Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

• \( 51 = 3 \times 17 \)  
  Factors: 1, 3, 17, 51

• \( 226 = 2 \times 113 \)  
  Factors: 1, 2, 113, 226

• \( 401 \) is a prime number  
  Factors: 1, 401
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and \( 23 \times 23 > 401 \))

• \( 1051 \) is a prime number  
  Factors: 1, 1051
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and \( 37 \times 37 > 1051 \))

Collecting data

a) Let's collect data on the months in which you were born.

How could we do it? (List the months and keep a tally.) T draws table on BB, as dictated by Ps and Ps draw one in Ex. Bks.

Ps dictate their birthday month at speed in order round class and T keeps a tally on BB while rest of Ps do the same in Ex. Bks.

Ps count up the tally marks and write the numbers below.

BB: e.g. for 29 Ps:

```
Jan Feb Mar Apr May Jun Jul Aug Sep Oct Nov Dec
||| | | | | | | | | | | | | | | | |
| 3 2 4 1 0 3 1 5 2 3 2 3 (29) |
```

T (Ps) think of questions to ask about the data. e.g.:

• Which month had most (least) birthdays? (August, May)
• What is the difference between their data? (5 – 0 = 5)
• Which number of birthdays is the most common? (3) etc.

b) i) Work out your age in months, then we will collect the data.

T (P) demonstrates how to calculate on BB first if necessary: 

e.g. Born on 12 April 1993; Today’s date: 5 December 2003

Age in months: \( 10 \times 12 + 8 = 120 + 8 = 128 \)

T draws a table on BB and Ps do the same in Ex. Bks. Ps dictate ages in order round class. T (P) keeps a tally and Ps do the same in Ex. Bks. Ps count up the tally marks and write the numbers below. We say that these numbers are the frequency of the data.

BB: e.g. for 29 Ps:

```
118 119 120 121 122 123 124 125 126 127 128 129 130 131 132 133
||| | | | | | | | | | | | | | |
| 1 0 1 2 2 1 3 4 2 3 5 2 0 1 1 1 |
```

Ps think of questions to ask about the data.
Activity 2 (Continued)

ii) Let's list the data in increasing order. Ps dictate to T. e.g.
BB: 118, 120, 121, 121, 122, 122, 123, 124, 124, 125, 125, 125, 125, 126, 126, 127, 127, 128, 128, 128, 128, 128, 129, 129, 131, 131, 132, 133 (months)
Which number is in the middle of the set of data? (126) (There are 29 numbers, so the middle number is the 15th.) T: We say that 126 is the median of the data.

iii) Let's calculate the difference between the greatest and smallest data. Ps dictate what T should write.
BB: e.g. 133 – 118 = 15 (months)
T: We say that the range of this data is 15 months.

iv) Which age is the most frequent? (128 months) T: We say that this is the mode of the data.

v) How could we work out the average age of the class? Ps make suggestions (e.g. add up the 29 numbers, then divide their sum by 29) or extra praise if a P suggests:
BB: (118 + 120 + 2 × 121 + 2 × 122 + 123 + 3 × 124 + 4 × 125 + 2 × 126 + 3 × 127 + 5 × 128 + 2 × 129 + 131 + 132 + 133) ÷ 29 = 3646 ÷ 29 = 125.7
T: We say that 125.7 is the mean of the data.

vi) Let's show the data in a graph.
If no computer is available, T could have axes already prepared and Ps come to BB to complete the graph. e.g.

BB: No. of pupils

<table>
<thead>
<tr>
<th>Age (months)</th>
<th>0</th>
<th>118</th>
<th>120</th>
<th>122</th>
<th>124</th>
<th>125</th>
<th>126</th>
<th>128</th>
<th>130</th>
<th>132</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. of pupils</td>
<td>1</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

T: We say that 125.7 is the mean of the data.

Notes

[Or T inputs the data on computer and uses a program to order them.]

BB: median middle value

range difference between greatest and smallest values

mode most frequent value

Agreement, praising
Ps use a calculator as a check.
BB: mean average value

[If possible, T shows the calculation of the mean on a computer (e.g. using Exel) and also gets the computer to draw a graph of the data, with a line at the mean.
Ps should experience how calculators and computers can help in ordering and calculating when dealing with large sets of data.]

Otherwise, T can use enlarged copy master or OHP

PbY6a, page 51

Q.1 Read: The heights of the 7 peaks in a mountain range are:
945 m, 1023 m, 1311 m, 996 m, 1286 m, 1504 m, 1150 m
Deal with one part at a time if class is not very able, otherwise set a time limit. Ps read questions themselves, do listing and calculations in Ex. Bks and write results in Pbs.
Review with whole class. Ps could show answers to b) to d) on slates or scrap paper on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

Solution:
a) Write the data in increasing order in your exercise book.
945 m, 996 m, 1023 m, 1150 m, 1286 m, 1311 m, 1504 m

Individual work, monitored, helped
Ps who have been to the top of a mountain tell class about it and/or T shows pictures of famous mountain ranges.
Differentiation by time limit
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Class dictates to T.
### Activity 3 (Continued)

b) **Calculate the difference between the highest and the lowest heights.**

The **range** of the sample is **559 m**.

c) **Calculate the average height of these 7 peaks.**

BB: \(\frac{945 + 1023 + 1311 + 996 + 1286 + 1504 + 1150}{7} = 8215 \div 7 = 1173.6 \text{ m}\)

The **mean** of the sample is **1173.6 m**.

d) **Find the middle value among the 7 heights.**

The **median** of the sample is **1150 m**.

### Lesson Plan 51

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>BB: 1504 m – 945 m = <strong>559 m</strong></td>
</tr>
</tbody>
</table>

[If possible, T checks the calculation of the mean, and shows a graph for the data, on a computer.]

(4th in ordered list of data)

Feedback for T

---

### Activity 4

**PhY6a, page 51**

Q. 2 Read: *These are the masses of 8 pumpkins.*

8.3 kg, 9.7 kg, 7.9 kg, 9.1 kg, 9.0 kg, 7.6 kg, 9.0 kg, 7.9 kg

Deal with one part at a time if class is not very able, otherwise set a time limit. Ps read questions themselves, do listing and calculations in Ex. Bks and write results in Pbs.

Review with whole class. Ps could show answers to b) to d) on slates or scrap paper on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) **Write the data in increasing order in your exercise book.**

7.6 kg, 7.9 kg, 7.9 kg, 8.3 kg, 9.0 kg, 9.1 kg, 9.0 kg

b) **Calculate the difference between the heaviest and the lightest pumpkin.**

The **range** of the sample is **2.1 kg**.

c) **Which is the most frequent value?**

The **mode** of the sample is **7.9 kg and 9.0 kg**.

d) **Calculate the average mass of the 8 pumpkins.**

BB: \(8.65 \text{ kg}\)

The **mean** of the sample is **8.65 kg**.

e) **Find the middle value among the masses.**

BB: \(\frac{8.3 + 9.0}{2} = \frac{17.3}{2} = 8.65 \text{ kg}\)

The **median** of the sample is **8.65 kg**.

Review the vocabulary. Ps explain in their own words what they understand by mean, mode, median, range, frequency.

---

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Lesson Plan 51

Y6

Activity

5

PbY6a, page 51

Q.3 Read: These were the scores of pupils in a class who took a mathematics test which had a maximum score of 50 marks.

How many Ps were in the class? (30)

Read: In your exercise book:
   a) write the data in a table  b) draw a bar chart.

First discuss (or suggest if Ps have no ideas) the form of the table (e.g. score on top row and number of Ps on bottom row) and the bar chart (e.g. scores on horizontal x-axis and number of pupils on vertical y-axis).

Set a time limit for drawing the table, then review quickly at BB and mistakes corrected before Ps draw the bar chart.

Review the bar chart at BB and mistakes corrected before dealing with the questions one at a time. Ps could show values on scrap paper or slates in unison. Ps responding correctly explain to Ps who were wrong. Agreed values written below words in Pbs.

Solution:

a)

<table>
<thead>
<tr>
<th>Marks</th>
<th>26</th>
<th>27</th>
<th>28</th>
<th>29</th>
<th>30</th>
<th>31</th>
<th>32</th>
<th>33</th>
<th>34</th>
<th>35</th>
<th>36</th>
<th>37</th>
<th>38</th>
<th>39</th>
<th>40</th>
<th>41</th>
<th>42</th>
<th>43</th>
<th>44</th>
<th>45</th>
<th>46</th>
<th>47</th>
<th>48</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pupils</td>
<td>1</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

b) [Bar chart image]

What is:
   i) the range  (48 – 26 = 22) marks
   ii) the mode (43 is the most frequent mark)
   iii) the mean of the data?  (39.6 marks)

Mean = \[\frac{[26 + 27 + 30 + 40 + 45 + 2 \times (29 + 32 + 35 + 38 + 47) + 3 \times (41 + 48) + 4 \times 44 + 5 \times 43]}{30}\]

= \[\frac{(168 + 2 \times 181 + 3 \times 89 + 176 + 215)}{30}\]

= \[\frac{(168 + 362 + 267 + 391)}{30}\]

= 1188 \div 30 = 39.6 (marks)

What is the median of the data? Ps dictate marks in increasing order round class and T writes on BB. Class points out errors. Agree that there are two middle marks: 41 and 43. (15th, 16th) Ps come to BB to calculate the mean of 41 and 43.

BB: \[\frac{41 + 43}{2} = \frac{84}{2} = 42\]

The median of the data is 42.

Extension

45 min
**Lesson Plan**

**Y6**

### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(52 = 2 \times 2 \times 13 = 2^2 \times 13\)  
  Factors: 1, 2, 4, 13, 26, 52

- 227 is a prime number  
  Factors: 1, 227

(as not exactly divisible by 2, 3, 5, 7, 11, 13, and 17 \(\times 17 > 227\))

- \(402 = 2 \times 3 \times 67\)  
  Factors: 1, 2, 3, 6, 67, 134, 201, 402

- \(1052 = 2 \times 2 \times 263 = 2^2 \times 263\)  
  Factors: 1, 2, 4, 263, 526, 1052

---

**Notes**

Individual work, monitored (or whole class activity)

BB: 52, 227, 402, 1052

Calculators allowed.

Reasoning, agreement, self-correction, praising

---

### Activity 2

**Pie Chart**

Listen carefully and note the data in your Ex. Bks.

In a school there are 720 pupils. There are 180 pupils in Year 3, 216 pupils in Year 4, 198 pupils in Year 5 and the rest are in Year 6.

a) What part of all the pupils in the school is each year group?

Ps come to BB or dictate what T should write. Class agrees/disagrees.

- Year 3: \(\frac{180}{720} = \frac{18}{72} = \frac{9}{36} = \frac{1}{4}\)
- Year 4: \(\frac{216}{720} = \frac{72}{240} = \frac{6}{20} = \frac{3}{10}\)
- Year 5: \(\frac{198}{720} = \frac{66}{240} = \frac{11}{40}\)
- Year 6: \(\frac{720 - (180 + 216 + 198)}{720} = \frac{720 - 594}{720} = \frac{126}{720} = \frac{42}{240} = \frac{7}{40}\)

b) Let’s complete this pie chart. BB:

Ps come to BB to fill in the missing items, explaining reasoning. Class agrees/disagrees

(Fill in items for 1 quarter first, then smallest part must be 7 fortieths, and the next smaller part must be 11 fortieths.)

- Year 3: \(\frac{180}{720} = \frac{10}{40}\)
- Year 4: \(\frac{216}{720} = \frac{12}{40}\)
- Year 5: \(\frac{198}{720} = \frac{11}{40}\)
- Year 6: \(\frac{720 - (180 + 216 + 198)}{720} = \frac{720 - 594}{720} = \frac{126}{720} = \frac{7}{40}\)

---

### Notes

Whole class activity

T repeats slowly, or has question written on BB or SB or OHT

Reasoning, agreement, praising

Elicit that dividing numerator and denominator of a fraction by the same number of times does not change its value.

[T might suggest finding the greatest common factor so that the simplification can be done in 1 step. e.g.

- 180 = \(2^2 \times 3^2 \times 5\)
- 720 = \(2^4 \times 3^2 \times 5\)
- HCF: \(2^2 \times 3^2 \times 5 = 180\)

Drawn on BB or use enlarged copy master or OHP

Elicit that:

\(\frac{1}{4} = \frac{10}{40}\)  
\(\frac{3}{10} = \frac{12}{40}\)

Discussion, reasoning, agreement, praising
**Lesson Plan 52**

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP:

Initial whole class discussion

Ps say what they know about parachute jumping. (T might have extra information prepared about competition target jumping.)

Agreement, praising

---

**Activity 3**

*PbY6a, page 52*

Q1 Read: *In a parachute target jumping competition, each competitor makes 8 jumps. The target is a circle with radius 16 cm. The scores range from 0 cm to 16 cm, depending on how far away from the centre of the target circle the parachutist lands. If the parachutist misses the target completely, the lowest score they can get is 16 cm.*

Clarify the context first. T could have a circular target already prepared to scale with a dot at the centre to give Ps an idea of how accurate the parachutists need to be. Some Ps (or the T) might know someone who has done a parachute jump.

BB: Distance from centre of target (cm)

---

[Bar chart showing scores]

Read: *The bar chart shows the scores of one competitor.*

Ask Ps to come to BB to explain what the bar chart means.

(8 jumps along x-axis and the height of each rectangle shows how far away from the centre of the target the parachutist landed.)

Set a time limit. Ps read questions themselves, do listing and necessary calculations in *Ex. Bks* and write answers in *Pbs*.

Review with whole class. Ps dictate numbers for a), then show answers for *b) to e*) on scrap paper or slates on command.

Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) Write the scores in decreasing order in your exercise book.

- 16 cm, 16 cm, 16 cm, 15 cm, 11 cm, 7 cm, 4 cm, 2 cm

b) What is the range of the data? (16 cm – 2 cm = 14 cm)

c) What is the mode of the data? (16 cm)

d) Calculate the mean value.

\[
\frac{3 \times 16 + 15 + 11 + 7 + 4 + 2}{8} = \frac{48 + 39}{8} = 10.875 \text{ (cm)}
\]

e) Calculate the mean of the two middle scores.

Median: \[\frac{15 + 11}{2} = \frac{26}{2} = 13 \text{ (cm)}\]

22 min
Q.2 Read: In a survey about television programmes, a quarter of the people questioned preferred nature programmes, an eighth preferred science programmes, three eighths preferred romantic films, an eighth preferred sports events and 40 people preferred game shows.

T asks several Ps what kind of television programmes they prefer.

Read: a) Draw a pie chart to show the data.

Ps say what to first and how to continue. T asks appropriate questions as necessary to direct Ps' thinking. E.g.

Draw a circle with compasses and mark its centre. Divide the circle into 8 equal parts (as the question mentions eights and quarters and 1 quarter equals 2 eighths), by drawing vertical and horizontal diameters with rulers, then dividing 2 right angles into two 45° angles using a protractor.

Set a time limit. (Ps could shade each part in a different colour.)

Review with whole class. BB:

Ps come to BB to draw the pie chart, explaining reasoning (or T has steps already prepared on SB or OHTs). Class agrees/disagrees. Mistakes corrected.

Deal with the questions one at a time or set a time limit.

Ps read questions themselves, do necessary calculations and write answers as sentences in Ex. Bks.

Review with whole class. Ps could show answers on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

Solutions:

b) What part of the number of people questioned preferred game shows?

Plan: \[ 1 - \left( \frac{1}{4} + \frac{1}{8} + \frac{3}{8} + \frac{1}{8} \right) = 1 - \left( \frac{2}{8} + \frac{5}{8} \right) = 1 - \frac{7}{8} = \frac{1}{8} \]

Answer: One eighth of the number of people questioned preferred game shows.

c) How many people were questioned in the survey?

Plan: \( \frac{1}{8} \rightarrow 40 \text{ people} \quad \frac{8}{8} \rightarrow 40 \times 8 = 320 \text{ (people)} \)

Answer: 320 people were questioned in the survey.

d) How many people preferred each of the 4 types of programmes?

Nature: \( \frac{1}{4} \text{ of } 320 = 320 \div 4 = 80 \text{ (people)} \)

Science/Sports: \( \frac{1}{8} \text{ of } 320 = 320 \div 8 = 40 \text{ (people)} \)

Romantic films: \( \frac{3}{8} \text{ of } 320 = 40 \times 3 = 120 \text{ (people)} \)

Answer: In the survey, 80 people preferred nature programmes, 40 people preferred science programmes, 120 people preferred romantic films and 40 people preferred sports events.
Q.3 Read: *The tables show the times when the sun rose and set in a certain place on the 21st day of each month over one year.*

a) Complete the tables to show the hours of daylight and darkness on each day.

Set a time limit or deal with one month at a time. Ps can do necessary calculations in Ex. Bks or on scrap paper. As there is not much room in the tables to write hours and minutes, T shows a short way of writing the times (see solution below).

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Who did the same? Who worked it out a in a different way? etc. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Date</th>
<th>21 Jan</th>
<th>21 Feb</th>
<th>21 Mar</th>
<th>21 Apr</th>
<th>21 May</th>
<th>21 Jun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunrise</td>
<td>07:23</td>
<td>06:41</td>
<td>05:46</td>
<td>04:45</td>
<td>04:02</td>
<td>03:46</td>
</tr>
<tr>
<td>Day-time</td>
<td>9; 5</td>
<td>10; 35</td>
<td>12; 11</td>
<td>13; 56</td>
<td>15; 19</td>
<td>15; 59</td>
</tr>
<tr>
<td>Night-time</td>
<td>14; 55</td>
<td>13; 25</td>
<td>11; 49</td>
<td>10; 04</td>
<td>8; 41</td>
<td>8; 1</td>
</tr>
</tbody>
</table>

b) Read: *Make a graph to show the hours of daylight.*

If possible, Ps say what to do first and how to continue. Agree on suitable ranges for the x and y axes. T works on BB and Ps work in Ex. Bks. T directs where necessary.

Once axes have been drawn and labelled, Ps come to BB to choose a column in table, say the date and the daylight hours, point to the relevant grid lines on the x and y axes with both hands, move their fingers along the grid lines until they meet and draw a dot or a cross. Ps do the same in Ex. Bks.

Is it correct to join up the dots? (Yes, as time is continuous.)

**Solution:**

![Graph](https://example.com/graph.png)

<table>
<thead>
<tr>
<th>Date</th>
<th>21 Jul</th>
<th>21 Aug</th>
<th>21 Sep</th>
<th>21 Oct</th>
<th>21 Nov</th>
<th>21 Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sunrise</td>
<td>04:08</td>
<td>04:47</td>
<td>05:29</td>
<td>06:10</td>
<td>06:57</td>
<td>07:29</td>
</tr>
<tr>
<td>Sunset</td>
<td>19:33</td>
<td>18:47</td>
<td>17:45</td>
<td>16:46</td>
<td>16:02</td>
<td>15:55</td>
</tr>
<tr>
<td>Day-time</td>
<td>8; 35</td>
<td>10; 0</td>
<td>11; 44</td>
<td>13; 24</td>
<td>14; 55</td>
<td>15; 34</td>
</tr>
<tr>
<td>Night-time</td>
<td>8; 26</td>
<td>9; 5</td>
<td>10; 36</td>
<td>11; 16</td>
<td>12; 16</td>
<td>13; 56</td>
</tr>
</tbody>
</table>

Erratum

In *Pbs*:
Sunset for 21 Oct should be 16:46, not 17:46.

Individual work, monitored, helped

Tables drawn on BB or use enlarged copy master or OHP

At a good pace

Reasoning, agreement, self-correction, praising

Show details of calculation on BB if problems or disagreement.

BB: e.g. 21 Jan column

Day-time:

\[16 \text{ h 28 min} - 7 \text{ h 23 min} = 9 \text{ h 5 min} \text{ (in table: 9; 5)}\]

Night-time:

\[24 \text{ h} - 9 \text{ h 5 min} = 14 \text{ h 55 min} \text{ (in table: 14; 55)}\]

Whole class activity but individual drawing with class kept together, monitored closely, helped, corrected (or very slow Ps could use copy of copy master to save time)

At a good pace

Discussion, agreement, praising, encouragement only

Encourage Ps to draw a curved line between the points.

What does the graph show us?

(June had the most hours of daylight and December had the least hours of daylight)

Ask Ps whether they think that the table or the graph shows this most clearly.
Deal with questions c) to e) one at a time. Ps read question, do calculation or listing and write the answer in Ex. Bks.
Review with whole class. Ps show result on scrap paper or slates on command. P answering correctly explains at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solutions:**

**c) Calculate the mean of the daylight hours.**

*Plan:* \((142 \text{ h } + 293 \text{ min}) \div 12 = 146 \text{ h } 53 \text{ min} \div 12 = 144 \text{ h } 17 \text{ min} 25 \text{ sec} = 12 \text{ h } 14 \text{ min} 25 \text{ sec}*

*Answer:* The mean of the daylight hours is 12 hours, 14 minutes and 35 seconds.

d) **Calculate the range of:**

i) **the day-time hours:** \(15 \text{ h } 59 \text{ min} – 8 \text{ h } 26 \text{ min} = 7 \text{ h } 33 \text{ min} \)

ii) **the night-time hours:** \(15 \text{ h } 34 \text{ min} – 8 \text{ h } 1 \text{ min} = 7 \text{ h } 33 \text{ min} \)

*Answer:* The range of the day-time hours is the same as the range of the night-time hours, 7 hours 33 minutes.

e) **Calculate the median of the daytime hours.**

Ordered listing: 8; 26, 9; 05, 9; 05, 10; 35, 10; 36, 12; 11, 12; 16, 13; 56, 14; 00, 15; 19, 15; 25, 15; 59

*Plan:* \((12 \text{ h } 11 \text{ min } + 12 \text{ h } 16 \text{ min}) \div 2 = 24 \text{ h } 27 \text{ min } \div 2 = 12 \text{ h } 13 \text{ min } 30 \text{ sec} \)

*Answer:* The median of the daytime hours is 12 hours, 13 minutes and 30 seconds.

**Extension**

What is the **mode** of the daylight hours? (9 hours 5 minutes) **45 min**

**Homework**

(Optional)

Ps mark the night-time hours on their graph and join up the points in a different colour from the daytime hours.

---

**Lesson Plan 52**

- **Notes**
  - Individual work, monitored, helped
  - Ps may use a calculator.
  - Responses shown in unison.
  - Reasoning, agreement, self-correction, praising

  \( (as \ 60 \times \frac{4}{12} = 240 \text{ (min)} \)

  \(173 \text{ min } \div 12 = 14 \text{ min } + (5 \text{ min } \div 12) \)

  \(= 14 \text{ min } + (300 \text{ sec } \div 12) \)

  \(= 14 \text{ min } + 25 \text{ sec} \)

  Even number of ordered data, so we calculate the mean of the 2 middle values (6th and 7th) to determine the median of all the data.

- Review before the start of **Lesson 53** and compare the 2 graph lines.
### Activity 1

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **53** is a prime number
  - Factors: 1, 53
  - (as not exactly divisible by 2, 3, 5, 7, and 11 × 11 > 53)

- **228** = 2 × 2 × 3 × 19 = 2² × 3 × 19
  - Factors: 1, 2, 3, 4, 6, 12, 19, 38, 57, 76, 114, 228

- **403** = 13 × 31
  - Factors: 1, 13, 31, 403

- **1053** = 3 × 3 × 3 × 13 = 3³ × 13
  - Factors: 1, 3, 9, 13, 27, 39, 81, 117, 351, 1053

### Activity 2

#### Extracting data from a graph

One winter’s day in a city in Germany, the outside temperature was taken every 2 hours. This graph shows how the temperature changed.

**BB:**

What does the graph tell us? (e.g. Day started off very cold, then gradually became warmer until it reached its highest temperature at 4 o’clock in the afternoon. Then it grew steadily colder and by midnight the temperature had fallen back almost to what it had been at the start.)

- **a)** Let’s write the data in a table. Ps suggest the form (e.g. time on top row and temperature on bottom row). When T has drawn the table, Ps come to BB in pairs, one to point to a marked dot and read out the corresponding values and the other to write the data in the table.

  **BB:**

<table>
<thead>
<tr>
<th>Time (hours)</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>-7</td>
<td>-7</td>
<td>-6</td>
<td>-4</td>
<td>-2</td>
<td>1</td>
<td>3</td>
<td>5</td>
<td>6</td>
<td>4</td>
<td>1</td>
<td>-2</td>
<td>-6</td>
</tr>
</tbody>
</table>

Is it correct to have joined up the dots when we only know exact the data for every 2 hours? (The points between the dots are not exact but are likely to be very close, as temperature during a day rises and falls gradually.)

Discussion involving several Ps. Accept and praise any valid piece of information shown by the graph.

- **At a good pace**
- **Reasoning, agreement,**
- **praising**

T explains or elicits the difference between the data in the table (discrete data) which shows the temperature at only certain times of the day, and the data shown by the graph line (continuous data), which shows the temperature at all times throughout the day.
b) Let’s list the temperatures in increasing order. Ps dictate what T should write.
   BB: –7, –7, –6, –6, –4, –2, 1, 1, 3, 4, 5, 6 (°C)

c) What is the range of the data?  Show me . . . now!  (13)
P with correct response explains at BB to Ps who were wrong.
   BB: Range: 6 – (–7) = 6 + 7 = 13 (°C)
   Who can explain what the range of a set of data means?
   (The range of a set of data is the difference between the greatest value and the smallest value.)

d) What was the average temperature that day?
   How can we calculate it? Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. What name do we give to the average value of a set of data? (the mean)
   BB:
   \[ [–7 \times 2 + (–6) \times 2 + (–4) \times (–2) \times 2 + 1 \times 2 + 3 + 4 + 5 + 6] \div 13 \]
   \[ = (–34 + 20) \div 13 = –14 \div 13 = 1.08 \] (°C) (the Mean)
   Who can explain what the mean value of a set of data is?
   Ps: e.g. The mean value is the sum of all the data divided by how many there are.
   T: The mean of a set of data is the value which can be substituted for each real value so that the total sum of the data is unchanged.

e) What is the mode of the data?  (–7, –6, –2 and 1)
   Who can explain what the mode of a set of data is?
   (The mode of a set of data is the most frequent value or values.)

f) What is the median of the data?  Show me . . . now!  (–2)
   (There are 13 values, so the middle value is 7th in the ordered list.)
   Who can explain what the median of a set of data is?
   (The mean of a set of data is the middle value when the data are listed in order. When there is an even number of data, the median is the mean of the two middle values.)

20 min

PbY6a, page 53

Q.1 Read: The pictogram shows the number of weddings in a certain city over one year.
   Here is another way of presenting data – using pictures.
   What does each heart in the drawing represent?  (500 weddings)
   What does half a heart represent?  (250 weddings)
   Who has been to a wedding? Ps briefly tell of their experiences.

   a) Read: Write the actual numbers in the table.
      Make sure that Ps understand that it is the number of weddings which is required, not the number of hearts! Set a time limit.
      Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected before Ps do parts b) and c) either one at a time or under a time limit.

   Individual work, monitored, helped
   Drawn on BB or use enlarged copy master or OHP
   Initial discussion to clarify the context.
   Involve several Ps.

   Reasoning, agreement, self-correction, praising
### Lesson Plan 53

#### Activity 3

(Continued)

**Solution:**

<table>
<thead>
<tr>
<th>Month</th>
<th>Weddings (in thousands)</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>2500</td>
</tr>
<tr>
<td>February</td>
<td>1750</td>
</tr>
<tr>
<td>March</td>
<td>3000</td>
</tr>
<tr>
<td>April</td>
<td>3750</td>
</tr>
<tr>
<td>May</td>
<td>5000</td>
</tr>
<tr>
<td>June</td>
<td>5250</td>
</tr>
<tr>
<td>July</td>
<td>5000</td>
</tr>
<tr>
<td>August</td>
<td>5500</td>
</tr>
<tr>
<td>September</td>
<td>4000</td>
</tr>
<tr>
<td>October</td>
<td>2500</td>
</tr>
<tr>
<td>November</td>
<td>2000</td>
</tr>
<tr>
<td>December</td>
<td>2750</td>
</tr>
</tbody>
</table>

\[ \heartsuit = 500 \text{ weddings} \]

Read: **b) Calculate in your exercise book the range of the monthly data.**

**c) Calculate in your exercise book the average number of weddings per month.**

Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected

**Solution:**

b) Range: \( 5500 - 1750 = 3750 \)

c) Mean: \[
\frac{(1750 + 2000 + 2500 \times 2 + 2750 + 3000 + 3750 + 4000 + 5000 \times 2 + 5250 + 5500)}{12} = 43000 \div 12 = 21500 \div 6 \approx 3583.3
\]

Elicit/point out that the mean makes no sense in real life, as you cannot have 0.3 of a wedding — it is an *average* value! (i.e. the value which can take the place of each real value so that the total will be the same.)

**d) What is the mode of the data?** (2500 and 5000)

**e) What is the median of the data?**

Ordered list (e.g. in thousands):

BB: 1.75, 2, 2.5, 2.5, 2.75, 3, 3.75, 4, 5, 5, 5.25, 5.5

Median: \[
(3 + 3.75) \div 2 = 6.75 \div 2 = 3.375 \text{ (thousands)}
\]

\[ \approx 3375 \]

---

#### Notes

Which month was most (least) popular?

Why do you think it is so?

Individual work, monitored, helped

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Ps might be allowed to use calculators here.

(\text{or} = 3583.3)

Whole class activity

T suggests writing just the number of thousands to keep the calculation simple.

Ps come to BB or dictate to T.

Class agrees/disagrees.

Praising
Week 11

Lesson Plan 53

Notes

Individual work, monitored
helped
[or do c) and d) with the whole class]

Drawn on BB or use enlarged
copy master or OHP

BB:

Initial discussion (and
demonstration if possible) to
clarify the context and terms.

Responses shown in unison.
Reasoning, agreement, self-
correction, praising

First relative frequency could
be done with the whole class
as a model for Ps to follow.

Elicit the fractions in decimal
form also.  e.g.

\[
\frac{7}{40} = \frac{7}{40} \div 4 = 0.175
\]

(T decides whether to let Ps
use calculators.)

The mean score is 3.125.

Whole class activity

Ps suggest what to do.

Median: \(\frac{3 + 3}{2} = \frac{6}{3} = 3\)
Y6

Activity

5  Experiment
Ps have spinners (or dice) on desks. Ps work in pairs, taking turns to spin the spinner 10 times each and keeping a tally of the numbers landed on. Ps draw a bar chart to show the data (using chart in Pbs as a model). Ps work out the mode, mean and median of their data. (Very able Ps could also work out the relative frequency of each outcome.) T chooses Ps to report their findings to class.
(Ps not finished could complete their drawing and/or calculations for homework.)

40 min

Notes

Spinners already prepared.
(Can use copy master)
Paired work (able with less able), monitored, helped, corrected
Have no expectations!
Praising, encouragement only
(Optional: review before start of Lesson 54)

6  PbY6a, page 53,
Q.3  Read: In this graph, you can see 6 connecting pairs of numbers.
   a) Make up a problem about these pairs of numbers so that the graph represents appropriate data and frequencies.
      Label the axes.
      Write the problem in your exercise book.
   Allow Ps a minute to think of a context, label the axes and think of a question to ask about it.
   T chooses several Ps say what their labels are for the axes and to read out their questions. Class decides whether or not the context, labels and questions are valid.
(e.g. scores of Ps in a class in a test out of 10 marks; number of birds seen on the lawn at 8.00 am each day for a week; length in cm of pencils in a pencil box; etc.)
   Read: b) Calculate in your exercise book:
      i)  the range of the data
      ii) the mode of the data
      iii) the mean of the data.
   Either the class decides on one of the contexts and works out the answers to part b) together, or Ps use own contexts to do their own calculations in Ex. Bks.
   Solution: e.g. using the test scores of Ps in a class: (25 Ps)

BB:

<table>
<thead>
<tr>
<th>No. of Pupils</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
</tbody>
</table>

i)  Range of marks:
     \[8 - 2 = 6\] (marks)

ii) Mode: \[4\] (marks)

iii) Mean:
     \[
     \frac{1 \times 8 + 2 \times 6 + 4 \times 3 + 5 \times 5 + 6 \times 2 + 7 \times 4}{25} = \frac{8 + 12 + 25 + 12 + 28}{25} = \frac{97}{25} = \frac{388}{100} = 3.88\] (marks)

45 min

Extension

What is the median of the data?
[Median is the 13th mark in the ordered list of 25 marks. First 6 marks are '2', next 4 marks are '3'; next 7 marks are '4', so the 13th mark is '4' and 4 is the median of the data.]
## Activity

### 1 Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3 \)
  Factors: 1, 2, 3, 6, 9, 18, 27, 54
- 229 is a prime number   
  Factors: 1, 229 
  (as not exactly divisible by 2, 3, 5, 7, 11, 13 and 17 × 17 > 229)
- \( 404 = 2 \times 2 \times 101 = 2^2 \times 101 \)
  Factors: 1, 2, 4, 101, 202, 404
- \( 1054 = 2 \times 17 \times 31 \)
  Factors: 1, 2, 17, 31, 34, 62, 527, 1054

### 2 Analysing data

The data in this table are from an international project on attainment in mathematics. The table shows the mean scores on a maths test out of 110 marks for pupils in 10 schools in a certain country.

<table>
<thead>
<tr>
<th>School code</th>
<th>01</th>
<th>02</th>
<th>03</th>
<th>04</th>
<th>05</th>
<th>06</th>
<th>07</th>
<th>08</th>
<th>09</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>School mean score</td>
<td>80</td>
<td>76</td>
<td>72</td>
<td>84</td>
<td>84</td>
<td>88</td>
<td>68</td>
<td>66</td>
<td>83</td>
<td>65</td>
</tr>
<tr>
<td>Number of pupils</td>
<td>78</td>
<td>84</td>
<td>34</td>
<td>66</td>
<td>82</td>
<td>76</td>
<td>19</td>
<td>6</td>
<td>12</td>
<td>20</td>
</tr>
</tbody>
</table>

Ps come to BB to explain the table. (School codes along top row, average score of pupils in each school in middle row, number of Ps in each school along bottom row.)

a) Let’s calculate the difference between the highest and lowest means. Ps dictate what T should write. Class agrees/disagrees.

T: We could say that the range of the means is 23 marks.

b) How many pupils did the test in that country?

BB: \( 78 + 84 + 34 + 66 + 82 + 76 + 19 + 6 + 12 + 20 = 477 \) (pupils)

c) How could we calculate the average score of all the pupils in the 10 schools? Ps suggest their ideas. T helps/directs if necessary.

[We do not know exactly what each P scored but the mean mark for each school can take the place of the real mark for each pupil in that school and the total score will be the same (e.g. as if the 78 Ps in School 01 each scored 80 marks). We can work out the total score of the Ps in each school by multiplying the school’s mean score by the number of pupils in that school who did the test.

Then add the total marks of each school and divide this sum by the total number of Ps tested to calculate the mean for all 10 schools, i.e. for that country.]

Ps dictate operations, then do the calculations in Ex. Bks (or use a calculator for the more difficult operations) and dictate results to T.

### Notes

Individual work, monitored (or whole class activity)

BB: 54, 229, 404, 1054

Calculators allowed.

Reasoning, agreement, self-correction, praising

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Discussion, agreement

Reasoning, agreement, praising

If a P suggests adding up the mean marks and dividing by 10, elicit that this would be the average of the means, not the average of all the scores.

Extra praise if a P thinks of what to do without help from T.
Activity 2 (Continued)

BB: Mean country score:

\[
\frac{80 \times 78 + 76 \times 84 + 72 \times 34 + 84 \times 66 + 84 \times 82 + 88 \times 76 + 68 \times 19 + 66 \times 6 + 83 \times 12 + 65 \times 20}{477}
\]

\[
= \frac{6240 + 6384 + 2448 + 5544 + 6888 + 6688 + 1292 + 396 + 996 + 1300}{477}
\]

\[
= \frac{38176}{477} \approx 80 \text{ (marks)}
\]

The mean score for all the pupils in the 10 schools in that country is 80 marks. Elicit that School 01 is the only school which has the same mean score as the country mean.

What are the range, mode and median of the Ps’ scores?

(We do not know the mark of every pupil so we cannot list them in increasing order. Therefore it is impossible to determine the range (difference between lowest and highest mark), mode (most frequent mark) or the median (middle mark) from the data given in the table.

Extension

What are the range, mode and median of the Ps’ scores?

(We do not know the mark of every pupil so we cannot list them in increasing order. Therefore it is impossible to determine the range (difference between lowest and highest mark), mode (most frequent mark) or the median (middle mark) from the data given in the table.

PbY6a, page 54

Q.1 Read: The table shows some data from an international project on attainment in mathematics.

The table shows the mean scores on a test out of 140 marks achieved by the pupils in 10 project schools in one of the countries in the project.

<table>
<thead>
<tr>
<th>School code</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
</tr>
</thead>
<tbody>
<tr>
<td>School mean score</td>
<td>89</td>
<td>94</td>
<td>80</td>
<td>107</td>
<td>95</td>
<td>117</td>
<td>87</td>
<td>77</td>
<td>90</td>
<td>85</td>
</tr>
<tr>
<td>Number of pupils</td>
<td>58</td>
<td>75</td>
<td>32</td>
<td>70</td>
<td>93</td>
<td>75</td>
<td>34</td>
<td>9</td>
<td>10</td>
<td>18</td>
</tr>
</tbody>
</table>

Do you think that this could be the same project as in the previous activity? (No, as the schools are coded differently and the test is out of 140 marks.)

Deal with one question at a time. Ps read question themselves and calculate in Ex. Bks. under a time limit

Review with whole class. T chooses a P to read out the question. Ps show result on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected

Solution:

a) Calculate the difference between the highest and lowest means. (40 marks)

b) What is the average of the school means (as if an equal number of pupils did the test in each school)? (92.1)

BB: Mean of the school means:

\[
\frac{89 + 94 + 80 + 107 + 95 + 117 + 87 + 77 + 90 + 85}{10}
\]

\[
= 921 \div 10 = 92.1 \text{ (marks)}
\]

c) How many pupils did the test in this country? (474)

BB: 58 + 75 + 32 + 70 + 93 + 75 + 34 + 9 + 10 + 18 = 474

Answer: In this country, 474 pupils did the test.
### Activity 3 (Continued)

d) **Calculate the mean score for the country, taking the number of children in each school into consideration.**

Mean country score:

\[
\frac{(89 \times 58 + 94 \times 75 + 80 \times 32 + 107 \times 70 + 95 \times 93 + 117 \times 75 + 87 \times 34 + 77 \times 9 + 90 \times 10 + 85 \times 18)}{474} = \frac{5162 + 7050 + 2560 + 7490 + 8835 + 8775 + 2958 + 693 + 900 + 1530}{474} = \frac{45953}{474} \approx 96.9 \text{ marks}
\]

**Answer:** The mean country score is 96.9 marks.

### Extension

- Why is the average country score of 96.9 marks more than the average of the school mean scores, 92.1 marks? (Because the larger schools have higher mean scores than the smaller schools.)
- When would the average country score be the same as the average of the school means? (If each school had the same number of Ps, or if each school had the same mean score)

### Question 2

**PbY6a, page 54**

**Q.2 Read:** John spun this spinner several times. He wrote down the number it stopped at each time. This is what he wrote.

BB: 0, 2, –3, –1, 2, 1, –2, 0, –2, 0, 2, 2, –3, –1, 1, 2, 0, –3, –2, 2, 1

How many times did John spin the spinner? (21)

Set a time limit. Ps read questions themselves and do any necessary calculations in Ex. Bks.

Review with whole class. T chooses a P to read out the question. Ps dictate part a) to T. Class agrees/disagrees. Ps show results for b) to e) on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) **Write the data in increasing order in your exercise book.**

BB: –3, –3, –3, –2, –2, –2, –1, –1, 0, 0, 0, 0, 1, 1, 1, 2, 2, 2, 2, 2

b) **Calculate the range of the data** \[2 – (–3) = 5\]

c) **What is the mode of the data?** (2)

(The mode of the data is 2, as 2 occurred most often.)

d) **Calculate the mean of the data.**

BB: Mean:

\[
\frac{(-3 \times 3 + -2 \times 3 + -1 \times 2 + 0 \times 4 + 1 \times 3 + 2 \times 6)}{21} = \frac{-9 + (-6) + (-2) + 0 + 3 + 12}{21} = -2 \div 21 = -\frac{2}{21} \approx -0.095
\]

The mean of the data is –0.095 (to nearest 1000th)

e) **What is the median of the data?** (0)

(The median of the data is the 11th value in the ordered list.)

