
</tr>
<tr>
<td>k) All rhombi are convex.</td>
<td>(T)</td>
</tr>
<tr>
<td>l) Not every parallelogram is a rhombus.</td>
<td>(T) [negative of d)]</td>
</tr>
</tbody>
</table>

---

Whole class activity

(Or individual trial first, then whole class review)

Or Ps show T and F, or ticks and crosses, on slates or scrap paper, or Ps choose the actions to be used.

Responses shown in unison.

At a good pace

In good humour!

Discussion, reasoning, agreement, praising

Feedback for T
**Lesson Plan**

### Y5

**R:** Perimeter and area of rectangles  
**C:** Special quadrilaterals  
**E:** Sets, definitions, logic statements

#### Activity

##### 1

**Special quadrilaterals**

T has various quadrilaterals drawn (stuck) on BB. T asks Ps to come to BB to select certain shapes and to say the criteria that they used to choose them. Class agrees/disagrees, or points out important properties not mentioned.

- trapeziums: (quadrilaterals with at least 1 pair of parallel sides)
- parallelograms: (opposite sides parallel)
- rectangles: (opposite sides equal and parallel and adjacent sides perpendicular)
- rhombi: (equal sides and opposite sides parallel)
- deltoids: (2 pairs of adjacent equal sides)
- squares: (equal sides, opposite sides parallel and adjacent sides perpendicular)

5 min

##### 2

**Sets**

T draws one set in a Venn diagram on BB and gradually builds it up.

- **BB:** Who can draw some shapes in it?  
- **Ps:** Who can draw some shapes in it?

This is the set of rhombi.  
Who can draw some shapes in it? BB: Where should the square go? Why?  
Ps rub out square and redraw in a better position if necessary.

Where should we draw the set for parallelograms? Ps come to BB or tell T how to draw it. Who can draw some shapes in it?

**BB:** e.g.

![Diagram](image)

T points to different areas in the diagram and asks Ps to describe the type of shapes which belong there. e.g. areas indicated in diagram:

1. Rectangles with different sides. Rectangles which are not squares.
2. Parallelograms which are not rectangles and not rhombi.

Extra praise if Ps suggest quadrilaterals as the next wider set (then polygons, then plane shapes) but there is no need to draw them all!

15 min

### Notes

Whole class activity  
Drawn/stuck on BB  
At a good pace  
Reasoning, agreement, praising  
Extra praise if Ps give correct definitions – but do not expect it!

T repeats reasoning in a more concise, clearer way where necessary.

Whole class activity  
(Or T has shapes stuck to side of BB and Ps stick them in the correct set)

Only 2 or 3 shapes need be drawn in each set.  
Elicit that a square is also a rectangle and a rhombus.

If necessary, T says the first statement as a model for Ps to follow.  
Class agrees/disagrees.  
Extra praise for creativity!

Encourage the use of not, and, or. Praising, encourage only
### Activity 3

**Trapeziums**

Study these 3 shapes. What is common to them all? (They are all trapeziums, i.e. quadrilaterals with at least 1 pair of parallel sides.)

BB:

```
        c          c          c
       / \        / \        / \  
      /   \      / \      /   \ 
     /     \    /   \    /     \ 
    d  1  b  d  2  b  d  3  b  a  a  a
   /   \ /   \ /   \ /   \ /   \ /   \ 
  a  a  a  a  a  a  a  a  a  a  a  a
```

What is different about each one? Elicit that:

1. has two right angles.
2. has two equal sides. \((d = b)\)
3. has all its sides of different lengths. (general trapezium)

What is the perimeter of each trapezium? Who can write a general plan for it? Ps come to BB or dictate to T. Class agrees/disagrees.

T: Side \(a\) or side \(c\) (i.e. the parallel sides) can be thought of as the base of the trapezium. Where would we measure the height of the trapezium? Agree that it is the shortest distance between \(a\) and \(c\), i.e. the perpendicular distance between them.

In your Ex. Bks. draw a rough sketch of the 3 types of trapezium, marking any parallel or equal sides and right angles and showing where you would measure its height.

---

### Activity 4

**Parallelograms**

What can you say about the quadrilateral ABCD? (It is a parallelogram, i.e. its opposite sides are parallel) Ps come to BB to mark on diagram.

Where should we measure the height of the parallelogram? Elicit that two height measurements are possible: the perpendicular distance between \(a\) and \(c\) or between \(b\) and \(d\).

