**Activity 1**

### The number 1

T writes '1' on BB. Tell me different ways of defining 'one'.

Ps dictate different ways and T writes them on BB. Class agrees/disagrees. e.g.

BB:  \( \frac{1}{3} = \frac{3}{9} = 23 - 22 = \frac{1}{8} + \frac{7}{8} = 3 \times \frac{1}{3} = 50\% \text{ of } 2, \text{ etc.} \)

T: 1 is the base unit among the numbers. Let's think about what its role is in the 4 operations. What are the 4 operations? (+, −, ×, ÷)

T writes a few calculations on BB for each type of operation and Ps dictate results, then discuss the general role of '1'. e.g.

BB:  \( 1 + 1 = 2, \ 2 + 1 = 3, \ 3 + 1 = 4, \ldots \)

\( 100 - 1 = 99, \ 44 - 1 = 43, \ 5 - 1 = 4, \ 1 - 1 = 0, \ldots \)

\( 5 \times 1 = 5, \ 1 \times 439 = 439, \ 1 \times \frac{5}{8} = \frac{5}{8}, \ 1 \times 0.7 = 0.7, \ldots \)

Elicit that:

- adding 1 to a whole number results in the next greater whole number;
- subtracting 1 from a whole number results in the next smaller whole number;
- multiplying or dividing a number by 1 results in that number.

4 min

### Factorising

Factorise each of these numbers in your exercise book and then list all its positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB to show their method of finding the prime factors. Who did the same? Who did it a different way? etc. Then Ps dictate all the positive factors for each number.

\[ \begin{align*}
176 &= 2 \times 2 \times 2 \times 11 \\
351 &= 3 \times 3 \times 39 \\
1001 &= 7 \times 11 \times 13
\end{align*} \]

\[ \begin{align*}
\text{Positive factors of: } 176 &:\ 1, 2, 4, 8, 11, 16, 22, 44, 88, 176 \\
351 &:\ 1, 3, 9, 13, 27, 39, 117, 351 \\
1001 &:\ 1, 7, 11, 13, 77, 91, 143, 1001
\end{align*} \]

Are there any common factors? (Only 1 is common to all three numbers; 11 is a common factor of 176 and 1001; 13 is a common factor of 351 and 1001)

10 min

**Notes**

Whole class activity

Encourage Ps to make use of all the operations and types of numbers that they know.

At speed

Involve the majority of Ps.

Agreement, praising

Extra praise for creativity

(or Ps dictate the operations)

Agreement, praising

T repeats clearly if necessary.

Individual work, monitored, (helped)

Numbers written on BB or SB or OHT

(T decides whether to allow the use of calculators.)

Discussion, agreement, self-correction, praising

If all Ps used the same method, T shows the other method too.

Whole class activity

Ps could join up the factor pairs.

Ps come to BB to point them out. Praising
**Activity**

**Place-value**

T writes the number 87 654 on BB and Ps read it out in unison.

Let’s complete this table about it. Ps come to BB to choose a digit and fill in its column. Class agrees/disagrees and helps where necessary.

(T could do first column as a model for Ps to follow if they are unsure.)

<table>
<thead>
<tr>
<th>BB: 87 654</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digit value</td>
</tr>
<tr>
<td>Place value</td>
</tr>
<tr>
<td>Actual value</td>
</tr>
<tr>
<td>In sum form</td>
</tr>
</tbody>
</table>

**Extension**

If I wrote the sum like this, can anyone explain it to the class?

BB: $87 654 = 8 \times 10^4 + 7 \times 10^3 + 6 \times 10^2 + 5 \times 10 + 4 \times 1$

If a P knows, allow him or her to explain, otherwise T directs P’s thinking. Elicit that:

BB: $10^2 = 10 \times 10 = 100$

We say this (T points to $10^2$) as ‘10 to the power 2’, or ‘10 squared’)

We say this (T points to $10^3$) as ‘10 to the power 3’, or ‘10 cubed’)

We say this (T points to $10^4$) as ‘10 to the power 4’, or ‘10 to the 4th’)

**Lesson Plan 1**

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class activity</td>
</tr>
<tr>
<td>Table drawn on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>(Ps could have copies on desks too.)</td>
</tr>
<tr>
<td>At a good pace.</td>
</tr>
<tr>
<td>Reasoning, agreement, praising</td>
</tr>
<tr>
<td>Ps could dictate bottom row in unison.</td>
</tr>
<tr>
<td>Whole class activity</td>
</tr>
<tr>
<td>Discussion, reasoning, agreement, praising</td>
</tr>
<tr>
<td>Extra praise if a P can explain without help from T.</td>
</tr>
</tbody>
</table>
| [What do you think these powers mean? ]
| BB: $10^1 = (10)$ (1 zero) |
| $10^0 = (1)$ (no zeros) |
| T tells it if no P can work it out or guess.] |

**4 PbY6a, page 1**

Q.1 Read: *Fill in the table for the number 249 358.*

Set a time limit. (More able Ps could also be asked to write the sum form using powers of 10 in their exercise book.)

Review with whole class. Ps come to BB to complete the table, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>249 358</th>
</tr>
</thead>
<tbody>
<tr>
<td>Digit value</td>
</tr>
<tr>
<td>Place value</td>
</tr>
<tr>
<td>Actual value</td>
</tr>
<tr>
<td>In sum form</td>
</tr>
</tbody>
</table>

If no P had time to try it, do the extension using powers of 10 with the whole class.

**Extension**

<table>
<thead>
<tr>
<th>249 358 = $2 \times 10^5 + 4 \times 10^4 + 9 \times 10^3 + 3 \times 10^2 + 5 \times 10 + 8 \times 1$ (or $\ldots + 5 \times 10^1 + 8 \times 10^0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 min</td>
</tr>
</tbody>
</table>
Activity 5  
PbY6a, page 1

Q2 Read: In your exercise book:

a) write these numbers as digits in a place-value table
   i) nine hundred and forty one thousand,
      two hundred and seventy six
   ii) five hundred and four thousand,
        eight hundred and twenty five
   iii) two hundred and ninety thousand and
        thirty eight
   iv) one hundred and six thousand and twenty seven

b) write each number in sum form.

How many place-value columns will you need?
(6: HTh, TTh, Th, H, T, U) How many rows will you need? (5)

Encourage Ps to use rulers to draw the table. Set a time limit of 4 minutes. If you have time, calculate the sum of the 4 numbers.

Review with whole class. T chooses a P to read out a number, then another P to come to BB to write the digits in the table and a 3rd P to write and say the numbers in sum form. Class agrees/disagrees. Mistakes discussed and corrected.

(T could also ask a 4th P to write the sum form as powers of 10, with the help of other Ps if necessary.)

Who had time to calculate the sum? If several Ps did it, ask them to show their results on scrap paper or slates in unison. If no P had time, do it now quickly with the whole class.

Solution:

\[
\begin{array}{cccccc}
\text{HTh} & \text{TTh} & \text{Th} & \text{H} & \text{T} & \text{U} \\
\hline
\text{i)} & 9 & 4 & 1 & 2 & 7 & 6 \\
\text{ii)} & 5 & 0 & 4 & 8 & 2 & 5 \\
\text{iii)} & 2 & 9 & 0 & 0 & 3 & 8 \\
\text{iv)} & 1 & 0 & 6 & 0 & 2 & 7 \\
\end{array}
\]

\text{Sum: 1 842 166}

i) \[941 276 = 9 \times 100 000 + 4 \times 10 000 + 1 \times 1000 + 2 \times 100 + 7 \times 10 + 6 \times 1\]
   \[= 9 \times 10^5 + 4 \times 10^4 + 1 \times 10^3 + 2 \times 10^2 + 7 \times 10 + 6 \times 1\]

ii) \[504 825 = 5 \times 100 000 + 0 \times 10 000 + 4 \times 1000 + 8 \times 100 + 2 \times 10 + 5 \times 1\]
   \[= 5 \times 10^5 + 0 \times 10^4 + 4 \times 10^3 + 8 \times 10^2 + 2 \times 10 + 5 \times 1\]

iii) \[290 038 = 2 \times 100 000 + 9 \times 10 000 + 0 \times 1000 + 0 \times 100 + 3 \times 10 + 8 \times 1\]
    \[= 2 \times 10^5 + 9 \times 10^4 + 0 \times 10^3 + 0 \times 10^2 + 3 \times 10 + 8 \times 1\]

iv) \[106 027 = 1 \times 100 000 + 0 \times 10 000 + 6 \times 1000 + 0 \times 100 + 2 \times 10 + 7 \times 1\]
   \[= 1 \times 10^5 + 0 \times 10^4 + 6 \times 10^3 + 0 \times 10^2 + 2 \times 10 + 7 \times 1\]

30 min

Notes

Individual work, monitored (helped)

T has place-value table already prepared on BB or SB or OHT, or uses enlarged copy master.

Very slow Ps could have a copy of the table to stick into their Ex. Bks, rather than having to draw it.

Differentiation by time limit

Discussion, agreement, self-correction, praising

Have no expectations for this!

Agreement, praising

Write on one line on BB if possible.

(or T has sum forms already prepared to save time and uncovers each line as it is dictated by a P)

Feedback for T
Activity

6 PbY6a, page 1

Q.3 Read:

a) What are these numbers? Write them in decreasing order in your exercise book.

b) Write the numbers in words.

Set a time limit of 3 minutes.

Review with whole class. T chooses a P to read each sum form, then Ps show the number as digits on scrap paper or slates on command. Ask Ps who made a mistake to read out the number they have actually written, then to correct their mistake in their PBs.

Ps dictate the numbers in order and T writes the inequality on BB and uncovers the number written in words. Class points out errors. Mistakes (including spelling and grammatical mistakes) corrected in PBs.

Solution:

a) i) \(2 \times 100 000 + 3 \times 10 000 + 8 \times 1000 + 1 \times 100 + 5 \times 10 + 6 \times 1 = 238 156\)

ii) \(7 \times 100 000 + 0 \times 10 000 + 9 \times 1000 + 4 \times 100 + 0 \times 10 + 0 \times 1 = 709 401\)

iii) \(7 \times 100 000 + 8 \times 1000 + 8 \times 100 + 5 \times 1 = 708 805\)

iv) \(9 \times 10 000 + 9 \times 100 + 9 \times 1 = 90 909\)

In order: \(709 401 > 708 805 > 238 156 > 90 909\)

b) i) \(238 156\) two hundred and thirty eight thousand, one hundred and fifty six

ii) \(709 401\) seven hundred and nine thousand, four hundred and one

iii) \(708 805\) seven hundred and eight thousand, eight hundred and five

iv) \(90 909\) ninety thousand, nine hundred and nine

35 min

7 PbY6a, page 1

Q.4 Read: Fill in the table for the amount £38 406.52.

What does the thick line in the table mean? (It separates the whole units from the parts of a unit, just like the decimal point.)

Set a time limit. (Ps finished quickly could write the sum form as powers of 10, otherwise do it as a whole class extension.)

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

<table>
<thead>
<tr>
<th>Digit value</th>
<th>3</th>
<th>8</th>
<th>4</th>
<th>0</th>
<th>6</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Place value</td>
<td>TTh</td>
<td>Th</td>
<td>H</td>
<td>T</td>
<td>U</td>
<td>t</td>
<td>h</td>
</tr>
<tr>
<td>Actual value</td>
<td>30 000</td>
<td>8000</td>
<td>400</td>
<td>0</td>
<td>6</td>
<td>(\frac{5}{8})</td>
<td>(\frac{2}{100})</td>
</tr>
<tr>
<td>In sum form</td>
<td>(3 \times 10 000 + 8 \times 1000 + 4 \times 100 + 6 \times 1 + 5 \times \frac{1}{10} + 2 \times \frac{1}{100})</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Extension

\[38 406.52 = 3 \times 10^4 + 8 \times 10^3 + 4 \times 10^2 + 6 \times 1 + \frac{5}{10} + 2 \times \frac{1}{100}\]

40 min

Notes

Individual work, monitored, (helped)

Written on BB or SB or OHT

Differentiation by time limit.

Ps finished first could come to BB to write the numbers as words, hidden from rest of class (or T could have the words already prepared and uncover them as they are dealt with in the review).

Discussion, agreement, self-correction, praising

Class applauds Ps with all numbers correct.

(Written on BB in one line)

Ps could say the inequality in unison.

Feedback for T

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit.

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

Praising, encouragement only

Have no expectations!

© CIMT, University of Exeter
Activity 8

PbY6a, page 1

Q.5  Read: Write the quantities in the table.

T points to each quantity in turn and chooses a P to read it out and say which type of measure it is (length, capacity, mass, money, length or distance).

Set a time limit of 2 minutes. Review with whole class. Ps come to BB to complete the table, explaining reasoning and saying the quantity as a decimal. Class points out errors. Mistakes discussed and corrected.

Solution:

<table>
<thead>
<tr>
<th>a) 1002 m 20 cm</th>
<th>b) 47 litres 83 cl</th>
<th>c) 50 kg 430 g</th>
<th>d) £602 75 p</th>
<th>e) 16 km 39 m</th>
</tr>
</thead>
<tbody>
<tr>
<td>Th</td>
<td>H</td>
<td>T</td>
<td>U</td>
<td>t</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>litres</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>7</td>
<td>8</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Th</td>
<td>H</td>
<td>T</td>
<td>U</td>
<td>t</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>2</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Th</td>
<td>H</td>
<td>T</td>
<td>U</td>
<td>t</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>0</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

- Who can write each decimal in sum form?
- Who can write the sum using powers of 10?

Ps come to BB or dictate to T. Class agrees/disagrees.

BB: e.g.

16.039 = 1 \times 10^1 + 6 \times 10^0 + 0 \times \frac{1}{10} + 3 \times \frac{1}{100} + 9 \times \frac{1}{1000}

= 1 \times 10^1 + 6 \times 10^0 + 0 \times \frac{1}{10^1} + 3 \times \frac{1}{10^2} + 9 \times \frac{1}{10^3}

- P points to an amount or a decimal on BB and chooses another P to round it to the nearest given unit (or the nearest 10, 100, tenth, etc.) Class points out errors.

45 min

Lesson Plan 1

Notes

Individual work, monitored helped
Drawn BB or use enlarged copy master or OHP
Discussion, reasoning, agreement, self-correction, praising

Whole class activity
At a good pace
Praising, encouragement only

Involves several Ps.
At speed. Praising only
**Y6**

**R:** Measures, quantities

**C:** Place value. Reading, writing, ordering, rounding numbers

**E:** Decimals (Powers of 10)

## Lesson Plan

### Activity 1

**Factorising**

Factorise each of these numbers in your exercise book and list the positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB to show their method of finding the prime factors, explaining reasoning. Who did the same? Who did it a different way? etc.

Let's list all its positive factors. Ps dictate to T in increasing order.

Elicit that:

- 2 is a **prime** number, as it has only 2 factors, itself and 1.
- 179 is also a **prime** number, as it is not exactly divisible by 2, 3, 5, 7, 11 and 13, and $17 \times 17 > 179$

Factors: 1, 179

- 352 = $2 \times 2 \times 2 \times 2 \times 11$ ($= 2^5 \times 11$)

Positive factors: 1, 2, 4, 8, 11, 16, 22, 32, 44, 88, 176, 352

- e.g. $1002 = 2 \times 3 \times 167$

Positive factors: 1, 2, 3, 6, 167, 334, 501, 1002

### Extension

**PbY6a, page 2**

**Q.1** Read: *List these numbers as digits in increasing order.*

one thousand, one, one hundred thousand, one hundred, ten thousand, ten, one million, ten million

Set a time limit of 2 minutes. Review with whole class. Ps come to BB to say and write the numbers. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

1 < 10 < 100 < 1000 < 10 000 < 100 000 < 1 000 000 < 10 000 000

How many zeros are to the right of the digit 1? [0, 1, 2, 3, 4, 5, 6, 7]

Let's use the number of zeros to help us write each number as a power of 10. Ps dictate what T should write if possible.

**BB:** $10^0 < 10^1 < 10^2 < 10^3 < 10^4 < 10^5 < 10^6 < 10^7$

Is 1 million a great or a small number? T asks several Ps what they think. T suggests that it is relative and depends on what it is being compared with, or what the context is.

For example:

- If you said every natural number from 1 to 1 million, on average you could say 4 numbers every 10 seconds and it would take you about 29 days and nights to do it – but one million drops of water could not fill a bath.
- One million years ago, mankind did not exist on Earth – but the Earth is about 4500 times older than 1 million years.

### Notes

Individual work, monitored

(or whole class activity if T prefers)

BB: 2, 179, 352, 1002

T decides whether to allow Ps to use a calculator.

Discussion, reasoning, agreement, self-correction, praising

BB: e.g. 352 2

176 2

88 2

44 2

22 2

11 11

11

Elicit that 1 is a factor of all natural numbers and 2 is the lowest common factor of all even natural numbers.

---

**Lesson Plan 2**

**Notes**

Individual work, monitored

(or whole class activity if T prefers)

BB: 2, 179, 352, 1002

T decides whether to allow Ps to use a calculator.

Discussion, reasoning, agreement, self-correction, praising

BB: e.g. 352 2

176 2

88 2

44 2

22 2

11 11

11

Elicit that 1 is a factor of all natural numbers and 2 is the lowest common factor of all even natural numbers.

---

**Extension**

**PbY6a, page 2**

**Q.1** Read: *List these numbers as digits in increasing order.*

one thousand, one, one hundred thousand, one hundred, ten thousand, ten, one million, ten million

Set a time limit of 2 minutes. Review with whole class. Ps come to BB to say and write the numbers. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

1 < 10 < 100 < 1000 < 10 000 < 100 000 < 1 000 000 < 10 000 000

How many zeros are to the right of the digit 1? [0, 1, 2, 3, 4, 5, 6, 7]

Let's use the number of zeros to help us write each number as a power of 10. Ps dictate what T should write if possible.

**BB:** $10^0 < 10^1 < 10^2 < 10^3 < 10^4 < 10^5 < 10^6 < 10^7$

Is 1 million a great or a small number? T asks several Ps what they think. T suggests that it is relative and depends on what it is being compared with, or what the context is.

For example:

- If you said every natural number from 1 to 1 million, on average you could say 4 numbers every 10 seconds and it would take you about 29 days and nights to do it – but one million drops of water could not fill a bath.
- One million years ago, mankind did not exist on Earth – but the Earth is about 4500 times older than 1 million years.

---

Individual work, monitored

Agreement, self-correction, praising

Feedback for T

T points to each number in turn and Ps shout out the number of zeros.

Agreement, praising

Ps write the powers of 10 below each number in *Pbs.*

Whole class discussion.

Involve several Ps.

Extra praise if a P suggests that it is relative, or if Ps think of their own examples without help from T.

---

© CIMT, University of Exeter
**Activity 3**

**PbY6a, page 2**

**Q.2** Read:

a) *Join up the equal numbers.*

b) *List the decimals in increasing order.*

Set a time limit. Ps come to BB to draw joining lines and explain reasoning. Class agrees/disagrees. Ps dictate the decimals in order. Mistakes discussed and corrected.

**Solution:**

a) 

\[
0.01 < \frac{1}{1000} < \frac{1}{100} < \frac{1}{10} < 0.0001 < 0.01 < 0.1
\]

b) \(0.0001 < 0.001 < 0.01 < 0.1\)

Ps say and write the fractions as powers of 10.

**BB:** \(\frac{1}{10^4} < \frac{1}{10^3} < \frac{1}{10^2} < \frac{1}{10}\)

---

**Extension**

Ps say and write the fractions as powers of 10.

**BB:** \(\frac{1}{10^4} < \frac{1}{10^3} < \frac{1}{10^2} < \frac{1}{10}\)

**Notes**

Individual work, monitored

Written on BB or use enlarged copy master or OHP

Agreement, self-correction, praising

---

**Activity 4**

**PbY6a, page 2**

**Q.3** Read: *Join up each number to the corresponding point on the number line.*

Set a time limit or deal with one part at a time.

Review with whole class. Ps come to BB to draw a joining line and say the number. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) 

\[
38 347 \approx 38 350; 38 342 \approx 38 340; 38 355 \approx 38 360
\]

b) 

\[
726 250 \approx 726 300; 726 225 \approx 726 200; 726 290 \approx 726 340
\]

a) T points to each number in turn. P_1 says the next smaller 10, P_2 says the next greater 10, P_3 says the correct rounding to the nearest 10. T writes approximation on BB and P_4 shows it on the number line. e.g.

**BB:** 38 347 = 38 350; 38 342 = 38 340; etc.

b) Repeat in a similar way for the next smallest and greatest 100.

**BB:** 726 250 = 726 300; 726 190 = 726 200; etc.

---

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit.

Reasoning, agreement, self-correction, praising

Elicit that in the number line in:

a) there is a 'tick' at every unit
b) there is a 'tick' at every ten.

**Notes**

Whole class activity

At speed round class

Class points out any errors

In good humour!

Praising
**Activity 5**

PbY6a, page 2

Q.4 Read:

a) Follow the pattern and complete the table.

b) Write an $\approx$ sign beside the correct rounding to the nearest whole hundred.

Set a time limit. (Ask Ps to add the numbers if they have time.) Review with whole class. Ps come to BB to choose a number and complete its row, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Next smaller hundred</th>
<th>Number</th>
<th>Next greater hundred</th>
</tr>
</thead>
<tbody>
<tr>
<td>26 400</td>
<td>26 482</td>
<td>$\approx$ 26 500</td>
</tr>
<tr>
<td>604 700</td>
<td>$\approx$ 604 719</td>
<td>604 800</td>
</tr>
<tr>
<td>140 300</td>
<td>$\approx$ 140 348</td>
<td>140 400</td>
</tr>
<tr>
<td>1 215 700</td>
<td>1 215 750</td>
<td>$\approx$ 1 215 800</td>
</tr>
<tr>
<td>499 400</td>
<td>499 499</td>
<td>$\approx$ 499 500</td>
</tr>
<tr>
<td>812 400</td>
<td>812 500</td>
<td>$\approx$ 812 600</td>
</tr>
</tbody>
</table>

Rounds to itself

Let's say the numbers in decreasing order. 33 min

**6 Rounding quantities**

Let's round these quantities. Ps come to BB to write the appropriate amount, explaining reasoning, then to read out the completed approximation. Class points out errors.

BB:

a) Round to the nearest 10 units:

- £78 326 $\approx$ £78 330
- 10 508.4 m $\equiv$ 10 510 m
- 2065 $\ell$ 51 cl $\equiv$ 2070 $\ell$
- 429 km 350 m $\equiv$ 430 km

b) Round to the nearest unit:

- £6710 65 p $\approx$ £6711
- 2356 m 48 cm $\equiv$ 2356 m
- 41.3 litres $\approx$ 41 $\ell$
- 18.38 kg $\approx$ 18 kg

- £580.27 $\approx$ £580.3
- 120.55 m $\equiv$ 120.6 m
- 66 litres 99 cl $\equiv$ 67.0 $\ell$
- 46 kg 87 g $\equiv$ 46.1 kg (as 46 kg 87 g = 46.087 kg)

40 min

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit

At a good pace

Reasoning, agreement, self-correction, praising

Agree that 812 500 is already a whole hundred.

Ps who calculated the sum could show their results on scrap paper or slates in unison; otherwise do the calculation with the whole class.

BB: Sum: 3 299 298

T chooses Ps at random.

Whole class activity

Written on BB or use enlarged copy master or OHP

At a good pace

Reasoning, agreement, praising

Feedback for T

**Extension**

Could any of the amounts be written in other ways? e.g.

- 41.3 litres = 41 litres 30 cl
- 10 508.4 m = 10.5084 km
- 18.38 kg = 18 kg 380 g

etc.
Activity 7

PbY6a, page 2

Q.5 Read: Write these numbers as decimals.

Set a time limit. Review with the whole class. Ps could show decimals on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Mistakes discussed and corrected.

Solution:

a) \(3 \times 1000 + 7 \times 10 + 5 \times 1 + 6 \times \frac{1}{10} + 2 \times \frac{1}{100} = 3075.62\)

b) \(1 \times 1000000 + 7 \times 10000 + 4 \times 1000 + 8 \times 100 + 1 + 3 \times \frac{1}{100} = 1074801.03\)

c) \(9 \times 100000 + 4 \times 100 + 6 \times 10 + 8 \times \frac{1}{10} + 3 \times \frac{1}{100} = 900460.83\)

Extension

Write each sum using powers of 10. Ps come to BB or dictate what T should write. Class points our errors.

a) \(3 \times 10^3 + 7 \times 10^1 + 5 \times 10^0 + 6 \times \frac{1}{10} + 2 \times \frac{1}{10^2}\)

b) \(1 \times 10^6 + 7 \times 10^4 + 4 \times 10^3 + 8 \times 10^2 + 1 \times 10^0 + 3 \times \frac{1}{10^2}\)

c) \(9 \times 10^5 + 4 \times 10^3 + 6 \times 10^1 + 8 \times \frac{1}{10} + 3 \times \frac{1}{10^2}\)

Notes

Individual work, monitored
Written on BB or SB or OHT
Reasoning, agreement, self-correction, praising
Feedback for T

Whole class activity
T helps where necessary
Praising, encouragement only

45 min
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **1** Factorising  
Factorise each of these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.  
Review with whole class. Ps come to BB to show their method of finding the prime factors, explaining reasoning. Who did the same? Who did it a different way? etc.  
Let's list all its positive factors. Ps dictate to T in increasing order.  
Elicit that:  
• \( \frac{3}{ } \) is a prime number. Factors: 1, 3  
• \( 178 = 2 \times 89 \) Factors: 1, 2, 89, 178  
• \( 353 \) is a prime number. Factors: 1, 353  
(as it is not exactly divisible by 2, 3, 5, 7, 11, 13 or 17, and \( 19 \times 19 > 353 \) )  
• \( 1003 = 17 \times 59 \) Factors: 1, 17, 59, 1003  
Who remembers the name we gave to a number which has only 2 factors apart from itself and 1? (178 and 2003 are nice numbers.) | Individual work, monitored, helped  
(or whole class activity)  
BB: 3, 178, 353, 1003  
Allow Ps to use a calculator.  
Discussion, reasoning, agreement, self-correction, praising |
| **2** Sequences 1  
In these sequences, each following term is 10 times the previous term. Let's continue the sequences for 5 more terms.  
Ps come to BB to write the terms, explaining reasoning. Class agrees/disagrees.  
BB:  
\[ a) \ 1, 10, (100, 1000, 10000, 100000, 1000000) \ [Rule: \times 10] \]  
Who could write the terms as powers of 10? Ps come to BB to write and say the numbers or dictate what T should write.  
BB: \( 10^0, 10^1, 10^2, 10^3, 10^4, 10^5, \ldots \)  
Elicit that the value of the power is the same as the number of zeros to the right of the digit '1'. Ask questions about the terms. e.g.  
What is the product of the 2nd and 4th terms? (10 \( \times \) 10000 = 100000)  
Which 2 terms have a product of 1000000? Are there other pairs? (10 \( \times \) 100 000 = 1000 \( \times \) 10000 = 10000 \( \times \) 1000 = 1 \( \times \) 10\(^6\) \ldots )  
\[ b) \ 3, (30, 300, 3000, 30 000, 300 000, \ldots ) \ [Rule: \times 10] \]  
\[ c) \ 0.007, (0.07, 0.7, 7, 70, 700, \ldots ) \ [Rule: \times 10] \]  
Who can explain what happens when we multiply a number by 10? T asks several Ps what they think. If necessary T repeats in a clear way.  
'When a natural number or a decimal number is multiplied by 10, each digit of the multiplicand moves to the next greater place value in the product.' | Whole class activity  
(or individual work in Ex. Bks first)  
Written on BB or SB or OHT  
At a good pace  
Reasoning, agreement, (self-correction), praising  
T helps with wording if necessary. (e.g. 'one hundred thousand is ten to the power five')  
[Extra praise if a P notices that \( 10^1 \times 10^4 = 10^5 \times 10^{1+3} \) but do not stress it at this stage.]  
Explanation, agreement, praising  
(i.e. each digit moves 1 place-value to the left) |
### Activity

#### Sequences 2

In these sequences, each following term is 1 tenth of the previous term. Let's continue the sequences for 5 more terms.

Ps come to BB to write the terms, explaining reasoning, or dictate what T should write. Class agrees/disagrees. T asks Ps to say and write the decimals as fractions too.

**BB:**

- **a)** 7 000 000, 700 000, (70 000, 7000, 70, 7, . . .)  
  \[ \text{Rule: } \div 10 \]

- **b)** 8000, (800, 80, 8, 0.8, 0.08, . . .)  
  \[ \text{Rule: } \div 10 \]

- **c)** 23 000, 2300, (230, 23, 2.3, 0.23, 0.023, . . .)  
  \[ \text{Rule: } \div 10 \]

Who can explain what happens when we divide a number by 10? T asks several Ps what they think. If necessary T repeats in a clear way.

‘When a natural number or a decimal number is divided by 10, each digit of the multiplicand moves to the next smaller place value in the product.’

---

### Notes

#### Whole class activity

- (or individual work in *Ex. Bks* first)
- Written on BB or SB or OHT
- At a good pace
- Reasoning, agreement, (self-correction), praising
- T helps with wording if necessary.

(e.g. ‘zero point two three’ – not ‘zero point twenty three’)

Explanation, agreement, praising

(i.e. each digit moves 1 place value to the right)

---

### PbY6a, page 3

**Q.1** Read: Write the number, then 10 times, 100 times and 1000 times its value in the place-value table. Complete the multiplications.

What do the letters in the table mean? (Hundred Thousands, Ten thousands, Thousands, Hundreds, Units, tenths and hundredths) T (P) explains the task, relating the number already in the table to the given product.

Set a time limit or deal with one part at a time.

Review with whole class. Ps come to BB to fill in the table and complete the relevant multiplications. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>HTh</th>
<th>TTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
<th>t</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 237</td>
<td>2</td>
<td>3</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>3</td>
<td>7</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. \( 1 \times 237 = 237 \)
2. \( 10 \times 237 = 2370 \)
3. \( 100 \times 237 = 23700 \)
4. \( 1000 \times 237 = 237000 \)

<table>
<thead>
<tr>
<th></th>
<th>HTh</th>
<th>TTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
<th>t</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>b) 65.2</td>
<td>6</td>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>5</td>
<td>2</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. \( 1 \times 65.2 = 65.2 \)
2. \( 10 \times 65.2 = 652 \)
3. \( 100 \times 65.2 = 6520 \)
4. \( 1000 \times 65.2 = 65200 \)

<table>
<thead>
<tr>
<th></th>
<th>HTh</th>
<th>TTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
<th>t</th>
<th>h</th>
</tr>
</thead>
<tbody>
<tr>
<td>c) 8.14</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1. \( 1 \times 8.14 = 8.14 \)
2. \( 10 \times 8.14 = 81.4 \)
3. \( 100 \times 8.14 = 814 \)
4. \( 1000 \times 8.14 = 8140 \)

---

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP

Brief whole-class discussion to clarify the task.

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Who can explain the effect of multiplying a natural number or a decimal by 1 (10, 100, 1000)?

T repeats each explanation in a clearer way if necessary. e.g.

‘When a natural number or a decimal is multiplied by 1, the number does not change.’

‘When a natural number or a decimal is multiplied by 10, each digit in the multiplicand moves to 2 greater place values in the product.’

Praising, encouragement only
**Lesson Plan 3**

### Activity

#### 5

**PbY6a, page 3**

Q.2 Read: *Write the number, then 1 tenth, 1 hundredth and 1 thousandth of its value in the place-value table. Complete the divisions.*

If necessary, T (P) explains the task, relating the number already in the table to the given product.

Set a time limit or deal with one part at a time.

Review with whole class. Ps come to BB to fill in the table and complete the relevant divisions. Class agrees or disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>a)</th>
<th>HTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
<th>t</th>
<th>h</th>
<th>th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>0</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

143,000 ÷ 1 = 143,000
143,000 ÷ 10 = 14,300
143,000 ÷ 100 = 1,430
143,000 ÷ 1,000 = 143

<table>
<thead>
<tr>
<th>b)</th>
<th>HTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
<th>t</th>
<th>h</th>
<th>th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td>1</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4510 ÷ 1 = 4510
4510 ÷ 10 = 451
4510 ÷ 100 = 45.1
4510 ÷ 1000 = 4.51

<table>
<thead>
<tr>
<th>c)</th>
<th>HTh</th>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
<th>t</th>
<th>h</th>
<th>th</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>7</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>7</td>
<td>2</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

726 ÷ 1 = 726
726 ÷ 10 = 72.6
726 ÷ 100 = 7.26
726 ÷ 1000 = 0.726

### Notes

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP

Brief discussion to clarify the task

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Who can explain the effect of dividing a natural number or a decimal by 1 (10, 100, 1000)?

T repeats each explanation in a clearer way if necessary. e.g.

‘When a natural number or a decimal is divided by 1, the number does not change.’

‘When a natural number or a decimal is divided by 10, each digit in the dividend moves to the next smaller place value in the quotient.’

(i.e. moves 1 place value to the right)

Whole class activity

Written on BB or SB or OHT

At a good pace

Reasoning, agreement, praising

Show in a place-value table if there is disagreement.

Feedback for T
Lesson Plan 3

Notes

Whole class activity
Written on BB or use enlarged copy master or OHP
At a good pace
Reasoning, agreement, praising

(smaller measuring unit → greater measuring number, and vice-versa)
(or Ps could show missing items on slates or scrap paper in unison on command.)

Feedback for T

T could ask one or two Ps to repeat the ‘rules’ in their own words.
Praising only

Activity 7

Converting quantities

Let’s fill in the missing numbers and units of measure. Ps come to BB to write the missing items, explaining reasoning. Class agrees/disagrees.

BB:

\[
\begin{align*}
\text{a) } & \quad 4.31 \text{ m} = \frac{4310}{\times 10} \frac{\text{mm}}{\div 1000} = 431 \text{ cm} \\
\text{b) } & \quad 81.6 \text{ cm} = \frac{816}{\times 100} \frac{\text{mm}}{\div 10} = 0.816 \text{ m}
\end{align*}
\]

After completing parts a) and b), ask Ps to try to explain what is happening.
Continue with c) to f), with Ps coming to BB or dictating to T, explaining reasoning. Class agrees/disagrees.

BB:

\[
\begin{align*}
\text{c) } & \quad 2.945 \text{ litres} = \frac{294.5}{\times 1000} \frac{\text{cl}}{\div 10} = 2945 \text{ ml} \\
\text{d) } & \quad 72.8 \text{ ml} = \frac{7.28}{\times 100} \frac{\text{cl}}{\div 100} = 0.0728 \text{ litres} \\
\text{e) } & \quad 5.26 \text{ kg} = \frac{5260}{\times 1000} \frac{\text{g}}{\div 1000} \\
\text{f) } & \quad 12406 \text{ g} = 12.406 \text{ kg}
\end{align*}
\]

T consolidates the ‘rules’ in a clear way. e.g.