**33 min**

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Q.3 Read: A river flowed through a city and on the wall of a certain bridge was marked the water levels of the river. The zero mark was set at 113 m above sea level. The level of the river was measured each week and the data are shown in the table.

What do you notice about the table? (River level is in cm and height above sea level is in metres.) Who can explain what is meant by river level and sea level? (The river bed at that bridge is 113 m above the level of the sea. The marks on the supports of the bridge show the depth of the river.)

Read: a) Calculate the heights (rounded if necessary) above sea level. Complete the table.

First elicit the calculation and rounding that should be done. (Round the river level to the nearest metre, then add to 113 m)

Set a time limit. Ps complete the bottom row of table, rounding and calculating mentally.

Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected before Ps do parts b) and c).

Solution:

<table>
<thead>
<tr>
<th>River level (cm)</th>
<th>265</th>
<th>183</th>
<th>95</th>
<th>–36</th>
<th>–110</th>
<th>–280</th>
<th>–196</th>
<th>–72</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height above sea level (m)</td>
<td>116</td>
<td>115</td>
<td>114</td>
<td>113</td>
<td>112</td>
<td>110</td>
<td>111</td>
<td>112</td>
</tr>
</tbody>
</table>

b) Read: Write the river levels in order in your exercise book.

BB: –280 cm, –196 cm, –110 cm, –72 cm, –36 cm, 95 cm, 183 cm, 265 cm

c) Calculate:

i) the mean of the data

Mean:

\[
\frac{(265 + 183 + 95) - (36 + 110 + 280 + 196 + 72)}{8} = \frac{543 - 694}{8} = \frac{-151}{8} \approx -18.9 \text{ (cm)}
\]

ii) the median of the data.

Median is the mean of the 4th and 5th values in the ordered list:

Median: \([–72 + (–36)] ÷ 2 = –108 ÷ 2 = –54 \text{ (cm)}\)

What other questions could be asked about the river level data?

e.g. What is the mode? (Accept all or none.)

What is the range? \([265 – (–280)] = 545 \text{ (cm)}\)
Y6

Activity

6

PbY6a, page 54, Q.4

Read: Which two numbers are missing from this data sample if its median is 2.6, its mode is 3.1 and its mean is 2.5? (The data are already in order.)

Allow Ps 2 minutes to think about it, then Ps who have an answer show missing numbers on slates or scrap paper on command. Ps with different responses explain reasoning at BB. Who agrees? Who thought another way? Class decides who is correct. Ps write agreed numbers in Pbs.

Solution:

1.1  1.4  2.1  [2.6]  3.1  3.1  4.1

Reasoning: e.g.

As the data are in order, the 1st missing number is the median, 2.6.
As 3.1 is the mode, there must be another 3.1 in the list, so 2nd missing number is 3.1.

Check using the mean.

Mean: \( \frac{1.1 + 1.4 + 2.1 + 2.6 + 3.1 + 3.1 + 4.1}{7} = \frac{17.5}{7} = 2.5 \)

45 min
Factorising 55, 230, 405 and 1055. Revision, activities, consolidation

**PbY6a, page 55**

**Solutions:**

Q.1  
   a) 4.26 m, 4.35 m, 4.88 m, 4.90 m, 5.06 m, 5.44 m, 5.71 m, 5.71 m  
   b) Range: 5.71 m – 4.26 m = **1.45 m**  
   c) Mode: 5.71 m  
   d) Mean:  
      \[
      \frac{(4.26 + 4.35 + 4.88 + 4.90 + 5.06 + 5.44 + 5.71 + 5.71)}{8} = 5.03875 = **5.04** (m) 
      \]
   e) Median: \( (4.90 + 5.06) \div 2 = 9.96 m \div 2 = **4.98 m** \)

Q.2  
   a) Minutes late: Mon: 19, Tue: 5, Wed: 1, Thu: 26, Fri: 12  
      Increasing order: 1, 5, 12, 19, 26 (minutes)  
   b) Range: 26 – 1 = **25** (minutes)  
      Median: **12 minutes**  
      Mean: \( (1 + 5 + 12 + 19 + 26) \div 5 = 63 \div 5 = **12.6** (min) \)  
      \( (= 12 \text{ min} 36 \text{ seconds}) \)  
   c) Creative activity.

Q.3  
   a)  
   \[
   \begin{array}{|c|c|c|c|c|c|c|c|}
   \hline
   \text{Type of bird} & \text{Sp} & \text{R} & \text{St} & \text{BT} & \text{Se} & \text{M} & \text{Bl} & \text{P} \\
   \hline
   \text{Number seen} & 20 & 8 & 18 & 12 & 15 & 5 & 8 & 6 \\
   \hline
   \end{array}
   \]
   b) Sparrow  
   c) Average no. per garden:  
      Sparrows: 2; Robins: 0.8, Starlings: 1.8; Blue TIts: 1.2,  
      Seagulls: 1.5, Magpies: 0.5, Blackbirds: 0.8, Pigeons: 0.6  
   d) Total seen: 20 + 8 + 18 + 12 + 15 + 5 + 8 + 6 = **92** (birds)  
   e) e.g. pie chart, line graph or pictogram  
   f) They might have counted the same birds more than once.  
      (e.g. a bird could land on the lawn, fly away and come back again within the hour.)

Q.4 Creative activity
Lesson Plan

56

Notes

Individual work, monitored
(or whole class activity)
BB: 56, 231, 406, 1056
Calculators allowed.
Reasoning, agreement, self-correction, praising
Whole class listing of the factors of 1056
e.g. 56

8 min

R: Calculation
C: Using language associated with probability to discuss events
E: Equally likely outcomes

Activity

1

Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:
• \(56 = 2 \times 2 \times 2 \times 7 = 2^3 \times 7\)
  Factors: 1, 2, 4, 7, 8, 14, 28, 56
• \(231 = 3 \times 7 \times 11\)
  Factors: 1, 3, 7, 11, 21, 33, 77, 231
• \(406 = 2 \times 7 \times 29\)
  Factors: 1, 2, 7, 14, 29, 58, 203, 406
• \(1056 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 11 = 2^5 \times 3 \times 11\)
  Factors: 1, 2, 3, 4, 6, 8, 11, 12, 16, 22, 24, 32, 44, 66, 88, 1056, 252, 264, 332, 352, 528, 666, 772, 1056

8 min

2

Probability 1

a) Tell me outcomes or events which are certain to happen.
Ps make suggestions and class agrees or disagrees. e.g.
• If you pick a card from a pack of cards:
  – its number will be even or odd (Certain)
  – its colour will be black or red (Certain)
  – it will be a diamond, a club, a heart or a spade. (Certain)
• This sentence has the letter 'e' in it. (Not a good example, as it is a fact, not an outcome.)
• The sun will rise in the East tomorrow. (Not a good example: it is certain unless something happens to the Earth – not impossible but very, very unlikely.) etc.

b) Tell me outcomes or events which are possible but not certain.
Ps make suggestions and class decides how likely or unlikely they are.
• If you pick a card from a pack of cards:
  – it will be black (Good example, as it has a 'fifty-fifty' or 'equal' chance of happening as of not happening.)
  – it will be a club (Possible but unlikely, as there are 4 different suites in a pack of cards)
  – it will be a number between 0 and 8. (Very likely)
• When you roll a dice, the number facing up will be at least a 3. (More likely than unlikely, as there are 6 possible numbers on a dice and 'at least a 3' will cover 4 of them.) etc.

c) Tell me outcomes or events which are impossible. e.g.
• If you roll a fair dice, you will score a number less than 1.
• The Sun will sink in the East this evening.
• The bonus number in the next National Lottery will be 50.

18 min

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**Activity**  

**PbY6a, page 56**

Q.1 Read: A bag contains 3 red and 5 green marbles. If you took out a marble with your eyes closed, what chance would you give to each of these outcomes? Join each outcome to the appropriate level of chance.

Set a time limit. Review with whole class. T chooses a P to read each statement and Ps come to BB to draw joining lines and explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- The marble taken out is red.
- The marble taken out is green.
- The marble taken out is red and green.
- If you take out a marble, put it back again, then take out a second marble, both marbles will be red.
- The marble taken out is red or green.

**Extension**

- What is the ratio of red marbles to green marbles? (3 : 8)
- What part of all the marbles are the red marbles? (3/8)
- What chance do you have of taking out a red marble? (3 chances out of 8; 3 eighths;)
- What can you say about the chance of taking out:
  - a green marble (5 chances out of 8; 5 eighths)
  - a marble which is red and green (No chance; 0 out of 8; 0 eighths = 0)
  - a marble which is red or green? (8 chances out of 8; 8 eighths = 1)

**Notes**

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
Reasoning, agreement, self-correction, praising
Feedback for T

Whole class discussion
T asks several Ps what they think. Class agrees/disagrees.
Elicit that in all cases, we are assuming that each of the 8 marbles has an equal chance of being chosen (i.e. the same size and shape, feel the same, have no distinguishing chips or scratches, etc.)
(Extra praise if a P mentions probability but otherwise do not mention it yet.)

**Extension**

Ps suggest 'likely' outcomes. Class agrees/disagrees.

**Notes**

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
Reasoning, agreement, self-correction, praising
Feedback for T

Whole class activity. Praising
**Y6**

**Activity**

5  

**PbY6a, page 56, Q.3**

Read: *In a summer camp, 4 Polish children, 4 Hungarian children and 2 Scottish children have formed a friendly group.*

Ask Ps to point out Poland, Hungary and Scotland on a world map. Who has been to any of these countries? Who has been to a summer camp? Ps tell class briefly of their experiences.

Read: *They are going on a boat trip and get on board the boat in a random order.*

How many children are in this group of friends? (10) What does random mean? (By chance, not planned, in any order)

a) Read: *Join each outcome to the matching level of chance.*

Deal with one outcome at a time. T asks a P to read the sentence, allows time for Ps to think about it, then Ps show C, L, E, U or I on scrap paper or slates on command. Ps with different responses explain reasoning to class. Class decides who is correct. P joins statement to the agreed chance on BB and rest of Ps in *Pbs.*

**Solution:**

i) *The first 5 children to get on board are Polish.* (I)

(Impossible, as there are only 4 Polish children)

ii) *The last child to get on board is Polish or Hungarian or Scottish.* (C)

(Certain, as no other nationality is possible)

iii) *The first child to get on board is Scottish.* (U)

(Only 2 of the 10 children are Scottish.)

iv) *The first 4 children to get on board are Polish, Hungarian, Polish and Scottish in that order.* (U)

(Possible, but unlikely)

v) *The first child to get on board is Hungarian.* (U)

(Unlikely, as only 4 out of the 10 children are Hungarian, but it is more likely than a Scottish child being first.)

b) Let’s see if we can be more exact about the levels of chance!

T reads each questions and Ps show answers on scrap paper or slates on command. Ps answering correctly explain reasoning. Class agrees/disagrees. Ps write agreed ratio or fraction beside relevant question in *Pbs.*

**Solution:**

i) *What part of the group is Scottish?*

\[
\frac{2}{10} = \frac{1}{5} = 0.2
\]

ii) *What chance is there that the first child on board is Scottish?*

\[
\frac{2}{10} = \frac{1}{5} = 0.2
\]

T: When we give a numerical value to the level of chance a certain event has of happening, we call it the *probability* of that event.

iii) *What is the probability of the first child on board being Polish or Hungarian?*

(as 8 out of the 10 children are either Polish or Hungarian, i.e. are not Scottish)

**Notes**

Whole class activity

(or individual work, monitored and reviewed as usual)

Short introduction to set the scene. (T could have some facts about the 3 countries and about summer camps already prepared in case no P has anything to tell.)

Written on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, praising

Compare iii) and v) and elicit that v) would be higher on the scale than iii).

Accept any correct form but elicit the decimal form too if no P shows it.

T shows how to write it in a mathematical way.

**BB:** probability

\[ p (P \text{ or } H) = \frac{8}{10} = \frac{2}{5} = 0.8 \]

or \[ p (\text{not } S) = 1 - 0.2 = 0.8 \]
### Activity 6

#### Probability 2

T asks a question and Ps show the answer on scrap paper or slates on command. Ps with different responses explain reasoning at BB. Class decides on the correct answer. T shows the correct notation on BB.

**a)** If you throw a fair dice 60 times how many times would you expect to get a '4'?  

(Reasoning: e.g. 6 possible outcomes: 1, 2, 3, 4, 5, or 6; only one outcome is '4', so on each throw the probability of getting a 4 is 1/6th.

\[ BB: \frac{1}{6} \text{ of 60} = \frac{60}{6} = 10 \]

so for 60 throws you would expect to get a '4' 10 times.)

**b)** If you throw a fair dice 120 times how many times would you expect to get a number which is 'at most 4'?  

(Reasoning: e.g. 6 possible outcomes: 1, 2, 3, 4, 5, or 6; 4 of them are 'at most 4', so on each throw the probability of getting 'at most 4' is 4 out of 6, or 4 sixths.

\[ BB: \frac{2}{3} \text{ of 120} = \frac{120}{3} \times 2 = 40 \times 2 = 80 \]

For 120 throws you would expect to get 'at least 4' 80 times.)

**c)** If you drew a card from a pack of playing cards and replaced it in the pack 100 times, how many times would you expect the card to be a 'heart'?  

(Reasoning: 4 possible outcomes: heart, diamond, spade or club; only 1 of them is a 'heart', so each time, the probability of getting a 'heart' is 1 out of 4, or \( \frac{1}{4} \) quarter.

\[ BB: \frac{1}{4} \text{ of 100} = \frac{100}{4} = 25 \]

For 100 times you would expect to get 'a heart' 25 times.)

**d)** If you toss a fair coin 100 times, how many times would you:

i) expect to get a 'head'?  

(Reasoning: 2 possible outcomes: head or tail, only 1 of them is a 'head', so for each toss, the probability of 'a head' is \( \frac{1}{2} \) half.

\[ BB: \frac{1}{2} \text{ of 100} = \frac{100}{2} = 50 \]

For 100 tosses you would expect to get 'a head' 50 times.)

ii) expect to get a 'head or a tail'?  

(Reasoning: 2 possible outcomes: head or tail, each with a probability of \( \frac{1}{2} \) half, so probability of 'a head or a tail' is \( \frac{2}{2} \) halves or 1, i.e. it is certain to happen.

For 100 tosses you would expect to get 'a head or a tail' 100 times.

iii) expect to get a 'head and a tail'?  

(Reasoning: It is impossible – there is no chance of getting 'a head and a tail' at the same time!)

\[ BB: \]

**Notes**

Whole class activity but individual calculation in Ex. Bks. or on scrap paper or slates

T repeats question slowly to give Ps time to think and calculate.

Responses shown in unison.

Reasoning, agreement, praising

\[ BB: p(4) = \frac{1}{6} = 0.16 \]

Elicit decimal form too.

\[ BB: p(\text{at most 4}) = \frac{4}{6} = \frac{2}{3} = 0.6 \]

\[ BB: p(\text{heart}) = \frac{1}{4} = 0.25 \]

\[ BB: p(\text{head or tail}) = \frac{2}{2} = 1 \]

\[ BB: p(\text{head and tail}) = \frac{0}{2} = 0 \]
Lesson Plan

Y6

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- \( 57 = 3 \times 19 \) Factors: 1, 3, 19, 57
- \( 232 = 2 \times 2 \times 2 \times 29 = 2^3 \times 29 \) Factors: 1, 2, 4, 8, 29, 58, 116, 232
- \( 407 = 11 \times 37 \) Factors: 1, 11, 37, 407
- \( 1057 = 7 \times 151 \) Factors: 1, 7, 151, 1057

2

Relative Frequency

a) Ps work in pairs to toss a coin 30 times and keep a tally of the outcomes in a table. Ps check that their totals sum to 30.

T chooses a pair to show their results on the BB. e.g.

BB:  
<table>
<thead>
<tr>
<th>Pupil data</th>
<th>Tally of 30 tosses</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>⌐▲▲▲▲</td>
<td>13</td>
</tr>
<tr>
<td>Tail</td>
<td>⌐▲▲▲▲</td>
<td>17</td>
</tr>
</tbody>
</table>

We say that the number of times an outcome happened is its frequency. What is the frequency of a Head (Tail)? (13, 17)

What part of these 30 tosses were Heads (Tails)? (Ask for the decimal form too. (Ps use calculators and round result appropriately.) Ps dictate to T. Class agrees/disagrees.

BB: Heads: \( \frac{13}{30} \approx 0.43 \) (43%) Tails: \( \frac{17}{30} \approx 0.57 \) (57%)

When we compare the frequency with the total number of outcomes, we call it the relative frequency. We can give the relative frequency as a fraction or a decimal like this (T points to those on BB) or as a percentage. Elicit that 'percent' means 'out of 100'. Ps say the fractions as percentages (H: 43%, T: 57%) and T writes on BB.

What is the frequency of the outcome 'Head or Tail'? (30)

What is the relative frequency of 'Head or Tail'? (\( \frac{30}{30} = 1 \rightarrow 100\% \))

Elicit that this is the outcome which is certain to happen.

b) Let's collect all the frequency data for the class and write it in this table. Each P in a pair is responsible for Heads or Tails. Ps dictate their results and keep running totals on their calculators (or T projects 2 calculators from computers onto a screen). T writes class frequencies in table on BB and Ps write them in table on their sheets (or drawn in Ex. Bks).

Elicit the relative frequencies in fraction, decimal and percentage form (using calculators and rounding result appropriately).
Lesson Plan 57

**Activity**

2  (Continued)

**Notes**

BB: e.g.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Head</td>
<td>379</td>
<td>(\frac{379}{750} = 0.505 \rightarrow 50.5%)</td>
</tr>
<tr>
<td>Tail</td>
<td>371</td>
<td>(\frac{371}{750} = 0.495 \rightarrow 49.5%)</td>
</tr>
</tbody>
</table>

*What is the frequency of a 'Head or a Tail'? (750)*

*What is the relative frequency of a 'Head or a Tail'? (1 \(\rightarrow\) 100%)*

d) If you toss a fair coin, what probability would you give to getting a Head (Tail)? Why?

(1 chance out of 2, or \(\frac{1}{2}\) or 0.5, as there are 2 possible outcomes and each outcome has an equal chance of happening)

Why are the relative frequencies in our experiment not exactly 1 half? (The more experiments we do, the closer they will get.)

c) If possible, show a computer simulation of the experiment projected on a screen. Ps suggest increasingly large values for \(n\) (the number of tosses) and read out the resulting frequencies for Heads and Tails.

Ps could calculate the relative frequencies using calculators first then T reveals them on the computer program.

Show that as the number of tosses increases, the relative frequencies get closer to the expected probability: a half.

\[17\text{ min}\]

3  **PbY6a, page 57**

Q.1  a) Read: *Throw a fair dice 60 times. Keep a tally of the outcomes. Write the frequency in the table and calculate the relative frequency of each outcome.*

Set a time limit of 5 minutes. Ps check that they have 60 tally marks before writing the frequencies and calculating the relative frequencies as fractions, decimals and percentages.

T asks P finished first to show their results on table on BB.

**Pupil data**

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Tally of 60 throws</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\square)</td>
<td>(\square)</td>
<td>8</td>
<td>(\frac{8}{60} = 0.13 \rightarrow 13%)</td>
</tr>
<tr>
<td>(\square)</td>
<td>(\square)</td>
<td>11</td>
<td>(\frac{11}{60} = 0.18 \rightarrow 18%)</td>
</tr>
<tr>
<td>(\square)</td>
<td>(\square)</td>
<td>10</td>
<td>(\frac{10}{60} = 0.17 \rightarrow 17%)</td>
</tr>
<tr>
<td>(\square)</td>
<td>(\square)</td>
<td>9</td>
<td>(\frac{9}{60} = 0.15 \rightarrow 15%)</td>
</tr>
<tr>
<td>(\square)</td>
<td>(\square)</td>
<td>12</td>
<td>(\frac{12}{60} = 0.2 \rightarrow 20%)</td>
</tr>
<tr>
<td>(\square)</td>
<td>(\square)</td>
<td>10</td>
<td>(\frac{10}{60} = 0.17 \rightarrow 17%)</td>
</tr>
</tbody>
</table>

\[n = 60\]

Individual (or paired, able with less able) work, closely monitored, helped, corrected

Table drawn on BB or use enlarged copy master or OHP

Ps use calculators to work out the decimal forms. (numerator divided by denominator)

Class points out any errors in two RH columns of table.

Ps with vastly different results from everyone else check their counting and calculations.

If correct, ask class what it might show. (The dice of that P could be unfair or biased, i.e. weighted to make a certain number occur more often.)
### Activity

<table>
<thead>
<tr>
<th>Lesson Plan 57</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td>Whole class activity</td>
</tr>
<tr>
<td>Table drawn on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>At a good pace, in good humour!</td>
</tr>
<tr>
<td>Reasoning, agreement, praising</td>
</tr>
<tr>
<td>Individual work, monitored Agreement, praising only</td>
</tr>
</tbody>
</table>

#### Extension

- **b)** Read: *Collect the data for the class and calculate the relative frequencies in your exercise book.*
  - T chooses 6 able Ps to keep a running total for each of the outcomes on their calculators while rest of Ps dictate their results. The 6 Ps write the frequencies on BB and class calculates the relative frequencies in the three different forms. Ps write them in Ex. Bks.

- **Read:** *Write a sentence about what you notice.*
  - Allow a minute or two for Ps to think and write their sentences, then T asks individual Ps to read what they wrote. Who wrote much the same? Who noticed something else? etc.

- **Elicit** that the frequencies are very similar and the relative frequencies are close to 1 sixth.

#### 4 Probability scale

**Let’s revise what we know about probability.** Ps tell class what they have learned and help T to draw a probability scale on BB.

**Elicit or tell** that halfway on the scale (i.e. probability of a half or 0.5 or 50%) is sometimes called ‘Evens’, especially when gambling!

**BB:**

<table>
<thead>
<tr>
<th>Impossible</th>
<th>Equal chance of happening as of not happening</th>
<th>Certain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.5</td>
<td>1</td>
</tr>
<tr>
<td>0%</td>
<td>50%</td>
<td>100%</td>
</tr>
</tbody>
</table>

*Whole class discussion and revision*

Discuss the connection with frequency and relative frequency.

Ps give examples of 'impossible', 'evens' and 'certain' events or outcomes.

Class agrees/disagrees.

Praising, encouragement only

#### 5 PbY6a, age 57

**Q.2 Read:** *What chance do you think each of these outcomes has of happening? Write its letter at the appropriate place below the probability scale.*

Set a time limit. Ps read the statements themselves and write the letters below the scale.

Review with whole class. Ps come to BB to write letters and explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected. T also asks Ps to give an exact probability if they can. (BB)

**Solution:**

- **A:** *If a card is picked at random from a full pack of playing cards, it will be a heart.*
  - (4 different suites: heart, diamond, club, spade, so 4 possible outcomes, each with an equal chance of happening)

**BB:**

\[
p (\text{heart}) = \frac{1}{4} = 0.25
\]
### Activity 5

(Continued)

**B:** When you throw a fair dice the score will **not** be less than 3.
(6 possible outcomes, each with an equal chance of happening and the statement involves 4 of them: 3, 4, 5, 6)

**C:** The next baby born in your local hospital will be a girl.
(2 possible outcomes, each with an equal chance of happening)

**D:** A card picked at random from a pack of playing cards will be black or red.
(2 colours, red and black, in a pack of cards, and the statement involves both of them, so the outcome is **certain**.)

**E:** The next Olympic Games will be held in 2007.
(Impossible, as there is an Olympic Games being held in 2004 and they are held every 4 years.)

**BB:**

<table>
<thead>
<tr>
<th></th>
<th>Impossible</th>
<th>Equal chance of happening as of not happening</th>
<th>Certain</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>A</td>
<td>0.5</td>
<td>B</td>
</tr>
<tr>
<td>E</td>
<td>0.25</td>
<td>C</td>
<td>0.67</td>
</tr>
<tr>
<td></td>
<td></td>
<td>D</td>
<td></td>
</tr>
</tbody>
</table>

### Notes

\[ p (s \geq 3) = \frac{4}{6} = \frac{2}{3} \approx 0.67 \]

\[ p (\text{girl}) = \frac{1}{2} = 0.5 \]

\[ p (\text{black or red}) = \frac{2}{2} = 1 \]

\[ p (2007) = 0 \]

### 6 PbY6a, page 57

**Q.3** Read: This probability scale shows the probabilities of 6 outcomes: A, B, C, D, E and F.

**BB:**

```
A  C  E  F  B  D
0  0.5 1
```

Set a time limit. Ps read questions themselves and write answers in Pbs.

Review with whole class. T chooses a P to read out the question and Ps show letters on scrap paper or slates on command.

Ps with correct responses explain reasoning at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) Which outcome is:
   i) **certain** to happen? (D)  ii) **impossible**? (A)
   iii) the **most unlikely** to happen but is not impossible? (C)

b) Which outcomes are **more likely** than C to happen? (EFBD)

c) Which outcome is **least likely** to happen but is not impossible? (C)

Ps think of questions to ask for the outcomes not mentioned.

E.g. 'Which outcome is as likely to happen as not to happen? (F)

### Notes

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Feedback for T

Whole class activity

Ps ask questions and class shows answers in unison.
## Activity

**PbY6a, page 57. Q.4**

T (P) reads out the question.

Ps show probabilities on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong.

Read: *In a bag there are 5 red, 2 green and 3 yellow marbles.*

If you take out 1 marble with your eyes closed, what is the probability that it will be:

a) red \[ p(\text{red}) = \frac{5}{10} = \frac{1}{2} = 0.5 \rightarrow 50\% \]

b) green \[ p(\text{green}) = \frac{2}{10} = \frac{1}{5} = 0.2 \rightarrow 20\% \]

c) yellow \[ p(\text{yellow}) = \frac{3}{10} = 0.3 \rightarrow 30\% \]

d) not red \[ p(\text{not red}) = \frac{5}{10} = \frac{1}{2} = 0.5 \rightarrow 50\% \]

e) not green \[ p(\text{not green}) = \frac{8}{10} = \frac{4}{5} = 0.8 \rightarrow 80\% \]

f) blue? \[ p(\text{blue}) = \frac{0}{10} = 0\] (Impossible outcome)

## Extension

- If you took a marble out of the bag and replaced it 200 times, how many times would you expect the marble to be:
  - red (100)
  - green (40)
  - yellow? (60)

- What is the ratio of the colours in the bag? \( r : g : y = 5 : 2 : 3 \)

---

## Notes

Whole class activity

(or T puts coloured marbles in a bag, shakes it, then asks the question, amending the words appropriately).

Responses shown in unison. Accept any correct form.

Reasoning: e.g.

a) ‘10 marbles in the bag, each with an equal chance of being picked;

5 of them are red, so red has 5 chances out of 10.’

(i.e. green or yellow)

(i.e. red or yellow)

Whole class activity

Reasoning, e.g.

i) Red: \( \frac{5}{10} \) out of \( \frac{100}{200} \) \( \times 20) \)

or \( \frac{1}{2} \) of 200 = 100
Lesson Plan

Y6

R: Calculations

C: Experiments. Frequency, relative frequency, probability

E: Equally likely outcomes (events)

Activity

1. **Factorisation**
   - Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.
   - Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
   - Elicit that:
     - $58 = 2 \times 29$  
       Factors: 1, 2, 29, 58
     - $233$ is prime number  
       Factors: 1, 233
       (as not exactly divisible by 2, 3, 5, 7, 11, 13 and $17 \times 17 > 233$)
     - $408 = 2 \times 2 \times 2 \times 3 \times 17 = 2^3 \times 17$  
       Factors: 1, 2, 3, 4, 6, 8, 12, 17
       408, 204, 136, 102, 68, 51, 34, 24
     - $1058 = 2 \times 23 \times 23 = 2 \times 23^2$  
       Factors: 1, 2, 23, 46, 529, 1058

2. **Review of probability scale**
   - a) T and Ps describe events or outcomes and class discusses whether their probabilities are rough (e.g. ‘The Head Teacher will come into the classroom in the next minute’ – unlikely but possible) or exact (e.g. ‘If I throw a dice, I will score a ‘6’ – 1 chance out of 6) and where their position would be on a probability scale (drawn vertically or horizontally on BB).
   - b) Each of the letters on this probability scale represents a certain outcome or event. Let’s think of an outcome to match each letter.

<table>
<thead>
<tr>
<th>BB:</th>
<th>A</th>
<th>F</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>G</th>
<th>E</th>
<th>H</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>0.5</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   - Ps make suggestions and give reasoning too. Who agrees? Who can think of another outcome for that letter? etc.
     - e.g.  
       A: If you roll a fair dice, your score will be less than 3 but more than 4.  
       (A is 0, so outcome is impossible.)
       B: If you take a card from a pack of cards, it will be a diamond.  
       (B is 1 quarter, so outcome should have 1 chance in 4 of happening)
       F: If you roll a fair dice you will score ‘2’.  
       (F is 1 sixth, so outcome should have 1 chance in 6 of happening)
       D: If you toss a fair coin, the side facing up will be a Head.  
       (D is 1 half, so outcome should have 1 chance in 2.)

   - Whole class activity
     - Involve several Ps.
     - Extra praise for creativity
     - Discussion, reasoning, agreement, praising

   - Drawn on BB or use enlarged copy master or OHP
     - Discussion, reasoning, agreement, praising
     - Extra praise for creativity.
     - (T could have outcomes already prepared for each letter in case Ps cannot think of any themselves.)

   - Feedback for T

   [Note that on the probability scale, every twelfth is marked.]
Activity

PbY6a, page 58

Q.1 Read: An opaque jar contains 7 red, 3 white and 5 black balls, all the same size. One of the balls is taken out at random and then replaced.

What does opaque mean? (cannot be seen through) What is the opposite of opaque? (transparent)

Set a time limit. Ps read questions themselves and write answers in Pbs.

Review with whole class. Ps could show answers on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong. Mistakes discussed and corrected. Ask for probabilities as decimals too.

Solution:

a) Which colour is the ball most likely to be? (red)

b) Which colour is the ball least likely to be? (white)

c) If you did the experiment 300 times, how many times would you expect the ball to be red? (140)

(7 chances out of 15, so 14 out of 30, so 140 out of 300)

d) What do you think is the probability of the ball being:

i) red \[ p(\text{red}) = \frac{7}{15} = 0.47 \]

ii) white \[ p(\text{white}) = \frac{3}{15} = \frac{1}{5} = 0.2 \]

iii) black \[ p(\text{black}) = \frac{5}{15} = \frac{1}{3} = 0.33 \]

Extension

Ps think of other outcomes and ask other Ps for their probabilities. e.g.

\[ p(\text{red or white}) = \frac{7 + 3}{15} = \frac{10}{15} = \frac{2}{3} = 0.67 \]

\[ p(\text{not white}) = 1 - 0.2 = 0.8, \text{ or } \frac{7 + 5}{15} = \frac{12}{15} = \frac{4}{5} = 0.8 \]

If possible use, and let Ps use, a computer simulation for very large frequencies to show that the more times the experiment is done, the closer the actual data are to the expected outcomes.

Notes

Individual work, monitored, helped

Ps give examples of opaque and transparent materials.

Differentiation by time limit

Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Lesson Plan 58

Q.2 Read: Toss two coins 40 times and write your results in this table.

Demonstrate the experiment with the whole class if necessary.

Set a time limit. P finished first could show his or her results in table on BB as a model for slower Ps. e.g.

BB:

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Tally of 40 tosses</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Heads</td>
<td>⬛♣</td>
<td>8</td>
<td>( \frac{8}{40} = \frac{1}{5} = 0.2 )</td>
</tr>
<tr>
<td>1H + 1T</td>
<td>⬛⃣</td>
<td>21</td>
<td>( \frac{21}{40} = 0.525 )</td>
</tr>
<tr>
<td>2 Tails</td>
<td>⬛⃣</td>
<td>11</td>
<td>( \frac{11}{40} = 0.275 )</td>
</tr>
</tbody>
</table>

\( n = 40 \) \( \sum = 1 \)

Individual or paired work (able with less able), monitored closely, helped, corrected

Table drawn on BB or use enlarged copy master or OHP

T asks 2 or 3 Ps to report their results to class.

Check that the relative frequencies sum to 1.

N.B. \( \sum \) is the symbol for 'sum'
(Continued)

b) Read: **Collect the class data and calculate the relative frequencies in your exercise book.**

T allocates an outcome to certain Ps. Ps stand and dictate results in order round class while chosen Ps keep a running total on calculators (or on computers). Totals for each outcome written in table on BB. Ps check that the total frequency is correct.

Ps calculate relative frequencies as decimals using calculators, rounding appropriately when required, and dictate them to T (P) who writes them in table on BB. Ps check that relative frequencies sum to 1.

e.g.

<table>
<thead>
<tr>
<th>Outcome</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Heads</td>
<td>302</td>
<td>0.252</td>
</tr>
<tr>
<td>1H + 1T</td>
<td>594</td>
<td>0.495</td>
</tr>
<tr>
<td>2 Tails</td>
<td>304</td>
<td>0.253</td>
</tr>
</tbody>
</table>

\[ n = 1200 \quad \Sigma = 1 \]

[If possible, T shows a computer simulation for large values of \( n \). Think about which values the relative frequencies are getting closer and closer to and why!]

c) Read: **What is the probability of each outcome?**

Set a short time limit. Ps write as fractions in Pbs.

Review with whole class. T says each outcome in turn and Ps show probabilities on scrap paper or slates on command. Ps responding correctly explain to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

\[ p(2H) = 0.25 = \frac{1}{4} \]
\[ p(1H + 1T) = 0.5 = \frac{1}{2} \]
\[ p(2T) = 0.25 = \frac{1}{4} \]

Elicit that the relative frequencies in the table point to the fact that we are twice as likely to get 1H + 1T as 2H or 2T. Why?

(There are really 4 possible outcomes: HH, HT, TH, TT because for each of the 2 possible outcomes on one coin there are 2 possible outcomes on the other coin and each outcome is equally likely. In the table, HT and TH have been combined, as the order is not important, so they are twice as likely to happen.)
Activity

5  
PbY6a, page 58
Q.3  a) Read: *If this spinner is spun, how often would you expect the pointer to come to rest on each of the numbers?*

Allow a minute for P's to think write the answer in Pbs.

A, what did you write? Why? Who agrees with A? Who wrote something else? etc. Elicit that the circle has been divided into 7 equal parts, so each of the 7 numbers has an equal chance of coming to rest at the pointer, i.e. 1 chance out of 7 (or a 1 in 7 chance). Mistakes corrected.

b) Read: *Calculate these probabilities.*

Set a short time limit. Review with whole class. P's dictate probabilities to T, explaining reasoning. Class agrees or disagrees. Mistakes discussed and corrected.

**Solution:**

i) \( p(\text{even number}) = \frac{3}{7} \)

ii) \( p(\text{odd number}) = \frac{4}{7} \)

iii) \( p(\text{x} > 5) = \frac{2}{7} \)

iv) \( p(\text{x} \leq 4) = \frac{4}{7} \)

38 min

6  
PbY6a, page 58
Q.4  Read: *A fair spinner is spun twice.*

If possible, T (P) demonstrates the experiment first.

a) Read: *List the possible outcomes if their order is important.*

P's list outcomes in Ex.Bks. Encourage a logical listing.

Review with whole class. P's come to BB or dictate to T. Class points out errors or omissions. P's correct their mistakes.

**Solution:**

1, 1; 1, 2; 1, 3; 1, 4;
2, 1; 2, 2; 2, 3; 2, 4;
3, 1; 3, 2; 3, 3; 3, 4;
4, 1; 4, 2; 4, 3; 4, 4 (16 possible outcomes)

b) Read: *If you repeated the experiment 160 times, how many times would you expect each of these outcomes to happen?*

Set a time limit. P's write answers in Pbs.

Review with whole class. P's could show number of times on slates or scrap paper on command. P's with different responses explain reasoning and class decides who is correct.

Mistakes discussed and corrected.

**Solution:**

i) 2, 2 1 chance out of 16, so 10 out of 160 (× 10)

ii) 1, 3 1 chance out of 16, so 10 out of 160

iii) 4, 2 in any order 2 out of 16, so 20 out of 160

38 min

Extension

What are the probabilities of these outcomes?

\[ p(2, 2) = p(1, 3) = \frac{1}{16} = 0.0625 \]  
(Ps use calculators to give decimal form too.)

\[ p(4, 2 \text{ or } 2, 4) = \frac{2}{16} = \frac{1}{8} = 0.125 \]

45 min

Lesson Plan 58

Notes

Individual work, monitored (helped)

(If possible, demonstrate the experiment using a spinner made from enlarged copy master)

BB:

Reasoning, agreement, self-correction, praising

Note that the probabilities of odd and even sum to 1.

P's suggest certain and impossible outcomes.

Individual work, monitored (helped)

Use enlarged copy master (stuck or drawn on BB)

BB:

Agreement, self-correction, praising

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Whole class activity

P's could suggest other outcomes and ask class to give their probabilities.
### Lesson Plan

**59**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>Factorisation</strong>&lt;br&gt;Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.&lt;br&gt;Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:&lt;br&gt;• <strong>59</strong> is a prime number Factors: 1, 50&lt;br&gt;(as not exactly divisible by 2, 3, 5, 7 and 11 (\times 11 &gt; 59))&lt;br&gt;• <strong>234</strong> = (2 \times 3 \times 3 \times 13 = 2 \times 3^2 \times 13)&lt;br&gt;Factors: 1, 2, 3, 6, 9, 13, 18, 26, 39, 78, 117, 234&lt;br&gt;• <strong>409</strong> is a prime number Factors: 1, 409&lt;br&gt;(as not exactly divisible by 2, 3, 5, 7, 11, 13, 17 and 19, and (23 \times 23 &gt; 409))&lt;br&gt;• <strong>1059</strong> = (3 \times 353) Factors: 1, 3, 353, 1059&lt;br&gt;(353 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and (19^2 &gt; 353))&lt;br&gt;---&lt;br&gt;<strong>8 min</strong></td>
</tr>
</tbody>
</table>
| **2** | **Probability**<br>a) Give me examples of outcomes which have these probabilities.<br>\[
\begin{align*}
\frac{1}{7} & \quad (\text{e.g. scoring '3' on a 7-number spinner}) \\
\frac{1}{4} & \quad (\text{e.g. drawing a 'diamond' from a pack of cards}) \\
\frac{1}{3} & \quad (\text{e.g. scoring '2 or 5' on a fair dice}) \\
50\% & \quad (\text{e.g. getting a 'Tail' when tossing an unbiased coin}) \\
\frac{2}{3} & \quad (\text{e.g. drawing a red marble from a bag containing 4 red and 2 white marbles}) \\
0.75 & \quad (\text{e.g. drawing a card which is not a diamond from a pack of cards}), \\
\end{align*}
\]
e tc.<br>b) T and Ps suggest events or outcomes and class considers their probabilities (as fractions, decimals and percentages when possible, or as 'likely' or 'unlikely' when an exact value is inappropriate).<br>---<br>**15 min** |
**Activity**

**PbY6a, page 59**

Q.1 a) Read: *If you toss 3 fair coins one after the other, what are the possible outcomes if the order in which they occur is taken into account?*

Set a time limit. Ps can work in Ex. Bks if they wish to draw a tree diagram or list the outcomes vertically.

Review with whole class. P dictates his/her outcomes to T. Who agrees? Who had more? Agree that there are 8 possible outcomes. e.g. Listing horizontally:

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

**Notes**

T should have a supply of coins in case some Ps wish to try out the experiment.

Reasoning, agreement, self-correction, praising

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

b) Read: *Calculate the probability of each of these outcomes.*

Set a time limit. Ask more able Ps to give the probabilities in 3 forms: fractions, decimals and percentages.

Review with whole class. Ps could show fractions on scrap paper or slates on command. Ps answering correctly explain reasoning at BB to Ps who were wrong. Mistakes discussed and corrected. Who wrote the fraction in another form? Ps come to BB or direct to T. Class agrees/disagrees.

**Solution:**

i) \[ p(3H) = \frac{1}{8} = 0.125 \rightarrow 12.5\% \]

ii) \[ p(2H + 1T) = \frac{3}{8} = 0.375 \rightarrow 37.5\% \]

iii) \[ p(1H + 2T) = \frac{3}{8} = 0.375 \rightarrow 37.5\% \]

iv) \[ p(3T) = \frac{1}{8} = 0.125 \rightarrow 12.5\% \]

**Extension**

If possible, demonstrate the experiment on a computer. Ps suggest increasing values for \(n\). Ask Ps to calculate the relative frequencies using calculators before revealing them on the program. Compare with the expected probabilities. Show that the greater the value of \(n\), the closer the relative frequencies are to the expected probabilities.