If we think of side \(a\) as the baseline, we can draw a perpendicular line from \(a\) to \(c\). T demonstrates on BB by extending lines \(a\) and \(b\) and using a set square to draw a line perpendicular to line \(a\). Let’s label it \(h_a\).

If we think of side \(b\) as the baseline, we can draw a perpendicular line from \(b\) to \(d\). T demonstrates on BB by extending lines \(b\) and \(d\) and using a set square to draw a line perpendicular to line \(b\). Let’s label it \(h_b\).

What is special about these parallelograms? (LH parallelogram is a rhombus, middle parallelogram is a rectangle and RH parallelogram is a square) How could we measure their heights? Ps come to BB to show on diagrams, with T’s help if necessary. Class agrees/disagrees.

BB:

```
   D     C
  / \     / \  
 /   \   /   \ 
D     C
```

Elicit that in the:

- **rhombus**: there only one height, \(h\), but it can be measured in 2 places. **rectangle**: \(h_a = b\) and \(h_b = a\); **square**: only 1 height: \(h = a\)

How could we calculate the perimeter of each type of parallelogram? Ps come to BB or dictate to T. Class agrees/disagrees.

---

### Notes

**Lesson Plan 77**

Whole class activity

- Drawn on BB or use enlarged copy master or OHP

Discussion, agreement, praising

- \(BB: \ P = a + b + c + d\)

After agreement, T draws measuring arrow on diagram with BB ruler and labels it \(h\). Elicit that in \(h = d\)

Individual work, monitored, helped, corrected

Praising, encouragement only

---

**Whole class activity**

- Drawn on BB or SB or OHT

**BB:**

```
   D     C
  / \     / \  
 /   \   /   \ 
D     C
```

\(P = a + b + c + d\)

T demonstrates and Ps could copy in Ex. Bks.

**BB:**

```
   D     C
  / \     / \  
 /   \   /   \ 
D     C
```

Drawn on BB or SB or OHT

(or Ps could have small shapes on desks)

At a good pace

Discussion, reasoning, agreement, praising

Elicit that:

- \(P_{\text{parallelogram}} = P_{\text{rectangle}}\)
- \(2 \times (a + b) = 2a + 2b\)
- \(P_{\text{rhombus}} = P_{\text{square}}\)
  - \(4 \times a = 4a\)
### Activity 5

**PbY5a, page 77**

**Q.1 Read:** Write the numbers of the trapeziums in the correct set.

Set a time limit of 1 minute. Review with whole class. Ps come to BB to write the numbers, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7\]

**Read:** List in your exercise book the common properties of these trapeziums:

a) 2, 3 and 5  
b) 3, 4, 5 and 7  
c) 3, 5, 6 and 7.

Deal with one set at a time. Ps come to BB to point to relevant shapes and say what they have in common. Who agrees? Who can think of something else? etc.

**Solution:**

a) 2, 3, 5: They have at least 2 right angles.

b) 3, 4, 5 and 7: They have 2 pairs of parallel sides. (parallelograms)

c) 3, 5, 6 and 7: They are symmetrical. Elicit what symmetrical means, then Ps come to BB to draw the lines of symmetry (or mirror lines). Ps do the same on diagrams in Pbs.

Is there a concave trapezium among these shapes? (No) Who can draw one on the BB? (Agree that it is impossible, as a trapezium has at least 1 pair of opposite sides which are parallel to one another, so all trapeziums are convex.)

---

**Notes**

30 min

---

### Activity 6

**PbY5a, page 77**

**Q.2 Read:** Make a set diagram for these parallelograms. Write the numbers of the parallelograms in the correct set.

Set a time limit of 2 minutes. Review at BB with whole class. Ps come to BB to draw sets, label them and write the numbers in the correct places. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7\]

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Agreement, self-correction, praising

Whole class activity

(or individual trial first if Ps wish)

At a good pace

Discussion, reasoning, agreement, praising

BB: symmetrical

(one half is the mirror image of the other half)

Allow Ps to try it if they think such a shape exists!