‘When converting a quantity to a new unit of measure which is 10 (100, 1000) times greater than the original unit of measure, then the measuring number is \textit{divided} by 10 (100, 1000) but the actual quantity does not change.’

‘When converting a quantity to a new unit of measure which is 1 tenth (1 hundredth, 1 thousandth) of the original unit of measure, then the measuring number is \textit{multiplied} by 10 (100, 1000) but the actual quantity does not change.’

45 min
### Activity 1

#### Factorising

Factorise each of these numbers in your exercise book and list the positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB to show their method of finding the prime factors, explaining reasoning. Who did the same? Who did it a different way? etc.

Let's list all its positive factors. Ps dictate to T in increasing order.

Elicit that:
- $4 = 2 \times 2$  
  Factors: 1, 2, 4  
  (4 is a square number and can be written as $2^2$)
- $179$ is a prime number  
  Factors: 1, 179  
  (as it is not exactly divisible by 2, 3, 5, 7, 11, 13; $17 \times 17 > 179$)
- $354 = 2 \times 3 \times 59$  
  Factors: 1, 2, 3, 6, 59, 118, 177, 354
- $1004 = 2 \times 2 \times 251$  
  Factors: 1, 2, 4, 251, 502, 1004

---

### Activity 2

#### Calculation strategies

How could you do these calculations in your head? Think of different ways to do it. Ps come to BB to explain their methods. Class decides whether it is valid. T shows any strategies not suggested by Ps and asks whether it is correct.

BB: e.g.

a) $17 \, 405 + 1385 = \underline{17 \, 405} + 1000 + 300 + 80 + 5 = \underline{18 \, 790}$

\[ \underline{18 \, 705} \]

\[ \underline{18 \, 785} \]

or $= 17 \, 400 + 1390 = 18 \, 790$

\[ = 17 \, 405 + 1000 + 400 – 15 = 18 \, 805 – 15 = 18790 \]

b) $5072 + 969 = \underline{5072} + 60 + 9 = 5032 + 9 = \underline{6041}$

\[ \underline{6072} – 31 = \underline{6041} \]

or $= 5072 + 1000 – 31 = 6072 – 31 = 6041$

\[ = 5072 + 1000 + 400 – 15 = 5472 + 400 – 15 = 5857 \]

\[ = 5800 + 7 + 400 – 15 = 6200 – 15 = 6185 \]

\[ = 6200 + 8 – 15 = 6285 \]

c) $73 \, 825 – 4167 = \underline{73 \, 825} – 4000 – 100 – 60 – 7 = \underline{69 \, 658}$

\[ \underline{69 \, 725} \]

\[ \underline{69 \, 665} \]

d) $243.6 – 8.8 = \underline{243.6} – 8 – 0.8 = 235.6 – 0.8 = \underline{234.8}$

\[ \underline{243.6} – 10 + 1.2 = 233.6 + 1.2 = 234.8 \]

\[ \underline{(243.6 + 1.2) – (8.8 + 1.2)} = 244.8 – 10 = 234.8 \]

---

**Lesson Plan**

**Notes**

Individual work, monitored, helped  
(or whole class activity)  
BB: 4, 179, 354, 1004  
T decides whether to allow Ps to use a calculator.  
Discussion, reasoning, agreement, self-correction, praising  

BB: e.g.

<table>
<thead>
<tr>
<th>354</th>
<th>2</th>
<th>1004</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>177</td>
<td>3</td>
<td>502</td>
<td>2</td>
</tr>
<tr>
<td>59</td>
<td>59</td>
<td>251</td>
<td>251</td>
</tr>
</tbody>
</table>

Whole class activity  
Written on BB or SB or OHT  
Discussion, reasoning, agreement, praising  
Involve as many Ps as possible  

Ask Ps to explain their methods using the correct names for the components:  
**Addition:** terms, sum

**Subtraction:** reductant, subtrahend, difference  
Extra praise for unexpected methods, e.g.

c) $73 \, 825 – 4167$

\[ = 73 \, 825 – 3125 – 1000 – 42 \]

\[ = 70 \, 700 – 1000 – 42 \]

\[ = 69 \, 700 – 42 \]

\[ = 69 \, 658 \]

Ps say which method they think is easiest.
Y6

**Activity**

3  

*PbY6a, page 4*

Q.1 Read: *Work out the calculation strategy and fill in the missing numbers.*

Set a time limit of 4 minutes. Review with whole class.
Ps come to BB to fill in the missing numbers, explaining the strategy. Who agrees? Who wrote a different number? Why? etc. Mistakes discussed and corrected.

*Solution:*

a) \[60\,419 + 897 = 60\,416 + \boxed{900} = 61\,316\]

b) \[5643 + 489 = 5643 + 500 - \boxed{11} = 6132\]

c) \[12\,345 - 678 = 12\,367 - 700 = 11\,667\]

\[\boxed{+ 22}\]

\[+ 22\]

d) \[9636 - 3482 = \frac{9636 - 3000 - 500 + 18}{6636} = 6154\]

\[\frac{6136}{\boxed{0.4}}\]

e) \[41.3 - 12.4 = 41.3 - 12 - \boxed{0.4} = 28.9\]

21 min

---

4  

*PbY6a, page 4*

Q.2 Read: *Work out the calculation strategy and fill in the missing numbers.*

Set 2 at a time, then review before dealing with the next 2.
Ps come to BB to fill in the missing numbers, explaining the strategy. Class agrees/disagrees. Mistakes discussed/corrected.

*Solution:*

a) \[628 \times 20 = 6280 \times 2 = 12\,560\]

b) \[135 \times 18 = \frac{135 \times 2 \times 3 \times 3}{270} = 2430\]

c) \[135 \times 18 = \frac{135 \times 20 - 270}{2700} = 2430\]

d) \[43 \times 51 = 43 \times 50 + 43 = 2193\]

e) \[305 \times 14 = 305 \times 10 + 305 \times 4 = 4270\]

\[\frac{3050}{1220}\]

\[\frac{760}{1520}\]

f) \[15.2 \times 25 = \frac{15.2 \times 100}{2} \div 2 = 380\]

\[\frac{1520}{\boxed{2}}\]

g) \[252 \div 6 = 252 \div 2 \div \boxed{3} = 42\]

32 min

---

**Notes**

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Discussion by time limit.

Discussion, reasoning using the correct names of the components, agreement, self-correction, praising

Elicit that:

- in a 2-term addition, increasing one term and reducing the other term by the same amount does not change the sum;
- in a subtraction, increasing or reducing the reductant and the subtrahend by the same amount does not change the difference.

Elicit that:

- in a 2-term addition, increasing one term and reducing the other term by the same amount does not change the sum;
- in a subtraction, increasing or reducing the reductant and the subtrahend by the same amount does not change the difference.

Discuss the names of the components in a) and g):

a) \[628 \times 20 = 12\,560\]

628 is the *multiplicand*, 20 is the *multiplier* and 12,560 is the *product*;

but \(20 \times 628 = 12\,560\)

where 20 is the *multiplicand* and 628 is the *multiplier*, so the factors can be exchanged and give the same product.

\[252 \div 6 = 42\]

252 is the *dividend*, 6 is the *divisor*, 42 is the *quotient*, and they cannot be exchanged. Also, e.g. \(152 \div 6 = 25\), r 2 where 2 is the remainder, so \(25 \times 6 + 2 = 152\)

© CIMT, University of Exeter
### Y6

#### Activity 5

**PbY6a, page 4**

**Q.3** Read: *Do these calculations in a clever way in your exercise book (or mentally if you can).*

- Set a time limit, or deal with one or two at a time then review before Ps continue with the next pair.
- Ps could show results on scrap paper or slates on command. Ps answering correctly come to BB to explain their thought processes. Who did the same? Who did it another way? Mistakes discussed and corrected.

**Solution:** e.g.

- a) $2087 - 1022 = 1087 - 22 = 1065$
- b) $249 + 63 + 151 + 27 = 400 + 90 = 490$
- c) $13 \times 4 \times 25 = 13 \times 100 = 1300$
- d) $1063 \times 29 \times 0 = 0$ (as zero times any number is zero)
- e) $8.2 \times 13 = 82 + 24 + 0.6 = 106.6$
- f) $3740 \div 170 = 374 \div 17 = 340 \div 17 + 34 \div 17 = 20 + 2 = 22$
- g) $998 \times 35 = 100 \times 35 - 2 \times 35 = 3500 - 70 = 34930$
- h) $28500 \div 170 = 28500 \div 17 + 34 \div 17 = 20 + 2 = 22$

---

#### Activity 6

**PbY6a, page 4**

**Q.4** Read: *Write a plan, convert the quantities where necessary, do the calculation and write the answer as a sentence in your exercise book.*

Let's see how many of these problems you can do in 3 minutes! Start . . . now! . . . Stop!

Review quickly with whole class. Ps show results on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. T chooses a P to say the answer in a sentence.

**Solutions:** e.g.

- a) *The sides of a triangle are 2.3 cm, 31 mm and 0.018 m long. What is the perimeter of the triangle?*

  **Plan:** $P = 2.3 \text{ cm} + 3.1 \text{ cm} + 1.8 \text{ cm} = 7.2 \text{ cm}$

  **Answer:** The perimeter of the triangle is 7.2 cm.

- b) *How many hours are in September?*

  **Plan:** $24 \times 30 = 240 \times 3 = 720$ (hours)

  **Answer:** There are 720 hours in September.

- c) *A car travels 20 m every second. How far does it travel in:*

  i) 1 minute  
  ii) 2 hours?

  **Plan:** i) $20 \times 60 = 1200 \text{ m} (=1.2 \text{ km})$

  ii) $1200 \times 120 = 12000 \times 12 = 144000 \text{ (m)} (=144 \text{ km})$

  **Answer:** The car travels 1.2 km in 1 minute and 144 km in 2 hours.

---

**Lesson Plan 4**

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual work, monitored, helped</td>
</tr>
<tr>
<td>Written on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>Responses shown in unison. Discussion, reasoning, agreement, self-correcting, praising</td>
</tr>
<tr>
<td>T shows any easier method not suggested by Ps. Feedback for T</td>
</tr>
</tbody>
</table>

(Reducing the dividend and divisor by the same amount does not change the quotient.)

---

Individual work, monitored, or could be a competition among groups of roughly equal ability, with the more able Ps helping the slower Ps in their group once they have finished their own work.) Responses shown in unison. Discussion, reasoning, agreement, self-correction, praising

- or $23 + 31 + 18 = 72 \text{ (mm)}$

- or $1.2 \text{ km} \times 120 = 144 \text{ km}$

BB: $2 \text{ h} = 120 \text{ min}$

© CIMT, University of Exeter
**Lesson Plan 4**

### Activity 6

(Continued)

**d)** If 750 g of meat costs £9.60 p, how much does 1 kg of meat cost?

- e.g. 750 kg = \(\frac{3}{4}\) kg → £9.60 p = 960 p
  
  \[ \frac{1}{4} \text{ kg} \rightarrow 960 \text{ p} \div 3 = 320 \text{ p} \]
  
  \[ 1 \text{ kg} \rightarrow 320 \text{ p} \times 4 = 1280 \text{ p} = £12.80 \]

**Answer:** One kilogram of meat costs £12.80.

Stand up if you had all 4 problems correct. Stand up if you made just 1 mistake. Let’s give them a clap!

### Homework

Factorise 5, 180, 355 and 1005 in your exercise book.

**Solution:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>5 (prime number)</td>
</tr>
</tbody>
</table>
| 180    | \(2 \times 2 \times 3 \times 3 \times 5\)  
  \(= 2^2 \times 3^2 \times 5\) |
| 355    | \(5 \times 71\) |
| 1005   | \(3 \times 5 \times 67\) |

Review before the start of **Lesson 5**.

(More able Ps could be asked to list all the positive factors too.)
Y6

Activity

Calculation practice, revision, consolidation

PbY6a, page 5

Notes

Lesson Plan 5

Solutions:

Q.1  
a)  $17\,083.26 < 111\,215.09 < 462\,590.5 < 1\,300\,450.46$

b)  i) to nearest 1000:

\[
17\,083.26 \approx 17\,000; \quad 1\,300\,450.46 \approx 1\,300\,000;
\]

\[
111\,215.09 \approx 111\,000; \quad 462\,590.5 \approx 463\,000
\]

ii) to nearest 100:

\[
17\,083.26 \approx 17\,100; \quad 1\,300\,450.46 \approx 1\,300\,000
\]

\[
111\,215.09 \approx 111\,200; \quad 462\,590.5 \approx 462\,600
\]

iii) to nearest 10:

\[
17\,083.26 \approx 17\,080; \quad 1\,300\,450.46 \approx 1\,300\,450
\]

\[
111\,215.09 \approx 111\,220; \quad 462\,590.5 \approx 462\,590
\]

iv) to nearest 1:

\[
17\,083.26 \approx 17\,083; \quad 1\,300\,450.46 \approx 1\,300\,450
\]

\[
111\,215.09 \approx 111\,215; \quad 462\,590.5 \approx 462\,591
\]

v) to nearest 0.1:

\[
17\,083.26 \approx 17\,083.3; \quad 1\,300\,450.46 \approx 1\,300\,450.5
\]

\[
111\,215.09 \approx 111\,215.1; \quad 462\,590.5 \approx 462\,590.5
\]

Q.2

\[
\begin{align*}
3 \text{ kg} & \approx \frac{3}{1000} \text{ tonne} \\
0.003 \text{ kg} & \approx \frac{3}{1000} \text{ litre} \\
0.03 \text{ m} & \approx \frac{3}{1000} \text{ km} \\
0.3 \text{ m} & \approx \frac{3}{1000} \text{ litre} \\
30 \text{ mm} & \approx \frac{3}{1000} \text{ tonne} \\
30 \text{ cl} & \approx \frac{3}{100} \text{ litre} \\
30 \text{ ml} & \approx \frac{3}{100} \text{ tonne}
\end{align*}
\]

Q.3  
a)  to the nearest 10 units:

\[
\begin{align*}
£503\,455 & \approx £503\,460 \\
7459.8 \text{ m} & \approx 7\,460 \text{ m} \\
300\,005 \text{ g} & \approx 4\,205\,29 \text{ kg} \\
15 \text{ litres} & \approx 20 \text{ litres} \\
83\,104.55 \text{ km} & \approx 83\,100 \text{ km}
\end{align*}
\]

b)  to the nearest unit:

\[
\begin{align*}
£611\,32\text{ p} & \approx £611 \\
88\,\text{ cm} & \approx 88\,\text{ cm} \\
4\,205\,29 \text{ kg} & \approx 4\,205\,3 \text{ kg} \\
1453.51 \text{ litres} & \approx 1454 \text{ litres} \\
83\,104\,\text{ km} & \approx 83\,100 \text{ km}
\end{align*}
\]

c)  to the nearest 10th:

\[
\begin{align*}
£1011\,54\text{ p} & \approx £1011.5 \\
1766.21 \text{ cm} & \approx 1766.2 \text{ cm} \\
4\,205.29 \text{ kg} & \approx 4\,205.3 \text{ kg} \\
1994.06 \text{ ml} & \approx 1994.1 \text{ ml} \\
83\,104\,\text{ km} & \approx 83\,105 \text{ km}
\end{align*}
\]

Q.4  
a)  i)  $51\,328 + 786 = 52\,114$

ii)  $41.84 + 62.79 + 103.06 = 207.69$

iii)  $35\,879 + 64\,121 = 100\,000$

b)  i)  $8574 - 1569 = 7005$

ii)  $9000 - 2456 = 6544$

iii)  $137.82 - 48.93 = 88.89$

c)  i)  $413 \times 600 = 247\,800$

ii)  $75 \times 16 = 300$

iii)  $5376 \times 11 - 1 = 59\,135$

d)  i)  $4254 \div 24 = 177.25$

ii)  $(7023 + 542) \div 5 = 7565 \div 5 = 1513$

iii)  $1269 \div 18 \times 2 = 1269 \div 9 = 141$

e)  i)  $(121 \div 11) \div 100 = 0.11$

ii)  $8151 \div 4 = 2037.75$

iii)  $(6000 - 4368) \div 8 = 1632 \div 8 = 204$

© CIMT, University of Exeter
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.5</td>
<td></td>
</tr>
</tbody>
</table>
| a) $5.44 \text{ cm} \div 8 = 0.68 \text{ (cm)}$  
  *Answer:* The length of each side of the octagon is 0.68 cm. |
| b) $60 \times 60 \times 24 \times 7 = 60 \times 1440 \times 7 = 86400 \times 7 = 604800 \text{ (sec)}$  
  *Answer:* There are 604 800 seconds in 1 week. |
| c) $2 \text{ h 36 min} \div 13 = 156 \text{ min} \div 13 = 12 \text{ min}$  
  *Answer:* Paula ran each mile in 12 minutes on average. |
| d) $1 \text{ kg} \rightarrow 1000000 \text{ flowers}$  
  $1 \text{ g} \rightarrow 1000000 \div 1000 = 1000 \text{ (flowers)}$  
  *Answer:* One thousand jasmine flowers are needed to produce one gram of jasmine oil. |
**Lesson Plan**

### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 6 = 2 \times 3 \)  
  Factors: 1, 2, 3, 6
- \( 181 \) is a prime number  
  Factors: 1, 181  
  (as it is not exactly divisible by 2, 3, 5, 7, 11, 13; \( 17 \times 17 > 181 \))
- \( 356 = 2 \times 2 \times 89 \)  
  Factors: 1, 2, 4, 89, 178, 356
- \( 1006 = 2 \times 503 \)  
  Factors: 1, 2, 503, 1006  
  (503 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and \( 23 \times 23 > 503 \))

**Notes**

Individual work, monitored, (helped)  
(or whole class activity)  
BB: 6, 181, 356, 1006

Calculators allowed

Discussion, reasoning, agreement, self-correction, praising

**Activity 2**

**Relay: Multiplication and division**

Everyone stand up! T says a multiplication or division and a P says the result. If the P answers correctly, he or she sits down. If not, the P remains standing and the next P answers. Ps still standing after one round of the class are given another multiplication or division. Continue until all Ps have answered at least one question correctly.

Ps who had difficulties should be given a copy of the multiplication table square and be asked to learn the facts which they do not know at home.

T notes the facts that certain Ps did not know and checks them regularly (and not only in maths lessons!).

**Notes**

Whole class activity  
At speed in order round class  
In good humour  
Some differentiation by questions  
Ps seated can ask some questions too.  
Feedback for T

**Activity 3**

**Place values**

Let’s complete the missing place values. Ps come to BB to say and write the place values. If a box is not large enough to write the whole word, Ps should use initial letters, as below. Class points out errors.

**Notes**

Whole class activity  
Written on BB or use enlarged copy master of OHP  
At a good pace  
Discuss which initial letters should be used.  
Agreement, praising

**Extension**

T points to a place-value and asks a P to write it with digits, then asks another P to write it as a power of 10.  
Praising, encouragement only
### Activity 4  
**PbY6a, page 6**

**Q.1** Read: *Write each addition in a shorter way, then calculate the result.*

Set a time limit. Review with whole class. Ps come to BB or dictate what T should write. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- a) $700 + 700 + 700 = 700 \times 3 = 2100$
- b) $45 + 45 + 45 + 45 + 45 = 45 \times 6 = 270$
- c) $7100 + 7100 + 7100 + 7100 = 7100 \times 5 = 35500$
- d) $600 + 600 + 600 + 600 + 600 + 600 = 600 \times 7$
  \[= 4200\]
- e) $10.5 + 10.5 + 10.5 = 10.5 \times 3 = 31.5$
- f) $0.3 + 0.3 + 0.3 + 0.3 + 0.3 + 0.3 + 0.3 + 0.3 + 0.3 = 0.3 \times 9$
  \[= 2.7\]

T: What does $78 \times 19$ really mean? A, what do you think? Who can explain it another way? And another? e.g.

- P$_1$: 78 is added to itself 19 times.
- P$_2$: It is an addition with 19 terms, and each term is 78.
- P$_3$: It is an addition with 78 terms, and each term is 19.
- T: Who can explain what $1.25 \times 43$ really means? (It is an addition with 43 terms and each term is 1.25)

**Notes**

Individual work, monitored (helped)

Written on BB or SB or OHT

Reasoning, agreement, self-correction, praising

---

### Activity 5  
**PbY6a, page 6**

**Q.2** Read: *Fill in the missing factors.*

What is a factor of a number? (A number which divides into that number exactly, or a number which multiplies another number to make the original number.)

Set a time limit or deal with one row at a time.

Review with the whole class. Ps come to BB or dictate to T, explaining reasoning with division. (e.g. $7 \times 8 = 56$, because $56 \div 7 = 8$) T helps with wording if necessary. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- a) $7 \times \fbox{8} = 56, \quad 7 \times \fbox{800} = 5600, \quad \fbox{0.8} \times 7 = 5.6, \quad 70 \times \fbox{80} = 5600$
- b) $\fbox{150} \times 5 = 750, \quad 5 \times \fbox{15} = 75, \quad 50 \times \fbox{15} = 750, \quad 50 \times \fbox{1.5} = 75$
- c) $60 \times \fbox{7} = 420, \quad \fbox{70} \times 60 = 4200, \quad 600 \times \fbox{7} = 4200, \quad 60 \times \fbox{0.7} = 42$
- d) $\fbox{125} \times 4 = 500, \quad \fbox{125} \times 40 = 5000, \quad \fbox{1250} \times 40 = 50000, \quad 40 \times \fbox{12.5} = 500$
- e) $4 \times \fbox{25} = 100, \quad 4 \times \fbox{250} = 1000, \quad \fbox{25} \times 40 = 1000, \quad 25 \times \fbox{40} = 1000\frac{25}{25} \times \fbox{40} = 100$
- f) $\fbox{8} \times 15 = 120, \quad \fbox{8} \times 150 = 1200, \quad 15 \times \fbox{80} = 1200, \quad \fbox{0.8} \times 150 = 120$

**Notes**

Individual work, monitored, Written on BB or use enlarged copy master or OHT

Reasoning, agreement, self-correction, praising

Differentiation by praising (Less able Ps deserve praise for getting at least one multiplication in each row correct.)

Extra praise for clever reasoning: e.g.

- d) $\fbox{4} \times 4 = 500$
- $\fbox{500} \times 2 = 250$
- $\fbox{250} \div 2 = 125$

or $500 \div 2 \div 2 = 250 \div 2 = 125$

© CIMT, University of Exeter
**Y6**

**Activity**

6  **Relationship between subtraction and division**

T writes a subtraction on the BB. How can we shorten this calculation?

BB: \(197 - 20 - 20 - 20 - 20 - 20 - 20 - 20 - 20 - 20\)

\[= 197 - 9 \times 20 = 197 - 180 = 17\]

(i.e. 17 remains when you subtract 9 lots of 20 from 197)

or ‘we could work out how many 20’s are in 197 by doing a division.’

BB: \(197 \div 20 = 9, r 17\)

\[-180\]

\[17\]

T: How many times did we take 20 away from 197 in the subtraction? (9)

What number remained? (17)

Compare the numbers in the division and the subtraction. What do you notice? Elicit that:

- the **dividend**, 197, in the division is the same as the reductant in the subtraction;
- the **divisor**, 20, in the division is the number which is being taken away lots of times in the subtraction;
- the **quotient**, 9, in the division is the number of times the 20 is taken away in the subtraction;
- the **remainder**, 17, in the division is the same as the difference in the subtraction;

What does this way of writing the information have to do with the division?

BB: \(197 = 9 \times 20 + 17\)


When would we use this form? (To check a division with a remainder.)

35 min

7  **PbY6a, page 6**

Q.3 Read: **Calculate the quotient and the remainder mentally.**

Look for easy ways of doing the division and remember to check your results mentally using reverse operations.

Set a time limit. Review with whole class. Ps come to BB to write quotient and remainder, explaining reasoning with reverse multiplication, and addition where relevant. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) i) \(64025 \div 2 = 32012, r 1\)   ii) \(64025 \div 30000 = 2, r 4025\)

b) i) \(1020000 \div 20000 = 51, r 0\)   ii) \(1020000 \div 4 = 255000, r 0\)

c) i) \(56000 \div 700 = 80, r 0\)   ii) \(56000 \div 800 = 70, r 0\)

d) i) \(710608 \div 100 = 7106, r 8\)   ii) \(710608 \div 1 = 710608, r 0\)

e) i) \(3240 \div 324 = 10, r 0\)   ii) \(3240 \div 0 \neq \) (Makes no sense)

**Checks:** e.g

a) i) \(32012 \times 2 + 1 = 64025\)   ii) \(2 \times 30000 + 4025 = 64025\)

etc.

40 min

**Notes**

Whole class activity

Ps dictate what T should write or T directs Ps’ thinking if nobody has any ideas.

Discussion, reasoning, agreement, praising

Ask Ps to use the correct names for the components of subtraction and division.

Ask several Ps what they think.

If necessary T shows the first relationship, then Ps should be able to point out the others.

Praising

Feedback for T

Individual work, monitored (helped)

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, reasoning, agreement, self-correction, praising

Extra praise for Ps who noticed that, e.g.

- \(1020000 \div 20000 = 102 \div 2 = 51\)

- \(56000 \div 700 = 560 \div 7 = 80\)

- any number divided by 1 is the number itself

- dividing by 0 is impossible!
### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY6a, page 6, Q.4</strong></td>
<td>Whole class activity</td>
</tr>
<tr>
<td>Read: Write a plan and do the calculation in your exercise book. Write only the result here.</td>
<td>T could repeat the question while monitoring Ps.</td>
</tr>
<tr>
<td>Deal with one question at a time. T chooses a P to read out the question, Ps do calculation in Ex. Bks, check it, then show result on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it another way? etc. Mistakes discussed and corrected in Ex. Bks and Pbs.</td>
<td>Allow 1 minute for each question.</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td>(or individual work under a time limit if Ps prefer)</td>
</tr>
<tr>
<td>a) Kate measured her heart beat as 72 beats in 1 minute. How many times did her heart beat in 9 minutes?</td>
<td>Responses shown in unison.</td>
</tr>
<tr>
<td>Plan: (72 \times 9 = 70 \times 9 + 2 \times 9 = 630 + 18 = 648) (beats)</td>
<td>Reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>Check: (648 \div 9 = 72\ ✓)</td>
<td>At a good pace</td>
</tr>
<tr>
<td>Answer: Kate's heart beat 648 times in 9 minutes.</td>
<td>T asks a P to say the answer in a sentence.</td>
</tr>
<tr>
<td>b) A farmer gathered the apples from his orchard and packed them in boxes. In a full box, there were 6 rows of 10 apples. How many apples could he pack in 50 such boxes?</td>
<td>Feedback for T</td>
</tr>
<tr>
<td>Plan: (10 \times 6 \times 50 = 60 \times 50 = 3000) (apples)</td>
<td></td>
</tr>
<tr>
<td>Check: (3000 \div 50 = 300 \div 5 = 60\ (= 6 \times 10)\ ✓)</td>
<td></td>
</tr>
<tr>
<td>Answer: The farmer could pack 3000 apples in the 50 boxes.</td>
<td></td>
</tr>
<tr>
<td>c) 49 000 bricks were used for a building. This was 70 times as many bricks as were used to build a kennel for the guard dog. How many bricks were used to build the kennel?</td>
<td></td>
</tr>
<tr>
<td>Plan: (49000 \div 70 = 4900 \div 7 = 700) (bricks)</td>
<td></td>
</tr>
<tr>
<td>Check: (700 \times 70 = 7000 \times 7 = 49000\ ✓)</td>
<td></td>
</tr>
<tr>
<td>Answer: 700 bricks were used to build the kennel.</td>
<td></td>
</tr>
<tr>
<td>Any questions not done or completed can be set for homework.</td>
<td>If so, review before the start of Lesson 7.</td>
</tr>
</tbody>
</table>

---

© CIMT, University of Exeter
**Lesson Plan**

**Notes**

- Individual work, monitored, (helped)
- (or whole class activity)
- BB: 7, 182, 357, 1007
- Calculators allowed
- Whole class activity
- At speed in order round class, or T chooses Ps at random.
- In good humour!
- Differentiation by question
- Ps seated can ask some questions too.
- T notes which facts are not known by which Ps and regularly checks them throughout the day.

**Y6**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **1** Factorising | Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:
  - 7 is a prime number Factors: 1, 7
  - 182 = 2 × 7 × 13 Factors: 1, 2, 7, 13, 14, 26, 91, 182
  - 357 = 3 × 7 × 17 Factors: 1, 3, 7, 17, 21, 51, 119, 357
  - 1007 = 19 × 53 Factors: 1, 19, 53, 1007 (nice) |
| **2** Multiplication and division tables | T says related multiplications and divisions. Ps say the results. Ps sit down if they answer correctly. If a P makes a mistake, he or she must stay standing and the next P corrects their mistake.
  - a) e.g. 8 × 9, 9 × 8, 3 × 70, 70 × 3, 7 × 30, etc.
  - Who can generalise the rule for multiplication?
    - BB: a × b = b × a (i.e. the factors are interchangeable)
  - b) 630 ÷ 7, 630 ÷ 70, 630 ÷ 90, 630 ÷ 9, etc.
    - Does the same rule apply for division? (No, a ÷ b ≠ b ÷ a) |
| **3** PbY6a, page 7 | Read: Calculate the sums, differences, products and quotients.
  - How many calculations are there? (3 × 3 × 4 = 36)
  - Set a time limit or deal with one part at a time. Review with whole class. Ps come to BB or dictate results to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed/corrected. Elicit that 1st answer in each row helps calculation of the others.
    - Solution:
      - a) 260 + 30 = 290
      - 2600 + 300 = 2900
      - 26 000 + 3000 = 29 000
      - 5260 + 30 = 5290
      - 52 600 + 300 = 52 900
      - 526 000 + 3000 = 529 000
      - 5260 + 430 = 5690
      - 52 600 + 4300 = 56 900
      - 526 000 + 43 000 = 569 000
      - b) 320 – 170 = 150
      - 3200 – 1700 = 1500
      - 32 000 – 17 000 = 15 000
      - 625 – 170 = 455
      - 6250 – 1700 = 4550
      - 62 500 – 17 000 = 45 500
      - 57 – 37 = 20
      - 585 – 385 = 200
      - 5899 – 3899 = 2000
      - c) 300 × 8 = 2400
      - 300 × 80 = 24 000
      - 300 × 800 = 2 400 000
      - 26 × 4 = 104
      - 2600 × 4 = 10 400
      - 260 × 400 = 1 040 000
      - 43 ÷ 7 = 60
      - 430 ÷ 70 = 60
      - 4300 ÷ 700 = 60
      - d) 60 ÷ 12 = 5
      - 600 ÷ 12 = 50
      - 60 000 ÷ 12 = 5000
      - 420 ÷ 7 = 60
      - 4200 ÷ 70 = 60
      - 4200 ÷ 7000 = 60
      - 78 ÷ 20 = 3, r 18
      - 7800 ÷ 200 = 39
      - 78 000 ÷ 20 000 = 3, r 18 000 |

© CIMT, University of Exeter
### Lesson Plan 7

#### Activity 4

**Formulae**

Study each formula and think of different numbers which could be written instead of the letters so that the equation is true. Ps suggest numbers and T writes on BB. Is it true for decimals and fractions too?

**BB:**

1. **a) \( a + b = b + a \) [To Ts only: Commutative law of addition]**
   
   e.g. \( 6247 + 503 = 503 + 6247 = 6750 \) ← Ps dictate
   
   \[
   \begin{align*}
   \frac{4}{5} + \frac{1}{10} & = \frac{1}{10} + \frac{4}{5} = \frac{1}{10} + \frac{8}{10} = \frac{9}{10} \\
   4.3 + 10.8 & = 10.8 + 4.3 = 15.1 \\
   6 + (– 9) & = (– 9) + 6 = – 3
   \end{align*}
   \]
   
   Can you think of a counter example where the formula is not true? (No, it is true for any two numbers.) Let's put it in a sentence.
   
   T shows the sentence and Ps say it in unison and write it in Pbs.
   
   **BB:** The two terms of any sum are interchangeable.
   
   T: Because this is true for all numbers and there is no example where it is not true, we say that this statement is a law of addition.

2. **b) \( a \times b = b \times a \) [To Ts only: Commutative law of multiplication]**
   
   e.g. \( 415 \times 11 = 11 \times 415 = 4565 \)
   
   \[
   \begin{align*}
   \frac{6}{8} \times 7 & = \frac{7}{8} \times \frac{6}{8} = \frac{42}{8} = \frac{21}{4} = \frac{51}{4} \\
   4.25 \times 6 & = 6 \times 4.25 = 25.5 \\
   – 12 \times 7 & = 7 \times (– 12) = – 84
   \end{align*}
   \]
   
   Can you think of a counter example where the equation is not true? (No, it is true for any two numbers.) Let's put it in a sentence.
   
   T shows the sentence (already prepared on BB or SB or OHT). Ps say it in unison and write it in Pbs.
   
   **BB:** The two factors of a product are interchangeable.

3. **c) \( a – b = b – a \)**

   Is this formula always true for subtraction? Some Ps might think it is never true and give many counter examples. e.g. \( 5 – 3 \neq 3 – 5 \)
   
   Can you think of an example where it is true? e.g. \( 5 – 5 = 5 – 5 \)
   
   Elicit that the equation is true when \( a = b \) but not true when \( a \neq b \).
   
   Can we say that this is a law of subtraction? (No, as generally it is not true, so there are many counter examples.)

4. **d) \( a \div b = b \div a \)**

   Deal with this in a similar way to subtraction. Ps give examples and counter examples e.g. \( 18 \div 3 \neq 3 \div 18 \) but \( 7 \div 7 = 7 \div 7 \).
   
   Elicit that the equation is true for natural numbers when \( a = b \neq 0 \) but not true when \( a \neq b \), so it is not a law of division. In fact, generally it is not true for natural numbers.

---

© CIMT, University of Exeter
T might also ask about the following formulae if Ps are interested.

e) \((a + b) + c = a + (b + c)\)

e.g. \((320 + 50.6) + 29.4 = 320 + (50.6 + 29.4) = 400\), etc.
Elicit that it is true for any 3 numbers, so it is a law.

f) \((a \times b) \times c = a \times (b \times c)\)

e.g. \((100 \times 6) \times 2 = 100 \times (6 \times 2) = 1200\), etc.
Elicit that it is true for any 3 numbers, so it is a law.

g) \(a \times (b + c) = a \times b + a \times c\)

e.g. \(6 \times (3.4 + 5.6) = 6 \times 3.4 + 6 \times 5.6 = 20.4 + 33.6 = 54\)
Elicit that it is true for any 3 numbers, so it is a law.

There is no need to tell Ps the names of these laws but ask Ps to explain them in a sentence in their own words.