**Notes**

Individual work, monitored, helped

T should have a supply of coins in case some Ps wish to try out the experiment.

Reasoning, agreement, self-correction, praising

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

or:

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

Ps can use calculators.

Responses shown in unison.

Reasoning, agreement, self-correction, praising

(HHT, HTH, THH)

(HTT, THT, TTH)

Whole class activity

(Use Probability Program 10 projected on to a wall/screen so that whole class can see.)

Discussion, agreement, praising
Q2 Read: In a game, a card is taken at random from a full pack of 52 playing cards. The card is then replaced in the pack and a second card is taken.

Demonstrate the experiment first. 2 Ps come to front of class to take a card, note it's suite and replace it in the pack.

a) Read: Draw a tree diagram to show all the possible outcomes.

How many outcomes are possible for each card? (4: Club, Diamond, Heart or Spade)

T starts off the tree diagram on BB, as dictated by Ps, then Ps copy and complete it in Ex. Bks. Set a short time limit.

Review with whole class. A, how many outcomes did you find? Who agrees? Who had more? Ps dictate them to T.

Agree that there are 16 possible outcomes. (For each of the 4 possible outcomes on the 1st card, there are 4 possible outcomes on the 2nd card, i.e. $4 \times 4 = 16$)

b) Read: Use it to help you calculate these probabilities.

Deal with one question at a time. T reads question and Ps show probability on scrap paper or slates on command. Ps with different responses explain reasoning by referring to diagram. Class decides who is correct. Ps write agreed probability in fraction form in Pb's, then calculate the decimal and percentage forms and dictate to T.

Solution:

i) Both cards are clubs.

\[ p(\text{C, C}) = \frac{1}{16} = 0.0625 \rightarrow 6.25\% \]

ii) Neither card is a club.

\[ p(\text{No C}) = \frac{9}{16} = 0.5625 \rightarrow 56.25\% \]

iii) Exactly 1 card is a club.

\[ p(\text{1 C}) = \frac{6}{16} = \frac{3}{8} = 0.375 \rightarrow 37.5\% \]

iv) At least 1 card a club.

\[ p(\text{at least 1 C}) = \frac{7}{16} = 0.4375 \rightarrow 43.75\% \]

Ps count the frequency on the diagram and compare with the total number of outcomes.

Ps use calculators to determine decimal and percentage forms.

Extension

T writes and explains operations to calculate the probabilities, and Ps check that the results match their counted probabilities.

i) Both cards are clubs.

\[
\begin{align*}
\text{1st card} & \quad \text{2nd card} \\
\text{BB:} & \quad p(\text{C, C}) = p(\text{C}) \quad \text{and} \quad p(\text{C}) \\
& \quad = \frac{1}{4} \times \frac{1}{4} \\
& \quad = \frac{1}{16}
\end{align*}
\]

This is only to familiarise Ps with the calculations used to determine probability. Do not expect Ps to be able to use them yet but involve Ps whenever possible.

Tell Ps that to multiply a fraction by a fraction, multiply the numerators to give the numerator of the product and the denominators to give the denominator of the product.
Activity

4

(Continued)

ii) Neither card is a club.

BB: \( p(\text{not C, not C}) = p(\text{not C}) \times p(\text{not C}) \)

\[ = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16} \]

iii) Exactly 1 card is a club.

BB: \( p(\text{C, not C}) \) or \( p(\text{not C, C}) \)

\[ = \frac{1}{4} \times \frac{3}{4} + \frac{3}{4} \times \frac{1}{4} \]

\[ = \frac{3}{16} + \frac{3}{16} = \frac{6}{16} = \frac{3}{8} \]

iv) At least 1 card is a club.

BB: \( p(\text{at least 1 C}) = 1 - \left( \frac{3}{4} \times \frac{3}{4} \right) = 1 - \frac{9}{16} = \frac{7}{16} \)

If possible, use a computer program to simulate the experiment.

Check the probabilities against the relative frequencies for large \( n \).

Notes

Lesson Plan 59

5 PbY6a, page 59

Q.3 Read: This spinner is fairly divided into 6 equal sectors but the possible outcomes do not have equal chances.

Why is that? (Numbers 1 and 2 occur twice but 6 and 3 occur only once.)

a) Read: List the possible outcomes.

Ps list them in \( Pbs \), then dictate to T. Agree that there are 4 different outcomes: 1 (in 2 ways), 2 (in 2 ways), 3, 6

b) Read: Calculate the probability of each outcome in your exercise book.

Set a short time limit. Ps write probabilities as fractions. Review with whole class. Ps come to BB or dictate what T should write, explaining reasoning by referring to the spinner. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

\[ p(\text{score of 1}) = \frac{2}{6} = \frac{1}{3}; \quad p(\text{score of 2}) = \frac{2}{6} = \frac{1}{3}; \]

\[ p(\text{score of 3}) = \frac{1}{6}; \quad p(\text{score of 6}) = \frac{1}{6} \]

Individual work, monitored, helped

Spinner stuck (drawn) on BB or use enlarged copy master

BB:

Reasoning, agreement, self-correction praising

Feedback for T

Extension

If possible, use a computer simulation for large values of \( n \) to check that the relevant frequencies match the expected probabilities.
**Activity 6**

*PbY6a, page 59, Q.4*

Read: *Imagine that the spinner in Question 3 is spun twice and the two numbers are added together. Calculate these probabilities in your exercise book and write them here.*

Ps discuss what to do first and how to continue. T helps or gives hints where necessary. If Ps decide to list all the possible outcomes first, T might suggest labelling the two '1's as 1A and 1B, and similarly the '2's.

**BB:** (Outcomes which match criteria for a) are underlined.)

<table>
<thead>
<tr>
<th>1A, 1A</th>
<th>1B, 1A</th>
<th>2A, 1A</th>
<th>2B, 1A</th>
<th>3, 1A</th>
<th>6, 1A</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A, 1B</td>
<td>1B, 1B</td>
<td>2A, 1B</td>
<td>2B, 1B</td>
<td>3, 1B</td>
<td>6, 1B</td>
</tr>
<tr>
<td>1A, 2A</td>
<td>1B, 2A</td>
<td>2A, 2A</td>
<td>2B, 2A</td>
<td>3, 2A</td>
<td>6, 2A</td>
</tr>
<tr>
<td>1A, 3</td>
<td>1B, 3</td>
<td>2A, 3</td>
<td>2B, 3</td>
<td>3, 3</td>
<td>6, 3</td>
</tr>
<tr>
<td>1A, 6</td>
<td>1B, 6</td>
<td>2A, 6</td>
<td>2B, 6</td>
<td>3, 6</td>
<td>6, 6</td>
</tr>
</tbody>
</table>

Then Ps come to BB to count the outcomes which match the given description. Class agrees/disagrees. Ps write agreed probabilities in *Pbs.*

**Solution:** (By counting relevant outcomes in the list)

a) *The total score is 5.* (see list) \( p \) (score 5) = \( \frac{4}{36} = \frac{1}{9} \)

b) *The total score is less than 5.* \( p \) (score < 5) = \( \frac{20}{36} = \frac{5}{9} \)

c) *The total score is an odd number.* \( p \) (score odd) = \( \frac{18}{36} = \frac{1}{2} \)

d) *The total score is a multiple of 3.* \( p \) (score a multiple of 3) = \( \frac{12}{36} = \frac{1}{3} \)

e) *The total score is greater than 4.* \( p \) (score > 4) = \( \frac{16}{36} = \frac{4}{9} \)

**Notes**

Whole class activity

Use enlarged copy master for *Y6 LP 59/5*

Discussion, reasoning, agreement, praising

Involve several Ps.

Ps list outcomes in *Ex. Bks.* too.

Or by reasoning:

For each of the 6 possible outcomes on the first spin, there are 6 possible outcomes on the 2nd spin, i.e. \( 6 \times 6 = 36 \) (outcomes)

Ask Ps to simplify fractions where possible.

(Reducing the numerator and denominator by the same number of times does not change the value of the fraction)
Y6

Activity

PbY6a, page 59, Q.5

Read: Dad wrote 3 different letters and addressed 3 envelopes. Then he heard the baby crying and went to see what was the matter. While he was out of the room his little daughter, who could not read, put a letter into each envelope and sealed it.

What is the probability that:

a) none of the letters was in the correct envelope
b) all the letters were in the correct envelopes?

(List all the possible outcomes in your exercise book to help you work it out.)

Discuss how listing the outcomes could be done. (e.g. Call the letters A, B and C and the matching envelopes a, b and c and list outcomes horizontally or vertically, or draw a tree diagram.)

BB: e.g.

\[
\begin{align*}
\text{Aa} & \rightarrow \text{Bb} \rightarrow \text{Cc} \\
\text{Aa} & \rightarrow \text{Bc} \rightarrow \text{Cb} \\
\text{Ab} & \rightarrow \text{Ba} \rightarrow \text{Cc} \\
\text{Ab} & \rightarrow \text{Bc} \rightarrow \text{Ca} \\
\text{Ac} & \rightarrow \text{Ba} \rightarrow \text{Cb} \\
\text{Ac} & \rightarrow \text{Bb} \rightarrow \text{Ca}
\end{align*}
\]

Agree that there are 6 possible outcomes.

Deal with one question at a time. T chooses a P to read the outcome, and Ps show probability on slates or scrap paper on command. Ps with different responses explain reasoning on tree diagram. Class decides who is correct.

Solution:

What is the probability that:

a) none of the letters was in the correct envelope?

\[
p (\text{none correct}) = \frac{2}{6} = \frac{1}{3}
\]

b) all the letters were in the correct envelope?

\[
p (\text{all correct}) = \frac{1}{6}
\]

Who can think of another question to ask about the letters? e.g.

What is the probability that 'at least one' letter will be in the correct envelope?

\[
\text{BB: } p (\text{at least 1 correct}) = \frac{4}{6} = \frac{2}{3}
\]

Extension

45 min

Lesson Plan 59

Notes

Whole class activity
(or individual or paired trial first if Ps wish, monitored)
Discussion, reasoning, agreement, (self-correction), praising

T suggests this if no P does so.

Ps come to BB to draw the tree diagram, with T's help.
Ps draw it in Ex.Bks too.
Agreement, praising

Responses shown in unison.
Discussion, reasoning, agreement, praising
Ps show the relevant 'paths' on the tree digram.

Give praise for the question and extra praise if Ps can answer it correctly.

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Activity
Factorising 60, 235, 405 and 1060. Revision, activities, consolidation

PbY6a, page 60
Solutions:
Q.1 a) The marble taken out is green. (Equally likely as unlikely)
   b) The marble taken out is red. (Unlikely)
   c) The marble taken out is either red or yellow. (Equally likely as unlikely)
   d) The marble taken out is not yellow. (Likely)
   e) The marble taken out is black. (Impossible)
   f) The marble taken out is not black. (Certain)

Q.2 a) once out of 8 times
   b) i) \( p(x \text{ is even}) = \frac{4}{8} = \frac{1}{2} \) [x can be 2, 4, 6 or 8]
   ii) \( p(x > 6) = \frac{2}{8} = \frac{1}{4} \) [x can be 7 or 8]
   iii) \( p(x > 8) = 0 \) [impossible]
   iv) \( p(x \text{ is prime}) = \frac{4}{8} = \frac{1}{2} \) [x can be 2, 3, 5 or 7]
   v) \( p(x \leq 6) = \frac{6}{8} = \frac{3}{4} \) [x can be 1, 2, 3, 4, 5 or 6]
   vi) \( p(x \leq 8) = \frac{8}{8} = 1 \) (certain)

Q.3 a) HHHH (4H)
   HHTT, HHTH, HTHH, THHH (3H + 1T)
   HHTT, HHTH, THTH, THHH, HTTH (2H + 2T)
   HHTT, HTTT, THTT, TTHH (1H + 3T)
   TTTT (4T)
16 different possible outcomes
   b) i) \( p(4H) = \frac{1}{16} \)
   ii) \( p(3H + 1T) = \frac{4}{16} = \frac{1}{4} \)
   iii) \( p(2H + 2T) = \frac{6}{16} = \frac{3}{8} \)
   iv) \( p(1H + 3T) = \frac{4}{16} = \frac{1}{4} \)
   v) \( p(4T) = \frac{1}{16} \)
   vi) \( p(3H + 2T) = 0 \) (impossible!)

Factors:

60 = 2\(^2\) × 3 × 5
Factors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

235 = 5 × 47
Factors: 1, 5, 47, 235

410 = 2 × 5 × 41
Factors: 1, 2, 5, 10, 41, 82, 205, 410

1060 = 2\(^2\) × 5 × 53
Factors: 1, 2, 4, 5, 10, 20, 53, 106, 212, 265, 530, 1060
(or set factorising as homework at the end of Lesson 59 and review at the start of Lesson 60)
(A prime number has only 2 different factors, itself and 1)
Q.4 Possible outcomes:
1, 1; 1, 2; 1, 3; 1, 4; 1, 5; 1, 6; 1, 7; 1, 8;
2, 1; 2, 2; 2, 3; 2, 4; 2, 5; 2, 6; 2, 7; 2, 8;
3, 1; 3, 2; 3, 3; 3, 4; 3, 5; 3, 6; 3, 7; 3, 8;
4, 1; 4, 2; 4, 3; 4, 4; 4, 5; 4, 6; 4, 7; 4, 8;
5, 1; 5, 2; 5, 3; 5, 4; 5, 5; 5, 6; 5, 7; 5, 8;
6, 1; 6, 2; 6, 3; 6, 4; 6, 5; 6, 6; 6, 7; 6, 8;
7, 1; 7, 2; 7, 3; 7, 4; 7, 5; 7, 6; 7, 7; 7, 8;
8, 1; 8, 2; 8, 3; 8, 4; 8, 5; 8, 6; 8, 7; 8, 8 (64)

Or by reasoning:
For each of the 8 possible outcomes on the first spin there are
8 possible outcomes on the 2nd spin: $8 \times 8 = 64$.

a) The total score is 4. Possible outcomes: 1, 3; 2, 2; 3, 1

$p$ (total score is 4) = $\frac{3}{64}$

T might show this calculation and ask Ps to explain it.

b) The total score is 4 or less.

Possible outcomes: 1, 1; 1, 2; 1, 3; 2, 1; 2, 2; 3, 1

$p$ (total score $\leq$ 4) = $\frac{6}{64} = \frac{3}{32}$

c) The total score is 16. Only one possible outcome: 8, 8

$p$ (total score is 16) = $\frac{1}{64}$

d) The total score is more than 4.

This is the opposite (or complement) of b).

$p$ (total score $> 4$) = $1 - \frac{3}{32} = \frac{29}{32}$

To multiply a fraction by a fraction:

- multiply the numerators to give the numerator of the product
- multiply the denominators to give the denominator of the product

i.e. \( \frac{1}{8} \times \frac{1}{8} \) + \ldots 6 times

\[
= \frac{1}{64} + \frac{1}{64} + \frac{1}{64} = \frac{3}{64}
\]
### Lesson Plan 61

#### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **61** is a prime number  
  Factors: 1, 61  
  (as not exactly divisible by 2, 3, 5, 7 and 11 × 11 > 61)
- **236** = 2 × 2 × 59 = 2² × 59  
  Factors: 1, 2, 4, 59, 118, 236
- **411** = 3 × 137  
  Factors: 1, 3, 137, 411
- **1061** is a prime number  
  Factors: 1, 1061  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37² > 1061)

8 min

#### Activity 2

**Revision of multiplication**

a) Let's see how good you are at multiplication! Ps come to BB to write products or dictate what T should write, explaining reasoning in detail. Class points out errors. What sign should we write in the circles? (=) Ps check that the quotients are equal to the products. Ps come to BB to continue the pattern for 2 or 3 more multiplications.

BB:

i) 64 × 4 = (256)
   64 × 2 = (128)  (Ps write missing signs.)
   64 × 1 = (64)
   64 × \(\frac{1}{2}\) = (32) \(\Downarrow\) 64 ÷ 2
   64 × \(\frac{1}{4}\) = (16) \(\Downarrow\) 64 ÷ 4
   64 × \(\frac{1}{8}\) = (8) \(\Downarrow\) 64 ÷ 8
   64 × \(\frac{1}{16}\) = (4) \(\Downarrow\) 64 ÷ 16
   etc.

ii) 43 × 100 = (4300)
   43 × 10 = (430)
   43 × 1 = (43)
   43 × 0.1 = (4.3) \(\Downarrow\) 43 ÷ 10
   43 × 0.01 = (0.43) \(\Downarrow\) 43 ÷ 100
   43 × 0.001 = (0.043) \(\Downarrow\) 43 ÷ 1000
   etc.

Whole class activity

Written on BB or SB or OHT

Involve as many Ps as possible.

At a good pace
Reasoning, agreement, praising

Discuss how the products change and compare with the reverse operation, division.

Elicit that:
- if the multiplier is reduced by 1 half (1 tenth), the product is also reduced by 1 half (1 tenth)
- multiplying by \(\frac{1}{n}\) is the same as dividing by \(n\)
- multiplying by 0.1, is the same as dividing by 10
- multiplying by 0.01, is the same as dividing by 100
- multiplying by 0.001, is the same as dividing by 1000.
b) What does $5 \times \frac{2}{3}$ mean? Let’s think of different ways to explain it.

Ps come to BB to draw diagrams and explain reasoning. Who agrees? Who can think of another way to explain it? T shows any not suggested by Ps and asks Ps what they think of it.

BB: e.g.

i) $5 \times \frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3} = 3 \frac{1}{3}$

ii) 5 jumps

iii) $5 \times \frac{2}{3}$ could also be thought of as $\frac{2}{3}$ of 5

T explains by reasoning:

If $\frac{3}{3}$ of 5 is 5,

then $\frac{1}{3}$ of 5 is $5 \div 3 = \frac{5}{3}$

and $\frac{2}{3}$ of 5 is $\frac{5}{3} \times 2 = \frac{10}{3} = 3 \frac{1}{3}$

c) How would you do these multiplications? Ps come to BB to show calculation and explain reasoning. Class agrees/disagrees.

i) $7 \times \frac{3}{4} = ?$ \[7 \times \frac{3}{4} = \frac{21}{4} = 5 \frac{1}{4}\]

ii) $a \times \frac{b}{c} = ?$ \[a \times \frac{b}{c} = \frac{a \times b}{c}\] (a, b and c are integers and c ≠ 0)

iii) $\frac{b}{c}$ of $a = ?$ \[a \div c \times b = \frac{a}{c} \times b = \frac{a \times b}{c}\]

Who can tell me the rules for multiplying an integer by a fraction?

• Multiply the integer by the numerator, then divide the product by the denominator, or
• divide the integer by the denominator then multiply the quotient by the numerator.

Point out/elicit that this long explanation can be summarised by the operations highlighted.

BB: $5 \times \frac{2}{3} = \frac{2}{3}$ of 5

Elicit that: $a \times \frac{b}{c} = \frac{b}{c}$ of $a$

T repeats in a clearer way if necessary.

Praising only
Activity

3

**PbY6a, page 61**

Q.1 Read: *Calculate the products.*

Set a time limit of 3 minutes. Ps write products in Pbs.

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. If problems or disagreement, draw diagrams on BB or use models (e.g. multilink cubes). Mistakes discussed and corrected.

**Solution:**

a) \(9 \times 2 = 18\)  
b) \(6 \times 3 = 18\)  
c) If \(a \times b = c\), then \(a \times \frac{b}{2} = \frac{c}{2}\)  
\(9 \times \frac{1}{2} = 4\)  
\(6 \times \frac{1}{3} = 2\)  
\(a \times \frac{b}{3} = \frac{c}{3}\)  
\(9 \times \frac{1}{4} = 2\)  
\(6 \times \frac{2}{3} = 4\)  
\(a \times \frac{b}{4} = \frac{c}{4}\)  
\(9 \times \frac{1}{8} = 1\)  
\(6 \times \frac{1}{6} = 1\)  
\(a \times \frac{b}{5} = \frac{c}{5}\)

Show interim steps if necessary. e.g. \(6 \times \frac{2}{3} = \frac{12}{3} = 4\)

What do you notice? Ps point out relationships and how the products are changing.

24 min

---

**Extension**

**PbY6a, page 61**

Q.2 Read: *Calculate the products.*

Set a time limit of 3 minutes. Ps write products in Pbs.

Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. If problems or disagreement, check with reverse operation, division. Mistakes discussed/corrected.

**Solution:**

a) \(25 \times 100 = 2500\)  
b) \(7 \times 2 = 14\)  
c) \(41 \times 0.3 = 12.3\)  
\(25 \times 10 = 250\)  
\(7 \times 0.2 = 1.4\)  
\(15 \times 0.3 = 4.5\)  
\(25 \times 1 = 25\)  
\(7 \times 0.6 = 4.2\)  
\(10 \times 0.3 = 3\)  
\(25 \times 0.1 = 2.5\)  
\(7 \times 0.1 = 0.7\)  
\(5 \times 0.3 = 1.5\)  
\(25 \times 0.01 = 0.25\)  
\(7 \times 0.05 = 0.35\)  
\(0 \times 0.3 = 0\)  
\(25 \times 0.001 = 0.025\)

Who can explain what \(25 \times 0.01\) means?

(e.g. Adding 0.01 to itself 25 times, or \(\frac{1}{100}\) of 25)

Who can explain what \(5 \times 0.3\) means?

(e.g. \(0.3 + 0.3 + 0.3 + 0.3 + 0.3\),

or \(\frac{3}{10}\) of 5 = \(\frac{5}{10} \times 3 = \frac{15}{10} = 1.5\)

---

Notes

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Discussion by time limit

Feedback for T

Ps point out relationships and how the products change.

30 min

---

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, agreement, self-correction, praising

Feedback for T

Ps point out relationships and how the products change.

Whole class discussion

Ps make suggestions and class agree/disagree.

If no P has an idea, T shows one of those opposite and asks class if it is correct.

Praising only
## Lesson Plan 61

### Activity 5

**PbY6a, page 61**

Q.3 **Read:** Calculate the quotients.

Set a time limit. Ps calculate mentally and write quotients in *Pbs*.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. If problems or disagreement, check with reverse operation, multiplication.

**Solution:**

\[
\begin{align*}
\text{a) } & \quad \frac{4}{5} \div 4 = \frac{1}{5}; \quad \frac{4}{5} \div 2 = \frac{2}{5}; \quad \frac{4}{5} \div 1 = \frac{4}{5} \\
\text{b) } & \quad \frac{5}{9} \div 1 = \frac{5}{9}; \quad \frac{5}{9} \div 2 = \frac{5}{18}; \quad \frac{5}{9} \div 4 = \frac{5}{36} \\
\text{c) } & \quad \frac{2}{3} \div 5 = \frac{2}{15}; \quad \frac{2}{3} \div 2 = \frac{1}{3}; \quad \frac{2}{3} \div 2 = \frac{5}{6}; \\
& \quad \frac{2}{3} \times 2 = 1\frac{1}{3} \text{ (Divide the integer first, then the numerator)}
\end{align*}
\]

**d) \quad 0.8 \div 4 = 0.2; \quad 2.4 \div 4 = 0.6; \quad 16.8 \div 8 = 2.1; \quad 0.8 \div 40 = 0.02**

Elicit the rule or law for dividing a fraction by an integer.

'To divide a fraction by an integer:

- divide the numerator if it is a multiple of the integer, or
- multiply the denominator by the integer.'

35 min

### Activity 6

**PbY6a, page 61**

Q.4 **Read:** Calculate in your exercise book.

Set a time limit. Ps write the whole operation and underline the result. Remind Ps to write the unit of measure too.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB to Ps who were wrong. Who agrees? Who did it another way? Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
\text{a) i) } & \quad \frac{1}{4} \text{ of } 240 \text{ kg } = \frac{240}{4} \times \frac{1}{4} = 60 \text{ kg} \\
& \quad \frac{1}{4} \times \frac{240}{4} = 60 \text{ kg} \\
\text{ii) } & \quad 240 \text{ kg } \times \frac{1}{4} = 60 \text{ kg} \\
\text{b) i) } & \quad \frac{1}{6} \text{ of } 240 \text{ kg } = \frac{240}{6} \times \frac{1}{6} = 40 \text{ kg} \\
& \quad \frac{1}{6} \times \frac{240}{6} = 40 \text{ kg} \\
\text{ii) } & \quad 240 \text{ kg } \times \frac{1}{6} = 40 \text{ kg} \\
\text{c) i) } & \quad \frac{3}{4} \text{ of } 240 \text{ kg } = \frac{240}{4} \times \frac{3}{4} = 60 \text{ kg } \times \frac{3}{4} = 180 \text{ kg} \\
& \quad \frac{240}{4} \times \frac{3}{4} = \frac{240 \times 3}{4} \text{ kg } = 180 \text{ kg} \\
& \quad \frac{240}{4} \times \frac{3}{4} = 180 \text{ kg} \\
\text{ii) } & \quad 240 \text{ kg } \times \frac{3}{4} = \frac{240 \times 3}{4} \text{ kg } = 180 \text{ kg} \\
& \quad \frac{240}{4} \times \frac{3}{4} = \frac{180}{4} \text{ kg } = 45 \text{ kg}
\end{align*}
\]

Ps explain in their own words. T repeats more clearly if necessary.

Extra praise if Ps show reduction of numerator and denominator by cancellation. If not, T shows it.

Extra praise if Ps realise that in each case, i) = ii)
Y6

Activity

(Continued)

d) i) \( \frac{5}{6} \) of 240 kg = \( 240 \text{ kg} \div 6 \times 5 = 40 \text{ kg} \times 5 = 200 \text{ kg} \)

ii) \( 240 \text{ kg} \times \frac{5}{6} = \frac{240 \times 5}{61} \text{ kg} = 200 \text{ kg} \)

e) i) \( \frac{9}{4} \) of 240 kg = \( 240 \text{ kg} \div 4 \times 9 = 60 \text{ kg} \times 9 = 540 \text{ kg} \)

ii) \( 240 \text{ kg} \times \frac{9}{4} = \frac{240 \times 9}{41} \text{ kg} = 540 \text{ kg} \)

f) i) 0.4 of 240 kg = \( 240 \text{ kg} \div 10 \times 4 = 24 \text{ kg} \times 4 = 96 \text{ kg} \)

(or \( 24 \text{ kg} \times 4 = 96 \text{ kg} \))

ii) \( 240 \text{ kg} \times 0.4 = 96.0 \text{ kg} = 96 \text{ kg} \)

\( 60 \) min

Lesson Plan 61

Notes

1) \( 40 \text{ kg} \times \frac{5}{6} = 200 \text{ kg} \)

2) \( 60 \text{ kg} \times \frac{9}{4} = 540 \text{ kg} \)

3) \( \frac{4}{10} \)

4) Individual work, monitored, helped

If possible, T has a sample or picture of sugar beet and gives information about where it is grown and how sugar is extracted (or Ps could find out information for homework).

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid method of solution.

or \( 12 \text{ kg} \)

or \( 18 \text{ kg} \)

(\( 18 \text{ hundredths} \rightarrow 12 \text{ kg} \))

or \( 216 \text{ kg} \)

(\( 216 \text{ kg} \)

4) Accept the answer in any correct form.

Q.5 Deal with one question at a time. Ps read question themselves, write a plan, calculate and check the result, then write the answer in a sentence in Ex. Bks.

Review with whole class. T chooses a P to read out the question. Ps show results on scrp paper or slates on command. P with correct answer explains reasoning on BB. Who did the same? Who did it a different way? etc. Mistakes discussed/corrected.

T asks a P to say the answer in a sentence.

Solution:

a) In a certain year, 1 kg of sugar beet contained \( \frac{9}{50} \) kg of sugar on average.

How much sugar was in 1200 kg of sugar beet that year?

Plan: \( 1200 \times \frac{9}{50} = \frac{2400 \times 9}{50} = 216 \text{ (kg)} \)

or \( \frac{9}{50} \) of 1200 kg = \( 1200 \text{ kg} \div 50 \times 9 \)

\( 24 \text{ kg} \)

\( = 120 \text{ kg} \div 5 \times 9 = 216 \text{ kg} \)

Answer: There were 216 kg of sugar in 1200 kg of sugar beet.

b) What is \( 3 \) sevenths of 5 and \( 3 \) fifths kilometres?

Plan: \( \frac{3}{7} \) of \( \frac{5}{3} \) km = \( \frac{5 \times 3}{5} \text{ km} \div 7 \times 3 = \frac{28}{5} \text{ km} \div 7 \times 3 \)

\( = \frac{4}{5} \text{ km} \times 3 = \frac{12}{5} \text{ km} = 2 \frac{2}{5} \text{ km} \)

or \( \frac{5}{3} \text{ km} \times \frac{3}{7} = 5600 \text{ m} \times \frac{3}{7} = \frac{5600 \times 3}{7} \text{ m} = 2400 \text{ m} \)

Answer: \( 3 \) sevenths of \( 5 \) and \( 3 \) fifths kilometres is \( 2 \) and \( 2 \) fifths kilometres.
Activity

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- \(62 = 2 \times 31\) Factors: 1, 2, 31, 62
- \(237 = 3 \times 79\) Factors: 1, 3, 79, 237
- \(412 = 2 \times 2 \times 103 = 2^2 \times 103\) Factors: 1, 2, 4, 103, 206, 412
- \(1062 = 2 \times 3 \times 3 \times 59 = 2 \times 3^2 \times 59\) Factors: 1, 2, 3, 6, 9, 18, 59, 118, 177, 354, 531, 1062

7 min

Calculations with fractions

a) Let’s do these calculations in different ways. Ps come to BB or dictate what T should write. Class agrees/disagrees. Who can think of another way to do it? If no P does so, T shows one and ask class if it is correct. Ps say which method they prefer and why. Agree that cancelling where possible is simpler.

BB: e.g.

i) \(\frac{16}{15} \times 5 = \frac{16 \times 5}{15} = \frac{80}{15} = \frac{16}{3} = \frac{5}{3}\) or \(\frac{16 \times 5}{15\cdot3} = \frac{16}{3}\)

ii) \(8 \times \frac{23}{24} = \frac{8 \times 23}{24} = \frac{8 \times 23}{24} = \frac{24 + 72 / 23}{3} = \frac{31}{3}\)

or \(= 24 + \frac{23}{24 \div 8} = 24 + \frac{23}{3} = 24 + \frac{72 / 23}{3} = \frac{31}{3}\)

iii) \(\frac{14}{23} \div 7 = \frac{14}{23} \div 7 = \frac{2}{3} \text{ or } \frac{2 \times 14}{23 \times 7} = \frac{2}{23}\)

iv) \(\frac{5}{3} \div 4 = \frac{5 \times 4}{3} = \frac{1 + 5}{3 \times 4} = \frac{1 + 5}{3 \times 4} = \frac{15}{12}\)

or \(\frac{17}{3} \div 4 = \frac{17}{3 \times 4} = \frac{17}{12} = \frac{15}{12}\)

b) Which number can be written instead of the letters so that the equation is true?

Ps come to BB or dictate what T should write. Class checks result by substituting the number for the letter in the equation.

BB: e.g.

i) \(a \times 2 = 8\) ii) \(\frac{15}{a} \times 4 = \frac{20}{3}\)

\(\frac{a}{7} = 8 \div 2 = 4\) \(\frac{15}{a} = \frac{20}{3} \div 4 = \frac{5}{3} = \frac{15}{9}\)

\(a = 4 \times 7 = 28\) \(a = \frac{28}{9}\)

Check: \(\frac{28}{7} \times 2 = 4 \times 2 = 8 \checkmark\) Check: \(\frac{15}{9} \times 4 = \frac{60}{9} = \frac{20}{3} \checkmark\)
(Continued) e.g.

iii) \( \frac{6}{64} \times b = \frac{3}{8} \)
iv) \( \frac{3}{4} \div c = \frac{3}{8} \)

\( \frac{3}{32} \times b = \frac{3}{8} \)
ii) \( \frac{3}{4} \div c = \frac{3}{8} \)

\( \frac{3}{32} \times b = \frac{12}{32} \)
\( 4 \times c = 8 \)

\( 3 \times b = 12 \)
\( c = 8 \div 4 = \frac{1}{2} \)

\( b = 12 \div 3 = 4 \)

Check: \( \frac{6}{64} \times 4 = \frac{6}{16} = \frac{3}{8} \)

\( \frac{3}{4} \div 2 = \frac{3}{4} \times 2 = \frac{3}{8} \)

or

v) \( d \div 12 = \frac{3}{10} \)
vi) \( \frac{15}{e} \div 3 = \frac{1}{4} \)

\( \frac{d}{5 \times 12} = \frac{3}{10} \)
\( \frac{5}{e} = \frac{1}{4} = \frac{5}{20} \)

\( d = 18 \)

\( e = 20 \)

Check: \( \frac{18}{5} \div 12 = \frac{18}{60} = \frac{3}{10} \)

\( \frac{15}{20} \div 3 = \frac{5}{20} = \frac{1}{4} \)

20 min

Q.1 Read: Solve the problem in your exercise book in the 3 ways shown below:
An express train is travelling at a steady speed of 105 km per hour. How far does it travel in:

i) \( \frac{4}{5} \) of an hour
ii) \( \frac{3}{4} \) hours?

Set a time limit or deal with one method at a time.
Review with whole class. Ps come to BB to explain their reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Which method did you like best? Why?

Solution:
a) Using proportion:

i) \( \frac{1}{5} \) hour → 105 km ÷ 5 = 21 km

\( \frac{4}{5} \) hour → 21 km × 4 = 84 km

ii) \( \frac{1}{4} \) hour → 105 km ÷ 4 = 26 \( \frac{1}{4} \) km

\( \frac{7}{4} \) hour → 26 \( \frac{1}{4} \) km × 7 = 182 km + \( \frac{7}{4} \) km

= 183 \( \frac{3}{4} \) km

Individual work, monitored, helped
Reasoning, agreement, self-correction, praising
Discuss the pros and cons of the different methods.
What other method could we have used?

Drawing diagrams e.g.

i) \( \frac{5}{3} \) → 105 km

\( \frac{1}{3} \) → 21 km

\( \frac{4}{3} \) → 84 km

ii) \( \frac{4}{3} \) → 105 km

\( \frac{1}{4} \) → 26 \( \frac{1}{4} \) km

\( \frac{7}{4} \) → 183 \( \frac{3}{4} \) km
### Activity 3 (Continued)

**b) Using 2 operations in one line**

i) \(105 \text{ km} \div 5 \times 4 = 21 \text{ km} \times 4 = 84 \text{ km}\)

ii) \(105 \text{ km} \div 4 \times 3 = 26.25 \text{ km} \times 3 = 183.75 \text{ km}\)

**c) Using a single multiplication**

i) \(\frac{105}{21} \times 4 = 84 \text{ km}\) (or \(105 \text{ km} \times 0.8 = 84 \text{ km}\))

ii) \(\frac{105}{3} \times \frac{7}{4} = \frac{735}{4} \text{ km} = 183 \frac{3}{4} \text{ km}\)

(or \(105 \text{ km} \times 1.75 = 183 \frac{3}{4} \text{ km}\))

*Answer:* The train travels 84 km in 4 fifths of an hour and 183 and 3 quarter km in 1 and 3 quarter hours.

---

### Notes

- Individual work, monitored (helped)
- Responses shown in unison.
- Class points out any errors or missed parts of explanation.

---

### Activity 4

**Q.2 Read:** What is the whole quantity if:

- a) 1 quarter of it is 18 m
- b) 1 fifth of it is 253 litres
- c) 0.1 of it is 31 km
- d) 0.01 of it is 27.6 kg?

*Calculate like this in your exercise book.*

- a) If \(\frac{1}{4}\) is 18 m, then \(\frac{4}{4}\) is . . . . . . m.

Set a time limit or deal with one at a time.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Mistakes discussed and corrected.

*Solution:*

a) If \(\frac{1}{4}\) is 18 m, then \(\frac{4}{4}\) is \(18 \text{ m} \times 4 = 72 \text{ m}\)

b) If \(\frac{1}{5}\) is 253 litres, then \(\frac{5}{5}\) is \(253 \text{ litres} \times 5 = 1265 \text{ litres}\)

c) If 0.1 is 31 km, then the whole is \(31 \text{ km} \times 10 = 310 \text{ km}\)

d) If 0.01 is 27.6 kg, then the whole is \(27.6 \text{ kg} \times 100 = 2760 \text{ kg}\)

---

### Activity 5

**Q.3 a) Read:** Three quarters of my money is £660.

*How much money do I have?*

T has BB already prepared with the method of solution example given in *Pbs*. Ps come to BB to fill in the missing amounts, explaining reasoning by referring to the diagrams. Class points out any errors or missed parts of explanation. After agreement, Ps write missing amounts in *Pbs* too.

BB: If \(\frac{3}{4}\) → £660

\[\text{then } \frac{1}{4} \rightarrow \frac{660}{3} = \frac{220}{1} \text{ \(\frac{1}{4}\)}\]

\[\text{and } \frac{4}{4} \rightarrow \frac{220 \times 4}{1} = \frac{880}{4} \text{ \(\frac{4}{3}\)}\]

*Whole class activity to start*

Written/drawn on BB or SB or OHT

Discussion, reasoning, agreement, praising

Who could write a plan for the solution on one line?

**BB:** £660 ÷ 3 ÷ 4

Who could write the plan as a single operation?

**BB:** £660 × \(\frac{4}{3}\)
T: The whole quantity is 4 thirds of 3 quarters of the quantity.
We say that 4 thirds is the reciprocal value of 3 quarters, and
3 quarters is the reciprocal value of 4 thirds.

Set the remaining questions as individual work under a time limit.
Deal with one at a time if necessary. Ps copy the method of solution
for part a) but can draw different diagrams if they wish.
Review with whole class. Ps read out the questions and Ps show
solutions on scrap paper or slates on command. Ps with correct
answers come to BB to write their solutions and draw diagrams,
explaining reasoning. Class agrees/disagrees. Mistakes discussed
and corrected. T chooses a P to say the answer in a sentence.
After each solution, ask Ps to write a plan in one line and a shorter
plan using one operation. Elicit the relevant reciprocal fractions.

Solutions:

b) How much does 1 metre of material cost if 4 fifths of a metre
costs £6.40?

If \( \frac{4}{5} \rightarrow £6.40 \)

then \( \frac{1}{5} \rightarrow £6.40 ÷ 4 = £1.60 \)

and \( \frac{5}{5} \rightarrow £1.60 \times 5 = £8 \)

Answer: One metre of material costs £8.

BB: $\frac{4}{3} \times \frac{3}{4} = 1$

reciprocal values

Individual work, monitored, helped
Differentiation by time limit
Responses shown in unison.
Reasoning, agreement, self-correction, praising

BB:

Plan: £6.40 ÷ 4 × 5

or £6.40 × \( \frac{5}{4} \)

[Reciprocals: \( \frac{4}{5} \times \frac{5}{4} = 1 \)]

Plan: \( \frac{4}{5} \times \frac{5}{4} = 1 \)

Plan: \( \frac{4}{5} \times \frac{5}{4} = 1 \)

Plan: \( \frac{4}{5} \times \frac{5}{4} = 1 \)

Answers:

\( \frac{4}{5} \times \frac{5}{4} = 1 \)

Plan: \( \frac{4}{5} \times \frac{5}{4} = 1 \)

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Plan: \( \frac{4}{5} \times \frac{5}{4} = 1 \)
Q.4 Read: Calculate the length of the adjacent side, then the perimeter and the area of the shape.

Deal with one question at a time. Ps draw diagrams, write plans and do calculations in Ex. Bks. Set a time limit.