Agreement, praising

---

Individual work, monitored, helped, corrected

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Feedback for T
### Lesson Plan 77

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<td><strong>Lesson Plan 77</strong></td>
</tr>
<tr>
<td><strong>Activity</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>(Continued)</td>
</tr>
<tr>
<td>Read: List in your exercise book the common properties of these trapeziums:</td>
<td>Whole class activity (or individual trial first if Ps wish)</td>
</tr>
<tr>
<td>a) 1, 4 and 6 b) 3, 4 and 7 c) 1, 3, 4, 6 and 7.</td>
<td>At a good pace</td>
</tr>
<tr>
<td>Deal with one part at a time. Ps come to BB to point to relevant shapes and say what they have in common. Class agrees/disagrees. Solution:</td>
<td>Discussion, reasoning, agreement, praising</td>
</tr>
<tr>
<td>a) 1, 4 nd 6: They have equal sides. (i.e. rhombi)</td>
<td><strong>Extension</strong></td>
</tr>
<tr>
<td>b) 3, 4 and 7: They have 4 right (equal) angles; their adjacent sides are perpendicular to one another. (i.e. rectangles)</td>
<td>Draw lines of symmetry where relevant [Only 2 and 5 are not symmetrical.]</td>
</tr>
<tr>
<td>c) 1, 3, 4, 6 and 7: Parallelograms which have equal sides or which have equal angles. (i.e. rhombus or rectangle)</td>
<td></td>
</tr>
<tr>
<td><strong>35 min</strong></td>
<td></td>
</tr>
<tr>
<td><strong>7</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Missing words</strong></td>
<td>Individual work, monitored</td>
</tr>
<tr>
<td>Copy and complete these sentences in your Ex. Bks. so that they are true and make sense. Set a time limit.</td>
<td>Written BB or SB or OHT</td>
</tr>
<tr>
<td>[Ps finished quickly write their own sentences about quadrilaterals and underline words they want the class to guess.]</td>
<td>Make sure that Ps realise that the sets of 5 dots are ellipses and show only that something is missing; they do not stand for single 5-letter words!</td>
</tr>
<tr>
<td>Review with whole class. T chooses Ps to read the whole sentence, stressing their own words. Who did the same? Who wrote something else? etc. Deal with all cases. Class decides which words are not valid. Mistakes corrected.</td>
<td>Agreement, self-correction, praising</td>
</tr>
<tr>
<td>BB:</td>
<td>Accept any suitable words in a) and b)</td>
</tr>
<tr>
<td>a) . . . . parallelogram is a trapezium. e.g. (Every)</td>
<td>In good humour!</td>
</tr>
<tr>
<td>b) A trapezium . . . . 4 equal sides. e.g. (can have)</td>
<td>Agreement, praising</td>
</tr>
<tr>
<td>c) A parallelogram which has 4 equal sides is called a . . . . (rhombus)</td>
<td>Extra praise for creativity!</td>
</tr>
<tr>
<td>d) A rhombus which has equal angles is called a . . . . (square)</td>
<td></td>
</tr>
<tr>
<td>Ps who wrote their own sentences come to front of class to read them aloud, saying ‘something’ instead of their underlined words. Class has to think of words which make the sentence true.</td>
<td></td>
</tr>
<tr>
<td><strong>40 min</strong></td>
<td></td>
</tr>
</tbody>
</table>
### Y5

#### Activity

**PbY5a, page 77**

| Q.3 | Read:  
|-----|------------------------------------------|
|     | a) **Label the sides with letters, using the same letter for equal sides.**  
|     | b) **Below each shape, write a plan for its perimeter.**  
|     | c) **Measure the sides, then calculate the perimeters in your exercise book.**  
|     | First revise the usual practice for labelling sides of quadrilaterals. (Start with $a$ on the bottom and move in a clockwise direction around the shape.) Set a time limit or deal with one part at a time.  
|     | Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed/corrected.  

**Solution:**

<table>
<thead>
<tr>
<th>i)</th>
<th><img src="image" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = a + b + c + d$</td>
</tr>
<tr>
<td></td>
<td>$= 21 + 14 + 9 + 12$</td>
</tr>
<tr>
<td></td>
<td>$= 56$ (mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>ii)</th>
<th><img src="image" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = a + a + c + d$</td>
</tr>
<tr>
<td></td>
<td>$= 15 + 15 + 6 + 12$</td>
</tr>
<tr>
<td></td>
<td>$= 48$ (mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>iii)</th>
<th><img src="image" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = 2(a + b)$</td>
</tr>
<tr>
<td></td>
<td>$= 2(25 + 12)$</td>
</tr>
<tr>
<td></td>
<td>$= 74$ (mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>iv)</th>
<th><img src="image" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = 2(a + b)$</td>
</tr>
<tr>
<td></td>
<td>$= 2(16 + 12)$</td>
</tr>
<tr>
<td></td>
<td>$= 56$ (mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>v)</th>
<th><img src="image" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = a + 2b + c$</td>
</tr>
<tr>
<td></td>
<td>$= 28 + 2 \times 15 + 9$</td>
</tr>
<tr>
<td></td>
<td>$= 67$ (mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>vi)</th>
<th><img src="image" alt="Diagram" /></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P = 2(a + b)$</td>
</tr>
<tr>
<td></td>
<td>$= 2(30 + 12)$</td>
</tr>
<tr>
<td></td>
<td>$= 84$ (mm)</td>
</tr>
</tbody>
</table>

#### Extension

Where would you measure their heights? Ps draw measuring lines and label them $h$.  