**Lesson Plan 7**

**Notes**

To Ts only:

- **Associative law for addition**
  - There are no counter examples.

- **Associative law for multiplication**
  - There are no counter examples.

- **Distributive law for multiplication**
  - There are no counter examples.

Individual work, monitored, (helped)

Written on BB or use enlarged copy master or OHP

Responses shown in unison.

(or T chooses a P to say whether a statement is true or false and asks who agrees/ disagrees. Why?)

Ps could write interim results above the relevant operations when they explain at BB.

Discussion, reasoning, agreement, self-correction, praising

Extra praise for Ps who realised that in c) iv), dividing 63 by 0 makes no sense.

Who had them all correct?

Let's give them a clap!

Who made just 1 mistake?

The person nearest them, give them a pat on the back.

There are no counter examples.

There are no counter examples.

There are no counter examples.

**5**  

**PbY6a, page 7**

Q.2  
**Read:** Colour the box if the statement is true.

*If it is not true, change the ‘=’ sign to ‘≠’.*

Set a time limit of 6 minutes.

Review with whole class. Ps could write T and F on slates and show to T on command. Ps with different responses explain reasoning to class, either by citing a law they have learned or by working out the value of each side of the equation. Class agrees/disagrees. Mistakes corrected. In some cases, Ps could asked what should be done to make the false statement true.

**Solution:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>368 + 152 = 152 + 368 □ 7230 – 430 ≠ 430 – 7230 □</td>
</tr>
<tr>
<td>b)</td>
<td>1230 × 21 = 21 × 1230 □ 460 ÷ 23 ≠ 23 ÷ 460 □</td>
</tr>
<tr>
<td>c)</td>
<td>290 ÷ 0 ≠ 0 – 290 □ 1 × 167 = 167 × 1 □ 0 ÷ 8 ≠ 8 ÷ 0 □</td>
</tr>
<tr>
<td>d)</td>
<td>(82 + 38) + 15 = 82 + (38 + 15) □ (670 + 130) – 100 = 670 + (130 – 100) □</td>
</tr>
<tr>
<td></td>
<td>(400 – 250) ÷ 50 ≠ 400 – (250 ÷ 50) □ (360 – 160) – 30 ≠ 360 – (160 – 30) □</td>
</tr>
<tr>
<td></td>
<td>400 – (250 ÷ 50) = 400 – 250 ÷ 50 □ 360 – (160 – 30) = 360 – 160 + 30 □</td>
</tr>
<tr>
<td>e)</td>
<td>(18 × 2) ÷ 4 ≠ 18 ÷ (2 × 4) □ (18 × 4) ÷ 2 ≠ 18 ÷ (4 × 2) □</td>
</tr>
<tr>
<td></td>
<td>(60 ÷ 3) × 5 ≠ 60 ÷ (3 × 5) □ (80 ÷ 4) ÷ 2 ≠ 80 ÷ (4 ÷ 2) □</td>
</tr>
<tr>
<td></td>
<td>60 ÷ (3 ÷ 5) ≠ 60 ÷ 3 ÷ 5 □ 80 ÷ (4 ÷ 2) ≠ 80 ÷ 4 ÷ 2 □</td>
</tr>
<tr>
<td>f)</td>
<td>7 × (15 + 25) = 7 × 15 + 7 × 25 □ 7 + (15 × 25) ≠ (7 + 15) × (7 + 25) □</td>
</tr>
</tbody>
</table>

**30 min**
Q.3 Read: Solve the problems in your exercise book. Write only the results here.

Deal with one question at a time. Set a time limit of 2 minutes per question. Ps read question themselves, write a plan, do the calculation and check it in Ex. Bks. then write the result in Pbs.

Review with whole class. Ps show results on scrap paper or slates on command. P answering correctly explains solution at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected. T asks a P to say the answer in a sentence.

Solution: e.g.

a) A tradesman bought 8 machines of the same type for £4400 in total. Later, he sold them for £5184.

How much profit did he make on each machine?

Plan: \((£5184 – £4400) \div 8 = £784 \div 8 = £98\)

Answer: He made a profit of £98 on each machine.

b) Six people attended a conference. The conference fee was £320 per person and the travel cost was £222 per person.

How much did their company have to pay altogether?

Plan: \((£320 + £222) \times 6 = £542 \times 6 = £3252\)

Answer: Their company had to pay £3252 altogether.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Factorising</strong></td>
<td><strong>8</strong></td>
</tr>
<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:</td>
<td></td>
</tr>
<tr>
<td>- 8 = 2 \times 2 \times 2 \quad \text{Factors: 1, 2, 4, 8 (cubic number: 2^3)}</td>
<td></td>
</tr>
<tr>
<td>- 183 = 3 \times 61 \quad \text{Factors: 1, 3, 61, 181 (a nice number)}</td>
<td></td>
</tr>
<tr>
<td>- 358 = 2 \times 179 \quad \text{Factors: 1, 2, 179, 358}</td>
<td></td>
</tr>
<tr>
<td>- 1008 = 2 \times 2 \times 2 \times 3 \times 3 \times 7</td>
<td></td>
</tr>
<tr>
<td>Factor pairs:</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>252</td>
</tr>
<tr>
<td>2</td>
<td>126</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>179</td>
</tr>
<tr>
<td>T shows the number as powers of its prime factors.</td>
<td></td>
</tr>
<tr>
<td><strong>Mental practice</strong></td>
<td><strong>20 min</strong></td>
</tr>
<tr>
<td>a) Let’s list the natural multiples of:</td>
<td></td>
</tr>
<tr>
<td>2: (2, 4, 6, 8, 10, 12, . . .)</td>
<td></td>
</tr>
<tr>
<td>3: (3, 6, 9, 12, 15, 18, 21, 24, . . .)</td>
<td></td>
</tr>
<tr>
<td>4: (4, 8, 12, 16, 20, 24, . . .)</td>
<td></td>
</tr>
<tr>
<td>5: (5, 10, 15, 20, 25, 30, . . .)</td>
<td></td>
</tr>
<tr>
<td>6: [6, 12, 18, 24, 30, . . .]</td>
<td></td>
</tr>
<tr>
<td>7: [7, 14, 21, 28, 35, 42, . . .]</td>
<td></td>
</tr>
<tr>
<td>8: [8, 16, 24, 32, 40, 48, . . .]</td>
<td></td>
</tr>
<tr>
<td>b) T says a multiplication or division table fact and Ps say result.</td>
<td></td>
</tr>
<tr>
<td>c) More complicated multiplications and divisions. T starts, ( P_1 ) answers then says a multiplication or division for ( P_2 ), ( P_2 ) answers then says a multiplication or division for ( P_3 ), and so on.</td>
<td></td>
</tr>
<tr>
<td><strong>PbY6a, page 8</strong></td>
<td><strong>Q.1 Read: Calculate the sums in a clever way.</strong></td>
</tr>
<tr>
<td>Set a time limit. Review with whole class. Ps come to BB to explain reasoning. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td>a) 275 + 99 + 25 + 34 + 66 = 300 + 100 + 99 = 499</td>
<td></td>
</tr>
<tr>
<td>b) 605 + 13 + 300 + 67 + 95 = 700 + 300 + 80 = 1080</td>
<td></td>
</tr>
<tr>
<td>c) 810 + 183 + 140 + 7 + 1860 = 810 + 190 + 2000 = 3000</td>
<td></td>
</tr>
<tr>
<td>d) 15 + 35 + 6666 + 50 + 3334 = 50 + 50 + 10 000 = 10 100</td>
<td></td>
</tr>
</tbody>
</table>

© CIMT, University of Exeter
### Activity

**PbY6a, page 8**

Q.2 Read: *Calculate the products in a clever way.*

Set a time limit. Review with whole class. Ps come to BB to explain reasoning. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>$5 \times 37 \times 25 \times 20 \times 4 = 100 \times 100 \times 37 = 10000 \times 37 = 37000$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>$25 \times 125 \times 4 \times 8 \times 7 = 100 \times 1000 \times 7 = 100000 \times 7 = 700000$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>$2 \times 25 \times 8 \times 20 \times 70 = 1000 \times 560 = 560000$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>$5 \times 40 \times 5 \times 20 \times 65 = 20000 \times 65 = 1300000$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

25 min

---

**Notes**

- **Lesson Plan 8**

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Individual work, monitored

Written on BB or SB or OHT

Discussion, reasoning, agreement, self-correction, praising

Extra praise for Ps who looked for factors which could be combined to make whole hundreds or thousands

or for d):

$(5 \times 20) \times (5 \times 20) \times 2 \times 65$

$= 100 \times 100 \times 130$

$= 10000 \times 130 = 1300000$

---

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Individual work, monitored

Written on BB or SB or OHT

Discussion, reasoning, agreement, self-correction, praising

Accept and praise any valid method of calculation but extra praise for Ps who noticed the easy ways shown opposite. T shows them if no P noticed them.

Feedback for T

---
Lesson Plan 8

Notes

Individual work, monitored
Written on BB or use enlarged copy master or OHT
Discussion, reasoning, agreement, self-correction, praising
Feedback for T

Activity

PbY6a, page 8

Q.4 Read: Calculate the results and compare them.

Set a time limit. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Who did the same? Who did it a different way? Mistakes discussed and corrected. What do you notice about the results? (Top and bottom calculations are the same and the two middle calculations are the same.) Why? Ps explain in own words.

Solution:

a) i) \(675 - (453 + 123) = 675 - 576 = 99\)
ii) \(675 - (453 - 123) = 675 - 330 = 345\)
iii) \(675 - 453 + 123 = 222 + 123 = 345\)
iv) \(675 - 453 - 123 = 222 - 123 = 99\)

b) i) \(480 \div (12 \times 4) = 480 \div 48 = 10\)
ii) \(480 \div (12 \div 4) = 480 \div 3 = 160\)
iii) \(480 \div 12 \times 4 = 40 \times 4 = 160\)
iv) \(480 \div 12 \div 4 = 40 \div 4 = 10\)

Agree that when when there are brackets, the operations inside the brackets should be done first.

35 min

PbY6a page 8

Q.5 Deal with 3 or 4 questions at a time, then review and discuss and correct mistakes before Ps continue with the next 3 or 4.

Ps could show results on scrap paper or slates on command. Ps with different results explain their reasoning at BB. Class decides who is correct. Where relevant, Ps show two ways to do the calculation and class decides which method is simpler.

Solution:

a) \(16 \times (26 + 30) = 16 \times 56 = 560 + 336 = 896\)
   or \((16 \times 26) + (16 \times 30) = 416 + 480 = 896\)

b) \(37 \times (200 - 100) = 37 \times 100 = 3700\)
   or \((37 \times 200) - (37 \times 100) = 7400 - 3700 = 3700\)

c) \((156 + 44) \times 5 = 200 \times 5 = 1000\)
   or \((156 \times 5) + (44 \times 5) = 780 + 220 = 1000\)

d) \((200 - 20) \times 45 = 180 \times 45 = 7200 + 900 = 8100\)
   or \((200 \times 45) - (20 \times 45) = 9000 - 900 = 8100\)

e) \((78 + 96) \div 6 = 174 \div 6 = 29\)
   or \((78 \div 6) + (96 \div 6) = 13 \div 16 = 29\)

f) \((160 - 75) \div 5 = 85 \div 5 = 17\)
   or \((160 \div 5) - (75 \div 5) = 32 - 15 = 17\)

g) \(750 \div (10 + 15) = 750 \div 25 = 150 \div 5 = 30\)

h) \(144 \div (72 - 48) = 144 \div 24 = 72 \div 12 = 6\)

i) \((430 + 220) \div 1 = 650 \div 1 = 650\)

j) \((220 + 430) \div 0 \neq \text{ (dividing by zero makes no sense) \}
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lesson Plan 8</td>
<td>Elicit that dividing by zero makes no sense, but zero divided by any number is zero.</td>
</tr>
</tbody>
</table>

*Y6*

(Continued)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>k)</td>
<td>(365 – 165) ÷ 1 = 200 ÷ 1 = 200</td>
</tr>
<tr>
<td>l)</td>
<td>(493 – 203) ÷ 0 ≠ anything</td>
</tr>
<tr>
<td>m)</td>
<td>(147 – 147) ÷ 29 = 0 ÷ 29 = 0</td>
</tr>
<tr>
<td>n)</td>
<td>300 ÷ (15 – 15) = 300 ÷ 0 ≠ anything</td>
</tr>
<tr>
<td>o)</td>
<td>4 × (12 × 25) = 4 × 25 × 12 = 100 × 12 = 1200</td>
</tr>
<tr>
<td>p)</td>
<td>8 × (45 ÷ 5) = 8 × 9 = 72</td>
</tr>
<tr>
<td>q)</td>
<td>350 ÷ (14 × 5) = 350 ÷ 70 = 35 ÷ 7 = 5</td>
</tr>
<tr>
<td>r)</td>
<td>600 ÷ (60 ÷ 4) = 600 ÷ 15 = 200 ÷ 5 = 40</td>
</tr>
<tr>
<td></td>
<td>or = 600 ÷ 60 × 4 = 10 × 4 = 40</td>
</tr>
<tr>
<td>s)</td>
<td>9 × (0 ÷ 3) = 9 × 0 = 0</td>
</tr>
<tr>
<td>t)</td>
<td>4 × (9 ÷ 0) ≠ anything</td>
</tr>
</tbody>
</table>

45 min
R: Doubling and halving. Relationship between $\times$ and $\div$

C: Calculations. Squares of multiples of 10 up to 100

E: Powers as positive integers. Word problems

**Lesson Plan**

**Week 2**

### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $9 = 3 \times 3$ (square number) Factors: 1, 3, 9
- $184 = 2 \times 2 \times 2 \times 23$ Factors: 1, 2, 4, 8, 23, 46, 92, 184
- 359 is a prime number (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17; 19 $\times$ 19 > 359) Factors: 1, 359
- 1009 is a prime number (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37 $\times$ 37 > 1009)

8 min

### Activity 2

**Multiplication and division practice**

a) T says a multiplication or division fact. Ps say the result.

b) T says a more complicated multiplication or division. Ps say the result. e.g. $60 \times 90, 400 \div 40, 640 \div 8$, etc.

c) Relay with the 4 operations. T says an operation, P1 says result and thinks of an operation for the next P to answer, etc. (Operations can be combined.)

15 min

### Activity 3

**PbY6a, page 9**

Q.1 Read: Fill in the missing numbers.

Set a time limit of 3 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning with reverse operation. Class agrees/disagrees. Mistakes corrected.

**Solution:**

a) $4 \times 7 = 28, \ 81 \div 9 = 9, \ 9 \times 6 = 54, \ 63 \div 7 = 9$

b) $5 \times 70 = 350, \ 560 \div 80 = 7, \ 90 \times 3 = 270, \ 480 \div 8 = 60$

c) $20 \times 60 = 1200, \ 3200 \div 80 = 40, \ 90 \times 50 = 4500, \ 1800 \div 60 = 30$

20 min

---

Notes

Individual work, monitored, (helped) (or whole class activity)

BB: 9, 184, 359, 1009

Calculators allowed

Discussion, reasoning, agreement, self-correction, praising

Elicit that:

- $3^2$ is read as '3 squared' or '3 to the power 2'
- $2^3$ is read as '2 cubed' or '2 to the power 3'

Whole class activity

At speed in order round class (or T chooses Ps at random)

If a P makes a mistake, the next P must correct it.

In good humour. Praising

Individual work, monitored (helped)

Written on BB or SB or OHT

Reasoning, agreement, self-correction, praising

Reasoning: e.g. $81 \div 9 = 9$, as $9 \times 9 = 81$

Elicit that 81 is a square number.
**Y6**

### Activity 4

**PbY6a, page 9**

**Q.2** Read: *Write the area of each square in cm² and in mm².*

How many mm are in 1 cm? (10)

How many mm squares are in 1 cm square? (10 × 10 = 100)

Set a time limit of 3 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning and referring to diagram. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

Let’s continue the sequence. Ps say what the side lengths of the following squares would be, then give their area in cm² and in mm². Class points out errors. Ps write the dimensions in Ex. Bks.

**BB:**

\[ a = 7 \text{ cm}, \quad A = 49 \text{ cm}^2 = 4900 \text{ mm}^2 \]

\[ a = 8 \text{ cm}, \quad A = 64 \text{ cm}^2 = 6400 \text{ mm}^2 \]

\[ a = 9 \text{ cm}, \quad A = 81 \text{ cm}^2 = 8100 \text{ mm}^2 \]

\[ a = 10 \text{ cm}, \quad A = 100 \text{ cm}^2 = 10000 \text{ mm}^2 \]

\[ a = 11 \text{ cm}, \quad A = 121 \text{ cm}^2 = 12100 \text{ mm}^2 \]

Let’s continue the sequence. Ps say what the side lengths of the following squares would be, then give their area in cm² and in mm². Class points out errors. Ps write the dimensions in Ex. Bks.

**BB:**

\[ a \times a = a^2 \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cm</td>
<td>2 cm</td>
<td>3 cm</td>
<td>4 cm</td>
<td>5 cm</td>
<td>6 cm</td>
</tr>
</tbody>
</table>

**Area of:**

- A: \(1 \text{ cm}^2 = 100 \text{ mm}^2\)
- B: \(4 \text{ cm}^2 = 400 \text{ mm}^2\)
- C: \(9 \text{ cm}^2 = 900 \text{ mm}^2\)
- D: \(16 \text{ cm}^2 = 1600 \text{ mm}^2\)
- E: \(25 \text{ cm}^2 = 2500 \text{ mm}^2\)
- F: \(36 \text{ cm}^2 = 3600 \text{ mm}^2\)

### Activity 5

**PbY6a, page 9**

**Q.3** Read: *Continue the sequences using your own rule.*

Set a time limit of 4 minutes. Ps write the terms and the rule that they used.

Review with the whole class. Ps come to BB or dictate to T, explaining their rule. Who used the same rule? Who used a different one? Class decides whether the rules are valid. Mistakes discussed and corrected.

**Solution:**

a) \(1, 4, 9, 16, (25, 36, 49, 64, 81, 100, 121, 144, 169, \ldots)\)

**Rule:** The square numbers in increasing order.

The majority of Ps will most likely give the terms above, but a P might say the rule in a different way. E.g.

**Rule:** The difference between terms is increasing by 2.

or

The natural numbers to the power 2 in increasing order:

\[ 1^2 = 1, \quad 2^2 = 4, \quad 3^2 = 9, \quad 4^2 = 16, \quad 5^2 = 25, \ldots \]

b) \(100, 400, 900, 1600, (2500, 3600, 4900, 6400, \ldots)\)

**Rule:** Difference between terms is increasing by 200.

i.e. Difference sequence is 300, 500, 700, 900, \ldots

or

\(100 = 10 \times 10, \quad 400 = 20 \times 20, \quad 900 = 30 \times 30, \ldots\)

or

\(10 \times 10, \quad 20 \times 20, \quad 30 \times 30, \quad (40 \times 40, \quad 50 \times 50, \ldots)\)

Elicit that this sequence is the same as b): the whole tens to the second power, or to the power 2, or squared.

### Extension

**Why do you think we call the numbers 1, 4, 9, \ldots ‘square’ numbers?**

(They could be the area of a square and their factors could be the length of a side.)

**BB:**

\[ a \times a = a^2 \]

© CIMT, University of Exeter
### Activity 6

**PbY6a, page 9**

Q.4 Read: *Calculate the required values in your exercise book.*

Deal with one at a time. T chooses a P to read out the question. Ps write a plan and do the calculation in Ex. Bks, then show the answer on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) *The area of a square is 10 000 cm². What length is each side? What is its perimeter?*

\[
A = 10 000 \text{ cm}^2 = 100 \text{ cm} \times 100 \text{ cm}; \quad a = 100 \text{ cm}
\]

\[
P = 4 \times 100 \text{ cm} = 400 \text{ cm}
\]

b) *The side of a square is 50 cm. What is its perimeter? What is its area?*

\[
P = 4 \times 50 \text{ cm} = 200 \text{ cm}
\]

\[
A = 50 \text{ cm} \times 50 \text{ cm} = 500 \text{ cm} \times 5 \text{ cm} = 2500 \text{ cm}^2
\]

c) *The side of a square is 25 cm. What is its perimeter? What is its area?*

\[
P = 4 \times 25 \text{ cm} = 100 \text{ cm}
\]

\[
A = 25 \text{ cm} \times 25 \text{ cm} = (500 + 125) \text{ cm}^2 = 625 \text{ cm}^2
\]

or

\[
A = 50 \text{ cm} \times 50 \text{ cm} \div 2 \div 2 = 2500 \text{ cm}^2 \div 2 \div 2
\]

\[
= 1250 \text{ cm}^2 \div 2 = 625 \text{ cm}^2
\]

d) *The perimeter of a square is 60 cm. What length is each side? What is its area?*

\[
a = 60 \text{ cm} \div 4 = 15 \text{ cm}
\]

\[
A = 15 \text{ cm} \times 15 \text{ cm} = (150 + 75) \text{ cm}^2 = 225 \text{ cm}^2
\]

or

\[
A = 30 \text{ cm} \times 30 \text{ cm} \div 2 \div 2 = 900 \text{ cm}^2 \div 2 \div 2
\]

\[
= 450 \text{ cm}^2 \div 2 = 225 \text{ cm}^2
\]

e) *The side of a square is 35 cm. What is its perimeter? What is its area?*

\[
P = 4 \times 35 \text{ cm} = 120 \text{ cm} + 20 \text{ cm} = 140 \text{ cm}
\]

\[
A = 35 \text{ cm} \times 35 \text{ cm} = (1050 + 175) \text{ cm}^2 = 1225 \text{ cm}^2
\]

or

\[
A = 70 \text{ cm} \times 70 \text{ cm} \div 2 \div 2 = 4900 \text{ cm}^2 \div 2 \div 2
\]

\[
= 2450 \text{ cm}^2 \div 2 = 1225 \text{ cm}^2
\]

f) *The perimeter of a square is 560 cm. What length is each side? What is its area?*

\[
a = 560 \text{ cm} \div 4 = 140 \text{ cm}
\]

\[
A = 140 \text{ cm} \times 140 \text{ cm} = 70 \text{ cm} \times 70 \text{ cm} \times 2 \times 2
\]

\[
= 4900 \text{ cm}^2 \times 2 \times 2
\]

\[
= 9800 \text{ cm}^2 \times 2 = 19600 \text{ cm}^2
\]

---

**Notes**

Individual calculation, but class kept together, then whole class review.

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid method of calculation but T shows the quick method of halving and doubling if no P suggests it.
Lesson Plan 9

Notes

Individual trial first, then whole class discussion to establish the rule.

Drawn on BB or use enlarged copy master or OHP

If no P is on the right track, T gives hints.

Extra praise for Ps who worked it out without help from T.

Discussion, reasoning, agreement, praising

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Y6

Activity

PbY6a, page 9

Q.5 Read: Work out Tommy’s method and use it to calculate the area of these rectangles.

BB:

Tommy’s method:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>7</td>
</tr>
<tr>
<td>17</td>
<td>8</td>
</tr>
<tr>
<td>19</td>
<td>9</td>
</tr>
<tr>
<td>57</td>
<td>114</td>
</tr>
<tr>
<td>57</td>
<td>228</td>
</tr>
<tr>
<td>57</td>
<td>456</td>
</tr>
<tr>
<td>912</td>
<td>1</td>
</tr>
<tr>
<td>912</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>855 m²</td>
</tr>
<tr>
<td></td>
<td>969 m²</td>
</tr>
<tr>
<td></td>
<td>1083 m²</td>
</tr>
</tbody>
</table>

Allow Ps to think about it for a couple of minutes and to discuss with their neighbours if they wish. Who thinks that they understand what Tommy has done? A, come and explain it to us. Who thought the same? Who has a different idea?

Elicit that Tommy has halved the a value, ignoring the remainder, and doubled the b value until the a value is 1, then he scored out the b value if the a value is an even number and added up the b values which are left. The sum is the area of the rectangle.

Use this method to work out the areas of the rectangles in questions a) to c) in your Ex. Bks. Set a time limit or deal with one rectangle at a time. (If Ps are struggling, stop individual work and continue as a whole class activity.)

Review with the whole class. Ps show areas on scrap paper or slates on command. Ps with correct result explain reasoning at BB. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) \( a = 16 \text{ m}, \ b = 57 \text{ m} \)

b) \( a = 18 \text{ m}, \ b = 57 \text{ m} \)

c) \( a = 20 \text{ m}, \ b = 57 \text{ m} \)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>16</td>
<td>8</td>
</tr>
<tr>
<td>18</td>
<td>9</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
</tr>
<tr>
<td>8</td>
<td>114</td>
</tr>
<tr>
<td>9</td>
<td>228</td>
</tr>
<tr>
<td>10</td>
<td>456</td>
</tr>
<tr>
<td>18</td>
<td>912</td>
</tr>
<tr>
<td>20</td>
<td>912</td>
</tr>
<tr>
<td>912</td>
<td>1</td>
</tr>
<tr>
<td>912</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>912 m²</td>
</tr>
<tr>
<td></td>
<td>1026 m²</td>
</tr>
<tr>
<td></td>
<td>1140 m²</td>
</tr>
</tbody>
</table>

Extension

Who can explain why Tommy’s method works? Ask several Ps what they think, or leave the question open for homework.

[Think of the a value as the number of rows of grid squares, and the b value as the number of squares in each row.

Any even number of rows can be halved and the extra squares added to the end of the rows left so that b is twice as long but the area stays the same. Because the grid squares in each new shorter but wider rectangle were contained in the previous rectangle, the even values of b are scored out so that these grid squares won’t be counted more than once.

If there is an odd number of rows, when a is halved there will be one row over which cannot be added evenly to b, so only these extra rows are added to the final row in the rectangle where a is 1.]

45 min

Homework

Factorise 10, 185, 360 and 1010 in your exercise book.

Solution:

\( 10 = 2 \times 5, \)  \( 185 = 5 \times 37, \)  \( 360 = 2^3 \times 3^2 \times 5, \)

\( 1010 = 2 \times 5 \times 101 \)
## Activity

Mental and written calculations. Activities, consolidation

**Practice Book Y6a, page 10**

### Solutions:

**Q.1**

a) \(410.5 + 410.5 + 410.5 + 410.5 = 1642\)

b) \(7063.6 – 20.4 – 30.2 = 7013\)

c) \(160 ÷ 100 × 5 = 1.6 × 5 = 8\)

d) \(12 × 12 + 2 × 10 × 10 = 144 + 200 = 344\)

e) \(5 × (32 + 110) ÷ 5 = 32 + 110 = 142\)

f) \(761 × 100 ÷ 5 ÷ 2 = 761 × 10 = 7610\)

g) \(7867 + 435 – 128 – 207 = 7867 + 435 – 335 = 7967\)

h) \(200.6 – 33.2 × 3 + 899 = 200.6 – 99.6 + 899 = 101 + 899 = 1000\)

**Q.2**

a) \(386 + 78 + 83 + 22 + 517 = 1086\)

b) \(106 – 43 + 54 – 117 = 0\)

c) \(1000 – 4 × 25 – 8.09 × 100 = 1000 – 100 – 809 = 1000 – 909 = 91\)

d) \(5792 – 76 + 300 – 16 = 6092 – 92 = 6000\)

e) \(140.5 + \frac{359}{4} = 160.5 + 339\)

f) \(280 ÷ 5 ÷ 14 × \frac{25}{4} = 100\)

**Q.3**

a) \(4.3, 12.9, 38.7, (116.1, 348.3, 1044.9, 3134.7, . . .) \times 3\)

b) \(250, 50, 10, (2, 0.4, 0.08, 0.016, 0.0032, . . .) ÷ 5\)

c) \(4575, 4470, 4365, (4260, 4155, 4050, 3945, . . .) ÷ 105\)

d) \(100.73, 120.80, 140.87, (160.94, 181.01, . . .) ÷ 20.07\)

**Q.4**

A: \(P = 4 \text{ cm}\)  B: \(P = 12 \text{ cm}\)  C: \(P = 28 \text{ cm}\)

D: \(P = 36 \text{ cm}\)  E: \(P = 52 \text{ cm}\)  F: \(P = 400 \text{ cm}\)

A ~ B ~ C ~ D ~ E ~ F (All squares are similar.)

**Q.5**

a) \(A = 30 \text{ cm} × 30 \text{ cm} = 900 \text{ cm}^2\)

b) \(a = 14.8 \text{ cm} ÷ 4 = 3.7 \text{ cm},\)

\[
A = 3.7 \text{ cm} × 3.7 \text{ cm} = 37 \text{ mm} × 37 \text{ mm} = 1369 \text{ mm}^2 = 13.69 \text{ cm}^2
\]

c) \(A = 121 \text{ cm}^2 = 11 \text{ cm} × 11 \text{ cm}; P = 11 \text{ cm} × 4 = 44 \text{ cm}\)

d) \(A = 1.69 \text{ cm}^2 = 169 \text{ mm}^2 = 13 \text{ mm} × 13 \text{ mm}\)

\[
a = 13 \text{ mm} = 1.3 \text{ cm}
\]

e) \(V = 125 \text{ cm}^3 = 5 \text{ cm} × 5 \text{ cm} × 5 \text{ cm}, e = 5 \text{ cm}\)
### Lesson Plan

#### R: Mental calculation strategies

**Activity 1**

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **11** is a prime number
- **186** = \(2 \times 3 \times 31\) Factors: 1, 2, 3, 6, 31, 62, 93, 186
- **361** = \(19 \times 19\) (square number) Factors: 1, 19, 361
- **1011** = \(3 \times 337\) Factors: 1, 3, 337, 1011 (nice)

(337 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17 and \(19 \times 19 > 337\), so 337 is a prime number)

---

#### Notes

Individual work, monitored, (helped)

(or whole class activity)

BB: 11, 186, 361, 1011

Calculators allowed

Discussion, reasoning, agreement, self-correction, praising

BB:

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>1011</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>186</td>
<td>3</td>
<td>337</td>
<td>337</td>
</tr>
</tbody>
</table>

---

#### Activity 2

**PbY6a, page 11**

**Q.1** Read: *Do the first calculation, then use the result to help you do the other calculations mentally.*

Set a time limit of 6 minutes. Review with the whole class. Ps come to BB to explain reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- a) \(72 \times 14 = 1008\)
  
  i) \(7248 + 8717 = 16065\)
  
  ii) \(7248 + 8617 = 15965\)
  
  iii) \(7248 + 8717 = 15965\)

- b) \(4372\)
  
  i) \(4372 - 1837 = 2535\)
  
  ii) \(4372 - 337 = 3535\)
  
  iii) \(4372 - 1237 = 2735\)

---

**PbY56, page 11**

**Q.2** Read: *Fill in the missing numbers so that the operations and inequalities are true.*

Set a time limit of 6 minutes or deal with one part at a time.

Review with whole class. Ps come to BB to explain how they worked out the missing digits. Who did the same? Who did it another way? etc. Class checks that the operations are correct. Mistakes discussed and corrected.

**Solution:**

- a) \(\frac{364}{238}\)
  
  i) \(\frac{364}{238} = 1.53\)
  
  ii) \(\frac{364}{238} = 1.53\)
  
  iii) \(\frac{364}{238} = 1.53\)

- b) \(\frac{888}{333} + 555\)
  
  i) \(\frac{888}{333} = 2.67\)
  
  ii) \(\frac{888}{333} = 2.67\)
  
  iii) \(\frac{888}{333} = 2.67\)

- c) \(900\)
  
  i) \(\frac{900}{10} = 90\)
  
  ii) \(\frac{900}{10} = 90\)
  
  iii) \(\frac{900}{10} = 90\)

---

Extra praise for Ps who noticed the relationships which made the calculations easier (relevant digits underlined)

---

© CIMT, University of Exeter
Lesson Plan 11

Activity

4 Multiplication 1
Let's estimate the product, then do the calculation. Here are different ways to do the first calculation. Who can fill in the missing digits?

Ps come to BB to complete the estimation, then do the calculations, explaining reasoning with place-value detail. Class points out errors. In c), T points out or elicits that the number of zeros at the RHS of the product should match the total number of zeros at the the RHS of the multiplicand and the multiplier.

BB:

a) \[3265 \times 3 = 3300 \times 3 = 9900\]

\[
\begin{array}{c}
\text{Th H T U} \\
3 & 2 & 6 & 5 \\
\times & 3 & \\
\hline
1 & 8 & 1 & 5 \\
9 & 7 & 9 & 5 \\
\end{array}
\]

b) \[8903 \times 6 = 9000 \times 6 = 54000\]

\[
\begin{array}{c}
\text{Th H T U} \\
8 & 9 & 0 & 3 \\
\times & 6 \\
\hline
5 & 3 & 4 & 1 & 8 \\
3 & 8 & 9 & 0 & 3 \\
\end{array}
\]

c) \[8903 \times 600 = 9000 \times 600 = 5400000\]

\[
\begin{array}{c}
\text{Th H T U} \\
8 & 9 & 0 & 3 \\
\times & 6 & 0 & 0 \\
\hline
3 & 8 & 3 & 7 & 7 \\
8 & 3 & 7 & 7 & 7 \\
\end{array}
\]

d) \[9803 \times 6 = 9579\]

\[
\begin{array}{c}
\text{Th H T U} \\
9 & 8 & 0 & 3 \\
\times & 6 \\
\hline
5 & 2 & 7 & 8 \\
9 & 5 & 7 & 9 \\
\end{array}
\]

e) \[9803 \times 9 = 7273\]

\[
\begin{array}{c}
\text{Th H T U} \\
9 & 8 & 0 & 3 \\
\times & 9 \\
\hline
8 & 3 & 7 & 7 \\
7 & 2 & 7 & 2 \\
\end{array}
\]

f) \[9803 \times 6 = 7273\]

\[
\begin{array}{c}
\text{Th H T U} \\
9 & 8 & 0 & 3 \\
\times & 6 \\
\hline
5 & 3 & 4 & 1 & 8 \\
9 & 7 & 9 & 5 \\
\end{array}
\]

Whole class activity
Written on BB or use enlarged copy master or OHP
At a good pace
Discussion, reasoning, checking with estimate, agreement, praising
T helps with reasoning if necessary, e.g. in b):
\[6 \times 3U = 18U = 1T + 8U\]
I write 8 in the units column in the answer and 1 below the tens column;
[6 \times \text{zero tens} = \text{zero}] + 1 \text{ten} = 1 \text{ten}.
I write 1 in the tens column in the answer.

6 \times 9H = 54H = 5\text{Th} + 4\text{H}
I write 4 in the hundreds column in the answer and 5 below the thousands column', etc.

In d) to f), Ps estimate each result mentally first before they do the calculation.
Pxs could also check results with a calculator.