Review with whole class. Ps come to BB to draw rough sketches and write solutions, explaining reasoning. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

**Solutions:**

a) The length of a rectangle is 48 mm and its adjacent side is 5 sixths as long.

\[ b = \frac{8}{5} \times 48 \text{ mm} = 40 \text{ mm} \]

(or \( b = \frac{48}{6} \times 5 = 40 \text{ mm} \))

\[ P = 2 \times (48 \text{ mm} + 40 \text{ mm}) = 2 \times 88 \text{ mm} = 176 \text{ mm} \]

\[ A = 48 \text{ mm} \times 40 \text{ mm} = 1920 \text{ mm}^2 \]

b) One side of a rectangle is 7.2 cm, which is 3 fifths of the length of its adjacent side.

\[ b = \frac{2.4}{3} \times 5 = 2.4 \times 5 = 12 \text{ cm} \]

(or \( b = \frac{7.2}{5} \times 5 = 12 \text{ cm} \))

\[ P = 2 \times (7.2 \text{ cm} + 12 \text{ cm}) = 2 \times 19.2 \text{ cm} = 38.4 \text{ cm} \]

\[ A = 7.2 \text{ cm} \times 12 \text{ cm} = 86.4 \text{ cm}^2 \]

c) One side of a rectangle is 25 m, which is 1.2 times the length of its adjacent side.

\[ b = \frac{10}{12} \times 25 \text{ m} = \frac{250}{12} \text{ m} = \frac{20}{6} \text{ m} \]

(or \( b = \frac{25}{12} \times \frac{5}{6} = \frac{125}{6} \text{ m} = \frac{20}{5} \text{ m} \))

\[ P = 2 \times (25 \text{ m} + \frac{5}{6} \text{ m}) = 2 \times 25 \frac{5}{6} \text{ m} \]

\[ = 90 \text{ m} + \frac{5}{3} \text{ m} = 91 \frac{2}{3} \text{ m} \]

\[ A = 25 \text{ m} \times 20 \frac{5}{6} \text{ m} = 25 \text{ m} \times \frac{125}{6} \text{ m} \]

\[ = \frac{3125}{6} \text{ m}^2 = 520 \frac{5}{6} \text{ m}^2 \]

**Notes**

Individual work, monitored, helped, or c) done with the whole class

Discussion, reasoning, agreement, self-correction, praising

Accept any valid method of solution. Extra praise if Ps calculated with fractions correctly.

BB:
Activity 1

Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:
- \( 63 = 7 \times 9 \) Factors: 1, 7, 9, 63
- \( 238 = 2 \times 7 \times 17 \) Factors: 1, 2, 7, 14, 17, 34, 119, 238
- \( 413 = 7 \times 59 \) Factors: 1, 7, 59, 513
- \( 1063 \) is a prime number (Not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and \( 37^2 > 1063 \))

6 min

Activity 2

Multiplying and dividing by fractions
a) i) What does \( \frac{44}{3} \times \frac{3}{4} \) actually mean?
Ps come to BB or dictate what T should write. Class points out errors.
BB: e.g. \( \frac{3}{4} + \frac{3}{4} + \ldots + \frac{3}{4} \) or \( \frac{3}{4} \) of 44 = 33
(44 terms)

ii) How can we do this multiplication? Who agrees? Who can do it a different way? Ps come to BB or dictate to T, explaining reasoning.
BB: e.g. \( 13 \times \frac{7}{4} = \frac{13 \times 7}{4} = \frac{91}{4} = 22 \frac{3}{4} \)
or \( 13 \times \frac{7}{4} = 13 + 4 \times 7 = \frac{13}{4} \times 7 = \frac{91}{4} = 22 \frac{3}{4} \)
or \( 13 \times 1 \frac{3}{4} = 13 \times 1.75 = 22.75 \)

iii) What does \( 120 \times \frac{a}{5} \) mean? Ps come to BB or dictate to T.
BB: \( \frac{a}{5} + \frac{a}{5} + \ldots + \frac{a}{5} \) or \( \frac{a}{5} \) of 120 or \( \frac{24}{5} \times \frac{120 \times a}{5} = 24 \times a \)

• How would we write this calculation?
BB: \( 120 \times \frac{a}{b} = \frac{120 \times a}{b} \) or \( 120 \times \frac{a}{b} = 120 \div b \times a \)

• How would we write this calculation? Elicit that:
BB: \( c \times \frac{a}{b} = \frac{c \times a}{b} = c \div b \times a = \frac{a}{b \div c} \) (if possible)

b) Let's do these divisions. Ps come to BB or dictate to T.
BB: i) \( \frac{4}{9} \div 2 = \frac{4 \div 2}{9} = \frac{2}{9} \) ii) \( \frac{4}{9} \div 3 = \frac{4}{9 \times 3} = \frac{4}{27} \)

6 min

Notes
Individual work, monitored (or whole class activity)
BB: 63, 238, 413, 1063
Calculators allowed.
Reasoning, agreement, self-correction, praising

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
92 & 9 & 4 & 3 & 7 & 1 & 1 & 1 \\
\hline
19 & 7 & 17 & 17 & 1 & 1 & 1 & 1 \\
\hline
119 & 7 & 17 & 17 & 1 & 1 & 1 & 1 \\
\hline
1063 & 7 & 17 & 17 & 1 & 1 & 1 & 1 \\
\hline
\end{array}
\]

Whole class activity
T writes each multiplication on BB as question is asked. Reasoning, agreement, praising
Agree that:
\( 44 \times \frac{3}{4} = \frac{3}{4} \times 44 \)
Reasoning, agreement, praising
Accept any valid method.
T asks 2 or 3 Ps which method they prefer and why

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
1 & 1 & 1 \times \frac{7}{3} \\
\hline
14 & 7 & 3 \times 1 \frac{3}{3} \\
\hline
5 & 2 & 5 \times 1 \frac{1}{3} \\
\hline
17 & 5 & 10 \times 1 \frac{1}{3} \\
\hline
2 & 2 \downarrow 7 & 5 \times 1 \frac{1}{3} \\
\hline
\end{array}
\]

T points out or elicits that \( a, b \) and \( c \) are integers and \( b \neq 0 \).
i.e. where \( b \) is a multiple of \( c \)

iii) \( \frac{a}{b} \div c = \frac{a \div c}{b} = \frac{a}{b \times c} \) (if possible)
Lesson Plan 63

**Activity**

2

(Continued)

Let’s put into words the rules for multiplying and dividing fractions by a positive integer. Ps say the rule in their own words. T repeats more clearly if necessary.

- To multiply a fraction by an integer, multiply the numerator by the integer or, if possible, divide the denominator by the integer.

- To divide a fraction by an integer, multiply the denominator by the integer or, if possible, divide the numerator by the integer.

---

**Notes**

Elicit that when:

- **Multiplying**, the fraction increases in value: i.e. either *number* of parts increases, or size of the parts increases;

- **Dividing**, the fraction decreases in value: i.e. either *number* of parts decreases, or size of parts decreases.

---

**Erratum**

In *Ps*, 2nd c) i) should be c) (ii)

---

**Y6**

**PbY6a, page 63**

Q.1 Read: *Do the calculations in your exercise book.*

Deal with one part (a, b, c, d) at a time. Set a short time limit.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees, disagrees. Mistakes discussed and corrected.

**Solution:**

a) i) $15 \times \frac{3}{4} = \frac{45}{4} \ m = 11 \frac{1}{4} \ m$

ii) $\frac{3}{4}$ of $15 \ m = 15 \ m \div 4 \times 3 = 3 \frac{3}{4} \ m \times 3$

\[= (9 + \frac{9}{4}) \ m = 11 \frac{1}{4} \ m\]

b) i) $3 \ \text{litres} \times \frac{5}{6} = (3 + \frac{15}{6}) \ \text{litres} = (3 + \frac{5}{2}) \ \text{litres}$

\[= (3 + 2 \frac{1}{2}) \ \text{litres} = 5 \frac{1}{2} \ \text{litres}\]

ii) $\frac{5}{6}$ of $3 \ \text{litres} = 3 \ \text{litres} \div 6 \times 11 = \frac{1}{2} \ \text{litre} \times 11$

\[= 5 \frac{1}{2} \ \text{litres}\]

c) *Do each multiplication as if both factors were whole numbers first, then write the decimal point in the correct place in the product.*

i) $5 \times 0.75$

ii) $37 \times 0.285$

iii) $16 \times 23.8$

<table>
<thead>
<tr>
<th>$0.75 \times 5$</th>
<th>$0.285 \times 3.7$</th>
<th>$2.38 \times 1.6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3.75$</td>
<td>$10.195$</td>
<td>$4.608$</td>
</tr>
</tbody>
</table>

Elicit that when a decimal is multiplied by an integer, the product has the same number of decimal digits as the original decimal. (The integer has no decimal digits.)

d) i) $\frac{2}{5} \div 3 = \frac{2}{15}$

ii) $\frac{4}{5} \div 6 = \frac{54}{5} \div 6 = \frac{9}{5} = 1 \frac{4}{5}$

iii) $23.8 \div 5 = 4.76$

---

Individual work, monitored, (helped)

Written on BB or SB or OHT

Discussion, reasoning, agreement, self-correction, praising

Extra praise if Ps realised that i) = ii), so there is no need to do the 2nd calculation!

---

Ps estimate mentally first. *e.g.*

i) $E: 0.8 \times 5 = 4.0$

ii) $E: 0.3 \times 40 = 12.0$

iii) $E: 20 \times 20 = 400$

then write each calculation as a multiplication of integers vertically in Ex. Bks, then write the decimal points in the correct places (and zero in the units column where required).

Ps show details on BB and explain reasoning with place-value detail, then check final product against estimate.

Feedback for T
## Lesson Plan 63

### Activity 4  
**PbY6a page 63**

**Q.2** Deal with one part at a time. Set a time limit. Ps read question themselves and do necessary calculations in *Ex. Bks.* or on scrap paper.

Review with whole class. Ps show answer on scrap paper or slates on command. (S or M; Y or N) Ps with different responses explain reasoning on BB. Class decides which answer is correct. Incorrect plan is written again correctly.

**Solution:**

a) *Sally and Mandy calculated 4 fifths of 345 plums in different ways.*

*Sally's plan:* \(345 \div 4 \times 5\)  
*Mandy's plan:* \(345 \times 0.8\)

Who was correct? Who was wrong? Write the incorrect plan again correctly.  
(M correct)  
\[
S: 345 \div 4 \times 5 \text{ should be } 345 \div 5 \times 4 = 276
\]

\[
M: 345 \times 0.8 \text{ (as } \frac{4}{5} = \frac{8}{10} = 0.8)\]

b) *Henry tried the same calculation but he wrote this plan.*  
*Was he correct?* (Yes)  
\[
H: 345 \times 4 \div 5 \text{ [as } (70 - 1) \times 4 = 280 - 4 = 276]\]

c) *Ronny tried it too and wrote another plan.*  
*Was he correct?* (Y)  
\[
345 \times 4 \div 5 = 1380 \div 5 = 276
\]

### Activity 5  
**PbY6a. page 63**

**Q.3** Read: *Write a plan, estimate, calculate, check your result and write the answer in a sentence.*

Tell class that a Linden Tree is what we call a Lime Tree. (If possible, T has a real branch with leaves and blossom to show to class, otherwise pictures will suffice.) Tell class that the blossom of a lime tree has a pleasant fragrance; in countries such as Germany and Hungary, people gather the blossom, hang it up to dry, then use it to perfume their homes. Why do you think the mass of the blossom decreases when it is dried? (The water in it evaporates.)

Set a time limit. Ps read question themselves and solve in *Ex. Bks.*

Review with whole class. T chooses a P to read each question and Ps show results on scrap paper or slates on command. P answering correctly explains at BB to Ps who were wrong. Who agrees? Who did it a different way? Mistakes discussed and corrected.

T chooses a P to say the answer in a sentence.
Solution:
When the blossom of a Linden Tree is dried, it loses 74 hundredths of its mass.

a) How much dried blossom can you get from 325 kg of fresh blossom?

Plan: \[
325 - \frac{74}{100} \times 325 = 325 - 3.25 \times 74
\]

or Mass left: \[
1 - \frac{74}{100} = \frac{26}{100}
\]

\[
\frac{26}{100} \times 325 = 325 \times \frac{26}{100} = 84.5 \text{ kg}
\]

Answer: You can get 84.5 kg of dried blossom from 325 kg of fresh blossom.

b) How much fresh blossom is needed to produce 390 kg of dried blossom?

Plan: \[
\frac{26}{100} \rightarrow 390 \text{ kg}
\]

or \[
\frac{1}{100} \rightarrow 390 \text{ kg} \div 26 = 15 \text{ kg}
\]

or \[
\frac{100}{100} \rightarrow 15 \text{ kg} \times 100 = 1500 \text{ kg}
\]

Answer: 1500 kg of fresh blossom is needed to produce 390 kg of dried blossom.

Q.4 Read: Alice and Ben are discussing a problem about which is the better buy.

One shop reduces the original price of an item costing £100 by 0.3. Another shop cuts 2 tenths off the original price of £100 then cuts 0.1 off the reduced price.

Alice thinks that the first shop has the better offer. Ben thinks that they are the same.


Set a time limit of 3 minutes. Ps do calculations if necessary and write answer in Ex. Bks.

Review with whole class. If you agree with Alice, stand up . . . now! T chooses Ps standing and Ps sitting to explain their reasoning. Class decides on the correct answer. Mistakes corrected.

Individual work, monitored

Responses shown in unison.
Reasoning, agreement, self-correction, praising
### Lesson Plan 63

**Y6**

**Activity**

6  
(Continued)

**Solution:** e.g.

1st shop  
Reduction: £100 \times 0.3 = £30  
Final price: £100 - £30 = £70

2nd shop  
1st reduction: \(\frac{2}{10}\) of £100 = £20  
New price: £100 - £20 = £80  
2nd reduction: £80 \times 0.1 = £8  
Final price: £80 - £8 = £72

**Answer:** Alice is right, because a reduction of 3 tenths of £100 is more than a reduction of 2 tenths of £100 and 1 tenth of £80.

---

7  
PbY6a, page 63

**Q.5**  
Read: *Solve these problems in your exercise book.*

Set a time limit or deal with one at a time.

Review with whole class. T chooses a P to read each question and Ps show answers on scrap paper or slates on command. Ps answering correctly explain reasoning at BB to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) The original price of an item was reduced by 0.14 and it now costs £192. What was its original price?

**Plan:**  
\[0.86 = \frac{86}{100} \Rightarrow \£192\]

\[\frac{1}{100} \Rightarrow \£192 \div 86 = \£2\]

\[\frac{100}{100} \Rightarrow \£2 \times 100 = \£200\]

**Answer:** The original price was £200.

b) A shop reduced the £60 price of a pair of shoes by 1 fifth, then later increased the reduced price by 1 quarter.

How much do the shoes cost now?

**Reduction:** \(\frac{1}{5}\) of £60 = £12;

New price: £60 - £12 = £48

**Increase:** \(\frac{1}{4}\) of £48 = £12

Final price: £48 + £12 = £60

**Answer:** The shoes now cost £60, the original price.
PbY6a, page 63, Q.6

Read: *The length of a room is 9 m. Its width is 2 thirds of its length and 1.5 of its height.*

Calculate:  
- a) its width and height  
- b) its surface area  
- c) its capacity.

Let's draw a diagram and write on the diagram what we know. Ps come to BB to draw a cuboid and label the edges. Class helps and corrects where necessary.

Other Ps come to BB to write and explain calculations for the values asked for in the questions. Class agrees/disagrees or suggests a better way to calculate. After agreement on the dimensions, Ps complete the diagram before doing parts b) and c).

**Solution:** e.g.

a) **Width:** \( \frac{2}{3} \) of 9 m = \( 9 \div 3 \times 2 = 3 \times 2 = 6 \text{ m} \)

\[
\text{Height: } 1.5 \text{ times } = \frac{3}{2} \rightarrow 6 \text{ m} \\
\frac{1}{2} \rightarrow 6 \div 3 = 2 \text{ m} \\
\frac{2}{2} \rightarrow 2 \times 2 = 4 \text{ m}
\]

The width of the room is 6 metres and its height is 4 metres.

b) **Surface area:**  
\[
2 \times (9 \times 6 + 6 \times 4 + 9 \times 4) \text{ m}^2 \\
= 2 \times (54 + 24 + 36) \text{ m}^2 \\
= 2 \times 114 \text{ m}^2 \\
= 228 \text{ m}^2
\]

c) **Capacity:**  
\[
\text{length} \times \text{width} \times \text{height} = 9 \times 6 \times 4 \text{ m} \\
= 54 \times 4 \text{ (m}^3) \\
= 216 \text{ m}^3
\]

**Erratum**

In *Pbs*, c) should be b) and d) should be c)

**Notes**

Whole class activity  
(or individual work if Ps wish, reviewed with whole class)  
Involve several Ps.  
At a good pace.  
Discussion, reasoning, agreement, praising 
BB:

Floor and ceiling: \( 9 \times 6 \text{ m} \)
2 small walls: \( 6 \times 4 \text{ m} \)
2 large walls: \( 9 \times 4 \text{ m} \)

Elicit that the capacity of a room is the volume of the space inside it.
Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:

- \( 64 = 2 \times 2 \times 2 \times 2 \times 2 = 2^6 \)  
  Factors: 1, 2, 4, 8, 16, 32, 64

- 239 is a prime number   
  Factors: 1, 239
  (Not exactly divisible by 2, 3, 5, 7, 11, 13, and 17 × 17 > 239)

- \( 414 = 2 \times 3 \times 3 \times 23 = 2 \times 3^2 \times 23 \)  
  Factors: 1, 2, 3, 6, 9, 18, 23, 46, 69, 138, 207, 414

- \( 1064 = 2 \times 2 \times 2 \times 7 \times 19 = 2^3 \times 7 \times 19 \)  
  Factors: 1, 2, 4, 7, 8, 14, 19, 28, 38, 56, 76, 133, 152, 266, 532, 1064

Multiplication
Let’s calculate the products and look at how they change.
Ps come to BB to write products and missing items, or dictate what T should write. Class points out errors. Ps tell class what they notice about relationships and connections.
BB:

a) \( 7.6 \times 100 = \boxed{760} \)  
   [If multiplier is divided by 10, product is also divided by 10.]
\( 7.6 \times 10 = \boxed{76} \)  
   [Multiplying by 0.1 (0.01) has the same effect as dividing by 10 (100).]
\( 7.6 \times 1 = \boxed{7.6} \)
\( 7.6 \times 0.1 = 0.76 = 7.6 \div 10 \)
\( 7.6 \times 0.01 = 0.076 = 7.6 \div 100 \)

b) \( 0.5 \times 4 = \boxed{2} \)  
   [The result of multiplying 0.5 by 0.25 has the same digits in the same order as the result of multiplying 5 by 25, but each digit is \((1 + 2 = 3)\) decimal places to the right.]
\( 0.5 \times 2 = \boxed{1} \)
\( 0.5 \times 1 = \boxed{0.5} \)
\( 0.5 \times 0.5 = 0.25 = 0.5 \div 2 \)
\( 0.5 \times 0.25 = 0.125 = 0.5 \div 4 \)

c) \( \frac{1}{2} \times 4 = \boxed{2} \)  
   [c) is the same as b) but written as fractions instead of decimals.]
\( \frac{1}{2} \times 2 = \boxed{1} \)
\( \frac{1}{2} \times 1 = \boxed{\frac{1}{2}} \)
\( \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{1}{2} \div 2 \)
\( \frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = \frac{1}{2} \div 4 \)
Activity 3

PbY6a, page 64

Q.1 Read: Calculate the products in your exercise book. Notice how they change.

Set a time limit. Ps write whole calculations in Ex. Bks and write sentences about what they notice.


Solution:

a) i) \(2.3 \times 50 = 23 \times 5 = 115\)
   
   ii) \(2.3 \times 5 = 11.5\)
   
   iii) \(2.3 \times 0.5 = 1.15\)
   
   iv) \(2.3 \times 0.005 = 0.0115\)

b) i) \(\frac{4}{7} \times 4 = \frac{16}{7} = 2\frac{2}{7}\)
   
   ii) \(\frac{4}{7} \times 1 = \frac{4}{7}\)
   
   iii) \(\frac{4}{7} \times \frac{1}{4} = \frac{1}{7}\)

Solution:

a) One of the sides of this unit square is divided into 4 equal parts and the adjacent side is divided into 5 equal parts.

b) Each grid rectangle is \(\frac{1}{20}\) of the area of the square.

c) Let’s calculate the area of the shaded rectangle in 3 ways.

i) \(A = \frac{3}{5} \times \frac{3}{4}\) of 1 = \(\frac{9}{20}\)

ii) \(A = \frac{3}{4} \times \frac{3}{5}\) of 1 = \(\frac{9}{20}\)

iii) \(A = \frac{3}{4} \times \frac{3}{5} = \frac{9}{20}\)

T: We can multiply two fractions by multiplying the 2 numerators and multiplying the 2 denominators.

Notes

Individual work, monitored, helped

Written on BB or SB or OHT

Differentiation by time limit.

Discussion, reasoning, agreement, self-correction, praising

Points to notice are in brackets.

b) [Multiplier is divided by 4, so product is divided by 4.]

[Multiplier divided by 10, so product is divided by 10, i.e. each digit is in the next smaller place value.]
Y6

Activity

5

PbY6a, page 64

Q.3 Read: Calculate the area of each of these rectangles, if a and b are two adjacent sides. (Draw a rough sketch first.)

Do part a) on BB with whole class first. Ps suggest what to do first and how to continue, with T directing Ps thinking where necessary. T (Ps) work on BB, Ps work at same time in Ex. Bks.

1. Draw a unit square. Then decide how many equal parts its length and width should be divided into to match the fractions in the question. (Rough diagram only is needed.) Shade the rectangle required and label the sides with its length and width.

2. Ps first work out the area by counting the grid rectangles in the diagram (as shown below diagram opposite). Then Ps write operations to calculate the area (similar to those in Q.2) and check the results against the counted area. T shows or elicits how to multiply the fractions and how reduction of the numerators and denominators makes the calculation simpler.

BB:

a) \[ A = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} \times 3 = \frac{1}{2} \text{ m}^2 \]

or \[ A = \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} \times 2 = \frac{1}{2} \text{ m}^2 \]

or \[ A = a \times b = \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2} \text{ m}^2 \]

T: To multiply two fractions, multiply the numerators and multiply the denominators. It is easier to simplify (or cancel) the numerators and denominators before doing the multiplication rather than afterwards, but both ways are correct.

Deal with b) to d) one at a time under a short time limit. Ps can use any method they like.

Review with whole class. Ps could show area on scrap paper or slates on command. Ps answering correctly explain reasoning at BB, drawing a diagram (or referring to T’s diagram). Who calculated in a different way? Mistakes discussed and corrected.

Solution:

b) \[ A = \frac{3}{4} \times \frac{1}{2} \text{ m}^2 = \frac{3}{8} \text{ m}^2 \]

By counting: \[ A = \frac{3}{8} \text{ m}^2 \]

c) \[ A = \frac{5}{2} \times \frac{3}{2} \text{ m}^2 = \frac{15}{4} \text{ m}^2 \]

By counting: \[ A = (2 + \frac{3}{2} + \frac{1}{4}) \text{ m}^2 = \frac{3}{4} \text{ m}^2 \]

d) \[ A = 1.8 \times 1.5 \text{ m} = 1.8 \times 1.5 \text{ m} = 1.8 + 0.9 \text{ m} = 2.7 \text{ m}^2 \]

By counting: \[ A = (1 + 0.8 + 0.5 + 0.4) \text{ m}^2 = 2.7 \text{ m}^2 \]

31 min

Notes

Whole class activity to start, with T and Ps working on BB and Ps in Ex. Bks.

Discussion, reasoning, agreement, checking, praising Involve as many Ps as possible.

BB:

Each grid rectangle is \( \frac{1}{12} \text{ m}^2 \)

6 of them are shaded, so

\[ A = \frac{6}{12} \text{ m}^2 = \frac{1}{2} \text{ m}^2 \]

T also shows:

\[ A = \frac{1}{3} \times \frac{2}{4} = \frac{1}{2} \text{ m}^2 \]

Individual work, monitored, helped, corrected (T could have diagrams already prepared to save time.)

Discussion, reasoning, checking, agreement, self-correction, praising

BB:

b) 

c) 

d)
**Activity**

**PbY6a, page 64**

Q.4 Read: *If Snail moves 4 fifths of a metre every minute, how far will he move in:*

a) 5 minutes  
b) 11 minutes  
c) \( \frac{1}{4} \) minute  
d) \( \frac{3}{4} \) minute  
e) \( 1 \frac{2}{3} \) minutes?

Set a time limit. Ps do calculations in Ex. Bks.

Review with whole class. Ps show answers on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Class agrees/disagrees Who worked it out another way? Mistakes discussed and corrected. T chooses a P to say the answer in a sentence.

**Solution:**

a) \( 1 \text{ min} \rightarrow \frac{4}{5} \text{ m}; \ 5 \text{ min} \rightarrow \frac{4}{5} \text{ m} \times \frac{5}{1} = \frac{4}{1} \text{ m} \)

b) \( 11 \text{ min} \rightarrow \frac{4}{5} \text{ m} \times 11 = \frac{44}{5} \text{ m} = \frac{8}{1} \text{ m} (= 8 \text{ m} 80 \text{ cm}) \)

c) \( \frac{1}{4} \text{ min} \rightarrow \frac{4}{5} \text{ m} \div 4 = \frac{1}{5} \text{ m} \) or \( \frac{4}{5} \text{ m} \times \frac{1}{4} = \frac{1}{5} \text{ m} \)

d) \( \frac{3}{4} \text{ min} \rightarrow \frac{1}{5} \text{ m} \times 3 = \frac{3}{5} \text{ m} \) or \( \frac{4}{5} \text{ m} \times \frac{3}{4} = \frac{3}{5} \text{ m} \)

e) \( \frac{2}{3} \text{ min} = \frac{5}{3} \text{ min} \rightarrow \frac{4}{5} \text{ m} \div 3 \times 5 = \frac{4}{15} \text{ m} \times \frac{5}{1} \)

\( = \frac{4}{3} \text{ m} = \frac{1}{3} \text{ m} \)

or \( 2 \times 4 \text{ m} + \frac{4}{5} \text{ m} = \frac{8}{1} \text{ m} 80 \text{ cm} \)

T shows the multiplication and cancellation if no P used it. (as we get the distance by multiplying the distance Snail moved in 1 minute by the number of minutes.)

\( \frac{4}{5} \text{ m} \times \frac{1}{3} = \frac{4}{15} \text{ m} = \frac{1}{3} \text{ m} \)

Individual work, monitored, helped
(Revert to whole class activity if majority of Ps are struggling.)

Written on BB or SB or OHT
Discussion, reasoning, agreement self-correction, praising
Do not worry if Ps miss an opportunity for simplification but ask other Ps to point it out if they can.

**Notes**

Individual work, monitored, helped
Written on BB or SB or OHT
Difference by time limit
Responses shown in unison.
Discussion, reasoning, agreement, self-correction, praising
Accept any correct method of solution.

Feedback for T
**Y6**

**Activity**

(Continued)

7

b) i) \(-\frac{3}{4} \times \frac{10}{9} \times \frac{2}{5} = -\frac{1}{3} \times 10 \times 2\)

\(\frac{1}{3} \times 2 = -\frac{1}{3}\)

ii) \(\frac{13}{25} \times \left(-\frac{5}{26}\right) = -\frac{13}{25} \times \frac{5}{26}\)

\(= -\frac{1}{10}\)

iii) \(-\frac{2}{5} \times \left(-\frac{5}{2}\right) = +\frac{2}{5} \times \frac{5}{2}\)

\(= \frac{1}{1}\)

c) i) \(1\frac{2}{3} \times 4\frac{1}{2} = \frac{5}{3} \times \frac{9}{2}\)

\(= \frac{15}{2} = 7\frac{1}{2}\)

ii) \(2\frac{1}{3} \times \left(-1\frac{2}{3}\right) = -\frac{7}{3} \times \frac{5}{3}\)

\(= -\frac{35}{9} = -3\frac{8}{9}\)

iii) \(15.2 \times 4.3 = \frac{152}{10} \times \frac{43}{10}\)

\(= 6536\frac{100}{100} = 65.36\)

**Notes**

**Lesson Plan 64**

**Notes**

Or for c) iii):

- \(\frac{11}{5}\)

- \(\frac{12}{2}\)

- \(\times \frac{4}{3}\)

- \(+\frac{1}{4}\)

- \(\frac{5}{6}\)

- \(\frac{6}{0}\)

- \(\frac{8}{10}\)

- \(\frac{6}{3}\)

- \(\frac{3}{6}\)

Elicit or remind Ps that to multiply 2 decimals, do the multiplication as if they were 2 whole numbers, then write the decimal point in the product so that it has the same number of decimal digits as the total number in the multiplicand and multiplier.

8

**Problem**

Listen carefully and note the data. Do the calculations in your Ex.Bk and show me your result when I say. I will give you 2 minutes!

Two sides of a rectangle are 2.3 cm and 5.4 cm. What is the area of the rectangle in centimetre squares?

If you have an answer, show me . . . now! (12.42 cm²)

P with correct answer explains reasoning on BB. Who agrees? Who did it another way? Mistakes discussed and corrected.

If you were correct, stand up! Let’s give them a clap!

**Solution:** e.g.

\(A = 2.3 \times 5.4 = 23 \times 54.mm\)

\(= 1242 \text{ mm}^2 = 12.42 \text{ cm}^2\)

or

\(A = 2.3 \times 5.4 = 23 \times 54\) \((\text{cm}^2)\)

\(= 1242 \text{ cm}^2 = 12.42 \text{ cm}^2\)

or \(A = 2.3 \times 5.4 = 12.42 \text{ cm}^2\)

**Answer:** The area of the rectangle is 12.42 cm².

**Notes**

Individual work, monitored, helped
(or whole class activity if time is short or Ps are tired)

T repeats question slowly to give Ps time to think and calculate.

Responses shown on scrap paper or slates in unison.

Discussion, reasoning, agreement, praising

Accept any method of solution which gives the correct answer.

T chooses a P to say the answer in a sentence.

Feedback for T

[Or set problem as optional homework and review before the start of Lesson 65.]

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Factorising 65, 240, 415 and 1065. Revision, activities, consolidation

**PbY6a, page 65**

**Solutions:**

Q.1  

a) \(372 \times 100 = 37200\)

\(372 \times 10 = 3720\)

\(372 \times 1 = 372\)

\(372 \times 0.1 = 37.2\)

\(372 \times 0.01 = 3.72\)

\(372 \times 0.001 = 0.372\)

b) \(9 \times 700 = 6300\)

\(9 \times 70 = 630\)

\(9 \times 7 = 63\)

\(9 \times 0.7 = 6.3\)

\(9 \times 0.07 = 0.63\)

\(9 \times 0.007 = 0.063\)

c) \(4.2 \times 50 = 210\) (= \(42 \times 5\))

\(4.2 \times 5 = 21\)

\(4.2 \times 0.5 = 2.1\)

\(4.2 \times 0.05 = 0.21\)

\(4.2 \times 0.005 = 0.021\)

\(0.42 \times 500 = 210\) (= \(42 \times 5\))

Q.2  

a) 1 hour \(\rightarrow\) 510 km

\(\frac{3}{5}\) hour \(\rightarrow\) 510 km \(\div\) \(3\) \(\times\) \(3\) = 102 km \(\times\) \(3\) = 306 km

b) \(\frac{1}{4}\) hours = \(\frac{5}{4}\) hour \(\rightarrow\) 510 km \(\div\) \(4\) \(\times\) \(5\)

\(\frac{1}{4}\) hour \(\rightarrow\) 510 km \(\div\) \(4\) \(\times\) \(5\)

\(=\) 127.5 km \(\times\) \(5\) = 637.5 km

Q.3  

a) \(P = 2 \times (\frac{3}{4} + \frac{3}{5})\) m = \(\frac{1}{2} \times \frac{12 + 15}{20}\) m = \(\frac{27}{10}\) m = 2.7 m

\(A = \frac{3}{4} \times \frac{3}{5}\) m = \(\frac{9}{20}\) m²

b) \(P = 2 \times (0.65 m + 1.2 m) = 2 \times 1.85 m = 3.7 m\)

\(A = 0.65 m \times 1.2 m = 0.780 m²\) (as \(65 \times 12 = 780\))

c) \(P = 2 \times (\frac{3}{4} + 0.32)\) m = \(2 \times (0.75 m + 0.32)\) m

\(= 2 \times 1.07 m = 2.14 m\)

\(A = \frac{3}{4} \times \frac{32}{100} m² = \frac{24}{100} m² = 0.24 m²\)

d) \(P = 2 \times (78.4 cm + 78.4 cm) = 2 \times 156.8 cm = 313.6 cm\)

\(A = 784 mm \times 784 mm = 614656 mm² = 6146.56 cm²\)

\(| 1 \times 784 | 1 \times 316 | 6 \times 720 | 5 \times 80 | 6 \times 146 \times 16 | (mm²) |

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Solutions (continued)

Q.4  
   a) 1 second → 8.4 m  
      5 seconds → 8.4 m × 5 = 42 m  
   b) 10 seconds → 8.4 m × 10 = 84 m  
   c) \( \frac{1}{4} \) min = 15 seconds → 8.4 m × 15  
      = 84 m + 42 m = 126 m  
   d) 1 min → 126 m × 4 = 504 m  
   e) 1 hour = 60 min → 504 m × 60 = 5040 m × 6  
      = 30 240 m

Q.5  
   a) Original price: £300  
      1st reduction: 10% of £300 = \( \frac{1}{10} \) of £300 = £30  
      New price: £300 – £30 = £270  
      2nd reduction: 10% of £270 = \( \frac{1}{10} \) of £270 = £27  
      New price: £270 – £27 = £243  
   b) Increase: 20% of £243 = \( \frac{1}{5} \) of £243 = £48.60  
      New price: £243 + £48.60 = £291.60  
      Increased price was less than £300.

Q.6  
   a) \( \frac{a}{8} \) × 5 = 40, \( \frac{a}{8} \) = 8, a = \( \frac{64}{1} \)  
   b) \( \frac{50}{b} \) × 8 = 40, \( \frac{50}{b} \) = 5, b = \( \frac{10}{1} \)  
   c) \( \frac{5}{8} \) × c = 40, c = 40 ÷ 5 × 8 = 8 × 8 = \( \frac{64}{1} \)  
   d) \( \frac{5}{8} \) ÷ 5 = d, d = \( \frac{25}{8} \) = 3 \( \frac{1}{8} \)

Q.7  
   a) i) 0.75 of 36.12 kg = \( \frac{3}{4} \) of 36.12 kg = \( \frac{3}{41} \) × 36.12 kg  
      = 27.09 kg  
   ii) \( \frac{1}{12} \) of 36.12 kg = 36.12 kg ÷ 12 = 3.01 kg  
   iii) \( \frac{3}{100} \) of 36.12 kg = 36.12 kg × 0.03 = 1.0836 kg  
   iv) \( \frac{5}{24} \) of 36.12 kg = \( \frac{5}{24} \) × 36.12 kg = 15.05 kg ÷ 2  
      = 7.525 kg  
   v) 40% of 36.12 kg = \( \frac{40}{100} \) of 36.12 kg  
      = 36.12 kg × 0.04 = 1.4448 kg

or 36.12 kg ÷ 4 × 3  
   = 9.03 kg × 3  
   = 27.09 kg

(3612 × 3 = 10 836 and there should be (2 + 2 = 4) decimal digits in the product)  
or 36.12 kg ÷ 4 × 5  
   = 6.02 kg ÷ 4 × 5  
   = 1.505 kg × 5 = 7.525 kg  
as 3612 × 4 = 14 448 and there should be (2 + 2 = 4) decimal digits in the product)
Activity

Solutions (continued) (Accept any valid method of solution.)

b) i) \(0.75 = \frac{3}{4} \rightarrow 36.12 \text{ kg}\)
    \[\frac{4}{4} \rightarrow 36.12 \text{ kg} \div 3 \times 4 = 12.04 \text{ kg} \times 4 = 48.16 \text{ kg}\]

ii) \(\frac{1}{12} \rightarrow 36.12 \text{ kg}\)
    \[\frac{12}{12} \rightarrow 36.12 \text{ kg} \times 12 = 433.44 \text{ kg}\]

iii) \(\frac{3}{100} \rightarrow 36.12 \text{ kg}\)
    \[\frac{100}{100} \rightarrow 36.12 \text{ kg} \div 3 \times 100 = 12.04 \text{ kg} \times 100 = 1204 \text{ kg}\]

iv) \(\frac{5}{24} \rightarrow 36.12 \text{ kg}\)
    \[\frac{24}{24} \rightarrow 36.12 \text{ kg} \div 5 \times 24 = 7.224 \text{ kg} \times 24 = 173.376 \text{ kg}\]

v) \(\frac{40}{100} \rightarrow 36.12 \text{ kg}\)
    \[\frac{100}{100} \rightarrow 36.12 \text{ kg} \div 40 \times 100 = 3612 \text{ kg} \div 40 = 361.2 \text{ kg} \div 4 = 90.3 \text{ kg}\]
### Activity

#### 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **66** = \(2 \times 3 \times 11\)  
  Factors: 1, 2, 3, 6, 11, 22, 33, 66
- **241** is a prime number  
  Factors: 1, 241  
  (Not exactly divisible by 2, 3, 5, 7, 11, 13, and \(17 \times 17 > 239\))
- **416** = \(2 \times 2 \times 2 \times 2 \times 2 \times 13\) = \(2^5 \times 13\)  
  Factors: 1, 2, 4, 8, 13, 16, 26, 32, 52, 104, 208, 416
- **1066** = \(2 \times 13 \times 41\)  
  Factors: 1, 2, 13, 26, 41, 82, 533, 1066

#### 2

**Multiplying by fractions and decimals**

a) What does \(\frac{2}{7} \times \frac{3}{5}\) mean? Ps come to BB or dictate to T. Class agrees/disagrees. T helps Ps with ideas if necessary. (e.g. diagram)

BB: e.g. \(\frac{2}{7} \times \frac{3}{5} = \frac{2}{7} \times \frac{3}{5} = \frac{6}{35}\)

How can we do the calculation? P comes to BB to write and explain. Who agrees? Who would do it another way?

BB: \(\frac{2}{7} \times \frac{3}{5} = \frac{2 \times 3}{7 \times 5} = \frac{6}{35}\)  
(No cancelling is possible.)

Deal with the following in a similar way.

b) What does \(\frac{4}{5} \times 2\frac{1}{3}\) mean?

BB: e.g. \(\frac{4}{5} \times \frac{7}{3} = \frac{4 \times 7}{5 \times 3} = \frac{28}{15} = 1 \frac{13}{15}\)

Calculation:

\(\frac{4}{5} \times \frac{7}{3} = \frac{28}{15}\)

or \(\frac{4 \times 7}{5 \times 3} = \frac{28}{15}\)

or \(\frac{28}{15}\)

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or \(\frac{28}{15}\)
e) What does $0.36 \times 71.5$ m mean? (0.36 of 71.5 m, or $\frac{36}{100}$ of 71.5 m)

Calculation:

- BB: $0.36 \times 71.5$ m $= \frac{36}{100} \times 715$ cm $= 25.74$ m
- or $0.36 \times 71.5$ m $= 71.5 \div 100 \times 36 = 0.715 \times 36 = 25.74$ m

Note:
Accept any valid method, including changing 71.5 m to 7150 cm.

Draw a diagram if necessary.

- $0.36 \times 71.5$ m $= \frac{36}{100} \times 715 = \frac{2574}{100} = 25.74$ m

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, reasoning, agreement, self-correction, praising

Accept and praise any correct calculation but T extends the discussion to show the details given in the solution opposite.

Elicit that to calculate the price of a certain length of material, multiply the price of 1 m by that length.
**Y6**

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<tr>
<td>Q.2 Read: Do the multiplications. <strong>Simplify</strong> the fractions first where possible.</td>
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What does simplify mean? (Change to a simpler form. i.e reducing or cancelling down numerators and denominators which have a common factor.) T (Ps) shows examples on BB.

Set a time limit or deal with one part at a time. Ps write the complete calculations in Ex. Bks.

Review with whole class. Ps come to BB to show and explain details of their calculations. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- **a)**
  1. \( \frac{2}{5} \times \frac{4}{7} = \frac{8}{35} \)
  2. \( \frac{2}{5} \times \frac{7}{4} = \frac{14}{20} = \frac{7}{10} \)
  3. \( \frac{5}{2} \times \frac{4}{7} = \frac{20}{14} = \frac{10}{7} \)
  4. \( \frac{5}{2} \times \frac{7}{4} = \frac{35}{8} = \frac{43}{8} \)

- **b)**
  1. \( \frac{1}{3} \times \frac{7}{4} = \frac{7}{18} \)
  2. \( \frac{5}{4} \times \frac{15}{7} = \frac{25}{28} = \frac{98}{28} \)
  3. \( \frac{14}{5} \times \frac{7}{15} = \frac{98}{75} = \frac{3}{25} \)
  4. \( \frac{6}{5} \times \frac{15}{7} = \frac{90}{70} = \frac{18}{14} \)

- **c)**
  1. \( \frac{1}{4} \times \frac{2}{5} \times \frac{3}{8} \times \frac{6}{15} = \frac{1}{10} \)
  2. \( \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} \times \frac{1}{6} = \frac{1}{6} \)

- **d)**
  1. \( \frac{2}{5} \times \frac{1}{3} = \frac{7}{14} \times \frac{1}{2} = \frac{7}{28} = \frac{1}{4} \)
  2. \( \frac{11}{4} \times \frac{2}{5} \times \frac{9}{20} = \frac{99}{40} = \frac{99}{40} \)
  3. \( \frac{2}{3} \times \frac{1}{7} = \frac{7}{21} \times \frac{3}{7} = \frac{3}{7} \)

Who had all the multiplications correct or made just 1 mistake? Let’s give them a round of applause!