**45 min**

---

### Lesson Plan 77

#### Notes

Individual work, monitored, helped  
Drawn on BB or use enlarged copy master or OHP  
Accept other letters too, of course, and any correct form of equation.  
Reasoning, agreement, self-correction, praising  
Accept small variations in lengths. Ps with wildly inaccurate lengths measure the sides again.  
(Can be completed for homework if necessary.)  
What name can we give to all the shapes? (trapeziums)  
Whole class activity or extra individual work for quicker Ps.
### Lesson Plan

**Y5**

| R: Trapeziums | C: Perpendicular/parallel lines. Reading axonometric diagrams | E: Drawing axonometric diagrams of solids. Illustrating 3 views |

### Activity

**1**  
**Parallel and perpendicular lines**

1. In your *Ex. Bks* draw a straight line and label it *a*.
2. Construct a line which is **perpendicular** to line *a* and label it *b*.
   (T demonstrates on BB using BB ruler and set square if necessary.)
3. Construct a line which is **parallel** to line *a* and label it *c*.
   What is the position of line *b* in relation to line *c*? (b ⊥ c)
4. Draw a line *d* so that a rectangle is formed.
   What can you say about line *d*? (d || b, d ⊥ a, d ⊥ c)

**Notes**

- **Ps have rulers and set squares.**
  Individual drawing, closely monitored, helped
  BB: e.g.  
  ![Diagram](image)
  Agreement,
  praising

**2**  
**PbY5a, page 78**

Q.1 Read: *The diagram shows part of a map.*

Who can explain what the scale means? Ask several Ps what they think. Elicit that the distance on the diagram compared with the real-life distance is in the **ratio** of 1 to 50 000.

If we use 1 mm as the unit of measure in the diagram, what length would it represent in real life? (50 000 mm)

Let’s write the scale another way. Ps dictate what T should write.

Now let’s measure the distances asked for. Where should we measure? (In a straight line between the 2 closest points, or the perpendicular distance between a point and a line)

Deal with one part at a time. Ps read the question, measure to the nearest mm, write length in *Pbs*, then calculate the distance in real life.


**Solution:**

**How far away is:**

- **a) the waterfall from the statue:**
  i) on the map: 15 mm
  ii) in real life: 15 × 50 m = 750 m

- **b) the waterfall from the road**
  i) on the map: 12 mm
  ii) in real life: 12 × 50 m = 600 m

- **c) the statue from the forest:**
  i) on the map: 16 mm
  ii) in real life: 16 × 50 m = 800 m

**Extension**

Ps think of other questions too, e.g. the distance of the statue (forest) from the road, the waterfall from the forest, etc.