Solution:

Q.3 Read: Estimate the result in your head first, then do the exact calculation.
Deal with one at a time or set a time limit. Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) \[
\begin{array}{c}
\text{Th H T U} \\
4 & 2 & 9 & 0 \\
\times & 3 & 6 & 0 & 3 \\
\hline
1 & 7 & 1 & 6 \\
1 & 8 & 1 & 4 \\
\end{array}
\]

b) \[
\begin{array}{c}
\text{Th H T U} \\
9 & 8 & 0 & 3 \\
\times & 1 & 2 & 3 & 4 \\
\hline
8 & 6 & 0 & 7 & 3 \\
1 & 8 & 1 & 4 & 0 \\
\end{array}
\]

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Reasoning with place-value detail, checking against estimate, agreement, self-correction, praising
If disagreement, check results with a calculator.

Feedback for T
**Lesson Plan 11**

**Notes**

Whole class activity
Place-value table already drawn on BB or SB or OHT

Reasoning with place-value detail, checking against estimate, agreement, praising

Discussion on the pros and cons of the various methods.

T stresses that Ps may use the method which they like best.

Ps need not use grids, as long as they line up the digits with the same place values in the correct columns.

---

### Multiplication 2

Let’s estimate the product, then do the calculation.

Ps come to BB to complete the estimation, do the calculation in a place-value table first, then in any other way they choose. Class points out errors. Who knows another way to do it? Come and show us. Is there another way? T could show any of those below not suggested by Ps and ask if it is correct. Which method do you like best? Why?

BB: e.g.

- 246 × 57 = 250 × 60
  
  = 200 × 60 + 50 × 60 = 12000 + 3000 = 15000

- 732 × 163 = 700 × 200 = 14000

**Solution:**

<table>
<thead>
<tr>
<th>Th</th>
<th>H</th>
<th>T</th>
<th>U</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
<td>2</td>
<td>1</td>
</tr>
</tbody>
</table>
+    | 2 | 0 | 0 |
+    | 1 | 0 | 0 |
+    |   |   |   |
| 1  | 4 | 0 | 2 |
| 2  | 4 | 2 | 1 |
| 2  | 0 | 0 | 0 |
| 1  | 0 | 0 | 0 |

**Lesson Plan 11**

**Notes**

Whole class activity
Place-value table already drawn on BB or SB or OHT

Reasoning with place-value detail, checking against estimate, agreement, praising

Discussion on the pros and cons of the various methods.

T stresses that Ps may use the method which they like best.

Ps need not use grids, as long as they line up the digits with the same place values in the correct columns.

---

**Individual work, monitored (helped)**

Written on BB or use enlarged copy master or OHP
(or set 1 row to do in class, and the 2nd row for homework)

Differentiation by time limit.

Reasoning with place-value detail, checking against estimate, agreement, self-correction, praising

If disagreement, check result with a calculator.

Accept any valid method of calculation.
<p>**Lesson Plan**</p>

**Week 3**

**Y6**

- **R:** Mental calculation
- **C:** Pencil and paper procedures: Multiplication, division (HTU ÷ (T) U)
- **E:** Larger numbers. Puzzles

### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **12** = 2 × 2 × 3 (= 2² × 3) Factors: 1, 2, 3, 4, 6, 12
- **187** = 11 × 17 Factors: 1, 11, 17, 187 (nice number)
- **362** = 2 × 181 Factors: 1, 2, 181, 362 (nice)
- **1012** = 2 × 2 × 11 × 23 (= 2² × 11 × 23) Factors: 1, 2, 4, 11, 22, 23, 44, 46, 92, 253, 506, 1012

**Notes**

Individual work, monitored, (helped)

(or whole class activity)

BB: 12, 187, 362, 1012

Calculators allowed

Reasoning, agreement, self-correction, praising

BB: 17 17 1012 2

362 2 253 11

181 181 23 23

1

Join up the factor pairs for 1012

### Activity 2

**Missing digits**

Which digits are missing from these calculations? Ps come to BB or dictate to T, explaining reasoning with place-value detail. Class checks completed calculation mentally and agrees/disagrees.

BB:

a) \[ \begin{array}{c}
+ \\
\end{array} \begin{array}{c}
4 & 4 & 4 & 4 \\
2 & 7 & 7 & 7 \\
1 & 2 & 2 & 2 & 1 \\
\end{array} \]

b) \[ \begin{array}{c}
- \\
\end{array} \begin{array}{c}
8 & 2 & 6 & 5 \\
4 & 3 & 2 & 1 \\
4 & 4 & 4 & 4 \\
\end{array} \]

**Notes**

Whole class activity

Written on BB or use enlarged copy master or OHP

Reasoning, checking, agreement, praising

### Activity 3

**PbY6a, page 12**

Q.1 Read: Fill in the missing digits so that the results are correct.

Set a time limit. Remind Ps to check their solutions.

Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) \[ \begin{array}{c}
+ \\
\end{array} \begin{array}{c}
1 & 8 & 7 & 6 & 5 \\
3 & 4 & 5 & 6 \\
1 & 2 & 2 & 2 & 1 \\
\end{array} \]

b) \[ \begin{array}{c}
+ \\
\end{array} \begin{array}{c}
9 & 7 & 5 & 1 \\
2 & 4 & 6 & 5 \\
1 & 2 & 2 & 2 & 1 \\
\end{array} \]

c) \[ \begin{array}{c}
- \\
\end{array} \begin{array}{c}
7 & 7 & 7 & 7 \\
1 & 3 & 3 & 3 \\
4 & 4 & 4 & 4 \\
\end{array} \]

b) \[ \begin{array}{c}
- \\
\end{array} \begin{array}{c}
8 & 1 & 9 & 0 \\
2 & 5 & 2 & 5 \\
5 & 5 & 5 & 5 \\
\end{array} \]

**Notes**

Individual trial, monitored (helped)

(or whole class activity if Ps are not very able)

Written on BB or use enlarged copy master or OHP

Discussion, reasoning, checking, agreement, self-correction, praising

© CIMT, University of Exeter
Q.2 Read: Fill in the missing digits. Check that your answers are correct.

Set a time limit. Ps check calculation mentally or in Ex. Bks.

Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning. Who thought the same? Who did it a different way? etc. Class checks results mentally and agrees/disagrees. Mistakes discussed and corrected.

Solution:

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>0</td>
</tr>
</tbody>
</table>

| 8  | 4  | 8  |
| 3  | 6  | 7  |
| 4  | 6  | 0  |

Lesson Plan 12

Notes

Individual trial, monitored (less able Ps helped)

(or whole class activity if Ps prefer)

Written on BB or use enlarged copy master or OHP

Discussion, reasoning with place-value detail, checking, agreement, self-correction, praising

Accept trial and error but extra praise for Ps who reasoned logically.

Individual work, monitored

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, reasoning with place-value detail, agreement, self-correction, praising

Extra praise if Ps notice that the multiplicand is the same in each calculation and that:

BB: 42 = 2 \times 21

105 = 5 \times 21

189 = 9 \times 21

so they could use the product in a) to determine easily the products in b) and c).

If not, T asks Ps what they notice about the multipliers.

6 Division

a) Let's divide 644 by 28 in different ways. A, come and show us one way. Is A correct? How can we check it? (with multiplication)

Who can show us another way? T could show any not suggested by Ps and ask Ps if it is correct. e.g.

BB: \[ 644 \div 28 = 161 \div 7 = 140 \div 7 + 21 \div 7 = 20 + 3 = 23 \]

or

\[ \begin{array}{c}
2 \ 8 \\
6 \\
4 \\
\hline
2 \\
8 \\
0 \\
1 \\
\end{array} \quad \begin{array}{c}
12 \ 3 \\
1 \\
6 \\
4 \\
\hline
2 \\
8 \\
6 \\
4 \\
\end{array} \quad \begin{array}{c}
2 \ 3 \\
6 \\
4 \\
\hline
2 \\
8 \\
6 \\
4 \\
\end{array} \]

Check:

\[ 23 \times 28 = 644 \]

Whole class activity

(If possible, use a squared board or OHT; if not possible, stress the importance of lining up the digits in the correct place-value columns.)

At a good pace

Discussion, reasoning with place-value detail, checking, agreement, praising

T asks Ps which method they like best and why.
Activity 6 (Continued)

b) Let’s divide 8253 by 8 using short division. Ps come to BB to write the calculation, explain reasoning with place-value detail and check with reverse multiplication and addition. Class points out errors.

BB:

\[
\begin{array}{c@{.}c@{.}c@{.}c}
8 & 8 & 2 & 2 \\
\hline
1 & 0 & 3 & 1 \\
2 & 1 & 5 \\
\end{array}
\]

Check: 1031 \times 8 + 5 = 8248 + 5 = 8253

T points to certain components and asks Ps to name them. (divisor, dividend, quotient, remainder)

30 min

Notes

At a good pace
Reasoning, checking, agreement praising

In unison

Lesson Plan 12

PbY6a, page 12


Set a time limit of 2 minutes. Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

\[
\begin{array}{c@{.}c@{.}c@{.}c}
5 & 7 & 9 & 3 \\
\hline
2 & 4 & 3 \\
\end{array}
\]  
\[
\begin{array}{c@{.}c@{.}c@{.}c}
6 & 3 & 9 & 4 \\
\hline
3 & 4 \\
\end{array}
\]  
\[
\begin{array}{c@{.}c@{.}c@{.}c}
9 & 8 & 1 & 0 \\
\hline
1 & 0 & 6 \\
\end{array}
\]  

Check:

657 \times 6 + 4 = 3946

1587 \times 5 + 3 = 7938

900 \times 9 + 6 = 8106

34 min

Notes

Individual work, monitored, (helped)
Written on BB or use enlarged copy master or OHP
Differentiation by short time limit
Reasoning, checking, agreement, self-correction praising
Feedback for T

PbY6a, page 12

Q.5 Read: Calculate the quotient and remainder. Check the results in your exercise book.

Set a time limit of 4 minutes. (Ps finished early could be asked to use another method of division in Ex. Bks.)

Review with whole class. Ps could show quotients and remainders on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

Solution:

\[
\begin{array}{c@{.}c@{.}c@{.}c}
2 & 7 & 3 & 8 \\
\hline
2 & 3 & 8 \\
\end{array}
\]  
\[
\begin{array}{c@{.}c@{.}c@{.}c}
6 & 3 & 9 & 4 \\
\hline
3 & 4 \\
\end{array}
\]  
\[
\begin{array}{c@{.}c@{.}c@{.}c}
9 & 8 & 1 & 0 \\
\hline
1 & 0 & 6 \\
\end{array}
\]  

Check:

295 \times 25 + 7 = 7382

334 \times 29 + 10 = 9696

41 \times 75 + 16 = 3091

40 min

Notes

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit and extra task
Reasoning with place-value detail, checking, agreement, self-correction, praising

Who had time to write a different type of division? Come and show us. Deal with all cases (e.g. subtracting known multiples, short division, horizontal division)

If disagreement, check results on a calculator.
## Activity 9

**PbY6a, page 12**

**Q.6** Read: *Fill in the digits which are missing from the dividend, then calculate the remainder.*

T allows Ps 3 minutes to try it. Review with whole class. Ps who have answers show remainders on scrap paper or slates on command. Ps with correct answers come to BB to explain their reasoning. Who agrees? Who did it another way? Mistakes discussed and corrected.

**Solution:**

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>b)</td>
<td>c)</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
21 & \quad 76 \div 35 = 76, \\
\quad & \quad \text{remainder 16} \\
\end{align*}
\]

\[
\begin{align*}
8 & \quad 16 \div 16 = 501, \\
\quad & \quad \text{(no remainder)} \\
\end{align*}
\]

\[
\begin{align*}
47 & \quad 01 \div 62 = 75, \\
\quad & \quad \text{remainder 51} \\
\end{align*}
\]

Reasoning: e.g.

\[
\begin{align*}
a) & \quad 76 \times 35 = 2280 + 380 = 2660; \quad 2660 + 16 = 2676 \\
b) & \quad 501 \times 16 = 8000 + 16 = 8016 \\
c) & \quad 75 \times 62 = 4500 + 150 = 4650; \quad 4650 + 51 = 4701 \\
\end{align*}
\]

**Notes**

Individual trial first, monitored (helped)
(or whole class activity if time is short)
Written on BB or SB or OHT
(If Ps are struggling, T gives hint about using reverse multiplication)
Responses shown in unison.
Reasoning, agreement, (self-correction), praising

**45 min**
Lesson Plan

13

Notes

Individual work, monitored, (helped)
(or whole class activity)
BB: 13, 188, 363, 1013
Calculators allowed
Reasoning, agreement, self-correction, praising

Week 3

Activity

1

Factorising

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- 13 is a prime number
- 188 = 2 × 2 × 47 = 2² × 47
- 363 = 3 × 11 × 11 = 3 × 11²
- 1013 is a prime number

(as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37 × 37 > 1013)

2

Long multiplication

Let's do this multiplication in different orders. T suggests the order each time. Ps come to BB to do the calculation or dictate what T should write, explaining reasoning with place-value detail, while rest of Ps write the calculation in Ex. Bks.

BB:

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(by 9H)

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(by 3U)

5 9 9 8 4 1

7 min

extra

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(by 9H)

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(by 3U)

5 9 9 8 4 1

10 min

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>9</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

(by 9H)

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

(by 3U)

5 9 9 8 4 1

etc.

<table>
<thead>
<tr>
<th>9</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
</table>

(by 9H)

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
</table>

(by 6H)

<table>
<thead>
<tr>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

(by 5T)

<table>
<thead>
<tr>
<th>6</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
</table>

(by 7U)

5 9 9 8 4 1

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
</table>

(by 9H)

<table>
<thead>
<tr>
<th>5</th>
<th>6</th>
<th>5</th>
</tr>
</thead>
</table>

(by 5T)

<table>
<thead>
<tr>
<th>6</th>
<th>3</th>
<th>9</th>
</tr>
</thead>
</table>

(by 7U)

5 9 9 8 4 1

Are any other orders possible? (Yes) How many different orders are there? T asks several Ps what they think and why. (Agree that 12 orders are possible, as for each of the two possible numbers as the multiplier, there are 6 different orders of multiplying by the 3 digits:

BB: 1, 2, 3; 1, 3, 2; 2, 1, 3; 2, 3, 1; 3, 1, 2; 3, 2, 1

Agree that any of the 12 ways will give the correct product (as long as the digits are written in the correct place-value column).

Discussion

Involve several Ps.

Extra praise for Ps who give the correct number of different ways and the correct reasoning to support it.
**Lesson Plan 13**

### Activity 3

*PbY6a, page 13*

**Q.1** Read: *Estimate the result in your head first, then do the multiplication in your exercise book.*

Set a time limit. Ps finished early can do calculations on BB (but out of sight of the other Ps).

Review with whole class. Ps come to BB or dictate to T (or go through their solutions on the BB), writing an estimate first.

Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:** e.g. (Accept any valid method)

- **a)**
  
  \[
  \begin{array}{c}
  11700 \\
  \times 7 \end{array} \\
  \begin{array}{c}
  11900 \\
  + 51000 \\
  \hline
  62900 \\
  \end{array}
  \]

- **b)**
  
  \[
  \begin{array}{c}
  2405 \\
  \times 37 \end{array} \\
  \begin{array}{c}
  1880 \\
  + 7215 \\
  \hline
  8895 \\
  \end{array}
  \]

- **c)**
  
  \[
  \begin{array}{c}
  177 \\
  \times 444 \end{array} \\
  \begin{array}{c}
  73 \\
  + 2777 \\
  \hline
  3449 \\
  \end{array}
  \]

Time: **20 min**

### Activity 4

*PbY6a, page 13*

**Q.2** Read: *Estimate the result in your head first, then do the division in your exercise book.*

Set a time limit. (Ps finished early can do calculations on BB.)

Review with whole class. Ps come to BB or dictate to T (or go through their solutions on the BB), writing an estimate first.

Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:** e.g. (Accept any valid method)

- **a)**
  
  \[
  \begin{array}{c}
  818 \\
  \div 5 \end{array} \\
  \begin{array}{c}
  163 \\
  \end{array}
  \]

- **b)**
  
  \[
  \begin{array}{c}
  7129 \\
  \div 587 \\
  \end{array} \\
  \begin{array}{c}
  1239 \\
  \end{array}
  \]

- **c)**
  
  \[
  \begin{array}{c}
  5428 \\
  \div 390 \\
  \end{array} \\
  \begin{array}{c}
  142 \\
  \end{array}
  \]

Time: **25 min**

**Notes**

Individual work, monitored, helped

Written on BB or use grids on copy master in LP13/2.

Differentiation by time limit

Reasoning, checking against estimate, agreement, self-correction, praising

If disagreement, check on a calculator.

Estimates: e.g.

- **a)**
  
  \[
  1700 \times 37 = 2000 \times 40 = 80000 \\
  \]

- **b)**
  
  \[
  2405 \times 370 = 2000 \times 400 = 800000 \\
  \]

- **c)**
  
  \[
  777 \times 444 = 800 \times 400 = 320000, etc. \\
  \]

Feedback for T
<table>
<thead>
<tr>
<th>Activity</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY6a, page 13</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Q.3** Read: *Solve these problems in your exercise book.*

Deal with one at a time. Ps read problem themselves, write a plan, estimate the result where appropriate, do the calculation, check it and write the answer in a sentence.

Review with whole class. Ps show answer on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solutions:**

a) A group of 6 people met a group of 11 people. Each member of the first group shook hands with each member of the second group. How many handshakes were there?

**Plan:** $6 \times 11 = 66$

**Answer:** There were 66 handshakes.

b) Lee measured his heartbeat as 72 beats in a minute. How many times did his heart beat in 21 minutes?

**Plan:** $72 \times 21$  

**C:** 7 2  

**E:** $70 \times 20 = 1400$  

**Answer:**  

Lee’s heart beat 1512 times in 21 minutes.

c) A butcher bought 57 kg of meat for £1026. How much did he pay per kg?

**Plan:** £1026 ÷ 57  

**C:** 5 7  

**E:** £1200 ÷ 60 = £20  

**Answer:** He paid £18 per kg.

d) A spare part for a car costs £63. How many such parts can the garage buy for £2696?

**Plan:** £26 966 ÷ £63  

**C:** 6 3 2 6 9 6  

**E:** £30 00 ÷ 60 = 50 (times)  

**Answer:**  

The garage can buy 42 parts (and there will be £50 left over).

---

**Notes**

Individual work, monitored but class kept together on the questions  
Set a time limit for each question  
Responses shown in unison  
Reasoning, agreement, self-correction, praising  
T chooses a P to say the answer in a sentence.  
Feedback for T
Activity 6

**PbY6a, page 13. Q.4**

Deal with one part at a time. T (or a P) describes the secret number, Ps do calculation mentally or in Ex. Bks. and show the number on slates or scrap paper on command. Ps with different answers explain solution on BB. Class decides who is correct. Correct answers written in Pbs.

**Solutions:**

What is the secret number if:

a) the product of the secret number and 40 is 2600? (65)

\[ x \times 40 = 2600, \text{ so } x = 2600 \div 40 = 260 \div 4 = 65 \]

b) it is the product of 60 and 2400? (144 000)

\[ x = 60 \times 2400 = 6 \times 24 000 = 144 000 \]

c) the quotient when the secret number is divided by 50 is 800? (40 000)

\[ x \div 50 = 800, \text{ so } x = 800 \times 50 = 40 000 \]

d) it is the quotient of 600 divided by 20? (30)

\[ x = 600 \div 20 = 60 \div 2 = 30 \]

e) the quotient of 7500 divided by the secret number is 50? (150)

\[ 7500 \div x = 50, \text{ so } x = 7500 \div 50 = 750 \div 5 = 150 \]

Notes

Whole class activity but individual calculation
In good humour!
T repeats slowly to give Ps time to think and calculate.
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Feedback for T

Individual work, monitored but class kept together on the questions
Questions written on BB or use enlarged copy master or OHP
T repeats questions slowly to give Ps time to think.
Responses shown in unison.
In good humour!
Discussion, reasoning, agreement, self-correction, praising
Ps show:

| £500 |
| ? |
| e.g. 65 | ? |
### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **14** = \(2 \times 7\)  
  Factors: 1, 2, 7, 14 (nice number)

- **189** = \(3 \times 3 \times 3 \times 7 = 3^3 \times 7\)
  Factors: 1, 3, 7, 9, 21, 27, 63, 189

- **364** = \(2 \times 2 \times 7 \times 13 = 2^2 \times 7 \times 11\)
  Factors: 1, 2, 4, 7, 13, 26, 28, 52, 91, 182, 364

- **1014** = \(2 \times 3 \times 13 \times 13 = 2 \times 3 \times 13^2\)
  Factors: 1, 2, 3, 6, 13, 26, 39, 78, 169, 338, 507, 1014

### Lesson Plan 14

#### Notes

Individual work, monitored, (helped)  
(or whole class activity)  
BB: 14, 189, 364, 1014

Calculators allowed  
Reasoning, agreement, self-correction, praising

BB:  
\[
\begin{array}{c|c|c}
364 & 2 & 1014 \\
182 & 2 & 507 \\
91 & 7 & 169 \\
13 & 13 & 13 \\
1 & & 1 \\
\end{array}
\]

8 min  

Listing of factors for 1014 can be done with the whole class. Ps join up with the whole class.

### Activity 2

**Problems**

If I read out a problem, what are the first steps you must take to solve it?  
(Listen carefully, picture the story in our heads and note down the important data in our Ex. Bks.)

What should you do after that?  (Write a plan, estimate, calculate, check calculation, check that the result makes sense in the context and write the answer as a sentence.) T reminds Ps of any steps not mentioned.

Deal with one problem at a time. T reads out problem. Ps solve it in Ex. Bks. and show results on scrap paper or slates on command.  
P answering correctly explains reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.  
T chooses a P to say the answer in a sentence.

#### a) 75 pupils in Year 5 and 69 pupils in Year 6 went on a school trip.  
How many pupils went on the trip altogether?

- **Plan:**  
  75 + 69 = 74 + 70 = 144 (pupils)

- **Answer:** 144 pupils went on the school trip.

#### b) If each pupil had to pay £23, what did they pay altogether?

- **Plan:**
  \[23 \times 144\]  
  or \[23 \times 75 + 23 \times 69 = £1725 + £1587 = £3312\]

- **Answer:** The pupils paid £3312 altogether.

#### c) A group of pupils from another school went to a concert. There were 73 Year 5 pupils and 22 members of the school choir. Some Year 5 pupils are in the school choir. How many pupils went to the concert?

As we do not know if the Year 5 pupils who are in the choir are included in the 22 choir members who went to the concert, we can only say that it is certain that:

- not more than 73 + 22 = 95 pupils went to the concert; and
- not less than 73 pupils went to the concert (if all 22 are Y5 Ps).

- **Answer:** At least 73, and not more than 95, pupils went to the concert.

Whole class discussion on how to solve word problems  
Involves several Ps.  
Agreement, praising

Whole class activity but individual calculation  
T repeats problem while walking around the class to give Ps time to think and calculate.  
Responses shown in unison.  
Reasoning, agreement, self-correction, praising

\[
\begin{array}{c|c|c|c|c}
1 & 4 & 4 & 4 & 4 \\
\times & 2 & 3 & & \\
& 4 & 3 & 2 & 2 \\
+ & 2 & 8 & 8 & 0 \\
& 3 & 3 & 1 & 2 \\
\end{array}
\]

T asks Ps with different responses to explain their reasoning.  
Extra praise for Ps who wrote ‘?’ or an inequality on slates.

Extra credit if the 22 are not Y5 Ps

BB: 73 \(\leq n \leq 95\)  
(where \(n\) is a natural number)
Q.1 Read: Write a plan, estimate, calculate and check in your exercise book. Write the result here.

Deal with one at a time. T chooses a P to read out the problem. Ps solve it in Ex. Bks under a time limit.

Review at BB with whole class. Ps could show result on scrap paper or slates on command. Ps with different responses explain reasoning on BB. Class points out errors and agrees which is correct. Mistakes discussed and corrected.

T chooses a P to say the answer in a sentence.

Solutions: e.g.

a) Ian wants to buy a boat. He has saved £1347. If the price of the boat is £2580, how much money does Ian still need to save?

Plan: £2580 – £1347

\[
\begin{array}{c|c|c|c}
\text{E:} & 2 & 5 & 8 \\
\text{C:} & 1 & 3 & 4 \\
\end{array}
\]

Answer: Ian still needs to save £1233.

b) A greengrocer sold 75 kg of apples on Monday, 45 kg of apples on Tuesday and 124 kg of apples on Wednesday.

i) How many more kg of apples did he sell on Wednesday than on Monday?

Plan: 124 kg – 75 kg = 49 kg

Answer: He sold 49 kg more apples on Wednesday.

ii) How much money did he get from selling apples on these three days if he sold the apples at £1.50 p per kg?

BB: £1.50 p = 150 p

Plan: \[(75 + 45 + 124) \times 150 = 244 \times 150 (\text{p})\]

\[
\begin{array}{c|c|c|c|c|c}
\text{C:} & 2 & 4 & 4 & \times & 1 & 5 & 0 \\
\text{E:} & 1 & 2 & 2 & 0 & 0 \\
\text{C:} & 2 & 4 & 4 & \times & 1 & 5 & 0 \\
\text{E:} & 3 & 6 & 6 & 0 & 0 \\
\end{array}
\]

Answer: He got £366 from selling the apples.

c) A firm ordered 750 tonnes of oil. The oil was delivered in a container truck. The truck could carry only 18 tonnes of oil, so it had to make several deliveries.

i) How many deliveries did the truck have to make?

Plan: 750 t ÷ 18 t = 41 (times), r 12 (tonnes)

Check: \[41 \times 18 + 12 = 410 + 328 + 12 = 750 \checkmark\]

But the truck would have to make a delivery for the 12 t.

Answer: The truck had to make 42 deliveries.

ii) How much oil was in the final delivery?

Answer: The final delivery would be 12 tonnes of oil. (assuming that the previous 41 deliveries were all full loads)
### Lesson Plan 14

#### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Continued)</td>
</tr>
<tr>
<td></td>
<td>d) Peter has saved £735, which is 5 times as much as the amount that Paul has saved. How much money has Paul saved? Plan: £735 ÷ 5 = £147  Check: £147 × 5 = £735  Answer: Paul has saved £147.</td>
</tr>
<tr>
<td></td>
<td>e) Ann has £214 in her bank account, which is one fifteenth of the money in Dave's account. How much is in Dave's account? Plan: £214 × 15  E: £200 × 15 = £3000  Answer: Dave has £3210 in his account.</td>
</tr>
<tr>
<td>4</td>
<td></td>
</tr>
<tr>
<td>PbY6a, page 14</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Q.2 Read: Write a plan, estimate, calculate and check your answer in your exercise book. Write the answer in a sentence here. Underline any data not needed in the calculation.</td>
</tr>
<tr>
<td></td>
<td>Set questions a) and b) under a time limit and review with the whole class as usual. Agree on the irrelevant data. Then do questions c) and d) with the whole class. Ps come to BB to write plans and do calculations. T helps or prompts as necessary. Ps write solutions in Ex. Bks. too. Agree on which data are irrelevant. Ps write answers as sentences in Pbs.</td>
</tr>
<tr>
<td></td>
<td>Solutions: e.g. a) Christopher bought a painting for £2600. Then he sold it 3 weeks later for £2800. After another 2 weeks he changed his mind and bought the painting back for £3100. After 1 week he sold the painting again for £3200. Did he make a profit or a loss on the painting and how much was it? Plan: – £2600 + £2800 – £3100 + £3200 = £6000 – £5700 = £300 or (2800 – 2600) + (3200 – 3100) = 200 + 100 = 300  Answer: He made a profit of £300 on the painting.</td>
</tr>
<tr>
<td></td>
<td>b) A box 15 cm deep holds 13 kg of tomatoes and a box 20 cm deep holds 17 kg of tomatoes. What is the total price of all the tomatoes in the 2 boxes if 1 kg of tomatoes costs £2.25? Plan: (13 + 17) × £2.25 = 30 × 225 p = 3 × 2250 p = 6750 p = £67.50  Answer: The total cost of the tomatoes is £67.50.</td>
</tr>
<tr>
<td></td>
<td>Individual work for a) and b), monitored, helped Questions written on BB or use enlarged copy master Ps read questions themselves and solve them, write answers as sentences in Pbs, then show results on scrap paper or slates in unison on command. Ps answering correctly explain solutions at BB. Class agrees/disagrees. Mistakes discussed and corrected. Praising Whole class activity for c) and d) Discussion, reasoning, checking, agreement Accept any correct method of solution. Praising, encouragement only</td>
</tr>
</tbody>
</table>
c) Kate made some jam from 25 kg of apricots and 7 kg of sugar. She lost 8 kg of fruit through boiling and then sieving to remove the stones and skin.

How much did it cost to make 1 kg of jam if 1 kg of apricots cost £1.28, 1 kg of sugar cost £1.10 and other costs (covers and labels) were £1.25?

BB: e.g.

Total mass of the jam: 25 kg + 7 kg – 8 kg = 24 kg
Total cost: £1.28 × 25 + £1.10 × 7 + £1.25 = £32 + £7.70 + £1.25 = £40.95

Cost per kg: £40.95 ÷ 24

= 4095 p ÷ 24 [Elicit that to round to the nearest whole penny, it is necessary to calculate to tenths of a penny.]

= 171 p

= £1.71

Answer: Each kg of jam cost about £1.71 to make.

d) A shopkeeper bought 120 kg of potatoes from one farmer for 76 p per kg and 59 kg from another farmer for 69 p per kg. He then sold all the potatoes at the same price so that he made a profit of 16 p per kg.

At what price did he sell the potatoes?

BB: e.g.

Total cost: 76 p × 120 + 69 p × 59 = 9120 p + 4071p

= 13191 p = £131.91

C:

\[
\begin{array}{c}
120 \\
\times 76 \\
\hline
760 \\
+ 9120 \\
\hline 9880 \\
\end{array}
\]

\[
\begin{array}{c}
59 \\
\times 69 \\
\hline
521 \\
+ 3931 \\
\hline 4452 \\
\end{array}
\]

Profit: 16 p × (120 + 59) = 16 p × 179 = 2864 p = £28.64

Total Income: £131.91 + £28.64 = £160.55

Selling price per kg: £160.55 ÷ 179 = 16055 p ÷ 179 = 90 p (to nearest penny)

[What does this tell us about the 16 p per kg profit? (It must be an approximate profit per kg, otherwise the income would be 90 p × 179 = 6110 p = £161.10, which is more than £160.55.) Elicit that the profit per kg has been rounded up to the nearest penny, as we do not have parts of a penny in real life!]

Answer: He sold the potatoes at 90 p per kg.

T: Questions c) and d) show that in real life, a problem does not always have an exact solution but a close approximation is often enough.

© CIMT, University of Exeter
Factorising 15, 190, 365 and 1015. Revision, activities, consolidation

**Activity**

**PbY6a, page 15**

**Solutions:**

**Q.1**

a) i) $5183 + 6599 = 11782$ iv) $5273 + 6698 = 11971$

b) i) $7405 – 2966 = 4439$ iv) $7505 – 3066 = 4439$

**Q.2 a)**

i) $56554 \div 33527 \times 385 \times 16 = 106199$

b) e.g. $13200 + 20$

**Q.3 a)**

i) $46121 + 3875 + 56203 = 106199$

b) $56560 \div 33527 \times 385 \times 16 = 106199$

e.g. $7885 \times 20$

**Q.4 a)**

i) $289742 – 148867 = 140875$

ii) $888 + 99\times 9 = 888 + 891 = 1779$

**R.i)** $305117 + 4999999 = 5305116$

**S.i)** $7013 + 35 + 9 + 2663 = 9720$

**T.5 a)**

$\text{Answer:}$ Tom had £15.80 left.

b) $54 \times 15 = 810$ (buttons)

**Notes**

$15 = 3 \times 5$

Factors: 1, 3, 5, 15

$190 = 2 \times 5 \times 19$

Factors: 1, 2, 5, 10, 19, 38, 95, 190

$365 = 5 \times 73$

Factors: 1, 5, 73, 365 (nice)

$1015 = 5 \times 7 \times 29$

Factors: 1, 5, 7, 29, 35, 145, 203, 1015

(or set factorising as homework at the end of Lesson 14 and review at the start of Lesson 15)

Tom should be included too, so the costs are for 4 people.
R: Multiples and factors. Odd and even numbers
C: Properties of natural numbers. Simple tests for divisibility
E: Problems

### Lesson Plan

#### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **16** = \(2 \times 2 \times 2 \times 2 = 2^4\)  Factors: 1, 2, 4, 8, 16
- **191** is a prime number  Factors: 1, 191
- **366** = \(2 \times 3 \times 61\)  Factors: 1, 2, 3, 6, 61, 122, 183, 366
- **1016** = \(2 \times 2 \times 2 \times 127 = 2^3 \times 127\)  Factors: 1, 2, 4, 8, 127, 254, 508, 1016

BB: 16, 191, 366, 1016

Calculators allowed

Reasoning, agreement, self-correction, praising

**BB:** e.g.

\[
\begin{array}{c}
366 \\
= 2 \times 122 \\
= 2 \times 2 \times 61 \\
= 2^2 \times 61
\end{array}
\]

T revises the concepts and vocabulary.

We say that 8 is a factor of 1016 because \(8 \times 127 = 1016\), or because 1016 divided by 8 equals 127 and there is no remainder (or the remainder is zero).

We say that 2 and 127 are prime factors of 1016 because they are factors which are prime numbers.

### Notes

Individual work, monitored, (helped)
(or whole class activity)
BB: 16, 191, 366, 1016

Calculators allowed
Reasoning, agreement, self-correction, praising

BB: 366

**e.g.**

\[
\begin{array}{c}
122 \\
= 2 \times 61
\end{array}
\]

### Activity 2

**Divisibility 1**

**a) i)** Let's list the multiples of 6 from the set of integers which are not negative. Ps dictate what T should write.

BB: 0, 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, . . .

T: We can say that 42 is a multiple of 6, or 42 is exactly divisible by 6, or 6 is a factor of 42.

Elicit/write the general formula for multiples of 6.

BB: \(6 \times n\), or \(6n\), where \(n\) is an integer which is not negative

**ii)** Let's list the multiples of 6 from the set of integers. Ps dictate what T should write.

BB: . . ., –36, –30, –24, –18, –12, –6, 0, 6, 12, 18, . . .

Elicit the general formula for such numbers.

BB: \(6 \times n\), or \(6n\), where \(n\) is an integer.

**b)** Let's list the multiples of other natural numbers from the set of integers. T says each number and Ps dictate its multiples then give a general formula for them using \(n\), where \(n\) is an integer, and check that the formula is correct by giving an example.