**Lesson Plan 66**

**Notes**

Individual work, monitored, helped
BB: To simplify
\[
\frac{2}{4} \times \frac{5}{10} = \frac{2}{5}
\]
\[
\frac{4}{10} \times \frac{5}{8} = \frac{10}{10} \times \frac{8}{2} = \frac{1}{4}
\]
or a shortcut:
\[
\frac{1}{10} \times \frac{5}{8} = \frac{1}{4}
\]
Written on BB or use enlarged copy master or OHP
D Differentiation by time limit
Reasoning, agreement, self-correction, praising

Review the rules for multiplying a fraction (or a mixed number) by a fraction:
- First change any mixed number to a fraction.
- Simplify the fractions where possible.
- Multiply the numerators to get the numerator of the product and multiply the denominators to get the denominator of the product.
- Simplify the resulting fraction and change to a mixed number if necessary.

**PbY6a, page 66**

Q.3 Read: Complete the plans and do the calculations.

T: If this amount of gold (T holds up a 1 cm cube) weighs 19.32 g, let’s see if you can work out the mass of these amounts of gold.

Set a time limit. Ps can use Ex. Bks if they need more space.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it another way? Mistakes discussed and corrected.

Individual work, monitored, helped
Written on BB or SB or use enlarged copy master or OHP
D Differentiation by time limit
R Responses shown in unison.
Reasoning, agreement, self-correction, praising
Accept any correct method but extra praise for Ps who used a single multiplication.

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### Activity

(Continued)

**Solution:**

a) 4 cm³ → $19.32 \times 4 = 77.28$ g

b) 15 cm³ → $19.32 \times 15 = 193.2$ g + 96.6 g = 289.8 g

c) 0.1 cm³ → $19.32 \times 0.1 = 1.932$ g  

\[= \frac{19.32}{10} \text{ g}\] 

(i.e. \(\frac{1}{10}\) of 19.32 g)

d) 0.7 cm³ → $19.32 \times 0.7 = 13.524$ g  

\[= \frac{19.32}{10} \times 7 = 1.932 \times 7 = \frac{13.524}{10}\]

e) 1.6 cm³ → $19.32 \times 1.6 = 30.912$ g  

\[
\begin{array}{c}
19.32 \\
\times 1.6
\end{array}
\]

\[= 30.912 \text{ g}\]

either  

\[
\begin{array}{c}
19.32 \\
\times 0.1
\end{array}
\]

either \(\frac{1}{10}\) of 19.32 g

\[= \frac{19.32}{10} = 1.932 \text{ g}
\]

either \(\frac{19.32}{10} \times 16 = 1.932 \times 16 = \frac{30.912}{10}\)

f) 72.1 cm³ → $19.32 \times 72.1 = 1392.972$ g  

\[= \frac{19.32}{10} \times 721 = 1.932 \times 721 = \frac{1392.972}{10}\]

Elicit that to multiply a decimal by a decimal, do the multiplication as if the decimals were whole numbers, then write the decimal point so that the product has the same number of decimal digits as the total in the two original decimals.

**Notes**

Ps point out relationships, e.g. multiplying by 0.1 is the same as dividing by 10, etc.

T might allow Ps to use a calculator but also show the long multiplications on BB or SB or an OHT and ask Ps to explain them. e.g.

**BB:**

\[
\begin{array}{c}
721 \\
\times 1932
\end{array}
\]

\[= 144422\]

\[= \frac{144422}{10} \text{ or } 14442.2\]

\[= 14442.2 \text{ g}\]

\[= 14442.2 \times 0.1 = 1444.22 \text{ g}\]

\[= \frac{14442.2}{10} \text{ or } 1444.22\]

Individual work, monitored, helped

Written on BB or SB or OHT

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

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Activity 7

PbY6a, page 66

Q.5 Set a time limit.

Ps read questions themselves, write a plan, estimate the result, do the calculations, check against estimate and write the answers as sentences in Ex. Bks.

Review with whole class. Ps with answers show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who agrees? Who did it a different way? etc. Mistakes discussed and corrected. T chooses a P to say each answer in a sentence.

Solution: e.g.

A car has already covered \( \frac{3}{5} \) of an 80 \( \frac{5}{8} \) km journey.

a) How far has it travelled?

Plan:

\[
\frac{3}{5} \text{ of } 80 \frac{5}{8} \text{ km} = \frac{3}{5} \times 80 \text{ km} + \frac{3}{8} \times \frac{1}{8} \text{ km}
\]

\[
= 48 \text{ km} + \frac{3}{8} \text{ km} = 48 \frac{3}{8} \text{ km}
\]

Answer: The car has travelled 48 and 3 eighths kilometres.

b) What part of the journey has still to be done?

Plan:

\[
1 - \frac{3}{5} = \frac{2}{5}
\]

Answer: Two fifths of the journey still has to be done.

c) How far does it still have to go?

Plan:

\[
80 \frac{5}{8} \text{ km} - 48 \frac{3}{8} \text{ km} = 32 \frac{2}{8} \text{ km} = 32 \frac{1}{4} \text{ km}
\]

Answer: The car still has 32 and a quarter kilometres to go.

Homework

A stick was 0.8 m long. First 3 quarters of its length was cut off, then half of the remaining length was cut off.

What length was the piece of stick left over?

Solution: e.g.

Plan:

\[
\frac{1}{2} \text{ of } \frac{1}{4} \text{ of } 0.8 \text{ m} = \frac{1}{2} \times \frac{1}{4} \times 0.8 \text{ m} = \frac{1}{8} \times \frac{8}{10} \text{ m}
\]

Check: 0.6 + 0.1 + 0.1 = 0.8 (m) √

Answer: The piece of stick left over was 0.1 m long.

Notes

Individual work, monitored
Differentiation by time limit
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Accept longer plans than those given but show or elicit these short plans too!
Draw a diagram if necessary.

E: \( \frac{1}{2} \) of 80 km = 40 km

BB: e.g. 80 \( \frac{5}{8} \) km

\[
\begin{array}{ccc}
& & & \text{travelled} \\
& & & \text{still to go} \\
\end{array}
\]

\[
\frac{3}{5} \to ? \quad ?
\]

Optional

(or extra task for able Ps during the lesson)

Review before the start of Lesson 67.

e.g. 0.8 m left

\[
\begin{array}{ccc}
& & \text{1st cut} \\
& & \text{2nd cut} \\
\end{array}
\]

\[
\begin{array}{ccc}
\frac{3}{4} \text{ of } 0.8 \text{ m} & \frac{1}{2} \text{ of } ?
\end{array}
\]
Lesson Plan 67

Notes

Individual work, monitored (or whole class activity)
BB: 67, 242, 417, 1067

Calculators allowed.
Reasoning, agreement, self-correction, praising
e.g.

<table>
<thead>
<tr>
<th>242</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>121</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
</tr>
<tr>
<td>1</td>
<td>11</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>417</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>127</td>
<td>139</td>
</tr>
<tr>
<td>11</td>
<td>139</td>
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<tr>
<td>22</td>
<td>22</td>
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<tr>
<td>8</td>
<td>8</td>
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<tr>
<td>21</td>
<td>21</td>
</tr>
<tr>
<td>22</td>
<td>22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1067</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>97</td>
<td>97</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Whole class activity
Written on BB or SB or OHT
At speed in order round class
In good humour!
Agreement, praising
Discussion on the rule
Feedback for T

Ps may use a calculator for d).

Activity

1  Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:

- 67 is a prime number Factors: 1, 67
  (Not exactly divisible by 2, 3, 5, 7, and 11 × 11 > 67)
- 242 = 2 × 11 × 11 = 2 × 11²
  Factors: 1, 2, 11, 22, 121, 242
- 417 = 3 × 139
  Factors: 1, 3, 139, 417
- 1067 = 11 × 97
  Factors: 1, 11, 97, 1067

2  Sequences

T has first few terms of each sequence written on BB. Ps dictate the following terms until T decides when to stop. If a P makes a mistake, the next P corrects it. Final P also gives the rule.

BB:

a) \( \frac{41}{50}, \frac{37}{50}, \frac{33}{50}, \left( \frac{29}{50}, \frac{25}{50}, \frac{21}{50}, \frac{17}{50}, \ldots \right) \) \[ Rule: -\frac{4}{50} \]

b) \( -\frac{7}{11}, -\frac{9}{22}, -\frac{2}{11}, -\left( \frac{1}{22}, \frac{3}{11}, \frac{11}{22}, \frac{8}{11}, \frac{21}{22}, \ldots \right) \) \[ Rule: +\frac{5}{22} \]

c) \( 15, 10, \frac{20}{3}, \frac{40}{9}, \left( \frac{80}{27}, \frac{160}{81}, \frac{320}{243}, \ldots \right) \) \[ Rule: \times \frac{2}{3} \]

(or \( \frac{6}{3}, \frac{4}{9}, \left( \frac{26}{27}, \frac{79}{81}, \frac{177}{245}, \ldots \right) \))

d) 2, −5, 12.5, −31.25, 78.125, −195.3125, . . . \[ Rule: \times (-2.5) \]

3  PbY6a, page 67

Q.1 Read: Do these calculations in your exercise book.

Simplify where possible.

Set a time limit. Ps write complete calculations in Ex. Bks.
Review with whole class. Ps come to BB to write and explain reasoning. Class agrees/disagrees. Mistakes discussed/corrected.

Solution:

a) \( \left( \frac{2}{3} + \frac{3}{4} \right) \times \frac{12}{19} = \frac{8 + 9}{19} \times \frac{12}{19} = \frac{17}{19} \)

b) \( \left( \frac{1}{3} - \frac{3}{9} \right) \times \frac{9}{18} = \frac{6 + 4 - 5}{18} \times \frac{9}{18} = \frac{5}{12} \times \frac{9}{18} = \frac{1}{2} \)

c) \( \frac{1}{3} \times \frac{1}{2} - \frac{3}{4} \times \frac{3}{5} = \frac{4}{5} \times \frac{5}{4} = \frac{1}{6} + \frac{1}{2} - \frac{3}{5} - \frac{3}{4} \)

\[ = \frac{10 + 30 - 36 - 45}{60} = \frac{-41}{60} \]
### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(Continued)</td>
</tr>
<tr>
<td>d) ( \left( 1 - \frac{1}{2} \right) \times \left( 1 - \frac{1}{3} \right) \times \left( 1 - \frac{1}{4} \right) = \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4} )</td>
<td>19 min</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Lesson Plan 67</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q. 2 Read:</strong> Write a plan, do the calculation and write the answer in a sentence.</td>
<td></td>
</tr>
<tr>
<td>Deal with one part at a time. Set a time limit. Ps solve the problems in Ex. Bks. Allow Ps to discuss it with their neighbours.</td>
<td></td>
</tr>
<tr>
<td>Review with whole class. Ps could show answers on scrap paper or slates on command. Ps answering correctly explain reasoning at BB to Ps who were wrong. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td>T chooses a P to say the answer in a sentence.</td>
<td></td>
</tr>
<tr>
<td><strong>Solutions:</strong> e.g.</td>
<td></td>
</tr>
<tr>
<td>a) Three pieces of ribbon were cut from a 16( \frac{1}{5} ) length.</td>
<td></td>
</tr>
<tr>
<td>The 1st piece was ( \frac{4}{5} ) m, the 2nd piece was ( 1 \frac{1}{2} ) m and the 3rd piece was 3 times as long as the 1st and 2nd pieces put together.</td>
<td></td>
</tr>
<tr>
<td>i) What length of the ribbon was cut off altogether?</td>
<td></td>
</tr>
<tr>
<td>Plan: ( \frac{4}{5} ) m + ( 1 \frac{1}{2} ) m + ( 3 \times \left( \frac{4}{5} ) m + ( 1 \frac{1}{2} ) m ) = ( 4 \times \left( \frac{4}{5} ) m + ( \frac{3}{2} ) m ) = ( 4 \times \frac{8 + 15}{10} ) m = ( \frac{2}{4} \times \frac{23}{10} ) m = ( \frac{46}{5} ) m = ( 9 \frac{1}{5} ) m</td>
<td></td>
</tr>
<tr>
<td>Answer: 9 and a fifth metres were cut off altogether.</td>
<td></td>
</tr>
<tr>
<td>ii) What length of ribbon was left?</td>
<td></td>
</tr>
<tr>
<td>Plan: ( 16 \frac{1}{5} ) m − ( 9 \frac{1}{5} ) m = ( 7 ) m</td>
<td></td>
</tr>
<tr>
<td>Answer: The piece of ribbon left was 7 metres long.</td>
<td></td>
</tr>
<tr>
<td>b) Rabbit ran 5 and 3 quarter kilometres in an hour. In the next two hours, he ran 5 and a quarter kilometres less than 3 times the distance he ran in the first hour. How far did Rabbit run altogether?</td>
<td></td>
</tr>
<tr>
<td>Plan: ( 5.75 ) km + ( 3 \times 5.75 ) km − ( 5 \frac{1}{4} ) km = ( 4 \times 5.75 ) km − ( 5.25 ) km = ( 23 ) km − ( 5.25 ) km = ( 17.75 ) km</td>
<td></td>
</tr>
<tr>
<td>Answer: Rabbit ran 17 and 3 quarter kilometres altogether.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Individual work, monitored, helped</td>
<td></td>
</tr>
<tr>
<td>Differentiation by time limit</td>
<td></td>
</tr>
<tr>
<td>Responses shown in unison. Discussion, reasoning, agreement, self-correction, praising</td>
<td></td>
</tr>
<tr>
<td>Accept any valid method of solution using fractions or decimals or converting the given unit of measure to a smaller unit.</td>
<td></td>
</tr>
</tbody>
</table>

| **Erratum** |  |
| In part b) in *Pbs*: \( 5 \frac{1}{4} \) m should be \( 5 \frac{1}{4} \) km. |  |

**Check:** \( 7 + 0.8 + 1.5 + 6.9 = 16.2 \) (m) √

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>or ( 4 \times (0.8 \text{ m} + 1.5 \text{ m}) ) = ( 4 \times 2.3 \text{ m} ) = ( 9.2 \text{ m} )</td>
<td></td>
</tr>
<tr>
<td>( 4 \times 5 \frac{3}{4} \text{ km} − 5 \frac{1}{4} \text{ km} ) = ( 23 \text{ km} − 5 \frac{1}{4} \text{ km} ) = ( 18 \text{ km} − \frac{1}{4} \text{ km} ) = ( 17 \frac{3}{4} \text{ km} )</td>
<td></td>
</tr>
</tbody>
</table>
**Y6**

**Activity**

<table>
<thead>
<tr>
<th>5</th>
<th><strong>PbY6a, page 67</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.3:</td>
<td>Read: <strong>Write as many different plans as you can. Calculate one of them.</strong></td>
</tr>
<tr>
<td></td>
<td>Deal with one part at a time. Set a short time limit. Ps write plans in Ex Bks, and use one to calculate the answer.</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who used a different plan? Come and explain what you did. Class agrees/disagrees, etc. Mistakes discussed and corrected.</td>
</tr>
<tr>
<td></td>
<td>Who wrote a different plan but did not use it? Ps come to BB or dictate to T. Class decides whether or not it is valid.</td>
</tr>
<tr>
<td></td>
<td><strong>Solution:</strong> e.g.</td>
</tr>
<tr>
<td></td>
<td>a) ( \frac{3}{5} ) of ( \frac{2}{4} ) km = ( \frac{3}{5} \times \frac{2}{4} ) km = ( \frac{3}{5} \times \frac{9}{4} ) km = ( \frac{27}{20} ) km = ( 1 \frac{7}{20} ) km</td>
</tr>
<tr>
<td></td>
<td>(or ( \frac{2}{4} ) km ( \div ) 5 ( \times ) 3, or ( \frac{2}{4} ) km ( \times ) 3 ( \div ) 5,</td>
</tr>
<tr>
<td></td>
<td>or ( 2.25 ) km ( \div ) 5 ( \times ) 3, or ( 2.25 ) km ( \times ) 0.6, etc.)</td>
</tr>
<tr>
<td></td>
<td>b) ( \frac{5}{8} ) of ( \£132.50 ) = ( 1.625 \times \£132.50 ) = ( £215.3125 )</td>
</tr>
<tr>
<td></td>
<td>= ( £215.31 ) (or ( £132.50 \div 8 \times 13 ), or ( £132.50 \times 13 \div 5 ),</td>
</tr>
<tr>
<td></td>
<td>or ( £132.50 + \frac{5}{8} \times £132.50 ), or ( \frac{13}{8} \times \£ \frac{265}{2} ), etc.)</td>
</tr>
<tr>
<td></td>
<td>= ( £ \frac{3445}{16} ) = ( £215 \frac{5}{16} )</td>
</tr>
<tr>
<td></td>
<td>c) ( \frac{4}{100} ) of ( 520 \frac{4}{5} ) kg = 0.04 ( \times ) 520.8 kg = 20.832 kg</td>
</tr>
<tr>
<td></td>
<td>(or ( 520 \frac{4}{5} ) kg ( \div ) 100 ( \times ) 4, or ( 520 \frac{4}{5} ) kg ( \div ) 25,</td>
</tr>
<tr>
<td></td>
<td>or ( \frac{4}{100} \times 520 \frac{4}{5} ) kg = ( \frac{1}{25} \times \frac{2604}{5} ) kg, etc.</td>
</tr>
<tr>
<td></td>
<td><strong>30 min</strong></td>
</tr>
</tbody>
</table>

**Notes**

Individual work, monitored, helped
Differentiation by time limit
(If class is not very able, Ps write only the plans and then, after review, the class chooses a plan and calculates the result together.)
Responses shown in unison.
Discussion, reasoning, agreement, self-correction, praising

**6**

**PbY6a, page 67**

| Q.4 | Read: **Write as many different plans as you can. Calculate one of them.** |
|     | Deal with this in a similar way to Q.2 but allow calculators only to check Ps’ calculations. Ask Ps to show and explain details of calculations on BB. Mistakes discussed and corrected. |
|     | **Solution:** e.g. (but accept any valid plan) |
| a) | 0.85 of \( \frac{2}{3} \) tonnes = \( \frac{17}{20} \times \frac{7}{3} \) t = \( \frac{119}{60} \) t = \( 1 \frac{59}{60} \) t |
| b) | 1.2 of \( £450.80 \) = \( £450.80 \times 1.2 \) = \( £540.96 \) |
|     | (= \( £450.80 \div 10 \times 12 \), etc.) |

**Notes**

Individual work, monitored, helped
Deal with one at a time.
Discussion, reasoning, agreement, self-correction, praising

**BB:**

\[
\begin{array}{c}
4 \ 5 \\
0 \ 8 \\
0 \ 0 \\
\end{array}
\]

\[
\begin{array}{c}
9 \ 0 \\
1 \ 6 \ 0 \\
\end{array}
\]

\[
\begin{array}{c}
4 \ 5 \ 0 \ 8 \ 0 \ 0 \\
5 \ 4 \ 0 \ 9 \ 6 \ 0 \\
1 \\
\end{array}
\]
**Y6**

<table>
<thead>
<tr>
<th>Activity</th>
<th>6 (Continued)</th>
</tr>
</thead>
</table>
| c) 0.09 of 72.6 m = 72.6 m × 0.09 = 6.534 m  
(= 72.6 m ÷ 10 × 9, etc.) |
| d) 0.1 of 0.1 of a litre = 0.1 litre × 0.1 = 0.01 litre  
[= 0.1 ÷ 10 = \( \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} \) (litre)] |

**Notes**

BB: \[
\begin{array}{c}
7 \\
2.46 \\
\times 0.09 \\
6.534 \\
\hline \\
25
\end{array}
\] (m)

Check that the result has the same number of decimal digits as the multiplicand and multiplier combined.

---

**PbY6a, page 67**

Q.5 Read: Find a rule. Complete the table.

Set a time limit. Ask Ps finished early to write other forms of the rule in Ex. Bks. or to think of data for additional columns.

Review with whole class. Agree on one form of the rule for a and for b. Ps come to BB or dictate to T, explaining reasoning.

Class agrees/disagrees. Mistakes discussed and corrected. Elicit other forms of the rule. T shows them if no P does so and asks Ps if they are correct. Ps suggest values for extra columns.

**Solution:**

\[
a = b \times 5 \div 2 = b \times \frac{5}{2} = 2.5 \times b \quad (= 2.5b)
\]

\[b = \frac{2}{5} \quad \text{of} \quad a = \frac{2}{5} \times a = a \times 2 \div 5 = 0.4 \times a \quad (= 0.4a)
\]

---

**PbY6a, page 67, Q.6**

Deal with one question at a time. T (P) reads out the question, and Ps calculate in Ex. Bks, then show result on scrap paper or slates on command. (Allow Ps to use a calculator if time is short.)

Ps with correct responses explain at BB to Ps who were wrong. Who did the same? Who calculated another way? etc. Ps who were wrong write the plan they understand best in Ex. Bks.

**Solution:**

\[a = \frac{2}{5} \quad \text{of} \quad a = \frac{2}{5} \times a = a \times 2 \div 5 = 0.4 \times a \quad (= 0.4a)
\]

---

**Erratum**

In PbS: ‘£38.50’ should be ‘£38.40’

---

**Lesson Plan 67**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit and by extra tasks

Reasoning, agreement, self-correction, praising

T shows that: e.g. 2 × a = 2a, 5 × b = 5b, etc.

Extra praise if a P suggests
\[a = b \div \frac{2}{5}
\]

but do not expect it yet!

Whole class activity but individual calculation

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Feedback for T or
\[\frac{£38.40}{3 \times 4} = £12.80 \times 4 = £51.20
\]

or \[51.20 \times 1.4 = £71.68
\]

or \[0.75 \times x = \frac{3}{4} \times x
\]
\[x = \frac{38.40}{3 \times 4} = \frac{51.20}{4}
\]

Extra praise if Ps realised this and did not calculate again.

---

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### Lesson Plan

**Y6**

**R:** Calculations  
**C:** Understanding percentage as the number of parts in every 100  
**E:** Expressing simple fractions as percentages

**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(68 = 2 \times 2 \times 17\) Factors: 1, 2, 4, 17, 34, 68
- \(243 = 3 \times 3 \times 3 \times 3 \times 3 = 3^5\)
  Factors: 1, 3, 9, 27, 81, 243
- \(418 = 2 \times 11 \times 19\)
  Factors: 1, 2, 11, 19, 22, 38, 209, 418
- \(1068 = 2 \times 2 \times 267 = 2^2 \times 267\)
  Factors: 1, 2, 4, 267, 534, 1068

7 min

**Activity 2**

**Percentage**

What does ‘per cent’ mean? (out of 100) Who can write 1 unit as a percentage? (BB: 1 unit = 100%)

Let’s see what you can remember about calculating with percentages.

a) What do these percentages mean?

i) 50% of 40 m  
\[\frac{50}{100} \text{ or } 0.5 \text{ of } 40 \text{ m } = 20 \text{ m}\]

ii) 10% of 36 kg  
\[\frac{10}{100} \text{ or } 0.1 \text{ of } 36 \text{ kg } = 3.6 \text{ kg}\]

iii) 70% of £420  
\[\frac{70}{100} \text{ or } 0.7 \text{ of } £420 = £420 \div 10 \times 7 = £294\]

iv) 1% of 440 000 people  
\[\frac{1}{100} \text{ or } 0.01 \text{ of } 440 000 \text{ people } = 4400 \text{ people}\]

v) 100% of 53 g  
\[\frac{100}{100} \text{ of } 53 \text{ g } = 53 \text{ g}\]

vi) 0% of 73 litres  
\[0 \text{ of } 73 \text{ litres } = 0 \text{ litres}\]

vii) 120% of £350  
\[\frac{120}{100} \text{ or } 1.2 \text{ of } £350 = £350 + \frac{1}{5} \text{ of } £350 = £350 + 70 = £420\]

viii) 300% of 51 cm²  
\[\frac{300}{100} \text{ of } 51 \text{ cm}^2 = 3 \times 51 \text{ cm}^2 = 153 \text{ cm}^2\]

Whole class activity

At a good pace.

Involve the majority of Ps

Ps can think of other examples too.

Feedback for T

Elicit that:

120% of £350 > £350
b) Show the position of these percentages on the number line. Ps come to BB to mark and label them. Class agrees/disagrees. Ask for equivalent fractions too.

BB: 5% 40% 50% 75% 100% 140%

17 min

3 PbY6a, page 68, Q.1

T reads out each part. Ps calculate mentally if they can (or in Ex. Bks) and show result on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Who did the same? Who calculated in a different way? etc. Elicit other methods of calculation if Ps all did the same. Mistakes discussed and corrected.

Elicit other equivalent forms of the fractions. (Decimals and percentages)

Solution: e.g.

a)  i) \( \frac{1}{100} \) of £500 = £500 \( \div \) 100 = £5 [\( \frac{1}{100} = 0.01 \rightarrow 1\% \)]

ii) \( \frac{9}{100} \) of 300 m = \( \frac{3}{100} \times \frac{9}{100} \) = 27 m [0.09 \rightarrow 9\%]

iii) \( \frac{17}{100} \) of 600 litres = 600 litres \( \times \) 0.17 = 102 litres [17\%]

b) If \( \frac{1}{100} \) can be written as 1\% (read as 'one per cent') what is 20\% of 16 km?

20\% of 16 km \( \rightarrow \) \( \frac{20}{100} \) of 16 km = 16 km \( \times \) 0.2 = 3.2 km

22 min
Lesson Plan 68

Notes

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
Reasoning, agreement, self-correction, praising
Feedback for T

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
Reasoning, agreement, self-correction, praising
Feedback for T

Individual work, monitored, helped
Written on BB or use enlarged copy master or OHP
Differentiation by time limit
Reasoning, agreement, self-correction, praising
Feedback for T

MEP: Primary Project

Week 14

Y6

Activity

4 PbY6a, page 68

Q.2 Read: Express these parts of a whole unit in two ways. Follow the example.

Set a time limit of 2 minutes. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

Which of the fractions are not in their simplest form? Ps come to BB to point them out and simplify them.

Solution:

a) \( \frac{1}{100} = 0.01 \rightarrow 1\% \)
b) \( \frac{125}{100} = 1.25 \rightarrow 125\% \)
c) \( \frac{8}{100} = 0.08 \rightarrow 8\% \)
d) \( \frac{2}{100} = 0.02 \rightarrow 2\% \)
e) \( \frac{67}{100} = 0.67 \rightarrow 67\% \)
f) \( \frac{100}{100} = 1 \rightarrow 100\% \)

26 min

5 PbY6a, page 68

Q.3 Read: Express these parts of a whole unit in two ways. Follow the example.

Set a time limit of 2 minutes. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected. Extra praise if Ps give the fractions in their simplest form, otherwise elicit them.

Solution:

a) \( \frac{68}{100} = 0.68 \rightarrow 68\% \)
b) \( \frac{5}{100} = 0.05 \rightarrow 5\% \)
c) \( \frac{1}{100} = 0.01 \rightarrow 1\% \)
d) \( \frac{11}{100} = 0.11 \rightarrow 11\% \)
e) \( \frac{242}{100} = 2.42 \rightarrow 242\% \)
f) \( \frac{103}{100} = 1.03 \rightarrow 103\% \)

30 min

6 PbY6a, page 68

Q.4 Read: Express these parts of a whole unit in two ways.

Set a time limit of 2 minutes. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected. Extra praise if Ps give the fractions in their simplest form, otherwise elicit them.

Solution:

a) \( 47\% \rightarrow \frac{47}{100} = 0.47 \)
b) \( 71\% \rightarrow \frac{71}{100} = 0.71 \)
c) \( 6\% \rightarrow \frac{6}{100} = 0.06 \)
d) \( 0\% \rightarrow \frac{0}{100} = 0 \)
e) \( 193\% \rightarrow \frac{193}{100} = 1.93 \)
f) \( 50\% \rightarrow \frac{50}{100} = 0.5 \)

34 min

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### Lesson Plan 68

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td>Whole class activity but individual calculation</td>
</tr>
<tr>
<td><strong>Lesson Plan 68</strong></td>
<td>Responses shown in unison.</td>
</tr>
<tr>
<td><strong>Activity</strong></td>
<td>Reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>7</td>
<td>Extra praise for equivalent values</td>
</tr>
<tr>
<td><strong>PbY6a, page 68, Q.5</strong></td>
<td>Feedback for T</td>
</tr>
<tr>
<td>T reads out each part. Ps calculate mentally (or in Ex. Bks) and show answer on scrap paper or slates on command. Ps with correct responses explain reasoning to Ps who were wrong. Who did the same? Who worked it out another way? etc. Ps who were wrong write correct equation in Ex. Bks.</td>
<td>or 713 kg × 0.01</td>
</tr>
<tr>
<td>Elicit equivalent values. (e.g. 7.13 kg = 7 kg 130 g = 7130 g; 0.36 m = 36 cm = 360 mm; etc.)</td>
<td>or 713 kg ÷ 100</td>
</tr>
<tr>
<td><strong>Solution:</strong> e.g.</td>
<td>etc.</td>
</tr>
<tr>
<td>a) 1% of 713 kg → ( \frac{1}{100} ) of 713 kg = 7.13 kg</td>
<td></td>
</tr>
<tr>
<td>b) 1% of 36 m → ( \frac{1}{100} ) of 36 m = 0.36 m</td>
<td></td>
</tr>
<tr>
<td>c) 1% of 58 907 m → ( \frac{1}{100} ) of 58 907 m = 589.07 m</td>
<td></td>
</tr>
<tr>
<td>d) 1% of 3 litres → ( \frac{1}{100} ) of 3 litres = 0.03 litres (= 3 cl = 30 ml)</td>
<td></td>
</tr>
<tr>
<td>e) 1% of 41.6 kg → ( \frac{1}{100} ) of 41.6 kg = 0.416 kg (= 416 g)</td>
<td></td>
</tr>
<tr>
<td>f) 1% of 0.4 km → ( \frac{1}{100} ) of 0.4 km = 0.004 km (= 4 m)</td>
<td></td>
</tr>
</tbody>
</table>

| 8 | Individual work, monitored, helped |
| **PbY6a, page 68** | [There is really no need for calculators but T decides whether to allow them for c).] |
| Q.6 Set a time limit of 3 minutes. Ps calculate in Ex. Bks. | Reasoning, agreement, self-correction, praising |
| Review with whole class. T chooses Ps to show solutions on BB and explain their reasoning. Class agrees/disagrees. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. | or e.g. |
| **Solution:** e.g. | c) £5.34 × 30 – £5.34 |
| a) 1% of £534 = £534 × 0.01 = £5.34 | = £160.2 – £5.34 = £154.86 |
| b) 7% of £534 = £534 × 0.07 = £5.34 × 7 = £37.38 | f) £534 – £534 × 0.1 |
| c) 29% of £534 = £534 × 0.29 = £5.34 × 29 = £154.86 | = £534 – £53.4 = £480.6 |
| d) 50% of £534 = £534 × 0.5 = £534 ÷ 2 = £267 | | |
| e) 110% of £534 = £534 × 1.1 = £534 + £53.4 = £587.4 | | |
| f) 90% of £534 = £534 × 0.9 = £480.6 | | |

| Extension | | |
| T demonstrates, then Ps try out, the quick ‘one button’ method on a calculator. | | |

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**Activity 9**

**PbY6a, page 68, Q.7**

T (P) asks each question. Ps calculate mentally (or on scrap paper or slates) and show result on command. P with correct answer explains reasoning. Who did the same? Who worked it out in a different way? etc. Ps who made mistakes write correct equations in Ex. Bks.

Ps could think of other similar questions to ask if there is time.

**Solution:**

What percentage is:

a) 50 km of 100 km

\[
\frac{50}{100} \rightarrow 50\% \quad [0.5]
\]

b) 10 litres of 100 litres

\[
\frac{10}{100} \rightarrow 10\% \quad [0.1]
\]

c) 3 kg of 100 kg

\[
\frac{3}{100} \rightarrow 3\% \quad [0.3]
\]

d) 6 m of 6 m

\[
\frac{6}{6} = \frac{100}{100} \rightarrow 100\% \quad [1]
\]

e) 100 km of 200 km

\[
\frac{100}{200} = \frac{50}{100} \rightarrow 50\% \quad [0.5]
\]

f) 30 kg of 1000 kg

\[
\frac{30}{1000} = \frac{3}{100} \rightarrow 3\% \quad [0.03]
\]

g) 50 litres of 500 litres

\[
\frac{50}{500} = \frac{10}{100} \rightarrow 10\% \quad [0.1]
\]

h) 70 m of 70 m?

\[
\frac{70}{70} = \frac{100}{100} \rightarrow 100\% \quad [1]
\]

---

**Notes**

Whole class activity but individual calculation

At a fast pace
In good humour!
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Ask for decimal form too.
Feedback for T
Y6

R: Calculations. Expressing fractions as a % and vice versa
C: Simple percentages and fractions of quantities
E: Problems

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- \( 69 = 3 \times 23 \)
- \( 244 = 2 \times 2 \times 61 = 2^2 \times 61 \)
- \( 419 \) is a prime number
- \( 1069 \) is a prime number

(As not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and \( 23^2 > 419 \))

(As not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 23, 29, 31, and \( 37^2 > 1069 \))

8 min

2

Fractions and percentages

a) T says a fraction, P says it as a percentage, giving interim steps when necessary, and shows its position on the number line. (BB)

BB:

<table>
<thead>
<tr>
<th>(1%)</th>
<th>(5%)</th>
<th>(68%)</th>
<th>(132%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{100} )</td>
<td>( \frac{5}{100} )</td>
<td>( \frac{68}{100} )</td>
<td>( \frac{132}{100} )</td>
</tr>
</tbody>
</table>

\( \frac{1}{50} (\rightarrow 2\%) \), \( \frac{6}{50} (\rightarrow 12\%) \), \( \frac{25}{50} (\rightarrow \frac{50}{100} \) → 50\%)

\( \frac{71}{50} (\rightarrow 142\%) \), etc. (Ps can think of some too!)

b) How can we express a fraction as a decimal? (Change it to an equivalent fraction with 10, (100, 1000) as the denominator first.)

T says a fraction. Ps say it as a decimal and as a percentage, giving interim steps where necessary. Class points out errors.

\( \frac{1}{20} \) \( (= \frac{5}{100} \rightarrow 5\%) \), \( \frac{7}{20} \) \( (= \frac{35}{100} \rightarrow 35\%) \),

\( \frac{39}{20} \) \( (= \frac{195}{100} \rightarrow 195\%) \), \( \frac{1}{10} \) \( (= \frac{1}{10} \rightarrow 10\%) \),

\( \frac{3}{10} \) \( (= \frac{0.3}{10} \rightarrow 30\%) \), \( \frac{11}{10} \) \( (= \frac{11}{10} \rightarrow 110\%) \),

\( \frac{3}{6} \) \( (= \frac{3.6}{10} \rightarrow 360\%) \), \( \frac{1}{5} \) \( (= \frac{2}{10} \rightarrow 20\%) \),

\( \frac{3}{5} \) \( (= \frac{6}{10} \rightarrow 60\%) \), \( \frac{8}{5} \) \( (= \frac{1.6}{10} \rightarrow 160\%) \),

\( \frac{1}{4} \) \( (= \frac{25}{100} \rightarrow 25\%) \), \( \frac{3}{4} \) \( (= \frac{75}{100} \rightarrow 75\%) \),

\( 2\frac{1}{2} \) \( (= \frac{2.25}{10} \rightarrow 225\%) \)

Individual work, monitored (or whole class activity)
BB: 69, 244, 419, 1069
Calculators allowed.
Reasoning, agreement, self-correction, praising

Whole class activity
Number line drawn on BB or use enlarged copy master or OHP
At a good pace
Ps mark approximate position with a cross or a dot and label as a %.
Reasoning, agreement, praising
Feedback for T

Or Ps might remember about dividing the numerator by the denominator.
If no P mentions it here, leave this method until part c).
At a good pace
Reasoning, agreement, praising
Ps can write details on BB if they cannot keep the steps in their head.

Notes

Individual work, monitored (or whole class activity)
BB: 69, 244, 419, 1069
Calculators allowed.
Reasoning, agreement, self-correction, praising

<table>
<thead>
<tr>
<th>244</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>122</td>
<td>2</td>
</tr>
<tr>
<td>61</td>
<td>61</td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

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Study these fractions. What do you notice about them?

[Their denominators are not factors of a multiple of 10 (or they form recurring decimals – extra praise if a P notices this.)]

How can we express them as decimals? (Divide the numerator by the denominator.)

Let’s express them as a decimal and as a percentage.

Do i) on BB with Ps’ help as an example for Ps to follow.

If not mentioned earlier by Ps, elicit now that a decimal in which a digit (or group of digits) keeps repeating to infinity is called a recurring decimal and a dot is written above the recurring digit.

Ps come to BB or dictate to T, with help from T and other Ps if necessary.

BB:

i) \( \frac{1}{3} = \left(1 \div 3 = 0.3 \rightarrow 33.3\% = 33.3\%\right) \)

ii) \( \frac{2}{3} = \left(2 \div 3 = 0.6 \rightarrow 66.6\% = 66.7\%\right) \) (or \(0.3 \times 2 = 0.6\))

iii) \( \frac{1}{9} = \left(1 \div 9 = 0.1 \rightarrow 11.1\% = 11.1\%\right) \)

iv) \( \frac{7}{9} = \left(7 \div 9 = 0.7 \rightarrow 77.7\% = 77.8\%\right) \) (or \(0.1 \times 7 = 0.7\))

BB: recurring decimal
e.g. \(3.3 = 3.333333\ldots\)

Reasoning, agreement, (correcting) praising

At a good pace

Ps check divisions with calculators.

T helps with rounding to the nearest tenth of a percent

Extension

What other fractions form recurring decimals?
(e.g. \(\frac{2}{9}, \frac{4}{9}, \frac{1}{7}, \frac{2}{7}\), etc.

BB: \(\frac{1}{7} = 1 \div 7 = 0.\overline{142857} \approx 14.3\%\)

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Whole class discussion of example to start

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Feedback for T

Q.1 Read: Express these percentages as fractions and decimals. Follow the example.

What has been done to the percentage in the example? (Written as hundredths, then simplified, then written as a decimal)

Set a time limit. Ps work in Pbs or in Ex. Bks if they need more space. Note what Ps do with i) and j).

Review with whole class. Ps come to BB to complete the statements, saying what they are doing loudly and clearly.

Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) \(8\% \rightarrow \frac{8}{100} = \frac{2}{25} = 0.08\)

b) \(3\% \rightarrow \frac{3}{100} = 0.03\)

c) \(15\% \rightarrow \frac{15}{100} = \frac{3}{20} = 0.15\)

d) \(50\% \rightarrow \frac{50}{100} = \frac{1}{2} = 0.5\)

e) \(25\% \rightarrow \frac{25}{100} = \frac{1}{4} = 0.25\)
### Lesson Plan 69

#### Activity

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td><strong>Lesson Plan 69</strong></td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>3</strong> (Continued)</td>
<td></td>
</tr>
<tr>
<td>f) 80% → ( \frac{80}{100} = \frac{4}{5} = 0.8 )</td>
<td></td>
</tr>
<tr>
<td>g) 75% → ( \frac{75}{100} = \frac{3}{4} = 0.75 )</td>
<td></td>
</tr>
<tr>
<td>h) 150% → ( \frac{150}{100} = \frac{3}{2} = 1 \frac{1}{2} = 1.5 )</td>
<td></td>
</tr>
<tr>
<td>i) 33(\frac{1}{3})% → ( \frac{33.3}{100} = 0.333\ldots = 0.\overline{3} )</td>
<td></td>
</tr>
<tr>
<td>j) 16.6% → ( \frac{16.6}{100} = 0.1666\ldots = 0.16 )</td>
<td></td>
</tr>
</tbody>
</table>

#### Notes

- Have no expectations for i) and j) yet. T helps if necessary.
- Extra praise for Ps who were able to do it on their own.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4</strong></td>
<td></td>
</tr>
<tr>
<td><strong>PbY6a, page 69</strong></td>
<td></td>
</tr>
<tr>
<td>Q2</td>
<td>Read: Express these fractions as decimals and percentages. Follow the example.</td>
</tr>
<tr>
<td>Elicit the two ways of forming a decimal from a fraction. (Write as an equivalent fraction with denominator a multiple of 10, or divide the numerator by the denominator.) (If Ps know what the decimal is, there is no need for them to write the equivalent fraction or do the division.) Set a time limit. Ps work in Pbs or in Ex. Bks. Review with whole class. Ps come to BB to complete the statements, saying what they are doing. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td>a) ( \frac{1}{5} = 0.2 \rightarrow 20% )</td>
<td>b) ( \frac{3}{5} = 0.6 \rightarrow 60% )</td>
</tr>
<tr>
<td>c) ( \frac{1}{2} = 0.5 \rightarrow 50% )</td>
<td>d) ( \frac{3}{2} = 1.5 \rightarrow 150% )</td>
</tr>
<tr>
<td>e) ( \frac{1}{8} = 0.125 \rightarrow 12.5% )</td>
<td>f) ( \frac{5}{8} = 0.725 \rightarrow 72.5% )</td>
</tr>
<tr>
<td>g) ( \frac{7}{10} = 0.7 \rightarrow 70% )</td>
<td>h) ( \frac{6}{10} = 0.6 \rightarrow 60% )</td>
</tr>
<tr>
<td>i) ( \frac{1}{20} = \frac{5}{100} = 0.05 \rightarrow 5% )</td>
<td></td>
</tr>
<tr>
<td>j) ( \frac{15}{20} = \frac{75}{100} = 0.75 \rightarrow 75% )</td>
<td></td>
</tr>
<tr>
<td>k) ( \frac{1}{3} = 1 \div 3 = 0.3 \rightarrow 33\frac{1}{3}% ) (Accept 33(\frac{1}{3})% too.)</td>
<td></td>
</tr>
<tr>
<td>l) ( \frac{2}{3} = 2 \div 3 = 0.6 \rightarrow 66.6% ) (Accept 66(\frac{2}{3})% too.)</td>
<td></td>
</tr>
</tbody>
</table>

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td><strong>30 min</strong></td>
<td></td>
</tr>
</tbody>
</table>

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**Lesson Plan 69**

**Notes**

Individual work, monitored, helped
(or whole class activity if time is short)

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

e.g.

BB: $120\%$ of $5\ m = 5\ m \times 1.2 = 6\ m$

---

**Activity**

5  

**PbY6a, page 69**

Q.3  
Read:  *Complete the table to show the different percentages of 5 metres in mm, cm and metres.*

Elicit that $1\ m = 100\ cm = 1000\ mm$. Set a time limit or deal with one column at a time. Ps can do calculations in *Ex. Bks.*

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Show details of calculations on BB if problems or disagreement. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Base unit: $5\ m$</th>
<th>100%</th>
<th>1%</th>
<th>10%</th>
<th>30%</th>
<th>60%</th>
<th>80%</th>
<th>120%</th>
</tr>
</thead>
<tbody>
<tr>
<td>In mm</td>
<td>5000</td>
<td>50</td>
<td>500</td>
<td>1500</td>
<td>3000</td>
<td>4000</td>
<td>6000</td>
</tr>
<tr>
<td>In cm</td>
<td>500</td>
<td>5</td>
<td>50</td>
<td>150</td>
<td>300</td>
<td>400</td>
<td>600</td>
</tr>
<tr>
<td>In m</td>
<td>5</td>
<td>0.05</td>
<td>0.5</td>
<td>1.5</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

**35 min**

6  

**PbY6a, page 69**

Q.4  
Read:  *A grocer had $1.8\ kg$ of curry powder in stock. He sold $2$ ninths of it on Monday and $30\%$ of it on Tuesday. How much curry powder did the grocer have left?*

Set a time limit. Ps solve it and write the answer in a sentence in *Ex. Bks.*

Review with whole class. Ps show answer on scrap paper or slates on command. P responding correctly explains reasoning at BB to Ps who were wrong. Who did the same? Who did it another way? Deal with all methods used. Mistakes discussed and corrected.

**Solution:**  

e.g.

**Monday**  

Amount sold:  

\[
\frac{2}{9} \text{ of } 1.8\ kg = 1.8\ kg \div 9 \times 2 = 0.2\ kg \times 2 = 0.4\ kg
\]

**Tuesday**  

Amount sold: $30\%$ of $1.8\ kg = 1.8\ kg \times 0.3 = 0.54\ kg$

Amount left:  

\[
1.8\ kg - (0.4\ kg + 0.54\ kg) = 1.8\ kg - 0.94\ kg = 0.86\ kg
\]

or on one line:

**Plan:**  

\[
1.8 - (1.8 \div 9 \times 2) - (1.8 \times 0.3)
\]

\[
= 1.8 - 0.4 - 0.54 = 1.4 - 0.54 = 0.86\ (kg)
\]

**Answer:** The grocer had $0.86\ kg$ of curry powder left.

**40 min**
### Activity 7

**PbY6a, page 69, Q.5**

Read: Write a work problem for each of these plans. Solve the problem and write the answer.

Allow Ps 2 minutes to think of a problem for each part. Ps can discuss with their neighbours if they wish. T chooses Ps to tell the class their problems and class decides whether or not they match the plans.

Class decides on the best context for each part and Ps come to BB to write the missing numbers, explaining reasoning. Class agrees/disagrees. T chooses a P to say the answer in the chosen context.

What do you notice about the 3 plans? (They are really the same calculation, using the same numerical values but with different units of measure.)

**Solution:**

- **a)** 100% = \(\frac{100}{100}\) → 380 km
  
  \(1\% = \frac{1}{100}\) → 3.8 km
  
  30% = \(\frac{30}{100}\) → 114 km

- **b)**

```
\[
\text{\fbox{380 kg}} \div \text{100} \times \text{30 km}
\]
```

- **c)**

```
\[
\text{380 litres} \times 0.3 = 114 \text{ litres}
\]
```

**Problems** e.g.

- **a)** John went on a 380 km train journey. The train was so full that John had to stand for 30% of the way. For how many km did John have to stand?

- **b)** The greengrocer bought 380 kg of potatoes and sold 30% of them. How many kg did he sell?

- **c)** A tank which could hold 380 litres of water was filled to 0.3 of its capacity. How much water was in the tank?

**45 min**
Factorising 70, 245, 420 and 1070. Revision, activities, consolidation

**PbY6a, page 70**

**Solutions:**

Q.1  
   a) $0.15 = \frac{15}{100} = \frac{3}{20} \rightarrow 15\%$
   b) $0.12 = \frac{12}{100} = \frac{3}{25} \rightarrow 12\%$
   c) $0.25 = \frac{25}{100} = \frac{1}{4} \rightarrow 25\%$
   d) $0.60 = \frac{60}{100} = \frac{3}{5} \rightarrow 60\%$
   e) $0.20 = \frac{20}{100} = \frac{1}{5} \rightarrow 20\%$
   f) $0.61 = \frac{61}{100} \rightarrow 61\%$
   g) $1.10 = \frac{110}{100} = \frac{11}{10} = 1 \frac{1}{10} \rightarrow 110\%$
   h) $0.05 = \frac{5}{100} = \frac{1}{20} \rightarrow 5\%$
   i) $0.375 = \frac{375}{1000} = \frac{3}{8} \rightarrow 37.5\%$
   j) $0.19 = \frac{19}{100} \rightarrow 19\%$
   k) $0.66 = \frac{66}{100} = \frac{33}{50} \rightarrow 66\%$
   l) $0.125 = \frac{125}{1000} = \frac{1}{8} \rightarrow 12.5\%$

Q.2  

<table>
<thead>
<tr>
<th>$x$</th>
<th>9</th>
<th>$-\frac{1}{2}$</th>
<th>-9</th>
<th>3</th>
<th>$-\frac{15}{15}$</th>
<th>1</th>
<th>12</th>
<th>$-\frac{24}{8}$</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>-6</td>
<td>1</td>
<td>6</td>
<td>$-\frac{2}{2}$</td>
<td>4</td>
<td>$-\frac{10}{10}$</td>
<td>$\frac{2}{2}$</td>
<td>$-\frac{8}{8}$</td>
<td>16</td>
</tr>
</tbody>
</table>

**Rule:** $x = y \times (-\frac{3}{2}); \ y = x \times (-\frac{2}{3})$

or $x = y \div 2 \times (-3); \ y = x \div 3 \times (-2)$

Q.3  

a) i) $1\%$ of 428 m = 4.28 m
   ii) $9\%$ of 428 m = 428 m $\times$ 0.09 = 38.52 m
   iii) $25\%$ of 428 m = $\frac{1}{4}$ of 428 m = 107 m

b) i) $1\%$ of 512 kg = 5.12 kg
   ii) $20\%$ of 512 kg = 512 kg $\times$ 0.2 = 102.4 kg
   iii) $19\%$ of 512 kg = 102.4 kg $- 5.12$ kg = 97.28 kg
Activity

Q.4  
\(a) \) 20 kg out of 100 kg is \(\frac{20}{100} \rightarrow 20\%\)

\(b) \) 5 km out of 25 km is \(\frac{5}{25} = \frac{20}{100} \rightarrow 20\%\)

\(c) \) 0 km out of 10 km \(\rightarrow 0\%\)

\(d) \) £43 out of £100 is \(\frac{43}{100} \rightarrow 43\%\)

\(e) \) 12 g out of 200 g is \(\frac{12}{200} = \frac{6}{100} \rightarrow 6\%\)

\(f) \) 7 mm out of 7 mm is the whole amount \(\rightarrow 100\%\)

Q.5  
\(a) \) i) \(\frac{1}{3} \times \frac{2}{9} = \frac{1}{6} \)  
 ii) \(\frac{3}{4} \times \frac{9}{2} = \frac{27}{8} = \frac{3}{8}\)

iii) \(\frac{4}{3} \times \frac{2}{9} = \frac{8}{27}\)  
 iv) \(\frac{2}{4} \times \frac{3}{2} = 6\)

\(b) \) i) \(\frac{4}{5} \times \frac{12}{5} = \frac{16}{25}\)  
 ii) \(\frac{3}{4} \times \frac{12}{3} = 9\)

iii) \(\frac{4}{5} \times \frac{5}{12} = \frac{1}{9}\)  
 iv) \(\frac{4}{5} \times \frac{5}{12} = \frac{25}{16} = 1\frac{9}{16}\)

\(c) \) i) \(\frac{1}{3} \times \frac{3}{5} \times \frac{5}{7} \times \frac{7}{9} = \frac{1}{9}\)

ii) \(\frac{1}{2} \times \frac{4}{8} \times \frac{8}{16} \times \frac{32}{64} \times \frac{128}{256}\)

\(\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}\)

Q.6  
Cut off: \(\frac{3}{5} + \frac{4}{5} + \frac{3}{5} \times 3 = \frac{7}{5} + \frac{9}{5} = \frac{16}{5} = 3\frac{1}{5} = 3.2\) m

Left: 6.5 m – 3.2 m = 3.3 m

Answer: 3.2 m were cut off the plank and 3.3 m were left.

Erratum

In Pbs: 'pice' should be 'piece'
### Activity 1: Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **71** is a prime number Factors: 1, 71
  
  (As not exactly divisible by 2, 3, 5, 7, and \(11^2 > 71\))
- **246** = \(2 \times 3 \times 41\) Factors: 1, 2, 3, 6, 41, 82, 123, 246
- **421** is a prime number Factors: 1, 421
  
  (As not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and \(23^2 > 421\))
- **1071** = \(3 \times 3 \times 7 \times 17 = 3^2 \times 7 \times 17\)
  
  Factors: 1, 3, 7, 9, 17, 21, 51, 63, 119, 153, 357, 1071

### Activity 2: Dividing by a Fraction

a) How could we work out what the whole quantity is if we know that 2 fifths of it is 3 quarters of a km? Ps make suggestions and class discusses them. T makes sure that the following is shown.

BB:
\[
\begin{align*}
\frac{2}{5} & \text{ of the quantity } \rightarrow \frac{3}{4} \text{ km} \\
\frac{1}{5} & \text{ of the quantity } \rightarrow \frac{3}{4} \text{ km} \div 2 = \frac{3}{8} \text{ km} \\
\frac{5}{5} & \text{ of the quantity } \rightarrow \frac{3}{4} \text{ km} \div 2 \times 5 = \frac{3}{4} \text{ km} \times \frac{5}{2} \\
&= \frac{15}{8} \text{ km} = 1 \frac{7}{8} \text{ km}
\end{align*}
\]

Check: \(\frac{2}{5} \text{ of } 1 \frac{7}{8} \text{ km} = \frac{1}{2} \frac{2}{5} \times \frac{15}{8} \text{ km} = \frac{3}{4} \text{ km} \checkmark\)

b) We could also work it out this way. Let the whole quantity be \(x\).

BB:
\[
\begin{align*}
\frac{2}{5} & \text{ of } x \text{ is } \frac{3}{4} \text{ km}, \text{ so } x \div 5 \times 2 = \frac{3}{4} \text{ km} \\
\rightarrow x & \div 5 = \frac{3}{4} \text{ km} \div 2 \\
\rightarrow x & = \frac{3}{4} \text{ km} \div 2 \times 5 \text{ or } x = \frac{3}{4} \text{ km} \times \frac{5}{2}
\end{align*}
\]

We call \(x = \frac{5}{2}\) the reciprocal value of \(\frac{2}{5}\). What is a reciprocal value?

(The numerator and denominator of the fraction are exchanged, or the number by which the fraction must be multiplied to make 1.)

<table>
<thead>
<tr>
<th>BB: Reciprocal value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\frac{2}{5} \times \frac{5}{2} = 1)</td>
</tr>
</tbody>
</table>
c) We could think of it this way too.

BB: \[ \frac{2}{5} \text{ of } x \text{ is } \frac{3}{4} \text{ km, so } x \times \frac{2}{5} = \frac{3}{4} \text{ km} \]

How can we work out the unknown factor if we know the other factor and the product? (Divide the product by the known factor.)
Ps dictate what T should write.

BB: \[ x = \frac{3}{4} \text{ km} \div \frac{2}{5} \]

But we have seen in a) that: \[ x = \frac{3}{4} \text{ km} \times \frac{5}{2} \]
(T highlights it.)

so BB: \[ \frac{3}{4} \text{ km} \div \frac{2}{5} = \frac{3}{4} \text{ km} \times \frac{5}{2} = x \]

Let's compare the two equations and think about what they actually mean. T directs Ps' thinking if necessary. Elicit that:

- dividing by 2 fifths means calculating the whole \( \left( \frac{5}{5} \right) \) amount from 2 fifths of it;
- dividing by 2 fifths can be replaced by multiplying by \( \frac{5}{2} \).

\[ 15 \text{ min} \]

3 \hspace{1cm} PbY6a, page 71

Q.1 Read: \hspace{1cm} Solve the problem in your exercise book.

A shopkeeper has bought 40 kg of beans and wants to put them into equal-sized packs.

How many packs could he make if each pack held:

\( a) \ 5 \text{ kg} \hspace{0.5cm} b) \ 2 \text{ kg} \hspace{0.5cm} c) \ 1 \text{ kg} \hspace{0.5cm} d) \ 1 \frac{1}{2} \text{ kg} \hspace{0.5cm} e) \ 1 \frac{1}{3} \text{ kg} ? \)

Set a time limit. Ps write operations and calculate results in Ex. Bks.
Review with whole class. Ps could show results on slates or scrap paper on command. Ps answering correctly explain reasoning at BB to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected. Ps point out relationships.

Solution:
\begin{align*}
\hspace{2cm} a) \ 40 \text{ kg} \div 5 \text{ kg} &= 8 \text{ (packs)} \\
\hspace{2cm} b) \ 40 \text{ kg} \div 2 \text{ kg} &= 20 \text{ (packs)} \\
\hspace{2cm} c) \ 40 \text{ kg} \div 1 \text{ kg} &= 40 \text{ (packs)} \\
\hspace{2cm} d) \ 40 \text{ kg} \div 1 \frac{1}{2} \text{ kg} &= 80 \text{ (packs)} \\
\hspace{2cm} &\hspace{2cm} \text{(Each pack contains half the amount, so number of packs is twice as many.)} \\
\hspace{2cm} &\hspace{2cm} \text{(i.e. } 40 \text{ packs } \times 2, \text{ since } 2 \times 1 \frac{1}{2} \text{ kg } = 1 \text{ kg}) \\
\hspace{2cm} e) \ 40 \text{ kg} \div 1 \frac{1}{3} \text{ kg} &= 120 \text{ (packs)} \\
\hspace{2cm} &\hspace{2cm} \text{(i.e. } 40 \text{ packs } \times 3, \text{ since } 3 \times 1 \frac{1}{3} \text{ kg } = 1 \text{ kg})
\end{align*}

\[ 20 \text{ min} \]
### Activity

#### Q.2 Read: Calculate the quotients.

Set a time limit or deal with one column at a time. Encourage Ps to calculate mentally if they can and just write results in Pbs.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning by giving the reverse operation and/or pointing out its relationship with a previous operation. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- a) $32 \div 4 = 8$
- b) $36 \div 9 = 4$
- c) $\frac{4}{5} \div 4 = \frac{1}{5}$

$32 \div 2 = 16$

$36 \div 3 = 12$

$\frac{4}{5} \div 2 = \frac{2}{5}$

$32 \div 1 = 32$

$36 \div 1 = 36$

$\frac{4}{5} \div 1 = \frac{4}{5}$

$32 \div \frac{1}{2} = 64$

$36 \div \frac{1}{3} = 108$

$\frac{4}{5} \div \frac{1}{2} = \frac{8}{5}$

$= 32 \times 2$

$= 36 \times 3$

$= \frac{4}{5} \times 2$

$32 \div \frac{1}{4} = 128$

$36 \div \frac{1}{9} = 324$

$\frac{4}{5} \div \frac{1}{4} = \frac{16}{5}$

$= 32 \times 4$

$= 36 \times 9$

$= \frac{4}{5} \times 4$

---

#### Q.3 Read: Solve the problems in your Ex Bks.

Deal with one question at a time. T chooses a P to read out the question and Ps calculate mentally or in Ex. Bks and show the answer on slates or scrap paper on command. P answering correctly explains reasoning at BB. Who thought the same? Who worked it out in a different way? etc. Mistakes discussed. T chooses a P to say the answer in a sentence in context. Ps write correct solutions in Ex. Bks.

**Solution:**

- a) Five metres of material cost £4.50. How much does 1 metre cost?
  
  **Plan:** £4.50 \(\div 5 = £0.90\)
  
  Elicit that this amount of money is the **unit cost** and we get it by dividing the cost of 5 metres by 5.)
  
  **Answer:** One metre of material costs 90 p.

- b) A car travelled 174 miles in 3 hours. How far did it travel in 1 hour?

  **Plan:** 174 miles \(\div 3 = 58\) miles

  Elicit that this is the average speed at which the car was travelling. We get it by dividing the total distance travelled by the time taken.

  **Answer:** The car travelled 58 miles in 1 hour.
c) A bee flies 30 metres in half a minute. How far does it fly in 1 minute?

**Plan:**

\[30 \text{ m} \times \frac{2}{1} = 60 \text{ m} \quad \text{or} \quad 30 \text{ m} \div \frac{1}{2} = 60 \text{ m}

(as there are 2 half minutes in every minute)

**Answer:** The bee flies 60 m in 1 minute.

d) What is the price of 1 kg of fruit if 1 quarter of a kg costs £2?

**Plan:**

\[£2 \times 4 = £8 \quad \text{or} \quad £2 \div \frac{1}{4} = £8

(as there are 4 quarter kg in every 1 kg)

**Answer:** The price of 1 kg of fruit is £8.

e) I bought 3 fifths of a kg of beef for £6. What was the price per kilogram?

**Plan:**

\[£6 \div 3 \times 5 = £2 \times 5 = £10 \quad \text{(Direct proportion)}

or \[£6 \times \frac{5}{3} = £10 \quad \text{(as} \frac{5}{3} \text{is the reciprocal of} \frac{3}{5})

Elicit that this must be equal to £6 \div \frac{3}{5}, following the patterns in c) and d).

**BB:** £6 \[\div 3 \times 5 = £6 \times \frac{5}{3} = £6 \div \frac{3}{5} = £10

**Answer:** The price of 1 kg of beef was £10.

- What have we been calculating in these problems? (Finding the unit quantity when we know a part of it.)
- How did we do it? (Divide by the part we know, or multiply by its reciprocal value.)

**T:** To divide by a fraction, multiply by its reciprocal value.

---

**STRING STRING**

**Lesson Plan 71**

**Notes**

T shows the division by a fraction if no P suggests it and asks if it is correct and why.

Elicit that its average speed is 60 m per minute.

Elicit the division and explanation from Ps this time.

T directs Ps' thinking if necessary.

Discussion, agreement, praising

T repeats clearly: To find the whole amount when we know the value of part of it, divide the value we know by the part we know.

**Individual work, monitored, helped**

(or whole class activity if Ps are unsure)

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Reasoning: e.g. a) i):

There are 2 halves in 1, so there are 6 halves in 3.

or \[3 \div \frac{1}{2} = 3 \times \frac{2}{1} = 6

(Multiply by the reciprocal.)
Activity 6

(Continued)

c) i) \(\frac{4}{9} \div \frac{2}{9} = \frac{4}{9} \times \frac{9}{2} = \frac{2}{2} = 2\) (or since \(4 \div 2 = 2\))

ii) \(\frac{4}{9} \div \frac{2}{3} = \frac{4}{9} \times \frac{3}{2} = \frac{2}{3}\) (or since \(\frac{2}{3} \times \frac{2}{3} = \frac{4}{9}\))

iii) \(5 \div \frac{5}{8} = \frac{5}{5} \times \frac{8}{5} = \frac{8}{5} = 1.6\) (or \(5 \div \frac{5}{8} = \frac{40}{8} \div \frac{5}{8} = \frac{8}{1}\))

d) i) \(\frac{2}{5} \div \frac{1}{2} = \frac{2}{5} \times 2 = \frac{4}{5}\)

ii) \(\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} = \frac{9}{8} = 1\frac{1}{8}\)

iii) \(\frac{8}{10} \div \frac{3}{10} = \frac{8}{10} \times \frac{10}{3} = \frac{8}{3} = 2\frac{2}{3}\)

What are the rules for dividing by a fraction? Elicit that:

- dividing by a fraction can be replaced by multiplying by its reciprocal value;
- when the dividend and divisor are fractions with the same denominator, ony the numerators need to be taken into account.

Lesson Plan 71

Notes

- 40 min

7

PbY6a, page 71.

Q.5 Read: Write different plans for each problem. Use one of them to solve the problem.

Deal with one question at a time. Ps read problem themselves, write different plans and solve the problem in Ex Bks, writing the answer in a sentence.

Review with whole class. Ps show results on scrap paper or slates on command. P answering correctly explains reasoning at BB. Who did the same? Who used a different plan? Deal with all cases. Mistakes discussed and corrected.

Who wrote a different plan from those on the BB but did not use it? Ps come to BB or dictate to T. Class decides whether the plan is valid. T chooses a P to say the answer in a sentence.

Solution:

a) In a class there are 15 girls, which is 6 tenths of the number of boys. How many pupils are in the class?

Plans: e.g. \(\frac{6}{10} \rightarrow 15\) (boys)

\[\frac{1}{10} \rightarrow 15 \div 6 = 2.5\) (boys)

\[\frac{10}{10} \rightarrow 2.5 \times 10 = 25\) (boys)

G + B = 15 + 25 = 40 (children)

or on one line: \(15 + (15 \div \frac{6}{10}) = 15 + (15 \times \frac{10}{6})\)

\[5 \times \frac{10}{6} = \frac{50}{6}\]

\[= 15 + 25 = 40\) (pupils)

Answer: There are 40 pupils in the class.

Individual work, monitored

T notes which Ps use division.

Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Ps write the plan they like best in Ex. Bks. if they did not think of it themselves.

or Number of boys:

\(15 \div 6 \times 10 = 150 \div 6\)

\[= 25\) (boys)

or \(15 \div \frac{6}{10} = 15 \div \frac{3}{5}\)

\[= \frac{15}{1} \times \frac{5}{3} = 25\) (boys)

or \(\frac{150}{10} \div \frac{6}{10} = 150 \div 6\)

\[= 25\) (boys)
Activity

7

(Continued)

b) If 150 km is 2 thirds of a journey, what is the length of the whole journey?

Plans: e.g. \( \frac{2}{3} \rightarrow 150 \text{ km} \)
\[
\frac{1}{3} \rightarrow 150 \text{ km} \div 2 = 75 \text{ km}
\]
\[
\frac{3}{3} \rightarrow 75 \text{ km} \times 3 = 225 \text{ km}
\]

or \( 150 \text{ km} \div 2 \times 3 = 75 \text{ km} \times 3 = 225 \text{ km} \)

or \( 150 \text{ km} \div \frac{2}{3} = \frac{75}{\frac{2}{3}} \)

\[ \frac{75}{\frac{2}{3}} = 150 \text{ km} \times \frac{3}{2} = 225 \text{ km} \]

Answer: The length of the whole journey is 225 km.

45 min
Lesson Plan

72

**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2\)
- \(247 = 13 \times 19\)
- \(422 = 2 \times 211\)
- \(1072 = 2 \times 2 \times 2 \times 2 \times 67 = 2^4 \times 67\)

**Notes**

Individual work, monitored (or whole class activity)

BB: 72, 247, 422, 1072

Calculators allowed.

Reasoning, agreement, self-correction, praising

- \(19\)
- \(3\)
- \(536\)
- \(2\)
- \(134\)
- \(2\)
- \(67\)

**Activity 2**

**Problem**

Listen carefully, note the data and think of different ways to solve this problem.

* A submarine went down to a depth of \(-\frac{2}{5}\) km. Its depth was \(\frac{2}{7}\) of the depth of the sea at that point. What was the depth of the sea?

Allow Ps a minute to think about it and write plans in Ex. Bks. Then Ps come to BB or dictate what T should write. Who agrees? Who can think of another way to solve it? T gives hints or directs Ps thinking towards the methods below if Ps do not suggest them. e.g.

a) \(\frac{2}{7}\) of sea depth \(\rightarrow -\frac{2}{5}\) km (using direct proportion)

\[
\frac{1}{7}\text{ of sea depth} \rightarrow -\frac{2}{5}\text{ km} \div 2 = -\frac{1}{5}\text{ km}
\]

\[
\frac{7}{7}\text{ of sea depth} \rightarrow -\frac{1}{5}\text{ km} \times 7 = -\frac{7}{5}\text{ km} = -1\frac{2}{5}\text{ km}
\]

b) Draw a diagram:

BB: e.g.

\[
\text{Depth} \rightarrow -1\frac{2}{5}\text{ km}
\]

c) Let the depth of the sea be \(x\).

i) \(x \div 7 \times 2 = -\frac{2}{5}\) km

\[
x = -\frac{1}{5}\text{ km} \times 7 = -\frac{7}{5}\text{ km} = -1\frac{2}{5}\text{ km}
\]

ii) \(\frac{2}{7}\) of \(x = -\frac{2}{5}\) km

\[
\rightarrow x \times \frac{2}{7} = -\frac{2}{5}\text{ km}
\]

(Elicit that to divide by a fraction, multiply by its reciprocal value.)

\[
x = -\frac{2}{5} \div \frac{2}{7} = -\frac{2}{5} \times \frac{7}{2} = -\frac{7}{5} = -1\frac{2}{5}\text{ km}
\]

Check: \(\frac{2}{7}\) of \(-1\frac{2}{5}\) km = \(\frac{2}{7} \times -\frac{7}{5}\) km = \(-\frac{2}{5}\) km √

Whole class activity

T repeats slowly and asks a P to repeat in own words to give Ps time to think.

Involve as many Ps as possible in the discussions.

Reasoning, agreement, praising only

b) Draw a diagram:

BB: e.g.

\[
\text{Depth} \rightarrow +1\text{ km}
\]

- \(\frac{2}{5}\) km
- \(-1\frac{2}{5}\) km

d) To calculate the whole depth from part of the depth, divide the known depth by the part.

So depth of the sea is:

\[
-\frac{2}{5}\text{ km} \div \frac{2}{7} = -\frac{2}{5} \times \frac{7}{2} = -\frac{7}{5} = -1\frac{2}{5}\text{ km}
\]
Activity

3  Meaning of division by a fraction

a) What does \( \frac{2 \frac{2}{3}}{\frac{3}{5}} \) really mean? (Calculating how many \( \frac{3}{5} \)s are in \( 2 \frac{2}{3} \) or the whole amount from \( \frac{3}{5} \)s of it.)

How can we do the calculation? P comes to BB or dictates to T, explaining reasoning. Class agrees/disagrees. e.g.

BB: \( \frac{2 \frac{2}{3}}{\frac{3}{5}} = \frac{8 \times 5}{3 \times 3} = \frac{40}{9} = \frac{4}{9} \) (the whole amount)

b) What does \( \frac{2 \frac{2}{3}}{0.6} \) mean? (Calculating the whole length from \( 0.6 \) of it.)

How can we do the calculation? Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. e.g.

BB: \( \frac{2 \frac{2}{3}}{0.6} \times \frac{5}{3} = \frac{40}{9} \times \frac{5}{3} = \frac{40}{9} \times \frac{5}{3} = \frac{50}{9} = \frac{10}{3} \) km

Check:

BB: \( \frac{4 \frac{4}{9}}{0.6} \times \frac{5}{3} = \frac{40}{9} \times \frac{5}{3} = \frac{40}{9} \times \frac{5}{3} = \frac{50}{9} = \frac{10}{3} \) km

c) Let’s calculate \( \frac{4.8}{0.8} \) km.

Ps come to BB or dictate what T should write. Who can think of another way to do it? T shows any of those below not suggested by Ps and asks class if it is correct. e.g.

BB: \( \frac{4.8}{0.8} \) km = \( \frac{480}{80} \) km = \( \frac{6}{1} \) km = \( 6 \) (times)

or \( \frac{4.8}{0.8} \) km = \( \frac{48}{8} \) km = \( \frac{48}{8} \) km = \( 6 \) (as \( 48 \div 8 = 6 \))

What about this method? Is it correct?

BB: \( \frac{4.8}{0.8} \) km = \( \frac{48}{8} \) km = \( \frac{48}{8} \) km = \( 6 \) (times)

Agree that if the dividend and divisor are enlarged (or reduced) by the same number of times (i.e. by a non-zero number), the quotient does not change.

18 min

4  PbY6a, page 72

Q.1 Read: Calculate the quotients. Notice how the quotient changes. Follow the pattern.

Set a time limit or deal with one part at a time.

Review with whole class. Ps come to BB or dictate what T should write. Class agrees/disagrees. Mistakes discussed and corrected.

What do you notice? Elicit that in:

a) the dividend stays the same but the divisor is reduced by 1 tenth, so the quotient increases by 10 times;

b) the dividend stays the same but the divisor is reduced by 1 half so the quotient increases by 2 times.

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Remind Ps to check their results with multiplication.

Reasoning, agreement, checking, self-correction, praising
MEP: Primary Project

Y6

Activity

(Continued)

Solution:

a) \[45 \div 100 = 0.45 \times 10\]
\[45 \div 10 = 4.5 \times 10\]
\[45 \div 1 = 45\]
\[45 \div 0.1 = 450\]
\[45 \div 0.01 = 4500\]

b) \[2.4 \div 4 = 0.6 \times 2\]
\[2.4 \div 2 = 1.2 \times 2\]
\[2.4 \div 1 = 2.4\]
\[2.4 \div 0.5 = 4.8\]
\[2.4 \div 0.25 = 9.6\]

If we did not have the number pattern to help us, how could we have calculated \(2.4 \div 0.25\)? Ps and T suggests ways. e.g.

BB:

\[
\begin{align*}
&\left(2.4 \div 0.25\right) \times 10 = 240 \div 25 = 9.6 \\
&\left(2.4 \div 0.25\right) \div 5 = 2.4 \times 4 = 9.6 \\
&\left(2.4 \div 0.25\right) \times 100 = 240 \div 25 = 9.6 \\
&\left(2.4 \div 0.25\right) \div 5 = 2.4 \times 4 = 9.6 \\
&\left(2.4 \div 0.25\right) \times 2 = 240 \div 25 = 9.6 \\
&\left(2.4 \div 0.25\right) \div 5 = 2.4 \times 4 = 9.6 \\
&\left(2.4 \div 0.25\right) \times 100 = 240 \div 25 = 9.6 \\
&\left(2.4 \div 0.25\right) \div 5 = 2.4 \times 4 = 9.6 \\
&\left(2.4 \div 0.25\right) \times 2 = 240 \div 25 = 9.6 \\
&\left(2.4 \div 0.25\right) \div 5 = 2.4 \times 4 = 9.6 \\
\end{align*}
\]

Notes

Review the meaning of the some of the divisions. e.g. \(45 \div 0.1\) means that we are calculating the whole quantity from 1 tenth of it.

or long division: \(240 \div 25\)

or

\[2.4 \div 0.25 = 2.4 \div \frac{1}{4} = 2.4 \times 4 = 9.6\]

Individual work, monitored, helped

First discuss a good layout for Ps to use in Ex. Bks.
(e.g. as given in the solution)

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising

(T might show long division in iii): \(101250 \div 135\)
as revision. Ps come to BB or dictate to T, explaining reasoning with place-value detail.)

Feedback for T

Extension

Which words are missing from this sentence?

BB: (already prepared)

The quotient does not change if we multiply or divide both the dividend and the divisor by the same non-zero number.

(Underlined words missing.)

Ps read completed sentence in unison and/or write in Ex. Bks.
### Lesson Plan 72

#### Activity

**PbY6a, page 72**

Q.3  Read: *Calculate the whole quantity from the given decimal part. Check your result.*

Set a time limit or deal with one question at a time. Ps write plans, do calculations, check results and write the answer in a sentence in Ex Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. T chooses a P to say the answer in a sentence.

**Solution:** e.g.

**a)** 0.3 of what length is 45 cm?

*Plan:* Length: \(45 \text{ cm} \div 0.3 = 450 \text{ cm} \div 3 = 150 \text{ cm}\)

or Let the length be \(x\).

\[0.3 \times x = 45 \text{ cm},\]

so \(0.3 \times 150 \text{ cm} = 45 \text{ cm}\)

*Check:* \(0.3 \times 150 \text{ cm} = 0.3 \times 150 \text{ cm} = 45 \text{ cm}\)

*Answer:* 0.3 of 150 cm is 45 cm.

**b)** 0.85 of the mass of a box is 3.4 kg. What is the mass of the box?

*Plan:* Mass: \(3.4 \text{ kg} \div 0.85 = 340 \text{ kg} \div 85 = 4 \text{ kg}\)

or Let the mass be \(y\).

\[0.85 \times y = 3.4 \text{ kg},\]

so \(0.85 \times 4 \text{ kg} = 3.4 \text{ kg}\)

*Check:* \(0.85 \times 4 \text{ kg} = 0.85 \times 4 \text{ kg} = 3.4 \text{ kg}\)

*Answer:* The mass of the box is 4 kg.

**c)** Mike invested some money. After 1 year his investment was worth £334.80, which was 1.08 of the original amount. How much money did Mike invest?

*Plan:* \(£334.80 \div 1.08 = £33 \text{ 480} \div 108 = £3720 \div 12 = £310\)

or Let the money invested be \(z\).

\[1.08 \times z = £334.80\]

so \(1.08 \times z = £334.80\)

\[z = £334.80 \div 1.08 = £33 \text{ 480} \div 108 = £310\]

*Check:* \(1.08 \times £310 = 1.08 \times £310\)

\[= £310 + £310 \times 0.08\]

\[= £310 + £24.80 = £334.80\)

*Answer:* Mike invested £334.80.

---

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**Lesson Plan 72**

### Activity 7

**PbY6a, page 72**

Q.4 Read: *Calculate the quotients (to 2 decimal digits). Check your results with a calculator.*

Set a time limit. Ps estimate first, then do calculations in Ex. Bks, and check against estimate then with a calculator (or with multiplication rather than a calculator).

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Consolidate the rule for reducing or expanding fractions and decimals.

**Solution:**

a) i) \( 5.3 \div 0.4 = 13.25 \)

ii) \( 15 \div 0.9 = 16.67 \)

iii) \( 44.8 \div 0.56 = 80 \)

b) i) \( 27.2 \div 8.5 = 3.2 \)

ii) \( 2.924 \div 3.4 = 0.86 \)

iii) \( 22.2 \div 99.9 = 0.22 \)

### Notes

Individual work, monitored, helped

Written on BB or SB or OHT

Reasoning, agreement, self-correction, praising

Feedback for T

### Extension

**PbY6a, page 72, Q.5**

Read: *Find a rule. Complete the table. Write the rule in different ways.*

Ask several Ps what they think the rule is and agree on one form of it.

Ps come to BB to choose a column and fill in the missing value, explaining reasoning. Ps complete tables in Pbs at the same time.

Who could write the rule another way? Ps come to BB or dictate to T. T could write some too and ask if they are true. Class checks suggested rules with values from the table.

Ps think of values for other columns in each table.

**Solution:**

a) \[
\begin{array}{cccccc}
\text{a} & 6 & 2 & 10 & 5 & 20 \\
\text{b} & 3.6 & 1.5 & 6 & 3 & -9 \\
\end{array}
\]

\( b = a \div 5 \times 3,\ b = a \times \frac{3}{5},\ b = \frac{3}{5} \text{ of } a \)

b) \[
\begin{array}{cccccccc}
\text{x} & 4.4 & 6.3 & 3.15 & 4.41 & 10.5 & 31.5 & 9.45 & -42 \\
\text{y} & 4 & 3 & 1.5 & 2.1 & 5 & 15 & 4.5 & -20 \\
\end{array}
\]

\( y = x \div 2.1,\ x = y \times 2.1,\ \frac{x}{y} = 2.1 \)

\( 45 \text{ min} \)
**Activity 1**

**Factorisation**
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

- 73 is a prime number Factors: 1, 73
  (as not exactly divisible by 2, 3, 5, 7 and 11 × 11 > 73)
- 248 = 2 × 2 × 2 × 31 = 2³ × 31
  Factors: 1, 2, 4, 8, 31, 62, 124, 248
- 423 = 3 × 3 × 47 = 3² × 47
  Factors: 1, 3, 9, 47, 141, 423
- 1073 = 29 × 37
  Factors: 1, 29, 37, 1073

**Notes**

Individual work, monitored (or whole class activity)
BB: 73, 248, 423, 1073
Calculators allowed.
Reasoning, agreement, self-correction, praising
e.g.

| 248 | 423 | 3 |
| 124 | 141 | 3 |
| 62  | 47  | 47 |
| 31  | 37  | 1 |
| 1073| 29  | 37 |
| 37  | 1   | 1 |

**Extension**

Ps do calculations on calculators and round to the nearest hundredth. (i.e. correct to 2 decimal places)

- 31
- 248 2
- 124 2
- 62 2
- 31 3
- 1 1
- 1073 29
- 37 37
- 37 1

**Activity 2**

**Dividing by a decimal**
What do you notice about these divisions? (Divisors are decimals)
What could we do to make the divisions easier? (Increase the divisor so that it is a whole number and if we increase the dividend by the same number of times, the quotient will be the same.)

By how many times should we increase the numbers? Elicit that it depends on the number of decimal digits in the divisor, but it should be a multiple of 10.

e.g. 1 decimal digit → increase by 10 times, i.e. multiply by 10¹
    2 decimal digits → increase by 100 times, i.e. multiply by 10², etc.

Ps dictate the easier divisions. Class agrees/disagrees.

BB:
- a) 156 ÷ 1.65 (= 15 600 ÷ 165) [Increase by 100 times]
- b) 156 ÷ 16.5 (= 1560 ÷ 165) [Increase by 10 times]
- c) 15.6 ÷ 1.65 (= 1560 ÷ 165) [Increase by 100 times]
- d) 1.56 ÷ 16.5 (= 15.6 ÷ 165) [Increase by 10 times]

**Notes**

Whole class activity
Written on BB or SB or OHT
Discussion involving several Ps, reasoning, agreement, praising
Feedback for T

**Extension**

Ps do calculations on calculators and round to the nearest hundredth. (i.e. correct to 2 decimal places)

**Activity 3**

**Solving equations**
Let's solve these equations. Ps come to BB or dictate what T should write, explaining reasoning. Ps can write calculation details at side of BB where necessary. Other Ps check that the solution makes the equation true by substituting the value for the letter.