---

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<table>
<thead>
<tr>
<th>Activity</th>
<th>Plane shapes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>In your <em>Ex. Bks</em>, draw freehand sketches of the shapes I describe and mark the important information on your diagram.</td>
</tr>
<tr>
<td></td>
<td>Can you tell me anything else about the shape you have drawn?</td>
</tr>
<tr>
<td></td>
<td>a) a triangle with 2 equal sides (isosceles)</td>
</tr>
<tr>
<td></td>
<td>b) a right-angled trapezium with all its sides equal (square)</td>
</tr>
<tr>
<td></td>
<td>c) a right-angled parallelogram with unequal sides (rectangle)</td>
</tr>
<tr>
<td></td>
<td>d) a trapezium which has exactly two right angles</td>
</tr>
<tr>
<td></td>
<td>Ps can describe some shapes too if there is time.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes</th>
<th>Individual drawing but class kept together on the tasks, monitored, helped</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T advises Ps to imagine it in their heads first, then draw the rough shape. There is no need for rulers or measuring!</td>
</tr>
<tr>
<td></td>
<td>Discussion, agreement self-correction, praising</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>Solids</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>T has various solids on desk in front of class (or drawn on BB)</td>
</tr>
<tr>
<td></td>
<td>a) How could we group these solids. Ps suggest various ways .</td>
</tr>
<tr>
<td></td>
<td>BB:</td>
</tr>
<tr>
<td></td>
<td>e.g. only plane faces, both plane and curved faces, only curved faces</td>
</tr>
<tr>
<td></td>
<td>e.g. here are the frames of some of the solids. Which frames match which solids? Ps come to front of class to match them up (or draw joining lines if using copy master). Class agrees/disagrees.</td>
</tr>
<tr>
<td></td>
<td>BB:</td>
</tr>
<tr>
<td></td>
<td>c) T holds up one solid at a time and Ps say what they know about it.</td>
</tr>
<tr>
<td></td>
<td>(e.g. name; number of faces, edges, vertices; parallel/perpendicular faces and edges; symmetry; equal edges (faces), convex, etc.)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes</th>
<th>Whole class activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Use real models and frames if possible, otherwise draw (stick) on BB or use enlarged copy master or OHP and Ps join up the matching shapes.</td>
</tr>
<tr>
<td></td>
<td>At a good pace</td>
</tr>
<tr>
<td></td>
<td>Agreement, praising</td>
</tr>
<tr>
<td></td>
<td>T notes Ps having difficulty.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>Different views 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>T has a large square of paper and an object for demonstration. Ps have smaller squares and objects. (e.g. wooden house, car, animal, etc.)</td>
</tr>
<tr>
<td></td>
<td>T explains how to fold the square into quarters, unfold, cut (or tear) along a fold line to the centre and fold one quarter below the other, as in diagram, to make a 3-plane screen for recording different views.</td>
</tr>
<tr>
<td></td>
<td>T demonstrates how to draw around the object's base to obtain the <em>top view</em>, hold the object agains the back square and draw around it to obtain the front view, and similarly against the side square for a <em>side view</em>.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes</th>
<th>Individual folding and drawing but T demonstrates and keeps class together at each step</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Ps might remember a similar task in Year 4.)</td>
</tr>
<tr>
<td></td>
<td>Use simple objects or models.</td>
</tr>
<tr>
<td></td>
<td>Ps should draw in pencil, so that mistakes can be rubbed out easily</td>
</tr>
<tr>
<td></td>
<td>T monitors, helps, corrects, praises</td>
</tr>
</tbody>
</table>

**[To Ts:** |

This device is known as a 3–plane (Monge) projection.]|

| Whole class review |
| Reasoning, agreement, praising only |
Different views 2
Ps make other 3–D screens and use them to draw front, side and top views of other objects or geometric solids. [e.g. sphere (ball or marble), cylinder, pencil, cuboid, egg, cone, pyramid, various prisms, etc.]
(T could have a selection laid out at various points in the classroom.)
T chooses some Ps to draw their 3 views on the BB and Ps have to guess what the real object is.

30 min

PbY5a, page 78
Q.2 Read: Each solid was cut from a cube with edges 3 units long. Draw how you would see it from the front, from the side and from above. Calculate its volume,
Deal with one part at a time, or set a time limit.
Review with whole class. Ps come to BB to draw their views. Who agrees? Who drew something different? etc.
If disagreement, confirm on a large model. Mistakes corrected.
In b), Ps might point out that the LH and RH side views are different, so 2 side views should be given. (There is room in Pbs for Ps to draw another grid.)

What is the surface area of each solid?

Solution:

a) Front view Side view Top view

Volume = $3 \times 3 \times 3 - 2 \times 3 = 27 - 6 = 21$ (unit cubes)
Area = $2 \times 9 + 2 \times 7 + 2 \times (5 \times 3)$
(sides) (front/back) (top/bottom)
= $18 + 14 + 30 = 62$ (unit squares)

b) Front view Side view Side view Top view

Volume = $5 \times 3 = 15$ (unit cubes) (5 columns of 3 cubes)
Area = $2 \times 5 + 2 \times 9 + 2 \times 9$
(top/bottom) (sides) (front/back)
= $10 + 18 + 18 = 46$ (unit squares)
**Y5**

**Activity**

8

**PbY5a, page 78**

Q.3 Read: **Draw a copy of each solid on the grid. Name the solid and count how many vertices, edges and faces it has.**

Deal with one shape at a time or set a time limit.

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Confirm on models if there is disagreement. Mistakes discussed and corrected.

Solution:

Let's draw 3 views of the pyramid. Ps come to BB. Class agrees/disagrees.