BB: Multiples of:

<table>
<thead>
<tr>
<th>N</th>
<th>Multiples</th>
<th>General formula: (n \times n = n)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0, 0, 0, . . .</td>
<td>(0 \times n = 0n = 0)</td>
</tr>
<tr>
<td>1</td>
<td>. . ., –3, –2, –1, 0, 1, 2, 3, . . .</td>
<td>(1 \times n = 1n = n)</td>
</tr>
<tr>
<td>2</td>
<td>. . ., –4, –2, 0, 2, 4, 6, 8, . . .</td>
<td>(2 \times n = 2n)</td>
</tr>
<tr>
<td>3</td>
<td>. . ., –9, –6, –3, 0, 3, 6, 9, . . .</td>
<td>(3 \times n = 3n)</td>
</tr>
<tr>
<td>4</td>
<td>. . ., –12, –8, –4, 0, 4, 8, 12, . . .</td>
<td>(4 \times n = 4n)</td>
</tr>
</tbody>
</table>

Whole class activity
At a good pace
Agreement, praising
Feedback for T

Elicit that non-negative integers are the natural numbers and zero.

T shows it if Ps cannot form it.

Examples: e.g. if \(n = 5\)

\[
\begin{array}{c}
0 \times 5 = 0 \\
1 \times 5 = 5 \\
2 \times 5 = 10 \\
3 \times 5 = 15 \\
4 \times 5 = 20
\end{array}
\]

© CIMT, University of Exeter
Activity 2
(Continued)
c) What is the rule for this sequence? Who agrees? Who can think of another way to say the rule?
BB: 5, 11, 17, 23, 29, 35, 41, 47, . . .
e.g. P1: Starting at 5 and increasing by 6, or +6
P2: The non-negative integers which give a remainder of 5 when divided by 6.
T: The general rule can be written like this.
BB: \(6n + 5\), where \(n = 0, 1, 2, 3, \ldots\)
d) This table shows the possible remainders after dividing by a natural number. What is the natural number? (6)
Let's put these numbers in the correct place in the table. Ps come to BB to write numbers in table, explaining reasoning and doing divisions at side of BB when necessary. Class agrees/disagrees.
BB: 5, 27, 300, 19, 43, 200, 64, 1111, 126, 449

<table>
<thead>
<tr>
<th>r = 0</th>
<th>r = 1</th>
<th>r = 2</th>
<th>r = 3</th>
<th>r = 4</th>
<th>r = 5</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>19</td>
<td>200</td>
<td>27</td>
<td>64</td>
<td>5</td>
</tr>
<tr>
<td>126</td>
<td>43</td>
<td></td>
<td></td>
<td></td>
<td>449</td>
</tr>
<tr>
<td>1111</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Who can tell me true statements about the table? e.g.
‘64 is not a multiple of 6’ or ‘6 is not a factor of 64’ or ‘64 is not exactly divisible by 6’

Notes

T gives hint about remainder if Ps cannot think of it.

Table drawn on BB or use enlarged copy master or OHP
At a good pace
Reasoning, agreement, praising

PbY6a, page 16
Q.1 Read: Show in the graphs the remainders obtained when whole numbers which are not negative and not greater than 15 are:
a) divided by 2  b) divided by 5.
Set a time limit of 3 minutes. Ps mark the remainders with dots.
Review with whole class. Ps come to BB to complete the graphs, explaining reasoning. Class agrees/disagrees. Mistakes corrected.

Solution:

Ps say what they notice about each graph. e.g.
a) Dots on the x axis show even numbers, or multiples of 2; dots on the ‘1’ horizontal grid line show odd numbers; a remainder greater than 1 is impossible after dividing by 2, so ‘2’ to ‘5’ on the y axis are not needed.
b) There are no dots on the ‘5’ grid line, as it is impossible to have a remainder greater than 4 after dividing by 5. Dividends with dots on the x axis are multiples of 5.
What are the possible remainders after dividing by 10?

Whole class discussion
Involve several Ps.
Agreement, praising

23 min
**Lesson Plan 16**

**Y6**

**Activity**

4  

*PbY6a, page 16*

Q.2  Read:  *Use the regular pentagon and decagon to help you to complete the table.*

What do the polygons have to do with the table?  Ps come to BB to explain, referring to diagram.  T prompts if necessary.  Elicit that the vertices on the pentagon (decagon) show the 5 (10) possible remainders after dividing by 5 (10).

Set a time limit.  Review with whole class.  Ps come to BB or dictate to T, explaining reasoning.  Class agrees/disagrees.  Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Divisible by 2</th>
<th>Divisible by 5</th>
<th>Divisible by 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>12</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>43</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>79</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>154</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>228</td>
<td>8</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>2430</td>
<td>9</td>
<td>9</td>
<td>9</td>
</tr>
<tr>
<td>2433</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2436</td>
<td>11</td>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>2437</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>2438</td>
<td>13</td>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>2439</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>2440</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Which natural numbers are exactly divisible by:

- 2  (even numbers, i.e. numbers which have units digit 0, 2, 4, 6 or 8)
- 5  (Numbers which have units digit 0 or 5)
- 10?  (Numbers which have units digit 0)

27 min

5  

*PbY6a, page 16*

Q.3  Read:  *Follow the pattern. Fill in the missing numbers and words.*

Do part a) as individual work under a time limit, then review and correct mistakes before doing part b) with the whole class.

Ps could write each missing word on scrap paper or slates and show on command.  T writes agreed words in sentence on BB, while Ps write them in *Pbs*.  Ps say the sentence together.

**Solution:**

a)  i) 7 = 0 \times 10 + 7  
ii) 704 = \frac{70}{10} \times 10 + 4

b)  When a natural number is divided by 10, 2 or 5, the remainder is the same as when its units digit is divided by 10, 2 or 5.

T has extra sentence already prepared on BB or OHT.  Ps come to BB or dictate the missing words.  Class agrees/disagrees.

c)  Natural numbers which are exactly divisible by:

i) 10 have units digit 0
ii) 5 have units digit 0 or 5
iii) 2 have units digit 0 or 2 or 4 or 6 or 8

33 min

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHT

Initial whole class discussion to clarify the task

BB:  *pentagon:* 5 vertices  
*decagon:* 10 vertices

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Feedback for T

Whole class activity

Agree that only the units digit needs to be taken into account when determining whether a number is exactly divisible by 2, 5 or 10.

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHT

Reasoning, agreement, self-correction, praising

Agree that we only need to look at the units digit because any whole 10 is exactly divisible by 2, 5 and 10.

Written on BB or SB or OHT

At a good pace

Agreement, praising.

Ps could write sentence in *Ex. Bks.* too.
### Lesson Plan 16

#### Notes

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Extra praise for Ps who noticed they could use the sentence in Q.3 b) with appropriate amendments.

Agree that we only need to look at the last 2 digits because any whole 100 is exactly divisible by 100, 4 and 25.

If Ps are not very able, T could have sentences written on BB or OHT with appropriate words missing for Ps to fill in (e.g. as underlined).

Praising, encouragement only

---

#### Y6

<table>
<thead>
<tr>
<th>Activity 6</th>
<th>PbY6a, page 16</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q.4</strong></td>
<td>Read: <em>Follow the pattern. Fill in the missing numbers. Write a sentence about what you notice.</em></td>
</tr>
<tr>
<td></td>
<td>Set a time limit. Ps write sentence in Ex. Bks.</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.</td>
</tr>
<tr>
<td></td>
<td>T asks several Ps to read out their sentences. Class decides whether or not it is true. Ps with incorrect or vague sentences write them again correctly.</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td>a) $7 = 0 \times 100 + 7$ b) $2176 = \frac{21}{73} \times 100 + \frac{76}{90}$</td>
<td></td>
</tr>
<tr>
<td>$33 = 0 \times 100 + 33$ \hspace{1cm} $7390 = \frac{73}{119} \times 100 + \frac{8}{50}$</td>
<td></td>
</tr>
<tr>
<td>$200 = 2 \times 100 + 0$ \hspace{1cm} $28408 = \frac{284}{119} \times 100 + \frac{8}{50}$</td>
<td></td>
</tr>
<tr>
<td>$375 = \frac{3}{119} \times 100 + \frac{75}{50}$ \hspace{1cm} $11950 = \frac{119}{119} \times 100 + \frac{50}{50}$</td>
<td></td>
</tr>
<tr>
<td>$524 = \frac{5}{119} \times 100 + \frac{24}{62}$ \hspace{1cm} $678462 = \frac{6784}{6784} \times 100 + \frac{62}{62}$</td>
<td></td>
</tr>
<tr>
<td>Divisible by 100, 4 and 25</td>
<td>Divisible by 100, 4 and 25</td>
</tr>
<tr>
<td>Sentences: e.g.</td>
<td></td>
</tr>
<tr>
<td>When a natural number is divided by 100, 4 or 25, the remainder is the same as when its last two digits are divided by 100, 4 or 25.</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td></td>
</tr>
<tr>
<td>Natural numbers which are exactly divisible by 100 end in 00.</td>
<td></td>
</tr>
<tr>
<td>or</td>
<td></td>
</tr>
<tr>
<td>Natural numbers which are exactly divisible by 25 end in 00 or 25 or 50 or 75.</td>
<td></td>
</tr>
</tbody>
</table>

---

#### 7 Divisibility 2

| a) Study these numbers. | |
| BB: 53, 504, 6402, 72331, 517, 966, 2040 | |
| Which of them have a remainder of: | |
| i) 1 when divided by 2 (53, 72331, 517, i.e. odd numbers) | |
| ii) 0 when divided by 2 (504, 6402, 966, 2040, i.e even nos.) | |
| iii) 0 when divided by 5 (2040) | |
| iv) 1 when divided by 5 (72331, 966) | |
| v) 2 when divided by 5? (6402, 517) | |
| After each description, Ps dictate the appropriate numbers. Class agrees/disagrees. | |
| b) T has a new list of numbers already prepared. | |
| BB: 0, 6, 8, 5, 25, 40, 50, 72, 78, 100, 102, 125, 722, 755, 2600, 14550, 64316, 80000 | |
| Which of them are exactly divisible by: | |
| i) 100 (100, 2600, 80000, i.e. numbers ending in 00) | |
| ii) 4 (8, 40, 72, 100, 2600, 64316, 80000) | |
| iii) 25? (25, 50, 100, 125, 2600, 14550, 80000) | |

---

© CIMT, University of Exeter
### Activity

#### 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **17** is a prime number  
  Factors: 1, 17
- **192** = $2^6 \times 3$  
  Factors: 1, 2, 3, 4, 6, 8, 12, 192, 96, 48, 32, 24, 16
- **367** is a prime number  
  Factors: 1, 367
- **1017** = $3^2 \times 113$  
  Factors: 1, 3, 9, 113, 339, 1017

#### 2

**Remainders 1**

a) If we divide each of these numbers by 5, what is the remainder? Ps come to BB to explain reasoning (or do a division) and to write the remainder in a circle below the numbers. Class points out errors.

BB: 315, 608, 512, 828, 41, 63, 15, 9, 3, 4

b) Let's do these operations. Ps come to BB or dictate what T should write. Class points out errors. If we divide each operation by 5, what is the remainder? Ps come to BB to divide each term and the result and to write the remainders below. What do you notice?

BB:

- **i) 315 + 608 = (923)**  
  $0 + 3 \rightarrow 3$  
  $3 \times 1 \rightarrow 3$
- **ii) 608 × 41 = (24 928)**  
  $0 \times 3 \rightarrow 0$
- **iii) 828 − 315 = (513)**  
  $3 - 0 \rightarrow 3$  
  $0 \times 3 \rightarrow 0$
- **iv) 315 × 63 = (19 845)**  
  $3 \times 9 = (4608)$  
  $2 \times 4 \rightarrow 3$
- **v) 512 \times 9 = (4608)**  
  $2 \times 4 \rightarrow 3$
- **vi) 828 \times 3 − 512 \times 4 = (2484 − 2048 = 436)**  
  $3 \times 3 - 2 \times 4 \rightarrow 4 - 3 \rightarrow 1$

Ps try to put what they noticed into sentences, with T's help. e.g.

- The result's remainder can be calculated from the remainders of the components.
- The sum's remainder is equal to the **sum** of the remainders of the addition terms.
- The difference's remainder is equal to the **difference** between the remainders of the reductand and subtrahend.
- The product's remainder is equal to the **remainder** from the product of the factors' remainders.
Remainders 2

a) If we divide each of these numbers by 3, what is the remainder? Ps come to BB to do calculation (or explain with reasoning, e.g. 414 is exactly divisible by 3 because 414 = 300 + 90 + 24 and each of these numbers is exactly divisible by 3), and to write the remainder in a circle below the number. Class points out errors. e.g.

BB: 10, 11, 12, 62, 63, 64, 414, 415, 416, 2843, 2844, 2845

What do you notice? (e.g. Every 3rd natural number is exactly divisible by 3, the next consecutive number has remainder 1 and the number after that has remainder 2.)

b) If no P has noticed, T shows a list of new numbers and asks what is the remainder if the sum of the digits making up each number is divided by 3.

BB: 1, 2, 3, 8, 9, 10, 16, 17, 18, 29, 30, 31

What do you notice? (When the sum of the digits is divided by 3, it has the same remainder as when the number itself is divided by 3.)

c) Let’s check if it is also true for larger numbers. Tell me any 4-digit number. (e.g. 1489) Who can write an addition to make 1489 using numbers that you know are multiples of 3? P comes to BB or dictates what T should write. Agree on the remainder. Another P does a short division at side of BB. A 3rd P adds up the 4 digits and divides the sum by 3. Class checks that all the remainders are equal. Repeat for other numbers suggested by Ps. BB: e.g. 1489 = 1200 + 270 + 18 + and 1 + 4 + 8 + 9 = 22

65 048 = 60 000 + 3000 + 1800 + 240 + 6 + 6 + 5 + 4 + 8 = 23

41 763 = 39 000 + 2700 + 63 ; 4 + 1 + 7 + 6 + 3 = 21

d) Without doing a division, tell me what the remainder will be if each of these numbers is divided by 3. T points to each number in turn and class shouts out the remainder.

BB: 1, 10, 100, 1000, 10 000, 100 000,

What is the remainder when they are divided by 9? (1, because 10 = 9 + 1, 100 = 99 + 1, 1000 = 999 + 1, etc.)

e) Let’s fill in the table to show the remainders when each of these numbers is divided by 9 and by 3. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees.

BB:

<table>
<thead>
<tr>
<th>Number</th>
<th>10</th>
<th>20</th>
<th>70</th>
<th>100</th>
<th>300</th>
<th>800</th>
<th>1000</th>
<th>4000</th>
<th>6000</th>
<th>9000</th>
<th>10 000</th>
<th>100 000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remainder after dividing by 3</td>
<td>(0)</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>5</td>
</tr>
<tr>
<td>Remainder after dividing by 9</td>
<td>(0)</td>
<td>1</td>
<td>2</td>
<td>7</td>
<td>1</td>
<td>3</td>
<td>8</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>
**Lesson Plan 17**

**Notes**

- Individual work, monitored, (helped)
- Written on BB or use enlarged copy master or OHP
- Reasoning, agreement, self-correction, praising
- If doing part b) separately, Ps could say what they noticed in part a) in their own words.

**Activity**

**PbY6a, page 17**

**Q.1** Read:

a) Follow the example and continue the pattern.

b) Complete the sentence about each part.

Deal with one part at a time or set a time limit. Quick Ps could continue the pattern in Ex. Bks.

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

In b), Ps could show missing numbers on slates or scrap paper on command. Class decides on the correct responses then says the completed sentence in unison.

**Solution**

a) i) \(1 = 0 + 1\)

\(10 = 9 + 1\)

\(100 = 99 + 1\)

\(1000 = 999 + 1\)

\(10000 = 9999 + 1\)

\(\ldots\)

Divisible by 9 and 3

\(\ldots\)

Divisible by 9 and 3

\(\ldots\)

Divisible by 9 and 3

ii) \(2 = 0 \times 2 + 2\)

\(20 = 9 \times 2 + 2\)

\(200 = 99 \times 2 + 2\)

\(2000 = 999 \times 2 + 2\)

\(20000 = 9999 \times 2 + 2\)

\(\ldots\)

Divisible by 9 and 3

\(\ldots\)

Divisible by 9 and 3

\(\ldots\)

Divisible by 9 and 3

iii) \(7 = 0 \times 7 + 7\)

\(70 = 9 \times 7 + 7\)

\(700 = 99 \times 7 + 7\)

\(7000 = 999 \times 7 + 7\)

\(70000 = 9999 \times 7 + 7\)

\(\ldots\)

Divisible by 9 and 3

\(\ldots\)

Divisible by 9 and 3

\(\ldots\)

Divisible by 9 and 3

**b) i)** When 1000 is divided by 9 or by 3, the remainder is the same as when 1 is divided by 9 or by 3.

**ii)** When 200 is divided by 2 or by 3, the remainder is the same as when 2 is divided by 2 or by 3.

**iii)** When 70 000 is divided by 9 or by 3, the remainder is the same as when 7 is divided by 9 or by 3.

27 min

**Q.2** Read:

Complete the tables to show the remainders when the numbers are divided by 9 and by 3.

Set a time limit of 3 minutes or deal with one table at a time.

Review with whole class. Ps come to BB to write numbers and explain reasoning. Accept divisions but extra praise if Ps can reason using the sum of the digits. e.g.

- \(8346 = 800 + 300 + 40 + 6\) gives remainder 0 and 21 divided by 3 gives remainder 0.

- \(74 \, 538 = 7 \times 9 + 4 \times 9 + 5 \times 9 + 3 \times 9 + 8\) gives 27 remainder 0.

- 100000

- 74 538 is a multiple of 3 and 9.

**Solution:**

**a)**

<table>
<thead>
<tr>
<th>Number</th>
<th>8000</th>
<th>3000</th>
<th>40</th>
<th>6</th>
<th>8346</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remainder after dividing by:</td>
<td>(9)</td>
<td>8</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>(3)</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

**b)**

<table>
<thead>
<tr>
<th>Number</th>
<th>70 000</th>
<th>4000</th>
<th>500</th>
<th>30</th>
<th>8</th>
<th>74 538</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remainder after dividing by:</td>
<td>(9)</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>(3)</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

32 min

**Note to Ts**

8346 = 800 + 300 + 40 + 6 and using similar arguments as in Q.1 above, the remainder for 800 is 8, for 300 is 3, etc. so when we add the digits we are really adding the remainders from each place value.]
**Lesson Plan 17**

**Notes**

Whole class activity

Sentences written on BB or use enlarged copy master or OHP

At a good pace

Agreement, praising

Elicit the remainders too.

a) $7000 \div 9 \rightarrow r\ 7$

$7000 \div 3 \rightarrow r\ 1$

b) $400 \div 9 \rightarrow r\ 4$

$400 \div 3 \rightarrow r\ 1$

c) $50 \div 9 \rightarrow r\ 5$

$50 \div 3 \rightarrow r\ 2$

T could also point out that the digits of 24 can also be added:

e) $24 \rightarrow 2 + 4 = 6$

f) $24 \rightarrow 2 + 4 = 6 \rightarrow 0$

---

Individual work, monitored, helped

Numbers written on BB or SB or OHT

Differentiation by time limit

Reasoning, agreement, self-correction praising

Accept and praise all valid points made in Ps’ sentences.

Extra praise for creative thinking!

Reasoning: e.g.

<table>
<thead>
<tr>
<th>555 555 555</th>
<th>$\rightarrow$ 45 $\rightarrow$ 9</th>
</tr>
</thead>
<tbody>
<tr>
<td>56 418</td>
<td>$\rightarrow$ 24 $\rightarrow$ 6</td>
</tr>
</tbody>
</table>

so divisible by 3 and by 9;

so divisible by 3 but not by 9; etc.

---

**Activity**

6

Completing sentences

Let’s fill in the words and numbers missing from these sentences.

Ps come to BB or dictate what T should write. Who agrees? Who thinks something else? Class agrees on which items are missing.

BB:

a) When 7000 is divided by 9 or by 3, the remainder is the same as when 7 is divided by 9 or by 3.

b) When 400 is divided by 9 or by 3, the remainder is the same as when 4 is divided by 9 or by 3.

c) When 50 is divided by 9 or by 3, the remainder is the same as when 5 is divided by 9 or by 3.

d) When 8 is divided by 9, the remainder is itself, but when 8 is divided by 3 the remainder is 2.

e) When 7458 is divided by 9, the remainder is the same as when $7 + 4 + 5 + 8 = 24$ is divided by 9, so the remainder is 6.

f) When 7458 is divided by 3, the remainder is the same as when $7 + 4 + 5 + 8 = 24$ is divided by 3, so the remainder is 0.

---

7

PbY6a, page 17

Q.3 Read:

*Circle in red the numbers which are exactly divisible by 9.*

*Underline in green the numbers which are exactly divisible by 3.*

*Write a sentence about what you notice in your exercise book.*

Set a time limit of 3 minutes. Review with the whole class.

Ps come to BB or dictate to T, explaining reasoning (by adding the digits and dividing their sum). Class agrees/disagrees. Mistakes discussed and corrected.

T chooses a few Ps to read out their sentences. Who wrote much the same? Who wrote something else? etc.

*Solution:*

<table>
<thead>
<tr>
<th>534</th>
<th>436</th>
<th>354</th>
<th>7155</th>
<th>435</th>
<th>643</th>
<th>5175</th>
<th>453</th>
<th>111</th>
<th>20</th>
<th>202</th>
</tr>
</thead>
<tbody>
<tr>
<td>44044</td>
<td>555 555 555</td>
<td>56 418</td>
<td>50 418</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Elicit that:

Numbers marked in **green** and **red** are multiples of both 3 and 9.

Numbers marked in **green** or **red** are multiples of 3.

---

37 min

42 min
Lesson Plan 17

PbY6a, page 17, Q.4

Read: Write the whole numbers from 10 to 30 in the Venn diagrams.
P5s come to BB one after another to write 10, 11, 12, . . . , 30 in the correct place in the diagrams and explain reasoning. Class points out errors. Ps write numbers in diagrams in Pbs at the same time.

Solution:

a) \[10 \leq n \leq 30\]
   \[\begin{array}{c}
   11 & 13 & 17 & 19 \\
   10 & 14 & 16 \\
   20 & 22 & 26 & 28 \\
   \text{Multiple of } 2
   \end{array}\]

b) \[10 \leq n \leq 30\]
   \[\begin{array}{c}
   10 & 11 & 13 \\
   14 \\
   12 & 15 & 18 & 21 & 24 & 30 \\
   \text{Divisible by } 3 \\
   \text{Divisible by } 9 \\
   23 & 25 & 27 & 29
   \end{array}\]

T asks Ps to come to BB to choose an area in the Venn diagram and say a true statement about it. Class agrees or disagrees with the statements.
e.g.
There are no whole numbers which are divisible by 9 but not by 3.
Whole numbers divisible by 2 and by 3 are also divisible by 6. etc.

45 min

Notes

Whole class activity
Drawn on BB or use enlarged copy master or OHP
At a fast pace
Reasoning, agreement, praising
Feedback for T

T repeats statement in a clearer way if necessary.
Praising, encouragement only
Extra praise for creativity!

© CIMT, University of Exeter
### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 18 = 2 \times 3 \times 3 = 2 \times 3^2 \)
  - Factors: 1, 2, 3, 6, 9, 18
- \( 193 \) is a prime number
  - Factors: 1, 193
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13 and \( 17 \times 17 \neq 193 \))
- \( 368 = 2 \times 2 \times 2 \times 2 \times 23 = 2^4 \times 23 \)
  - Factors: 1, 2, 4, 8, 16, 23, 46, 92, 184, 368
- \( 1018 = 2 \times 509 \) (nice)
  - Factors: 1, 2, 509, 1018
  - (as 509 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17 or 19, and \( 23 \times 23 > 509 \))

7 min

### Problem 1

Listen carefully, note the data in your Ex. Bks and think about how you would solve this problem. Discuss it with your neighbour if you wish.

We have 60 green marbles and 72 pink marbles. We want to put them into bags so that each bag has the same number of green marbles and the same number of pink marbles.

What is the greatest number of bags that we can make and how many green and pink marbles would be in each bag?

Discuss what to do first and how to continue. T directs Ps thinking in a systematic way if necessary. Ps come to BB or dictate what T should write. Ps could write solution in Ex. Bks too. e.g.

- Into how many equal parts could the green marbles be divided?
  - BB: Green: \( 1, 2, 3, 4, 5, 6, 10, \frac{12}{12}, 15, 20, 30, 60 \)
    - (i.e. the factors of 60)
  - BB: Pink: \( 1, 2, 3, 4, 6, 8, 9, \frac{12}{12}, 18, 24, 36, 72 \)
    - (i.e. the factors of 72)
  - Which factors are common to both numbers? T underlines the numbers dictated by Ps.
  - Which is the greatest common factor? (12) T circles it.
  - Agree that each of the two kinds of marbles could be divided into at most 12 equal parts.

If we made 12 bags, how many marbles of each colour would be in each bag?

- BB: Green: \( 60 \div 12 = 5 \)
- Pink: \( 72 \div 12 = 6 \)

Who could say the answer in a sentence? e.g.

**Answer:** The greatest number of bags that we could make is 12 and each bag would contain 5 green marbles and 6 pink marbles.

13 min

---

© CIMT, University of Exeter
Lesson Plan 18

Notes
Individual work, monitored (helped)
Diagrams drawn on BB or use enlarged copy master or OHP
Discussion, reasoning, agreement, self-correction, praising
(T could have completed Venn diagram already prepared to save time.)

Extra questions: e.g.
• Factors of 20 but not factors of 30. (4, 20)
• Factors of 20 or factors of 30. (1, 2, 3, 4, 5, 6, 10, 15, 20, 30)
• Factors of 20 and of 30. (1, 2, 5, 10)

[To Ts only:
Numbers which have only 1 as their common factor are known as relative primes or are said to be coprime.]

Whole class activity
Discussion, reasoning, agreement, praising
Agree that we cannot list all the multiples as there is no room. Elicit or remind Ps about using an ellipsis (…) to show the multiples which are not written down.
Ps show on scrap paper or slates in unison on command. Praising
Q.2 Deal with one part at a time. Set a time limit of 3 minutes. Review with whole class. Ps could show the smallest common multiple on slates or scrap paper on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

In a) T labels different areas of the diagram (a, b, c, d in diagram below) and Ps describe the numbers in each set.

In b) elicit that the list of numbers is never ending, so Ps can stop when they run out of space and write an ellipsis to represent the numbers not shown. [Also in a) if space runs out]

**Solution:**

a) Write the whole numbers from 0 to 72 in the Venn diagram.

\[
\begin{array}{c}
\text{Multiples of 6} \\
0, 12, 24, 36, 48, 60, 72 \\
\text{Multiples of 8} \\
0, 8, 16, 24, 32, 40, 48, 56, 64 \\
\text{Common multiples of 6 and 8} \\
0, 24, 48, 72 \\
\end{array}
\]

List the common multiples of 6 and 8. (0, 24, 48, 72) (i.e. the multiples of 24 which are not negative)

What is the smallest common multiple of 6 and 8 which is not negative? (0)

T: What is the smallest positive common multiple of 6 and 8? (24)

b) Write some integers which are not negative in each part of the Venn diagram.

\[
\begin{array}{c}
\text{Multiples of 5} \\
0, 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, \ldots \\
\text{Multiples of 8} \\
0, 8, 16, 24, 32, 40, 48, 56, \ldots \\
\end{array}
\]

List the common multiples of 5 and 8 which are not negative. (0, 40, 80, 120, \ldots) (i.e. the non-negative multiples of 40)

What is the smallest positive common multiple of 5 and 8? (40)

© CIMT, University of Exeter
**Problem 2**

Listen carefully, note the data in your *Ex. Bks* and try to solve this problem. Set a time limit of 3 minutes.

*A bus leaves the station every 6th minute and a train leaves the station every 8th minute. At exactly mid-day, a bus and a train leave the station at the same time.*

*At what times after that will a bus and a train leave the station together?*

Ps who have an answer, or know what to do come to BB to explain their reasoning. Who agrees? Who would do it another way? etc. If no P is on the right track, T gives hints. Ps write solution in *Ex. Bks* too. e.g.

**BB:**

*Bus:* 6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, . . . (min)  
(i.e. the positive multiples of 6)

*Train:* 8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, . . . (min)  
(i.e. the positive multiples of 8)

Elicit that they will leave together every 24 minutes after 12 noon. Ps dictate the exact times and T writes them on BB.

**BB:** 12:24, 12:48, 13:12, 13:36, 14:00, 14:24, 14:48, . . .

**Answer:** The bus and train will leave the station every 24 minutes after mid-day.

---

**Lesson Plan 18**

**Notes**

Individual trial first, monitored (helped)

T repeats slowly to give Ps time to think and discuss.

Discussion, reasoning, self-correction, praising

Extra praise for Ps who solve the problem without help.

Ps underline the **common multiples** (i.e. the positive multiples of 24)

or \( 6 = \frac{2}{6} \times 3 \)

or \( 8 = \frac{2}{4} \times \frac{2}{2} \times 2 \)

Smallest common multiple:

\[ 2 \times 2 \times 2 \times 3 = 24 \]

Agree on the correct form of words for the answer.

---

**Problem 2b**

*Which natural number is the smallest common multiple of:*

i) 1 and 8 (8)  
ii) 16 and 24 (48)  
iii) 8 and 15 (120)

Reasoning: e.g.

iv) multiples of 15: 15, 30, 45, . . .  
so smallest common multiple of 15 and 45 is 45.
**Lesson Plan**

**Week 4**

**Activity 1**

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **19** is a prime number  
  Factors: 1, 19
- **194** = \(2 \times 97\) (nice)  
  Factors: 1, 2, 97, 194
  (and 97 is not exactly divisible by 2, 3, 5 or 7 and \(11 \times 11 > 97\))
- **369** = \(3 \times 3 \times 41\) = \(3^2 \times 41\)  
  Factors: 1, 3, 9, 41, 123, 369
- **1019** is a prime number  
  Factors: 1, 1019
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and \(37 \times 37 > 1019\))

**Notes**

Individual work, monitored, helped
(or whole class activity)

BB: 19, 194, 369, 1019

Calculators allowed

Reasoning, agreement, self-correction, praising

Feedback for T

Elicit that only one counter example is needed to prove that a statement is false.

---

**Activity 2**

**PbY6a, page 19**

Q.1 Read: Write T if the statement is true and F if it is false. Write examples or counter examples in your exercise book.

Set a time limit of 5 minutes. Review with whole class.

T chooses a P to read out the statement and Ps show T or F on scrap paper or slates on command. Ps with different responses give examples or counter examples and class decides who is correct. Mistakes corrected in Pbs.

**Solution:**

a) If a natural number is a multiple of 10, it is also a multiple of 5.  
(e.g. 20, 70, 100, and there are no counter examples)

(approved) (T)

b) If a natural number is exactly divisible by 5, it is a multiple of 10.  
(e.g. 15 is exactly divisible by 5, but is not a multiple of 10)

(approved) (F)

c) If a natural number is exactly divisible by 5 and by 2, it is a multiple of 10.  
(e.g. 10, 40, divisible by 5, 2 and 10; no counter examples)

(approved) (T)

d) If a natural number is a multiple of 9, it is also a multiple of 3.  
(e.g. 18 is a multiple of 9 and of 3; no counter examples)

(approved) (T)

e) If a natural number is a multiple of 3, it is also a multiple of 9.  
(e.g. 12 is a multiple of 3, but not of 9)

(approved) (F)

f) If a natural number is exactly divisible by 3 and by 5, it is also a multiple of 15.  
(e.g. 30 is divisible by 3, 5 and 15; no counter examples)

(approved) (T)

g) If a natural number is a multiple of 4 and of 6, it is also a multiple of 24.  
(e.g. 36 is a multiple of 4 and of 6, but not of 24)

(approved) (F)

---

© CIMT, University of Exeter
Lesson Plan 19

Notes

Whole class activity
(or individual trial first if Ps wish)
At a good pace
Ps first reason the type of numbers that are possible.
Agreement, (self-correction), praising

Elicit that all whole hundreds are multiples of 4.

Ellipsis represents the numbers not shown.

Elicit that all whole hundreds are divisible by 25.

Elicit that no numbers are possible, as all multiples of 25 are also multiples of 5.
Activity

3

(Continued)

d) Complete the numbers so that each number is divisible by 5 and by 4.

Elicit that the numbers will also be divisible by \(4 \times 5 = 20\).

<table>
<thead>
<tr>
<th>72</th>
<th>43</th>
<th>0</th>
<th>0</th>
<th>6</th>
<th>5</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>0</td>
<td></td>
<td>None</td>
<td>None</td>
<td>Any digit, but the thousand digit ≠ 0.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>0</td>
<td></td>
<td>possible</td>
<td>possible</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[\text{20 min}\]

Lesson Plan 19

Notes

[4 and 5 are relative primes, i.e. they have only one common factor, 1 so their smallest common multiple is \(4 \times 5 = 20\)].

Elicit that any whole 100 is divisible by 4 and by 5.

Individual work, monitored, helped
Numbers written on BB or SB or OHT
Agreement, praising
If done under a time limit, ask more able Ps to find as many numbers as they can in their Ex. Bks.
Encourage a logical listing.
Agreement, self-correction, praising
Extra praise for Ps who found all the possible numbers without help.

PbY6a, page 19

Q.3 Read: Below each number, write the remainder when it is divided by 6.

Ps dictate the remainders and T writes on BB, Ps in Pbs.

BB: 24 25 26 27 28 29 30
    (0) (1) (2) (3) (4) (5) (0)

Deal with one question at a time or set a time limit. Ps read the questions themselves and write the missing numbers in Pbs.
(Ps can check their answers in their Ex. Bks.)
Review with the whole class. Ps dictate their numbers. Who agrees? Who found different numbers? Are any other numbers possible? Agree on the necessary criteria. Ps dictate other possible numbers and T writes on BB. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:
Select from these 2-digit numbers:

a) two numbers so that their sum is divisible by 6

Choose numbers with remainders which sum to a multiple of 6.

\[\text{e.g. } 24 + 30 = 54 \text{ or } 25 + 29 = 54 \text{ or } 26 + 28 = 54\]

\[\text{(0) + (0) } \rightarrow (0) \text{ or (1) + (5) } \rightarrow (0) \text{ or (2) + (4) } \rightarrow (0)\]

b) two numbers so that their difference is divisible by 6

Choose numbers with remainders which are 0.

\[\text{(as 0 is the only possible difference in remainders which is a multiple of 6)}\]
\[24 - 30 = 6\]
\[\text{(0) - (0) } \rightarrow (0)\]

c) two numbers so that their product is divisible by 6

Choose numbers with remainders whose product is a multiple of 6, including zero.

\[\text{e.g. } 24 \times \text{ any of the numbers } \text{ or } 30 \times \text{ any of the numbers}\]
\[\text{(0) } \times ? \rightarrow (0) \text{ or (0) } \times ? \rightarrow (0)\]

\[\text{or } 26 \times 27 = 702 \text{ or } 27 \times 28 = 756\]
\[\text{(2) } \times (3) \rightarrow (0) \text{ or (3) } \times (4) \rightarrow (0)\]
Activity

4 (Continued)

d) **three numbers so that their sum is divisible by 6**

Choose numbers with remainders which sum to a multiple of 6.