BB:
- a) \( x ÷ 1.1 = 13.2 \), \( x = 13.2 \times 1.1 = 14.52 \)
  Check: \( 14.52 ÷ 1.1 = 14.52 ÷ 11 = 132 \) ✔
- b) \( x \times 1.1 = 13.2 \), \( x = 13.2 ÷ 1.1 = 132 ÷ 11 = 12 \)
  Check: \( 12 \times 1.1 = 13.2 \) ✔
- c) \( 2.3 \times x - 2 \times x = 2.4 \), \( 0.3 \times x = 2.4 \), \( x = 2.4 ÷ 0.3 = 24 ÷ 3 = 8 \)
  Check: \( 2.3 \times 8 - 2 \times 8 = 18.4 - 16 = 2.4 \) ✔

**Notes**

Whole class activity
Written on BB or SB or OHT
At a good pace
Reasoning, agreement, checking, praising
BB: e.g.

<table>
<thead>
<tr>
<th>1</th>
<th>3</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>\times 1.1</td>
<td>11</td>
<td>13.2</td>
</tr>
<tr>
<td>1 1 3 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 3 2</td>
<td></td>
<td></td>
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<tr>
<td>+ 1 3 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 4 5 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- 15 min
**Activity 4**

*PbY6a, page 73*

Q.1 Read: *Do the multiplications and divisions. In each row use the 1st result to help with the rest.*

What must you check when multiplying a decimal by a decimal? (The product should have the same number of decimal digits as the multiplicand and multiplier combined.)

Set a time limit. Ps work in *Ex. Bks*. Encourage Ps to check results by estimating first, or afterwards with reverse operations.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Who agrees? Who has a different answer? etc. Show details on BB if problems or disagreement.

**Solution:**

a) i) $35.4 \times 0.1 = 3.54$  
   ii) $35.4 \times 0.01 = 0.354$  
   iii) $0.354 \times 0.1 = 0.0354$

b) i) $63.5 \times 24 = 1524$  
   ii) $63.5 \times 2.4 = 152.4$  
   iii) $6.35 \times 2.4 = 15.24$

c) i) $8.4 \div 6 = 1.4$  
   ii) $8.4 \div 0.6 = (84 \div 6) = 14$  
   iii) $0.84 \div 0.06 = (84 \div 6) = 14$

20 min

---

**Notes**

Individual work, monitored, helped  
(Written on BB or SB or OHT  
Differentiation by time limit  
Discussion, reasoning, agreement, self-correction, praising  
Feedback for T

BB: e.g.

```
6 3 5
\times 2 4
1 2 7 0 0
1 5 2 4 0
1
```

---

**Activity 5**

*PbY6a, page 73*

Q.2 Read: *Fill in the missing numbers.*

Set a time limit. Ps do necessary calculations in *Ex. Bks.* and write results in *Pbs.*

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Who agrees? Who has a different answer? etc. Show details of calculations on BB if problems or disagreement.

**Solution:**

a) i) $63 \div 7 = 9$  
   ii) $6.3 \div 7 = 0.9$  
   iii) $63 \div 70 = 0.9$

b) i) $35 \div 7 = 5$  
   ii) $3.5 \div 7 = 0.5$  
   iii) $350 \div 70 = 5$

c) i) $1000 \div 4 = 250$  
   ii) $10 \div 4 = 2.5$  
   iii) $100 \div 0.4 = 250$

d) i) $18 \times 30 = 540$  
   ii) $180 \times 0.3 = 54$  
   iii) $0.18 \times 30 = 5.4$

25 min

Individual work, monitored, helped  
Written on BB or use enlarged copy master or OHP  
Reasoning, agreement, self-correction, praising  
Ps point out relationships.  
Elicit that, e.g. if the divisor increases by 10 times and the dividend stays the same, the quotient decreases by 10 times, etc.
Activity 6

PbY6a, page 73

Q.3 Read: Do the multiplications. Check your results with a calculator.

Who can explain how to multiply decimals? (Do the multiplication as if the two factors were whole numbers, then write the decimal point in the product so that it has the same number of decimal digits as the two factors combined.)

Set a time limit. Ps work in Pbs. Remind Ps to estimate result first by rounding appropriately.

Review with whole class. Ps come to BB to estimate and do the calculations. Class checks results on calculators. Mistakes discussed and corrected.

Agree that it does not matter if the same place values (and decimal points) in the multiplicand and multiplier are not lined up, as long as the number of decimal digits in the product matches the total number of decimal digits in the two factors.

Solution:

\[
\begin{align*}
&\text{a)} \quad 2178 \times 3 \quad \Rightarrow \quad \text{Correct: } 6534 \quad \text{(2.5\%) } \\
&\text{b)} \quad 202 \times 10 \quad \Rightarrow \quad \text{Correct: } 2020 \quad \text{(0\%)} \\
&\text{c)} \quad 0.0 \times 2175 \quad \Rightarrow \quad \text{Correct: } 0.0 \quad \text{(0\%)}
\end{align*}
\]

\[
\begin{align*}
&\text{e.g.} \quad (E: \quad 20 \times 30 = 600) \quad (E: \quad 70 \times 2 = 140) \quad (E: \quad 48 \div 4 = 12)
\end{align*}
\]

\[
\begin{align*}
&\text{30 min}
\end{align*}
\]

Lesson Plan 73

Q.4 Read: Do the divisions. Check with a calculator.

What has already been done in part a)? (Divisor and dividend have been increased by 10 times to make the divisor a whole number.) Deal with one at a time. Set a time limit.

Review with whole class. Ps come to BB to work through the divisions, explaining reasoning. Class points out errors. T helps Ps to check result on a calculator. Elicit that, e.g. in a), the remainder is 16 hundredths, i.e. 0.16, not 16!

Check: e.g. a) \(17.87 \div 3.2 \approx 5.728\) (as 0.728 is 0.728)

Mistakes discussed and corrected.

Solution:

\[
\begin{align*}
&\text{a)} \quad 57.2 \div 3.2 \quad (\approx 17.878) \\
&\text{b)} \quad 71.34 \div 6.3 \quad (\approx 11.32) \\
&\text{c)} \quad 5.6 \div 0.06 \quad (\approx 93.33)
\end{align*}
\]

\[
\begin{align*}
&\text{Extension} \\
&\text{Discuss what the correct rounding to 2 decimal digits should be and check by doing the original division on a calculator. (If possible, use a computer calculator projected onto a screen or whiteboard so that the whole class can see.)}
\end{align*}
\]

\[
\begin{align*}
&\text{37 min}
\end{align*}
\]

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

T repeats in a clearer way if necessary.

Responses shown in unison. Reasoning, agreement, checking, self-correction, praising

\[
\begin{align*}
&\text{c)} \quad \text{Extra praise if a P notices an easier method of calculation:} \\
&\quad 50.2 \div 4 = 12.55 \\
&\quad \text{(as 0.25 is } 1 \text{ quarter)}
\end{align*}
\]

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PbY6a, pge 73

Q.5 Read: Solve the problems in your exercise book.

Deal with one at a time. Set a time limit. Ps read question themselves, draw a diagram, write a plan, do the calculation and write the answer in a sentence in Ex. Bks.

Review with whole class. Ps could show result on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

Solution: e.g.

a) One side of a rectangle is 5.7 cm and its adjacent side is 1.2 times longer. What is the area of the rectangle?

BB:

\[
\begin{align*}
\text{Area} &= a \times b \\
&= 5.7 \text{ cm} \times 1.2 \\
&= 6.84 \text{ cm}^2
\end{align*}
\]

Answer: The area of the rectangle is 38.988 cm².

b) 2.5 times the length of one side of a rectangular garden is 24 m. 0.75 of the adjacent side is 15.6 m. What is the area of the garden?

BB:

\[
\begin{align*}
\text{Area} &= a \times b \\
&= 9.6 \text{ m} \times 20.8 \text{ m} \\
&= 199.68 \text{ m}^2
\end{align*}
\]

Answer: The area of the garden is 199.68 m².

c) Which quantity is more: 0.75 of 96 kg or 2 thirds of 48 kg?

Plan:

\[
\begin{align*}
\text{0.75 of 96 kg} &= \frac{3}{4} \times 96 \text{ kg} = 72 \text{ kg} \\
\text{2 thirds of 48 kg} &= \frac{2}{3} \times 48 \text{ kg} = 32 \text{ kg}
\end{align*}
\]

Extra praise for Ps who realised that no calculations are needed.

\[
\frac{2}{3} = \frac{8}{12} < \frac{3}{4} = \frac{9}{12}
\]

48 kg < 96 kg
### Lesson Plan

#### Activity

<table>
<thead>
<tr>
<th>1</th>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:</td>
<td></td>
</tr>
<tr>
<td><strong>74</strong></td>
<td>$2 \times 37$</td>
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<tr>
<td><strong>249</strong></td>
<td>$3 \times 83$</td>
</tr>
<tr>
<td><strong>424</strong></td>
<td>$2 \times 2 \times 2 \times 53 = 2^3 \times 53$</td>
</tr>
<tr>
<td><strong>1074</strong></td>
<td>$2 \times 3 \times 179$</td>
</tr>
</tbody>
</table>

6 min

<table>
<thead>
<tr>
<th>2</th>
<th>Creating problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Let’s make up problems which can be solved by these plans. T writes the plan on BB, allows a minute for Ps to think about it, then Ps tell class their problems. Class chooses the context they like best and Ps come to BB do the calculation and say the answer in context. Class helps and corrects as necessary. e.g.</td>
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<tr>
<td>a) BB: <strong>£400 ÷ 5 ÷ 8</strong></td>
<td>$[= \text{£80 ÷ 8} = \text{£640}]$</td>
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<td>b) BB: <strong>600 m ÷ 0.25</strong></td>
<td>$[= 6 \text{ m} \times 25 = 150 \text{ m}]$</td>
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<tr>
<td>c) BB: <strong>44 kg ÷ 0.4</strong></td>
<td>$(= 440 \text{ kg} ÷ 4 = 110 \text{ kg})$</td>
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12 min

<table>
<thead>
<tr>
<th>3</th>
<th>PbY6a, page 74</th>
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</thead>
<tbody>
<tr>
<td>Q.1 Read: <em>Solve the problems in your exercise book.</em> Set a time limit or deal with one at a time. Ps read problems themselves, write plans, do the calculations, check them and write the answers in sentences. Review with whole class. Ps could show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class decides who is correct. Who worked out the correct answer in another way? etc. Mistakes discussed and corrected.</td>
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<tr>
<td>Individual work, monitored, helped</td>
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<td>Responses shown in unison. Reasoning, agreement, checking, self-correction, praising</td>
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<tr>
<td>Accept any valid method of calculation.</td>
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### Lesson Plan 74

#### Notes

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</tbody>
</table>

What do you think a rectangle with these measurements could be?
(e.g. a section of a road)

---

### Activity

#### Continued

**Solution:** e.g.

a) The product of two numbers is 367.2. One of the numbers is 3.6. What is the other number?

**Plan:**  
\[ x = \frac{367.2}{3.6} = 102 \]

**Check:**  
\[ 102 \times 3.6 = 367.2 \checkmark \]

**Answer:** The other number is 102.

b) The area of a rectangle is \( 304 \frac{1}{5} \text{ m}^2 \). The length of one of the sides is \( 3 \frac{3}{5} \text{ m} \). What is the length of the adjacent side?

**Plan:**  
\[ b = \frac{304 \frac{1}{5}}{3 \frac{3}{5}} = \frac{304 \text{ m}^2}{3.6 \text{ m}} = 338 \text{ m}^2 \div 4 \text{ m} = 84.5 \text{ m} \]

**Check:**  
\[ 84.5 \text{ m} \times 3.6 \text{ m} = 338 \text{ m} \times 0.9 \text{ m} = 304.2 \text{ m}^2 \checkmark \]

**Answer:** The length of the adjacent side is 84.5 m.

---

### PbY6a, page 74

**Q.2** Read: In your exercise book, calculate these parts of \( 560 \text{ km}^2 \).

Set a time limit. Ps may use any valid method.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

a) \( \frac{3}{4} \) of \( 560 \text{ km}^2 \)  
\[ \frac{3}{4} \times 560 \text{ km}^2 = 420 \text{ km}^2 \]

b) \( 1\frac{3}{5} \) of \( 560 \text{ km}^2 \)  
\[ \frac{8}{5} \times 560 \text{ km}^2 = 896 \text{ km}^2 \]

c) 0.52 of \( 560 \text{ km}^2 \)  
\[ 560 \text{ km}^2 \times 0.52 = 56 \text{ km}^2 \times 5.2 = 291.2 \text{ km}^2 \]

d) 48% of \( 560 \text{ km}^2 \)  
\[ \frac{48}{100} \text{ of } 560 \text{ km}^2 = 560 \text{ km}^2 \times 0.48 = 56 \text{ km}^2 \times 4.8 = 268.8 \text{ km}^2 \]
**Lesson Plan 74**

### Activity

#### PbY6a, page 74

**Q.3** Read: **Write an operation to calculate the whole quantity if:**

- a) \( \frac{4}{5} \) of it is 48 kg
- b) \( \frac{1}{2} \) of it is 120 m
- c) 1.6 of it is 50 tonnes
- d) 96% of it is 33.6 g

Set a time limit. Ps write operations and results in *Pbs*. (Necessary calculations can done in *Ex Bks.*)

Review with whole class. Ps come to BB to write operations and do calculation, explaining reasoning. Class agrees/disagrees. Who did the same? Who wrote a different operation? etc. Mistakes discussed and corrected.

**Solution:**

- a) Whole quantity: \( 48 \text{ kg} \div \frac{4}{5} = 48 \text{ kg} \times \frac{5}{4} = 60 \text{ kg} \)
- b) Whole quantity: \( 120 \text{ m} \div \frac{1}{2} = 120 \text{ m} \div \frac{5}{2} = 24 \text{ m} \times \frac{2}{5} = 48 \text{ m} \)
- c) Whole quantity: \( 50 \text{ t} \div 1.6 = 500 \text{ t} \div 16 = 125 \text{ t} \div 4 = 31.25 \text{ t} \)
- d) Whole quantity: \( 33.6 \text{ g} \div 0.96 = 3360 \text{ g} \div 96 = 420 \text{ g} \div 12 = 35 \text{ g} \)

(as 96% means 96 out of 100 or \( \frac{96}{100} \) or 0.96)

Elicit again that:
- to calculate the whole quantity when we know part of it, we divide the quantity we know by the part we know.
- to divide by a fraction, multiply by its reciprocal value.

---

**Notes**

Individual work, monitored, (helped)

Differentiation by time limit

Ask Ps to check their results (mentally or in *Ex Bks.*)

Reasoning, agreement, self-correction, praising

Feedback for T

Checks: e.g.

- a) \( \frac{4}{5} \) of 60 kg = 48 kg
- b) \( \frac{1}{2} \) of 48 m = 96 + 24 = 120 (m)
- c) 1.6 of 31.25 t = 31.25 t + 31.25 t \times 0.6 = 31.25 t + 18.75 t = 50 t
- d) 0.96 of 35 g = 35 g – 0.04 \times 35 g = 35 g – 1.4 g = 33.6 g

---

#### PbY6a, page 74

**Q.4** Read: **Solve the problems in your exercise book. Write an equation first.**

Deal with one at a time or set a time limit.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps anwering correctly explain at BB to *Pbs* who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- a) \( A \) is \( \frac{5}{6} \) of 12 \( \frac{2}{5} \) kg. 2.5 of \( B \) is 25 \( \frac{2}{5} \) kg. Which is more, \( A \) or \( B \)?

  \[
  A = \frac{5}{6} \times 12 \frac{2}{5} \text{ kg} = \frac{5}{6} \times \frac{62}{5} \text{ kg} = \frac{31}{3} \text{ kg} = 10 \frac{1}{3} \text{ kg}
  \]

  \[
  B = 25 \frac{5}{6} \text{ kg} \div 2.5 = \frac{155}{6} \text{ kg} \div \frac{5}{2} = \frac{155}{6} \text{ kg} \times \frac{2}{5} = \frac{31}{3} \text{ kg} = 10 \frac{1}{3} \text{ kg}
  \]

**Answer:** neither \( A \) nor \( B \) is more, as \( A = B \).
Y6

Activity 6

(Continued)

b) \(\frac{3}{5}\) of \(x\) is 60, \(x\) = ?
\[x = 60 \div \frac{3}{5} = 60 \times \frac{5}{3} = \frac{300}{3} = 100\]

c) 0.75 of \(y\) is 60, \(y\) = ?
\[y = 60 \div 0.75 = 60 \div \frac{3}{4} = \frac{240}{3} = 80\]

d) \(z\) is 0.4 of 60, \(z\) = ?
\[z = 60 \times 0.4 = 6 \times 4 = 24\]

PbY6a, page 74

Q.5 Read: Do the calculations in your exercise book.

Set a time limit or deal with one at a time. (T might allow the use of calculators for one or two of them if time is short.)

Review with whole class. T asks several Ps for their answers. Ps with different answers explain reasoning on BB. Class points out errors and decides who is correct. Who had the correct answer but did the calculation another way? Mistakes discussed and corrected.

Solution:

a) \((17.75 + 29.8) \div 3 \times 7 = \left(46 + \frac{15 + 16}{20}\right) \times \frac{7}{3}\)
\[= \left(46 + \frac{31}{20}\right) \times \frac{7}{3} = \frac{1111}{20} \times \frac{7}{3} = \frac{9877}{60} = 164.62\]

b) \(6.7 + 3.2 \div \frac{9}{11} = 6.79 \times \frac{11}{9} = 7.43\)

c) 35.22 – 4 \times 3.15 + 0.75 \div 3 = 35.22 – 12.6 + 0.25
\[= 35.47 – 12.6 = 22.87\]

d) 3.71 + (10.29 \div 7 – 0.25) \times 8 = 3.71 + (1.47 – 0.25) \times 8
\[= 3.71 + 1.22 \times 8 = 3.71 + 9.76 = 13.47\]

Lesson Plan 74

Notes

Check: \(\frac{3}{5}\) of 100
\[= \frac{3}{5} \times 100 = 60\]

Check: 0.75 of 80 = \(\frac{3}{4}\) \times 80
\[= 60\]

Check: \(\frac{24}{60} = \frac{4}{10} = 0.4\]

Individual work, monitored, helped
Written on BB or SB or OHT
Differentiation by time limit
Discussion, reasoning, agreement, praising

or

a) \((17.75 + 29.8) \div 3 \times 7\)
\[= 47.55 \div 3 \times 7 = 15.85 \times 7 = 110.95\]

b) \((6 \times 7 + 3 \div \frac{2}{10}) \div \frac{91}{11}\)
\[= 9 \times \frac{91}{10} \times \frac{11}{9} = \frac{9911}{90} \times \frac{11}{9} = \frac{121}{10} = 12 \frac{1}{10}\]

Review order of operations: operations in brackets first, then multiplication or division, then addition or subtraction.
**Lesson Plan 74**

**Notes**

Whole class activity  
(or individual trial first if Ps wish)  
Discussion, reasoning, agreement, checking, praising  
If no P thinks of the method shown opposite, T gives hints or suggests it and asks Ps what they think about it.  
Extra praise if a P thinks of it without help from T.  

Or  
Let \( a \) be the first number and \( b \) be the 2nd number.  
\[ a + b = 18 \frac{1}{2}, \]
but \( a = 4 \times b \),  
so \( 4 \times b + b = 18 \frac{1}{2} \)  
\[ 5 \times b = 18 \frac{1}{2}, \]
etc.

or let the two numbers be \( a \) and \( b \),  
\[ a - b = 18.5 \]
but \( a = 6 \times b \)  
so \( 6 \times b - b = 18.5 \)  
\[ 5 \times b = 18.5, \]
etc.  
[T might point out that:  
\( 5 \times b \) can be written as \( 5b \),  
\( 6 \times y \) can be written as \( 6y \), etc.]

---

### Activity

**PbY6a, page 74, Q.6**

Deal with one question at a time. T chooses a P to read out the question. Ps suggest what to do first and how to continue. Class helps, corrects or suggests an easier method of solution. T intervenes only if necessary. Ps could write solution in Ex. Bks too.

**Solution:**

a) *The sum of two numbers is \( 18 \frac{1}{2} \). The first number is 4 times the second number. What are the two numbers?*

e.g. Let the 2nd number be \( x \), then the first number is \( 4 \times x \)

2nd number: \( x + 4 \times x = 18 \frac{1}{2} \)  
\[ 5 \times x = 18 \frac{1}{2}, \]
\[ x = 18 \frac{1}{2} \div 5 = \frac{37}{2} \div 5 = \frac{37}{10} = 3 \frac{7}{10} \]

1st number: \( x \times 4 = 3 \frac{7}{10} \times 4 = 12 \frac{28}{10} = 14 \frac{8}{10} \)

Check: \[ 14 \frac{8}{10} + 3 \frac{7}{10} = 17 \frac{15}{10} = 18 \frac{5}{10} = 18 \frac{1}{2} \]

Answer: The first number is 14.8 and the 2nd number is 3.7.

b) *The difference between two numbers is 18.5. The larger number is 6 times the smaller number. What are the two numbers?*

e.g. Let the smaller number be \( y \), then the larger number is \( 6 \times y \)

Smaller number: \( 6 \times y - y = 10.5, \)  
\[ 5 \times y = 10.5, \]
\[ y = 10.5 \div 5 = 2.1 \]

Larger number: \( y \times 6 = 2.1 \times 6 = 12.6 \)

Check: \( 12.6 - 2.1 = 10.5 \)  

Answer: The two numbers are 12.6 and 2.1.

---

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Factorising 75, 250, 425 and 1075. Revision, activities, consolidation

**PbY6a, page 75**

**Solutions:**

Q.1  
a) \(24 \text{ kg} \div 2 \text{ kg} = 12\) (packs)

b) \(24 \text{ kg} \div 1 \text{ kg} = 24\) (packs)

c) \(24 \text{ kg} \div \frac{1}{2} \text{ kg} = 24 \times 2 = 24\) (packs)

d) \(24 \text{ kg} \div \frac{1}{3} \text{ kg} = 24 \times 3 = 72\) (packs)

e) \(24 \text{ kg} \div \frac{1}{4} \text{ kg} = 24 \times 4 = 96\) (packs)

f) \(24 \text{ kg} \div \frac{1}{5} \text{ kg} = 24 \times 5 = 120\) (packs)

g) \(24 \text{ kg} \div \frac{1}{10} \text{ kg} = 24 \times 10 = 240\) (packs)

Q.2  
a) \(40 \div 4 = 10\)

b) \(45 \div 9 = 5\)

c) \(\frac{3}{5} \div 9 = \frac{1}{15}\)

\[
\begin{align*}
40 & \div 2 = 20 \\
45 & \div 3 = 15 \\
40 & \div 1 = 40 \\
45 & \div 1 = 45 \\
40 & \div \frac{1}{2} = 80 \\
45 & \div \frac{1}{3} = 80 \\
40 & \div \frac{1}{4} = 160 \\
45 & \div \frac{1}{9} = 160
\end{align*}
\]

d) \(\frac{3}{5} \div 3 = \frac{1}{5}\)

e) \(\frac{3}{5} \div 1 = \frac{3}{5}\)

\[
\begin{align*}
40 & \div \frac{1}{2} = 80 \\
45 & \div \frac{1}{3} = 80 \\
45 & \div \frac{1}{9} = 160
\end{align*}
\]

Q.3  
a) \(\frac{8}{2} \times \frac{1}{3} = \frac{4}{3}\)

b) \(\frac{8}{2} \times \frac{1}{6} = \frac{4}{3}\)

c) \(\frac{8}{2} \times \frac{1}{6} = \frac{4}{3}\)

Q.4  
a) \(27.8 \times 0.1 = 2.78, \ 27.8 \times 0.001 = 0.0278\)

\[
2.78 \times 0.01 = 0.0278
\]

b) \(42.5 \times 12 = 510, \ 42.5 \times 1.2 = 51, \ 42.5 \times 0.12 = 0.51\)

\[
42.5 \times 12 = 510, \ 42.5 \times 1.2 = 51, \ 42.5 \times 0.12 = 0.51
\]

c) \(7.8 \div 6 = 1.3, \ 7.8 \div 0.6 = 13, \ 0.78 \div 0.06 = 13\)
Q.5  a) \( b = 20.8 \text{ cm}^2 \div 6.5 \text{ cm} = 208 \text{ cm}^2 \div 65 \text{ cm} = 3.2 \text{ cm} \)
   \textit{Answer:} the length of the adjacent side is 3.2 cm.

b) \( A = 6.5 \text{ cm} \times (19.4 \text{ cm} \div 2 - 6.5 \text{ cm}) \)
   \textit{= 6.5 \text{ cm} \times (9.7 \text{ cm} - 6.5 \text{ cm})} \)
   \textit{= 6.5 \text{ cm} \times 3.2 \text{ cm} = 20.8 \text{ cm}^2} \)
   \textit{Answer:} The area of the rectangle is 20.8 cm².

c) \( a = 7.2 \text{ m} \div 1.5 = 72 \text{ m} \div 15 = 24 \text{ m} \div 5 = 4.8 \text{ m} \)
   \( b = 3.3 \text{ m} \div 0.6 = 33 \text{ m} \div 6 = 5.5 \text{ m} \)
   \( A = 4.8 \text{ m} \times 5.5 \text{ m} = 26.4 \text{ m}^2 \)
   \textit{Answer:} The area of the lawn is 26.4 m².

Q.6  Let the 1st number be \( x \), then the second number is \( 3 \times x \) (or \( 3x \))
\( x + 3 \times x = 12.8 \) (or \( x + 3x = 12.8 \))
\( 4 \times x = 12.8 \) (or \( 4x = 12.8 \))
\( x = 12.8 \div 4 = 3.2 \)

\textit{1st number:} 3.2
\textit{2nd number:} 3.2 \times 3 = 9.6 \quad \textit{Check:} 3.2 + 9.6 = 12.8 \checkmark

\textit{Answer:} The two numbers are 3.2 and 9.6.
### Lesson Plan

#### Y6

**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(76 = 2 \times 2 \times 19 = 2^2\) Factors: 1, 2, 4, 19, 38, 76
- \(251\) is a prime number Factors: 1, 251

(as not exactly divisible by 2, 3, 5, 7, 11, 13 and \(17^2 < 251\)

- \(426 = 2 \times 3 \times 71\) Factors: 1, 2, 3, 6, 71, 142, 213, 426
- \(1076 = 2^2 \times 269\) Factors: 1, 2, 4, 269, 538, 1076

#### Activity 2

**Percentage**

a) *How could we work out what 34% of £750 is?*

Ps come to BB or dictate what T should write, explaining reasoning. Class agrees/disagrees. (T shows any method not suggested by Ps and asks class if it is correct.)

**BB:**

i) \(100\% \rightarrow £750\)

\[1\% \rightarrow £750 \div 100 = £7.50\]

\[34\% \rightarrow £7.50 \times 34 = £255\]

ii) \(34\% \text{ of } £750 \rightarrow £750 \div 100 \times 34 = £7.50 \times 34 = £255\)

iii) \(£750 \times \frac{34}{100} = \frac{15}{50} \times 17 = £15 \times 17 = £255\)

iv) \(£750 \times 0.34 = £75 \times 3.4 = £255\)

b) *What percentage are we calculating if we use these plans?*

Ps say the percentages then come to BB to complete the calculations, explaining reasoning. Class agrees/disagrees.

**BB:**

i) \(450 \div 4 \times 3 (= 112.5 \times 3 = 337.5\) m\)

[75% of 450 m. as we are calculating \(\frac{3}{4} = \frac{75}{100} \rightarrow 75\%\)]

ii) \(20.8 \div 100 \times 61 (= 0.208 \times 61 = 12.688\) kg\)

[61% of 20.8 kg, as we are calculating \(\frac{61}{100} \rightarrow 61\%\)]

iii) \(0.91 \times \frac{7}{5} (= 6.37\) km\)

[140% of 0.91 km, as \(\frac{7}{5} = \frac{14}{10} = 1.4 \rightarrow 140\%\)]

iv) \(615 \times 0.11 (= 67.65\) cm\)

[11% of 615 cm, as 0.11 = \(\frac{11}{100} \rightarrow 11\%\)]

**Notes**

Individual work, monitored (or whole class activity)

BB: 76, 251, 426, 1076 Calculators allowed.

Reasoning, agreement, self-correction, praising

\[\begin{array}{c}
76 & 251 & 426 & 1076 \\
38 & 3 & 3 & 1 \\
19 & 19 & 269 & 269 \\
1 & 2 & 2 & 1 \\
\end{array}\]

Whole class activity

At a good pace

Involve several Ps.

Discussion, reasoning, agreement, praising

Details: e.g.

\[\begin{array}{c}
7.5 \times 1.0 \\
3.0 \times 0.1 \\
2.25 \times 0.1 \\
1.05 \times 1.7 \\
2.5 \times 0.1 \\
\end{array}\]

Written on BB or SB or OHT

Ps do necessary calculations at side of BB if they cannot do them mentally.

Involves several Ps.

Reasoning, agreement, praising

\[\begin{array}{c}
0.2 \times 0.8 \\
2.0 \times 1.2 \\
1.2 \times 0.8 \\
\end{array}\]

Feedback for T
**Activity

2**

(Continued)

c) Listen carefully and think about how you would solve this problem.

55% of a distance is 275 m. What is the whole distance?

A, what do you think we should do? Who agrees? Who can think of another way to do it? etc. And another? (Elicit the 4 methods shown below.) Which method do you like best? Why?

BB: e.g.

i) 55% \[\rightarrow\] 275 m

1% \[\rightarrow\] 275 m \(\div\) 55 = 55 m \(\div\) 11 = 5 m

100% \[\rightarrow\] 5 m \(\times\) 100 = 500 m

ii) 275 m \(\div\) 55 \(\times\) 100 = 5 m \(\times\) 100 = 500 m

iii) 275 m \(\div\) \(\frac{55}{100}\) = 275 m \(\div\) \(\frac{11}{20}\) = \(\frac{275\text{m} \times 20}{11}\) = 500 m

iv) 275 m \(\div\) 0.55 = 27500 m \(\div\) 55 = 5500 m \(\div\) 11 = 500 m

**Notes**

Discussion, reasoning, agreement, praising

Elicit that reducing or increasing the dividend and divisor by the same number of times does not change the quotient.

T shows any method which Ps miss.

---

**Lesson Plan 76**

**Notes**

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Feedback for T

Extra praise if Ps notice that a) and b), and c) and d), are the same calculations:

\[
\frac{4}{5} = \frac{8}{10} = 0.8 \rightarrow 80\%
\]

\[
\frac{3}{4} = \frac{75}{100} = 0.75 \rightarrow 75\%
\]

---

**Y6**

**PbY6a, page 76**

**Q.1** Read: Solve the problems in your exercise book.

Set a time limit or deal with one at a time.

Review with whole class. Ps show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB.

Class points out errors and decides who is correct. Who had the correct answer but used a different plan? Mistakes discussed and corrected.

Solution: e.g. (accept any valid method)

a) Calculate \(\frac{4}{5}\) of 89.6 m

**Plan:** 89.6 m \(\div\) 5 \(\times\) 4 = 17.92 \(\times\) 4 = 71.68 m

b) Calculate 80% of 89.6 m.

**Plan:** 89.6 m \(\times\) 0.8 = 71.68 m

c) \(\frac{3}{4}\) of a quantity is 720 kg. What is the whole quantity?

**Plan:** 720 kg \(\div\) \(\frac{3}{4}\) = \(\frac{240}{41}\) kg \(\times\) \(\frac{4}{1}\) = 960 kg

**Check:** \(\frac{3}{4}\) of 960 kg = \(\frac{3}{41}\) \(\times\) 960 kg = 720 kg

**Answer:** The whole quantity is 960 kg.

d) 75% of a quantity is 720 kg. What is the whole quantity?

**Plan:** 720 kg \(\div\) 75 \(\times\) 100 = 9.6 kg \(\times\) 100 = 960 kg

**Check:** 75\% of 960 kg = 960 kg \(\times\) 0.75 = 720 kg

**Answer:** The whole quantity is 960 kg.
Lesson Plan 76

Notes

Individual work, monitored (helped)
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Make sure that the plans shown opposite in a) and c) are discussed for each question.

\[
\frac{75}{100} = \frac{3}{4}
\]

Whole class activity
Drawn on BB or use enlarged copy master or OHP
At a good pace
Involve several Ps.
Discussion, reasoning, agreement, praising

Praising, encouragement only

Whole class activity
Ps label diagram in Pbs too.

(To save time, T could have basics of diagram already prepared on BB or SB or OHT and Ps dictate what to write in the boxes and on the arrows.)

Agreement, praising

T could ask 2 or 3 Ps to repeat the 'rules' (with help if necessary).

---

**Y6**

### Activity

**PbY6a, page 76**

Q.2  **Read:** *In your exercise book, calculate:*

- a) 15% of 800
- b) 75% of 4000
- c) 20% of 350
- d) 100% of 26.3

Set a time limit. Review with whole class. Ps show amounts on scrap paper or slates on command. Ps answering correctly come to BB to explain reasoning. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

- a) 15% of 800 = \( \frac{800}{100} \times 15 = 8 \times 15 = 120 \)
- b) 75% of £4000 = \( £4000 \times \frac{3}{4} = £3000 \)
- c) 20% of 350 = \( 350 \times 0.2 = 35 \times 2 = 70 \)
- d) 100% of 26.3 = \( 26.3 \times 1 = 26.3 \)

---

**PbY6a, page 76, Q.3**

**Read:** *What is 19% of 600? Fill in the items which are missing from the diagram.*

Allow Ps a minute to think about it, then Ps come to BB to fill in an item and explain their reasoning. Who agrees? Who thinks it should be something else? Why? etc. When class has agreed on the item, Ps also write it in Pbs.

**Solution:**

\[
\begin{align*}
\text{Whole amount} & \rightarrow 600 \\
\text{19\% of it} & \rightarrow 114 \\
\text{1\% of it} & \rightarrow 6 \\
\text{\% rate} & \rightarrow 19 \\
\text{\% value} & \rightarrow 0.19
\end{align*}
\]

Who can explain what the diagram shows? Ps explain in their own words. Class agrees/disagrees.

**Extension**

T: (pointing to relevant numbers and labelling them on diagram on BB)

In this diagram, 600 is the **whole amount**, 19 is the **percentage rate** and 114 is the **percentage value**.

Let’s draw a general diagram to show how we can calculate any percentage value from any amount. Ps dictate what T should draw and write, or if Ps have no ideas, T starts and involves Ps where possible.

**BB:**

\[
\begin{align*}
\text{Whole amount} & \rightarrow \times \% \text{ rate} \\
\text{1\% value} & \rightarrow \% \text{ value} \\
\% \text{ rate} & \rightarrow \% \text{ value} \\
\% \text{ value} & \rightarrow \frac{\% \text{ rate}}{100}
\end{align*}
\]

T reviews. To calculate a certain percentage of a whole amount:

- calculate the value of 1\% by dividing by 100, then multiply by the percentage rate; or
- multiply the whole amount by 1 hundredth of the percentage rate.

---

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**Activity**

**PbY6a, page 76**

Q.4 Read: *In your exercise book, calculate the whole quantity if:*

- a) 50% of it is 43
- b) 17% of it is 595
- c) 120% of it is 156
- d) 100% of it is 36.25
- e) \(33\frac{1}{3}\)% of it is 33
- f) 150% of it is 300.

Set a time limit. Review with whole class. Ps show amounts on scrap paper or slates on command. Ps answering correctly come to BB to explain reasoning. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

- a) Whole amount: \(43 \div 0.5 = 86\)
  
- b) Whole amount: \(595 \div 0.17 = 3500\)
  
- c) Whole amount: \(156 \div 1.2 = 130\)
  
- d) Whole amount: \(36.25 \div 0.5 = 72.5\)
  
- e) Whole amount: \(33 \div 0.3 = 110\)

**Extension**

Ps point out the whole amount, the % rate and the % value in each question.

---

**Notes**

Individual work, monitored, helped

Differentiation by time limit

If class is not very able, deal with one at a time.

T notes what Ps do in part e).

Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

- \(43 \div 0.5 = 43 \times 2 = 86\)

Make sure that the plans which are not in brackets are shown and discussed.

- (It is already the whole amount.)

Elicit that we cannot divide by a decimal in this case as

- \(33\frac{1}{3}\)% → \(\frac{1}{3} = 0.3\)

We could divided by 0.3 but the result would not be exact.

Encourage Ps to learn the equivalent forms of 1 third.

Whole class activity

Agreement, praising
Activity

7  

PbY6a. page 76, Q.5

Read: If 19% of a quantity is 114, what is the whole quantity?

Fill in the missing items.

Allow Ps a minute to think about it, then Ps come to BB to fill in an item and explain their reasoning. Who agrees? Who thinks it should be something else? Why? etc. When class has agreed on the item, Ps also write it in Pb. Extra praise if Ps notice that it is the reverse of Q.3.

Solution:

\[ \frac{114}{0.19} = \frac{\text{Whole amount}}{\text{19\% of it}} \]

Who can explain what the diagram shows? Ps explain in their own words. Class agrees/disagrees. Elicit which value is the whole amount, the % rate and the % value. Ps write it in Pb too.

Extension

Let's draw a general diagram to show how we can calculate the whole amount from any percentage value. Ps dictate what T should draw and write, or if Ps have no ideas, T starts and involves Ps where possible.

BB:

\[
\begin{align*}
\text{Whole amount} & \quad \times 100 \quad \div \% \text{ rate} \\
\text{1\% value} & \quad \% \text{ value} \\
\% \text{ rate} & \quad \div 100
\end{align*}
\]

Elicit that to calculate the whole amount from a certain percentage of it:

- divide the percentage value by the percentage rate then multiply by 100, or
- divide the percentage value by 1 hundredth of the percentage rate.

Notes

Lesson Plan 76

Whole class activity

Drawn on BB or use enlarged copy master or OHP

At a good pace

Involve several Ps.

Discussion, reasoning, agreement, praising

Individual work, monitored

Competition, in good honour!

Reasoning, agreement, checking, self-correcting, praising

Class applauds first P to have correct answer.

E: e.g. 700 \times 2 = 1400 (m²)

(as 650 m² \approx 700 m² and 40\% \approx 50\%)
(Continued)

b) The population of a city has risen by 2% over the past year and there are now 3100 more people.

What was the population of the city at this time last year?

**Plan:**

\[
3100 \div 0.02 = 310\,000 \div 2 = 155\,000
\]

or

\[
2\% \rightarrow 3100 \div 2 = 1550
\]

100% → 1550 × 100 = 155 000

**Check:** 2% of 155 000 = 155 000 × 0.02 = 3100 ✓

**Answer:** This time last year the population was 155 000.

---

**Extension**

What is the population now?

(155 000 + 3100 = 158 100)
### Activity 1

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- $77 = 7 \times 11$ Factors: 1, 7, 11, 77
- $252 = 2 \times 2 \times 3 \times 3 \times 7 = 2^2 \times 3^2 \times 7$
  Factors: 1, 2, 3, 4, 6, 7, 9, 12, 14, 252, 262, 84, 63, 42, 36, 28, 21, 18
- $427 = 7 \times 61$ Factors: 1, 7, 61, 427
- $1077 = 3 \times 359$ Factors: 1, 3, 359, 1077

**7 min**

### Activity 2

#### Ratio and percentage

a) What is the ratio of circles to squares? BB: ○○○ □□□□

(3 to 4, or 3 : 4 or $\frac{3}{4} = 0.75$)

What part of 4 is 3? ($\frac{3}{4}$ or 0.75 or 75%)

T: A fraction can mean a division (e.g. $3 \div 4$), it can be a quotient (e.g. $\frac{3}{4}$), or it can be a ratio (e.g. $\frac{3}{4}$)

What part of all the shapes are the circles? ($\frac{3}{7} \approx 0.429 \rightarrow 42.9\%$)

What is the ratio of squares to circles? (4 to 3 or 4 : 3 or $\frac{4}{3} = 1.3$)

What part of 3 is 4? ($\frac{4}{3}$ or $1\frac{1}{3}$ or $133\frac{1}{3}\%$ or 133.3%)

b) Let’s express these fractions or ratios as percentages.

Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees.

BB:

i) $\frac{2}{5} = \left(\frac{40}{100} \rightarrow 40\%\right)$  
ii) $6 \div 10 = \left(\frac{6}{10} = \frac{60}{100} \rightarrow 60\%\right)$

iii) $0.3 = \left(\frac{30}{100} \rightarrow 30\%\right)$ iv) $\frac{7}{4} = \left(1\frac{3}{4} = \frac{7}{4} = \frac{175}{100} \rightarrow 175\%\right)$

v) $4 : 9 = \left(\frac{4}{9} = 0.4 \rightarrow 44.4\%\right)$ vi) $10 : 5 = \left(2 \rightarrow 200\%\right)$

**18 min**

### Notes

**Individual work, monitored (or whole class activity)**

BB: 77, 252, 427, 1077

Calculators allowed.