[Extra praise if a P points out that, e.g. in b), the cube's base should be 4 unit squares but in the diagram it is shown as covering 2 unit squares. Discuss the need to shorten depth when representing 3-D shapes in 2 dimensions, otherwise the diagram would not look like a cube. (T could redraw the cube with a base of 4 unit squares to show how wrong it looks.)]

<table>
<thead>
<tr>
<th>Name</th>
<th>Cuboid</th>
<th>v = 8</th>
<th>e = 12</th>
<th>f = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name</td>
<td>Cube</td>
<td>v = 8</td>
<td>e = 12</td>
<td>f = 6</td>
</tr>
<tr>
<td>Name</td>
<td>Cuboid</td>
<td>v = 8</td>
<td>e = 12</td>
<td>f = 6</td>
</tr>
<tr>
<td>Name</td>
<td>Pyramid</td>
<td>v = 5</td>
<td>e = 8</td>
<td>f = 5</td>
</tr>
</tbody>
</table>

**Extension**

45 min

**Lesson Plan 78**

**Notes**

Individual work, monitored, helped, (corrected)

Drawn on BB or use enlarged copy master or OHP

If possible, T has already prepared models of the solids.

Reasoning, agreement, self-correction, praising

What do you notice about the results?

(All the cuboids, whatever their size or shape, have the same number of vertices, edges and faces.)

Which faces (edges) are parallel (perpendicular)?

Ps come to BB to show them on diagram.

BB: [front | side]

[To Ts: Such diagrams are known as axonometric diagrams.]
### Lesson Plan

#### Week 16

<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
</table>

#### R: Revision

#### C: Practice: trapeziums, triangles; parallel, perpendicular

#### E: Sets of points

### 1 Sets of triangles

What could the labels of each set mean? Ask several Ps what they think and why. Ps come to BB to explain.

**BB:**

*\( A = \{\text{Triangles}\} \)
*\( B = \{\text{Isosceles triangles}\} \)
*\( C = \{\text{Equilateral triangles}\} \)

Could we have grouped the triangles in another way?

*\( D = \{\text{acute-angled triangles}\} \)
*\( E = \{\text{right-angled triangles}\} \)
*\( F = \{\text{obtuse-angled triangles}\} \)

Where would we put a triangle which has parallel sides? (There is no such triangle!) Agree that each pair of sides in a triangle must meet at a point, so they cannot be parallel.

---

#### 2 Polygons

How could we put the polygons into two groups? T asks several Ps what they think. Ps come to BB to explain reasoning (moving shapes into appropriate groups if stuck on BB). Class agrees/disagrees.

**BB:**

*Triangles* or *quadrilaterals*

- Equal sides or No equal sides
- Parallel sides or No parallel sides
- Perpendicular sides or No perpendicular sides
- Symmetrical or Not symmetrical (asymmetrical)

How could we put the quadrilaterals into groups?

- **e.g.** by name:
  - Trapeziums: 1, 4, 5, 6, 7 (at least 1 pair of \( \parallel \) lines)
  - Parallelograms: 1, 4, 5, 6 (2 pairs of \( \parallel \) lines)
  - Rectangles: 1, 4
  - Rhombi: 1, 5
  - Square: 1
  - Deltoids: 1, 3, 5

---

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### Activity 3

**Solids**

T has diagrams drawn on BB and models of solids on desk.

**BB:**

- cuboid
- pyramid
- triangular-based prism
- square-based cuboid
- cube

T holds up one model at a time and Ps say its name and show which diagram matches it. They also say the main properties that they know.

T reminds Ps about the name of solid 3 if Ps have forgotten it and explains what a **prism** is. (A prism is a polyhedron which has at least 2 faces which are equal and parallel.) Which of the other solids are prisms? (1, 4, 5)

How could we group them? Ps suggest ways. e.g.

- Cuboids: 1, 4, 5
- Square-based: 2, 4, 5
- Prisms: 1, 3, 4, 5
- Pyramid: 2

Who can think of other questions to ask about the solids? (e.g. number of faces, edges, vertices; type of faces; finding parallel/perpendicular edges or faces; etc.)

T elicits or points out that in a cuboid:

- any two adjacent edges are perpendicular
- at any vertex, any two of the three edges are perpendicular.

15 min

### Activity 4

**PbY5a, page 79**

Q.1 Read: **Draw lines through the two parallel lines to make different trapeziums.**

**Make one of the shapes a special trapezium.**

Set a time limit. Ask Ps to write the name of any special trapezium inside the shape if they know it.