- e.g. $24 + 25 + 29 = 78$ or $24 + 26 + 28 = 78$
  - $(0) + (1) + (5) \to (0)$
  - $(0) + (2) + (4) \to (0)$

  or $25 + 26 + 27 = 78$ or $25 + 29 + 30 = 84$
  - $(1) + (2) + (3) \to (0)$
  - $(1) + (5) + (0) \to (0)$

or $26 + 28 + 30 = 84$
  - $(2) + (4) + (0) \to (0)$

e) **three numbers so that their sum is not divisible by 6**

Choose numbers with remainders which do not add up to a multiple of 6.

- e.g. $24 + 25 + 26 = 75$ or $25 + 26 + 27 = 76$
  - $(0) + (1) + (2) \to (3)$
  - $(0) + (1) + (3) \to (4)$

  or $25 + 26 + 28 = 77$ or $24 + 25 + 30 = 79$, etc.
  - $(0) + (1) + (4) \to (5)$
  - $(0) + (1) + (0) \to (1)$

  or $25 + 26 + 28 = 79$ or $25 + 26 + 29 = 80$
  - $(1) + (2) + (4) \to (1)$
  - $(1) + (2) + (5) \to (2)$

  or $26 + 27 + 29 = 82$ or $26 + 27 + 30 = 83$, etc.
  - $(2) + (3) + (5) \to (4)$
  - $(2) + (3) + (0) \to (5)$

  or $27 + 28 + 30 = 85$ or $28 + 29 + 30 = 87$
  - $(3) + (4) + (0) \to (1)$
  - $(4) + (5) + (0) \to (3)$

- or $26 + 28 + 30 = 84$
  - $(2) + (4) + (0) \to (0)$

(There are 28 different combinations. Interested Ps might like to find them all for homework.)

f) **three numbers so that their product is divisible by 6**

Choose numbers with remainders whose product is a multiple of 6.

- e.g. $24 \times$ any other two or $30 \times$ any other two
  - $(0) \times ? \times ? \to (0)$
  - $(0) \times ? \times ? \to (0)$

  or $25 \times 26 \times 27$ or $25 \times 27 \times 28$
  - $(1) \times (2) \times (3) \to (0)$
  - $(1) \times (3) \times (4) \to (0)$

  or $26 \times 27 \times 28$ or $26 \times 27 \times 29$
  - $(2) \times (3) \times (4) \to (0)$
  - $(2) \times (3) \times (5) \to (0)$

- or $27 \times 28 \times 29$
  - $(3) \times (4) \times (5) \to (0)$

(There are 28 different combinations. Interested Ps might like to find them all for homework.)

g) **three numbers so that their product is not divisible by 6**

Choose numbers with remainders whose product is not a multiple of 6.

- e.g. $25 \times 26 \times 28$ or $25 \times 26 \times 29$
  - $(1) \times (2) \times (4) \to (2)$
  - $(1) \times (2) \times (5) \to (4)$

  or $25 \times 27 \times 29$ or $25 \times 28 \times 29$
  - $(1) \times (3) \times (5) \to (3)$
  - $(1) \times (4) \times (5) \to (2)$

- or $26 \times 28 \times 29$
  - $(2) \times (4) \times (5) \to (4)$

35 min
PbY6a, page 19. Q.4 
Read: Complete the numbers so that the result of each operation is exactly divisible by 7.

Set a time limit of 6 minutes. Remind Ps about using remainders to work out the digits, as in previous question. Ps can write necessary calculations and check their results in Ex. Bks.

Review with whole class. Ps could show missing digits on scrap paper or slates on command. Ps with different numbers explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

Solution:

a) $1237 + 732$ (\(= 1974\))

\(C:\frac{1 + 2 + 3 + 7}{2 + 4 + 5}\) or $1237 = 700 + 490 + 42 + 5$

b) $1237 - 732$ (\(= 504\))

c) $1237 \times 14$ (\(= 17318\))

\(\left(5\right) - \left(5\right) \rightarrow (0)\)

\(\left(5\right) \times \left(0\right) \rightarrow (0)\)

d) $1237 + 40 + 46$ (\(= 1323\))

\(\left(5\right) + \left(5\right) + \left(4\right) \rightarrow (0)\)

\(\left(5\right) + \left(0\right) + (2) \rightarrow (0)\)

or $1237 + 42 + 16$ (\(= 1295\))

\(\left(5\right) + \left(0\right) + (2) \rightarrow (0)\)

\(\left(5\right) + \left(2\right) + (0) \rightarrow (0)\), etc.

\(39\) min

PbY6a, page 19. Q.5
Read: Decide on the answers by trials or by reasoning but without doing a calculation.

Deal with one part at a time. T reads out each question, gives Ps a minute to think about it and try it in Ex. Bks. then Ps write Y or N (or a number for e)) on scrap paper or slates on command. Ps with different responses explain their reasoning with examples or counter examples. Class decides who is correct. Ps write agreed Yes or No beside questions in Pbs.

Solution:

a) Could the product of 2 successive natural numbers be 999? (No)

(The product of an even and an odd number is even.)

b) Could the sum of 2 successive natural numbers be 2000? (No)

(The sum of an even and an odd number is odd.)

c) Could the sum of 3 successive natural numbers be 2001? (Yes)

(The sum is exactly divisible by 3, and $3x = (x - 1) + (x) + (x + 1)$

d) Could the products of the digits of a natural number be:

i) 26 (No) (As 26 $= 2 \times 13$, and 13 is a 2-digit number)

ii) 35? (Yes) (As 35 $= 7 \times 5$)

\(e\) How many zeros are there at the end of the result of:

$20 \times 21 \times 22 \times 23 \times 24 \times 25$? (3)

(e.g. factor 10 is in 20, factor 4 is in 24, and $10 \times 4 \times 25 = 1000$

but none of the factors left can make another 10.)

\(f\) Can 4 natural numbers have different remainders when divided by 3? (No) (The only possible remainders after division by 3 are 0, 1 or 2)

Whole class activity

Or use pre-agreed actions for Yes and No.

In good humour!

Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

\(e\) e.g. 7 \times 8 = 56

\(e\) e.g. 15 + 16 = 31

\(2001 \div 3 = 667\)

\(666 + 667 + 668 = 2001\)

\(ii\) 75, 57, 175, 571, 715, 751, 1175, 1115, 1117, etc.

So product must be a whole thousand. (127 512 000)

At least 2 of the 4 numbers would have the same remainder.
Factorising 20, 195, 370 and 1020. Revision, activities, consolidation

\textit{PbY6a, page 20}

\textbf{Solutions:}

\textbf{Q.1} 

\begin{enumerate}
  \item \hspace{0.5cm} a) \hspace{0.5cm} 70 = 2 \times 5 \times 7
    \\hspace{0.5cm} Factors: \hspace{0.5cm} 1, 2, 5, 7, 10, 14, 35, 70
    \\hspace{0.5cm} 84 = 2 \times 2 \times 3 \times 7
    \\hspace{0.5cm} Factors: \hspace{0.5cm} 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
    \\hspace{0.5cm} Or greatest common factor: \hspace{0.5cm} 2 \times 7 = 14
    \\hspace{0.5cm} \textit{Answer:} \hspace{0.5cm} The greatest number of bags we can make is 14.
  
  \item \hspace{0.5cm} b) \hspace{0.5cm} Green: \hspace{0.5cm} 70 \div 14 = 5
    \\hspace{0.5cm} Blue: \hspace{0.5cm} 84 \div 14 = 6
    \\hspace{0.5cm} \textit{Answer:} \hspace{0.5cm} There will be 5 green marbles and 6 blue marbles in each bag.
\end{enumerate}

\textbf{Q.2} 

\begin{enumerate}
  \item \hspace{0.5cm} a) \hspace{0.5cm} Exactly divisible by 3: \hspace{0.5cm} 8 764 425, 589 641
  \hspace{0.5cm} b) \hspace{0.5cm} Multiples of 5: \hspace{0.5cm} 36 520, 8 764 425
  \hspace{0.5cm} c) \hspace{0.5cm} Exactly divisible by 4: \hspace{0.5cm} 930 476, 36 520

  \item \hspace{0.5cm} i) \hspace{0.5cm} multiples of 3: \hspace{0.5cm} e.g. \hspace{0.5cm} (but many others possible)
    \hspace{0.5cm} a) \hspace{0.5cm} 3240 \hspace{0.5cm} b) \hspace{0.5cm} 1674 \hspace{0.5cm} c) \hspace{0.5cm} 13 494 \hspace{0.5cm} d) \hspace{0.5cm} 2940
  
    \item \hspace{0.5cm} ii) \hspace{0.5cm} exactly divisible by 4 \hspace{0.5cm} e.g. \hspace{0.5cm} (but many others possible)
    \hspace{0.5cm} a) \hspace{0.5cm} 1144 \hspace{0.5cm} b) \hspace{0.5cm} 7676 \hspace{0.5cm} c) \hspace{0.5cm} 13 428 \hspace{0.5cm} d) \hspace{0.5cm} 3980
  
    \item \hspace{0.5cm} iii) \hspace{0.5cm} a multiple of 5 but not a multiple of 4: \hspace{0.5cm} e.g.
    \hspace{0.5cm} a) \hspace{0.5cm} 8545 \hspace{0.5cm} b) \hspace{0.5cm} 3675 \hspace{0.5cm} c) \hspace{0.5cm} 13 415 \hspace{0.5cm} d) \hspace{0.5cm} 6930
  
    \item \hspace{0.5cm} iv) \hspace{0.5cm} a multiple of 5 and a multiple of 4: \hspace{0.5cm} (i.e. a multiple of 20)
    \hspace{0.5cm} a) \hspace{0.5cm} 1040 \hspace{0.5cm} b) \hspace{0.5cm} impossible \hspace{0.5cm} c) \hspace{0.5cm} 13 400 \hspace{0.5cm} d) \hspace{0.5cm} 7960

  \item \hspace{0.5cm} a) \hspace{0.5cm} \begin{align*}
  \text{GCF: } & 2 \times 2 \times 3 \times 5 = 60 \\
  \text{LCM: } & 2 \times 2 \times 3 \times 5 \times 7 = 420
\end{align*}

  \item \hspace{0.5cm} b) \hspace{0.5cm} Greatest common factor of 24 and 40: \hspace{0.5cm} 8

  \item \hspace{0.5cm} c) \hspace{0.5cm} Lowest common multiple of 24 and 40: \hspace{0.5cm} 120
    \hspace{0.5cm} Multiples of 24: \hspace{0.5cm} 24, 48, 72, 96, 120, 144, 172, \ldots
    \hspace{0.5cm} Multiples of 40: \hspace{0.5cm} 40, 80, 120, \ldots

  \item \hspace{0.5cm} a) \hspace{0.5cm} Bus: \hspace{0.5cm} 10, 20, 30, 40, 50, 60, 70, 80, \ldots \hspace{0.5cm} (min)
    \hspace{0.5cm} Train: \hspace{0.5cm} 12, 24, 36, 48, 60, \ldots \hspace{0.5cm} (min)
    \hspace{0.5cm} \textit{Answer:} \hspace{0.5cm} Charlie will have to wait for 1 hour.

  \item \hspace{0.5cm} b) \hspace{0.5cm} False. \hspace{0.5cm} e.g. \hspace{0.5cm} 15 is a multiple of 3, but is not a multiple of 9.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Factorising</strong></td>
<td></td>
</tr>
<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:</td>
<td></td>
</tr>
<tr>
<td>• (21 = 3 \times 7 ) (nice) Factors: 1, 3, 7, 21</td>
<td></td>
</tr>
<tr>
<td>• (196 = 2 \times 2 \times 7 \times 7 = 2^2 \times 7^2)</td>
<td></td>
</tr>
<tr>
<td>(= 2 \times 2 \times 7 \times 7 = 14 \times 14 = 14^2) (It is a square number.) Factors: 1, 2, 4, 7, 14, 28, 49, 98, 196</td>
<td></td>
</tr>
<tr>
<td>• (371 = 7 \times 53) (nice) Factors: 1, 7, 53, 371</td>
<td></td>
</tr>
<tr>
<td>• (1021) is a prime number Factors: 1, 1021</td>
<td></td>
</tr>
<tr>
<td>(as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37 (\times 37 &gt; 1021))</td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Sequences</strong></td>
<td>Whole class activity</td>
</tr>
<tr>
<td>a) The first term of a sequence is 7100 and it is decreasing by 340. Let's list the numbers in the sequence. Ps dictate what T should write. Class points out errors. BB: 7100, (6760, 6420, 6080, 5740, 5400, 5060, 4720, ...)</td>
<td></td>
</tr>
<tr>
<td>T has many terms prepared and decides when Ps should stop.</td>
<td></td>
</tr>
<tr>
<td>b) Let's have a competition! This time when I describe the sequence, let's see who can write the most terms in your Ex. Bks in 1 minute. The first term is 240 and it is increasing by 170. Start... now! ... Stop! Everyone stand up! Ps read out the terms in order round class. Ps check their own terms against those of other Ps. If a P made a mistake or missed out a term or reaches the end of their terms, they sit down. P(s) left standing are the winners. Let's give them a clap! BB: 240, (410, 580, 750, 920, 1090, 1260, 1430, 1600, 1770, 1940, 2110, 2280, 2450, 2620, 2790, 2960, 3130, ...)</td>
<td></td>
</tr>
<tr>
<td>Winner explains how they wrote so many terms correctly. (e.g. it is easier to add 200 and subtract 30 than add 170)</td>
<td></td>
</tr>
<tr>
<td><strong>3</strong></td>
<td>Individual work, monitored</td>
</tr>
<tr>
<td><strong>PbY6a, page 21</strong></td>
<td>Written on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>Q.1 Read: Practise addition and subtraction. Check your results. How can you check your additions and subtractions? (Calculate additions in opposite direction; add difference to subtrahend or subtract difference from reductant, or use a calculator.) Set a time limit of 6 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Who had all 6 correct? The person nearest them give them a pat on the back!</td>
<td></td>
</tr>
</tbody>
</table>

**Lesson Plan 21**

**Notes**

Individual work, monitored, (helped) (or whole class activity) BB: 21, 196, 371, 1021 Calculators allowed Reasoning, agreement, self-correction, praising Whole class listing of the factors of 196. Ps could join up the factor pairs. Feedback for T 196 2 371 7 49 7 53 53 7 7 1 1 Individual work, monitored In good humour! Whole class activity T chooses Ps at random. Ps calculate mentally if possible. Praising, encouragement only Individual work, monitored In good humour! At speed in order round class. If a P saying the next term makes a mistake the next P must correct it and the first P sits down. Praising, encouragement only Individual work, monitored Written on BB or use enlarged copy master or OHP Differentiation by time limit Reasoning, agreement, self-correction, praising Ps who made several mistakes could do them again as homework.

© CIMT, University of Exeter
Activity
(Continued)
Solution:

3

Solution:

Q.2 Read: Practise multiplication and division in your exercise books. Check your results.

Set a time limit. Ps check results with reverse operations, or by using another method of calculation (or with a calculator).

Review with whole class. Ps come to BB or dictate to T, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

5

Q.3 Read: Write a plan and do the calculations in your exercise book. Write only the results here.

Set a time limit or deal with one question at a time. Ps read question themselves and solve them in Ex. Bks.

Review with whole class. Ps show results on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Who agrees? Who did it a different way? etc.

Mistakes discussed and corrected. Ps write agreed results in Pb s. T chooses a P to say the answer in a sentence.

Solutions:

5

Q.3 Read: Write a plan and do the calculations in your exercise book. Write only the results here.

Set a time limit or deal with one question at a time. Ps read question themselves and solve them in Ex. Bks.

Review with whole class. Ps show results on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Who agrees? Who did it a different way? etc.

Mistakes discussed and corrected. Ps write agreed results in Pb s. T chooses a P to say the answer in a sentence.

Solutions:

28 min

Lesson Plan 21

Notes

Individual work, monitored
Copy master could be used by slower Ps to save time, or by T or Ps in the review.
(T could have solutions already prepared and uncover each part as it is dealt with, or Ps finished early could write solutions on BB or OHT hidden from class.)

Differentiation by time limit
Reasoning, agreement, self-correction, praising
Accept any valid method of calculation but also show the vertical multiplication and division opposite.

Feedback for T

Individual work, monitored (helped)

Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Feedback for T
b) Which quantity is 2684 kg more than 15 tonnes 46 kg?

Plan:

\[ 15 \text{ t } 46 \text{ kg} + 2684 \text{ kg} = 15046 \text{ kg} + 2684 \text{ kg} \]
\[ = 17730 \text{ kg} = 17 \text{ t } 730 \text{ kg} \]

Answer: The quantity is 17 tonnes 730 kg.

c) A 324 mm length was cut from an iron bar and 3 m 28 cm was left. What was the length of the bar before it was cut?

Plan:

\[ 3 \text{ m } 28 \text{ cm} + 324 \text{ mm} = 3 \text{ m } 28 \text{ cm} + 32 \text{ cm} 4 \text{ mm} \]
\[ = 3 \text{ m } 60 \text{ cm} 4 \text{ mm} \]

Answer: The length of the iron bar was 3 m 60 cm 4 mm before it was cut.

d) Which quantity is 24 times as much as 36 litres 50 cl?

Plan:

\[ 36 \text{ litres } 50 \text{ cl} \times 24 = 87600 \text{ cl} \]
\[ = 876 \text{ litres} \]

Answer: The quantity is 876 litres.

e) Which quantity is one 24th of 8 km 400 m?

Plan:

\[ 8 \text{ km } 400 \text{ m} \div 24 = 8400 \text{ m} \div 24 = 350 \text{ m} \]

Answer: The quantity is 350 m.

---

6  

**PbY56a, page 21**

Q.4 Read: Solve these problems in your exercise book.

Deal with one question at a time. T chooses a P to read out the question and Ps solve it in Ex. Bks if they can under a short time limit. Ps with answers show results on scrap paper or slates on command. Ps with correct answers explain their solutions. Who did the same? Who did it a different way? etc. (If no P has the right answer, T gives starting hint and class solves it together.) Mistakes discussed and corrected (or Ps write correct solutions in Ex. Bks.)

Solutions:

a) A natural number ends in zero. If we leave off the zero we get another number. The sum of these two numbers is 5445. What was the original number?

e.g. Let the 2nd number be \( x \), then the 1st number is 10\( x \).

\[
\begin{align*}
x + 10x &= 11x = 5445 \\
\text{Check:} \\
11x &= 5445 \div 11 = 495 \\
10x &= 495 \times 10 = 4950 \\
\text{Answer: The original number was 4950.}
\end{align*}
\]

b) The difference between a number ending in zero and a second number, formed by leaving off the zero of the first number, is 5445. What was the first number?

e.g. Let the 2nd number be \( x \), then the 1st number is 10\( x \).

\[
\begin{align*}
10x - x &= 9x = 5445 \rightarrow x = 5445 \div 9 = 605 \\
\text{So } 10x &= 605 \times 10 = 6050 \\
\text{Answer: The first number was 6050.}
\end{align*}
\]
MEP: Primary Project

Y6

Lesson Plan 21

(Continued)

c) Is it possible that the product of two consecutive natural numbers ends in:
   i) 4  ii) 8  iii) 6?

BB: 1 \times 2 = 2, 2 \times 3 = 6, 3 \times 4 = 12, 4 \times 5 = 20,
   5 \times 6 = 30, 6 \times 7 = 42, 7 \times 8 = 56, 8 \times 9 = 72

Agree that the product of 2 consecutive natural numbers can end in 0, 2 or 6.

Answer: The product of 2 consecutive natural numbers can end in 6 but not in 4 or 8.

d) Calculate the sum of the digits in the number 38 516 then subtract it from 38 516. Is the difference divisible by 9?
   Try it with other natural numbers.

BB: 3 + 8 + 5 + 1 + 6 = 23  \quad 38 516 – 23 = 38 493

38 493 is exactly divisible by 9 because:
   3 + 8 + 4 + 9 + 3 = 27, which is a multiple of 9.

Try, e.g. 8357: 8 + 3 + 5 + 7 = 23; 8357 – 23 = 8334

8334 is exactly divisible by 9 because:
   8 + 3 + 3 + 4 = 18, which is a multiple of 9.

After trying other numbers and getting the same result, ask Ps to explain why it is true for all natural numbers. e.g.
'The number, and the sum of its digits, have the same remainder when divided by 9, so their difference has no remainder and must be divisible by 9.'

e) I thought of a 5-digit natural number. When I wrote a ’4’ in front of it, the 6-digit number I made was 4 times as much as the number I would have made if I had written the 4 at the end of the 5-digit number.

What was the number that I first thought of?

We know that:

BB: As 4 \times 4 = 16, the last digit of the number must be 6.

As 4 \times 6 = 24, 24 + 1 (carried over from the 16) = 25, the next digit must be 5.

As 4 \times 5 = 20, 20 + 2 (carried over from the 25) = 22, the next digit must be 2.

As 4 \times 2 = 8, 8 + 2 (carried over from the 22) = 10, the next digit must be 0.

As 4 \times 0 = 0, 0 + 1 (carried over from the 10) = 1, the final digit must be 1.

Answer: The number that I first thought of was 10256.

9 \times 10 = 90, 10 \times 11 = 110,
11 \times 12 = 132, \ldots

and the pattern continues for all natural numbers.

Check:

\[
\begin{array}{c|c|c|c|c|c|c}
& 4 & 2 & 7 & 9 & 9 & 5 \\
\hline
2 & 7 & 8 & 5 & 1 & 1 & 6 \\
\hline
23 & 9 & 3 & 8 & 5 & 1 & 16 \\
\hline
\end{array}
\]

23 ÷ 9 = 2, r 5

38 516 (= 4279 \times 9 + 5)
– 23 (= 2 \times 9 + 5)

38493 (= 4277 \times 9) + 0

If no P has an idea which is on the right track, T helps Ps to write down the known elements in the calculation in the form opposite, then Ps might realise how it should be continued.

Or a bright P might suggest using algebra: e.g.

Let the 5 digit number be \( x \).

\[
\begin{align*}
40000 + x &= 4 \times (10x + 4) \\
40000 + x &= 40x + 16 \\
399984 + x &= 40x \\
399984 &= 39x \\
399984 &= 39x
\end{align*}
\]

\[
\begin{align*}
x &= 399984 \div 39 \\
&= 10256
\end{align*}
\]
Lesson Plan

22

Notes

Individual work, monitored, (helped)
(or whole class activity)
BB: 22, 197, 372, 1022
Calculators allowed
Reasoning, agreement, self-correction, praising
Whole class listing of the factors of 372. Ps could join up the factor pairs.
Feedback for T

<table>
<thead>
<tr>
<th>7 min</th>
</tr>
</thead>
</table>

| 1 |

Activity

Factorising

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:
- \( 22 = 2 \times 11 \) (nice) Factors: 1, 2, 11, 22
- \( 197 \) is a prime number Factors: 1, 197
(as not exactly divisible by 2, 3, 5, 7, 11, 13, and \( 17 \times 17 > 197 \))
- \( 372 = 2 \times 2 \times 3 \times 31 = 2^2 \times 3 \times 31 \)
Factors: 1, 2, 3, 4, 6, 12, 31, 62, 93, 124, 186, 372
- \( 1022 = 2 \times 7 \times 73 \)
Factors: 1, 2, 7, 14, 73, 146, 511, 1022

7 min

| 2 |

Quantities

Let’s put these quantities in increasing order. Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. Elicit what the decimal point means (separates the whole units from the parts of a unit) and discuss the relationships among the units of measure. e.g.

a) BB: £34 75 p £347.5 £347 50 p £34.8 £34 80 p £347 70 p £34.75 £347.40
Ps: £34 75 p = £34.75 < £34.8 = £34 80 p < £347.40 < £347.5 = £347 50 p < £347 70 p
Relationships: e.g. £34 75 p = £3.75 = \( \left( \frac{34 + \frac{75}{100}}{34 + \frac{80}{100}} \right) \)

b) BB: 1543 mm 230 cm 12.65 m 1.5 m 2200 mm 1.641 m
Ps:
1.5 m < 1543 mm < 1.641 m < 2200 mm < 230 cm < 12.65 m
(1.543 m) (2.2 m) (2.3 m)

13 min
## Lesson Plan 22

### Activity

#### 3 PbY6a, pge 22

**Q.1 Read:** Do the addition and subtraction in millimetres, centimetres and metres.

Set a time limit of 4 minutes. Remind Ps to check their results. Review with whole class. Ps come to BB or dictate to T, explaining reasoning with place-value detail and pointing out the relationships among the units of measure. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>652,418 mm</td>
<td>652,418 m</td>
<td>652,418 m</td>
</tr>
<tr>
<td></td>
<td>1043,706 mm</td>
<td>1043,706 cm</td>
<td>1043,706 m</td>
</tr>
<tr>
<td></td>
<td>1789,162 mm</td>
<td>1789,162 cm</td>
<td>1789,162 m</td>
</tr>
<tr>
<td></td>
<td>1094,283 mm</td>
<td>1094,283 cm</td>
<td>1094,283 m</td>
</tr>
<tr>
<td></td>
<td>2310,978 mm</td>
<td>2310,978 cm</td>
<td>2310,978 m</td>
</tr>
</tbody>
</table>

**Extension**

What is the sum (difference) rounded to the nearest km?

Elicit that 1 km = 1000 m = 100 000 cm = 1000 000 mm, so both results are approximately equal to to 2 km, but the sum rounds up and the difference rounds down.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>642,5 m</td>
<td>642,5 km</td>
<td>642,5 km</td>
</tr>
<tr>
<td></td>
<td>802,6 km</td>
<td>802,6 km</td>
<td>802,6 km</td>
</tr>
<tr>
<td></td>
<td>35,0 km</td>
<td>35,0 km</td>
<td>35,0 km</td>
</tr>
<tr>
<td></td>
<td>1015 m</td>
<td>1015 km</td>
<td>1015 km</td>
</tr>
<tr>
<td></td>
<td>8457,5 km</td>
<td>8457,5 km</td>
<td>8457,5 km</td>
</tr>
</tbody>
</table>

**Notes**

- Individual work, monitored (helped)
- Written on BB or use enlarged copy master or OHP
- Differentiation by time limit
- Discussion, reasoning, agreement, self-correction, praising

Feedback for T整

### 4 PbY6a, pge 22

**Q.2 Read:** Do the addition and subtraction in metres and kilometres.

Set a time limit of 4 minutes. Remind Ps to check their results. Review with whole class. Ps come to BB or dictate to T, explaining reasoning with place-value detail and measuring units. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>802,6 km</td>
<td>802,6 km</td>
<td>802,6 km</td>
</tr>
<tr>
<td></td>
<td>210,875 m</td>
<td>210,875 m</td>
<td>210,875 m</td>
</tr>
<tr>
<td></td>
<td>2211,93 m</td>
<td>2211,93 m</td>
<td>2211,93 m</td>
</tr>
<tr>
<td></td>
<td>8457,5 km</td>
<td>8457,5 km</td>
<td>8457,5 km</td>
</tr>
</tbody>
</table>

Elicit that to add or subtract decimal numbers, calculate in the same way as natural numbers, keeping digits with the same place-value and the decimal points lined up.

Which zeros are not really needed? (Those at end of digits after the decimal point. e.g. 802.600 = 802.6, 0.710 = 0.71, 35.000 = 35) Ps might think that it is easier to line up the digits correctly if we write zeros in the 'gaps'.

**Notes**

- Individual work, monitored (helped)
- Written on BB or use enlarged copy master or OHP
- Reasoning, agreement, self-correcting, praising

Feedback for T

T reminds Ps that if a decimal is less than 1 unit, zero must be written in the units column (e.g. 0.710)

T points to certain place-value columns and Ps say their values.
Lesson Plan 22

**Activity 5**  
*PbY6a, page 22*

Q.3 Read: *Do the multiplication and division in millimetres, centimetres and metres.*

Deal with one part at a time. Set a time limit for row a).

Review with whole class. Ps come to BB or dictate to T, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that to multiply a decimal by a natural number, do the calculation in the same way as multiplying a natural number, then write the decimal point in the answer so that there are the same number of decimal digits after the decimal point as there are in the multiplicand.

Set a time limit for row b) and review as for a).

Elicit that to divide a decimal by a natural number, do the division in the normal way but write a decimal point in the quotient when we reach the decimal point in the dividend.

**Solution:**

\[
\begin{align*}
a) & \quad \begin{array}{c|c|c}
6 & 8 & 4.2 \text{ mm} \\
\times & 7 & \\
\hline
4 & 7 & 8.9.4 \\
5 & 2 & 1 \\
\hline
8 & 4.4 & 9 \\
2 & 5
\end{array} \\
& \quad \begin{array}{c|c|c}
8 & 3.2 \text{ cm} \\
\times & 7 & \\
\hline
5 & 7 & 8.9.4 \\
4 & 7 & 8.9.4 \\
\hline
0 & 8 & 4.9 \\
2 & 5
\end{array} \\
& \quad \begin{array}{c|c|c}
6 & 8 & 4.2 \text{ m} \\
\times & 7 & \\
\hline
4 & 7 & 8.9.4 \\
5 & 2 & 1 \\
\hline
3 & 7 & 8.9.4 \\
1 & 7
\end{array}
\end{align*}
\]

Solution:

\[
\begin{align*}
a) & \quad 684.9 \text{ mm} \\
& \quad 5094 \text{ cm} \\
& \quad 5094 \text{ m}
\end{align*}
\]

Who can tell us the rule for multiplying a decimal by a 2-digit natural number? (Do the multiplication in the same way as multiplying a natural number by a 2-digit number, then write the decimal point in the answer so that there is the same number of decimal digits after (i.e. on RHS of) the decimal point as there is in the multiplicand.)

**Solution:**

\[
\begin{align*}
a) & \quad 405.3 \text{ cm} \\
& \quad 32199 \text{ cm} \\
& \quad 786654 \text{ cm}
\end{align*}
\]

Discuss the case of 5094 ÷ 6

(The sum of the digits is 5 + 9 + 4 = 18, so it is exactly divisible by 3, and as 5094 is even and there is no remainder after division by 3, the number is also divisible by 2 × 3, i.e. divisible by 6.)

**Notes**

Individual work, monitored (helped)

Written on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

Individual work, monitored, helped

(or whole class activity for a))

Ps have squared *Ex. Bks* or grid sheets or less able Ps have copies of copy master.

Reasoning, agreement, checking on a calculator, self-correction, praising

(Accept any valid method of multiplication, including exchanging the units:

\[
\text{e.g. } 405.3 \text{ cm} = 4053 \text{ mm} \\
\text{but make sure that the result is changed back to the original unit of measure.)}
\]

T helps Ps to put the rule into words.
Activity

7  

PbY6a, page 22

Q.5 Read: Do the divisions in your exercise book. Check your results with a calculator.

Set a time limit or deal with one at a time or do part a) with the whole class first as a model for Ps to follow.

Review at BB with whole class. Ps come to BB to write the calculation, explaining reasoning with place-value detail.

Class agrees/disagrees. Check result with a calculator.

Mistakes discussed and corrected.

Solution:

Who can tell us the rule for dividing a decimal by a 2-digit natural number? (Do the division in the same way as dividing a natural number by a 2-digit number, but write a decimal point in the quotient when we come to the decimal point in the dividend.)

42 min

8  

PbY6a, page 22, Q.6

Read: Do the division in millimetres here, then in centimetres and metres in your exercise book.

3 Ps come to BB, one to do the division in mm, one to do it in cm and one to do it in metres. Ps estimate first, then do the divisions, explaining reasoning with place-value details and check their results against their estimates. Class points out errors.

Ps write the divisions in Pbs and Ex. Bks. at the same time.

Solution:

\[ E: \ 5000 \div 25 = 200 \]

\[ E: \ 500 \div 25 = 20 \]

\[ E: \ 5 \div 25 = 0.2 \]

(5 \div 25 = 1 \div 5 = \frac{1}{5} = 0.2)

Whole class activity but Ps work in Ex. Bks. too

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

In a), accept:

4586 mm \div 25 = 183 mm,

and 11 mm will be left over.

(If time is short, do division in mm with the whole class and set divisions in cm and m for homework, then review before start of Lesson 23.)
**R:** Decimal notation  
**C:** Pencil/paper methods. Natural numbers and decimals  
**E:** Word problems

### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 23 \) is a prime number  
  Factors: 1, 23
- \( 198 = 2 \times 3 \times 3 \times 11 = 2 \times 3^2 \times 11 \)
  Factors: 1, 2, 3, 6, 9, 11, 18, 22, 33, 66, 99, 198
- \( 373 \) is a prime number  
  (as not exactly divisible by 2, 3, 5, 7, 9, 11, 13, 17, 19 and 23 \( \times 23 > 373 \))
  Factors: 1, 3, 11, 31, 33, 93, 341, 1023

\[ 1023 = 3 \times 11 \times 31 \]
Factors: 1, 3, 11, 31, 33, 93, 341, 1023

### Activity 2

**Adding and subtracting decimals**

Listen carefully and write these decimal numbers in your Ex. Bk one below the other, keeping digits with the same place values lined up.

T dictates the numbers, saying each number twice.

Now add them up and show me your result when I say. P with correct result comes to BB to do the addition, reasoning with place-value detail. Mistakes discussed and corrected.

Similarly for:

\[
\begin{align*}
\text{a)} & \quad 27.56, \quad 93.85, \quad 49.27 \\
\text{b)} & \quad 409.8 + 17.63 + 8.207 \\
\text{c)} & \quad 86.3 - 47.8 \\
\text{d)} & \quad 30.4 - 8.63 \\
\text{e)} & \quad 25 - 0.36
\end{align*}
\]

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Individual work, monitored, (helped)
(or whole class activity)

BB: 23, 198, 373, 1023
Calculators allowed
Reasoning, agreement, self-correction, praising

Whole class listing of the factors of 198. Ps could join up the factor pairs.

Feedback for T

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Individual work, monitored but Ps kept together on tasks.
Ps should have squared Ex.Bks or grid sheets.

Encourage Ps to check additions mentally by adding in opposite direction and to check subtractions with reverse addition or another subtraction.

Responses shown in unison on scrap paper or slates.
Reasoning, agreement, self-correction, praising

(Ps might find it easier to write zeros in the gaps in the place-value columns on RHS of decimal point.)
**Y6**

### Activity

#### PbY6a, page 23

**Q.1** Read: *Practise addition in your exercise book. Check your results.*

Deal with one at a time or set a time limit. Advise Ps to estimate result first by rounding the terms appropriately to give them an idea of whether their answer could be correct. Ps write vertical additions, keeping digits with the same place-value and the decimal points lined up, then Ps check their results by adding in the opposite direction.

Review with whole class. Ps come to BB to write vertical additions and explain reasoning with place-value details.