Review with whole class. T has a sample already prepared to show to class and elicits the name of each special trapezium. e.g.

**BB:**

<table>
<thead>
<tr>
<th>square</th>
<th>rectangle</th>
<th>trapezium</th>
<th>rhombus</th>
<th>trapezium</th>
<th>parallelogram</th>
<th>trapezium</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>right-angled</td>
<td></td>
<td>general</td>
<td>equal legs</td>
</tr>
</tbody>
</table>

Who drew all these trapeziums? Let's give them a clap! T asks one or two Ps which types they did not draw.

Who can say a true statement about trapeziums? T gives first one as an example if Ps cannot think of any. (e.g. Every square is a rectangle. Not all trapeziums are rectangles. etc.)

20 min

**Notes**

- Whole class activity
- Drawn on BB or use enlarged copy master or OHP
- If models are not possible, use frames instead.

- Agreement, praising
- T helps with vocabulary where necessary.

- BB: **polyhedron**
  - (solid with many plane faces)
  - **prism**
  - (polyhedron with at least two faces equal and parallel)

- Extra praise for creativity!
  - (Cuboids: 8 vertices, 6 faces, 12 edges)

- Demonstrate on large cube.

- Individual work, monitored (helped)
  - Ps should use rulers.

- Or Ps finished first show their trapeziums on BB and Ps point out the types that they have missed.

- Agreement, praising

- At speed. T chooses Ps at random. Class points out incorrect statements. Praising
### Activity 5

**PbY5a, page 79**

Q.2 Read: **Draw the set of points on the same plane which are 2 cm from:**

- **a) this ray**
- **b) this line segment.**

T advises Ps to measure with a ruler at different places and mark lots of individual dots first until they have an idea of the shape to draw. Then they use ruler and compasses to draw the lines accurately. Set a time limit.

Review with whole class. Ps come to BB to draw the lines using BB instruments, or tell T what to draw. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:** (reduced to scale)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram a)" /></td>
<td><img src="image2" alt="Diagram b)" /></td>
</tr>
</tbody>
</table>

**Extension**

Imagine the set of points in space. What shapes would they form?

- **a) circular tube with radius 2 cm, joined to the surface of a semi-sphere with radius 2 cm at one end and stretching to infinity at the other.**
- **b) a ‘capsule’ shape, i.e. as a) but the surface of a semi-sphere at each end.**

### Sets of points

Draw a 5 cm square in the middle of a page in your Ex. Bks. T draws a larger version on BB.

Now let's draw the set of points in the same plane which are 2 cm away from the square.

First Ps measure lots of dots 2 cm away from different sides of the square to get an idea of the shape of the set of points.

If Ps suggest drawing a larger square around the smaller one, do so but then elicit/point out that the corner points are more than 2 cm from the original square. What should we do? (Draw straight lines equal and parallel to each side, then use compasses to draw arcs with radius 2 cm and centres at each corner point to join up the straight lines.)

T (P) works on BB and Ps work in Ex Bks. Elicit that each arc is 1 quarter of a circle with radius 2 cm.

Are any other points possible? (No, as we are thinking of the square as being the points on its perimeter and all the points inside it)

### Extension

If I asked you to draw the set of points which are 2 cm from the perimeter of the 5 cm square, are any other points possible?

(Pe: Points inside the 5 cm square) T shows a diagram already prepared. Do you think that this is correct? Some Ps might think that the inner square should have curved corners too but T shows that the shortest distance of any point on the perimeter of the inner square is 2 cm away from the 5 cm perimeter. P measure and mark dots to confirm.

### Notes

**Lesson Plan 79**

- Individual work, monitored, helped with use of compasses
- Or encourage more able Ps to try to visualise the shape in their heads first.

- Discussion, reasoning, agreement, self-correction, praising
- (Or T has diagrams already prepared and uncovers each part as Ps describe it.)

Again, ask Ps to imagine it in their heads.

Discussion, agreement, praising

If possible, T has real models prepared to show to class.

**Whole class activity, but individual drawing in Ex. Bks**

BB:

Discussion, demonstration, agreement, praising

BB:
### Lesson Plan 79

#### Activity 7

**PbY4a, page 79**

Q.3 Read:  
- *a) Can this net be folded to make a cube?*  
- *b) Complete each net so that it can be folded to make a solid.*

Set a time limit. If possible, Ps have large grids and scissors on desks so that they can check their drawings by cutting and folding.  
Review at BB with whole class.  
- a) Stand up if you think the net in a) makes a cube! (Challenge Ps who stand to fold the net – they will have to agree that it can’t be done!)  
- b) A, come and show us your net. A holds up net and completes the drawing on BB. Who drew the same? Who drew a different net? Ps confirm that their nets are correct by folding.  

**Solution:**  
e.g.  
- i)  
  ![cube net](image1)  
  or  
  ![etc. net](image2)  
- ii)  
  ![pyramid net](image3)  
  or  
  ![etc. net](image4)

**Notes**

Individual work, monitored (or whole class activity if Ps cannot make their own nets)  
T has large nets already prepared and various solids on desk.  
Drawn on BB or SB or OHT  
BB:  

Demonstration, agreement, self-correction, praising  
Deal with all cases.  
Which of these solids do the nets represent? Ps choose from those on T’s desk.  
What can you say about them? (name, number of vertices, faces, edges etc.)

**Erratum**

In b) in *Pbs*: Side view: there should not be a horizontal line at top of grid.

**8**

**PbY5a, page 79**

Q.4  
- a) Read: *A thick black line has been drawn on the surface of a transparent glass cube. Draw the 3 views of the line.*  
  First elicit meaning of transparent. T could have a glass or plastic cube to show to class. Imagine that you are looking at the cube from the front, side and top. Draw what you see.  
  Set a time limit. Review with whole class. Ps come to BB to draw the different views. Class agrees/disagrees. If possible, T confirms by showing a prepared model.  
  Mistakes corrected.  

**Solution:**  
e.g.  
- Front view  
- Side view  
- Top view

b) Read: *Draw a line on the surface of the glass cube to match the 3 views shown below:*  
Set a time limit. Review with whole class. Ps come to BB to draw lines on diagram. Who agrees? Who drew the lines in another position? T confirms on pre-prepared cube (or Ps demonstrate solution by drawing lines on large cube in front of class.) Mistakes corrected.  

**Solution:**  
e.g.  
- Front view  
- Side view  
- Top view

c) Who can draw the line on this cube? (Very difficult!)  

**Notes**

Individual work, monitored, helped, corrected  
Drawn on BB or use enlarged copy master or OHP  
If possible, Ps have plastic cubes for trials.  
Discussion, agreement, self-correction, praising  

**Erratum**

b) done as a whole class activity  
Encourage Ps to visualise the cube in their heads and imagine how it would look from the 3 directions.  
Extra praise for Ps who drew correct line without needing to use a model  
(Side view from RHS)

**Extension**

Extra optional task as individual work for the more able Ps  
Ps draw cube/views in *Ex. Bks.*
**Y5**

**Activity**

Revision, consolidation, activities

*PbY5a, page 80*

**Solutions:**

**Q.1**

![Hexomino Diagram]

**Q.2**

a) 9-unit squares: 1  
4-unit squares: 4  
1-unit squares: 9  
Total: **14** squares

b) 6-unit rectangles: 1  
4-unit rectangles: 2  
3-unit rectangles: 2  
2-unit rectangles: 7  
1-unit rectangles: 6  
Total: **18** rectangles

**Q.3**

There are 35 different hexominoes but only 11 of them can form a net for a cube. Encourage a systematic strategy to find all the possibilities. E.g. squares joined in a row:

<table>
<thead>
<tr>
<th>Pattern</th>
<th>Number possible</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="hexomino-shaded.png" alt="Shaded Hexomino" /></td>
<td>1</td>
</tr>
<tr>
<td><img src="pattern.png" alt="Pattern" /></td>
<td>3</td>
</tr>
<tr>
<td><img src="another-pattern.png" alt="Another Pattern" /></td>
<td>5</td>
</tr>
<tr>
<td><img src="yet-another-pattern.png" alt="Yet Another" /></td>
<td>8</td>
</tr>
<tr>
<td><img src="yet-another-pattern-2.png" alt="Yet Another Pattern" /></td>
<td>3</td>
</tr>
<tr>
<td><img src="yet-another-pattern-3.png" alt="Yet Another Pattern" /></td>
<td>13</td>
</tr>
<tr>
<td><img src="yet-another-pattern-4.png" alt="Yet Another Pattern" /></td>
<td>Total: 35</td>
</tr>
</tbody>
</table>

Dashed lines are corrections to the lines crossed out.

Shaded hexominoes can be used as the net for a cube.

Hexominoes in *Pb*s labelled.