Class points out errors. Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
\text{a) } & \quad \begin{array}{c}
3826 \\
+ \quad 8519 \\
\hline
12345
\end{array} \\
\text{b) } & \quad \begin{array}{c}
3826 \\
+ \quad 8519 \\
\hline
12345
\end{array} \\
\text{c) } & \quad \begin{array}{c}
03826 \\
+ \quad 08519 \\
\hline
112345
\end{array}
\end{align*}
\]

**Q.2** Read: *Practise subtraction in your exercise book. Check your results.*

Deal with one at a time or set a time limit. Ps estimate result first by rounding dividend and subtrahend to the whole numbers. Ps check their results with reverse addition, or by another subtraction.

Review with whole class. Ps come to BB to write vertical subtractions and explain reasoning with place-value details.

Class points out errors. Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
\text{a) } & \quad \begin{array}{c}
10836 \\
- \quad 1478 \\
\hline
7358
\end{array} \\
\text{b) } & \quad \begin{array}{c}
61053 \\
- \quad 5040 \\
\hline
1063
\end{array} \\
\text{c) } & \quad \begin{array}{c}
81230 \\
- \quad 41394 \\
\hline
3986
\end{array} \\
\text{d) } & \quad \begin{array}{c}
10810 \\
- \quad 77102 \\
\hline
3098
\end{array} \\
\text{e) } & \quad \begin{array}{c}
25310 \\
- \quad 24310 \\
\hline
40
\end{array} \\
\text{f) } & \quad \begin{array}{c}
56710 \\
- \quad 46710 \\
\hline
999
\end{array}
\end{align*}
\]

Check:

\[
\begin{align*}
\text{e.g. } & \quad \begin{array}{c}
46710 \\
+ \quad 46710 \\
\hline
93420
\end{array}
\end{align*}
\]

**Notes**

Individual work, monitored, (helped)

Less able Ps could use copy master.

Differentiation by time limit T could have grids already prepared on BB or SB or OHT to make it easier for Ps to show their solutions (or use enlarged copy master).

(Ps might find it easier to write zeros in the gaps in the place-value columns on RHS of decimal point.)

Reasoning, agreement, checking, self-correction, praising

Feedback for T

---

**Lesson Plan 23**

© CIMT, University of Exeter
Lesson Plan 23

Notes

Individual work, monitored (helped)

Responses shown in unison.
Reasoning, agreement, self-correction, praising
Accept any correct form of the answer.
Feedback for T

e.g.

\[ C: \begin{array}{c}
3 \quad 7 \\
7 \quad 0 \\
0 \\
\hline
3 \quad 2 \quad 5 \\
1 \quad 7 \quad 2 \\
0 \\
\end{array} \hspace{1cm} \begin{array}{c}
10 \\
10 \\
10 \\
\end{array} \]

\[ C: \begin{array}{c}
4 \quad 3 \\
3 \quad 5 \\
0 \\
\hline
4 \quad 5 \quad 2 \quad 4 \\
1 \\
\end{array} \hspace{1cm} \begin{array}{c}
10 \\
10 \\
10 \\
\end{array} \]

Individual work, monitored (helped)

Written on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

BB: e.g.

\[ \begin{array}{c}
1 \quad 5 \\
5 \\
1 \\
\hline
1 \quad 5 \\
4 \\
0 \\
\end{array} \hspace{1cm} \begin{array}{c}
1 \quad 3 \quad 4 \\
6 \\\n1 \\
\end{array} \]

\[ \begin{array}{c}
2 \quad 4 \\
6 \\
3 \\
\hline
2 \quad 4 \\
6 \\
3 \\
\end{array} \hspace{1cm} \begin{array}{c}
10 \\
10 \\
10 \\
\end{array} \]
### Activity

#### Y6

**PbY5a, page 23**

**Q.5** Read: Write a plan, calculate, check and write the answer as a sentence in your exercise book.

Deal with one at a time or set a time limit. Ps read question themselves and solve it in Ex. Bks. Remind Ps to estimate the result first by rounding appropriately.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

**Solutions:** e.g.

**a)** 0.42 kg of prunes can be made from 1 kg of plums.

What amount of prunes can be made from 78 kg of plums?

Plan:  

\[
1 \text{ kg} \rightarrow 0.42 \text{ kg} \\
78 \text{ kg} \rightarrow 0.42 \text{ kg} \times 78
\]

\[= 32.76 \text{ kg (prunes)}\]

Answer: 32.76 kg of prunes can be made from 78 kg of plums.

**b)** How long is each side of a regular octagon if its perimeter is 341.8 cm?

Plan: 

\[P = 8 \times a, \text{ so } a = P \div 8\]

(A regular octagon has 8 equal sides.)

\[= 42.725 \text{ cm}\]

Answer: Each side of the octagon is 42.725 cm long.

**c)** The area of a rectangle is 63.6 cm². The length of one of its sides is 12 cm. What is the length of the adjacent side?

Plan: 

\[A = a \times b, \text{ so } a = A \div b\]

\[= 63.6 \div 12\]

\[= 5.3 \text{ (cm)}\]

Answer: The length of the adjacent side is 5.3 cm.

---

**Notes**

Individual work, monitored, (helped)

Responses shown in unison. Discussion, reasoning, agreement, self-correction, praising

T could have real plums and prunes to show to class.

Why do the plums produce a smaller amount of prunes?

(They are dried so lose much of their water.)

---

**Lesson Plan 23**

**Week 5**

Individual work, monitored, (helped)

Responses shown in unison. Discussion, reasoning, agreement, self-correction, praising

T could have real plums and prunes to show to class.

Why do the plums produce a smaller amount of prunes?

(They are dried so lose much of their water.)

---

Whole class activity (or individual work, monitored, helped if Ps wish and there is time)

Squared grids drawn on BB or use enlarged copy master

Discussion, reasoning, agreement, (self-correction), praising

If time is very short, do part a) in class and set b) and c) for homework. Review before the start of Lesson 24.
### Activity 1

**Factorising**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **24**  =  $2 \times 3 \times 2$  
  Factors: 1, 2, 3, 4, 6, 8, 12, 24  
  e.g.  
  

[![Factorisation Diagram](attachment:image.png)](attachment:image.png)

- **199** is a prime number  
  Factors: 1, 199  
  (as not divisible by 2, 3, 5, 7, 11, 13 and 17, and $17 \times 17 > 199$)  
  e.g.  

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>199</td>
<td>1024</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>256</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

- **374**  =  $2 \times 11 \times 17$  
  Factors: 1, 2, 11, 17, 22, 34, 187, 374  
  e.g.  

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>128</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>64</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

- **1024**  =  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$  
  $= \frac{2^{10}}{32} \times \frac{2^{10}}{32} = 2^{10}$  
  Factors: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512, 1024  

**Individual work, monitored, (helped)  
(whole class activity)**  
BB: 24, 199, 374, 1024  
Calculators allowed  
Reasoning, agreement, self-correction, praising  
Whole class listing of the factors of 1024. Ps could join up the factor pairs.

Feedback for T  
- **374**  =  2  
  e.g.  

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>174</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>17</td>
<td></td>
</tr>
</tbody>
</table>

1024 is a **square** number.

### Activity 2

**Sequences**

a) Let's list the next 5 odd numbers after 999 996. Ps come to BB or dictate what T should write. Class agrees/disagrees.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BB:</td>
<td>999 997, 999 999, 1000 001, 1000 003, 1000 005</td>
</tr>
</tbody>
</table>

Do the numbers form a sequence? (Yes) Who can describe this sequence? (The first term is 999 997 and it is increasing by 2.)

b) Let's list the 5 even numbers before 100 005. Ps dictate to T.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BB:</td>
<td>100 004, 100 002, 100 000, 99 998, 99 996</td>
</tr>
</tbody>
</table>

Who can describe this sequence? (The first term is 100 004 and it is decreasing by 2.)

c) T describes a sequence and gives Ps 1 minute to write as many terms as they can in their **Ex. Bks.**

Review with whole class. Ps stand up and say the terms in order round the class. If a P makes a mistake or misses a term or comes to the end of their terms, they sit down. Ps left standing are given a clap or 3 cheers or a pat on the back.

i) The first term is 0 and it is increasing by 3.5.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BB:</td>
<td>0, 3.5, 7, 10.5, 14, 17.5, 21, 24.5, 28, 31.5, 35, ...</td>
</tr>
</tbody>
</table>

ii) The first term is 85.2 and it is decreasing by 2.7.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>BB:</td>
<td>85.2, 82.5, 79.8, 77.1, 74.4, 71.7, 69, 66.3, 63.6, ...</td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>7 min</td>
<td></td>
</tr>
</tbody>
</table>

---

**Lesson Plan**

**Notes**

Individual work, monitored, (helped)  
(whole class activity)

BB: 24, 199, 374, 1024  
Calculators allowed  
Reasoning, agreement, self-correction, praising  
Whole class listing of the factors of 1024. Ps could join up the factor pairs.

Feedback for T  

At a good pace  
Reasoning, agreement, praising  
Individually, monitored

At a fast pace  
In good humour!  
Agreement, self-correction, praising
### Lesson Plan 24

#### Activity 3

**PbY6a, page 24**

**Q.1** Read: **Write the first term and the next 5 terms of each sequence in your exercise book.**

Deal with one at a time. T chooses a P to read out the description, then allows Ps a minute to write the terms. Encourage mental calculation where possible.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) *Its first term is 8346 and it is increasing by 520.*

8346, 8866, 9386, 9906, 10426, 10946, ...

b) *Its first term is 24080 and it is decreasing by 5200.*

24080, 18880, 13680, 8480, 3280, –1920, ...

c) *Its first term is 13.3 and it is decreasing by 3.2.*

13.3, 10.1, 6.9, 3.7, 0.5, –2.7, ...

20 min

**Q.2** Read: **Work out a rule and continue each sequence for 5 more terms. Write the rule you used.**

Deal with one at a time. Set a time limit of 1 minute each.

Review at BB with whole class. Ps come to BB or dictate to T, explaining reasoning and stating the rule that they used. Who agrees? Who used a different rule? Class decides whether or not it is valid. Mistakes discussed and corrected.

**Solution:**

a) 10 638, 10 794, 10 950, (11 106, 11 262, 11 418, 11 574, 11 730, ...)  
   [Rule: + 156]

b) 410.7, 390.1, 369.5, (348.9, 328.3, 307.7, 287.1, 266.5, ...)  
   [Rule: – 20.6]

c) 0.2, 0.3, 0.5, 0.8, 1.2, (1.7, 2.3, 3.0, 3.8, 4.7, ...) 
   0.1 0.2 0.3 0.4 0.5 0.6 0.7 0.8 0.9 ...  
   [Rule: Difference between terms is increasing by 0.1]

d) 1.2, 2.4, 3.6, 4.8, (6.0, 7.2, 8.4, 9.6, 10.8, ...)  
   [Rule: +1.2, or \( n \times 1.2 \), where \( n \) is the position of the term in the sequence, i.e. its ordinal number]  
   What would the 100th term be? Ps show on slates. (120)

e) 10.24, 5.12, 2.56, (1.28, 0.64, 0.32, 0.16, 0.08, ...)  
   [Rule: Each following term is half of the previous term, or \( \div 2 \)]

20 min

#### Notes

Individual work, monitored, helped  
Reasoning, agreement, self-correction, praising  
Feedback for T

Individual work, monitored, helped  
Written on BB or SB or OHT  
Discussion, reasoning, agreement, self-correction, praising  
Accept any valid rule.

(or increasing by 156)

(or decreasing by 20.6)

(or rule for difference sequence is \(+0.1\))

\e.g. 3rd term: \( n = 3 \), so term is \( 3 \times 1.2 = 3.6 \)

8th term: \( n = 8 \), so term is \( 8 \times 1.2 = 9.6 \)
Lesson Plan 24

### Activity 5

**PbY6a, page 24, Q.3**

Deal with one part at a time. T chooses a P to read out the description. Ps write the smallest number on one side of their slates and the greatest number on the other side, then show to T on command. Ps with different responses explain their reasoning to class and class decides who is correct. Ps write agreed numbers in Ex. Bks. or PbS.

**Solution:**

In your exercise book, write the smallest and the greatest:

a) whole number which can be rounded to:
   i) 600 as the nearest 100
      
   
   
   
   550, 649
   

   ii) 4000 as the nearest 1000
      
   
   
   
   3500, 4499
   

   iii) 5 million as the nearest million.
      
   
   
   
   4 500 000, 5 499 999
   

b) number which can be rounded to:
   i) 7 as the nearest unit
      
   
   
   
   6.5 ≤ a < 7.5
   

   Agree that there is no greatest number as decimal place values can go on and on to infinity: 7.499 999 999 999 999 . . .
   

   ii) 0.8 as the nearest tenth.
      
   
   
   
   0.75 ≤ e < 0.85
   

   Again, there is no greatest number. E.g. 0.849 999 999 999 . . .
   

   Another digit 9 can be written in the next smaller place-value column, and in the next and in the next . . . to infinity.
   
   

   

### Activity 6

**PbY6a, page 24, Q.4**

Read: Write the name of the operation in the box and complete the equations.

Set a time limit. (If Ps are unsure what to do, complete part a) with the whole class first.)

Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) 6.7 + 10.8 = (17.5)  
   
   
   
   a + b = c,  a = (c – b)  b = (c – a)
   

   **Addition**
   

b) 8.25 – 4.6 = (3.65)  
   
   
   
   a – b = c,  a = (c + b)  b = (a – c)
   

   **Subtraction**
   

c) 14.3 × 5 = (71.5)  
   
   
   
   a × b = c,  a = (c ÷ b)  b = (c ÷ a)
   

   **Multiplication**
   

d) 42.6 ÷ 3 = (14.2)  
   
   
   
   a ÷ b = c,  a = (c × b)  b = (a ÷ c)
   

   **Division**
   

Ask Ps to point to and name the components of the operations.

**Addition:** term a + term b = sum

**Subtraction:** reductant – subtrahend = difference

**Multiplication:** factor a × factor b = product

or multiplicand × multiplier = product

**Division:** dividend ÷ divisor = quotient

What other component of a division is not shown here? (Remainder, when a division is not exact.)

### Notes

Whole class activity

(or individual work under a time limit, monitored and reviewed as usual)

Responses shown in unison

Discussion, reasoning, agreement, praising

Extra praise for Ps who answered with an inequality or realised that there is no 'greatest' number.

If nobody realised, choose the greatest number shown by Ps. Can you think of a greater one . . . and a greater one . . . and a greater one . . . ?

### Notes

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

Whole class revision

T points to various equations and Ps say it using the names of the components, e.g.

a) term b = sum – term a

b) reductant = difference + subtrahend

c) factor a = product ÷ factor b, etc.
### Activity

#### PbY6a, page 24

**Q.5 Read:** Which numbers do the letters represent?

Solve the equations and check your solutions.

How can you check your solution? (By writing the number instead of the letter in the equation and checking that both sides are equal.) Deal with one at a time or one row at a time or set a time limit. Ps write necessary calculations, checks and solutions in Ex. Bks.

Review with whole class. Ps could show numbers on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it another way? etc. Check results by substituting the number for the letter in the equation. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Letter</th>
<th>Equation</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>1.75 + x = 7.1</td>
<td>( x = 7.1 - 1.75 = 5.35 ) Check: ( 1.75 + 5.35 = 7.1 ) ✓</td>
</tr>
<tr>
<td>b)</td>
<td>6 + (x + 0.5) = 8</td>
<td>( x + 0.5 = 8 - 6 = 2 ) ( x = 2 - 0.5 = 1.5 ) Check: ( 6 + 1.5 + 0.5 = 8 ) ✓</td>
</tr>
<tr>
<td>c)</td>
<td>y – 5.02 = 3.8</td>
<td>( y = 3.8 + 5.02 = 8.82 ) Check: ( 8.82 - 5.02 = 3.8 ) ✓</td>
</tr>
<tr>
<td>d)</td>
<td>15 – z = 8.4</td>
<td>( z = 15 - 8.4 = 6.6 ) Check: 15 - 6.6 = 8.4 ✓</td>
</tr>
<tr>
<td>e)</td>
<td>8 – (u + 5) = 2.6</td>
<td>( u + 5 = 8 - 2.6 ) ( u = 5.4 - 5 = 0.4 ) Check: 8 - (0.4 + 5) = 2.6 ✓</td>
</tr>
<tr>
<td>f)</td>
<td>(9.3 – v) + 5 = 5</td>
<td>( 9.3 - v = 5 - 5 = 0 ) ( v = 9.3 ) Check: (9.3 – 9.3) + 5 = 5 ✓</td>
</tr>
<tr>
<td>g)</td>
<td>7.2 \times x = 14.4</td>
<td>( x = 14.4 \div 7.2 ) ( x = 144 \div 72 = 2 ) Check: 7.2 \times 2 = 14.4 ✓</td>
</tr>
<tr>
<td>h)</td>
<td>y \times 10 = 12</td>
<td>( y = 12 \div 10 = 1.2 ) Check: 1.2 \times 10 = 12 ✓</td>
</tr>
<tr>
<td>i)</td>
<td>( x \div 42 = 1.5 )</td>
<td>( x = 1.5 \times 42 ) ( x = 42 \div 21 = \frac{63}{7} ) Check: ( 63 \div 42 = 9 \div 6 \div 6 = 1.5 ) ✓</td>
</tr>
<tr>
<td>j)</td>
<td>( 720 \div y = 120 )</td>
<td>( y = 720 \div 120 ) ( y = 72 \div 12 = 6 ) Check: ( 720 \div 6 = 120 ) ✓</td>
</tr>
<tr>
<td>k)</td>
<td>( z \times 0.1 = 5 )</td>
<td>( z = 5 \div 0.1 ) ( z = 50 \div 1 = 50 ) Check: ( 50 \times 0.1 = 5 \times 1 = 5 ) ✓</td>
</tr>
<tr>
<td>l)</td>
<td>( 96 \div u = 10 )</td>
<td>( u = 96 \div 10 = \frac{96}{10} ) ( u = 960 \div 96 = 10 ) Check: ( 96 \div 9.6 = 10 ) ✓</td>
</tr>
</tbody>
</table>

**Notes**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>45 min</td>
</tr>
</tbody>
</table>

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison. Discussion, reasoning using relationships between the components (or by balancing the equation), checking, agreement, self-correction, praising

or subtract 1.75 from each side:

\[ 1.75 - 1.75 + x = 7.1 - 1.75 \]

\[ x = 5.35 \] ✓

or add 5.02 to each side

or add \( z \) to each side, then subtract 8.4 from each side

or add \( u + 5 \) to each side, then subtract 2.6, then 5 from each side

or subtract 5 from each side, then add \( v \) to each side

or multiply each side by 10, then divide each side by 72

or divide each side by 10

or multiply each side by 42

or multiply each side by \( y \), then divide each side by 120

or multiply each side by 10

or multiply each side by \( u \), then divide each side by 10

© CIMT, University of Exeter
Factorising 25, 200, 375 and 1025. Revision, activities, consolidation

PbY6a, page 25

**Solutions:**

**Q.1**

a) $25 = 5 \times 5 = 5^2$ (square no.)

Factors: 1, 5, 25

b) $200 = 2^3 \times 5^2$

Factors: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200

$375 = 3 \times 5^3$

Factors: 1, 3, 5, 15, 25, 75, 125, 375

$1025 = 5^2 \times 41$

Factors: 1, 5, 25, 41, 1025

(Or set factorising as homework at the end of Lesson 24 and review at the start of Lesson 25)

**Q.2**

a) $9043$  

b) $4381$  

c) $3895$  

d) $5471$  

e) $2548$  

**Q.3**

a) $8.096 < 65.725 < 72.94 < 150.3$

b) $65.725 = 66$

$72.94 = 73$

$150.3 = 150$

**Notes**

$25 = 5 \times 5 = 5^2$ (square no.)

Factors: 1, 5, 25

$200 = 2^3 \times 5^2$

Factors: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200

$375 = 3 \times 5^3$

Factors: 1, 3, 5, 15, 25, 75, 125, 375

$1025 = 5^2 \times 41$

Factors: 1, 5, 25, 41, 1025

(Or set factorising as homework at the end of Lesson 24 and review at the start of Lesson 25)

Continue the division until there is no remainder.
Y6

Activity

(Continued)

Q.4  a) \( x = (100 - 15) \div 2 \div 5 = 85 \div 10 = 8.5 \)
    Answer: I am thinking of the number 8.5.
    b) \( x = (1 + 0.15) \times 2 \div 100 = 1.15 \times 2 \div 100 \)
        = \( 2.3 \div 100 = 0.023 \)
    Answer: I am thinking of the number 0.023.

Q.5  a) \( 0.332 + a = 10, \ a = 10 - 0.332 = 9.668 \)
    b) \( 5 \times b - 4.07 = 5 \)
        \( 5 \times b = 5 + 4.07 = 9.07 \)
        \( b = 9.07 \div 5 = 1.814 \)
    c) \( c - 92.7 = 3.8, \ c = 3.8 + 92.7 = 96.5 \)
    d) \( d \div 100 = 0.054, \ d = 0.054 \times 100 = 5.4 \)
    e) \( 8 \times (e \div 10) = 2.5 \)
        \( e \div 10 = 2.5 \div 8 = 0.3125 \)
        \( e = 0.3125 \times 10 = 3.125 \)
    f) \( (76.4 - f) + 5 = 80 \)
        \( 76.4 - f = 80 - 5 = 75 \)
        \( f = 76.4 - 75 = 1.4 \)
    g) \( 0.1 \times 100 < g \leq 1.5 \times 10 \)
        \( 10 < g \leq 15 \)
    h) \( h \div 10 < 2.2 - h \)
        Multiply each side by 10:
        \( h < (2.2 - h) \times 10 \)
        \( h < 22 - 10h \)
        Add \( 10h \) to each side:
        \( h + 10h < 22 \)
        \( 11h < 22 \)
        \( h < 2 \)

Notes
**Week 6**

**Lesson Plan**

**26**

**Notes**

Individual work, monitored, (helped)
(or whole class activity)
BB: 26, 201, 376, 1026
Calculators allowed.
Reasoning, agreement, self-correction, praising
Whole class listing of the factors for 1026.
Feedback for T

**Activity**

1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(26 = 2 \times 13\) Factors: 1, 2, 13, 26
- \(201 = 3 \times 67\) Factors: 1, 3, 67, 201
- \(376 = 2 \times 2 \times 2 \times 47 = 2^3 \times 47\) Factors: 1, 2, 4, 8, 47, 94, 188, 376
- \(1026 = 2 \times 3 \times 3 \times 3 \times 3 \times 19 = 2 \times 3^3 \times 19\) Factors: 1, 2, 3, 6, 9, 18, 19, 27, 57, 1026, 513, 342, 171, 54, 38, 8

8 min

2

**Negative numbers**

T has first 3 rows of each column below written on BB. What are the results of these operations? Ps come to BB or dictate what T should write. Class points out errors. Let’s continue the sequences.

Ps come to BB or dictate what T should write, reasoning with reverse operations. Class agrees/disagrees.

BB:

| a) 5 + 4 = (9) | b) –5 + 9 = (4) | c) 5 – 9 = (–4) | d) –5 – 3 = (–8) |
| 5 + 2 = (7) | –5 + 7 = (2) | 5 – 7 = (–2) | –5 – 1 = (–6) |
| 5 + 0 = (5) | –5 + 5 = (0) | 5 – 5 = (0) | –5 – (–1) = (–4) |
| 5 + (–2) = 3 | –5 + 3 = (–2) | 5 – 3 = 2 | –5 – (–3) = (–2) |
| 5 + (–4) = 1 | –5 + 1 = (–4) | 5 – 1 = 4 | –5 – (–5) = (0) |
| 5 + (–6) = –1 | –5 + (–1) = –6 | 5 – (–1) = 6 | –5 – (–7) = (2) |
| 5 + (–8) = –3 | –5 + (–3) = –8 | 5 – (–3) = 8 | –5 – (–9) = (4) |
| … | … | … | … |

What do you notice? Ps or T points out operations which give the same results or which are reverse operations.

15 min

3

**PbY6a, page 26**

Q.1 Read: *Join up each number to the corresponding point on the number line.*

Set a time limit. Review with whole class. Ps come to BB to draw lines, explaining why they chose the points. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
-4.3 & \quad -3 & \quad -2 & \quad -1 & \quad 0 & \quad 1 & \quad 2 & \quad 3 & \quad 4 & \quad 5 & \quad 6 & \quad 7 & \quad 8 & \quad 9 & \quad 10 & \quad 11 & \quad 12 & \quad 13 \\
-4.3 & \quad -3.2 & \quad -2.5 & \quad -2 & \quad -1.5 & \quad -1 & \quad -0.5 & \quad 0 & \quad 0.5 & \quad 1 \quad 1.5 \quad 2 \quad 2.5 \quad 3 \quad 3.5 \quad 4 \quad 4.5 \quad 5 \quad 5.5 \quad 6 \quad 6.5 \quad 7 \quad 7.5 \quad 8 \quad 8.5 \quad 9 \quad 9.5 \quad 10 \quad 10.5 \quad 11 \quad 11.5 \quad 12 \quad 12.5 \quad 13
\end{align*}
\]

- Let’s list the numbers in increasing order.
- Let’s compare pairs of numbers. Ps suggest the pairs. e.g. –4.3 with –2.5: BB: –4.3 < –2.5 etc. 1.8
- Let’s write operations about it. e.g. BB: \(-2.5 – (–4.3) = 1.8\)

21 min

© CIMT, University of Exeter
Q.2 Read: Solve the problems in your exercise book. Check your answer in context.

Deal with one at a time or set a time limit. Ps picture the problem in their heads, write a plan, calculate the result and check it by picturing the problem again or by imagining the numbers on a number line. Then Ps write the answer in a sentence in Ex. Bks. Review with the whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. Ps tell the story of the problem using their own words.

Solutions: e.g.

a) Rob has £64.50 p but is also £18.50 in debt.
What is his balance?
Plan: £64.50 + (– £18.50) = £64.50 – £18.50 = £46
Answer: Rob’s balance is £46.

b) Ted has £64.50 p but is also £108.50 in debt.
What is his balance?
Plan: £64.50 + (– £108.50) = £64.50 – £108.50 = –£44
Answer: Rob’s balance is –£44.

c) The highest point of a bridge is 2.40 m above a river.
The river is 3.70 m deep at that point.
How far would a coin fall from the highest point on the bridge to the bottom of the river?
Plan: 2.40 m + 3.70 m = 6.10 m ( = 6.1 m)
or 2.40 m – (– 3.70 m) = 6.10 m
Answer: The coin would fall 6.1 metres.

d) A farmer had a bank balance of – £2500 before he harvested and sold his crops.
After the harvest, his bank balance was – £1300.
Was the farmer’s bank balance better or worse after the harvest and by how much?
Plan: – £1300 > – £2500,

– £1300 – (– £2500) = – £1300 + £2500 = + £1200
Answer: The farmer’s balance was better by £1200 after the harvest.

28 min
Q.3 Read: Practise addition in your exercise book.

Set a time limit or deal with one row at a time. Ps write the whole operations in Ex. Bks. and underline the results.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning with reverse operation, or using the cash and debt model to explain the simplest addition in each part and relating it to the others. Class agrees/disagrees. Mistakes discussed and corrected. Show the simplest additions on the class number line if problems or disagreement, or as consolidation.

**Solution:**

a) \((+ 11) + (– 7) = 11 – 7 = 4\)
\((+110) + (– 70) = 40\)
\((+ 1.1) + (– 0.7) = 0.4\)

b) \((+ 6) + (– 15) = 6 – 15 = –9\)
\((+ 60) + (– 150) = –90\)
\((+ 0.6) + (– 1.5) = –0.9\)

c) \((– 23) + (– 41) = –64\)
\((– 230) + (– 410) = –640\)

d) \(15 + (– 80) = 15 – 80 = –65\)
\(150 + (– 800) = –650\)
\(1.5 + (– 8) = –6.5\)

e) \(– 28 + 36 = 8\)
\(– 280 + 360 = 80\)
\(– 2800 + 3600 = 800\)
\(2.8 + 3.6 = 6.4\)

Who remembers what the absolute value of a number is? Allow Ps to explain if they can, otherwise T points to a pair of opposite numbers on the number line (e.g. – 5 and + 5) and asks Ps what their distance is from zero. (5) This is their absolute value.

We write it like this: BB: \[| – 5 | = | + 5 | = 5\]

and read it as 'the absolute value of – 5 equals the absolute value of + 5 which equals 5', as they are both 5 units from zero.

Who can write the absolute value of – 3.6 (4.1, – 100, 0)?

BB: \[| – 3.6 | = 3, | 4.1 | = 4.1, | – 100 | = 100, | 0 | = 0\]

Let's try to think of rules about adding numbers using their absolute value. Ps make suggestions and T repeats in a clearer way if necessary, or T says a rule for addition and Ps formulate another one. e.g.

- We can add a positive and a negative number by subtracting the smaller absolute value from the greater absolute value and keeping the sign of the number with the greater absolute value.
- We can add two or more numbers with the same sign by adding their absolute values and keeping their common sign.
### Y6

#### Activity

<table>
<thead>
<tr>
<th>PbY6a, page 26</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q.4</strong> Read: Practise subtraction in your exercise book.</td>
</tr>
<tr>
<td>Set a time limit or deal with one part at a time. Ps write the whole operations in Ex. Bks. and underline the results.</td>
</tr>
<tr>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning with the cash and debt model [especially for a), c), g and h]), or using comparison (e.g. of temperatures), or showing steps along a number line.</td>
</tr>
<tr>
<td>Class agrees/disagrees. Mistakes discussed and corrected.</td>
</tr>
<tr>
<td>Check each result with reverse addition, demonstrating on a roughly drawn segment of the number link where necessary.</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
</tr>
<tr>
<td>a) ((+ 18) - (+ 5) = 13) ((+ 1.8) - (+ 0.5) = 1.3)</td>
</tr>
<tr>
<td>b) ((+ 7) - (+ 32) = -25) ((+ 0.7) - (+ 3.2) = -2.5)</td>
</tr>
<tr>
<td>c) ((- 43) - (- 15) = -28) ((- 4.3) - (- 1.5) = -2.8)</td>
</tr>
<tr>
<td>d) ((- 6) - (- 21) = 15) ((- 0.6) - (- 2.1) = 1.5)</td>
</tr>
<tr>
<td>e) ((+ 65) - (- 20) = 85) (6.5 - (- 2.0) = 8.5)</td>
</tr>
<tr>
<td>f) ((- 40) - (+ 32) = -72) (- 4 - (+ 3.2) = -7.2)</td>
</tr>
<tr>
<td>g) ((- 33) - 0 = -33) (- 3.3 - 0 = -3.3)</td>
</tr>
<tr>
<td>h) 0 - (+ 81) = -81 (0 - (+ 8.1) = -8.1)</td>
</tr>
<tr>
<td>i) 0 - (- 16) = 16 (0 - (- 1.6) = 1.6)</td>
</tr>
<tr>
<td>j) + 75 - (+ 75) = 0 (- 7.5 - (- 7.5) = 0)</td>
</tr>
<tr>
<td>T points to various numbers and asks Ps for their absolute values.</td>
</tr>
</tbody>
</table>

**Lesson Plan 26**

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
Discussion, reasoning, agreement, self-correction, praising

Reasoning: e.g.

a) I was given 18 p but then I lost 5 p of it and had only 13 p left. 

or 18 is greater than 5 by 13.

**Check:** \(13 + (+ 5) = 18\)

b) \(+ 7 - (+ 32) = -25\), because if we start at 7 and move 32 units in a negative direction, we land on -25, or 7°C is lower than 32°C by 25°C.

**Check:** \(-25 + 32 = +7\)

etc.

Individual work, monitored, helped
Written on BB or SB or OHT
Differentiation by time limit
Discussion, reasoning, agreement, self-correction, praising

Checks done with reverse addition for the original subtraction!
### Lesson Plan 26

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 (Continued)</td>
<td><strong>Check:</strong></td>
</tr>
<tr>
<td>i) (0 - (+53) = 0 + (-53) = -53)</td>
<td>(-53 + 0 = -53)</td>
</tr>
<tr>
<td>j) (0 - (-5.3) = 0 + (+5.3) = 5.3)</td>
<td>(5.3 + (-5.3) = 0)</td>
</tr>
<tr>
<td>k) (-72 - (-8) = -72 + (+8) = -64)</td>
<td>(-64 + (-8) = -72)</td>
</tr>
<tr>
<td>l) (12.6 - (+40.8) = 12.6 + (-40.8) = -28.2)</td>
<td>(-28.2 + 40.8 = 12.6)</td>
</tr>
</tbody>
</table>

Who can put what we have done into words? Ps suggests 'rules' and T repeats in a clearer way if necessary. e.g.

- Instead of subtracting a positive number, we can add its opposite negative number.
- Instead of subtracting a negative number, we can add its opposite positive number.

or

- Instead of subtracting a number, we can add its **opposite** number.

(Which combines the two rules)

**45 min**

**Notes**

Whole class discussion

T prompts if necessary.

Praising, encouragement only
### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $27 = 3 \times 3 \times 3 = 3^3$ (cubic number) Factors: 1, 3, 9, 27
  - Who could write the factors as powers of 3? Ps come to BB or T reminds Ps what 'power' means. (The number of times a number is multiplied by itself)
  - BB: $3^0, 3^1, 3^2, 3^3$

- $202 = 2 \times 101$ (nice) Factors: 1, 2, 101, 202
- $377 = 13 \times 29$ (nice) Factors: 1, 13, 29, 377
- $1027 = 13 \times 79$ (nice) Factors: 1, 13, 79, 1027

What do you notice about these numbers? (They all have 4 factors.)

T tells Ps that a number which has exactly 4 factors is either a 'nice' number (i.e. it has only 2 factors other than itself and 1) or a cubic number (i.e. it has 3 equal factors which could be the edge lengths of a cube).

#### 7 min

### Activity 2

**Operations with integers**

T has operations written on BB. Let's write the addition or subtraction in a simpler way before we do the calculations. Ps come to BB or dictate what T should write and say the result too. Class agrees/disagrees.

**BB:**

- a) $(+ 6) – (– 5) = [6 + 5 = 11]$ (Addition instead of subtraction)
- b) $(– 4) – (+ 5) = [– 4 – 5 = – 9]$ (Omitting the positive sign)
- c) $(– 8) + (– 3) = [– 8 – 3 = – 11]$ (Subtraction instead of addition)
- d) $(+ 80) + (– 30) = [80 – 30 = 50]$ etc.
- e) $(+ 8.8) – (+ 3.3) = [8.8 – 3.3 = 5.5]$
- f) $(- 50) – (+ 22) = [- 50 – 22 = –72]$

#### 10 min

### Activity 3

**PbY6a, page 27**

Q.1 Read: Write each addition and subtraction in a simpler form before doing the calculation.

Set a time limit or deal with 2 or 3 rows at a time. Ask Ps to check their results mentally with reverse operations.

Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- a) i) $(+ 83) + (+ 36) = 83 + 36 = 119$
  - Elicit the relationship between the numbers in i) and ii).
- b) i) $(+ 100) + (– 70) = 100 – 70 = 30$
  - ii) $(+ 1) – (+ 0.7) = 1 – 0.7 = 0.3$
- c) i) $(+ 26) + (– 82) = 26 – 82 = – 56$
  - ii) $(+ 2.6) – (+ 8.2) = 2.6 – 8.2 = – 5.6$

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Reasoning (with cash and debt model or using comparison, or showing on a number line or with the reverse operation, or with the rule), agreement, self-correction, praising

Who had them all correct?

Who made just 1 mistake?

Let's give them a round of applause!
### Activity 3 (Continued)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>d)</strong></td>
<td></td>
</tr>
</tbody>
</table>
| i) $(- 49) + (+ 94) = - 49 + 94 = 45$  
| ii) $(- 4.9) - (- 9.4) = - 4.9 + 9.4 = 4.5$  |
| **e)** | 
| i) $(- 35) + (- 53) = - 35 - 53 = - 88$  
| ii) $(- 3.5) + (+ 5.3) = - 3.5 - 5.3 = - 8.8$  |
| **f)** | 
| i) $0 + (+ 42) = 0 + 42 = 42$  
| ii) $0 - (- 4.2) = 0 + 4.2 = 4.2$  |
| **g)** | 
| i) $0 + (- 27) = 0 - 27 = - 27$  
| ii) $0 - (+ 2.7) = 0 - 2.7 = - 2.7$  |
| **h)** | 
| i) $48 + (- 48) = 48 - 48 = 0$  
| ii) $4.8 - (+ 4.8) = 4.8 - 4.8 = 0$  |

20 min

### Notes

**Lesson Plan 27**

- **Q.2** Read: *Do the calculations in a clever way in your exercise book.*
  - Allow a time limit of 5 minutes. T monitors thoroughly and notes Ps who have grouped the terms in different ways.
  - Review with whole class. T calls Ps to front of class to show their different groupings and explain reasoning. Rest of Ps say which method they like best and why. Mistakes discussed and corrected.
  - **Solution:** e.g.
    - **a)** $45 - 39 + 14 - 15 + 26 - 11$
      
      $= (45 - 15) + (14 + 26) + (- 39 - 11) = 30 + 40 - 50 = 20$
      
      or $= (45 + 14 + 26) - (15 + 39 + 11) = 85 - 65 = 20$
      
    - **b)** $63 - 98 + 37 - 32 + 27 - 37$
      
      $= (63 + 27) - 98 - 32 + (37 - 37) = 90 - 130 + 0 = - 40$
      
      or $= (63 + 37 + 27) - (98 + 32 + 37) = 127 - 167 = - 40$
      
    - **c)** $207 - 57 - 140 + 10 + 23 - 48$
      
      $= 207 - (57 + 140 + 10) + (23 - 48) = 207 - 207 - 25 = - 25$
      
      or $= (207 - 57) - (140 + 10) + 23 - 48 = 150 - 150 - 25 = - 25$
      
    - **d)** $- 200 - 50 - 102 + 300 + 64$
      
      $= (300 + 64) - (200 + 50 + 102 + 42) = 364 - 394 = - 30$
      
      or $= (- 200 - 50 + 300) - (102 + 42) + 64 = 50 - 144 + 64$
      
      $= 50 - 80 = - 30$
      
    - **e)** $1416 - 234 - 172 + 584 - 628$
      
      $= (1416 + 584) - (172 + 628) - 234 = 2000 - 800 - 234$
      
      $= 1200 - 234 = 966$
      
    - **f)** $1000 - 2450 + 1550 - 56 - 944$
      
      $= (1000 + 1550) - 2450 - (56 + 944) = 2550 - 2450 - 1000$
      
      $= 100 - 1000 = - 900$
      
    - **g)** $- (4 - 6) - (- 5) = - (- 2) + 5 = 2 + 5 = 7$
      
    - **h)** $5 - (- 9 - 14) = 5 - (- 23) = 5 + 23 = 28$
    
      or $= 5 + (9 + 14) = 5 + 23 = 28$

28 min

- **Individual work, monitored, helped**
- **Written on BB or use enlarged copy master or OHP**
- **Discussion, reasoning, agreement, self-correcting, praising**
- **Accept any method which gives the correct answer (e.g. calculating from left to right) but give extra praise to Ps who noticed simpler groupings.**
- **If no P has noticed them, T shows them and asks Ps what they think of them.**
- **Feedback for T**

- **Elicit that:** subtracting a negative number is the same as adding the opposite positive number.
Q.3 Read: Find a rule and complete the table. Draw axes in your exercise book and plot the points.

Deal with one table at a time. Ps complete the table first, then T reviews with whole class. Ps discuss and agree on the rule and correct any mistakes in their tables.

Discuss how to draw the axes (x-axis is horizontal and y-axis is vertical). Revise how to plot the points using the x and y values in the table. (e.g. $x = -10$, $y = 10$ gives a point with coordinates $(-10, 10)$). Agree on the range of values needed for each axis (e.g. $-16$ to $+16$) and that there should be a grid line at every unit (or every 0.5 of a unit). Also elicit or remind Ps that the point $(0, 0)$ is the origin.

T chooses Ps to work on BB while rest of Ps work in Ex. Bks. keeping the class together in plotting each point for the first graph at least. Class points out any errors made on BB.

Do you think it is correct to join up the dots? (Yes, as the $x$ and $y$ values can be any value between the dots too, i.e. any decimal number between the whole numbers.)

What do you notice about the graph lines? [e.g. in a), no dots below x-axis, i.e. in 3rd and 4th quadrants, as $y$ values cannot be negative; in b), no dots in 1st and 3rd quadrants, as $x$ and $y$ values cannot have the same sign; the parts of graphs a) and b) in the 2nd quadrant are the same]

Solution:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-15$</td>
<td>$-12$</td>
</tr>
<tr>
<td>$-10$</td>
<td>$-6$</td>
</tr>
<tr>
<td>$-2.5$</td>
<td>$-1$</td>
</tr>
<tr>
<td>$0$</td>
<td>$1$</td>
</tr>
<tr>
<td>$2$</td>
<td>$5.5$</td>
</tr>
<tr>
<td>$10$</td>
<td>$14$</td>
</tr>
<tr>
<td>$10$</td>
<td>$14$</td>
</tr>
<tr>
<td>$15$</td>
<td>$15.5$</td>
</tr>
</tbody>
</table>

Rule: $y = \text{absolute value of } x$, or $y = |x|$

Elicit that the absolute value of a number is its distance from 0.

T reminds Ps that each quarter of the coordinate grid is called a quadrant and they are numbered from the top right quadrant in an anti-clockwise direction.
**Y6**

**Activity**

5 (Continued)

b) \[ y = \text{the opposite of } x, \text{ or } y = -x \]

T might also suggest: \[ y = (-1) \times x \text{ or } y = -1 \times x \]

---

**Lesson Plan 27**

**Notes**

**Extension**

If this line was reflected in the \( y \) axis, what equation would its image show?

\( y = x \)

---

**Q.4 Read:** Write each multiplication as an addition in your exercise book and calculate the sum.

Set a time limit of 2 minutes. Review with whole class. Ps come to BB or dictate what T should write. Class agrees/ disagrees. Mistakes discussed and corrected.

**Solution:**

a) \((+ 7) \times 3 = 7 + 7 + 7 = 21\)

b) \((-7) \times 3 = -7 + (-7) + (-7) = -21\)

c) \((+3) \times 6 = 3 + 3 + 3 + 3 + 3 + 3 = 18\)

d) \((-3) \times 6 = -3 + (-3) + (-3) + (-3) + (-3) + (-3) = -18\)

Which number is the multiplicand? (1st factor) Which number is the multiplier? (2nd factor) What would happen if we made the negative number the multiplier? (The results would be the same, as in multiplication the factors can be exchanged.)

What is 3 multiplied by \(-7\)? What is 6 multiplied by \(-3\)?

What does multiplying by \(-7\) really mean? (7 times the opposite of the multiplicand) Agree that:

BB: \(3 \times (-7) = -21\)

\(6 \times (-3) = -18\)
**Activity 7**

*PbY56a, page 27*

Q.5 Read: *Look at how the product changes. Continue the pattern in your exercise book.*

Set a time limit of 3 minutes. Ps write products in *Pbs*, then continue the pattern of multiplications in *Ex. Bks* if they have time.

Review with whole class. Ps dictate the results, explaining reasoning with the rule of the pattern. Class agrees/disagrees. Ps who had time to write extra multiplications in *Ex. Bks* read them out and T writes them on BB. Class points out any errors.

**Solution:**

<table>
<thead>
<tr>
<th>Positive × Positive</th>
<th>Positive × Negative</th>
<th>Negative × Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>(+8) × (+3) = 24</td>
<td>(−8) × (+3) = −24</td>
<td></td>
</tr>
<tr>
<td>(+8) × (+2) = 16</td>
<td>(−8) × (+2) = −16</td>
<td></td>
</tr>
<tr>
<td>(+8) × (+1) = 8</td>
<td>(−8) × (+1) = −8</td>
<td></td>
</tr>
<tr>
<td>(+8) × (0) = 0</td>
<td>(−8) × (0) = 0</td>
<td></td>
</tr>
<tr>
<td>(+8) × (−1) = −8</td>
<td>(−8) × (−1) = 8</td>
<td></td>
</tr>
<tr>
<td>[(+8) × (−2) = −16</td>
<td>(−8) × (−2) = 16</td>
<td></td>
</tr>
<tr>
<td>(+8) × (−3) = −24</td>
<td>(−8) × (−3) = 24</td>
<td></td>
</tr>
<tr>
<td>(+8) × (−4) = −32</td>
<td>(−8) × (−4) = 32</td>
<td></td>
</tr>
</tbody>
</table>

Who can tell me the rules for multiplying by negative numbers? Ps suggest rules in their own words and T repeats more clearly or concisely if necessary. e.g.

- The product of a positive and a negative number is **negative**.
- The product of 2 negative numbers is **positive**.
- Multiplying by a negative number means multiplying the opposite of the multiplicand by the absolute value of the multiplier.

**Notes**

- Individual work, monitored, helped
- Written on BB or SB or OHT
- Differentiation by time limit
- Reasoning, agreement, self-correction, praising
- Extra praise for Ps who wrote more multiplications correctly!

<table>
<thead>
<tr>
<th>Positive × Positive</th>
<th>Positive × Negative</th>
<th>Negative × Negative</th>
</tr>
</thead>
<tbody>
<tr>
<td>(−25) × (−3) = 75</td>
<td>(−25) × (−2) = 50</td>
<td></td>
</tr>
<tr>
<td>(−25) × (−1) = 25</td>
<td>(−25) × 0 = 0</td>
<td></td>
</tr>
<tr>
<td>(−25) × (−1) = −25</td>
<td>(−25) × (−2) = −50</td>
<td></td>
</tr>
<tr>
<td>(−25) × (−3) = −75</td>
<td>(−25) × (−4) = −100</td>
<td></td>
</tr>
</tbody>
</table>

Whole class discussion

Praising, encouragement only

Ps repeat the rules in unison.

(General rule which combines the 2 rules above it.)
**R:** Calculations  
**C:** Understanding multiplication and division of integers  
**E:** Multiplying by a decimal

### Activity

#### 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **28** = $2 \times 2 \times 7 = 2^2 \times 7$  
  Factors: 1, 2, 4, 7, 14, 28
- **203** = $7 \times 29$ (nice)  
  Factors: 1, 7, 29, 203
- **378** = $2 \times 3 \times 3 \times 3 \times 7 = 2 \times 3^3 \times 7$  
  Factors: 1, 2, 3, 6, 7, 9, 14, 18, 378, 189, 126, 63, 54, 42, 27, 21
- **1028** = $2 \times 2 \times 257 = 2^2 \times 7$  
  (and 257 is not divisible by 2, 3, 5, 7, 11 or 13 and $2 \times 17 > 257$)

#### 2

**PbY6a, page 28**

**Q.1** Read: *Work out a rule and complete the table.*

Set at time limit of 3 minutes. Remind Ps to check their rule using values from the table.

Review with whole class. Ps come to BB or dictate to T, giving the rule that they used. Who agrees? Who used a different rule? etc. Mistakes discussed and corrected.

Who can write the rule in a different way? Ps (and T) suggest some and class decides whether or not they are valid by substituting values from the table.

**Solution:**

<table>
<thead>
<tr>
<th>a</th>
<th>25</th>
<th>8</th>
<th>– 12</th>
<th>9</th>
<th>– 10</th>
<th>3.1</th>
<th>– 10.5</th>
<th>0.3</th>
<th>0</th>
<th>– 1.2</th>
<th>15</th>
<th>– 1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>–100</td>
<td>–32</td>
<td>48</td>
<td>–36</td>
<td>40</td>
<td>–12.4</td>
<td>400</td>
<td>42</td>
<td>–1.2</td>
<td>0</td>
<td>48</td>
<td>– 6</td>
</tr>
</tbody>
</table>

**Rule:** 

- **a** = the opposite of $b \div 4$, $b = 4 \times$ the opposite of $a$
- or $b = 4 \times (-a)$, $b = (-4 \times a)$, $b = (-4) \times a$, $b = -4a$
- or $a = (-b) \div 4$, $a = - (b \div 4)$, $a = b \div (-4)$, $a = -b \div 4$, $[a = \frac{-b}{4}, a = \frac{-4}{b}]$

Let’s look at the form of the rule where the divisor is a negative number. Let’s write it using the values from the 2nd column. Ps dictate what T should write.

**BB:** $a = b \div (-4) \rightarrow 8 = (-32) \div (-4)$

What does dividing by $-4$ mean? Ps say what they think and T repeats in a clear way if necessary. Elicit that:

dividing by $-4$ means dividing the **opposite** of the dividend by 4.

**Notes**

- Individual work, monitored, (helped)  
- (or whole class activity)
- **BB:** 28, 203, 378, 1028
- Calculators allowed
- Reasoning, agreement, self-correction, praising

**Whole class listing of of the factors of 378**

<table>
<thead>
<tr>
<th>e.g.</th>
<th>28</th>
<th>378</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>7</td>
<td>189</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>63</td>
<td>3</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>21</td>
<td>3</td>
</tr>
</tbody>
</table>

| 203 | 7 | 514 | 2 | 7 | 7 |
| 29 | 29 | 257 | 1 |

**Bold** numbers are issing.

- Not all forms need to be shown.

**Whole class discussion**

- Involve severalPs.

- Praise all contributions.
- Ps repeat the rule in unison.
### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>PBY6a, page 28</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>Q.2 Read:</td>
</tr>
<tr>
<td></td>
<td>Solve the problems in your exercise book.</td>
</tr>
<tr>
<td></td>
<td>Write only the results here.</td>
</tr>
</tbody>
</table>

Deal with one at a time. Ps read question themselves, picture the problem in their heads and think whether the result will be more or less, write a plan, do the calculation and write the result in Pbs.

Review with whole class. Ps show results on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

T chooses a P to say the answer in a sentence.

**Solution:**

a) The temperature was \( 9^\circ C \). It fell by \( 6^\circ C \), then by \( 5^\circ C \), then it rose by \( 2^\circ C \) and rose again by \( 5^\circ C \). What is the temperature now?

**Plan:** \( 9 - 6 - 5 + 2 + 5 = 16 - 11 = 5^\circ C \)

**Answer:** The temperature now is \( 5^\circ C \).

b) James had £100 in cash but owed £20. Then £10 of this debt was cancelled. What is his balance now?

**Plan:** £100 + (– £20) – (– £10) = £80 + £10 = £90

**Answer:** His balance is now £90.

c) Sue had £100 in cash but was £120 in debt. She spent another £40. What is her balance now?

**Plan:** (£) 100 + (– 120) + (– 40) = 100 – 120 – 40 = –60 or (£) (100 – 120) – 40 = –20 – 40 = –60

**Answer:** Her balance is now –£60.

d) The temperature is falling steadily by \( 2^\circ C \) every hour. It is now \( 0^\circ C \).

i) What will the temperature be in 3 hours' time?

**Plan:** \( 0 + (– 2) \times 3 = 0 + (– 6) = 0 – 6 = –6^\circ C \)

**Answer:** In 3 hours' time the temperature will be –6°C.

ii) What was the temperature 4 hours ago?

**Plan:** \( 0 – (– 2) \times 4 = 0 – (– 8) = 0 + 8 = 8^\circ C \)

**Answer:** Four hours ago, the temperature was 8°C.

Revise the rules or laws for addition, subtraction and multiplication of negative integers. Ps say them in their own words and T repeats more clearly if necessary. Ps also give examples for each type. e.g.

- When adding a positive and a negative integer, subtract the smaller from the greater absolute value and take the sign of the greater absolute value.
- When subtracting a negative integer, add its opposite value.
- When multiplying by a negative integer, multiply the opposite of the multiplicand by the absolute value of the multiplier.

<table>
<thead>
<tr>
<th>Lesson Plan 28</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

© CIMT, University of Exeter
### Activity 4

**PbY6a, page 28**

**Q.3** Read: *Note how the quotient changes. Check with reverse multiplication.*

Set a time limit or deal with one column at a time.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning by pointing out the pattern or with reverse multiplication. Class agrees/disagrees. Mistakes discussed and corrected. Ps might notice relationships among the divisions.

_Solution:_

a) \((+ 27) \div (+ 3) = 9\)

b) \((+ 27) \div (- 3) = -9\)

\((+ 18) \div (+ 3) = 6\)

\((+ 18) \div (- 3) = -6\)

\((+ 9) \div (+ 3) = 3\)

\((+ 9) \div (- 3) = -3\)

\(0 \div (+ 3) = 0\)

\((0) \div (- 3) = 0\)

\((- 9) \div (+ 3) = -3\)

\((- 9) \div (- 3) = 3\)

\((- 18) \div (+ 3) = -6\)

\((- 18) \div (- 3) = 6\)

\((- 27) \div (+ 3) = -9\)

\((- 27) \div (- 3) = 9\)

What are the rules or laws for dividing by an integer? Ps say the rules in their own words and T writes on BB. Elicit that:

BB: • positive \(\div\) positive \(\rightarrow\) positive

• positive \(\div\) negative \(\rightarrow\) negative

• negative \(\div\) positive \(\rightarrow\) negative

• negative \(\div\) negative \(\rightarrow\) positive

[Similar to the rules for 2-factor multiplication]

---

### Notes

**Lesson Plan 28**

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Reasoning, e.g.

\((- 18) \div (+ 3) = -6\), because

\(-6 \times 3 = -18\), etc.

Agreement, self-correction, praising

\(8 \div (- 2) = -4\) \(\div 2\)

\(4 \div (- 2) = -2\) \(\div 2\)

\(2 \div (- 2) = -1\) \(\div 2\)

\(0 \div (- 2) = 0\)

\(-2 \div (- 2) = 1\) \(\times 2\)

\(-4 \div (- 2) = 2\) \(\times 2\)

\(-8 \div (- 2) = 4\) \(\times 2\)

Whole class discussion

Agreement, praising

Ps could write rules in _Ex Bks_ too.

---

**Activity 5**

**PbY6a, page 28**

**Q.4** Read: *Fill in the missing numbers.*

Set a time limit. Remind Ps to check their solutions.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning using the correct names of the components.

\[\text{e.g. } \text{multiplicand} \div \text{product} \div \text{multiplier}\]

\[\text{dividend} \div \text{quotient} \div \text{divisor} \div \text{dividend} \div \text{quotient}, \text{etc.}\]

Class agrees/disagrees. Mistakes discussed and corrected.

_Solutions:_

a) \([-9] \times (-5) = 45, \quad -2.5 \times \left[\frac{5}{2}\right] = 12.5, \quad \left[-32\right] \times 3 = -96, \quad \left[\frac{4}{3}\right] \times (-7) = -28\]

b) \(200 \div 40 = \left[\frac{5}{4}\right], \quad -36 \div (+4) = -9, \quad -60 \div (-12) = \left[\frac{5}{6}\right], \quad 48 \div (+8) = -6\]

c) \([28] \div (+7) = -4, \quad [-66] \div (-6) = 11, \quad \left[\frac{6}{3}\right] \div 5 = 1.2, \quad [20] \div (-3) = -40\]

d) \((-75) \div \left[\frac{3}{2}\right] = -25, \quad (-39) \div \left[-3\right] = 13, \quad 42 \div \left[\frac{3}{2}\right] = 1.4, \quad 150 \div [-3] = -50\]
<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 28</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td><strong>Notes</strong></td>
</tr>
</tbody>
</table>

**PbY6a, page 28**

Q.5 Read: *Calculate the result in 2 different ways where possible in your exercise book.*

Set a time limit or deal with one row at a time (or do first 2 rows in class and set the 3rd row for homework). (If necessary do part a) with the whole class first as a model for Ps to follow.)

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Ps say which method they think is easier.

**Solutions:**

a) \((-8 + 5) \times 7 = -3 \times 7 = -56 + 35 = \boxed{-21}\)

b) \((-15 - 8) \times 4 = -23 \times 4 = -60 - 32 = \boxed{-92}\)

c) \((-7 + 5) \times (-9) = -2 \times (-9) = 63 - 45 = \boxed{18}\)

d) \((-28 + 14) \div 7 = -14 \div 7 = -4 + 2 = \boxed{-2}\)

e) \((-18 - 12) \div 3 = -30 \div 3 = -6 - 4 = \boxed{-10}\)

f) \((-8 + 20) \div (-4) = 12 \div (-4) = 2 - 5 = \boxed{-3}\)

g) \((-21 + 21) \div 13 = 0 \div 13 = \boxed{0}\)

h) \((-12 + 5) \div 0 \neq \text{anything (as it does not make sense to divide by zero)}\)

i) \((15 - 30) \div (-1) = -15 \div (-1) = -15 + 30 = \boxed{15}\)

j) \(-66 \div (24 - 18) = -66 \div 6 = \boxed{-11}\)

k) \(-80 \div (-6 + 16) = -80 \div 10 = \boxed{-8}\)

l) \(13 \div (-7 + 8) = 13 \div 1 = \boxed{13}\)

**45 min**

Individual work, monitored helped

(or whole class activity if time is short, with Ps coming to BB or dictating to T)

Written on BB or use enlarged copy master or oHP

Discussion, reasoning, agreement, self-correction, praising

In j), k) and l), only one way of calculating is possible.
Y6

R: Calculations
C: Mental calculations with integers (with or without models)
E: Problems

Activity
1

Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 29 is a prime number
  - Factors: 1, 29

- 204 = 2 × 2 × 3 × 17 = 2² × 3 × 17
  - Factors: 1, 2, 3, 4, 6, 12, 17, 34, 51, 68, 102, 204

- 379 is a prime number
  - Factors: 1, 379
  (As not exactly divisible by 2, 3, 5, 7, 11, 13, 17 or 19, and 23 × 23 > 379)

- 1029 = 3 × 7 × 7 × 7 = 3 × 7³
  - Factors: 1, 3, 7, 21, 49, 147, 343, 1029

2

Erratum

In e) in PbY6a, page 29, 'number' should be 'numbers'

PbY6a, page 29, Q.1

Read: Complete the sentences so that they are well-known laws.

Deal with one sentence at a time. T chooses a P to read out the sentence, saying 'something' at each missing word.

Allow Ps a minute to think about it ad discuss with their neighbours if they wish, then Ps come to BB to write missing words and read out the completed sentence. Who agrees? Who thinks it should be something else? Ps write examples of operation on BB as a check. Class agrees on wording and Ps write missing words in sentence in Pb.

Class reads completed sentence in unison.

Solution:

a) The sum of two (or more) negative numbers is negative and its absolute value is the sum of the numbers' absolute values.

b) To add a positive and a negative number, calculate the difference of the absolute values and take the sign of the number which has the greater absolute value.

c) To multiply by a negative number, multiply the opposite number of the multiplicand by the opposite positive number.

d) The product of a negative and a positive number is negative and its absolute value is equal to the product of their absolute values.

e) The product or quotient of two negative numbers is positive.

What additional information could be written at the end of this sentence? T asks 2 or 3 Ps what they think.

(… and its absolute value is the product or quotient of the two numbers' absolute values.)

Lesson Plan

29

Notes

Individual work, monitored,
(helped)
(or whole class activity)
BB: 29, 204, 379, 1029
Calculators allowed
Reasoning, agreement, self-
correction, praising
Ps join up the factor pairs for 204 and 1028.

| 204 | 1029 |
| 2 | 343 | 3 |
| 102 | 49 |
| 51 | 3 |
| 17 | 7 | 7 |
| 1 | 1 |

8 min

15 min

Whole class activity
Written on BB or use enlarged copy master or OHP
At a good pace
In good humour!
Discussion, reasoning,
checking, agreement, praising

Missing words underlined.

e.g. –3 + (– 5) = –8

–4 + (– 12) = –2

8 × (– 6) = –48

–7 × (– 5) = 35

–9 × 3 = 9 × (– 3) = –27

–100 × (– 10) = 1000

–100 ÷ (– 10) = 10

© CIMT, University of Exeter
Activity

Q.2 Read: Practise calculation in your exercise book.

Deal with one part at a time (a, b, c, d, e). Ps write whole operation in Ex. Bks. Set a short time limit.

Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning with the rule or with reverse operation. Class agrees/disagrees. Mistakes discussed and corrected. If problems or disagreement, use cash and debt model or comparison or draw diagrams on BB.

Solution:

a) i) \((+ 12.3) + (- 24) = 12.3 - 24 = -11.7\)
   ii) \((- 2300 + (1100) = -2300 + 1100 = -1200\)
   iii) \(6.5 + (-2.3) + (+5) + (-9.2) = 11.5 - 11.5 = 0\)

b) i) \(4.7 - (+5.3) = 4.7 - 5.3 = -0.6\)
   ii) \((-210 - (+120) = -210 - 120 = -330\)
   iii) \(6.8 - (-2) = 6.8 + 2 = 8.8\)
   iv) \(-40 - (-50) = -40 + 50 = 10\)

c) i) \(+8.1 \times (-6) = -8.1 \times 6 = 48.6\)
   ii) \(-150 \times 9 = -1350\)
   iii) \(-10.5 \times (-5) = 52.5\)
   iv) \(-2 \times 3 \times (-1) \times (+4) \times (-5) = 6 \times -20 = -120\)

d) i) \(3 \div (-2) = -3 \div 2 = -1.5\)
   ii) \((-105) \div 21 = -5\)
   iii) \(-8.4 \div (-7) = 1.2\)
   iv) \(-123 \div 1 = -123\)
   v) \(41.3 \div (-1) = -41.3\)

e) i) \((-3) \times (-3) = 9\)
   ii) \((-3) \times (-3) \times (-3) = 9 \times (-3) = -27\)

How could we write 9 and -27 as powers of (-3)? Ps come to BB to write and say the numbers, or T reminds Ps about powers (the number of times a number is multiplied by itself).

BB: 9 = \((-3) \times (-3) = (\text{-3})^2\) [square number],
read as ‘-3 to the power 2’ or ‘-3 squared’.

-27 = \((-3) \times (-3) \times (-3) = (\text{-3})^3\) [cubic number]
read as ‘-3 to the power 3’, or ‘-3 cubed’.

What are the values of these numbers? Elicit that:

BB:
iii) \((-3)^2 = [(-3) \times (-3) \times (-3) \times (-3)]
   = (-3)^2 \times (-3)^3 = 9 \times 9 = 81\)
iv) \((-4)^3 = [(-4) \times (-4) \times (-4) = (-4)^2 \times (-4)
   = 16 \times (-4) = -64\)

Notes

Individual work, monitored, (helped)

Written on BB or use enlarged copy master or OHP

At a good pace

Discussion, reasoning, agreement, praising

Feedback for T

Reasoning: e.g.
a) with cash and debt model
I have £12.30 in cash but am £24 in debt, so my balance is –£11.70.

b) by comparison:
4.7 is less than 5.3 by 0.6

c) iv) and e):
Point out that the product of an odd number of negative factors is negative:
\(-2 \times -1 \times -5 = 2 \times -5 = -10\)

but the product of an even number of negative factors is positive: \(-3 \times -3 = 9\)

Whole class discussion

Involve many Ps.

Extra praise if a P points out that 9 is also equal to \(3^2\), so a square number can have positive or negative equal factors.

Further examples:
\(1^2 = 1, \quad 1^9 = 1\)
\((-1)^2 = 1, \quad (-1)^9 = -1\)
**Activity 4**

**Cartesian coordinates**

Let’s draw a Cartesian coordinate system and plot these points on it.

BB: A (– 2, 2), B (– 2, 4), C (– 4, 4), D (– 6, 2)

Discuss how to draw the x and y axes and what their range should be (e.g. from – 6 to 6). T (or P) works on BB and Ps work in Ex. Bks at the same time.

Ps come to BB to plot and label the points, pointing to the x and y values and moving their fingers along the appropriate grid lines until they meet. Class agrees/disagrees. If we join up the points, what shape have we made? (trapezium ABCD)

Let’s change the coordinates and see what happens to the shape. T gives instructions and Ps draw new shapes on the grid on BB, while rest of Ps work in Ex. Bks.

a) Multiply the x-coordinate of each point by – 1 but keep the y-coordinate the same. Ps dictate the new coordinates.

BB: A’ (2, 2), B’ (2, 4), C’ (4, 4), D’ (6, 2)

Ps come to BB to plot the new points, label them A’, B’, C’ and D’, and join them up to make trapezium A’B’C’D’.

What can you say about the two shapes? (They are the same size and shape, i.e. they are congruent.)

How can you get from ABCD to A’B’C’D’? (Reflection in the y-axis)

Repeat for other transformations of ABCD.

b) Multiply the y-coordinate by (– 1) but keep the x-coordinate the same.

BB: A” (– 2, – 2), B” (– 2, – 4), C” (– 4, – 4), D” (– 6, – 2)

(Reflection in the x-axis, ABCD and A’B’C’D’ are congruent)

c) Multiply both coordinates by (– 1).

BB: A” (– 2, – 2), B” (– 2, – 4), C” (– 4, – 4), D” (– 6, – 2)

(Rotation by 180° around O, ABCD and A”B”C”D” are congruent)

**Extension**

d) Multiply both coordinates by (– 2.)

BB: A* (4, – 4), B* (4, – 8), C* (8, – 8), D* (12, – 4)

(Enlargement in the origin by a scale factor of – 2. Similar.)

e) Divide both coordinates by (– 2)

BB: A (1, – 1), B (1, – 2), C (2, – 2), D (3, – 1)

(Reduction in the origin by scale factor \( \frac{1}{2} \). Similar shapes)

**Notes**

Whole class activity but individual drawing

Grid drawn on BB or use enlarged copy master or OHP

(Slow Ps could use copy of master to save time drawing the axes)

Discuss how to draw the axes, plot the points and label them. T reminds Ps about any aspects they have forgotten.

After each shape has been drawn, discuss the relationships of the new shape to the original shape ABCD and what translation would move the original shape to the new position.

Discussion, agreement, praising

T reminds Ps of names of transformations where necessary and writes them on BB.

Elicit that shapes with opposite coordinate values are mirror images of each other, reflected in the origin, i.e. are in diagonally opposite quadrants in the grid.

What would happen if you divided both coordinates by (– 1)?

(Same result as multiplying by 1: each coordinate would have the opposite value.)

Whole class activity

Draw trapezium ABCD on a new grid or use new enlarged copy master or OHP

Ps come to BB to write the coordinates and plot the points.

Discuss the relationships and transformations.

Praising, encouragement only

(\( \overline{A} \) is A b’ar)
**Activity 5**  
*PbY6a, page 29*

Q.3 Read: *Fill in the tables according to the given rules.*

Set a time limit. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) **Rule:** \( y = (-2) \times x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-6)</th>
<th>(-5)</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>( 0)</th>
<th>( 1)</th>
<th>( 2)</th>
<th>( 3)</th>
<th>( 4)</th>
<th>( 5)</th>
<th>( 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>12</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
</tr>
</tbody>
</table>

b) **Rule:** \( y = (-2) \times x + 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-6)</th>
<th>(-5)</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
<th>( 0)</th>
<th>( 1)</th>
<th>( 2)</th>
<th>( 3)</th>
<th>( 4)</th>
<th>( 5)</th>
<th>( 6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>15</td>
<td>13</td>
<td>11</td>
<td>9</td>
<td>7</td>
<td>5</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

Read: *In your exercise book, draw a coordinate grid.*

On it plot the \((x, y)\) points for both tables. Use a different colour for each table.

Agree on the range necessary for each axis, then Ps draw axes and plot points in *Ex. Bks*, while Ps come out one after the other to plot a point on the grid on the BB.

Is it correct to join up the points? (Yes, because \(x\) and \(y\) could be any value between the dots too, i.e. they could be fractions or decimals.) Ps use rulers to join up dots of the same colour.

**What do you notice?** (The two lines are parallel; the graph line for a) passes through the origin; the graph line for b) is 3 units higher on the \(y\)-axis).

**Solution:**

![Graph of equations](image)

Elicit the short form of each equation, then a P writes it beside the relevant line.

T points to each line in turn and class says its equation in unison.
Y6

Activity

Factorising 30, 205, 380 and 1030. Revision, activities, consolidation

PbY6a, page 30

Solutions:

Q.1

a) \(55 - 0.5 = 54.5\) 

b) \(16 - 4.3 = 11.7\) 

c) \(-76 - (-2.8) = -73.2\) 

d) \(-32 - (-0.5) = -31.5\) 

e) \(84 - (-11.5) = 95.5\) 

f) \(-90 - 5.6 = -95.6\) 

g) \(-11 - 0.11 = -11.11\) 

h) \(0.44 - 6.9 = -6.46\) 

Q.2 and Q.3

30 = \(2 \times 3 \times 5\)

Factors: 1, 2, 3, 5, 6, 10, 15, 30

205 = \(5 \times 41\) (nice)

Factors: 1, 2, 4, 5, 10, 19, 20, 38, 76, 95, 190, 380

380 = \(2^2 \times 5 \times 19\)

Factors: 1, 2, 4, 5, 10, 19, 20, 38, 76, 95, 190, 380

1030 = \(2 \times 5 \times 103\)

Factors: 1, 2, 5, 10, 103, 206, 515, 1030

(or set factorising as homework at the end of Lesson 29 and review at the start of Lesson 30)

Extra praise if Ps notice that:

in Q.2:
The lines \(y = x \div 2\) and \(y = x - 0.2\) meet at the point with coordinates (0.4, 0.2).

in Q.3:
The line \(y = x - 0.3\) is parallel to line \(y = x - 0.2\) and is 0.1 below it on the y-axis.

The lines \(y = x - 0.3\) and \(y = x \div 2\) meet at the point with coordinates (0.6, 0.3).