Reasoning, agreement, self-correction, praising

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>427</td>
<td>7</td>
<td>61</td>
<td>61</td>
<td></td>
</tr>
<tr>
<td>252</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
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<tr>
<td>126</td>
<td>2</td>
<td>1</td>
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<tr>
<td>63</td>
<td>3</td>
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<td>21</td>
<td>3</td>
<td></td>
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<td>7</td>
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<td>1077</td>
<td>3</td>
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<td>359</td>
<td>359</td>
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<tr>
<td>1</td>
<td>1</td>
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</tr>
</tbody>
</table>

Whole class activity

Shapes drawn (stuck) on BB

Ps come to BB or dictate what T should write. Class agrees/disagrees.

Involve many Ps.

Praising, encouragement only

Elicit that to change a fraction to a decimal:
- if possible, change to an equivalent fraction with a denominator which is a multiple of 10,
- or if this is not possible divide the numerator by the denominator.

Written on BB or SB or OHT

Accept any valid reasoning.

Agreement, praising

T helps or gives hints if necessary.

Allow the use of calculators:

$26 \div 43 = 0.6046511627...$

Agree on an appropriate rounding (e.g. to 3 d.p.)
Y6

Activity

3  PbY6a, page 77

Q.1  Read: Complete the table to show the different percentages of 160 kg in kg and grams.

Who can explain what the table means? (160 kg is the whole amount, the top row shows different percentage rates and the middle and bottom rows show the percentage values in kg and g.)

Set a time limit. Ps can do necessary calculations in Ex. Bks.

Review with whole class. Ps come to BB to fill in missing items, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Ps show details at side of BB if problems or disagreement.

Solution:

<table>
<thead>
<tr>
<th>160 kg</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
<th>125%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.6</td>
<td>8</td>
<td>16</td>
<td>40</td>
<td>80</td>
<td>120</td>
<td>160</td>
<td>200</td>
</tr>
<tr>
<td>in g</td>
<td>1600</td>
<td>8000</td>
<td>16000</td>
<td>40000</td>
<td>80000</td>
<td>120000</td>
<td>160000</td>
<td>200000</td>
</tr>
</tbody>
</table>

What is the general rule for the table?

(percentage value = whole amount ÷ 100 × percentage rate
or = whole amount × \( \frac{\text{percentage rate}}{100} \))

Extension

23 min

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Extra praise for Ps who notice relationships which make the calculations easier:

e.g. 25% of 160 kg = 5 × 5% of 160 kg = 5 × 8 kg = 40 kg

or = 160 kg ÷ 4 = 40 kg etc.

Whole class activity

Check with values in the table.

Praising

4  PbY6a, page 77

Q.2  Read: Complete the table to show the different percentages of 0.5 m in km and metres.

Elicit that the task is the same as in Q.1 but the whole amount this time is part of a km. Elicit that 0.5 km = 500 m.

Set a time limit. Ps do necessary calculations in Ex. Bks.

Review with whole class. Ps come to BB to fill in missing items, explaining reasoning. Who agrees? Who worked it out another way? etc. Mistakes discussed and corrected. Ps show details of calculations at side of BB if problems or disagreement.

Solution:

<table>
<thead>
<tr>
<th>0.5 km</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>75%</th>
<th>100%</th>
<th>125%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.005</td>
<td>0.025</td>
<td>0.05</td>
<td>0.125</td>
<td>0.25</td>
<td>0.375</td>
<td>0.5</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in km</td>
<td>0.5</td>
<td>0.025</td>
<td>0.05</td>
<td>0.125</td>
<td>0.25</td>
<td>0.375</td>
<td>0.5</td>
<td>0.625</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in m</td>
<td>5</td>
<td>25</td>
<td>50</td>
<td>125</td>
<td>250</td>
<td>375</td>
<td>500</td>
<td>625</td>
</tr>
<tr>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
</tbody>
</table>

Agree that the general rule is the same as the previous table.

28 min
**Activity 5**  
*PbY6a, page 77*

**Q.3** Read: *Complete the table to show the different percentages of a right angle, a straight angle and a whole angle (in°).*

Elicit that a whole angle = 360°, a straight angle = 180° and a right angle = 90°. Ask Ps to draw them on BB or show the turns. 

T writes the unit of measure beside each type of angle in table on BB and Ps write it in *Pbs* too. Set a time limit.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning and showing details of calculations where necessary. Who agrees? Who worked it out another way? etc. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Angle</th>
<th>1%</th>
<th>5%</th>
<th>10%</th>
<th>25%</th>
<th>50%</th>
<th>70%</th>
<th>90%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right (°)</td>
<td>0.9</td>
<td>4.5</td>
<td>9</td>
<td>22.5</td>
<td>45</td>
<td>63</td>
<td>81</td>
<td>90</td>
</tr>
<tr>
<td>Straight (°)</td>
<td>1.8</td>
<td>9</td>
<td>18</td>
<td>45</td>
<td>90</td>
<td>126</td>
<td>162</td>
<td>180</td>
</tr>
<tr>
<td>Whole (°)</td>
<td>3.6</td>
<td>18</td>
<td>36</td>
<td>90</td>
<td>180</td>
<td>252</td>
<td>324</td>
<td>360</td>
</tr>
</tbody>
</table>

Details: e.g. 90% of 360° = \(360° \times \frac{90}{100} = 324°\)

---

**6**  
*PbY6a, page 77*

**Q.4** Read: *Write the whole length in the table if 3.5 m is the given percentage.*

What is different about this table compared with the previous tables? (In the previous tables we had to calculate the percentage values but in this table we have to calculate the whole amount.)

Do one or two columns with the whole class first if necessary, otherwise set a time limit. Ps calculate mentally or in Ex. Bks.

Review with whole class. Ps come to BB to fill in a value and explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>If 3.5 m is:</th>
<th>1%</th>
<th>2%</th>
<th>4%</th>
<th>5%</th>
<th>10%</th>
<th>20%</th>
<th>25%</th>
<th>50%</th>
<th>100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>the whole length is:</td>
<td>350 m</td>
<td>175 m</td>
<td>87.5 m</td>
<td>70 m</td>
<td>35 m</td>
<td>17.5 m</td>
<td>14 m</td>
<td>7 m</td>
<td>3.5 m</td>
</tr>
</tbody>
</table>

Accept both decimal and fraction forms.

**Extension**

What is the general rule for completing the table?

(whole amount = percentage value ÷ percentage rate × 100)

---
Q.5 Read: Solve the problems in your exercise book. Estimate, calculate and check each result.

First discuss what gross income and net income mean and what kind of taxes are normally deducted (taken out) before anyone receives their pay. Allow Ps to explain if they can, otherwise T explains (showing a real example of a pay slip if possible).

Why does everyone who is earning money need to pay taxes to the Government? Ps make some suggestions.

Deal with one question at a time. T chooses a P to read out the question. Ps write plans, estimate, calculate and check result then write the answer as a sentence in Ex. Bks.

Review with whole class. Ps show results on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Mistakes discussed and corrected.

Solution: e.g.

a) How much does Mr. Smith earn per month if £3599 goes into his bank account after 41% has been deducted in taxes from his gross income?

Plan: 100% – 41% = 59% → £3599

100% → £3599 ÷ 59 × 100 = £61 × 100 = £6100

or £3599 ÷ 0.59 = £359 900 ÷ 59 = £6100

Check: 41% of £6100 = £6100 × 0.41 = £2501

£6100 – £2501 = £3599 ✔

Answer: Mr. Smith earns £6100 per month.

b) Mr. Smith spends 60% of his net income on household bills and food. How much does he have left each month to spend on other things?

Plan: Has left: 100% – 60% = 40%

40% of £3599 = £3599 ÷ 100 × 40

= £3599 ÷ 10 × 4 = £359.90 × 4

= £1439.60

Check: 40% of net income = £1439.60

100% of net income = £1439.60 ÷ 0.4

= £1439.60 ÷ 4 = £3599 ✔

Answer: Mr. Smith has £1439.60 left each month.
c) Mr. Smith saves 25% of the money he has left each month (after paying his household bills and food) for his family's yearly holiday.

**How much does he save each year for the family holiday?**

**Plan**: Amount saved each month: 25% of £1439.60

Amount saved each year: 25% of £1439.60 × 12

\[= \frac{1439.60}{4} \times 3\]

\[= £4318.80\]

**Check**: £4318.80 ÷ 12 × 4 = £4318.80 ÷ 3

\[= £1439.60 \checkmark\]

**Answer**: Mr. Smith saves £4318.80 each year for the family holiday.

d) The original price of a holiday was increased by 25% and its new price is £960. What was the original price of the holiday?

**Plan**: New price: 125% of original price

\[125\% \rightarrow £960\]

\[1\% \rightarrow £960 \div 125\]

\[100\% \rightarrow £960 \div 125 \times 100\]

\[= £7.68 \times 100\]

\[= £768\]

or 100% → £960 ÷ 1.25 = £96 000 ÷ 125 = £768

**Check**: £768 + £768 ÷ 4 = £768 + £192 = £960 \checkmark

**Answer**: The original price of the holiday was £768.

E: £1000 ÷ 125 × £100

\[= £8 \times 100 = £800\]

or Let original price be \(x\).

\[1 \frac{1}{4} \times x = £960\]

\[x = £960 \div \frac{1}{4}\]

\[= £960 \div \frac{5}{4}\]

\[= £960 \times \frac{4}{5}\]

\[= £768\]
Lesson Plan
78

Notes

Individual work, monitored
(or whole class activity)
BB: 78, 253, 428, 1078

Calculators allowed.
Reasoning, agreement, self-correction, praising

e.g. 78 2
39 3
13 13
1078 2
428 2
539 7
214 2
77 7
107 107
11 11
1 1

Whole class activity
Involve many Ps.
At a good pace
Reasoning, agreement, praising
Feedback for T

P comes to BB or dictates what
to BB to write the calculation,
with T's help where necessary.

Extra praise if a P thinks of this.

Discussion, reasoning,
agreement, praising

or

or

If no P suggests it, T mentions
using ratio and helps Ps to
write it as shown.

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**Activity**

3  

Erratum: In *Pbs*, there should be an 'a' before 'sentence'.

**PbY6a, page 78**

Q.1 Read: Write a plan. Estimate, calculate and check the result. Write the answer as a sentence.

Set a time limit of 4 minutes. Ps read questions themselves and solve them in *Ex. Bks*.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class points out errors and agrees on correct answer. Who had the correct answer but worked it out in a different way? Mistakes discussed and corrected. T chooses Ps to say the answers as sentences.

**Solution:**

a) 10% of an amount is £142.80. What is 93% of the same amount?

\[ E: \ 143 \times 10 = 1430 \text{ (as 93\% = 100\%)} \]

**Plan:** £142.80 ÷ 10 × 93 = £14.28 × 93 = £1328.04

or Let the amount be \( x \).

\[ x \times 0.1 = £142.80, \quad x = £142.80 \div 0.1 = £1428, \]

93\% of £1428 = £1428 × 0.93 = £1328.04

or \( \frac{14.28}{10} \times \frac{93}{10} = 14.28 \times 93 = £1328.04 \)

or \( 93 : 10 = \frac{93}{10} = 9.3, \quad £142.80 \times 9.3 = £1328.04 \)

or \( \frac{142.80}{10} \times \frac{93}{10} = 14.28 \times 93 = £1328.04 \)

or \( \frac{93}{10} : 10 = \frac{93}{10} = 9.3, \quad £142.80 \times 9.3 = £1328.04 \)

**Answer:** 93% of the amount is £1328.04.

b) I am thinking of a number. \( \frac{1}{3} \) of my number is \( \frac{7}{15} \).

What is 2 \( \frac{1}{4} \) times my number?

\[ E: \ \frac{8}{5} \rightarrow 16, \quad \frac{1}{5} \rightarrow 2, \quad \frac{5}{5} \rightarrow 10, \quad 2 \times 10 = 20 \]

**Plan:** Let the number be \( x \).

\[ x \times \frac{8}{5} = \frac{15}{15} \cdot \frac{7}{7} = \frac{232}{15} \]

or \( \frac{232}{15} \times \frac{5}{5} \times \frac{5}{5} = \frac{29}{3} \]

or \( \frac{232}{15} \times \frac{5}{5} \times \frac{5}{5} = \frac{29}{3} = \frac{21}{3} \)

or \( \frac{232}{15} \times \frac{5}{5} \times \frac{5}{5} = \frac{29}{3} = \frac{21}{3} \)

or \( \frac{92}{3} \times \frac{9}{3} = \frac{87}{4} = \frac{21}{4} \)

or \( \frac{92}{3} \times \frac{9}{3} = \frac{87}{4} = \frac{21}{4} \)

or \( 9 \times \frac{3}{3} = \frac{29}{3} \times \frac{3}{3} = \frac{87}{4} = \frac{21}{4} \)

or \( 9 \times \frac{3}{3} = \frac{29}{3} \times \frac{3}{3} = \frac{87}{4} = \frac{21}{4} \)

**Answer:** Two and a quarter times my number is 21 and 3 quarters.

**Notes**

Individual work, monitored, helped

Differentiation by time limit. [Only a) is expected from the majority of the class in the 4 minutes allowed; b) is an advanced problem for the more able Ps.]

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid method, but if Ps prefer any of the methods shown to their own method, they can write it in *Ex. Bks*.

If no P used ratio, T shows it and asks class if it is correct.

**BB:**

\[ \frac{1}{4} \times \frac{2}{8} = \frac{1}{8} \]

\[ \frac{4}{2} \times \frac{8}{4} = \frac{1}{2} \]

\[ \frac{3}{2} \times \frac{8}{4} = \frac{1}{1} \]

Individual work, monitored, helped

Differentiation by time limit.

[Only a) is expected from the majority of the class in the 4 minutes allowed; b) is an advanced problem for the more able Ps.]

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid method, but if Ps prefer any of the methods shown to their own method, they can write it in *Ex. Bks*.

If no P used ratio, T shows it and asks class if it is correct.

**BB:**

\[ \frac{1}{4} \times \frac{2}{8} = \frac{1}{8} \]

\[ \frac{4}{2} \times \frac{8}{4} = \frac{1}{2} \]

\[ \frac{3}{2} \times \frac{8}{4} = \frac{1}{1} \]

21 min
### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 78</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4</strong></td>
<td></td>
</tr>
<tr>
<td><strong>PbY6a, page 78</strong></td>
<td></td>
</tr>
</tbody>
</table>
| **Q.2**  | Read: The mass of a solid object is 144.5 g and its volume is 17 cm³.  

   a) What is the mass of 1 cm³ of the material from which the object is made?  
   b) What is the mass of 1 m³ of the same material?  

<table>
<thead>
<tr>
<th>Notes</th>
<th></th>
</tr>
</thead>
</table>
| **Notes** | Individual work, monitored, helped  
Expect part a) from all Ps but part b) only from more able Ps within the given time limit  
Responses shown in unison.  
Reasoning, agreement, self-correcting, praising  
Feedback for T |
| **Extension** | What could the object be? (e.g. a cuboid measuring 17 cm by 1 cm by 1 cm) What material might it be made from? (e.g. metal) Ps could measure and weigh different objects that they think might be close in volume and mass.  
 | | **5** PbY6a, page 78 |
| **Q.3**  | Read: In the 2003 Athletics World Championship in Paris, Felix Sanche (from the Dominican Republic) won the 400 m men's hurdles in a time of 47.25 seconds.  

   a) Joey Woody (from the USA) came second.  
   His time was 1.0197 of the winner's time.  What was Joey Woody's time?  
   b) Periklis Iakovakis (from Greece) was third in a time of 48.24 seconds.  What percentage was this of the winner's time?  

Talk briefly about Paris, France, the Dominican Republic and Greece. Elicit/show their positions on a globe or world map.  
Set a time limit of 3 minutes. Let Ps use calculators.  
Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.  
Solution: e.g.  
a) 17 cm³ → 144.5 g  
1 cm³ → 144.5 g ÷ 17 = 8.5 g (1 million)  
b) 1 m³ = 100 cm × 100 cm × 100 cm = 1 000 000 (cm³)  
1 m³ → 8.5 g × 1 000 000 = 8 500 000 g = 8500 kg  
= 8.5 tonnes

<table>
<thead>
<tr>
<th>Notes</th>
<th></th>
</tr>
</thead>
</table>
| **Extension** | Whole class discussion  
(T could have an object or objects already prepared.) |
| **5** PbY6a, page 78 |                |
| **Q.3**  | Read: In the 2003 Athletics World Championship in Paris, Felix Sanche (from the Dominican Republic) won the 400 m men's hurdles in a time of 47.25 seconds.  

   a) Joey Woody (from the USA) came second.  
   His time was 1.0197 of the winner's time.  What was Joey Woody's time?  
   b) Periklis Iakovakis (from Greece) was third in a time of 48.24 seconds.  What percentage was this of the winner's time?  

Talk briefly about Paris, France, the Dominican Republic and Greece. Elicit/show their positions on a globe or world map.  
Set a time limit of 3 minutes. Let Ps use calculators.  
Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. Ps say answers in sentences.  
Solution: e.g.  
a) 1.0197 of 47.25 sec = 47.25 sec × 1.0197 = 48.18 sec  
   Answer: Joe Woody's time was 48.18 seconds.  

b) 48.24 sec : 47.25 sec = 4824 : 4725  
\[
\frac{4824}{4725} \approx 1.02 \text{ (to 2 d.p.)} \rightarrow 102\% 
\]

   Answer: Periklis Iakovakis's time was about 102% of the winner's time.  

<table>
<thead>
<tr>
<th>Notes</th>
<th></th>
</tr>
</thead>
</table>
| **Extension** | Individual work, monitored, helped  
Expect part a) from all Ps but part b) only from more able Ps within the given time limit  
Responses shown in unison.  
Reasoning, agreement, self-correcting, praising  
Feedback for T |

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Lesson Plan 78

**Notes**

Ps have calculators on desks.

Whole class activity but Ps work in Pbs at same time.

Written on BB or use enlarged copy master or OHP

Discussion about context and meaning of table. Involve several Ps. Ps tell of their own experiences of different currencies.

BB: **currency**
current type of money in use

Discussion, agreement on which operation to use.

Reasoning, agreement, praising

When converting $ (and JPY) to £s, T might show and explain how to use the $ button on the calculator.

BB: \[ \frac{1}{x} \rightarrow \text{reciprocal} \]

At a good pace.

Reasoning, agreement, praising

**Note:**

Ps might have calculators which do not show a sufficient number of decimal digits and might give the result as, e.g. 7.8217795 E-3 which means that each digit should be moved 3 decimal places to the right, i.e. 0.0078217795 \( (\approx 0.00782) \)

(Converting 6 to 5 decimal places is given here but any suitable rounding is acceptable.)

Practice for Ps in using calculators accurately and in rounding results appropriately

---

**Activity 6**

*PbY6a, page 78, Q.4*

Who knows what teletext is? (T explains if necessary or even better, shows it on a television or projects it onto a screen.)

What are exchange rates? When do we use them? Relate to holidays abroad, football transfers between foreign teams, buying and selling goods from other countries, etc.

Ps say what they know about the countries mentioned in the table and show where they are on a world map.

Read: *On the 1st of September 2003, these exchange rates were shown on teletext. Fill in the missing rates. Use a calculator.*

Which column in the table shows the exchange rates? (LH column)

What do they mean? (e.g. If you changed £1 into Euros on that day, you would have got 1.429 Euros in exchange.) Elicit/tell that the type of money used in a certain country at a certain time is called its **currency**.

What do we have to calculate in the middle column? (How much money you would get for 1 American dollar if you changed it into other currencies)

How can we work them out? Ps make suggestions but if no P is correct T explains. Ps do calculation on calculator and agree on an appropriate rounding (e.g. correct to 3 decimal places, as in LH column of table).

BB: 1.567 $ = £1

\[ 1 \text{ $} = \frac{\text{£1}}{1.567} = \text{£0.638} \text{ (correct to 3 decimal places)} \]

\[ 1.567 \text{ $} = \text{£1} = 1.429 \text{ €} \]

\[ 1 \text{ $} = 1.429 \text{ €} \div 1.567 = 0.912 \text{ €} \text{ (to 3 d.p.) etc.} \]

Ps do calculations on calculators and come to BB to write results in the table, saying the whole operation. Class agrees/disagrees. Ps write amounts in tables in Pbs too.

When middle column is completed, ask Ps how to calculate RH column. Agree on an appropriate rounding.

BB: 182.695 JPY = £1

\[ 1 \text{ JPY} = \frac{\text{£1}}{182.695} = \text{£0.00547} \text{ (e.g. to 5 d.p.)} \]

\[ 182.695 \text{ JPY} = \text{£1} = 1.429 \text{ €} \]

\[ 1 \text{ JPY} = 1.429 \text{ €} \div 182.695 = 0.00782 \text{ €} \text{ (to 5 d.p.) etc.} \]

**Solution:**

<table>
<thead>
<tr>
<th>Key: £ = GBP (British Pound), € = Euro, $ = USD (Dollar), JPY = Japanese Yen, CHF = Swiss Franc, SEK = Swedish Krona</th>
</tr>
</thead>
<tbody>
<tr>
<td>£1 = 1.429 €</td>
</tr>
<tr>
<td>£1 = 1.567 $</td>
</tr>
<tr>
<td>£1 = 2.196 CHF</td>
</tr>
<tr>
<td>£1 = 13.111 SEK</td>
</tr>
<tr>
<td>£1 = 182.695 JPY</td>
</tr>
</tbody>
</table>

37 min
### Lesson Plan 78

#### Activity 7

**PbY6a, page 78**

Q.5 Deal with one question at a time. Set a time limit. Ps read question themselves, write a plan, do calculation, and write the answer in a sentence in Ex. Bks.

Review with the whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who agrees? Who did it another way? etc. Mistakes discussed and corrected. T chooses a P to say the answer in a sentence.

Accept any correct method but also show the short methods given below.

**Solution:** e.g.

a) A factory smelts iron from iron ore. Iron makes up only 62% of iron ore. How much iron can the factory smelt from 25.7 tonnes of iron ore?

**Plan:** 62% of 25.7 tonnes = 25.7 t × 0.62 = 15.934 tonnes

**Answer:** The factory can smelt 15.934 tonnes of iron from 25.7 tonnes of iron ore.

b) i) The original price of a machine was £700. The shop reduced the price by 10%, then cut the reduced price by 20%. What does the machine cost now?

**Plan:** 80% of (90% of £700)

\[= 0.8 \times (0.9 \times £700)\]

\[= 0.8 \times £630 = £504\]

**Answer:** The machine now costs £504.

ii) Another shop had the same machine and cut 20% off the £700 first, then cut 10% off the reduced price. Is the machine cheaper in this shop?

**Plan:** 90% of (80% of £700)

\[= 0.9 \times (0.8 \times £700)\]

\[= 0.9 \times £560 = £504\]

**Answer:** No, the machine costs the same in both shops.

c) The price of a television was cut by 10%, then by 10% of the reduced price. The television now costs £243. What was its original price?

**Plan:** Let the original price be \(x\).

\[90\% \text{ of } (90\% \text{ of } x) = £243\]

\[0.9 \times (0.9 \times x) = £243\]

\[0.81 \times x = £243\]

\[x = £243 \div 0.81 = £24300 \div 81 = £300\]

**Answer:** The original price was £300.

---

**Notes**

Individual work, monitored, helped
(or some done with whole class if time is short)

In a) T elicits/explains the context first. (Iron ore occurs naturally and contains a mixture of pure iron and other substances. The iron ore is heated to a high temperature so that the pure metal melts and can be collected separately.)

Results shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

(A reduction of 10% means a selling price of 90%.
A reduction of 20% means a selling price of 80%.)

Agree that:

\[80\% \text{ of } 90\% = 90\% \text{ of } 80\%\]

as \[0.8 \times 0.9 = 0.9 \times 0.8\]

\[\text{or } (£243 \div 0.9) \div 0.9 = (£2430 \div 9) \div 0.9\]

\[= £270 \div 0.9\]

\[= £2700 \div 9 = £300\]
### Activity 1

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **79** is a prime number, Factors: 1, 79
  (as not exactly divisible by 2, 3, 5 or 7 and $11^2 > 79$)
- **254** = $2 \times 127$, Factors: 1, 2, 127, 254
- **429** = $3 \times 11 \times 13$, Factors: 1, 3, 11, 13, 33, 39, 143, 429
- **1079** = $13 \times 83$, Factors: 1, 13, 83, 1079

#### 7 min

### Activity 2

#### Comparison

Which is more? Let’s fill in the missing signs.

Ps come to BB to write the missing sign and explain reasoning. Class agrees/disagrees or points out another way to determine the sign. Where possible, encourage Ps to explain without calculating each side.

BB:

- **a)** $2 \times \frac{3}{4} \quad \geq \quad \frac{2}{3} + \frac{3}{4}$
  
  
  $\frac{3}{4} + \frac{3}{4} > \frac{2}{3} + \frac{3}{4}$, as $\frac{3}{4} > \frac{2}{3}$

- **b)** $\frac{1}{2} - \frac{1}{3} \quad \geq \quad 1 - \frac{5}{6}$
  
  
  $\frac{3}{6} = \frac{1}{2}$, $1 - \frac{5}{6} = \frac{1}{6}$

- **c)** $6 - \frac{1}{6} \quad \geq \quad 5.6$
  
  
  $\frac{5}{6} > \frac{5}{6}$, as $\frac{5}{6} = \frac{25}{30} > \frac{6}{10} = \frac{18}{30}$

- **d)** $0.8 + (0.45 - 0.5) \quad \geq \quad 0.8 + 0.45 - 0.5$ [LHS brackets not needed]

- **e)** $2 - (1.1 - 0.2) \quad \geq \quad 2 - 1.1 - 0.2$ [2 - 1.1 + 0.2 > 2 - 1.1 - 0.2]

- **f)** $12 \times 0.6 \quad \leq \quad 12 \times \frac{2}{3}$
  
  
  $[0.6 < 0.6]$

- **g)** $6 \% \text{ of } £500 \quad \leq \quad 5 \% \text{ of } £600$
  
  
  $[6 \times £5 = 5 \times £6 = £30]$

#### 15 min

### Activity 3

#### PbY6a, page 79

**Q.1** Read: Practise addition and subtraction.

Set a time limit or deal with one row at a time. Ps write results in Exs if they can calculate mentally, or do calculations in Exs Bks if they need more space.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning where necessary. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- **a)** i) $\frac{5}{9} + \frac{2}{9} = \frac{7}{9}$
  
  ii) $\frac{8}{15} - \frac{3}{15} = \frac{5}{15} = \frac{1}{3}$
  
  iii) $\frac{4}{7} + \frac{5}{7} = \frac{9}{7} = \frac{7}{7}$

Individual work, monitored (or whole class activity)

BB: 79, 254, 429, 1079

Calculators allowed.

Reasoning, agreement, self-correction, praising

Whole class activity

Written on BB or use enlarged copy master or OHP

At a good pace

Involve many Ps.

Reasoning, agreement, praising

Extra praise for clever reasoning

Feedback for T

(Revision of operations with fractions and decimals and use of brackets)
Elicit that:
• when adding or subtracting fractions with unequal denominators, change them to equivalent fractions which have the lowest common denominator (i.e. the denominator is the lowest common multiple of the two original denominators);  
• when adding or subtracting mixed numbers, add or subtract the whole numbers first, then add or subtract the fractions;  
• when adding or subtracting decimals, it is easier to write the calculation vertically, with equal place-values lined up.

Individual work, monitored (helped)  
Written on BB or use enlarged copy master or OHP  
Differentiation by time limit  
Reasoning, agreement, self-correction, praising  
Deal with alternative methods as in b) i).

Elicit that:  
• to multiply a fraction by a whole number, multiply the numerator or, where possible, divide the denominator;  
• to divide a fraction by a whole number, divide the numerator where possible, or multiply the denominator;  
• to multiply a mixed number by a whole number, multiply the whole number first, then multiply the fraction, or first write as a single fraction;  
• to multiply a fraction by a fraction, first simplify where possible, then multiply the numerators and multiply the denominators  
• to divide a fraction by a fraction, multiply by the divisor’s reciprocal value.
### Activity 4 (Continued)

| e) i) $3 \div \frac{4}{5} = 3 \times \frac{5}{4} = \frac{15}{4} = 3\frac{3}{4}$  
   | ii) $2 \frac{1}{5} \times \frac{1}{2} = \frac{11}{5} \times \frac{11}{2} = \frac{121}{10} = 12\frac{1}{10}$  
   | iii) $9 \div \frac{2}{3} = 9 \times \frac{3}{11} = \frac{27}{11} = 2\frac{5}{11}$  
   | iv) $5 \frac{1}{7} \div \frac{3}{5} = \frac{36}{7} \times \frac{47}{14} = \frac{72}{47} = 1\frac{25}{47}$  
   | f) i) $0.8 \times 0.3 = 0.24$  
   | ii) $2.4 \div 0.3 = 8$  
   | iii) $11.4 \times 0.7 = 7.98$  
   | iv) $0.84 \div 1.2 = 0.7$ |

### Extension

**PbY6a, page 79**

**Q.3** Deal with one question at a time. Set a short time limit.

Ps read question themselves, solve it in Ex. Bks, then show result on scrap paper or slates on command.

Ps with different answers explain reasoning at BB. Class points out mistakes and agrees on the correct answer. Who thought the same? Who worked it out in another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

**a) Calculate:**

\[
\left(\frac{14}{4} - \frac{3}{5}\right) \div \frac{1}{7} = \left(\frac{5 - 16}{20}\right) \div \frac{8}{7} = \left(\frac{5 - 1}{20}\right) \times \frac{7}{8} = \frac{19}{20} \times \frac{7}{8} = 4\frac{93}{160} = 4\frac{53}{160}
\]

**b) Which decimal is an equal distance from both $-2\frac{1}{2}$ and $\frac{1}{2}$ on the number line?**

By calculation: $\left[\frac{1}{2} + (-2\frac{1}{2})\right] \div 2 = -2 \div 2 = -1$.

The only possible number is $-1$, which is a whole number but it can be written in decimal form as $-1.0$.

**c) What is the price of 1 kg of apples if the price of $\frac{1}{2}$ g is £3.20?**

**Plan:** £3.20 ÷ 2.5 = £32 ÷ 25 = £1.28

or £3.20 ÷ 5 ÷ 2 = £0.64 × 2 = £1.28

**Answer:** The price of 1 kg of apples is £1.28.

---

**Notes**

- to multiply or divide by a mixed number, first change it to a single fraction, then multiply or divide as normal;
- to multiply a decimal by a decimal, do the multiplication as if they were both whole numbers, then write the decimal point in the product so that it has the same number of decimal digits as the two factors combined;
- to divide by a decimal, increase the dividend and divisor by the same number of times so that the divisor is a whole number.
Activity

5 (Continued)

d) Linda had £500 in her bank account. She spent 18% of it. How much money does she have left?

Plan: Spent: 18%, so has left: 100% – 18% = 82%

82% of £500 = £500 × 0.82 = £5 × 82 = £410

Answer: Linda has £410 left.

Lesson Plan 79

Notes

or Spent: 18% of £500

= £500 × 0.18

= £90

Has left: £500 – £90 = £410

Individual work, monitored, helped

Results shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Feedback for T

or

First spent:

\[ \frac{2}{5} \text{ of } £2000 = £800 \]

Then spent:

\[ \frac{3}{4} \text{ of } £1200 = £900 \]

£2000 – £800 – £900 = £300

or

\[ \frac{5}{6} \rightarrow £1400 \]

\[ \frac{1}{6} \rightarrow £1400 \div 5 = £280 \]

\[ \frac{6}{6} \rightarrow £280 \times 6 = £1680 \]

or

\[ \frac{3}{4} \text{ of } £2400 = £1800 \]

£1800 ÷ 2 × 3 = £2700

PbY6a, page 79

Q. 4 Deal with one question at a time. Set a time limit. Ps read question themselves, write a plan, do calculation and write the answer as a sentence in Ex. Bks.

Review with the whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who agrees? Who did it another way? etc. Mistakes discussed and corrected. T chooses a P to say the answer in a sentence.

Accept any correct method but also show the short methods given below.

Solution: e.g.

a) I had £2000. First I spent 2 fifths of it, then I spent 3 quarters of what was left. How much money do I have now?

Plan: Have left: \( \frac{1}{4} \) of (\( \frac{3}{5} \) of £2000) = \( \frac{1}{4} \times \frac{3}{5} \times £2000 \)

= \( \frac{3}{20} \times £2000 = £300 \)

Answer: I now have £300.

b) The sum of 1 third of my money and half of my money is £1400. How much money do I have?

Plan: \( \frac{1}{3} + \frac{1}{2} = \frac{2 + 3}{6} = \frac{5}{6} \rightarrow £1400 \)

Whole amount: £1400 ÷ \( \frac{5}{6} \) = £1400 × \( \frac{6}{5} \)

= £1680

Answer: I have £1680.

c) Two thirds of my money is the same as 3 quarters of Joe’s money. If Joe has £2400, how much do I have?

Plan: Let my money be \( x \).

\( \frac{2}{3} \) of \( x \) = \( \frac{3}{4} \) of £2400

\( \frac{2}{3} \times x = \frac{3}{4} \times £2400 \)

\( \frac{600}{4} \)

\( x = £1800 \div \frac{2}{3} = £1800 \times \frac{3}{2} = £2700 \)

Answer: I have £2700.

35 min

41 min
### Lesson Plan 79

**Notes**

Whole class activity
(or individual work under a

time limit if Ps wish)

Involve many Ps.

At a good pace

Reasoning, checking,

greement, praising

Feedback for T

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| a) | $1 \frac{2}{5} + x = 4$, $x = 4 - \frac{2}{5} = 3 - \frac{2}{5} = \frac{23}{5}$
| Check: | $1 \frac{2}{5} + 2 \frac{3}{5} = 4$ ✓
| b) | $y - 2.91 = 3.3$, $y = 3.3 + 2.91 = 6.21$
| Check: | $6.21 - 2.91 = 3.3$ ✓
| c) | $u \times \frac{2}{3} = 1 \frac{3}{4}$, $u = \frac{13}{4} \div \frac{2}{3} = \frac{7}{4} \times \frac{3}{2} = \frac{21}{8} = 2 \frac{5}{8}$
| Check: | $2 \frac{5}{8} \times \frac{2}{3} = \frac{16}{8} \times \frac{1}{3} = \frac{7}{4} = 1 \frac{3}{4}$ ✓
| d) | $v \div 1.5 = 6.3$, $v = 6.3 \times 1.5 = 6.3 + 3.15 = 9.45$
| Check: | $9.45 \div 1.5 = 94.5 \div 15 = 18.9 + 3 = 6.3$ ✓
| e) | $4 \frac{1}{5} \div t = 6 \frac{2}{5}$
| $t = 4 \frac{1}{5} \div 6 \frac{2}{5} = \frac{21}{5} \div \frac{32}{5} = 21 \times \frac{5}{32} = \frac{21}{32}$
| Check: | $4 \frac{1}{5} \div 21 \frac{32}{5} = \frac{21}{5} \div \frac{32}{21} = \frac{32}{5} = 6 \frac{2}{5}$ ✓
| f) | $2 \times z + 3 \times z = 12.5$, $5 \times z = 12.5$, $z = 12.5 \div 5 = 2.5$
| Check: | $2 \times 2.5 + 3 \times 2.5 = 5 + 7.5 = 12.5$ ✓

---

**Activity 7**

*PhY6a, page 79, Q.5*

Read: *Solve the equations.*

Ps come to BB to calculate solutions, explaining reasoning. Class checks by mentally substituting the resulting value for the letter.

**Solution:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
</table>
| a) | $1 \frac{2}{5} + x = 4$, $x = 4 - \frac{2}{5} = 3 - \frac{2}{5} = \frac{23}{5}$
| Check: | $1 \frac{2}{5} + 2 \frac{3}{5} = 4$ ✓
| b) | $y - 2.91 = 3.3$, $y = 3.3 + 2.91 = 6.21$
| Check: | $6.21 - 2.91 = 3.3$ ✓
| c) | $u \times \frac{2}{3} = 1 \frac{3}{4}$, $u = \frac{13}{4} \div \frac{2}{3} = \frac{7}{4} \times \frac{3}{2} = \frac{21}{8} = 2 \frac{5}{8}$
| Check: | $2 \frac{5}{8} \times \frac{2}{3} = \frac{16}{8} \times \frac{1}{3} = \frac{7}{4} = 1 \frac{3}{4}$ ✓
| d) | $v \div 1.5 = 6.3$, $v = 6.3 \times 1.5 = 6.3 + 3.15 = 9.45$
| Check: | $9.45 \div 1.5 = 94.5 \div 15 = 18.9 + 3 = 6.3$ ✓
| e) | $4 \frac{1}{5} \div t = 6 \frac{2}{5}$
| $t = 4 \frac{1}{5} \div 6 \frac{2}{5} = \frac{21}{5} \div \frac{32}{5} = 21 \times \frac{5}{32} = \frac{21}{32}$
| Check: | $4 \frac{1}{5} \div 21 \frac{32}{5} = \frac{21}{5} \div \frac{32}{21} = \frac{32}{5} = 6 \frac{2}{5}$ ✓
| f) | $2 \times z + 3 \times z = 12.5$, $5 \times z = 12.5$, $z = 12.5 \div 5 = 2.5$
| Check: | $2 \times 2.5 + 3 \times 2.5 = 5 + 7.5 = 12.5$ ✓

---

T reminds Ps that when a letter is used to represent an unknown number, the multiplication sign can be omitted:

**BB:** $2 \times z + 3 \times z$

$= 2z + 3z = 5z$
Factorising 80, 255, 430 and 1080. Revision, activities, consolidation

PbY6a, page 80

**Notes**

80 = $2^4 \times 5$
Factors: 1, 2, 4, 5, 8, 10, 16, 20, 40, 80

255 = $3 \times 5 \times 17$
Factors: 1, 3, 5, 15, 17, 51, 85, 255

430 = $2 \times 5 \times 43$
Factors: 1, 2, 5, 10, 43, 86, 215, 430

1080 = $2^3 \times 3^3 \times 5$
Factors: 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 27, 40, 45, 54, 60, 72, 90, 108, 120, 135, 180, 216, 270, 360, 540, 1080

(or set factorising as homework at the end of Lesson 79 and review at the start of Lesson 80)

### Q.1

<table>
<thead>
<tr>
<th>Number</th>
<th>Whole Unit</th>
<th>1% of it</th>
<th>21% of it</th>
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### Q.2

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<td>27,000</td>
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<td>72,000</td>
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</tr>
</tbody>
</table>

### Q.3

a) $\frac{5}{6}$ of 45.6 kg = $45.6 \div 6 \times 5 = 7.6 \times 5 = 38$ kg

b) 70% of 45.6 kg = $45.6 \times 0.7 = 31.92$ kg

c) $\frac{5}{8}$ → 450 m, $\frac{8}{8}$ → $450 \div \frac{5}{8} = \frac{90}{8} \times \frac{8}{5} = 720$ m

d) 62.5% → 450 m,

Whole amount: $450 \div 0.625 = 450000 \div 625 = 720$ m

(Extra praise for Ps who realise that 62.5% → $\frac{5}{8}$; so d) is actually c) in percentage form.)

### Q.4

a) 25% → £81, 100% → £81 × 4 = £324

b) $66\frac{2}{3}$% → 120 kg, 100% → $120 \div \frac{2}{3} = 180$ kg

c) 125% → 12.5 km, 100% → $12.5 \div 1.25 = 10$ km

d) 200% → £47, 100% → £47 ÷ 2 = £23.50

e) 19% → 95 m, 100% → $95 \div 0.19 = 9500 \div 19 = 500$ m

f) 140% → 210 km,

100% → $210 \div 1.4 = 2100 \div 14 = 150$ km

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**Erratum**

In d) in PbY6a: ‘£47 kg’ should be ‘£47’
Solutions (Continued)

Q.5  

a) \( \frac{3}{4} + x = 5 \), \( x = 5 - \frac{3}{4} = 2 \frac{1}{4} \)

b) \( y + 2.81 = 3.21 \), \( y = 3.21 - 2.81 = 0.4 \)

c) \( u \times \frac{4}{3} = 6 \frac{2}{3} \), \( u = \frac{20}{3} \div \frac{1}{3} = \frac{20}{3} \times 3 = \frac{20}{1} = 5 \)

d) \( v \div 0.5 = 4.7 \), \( v = 4.7 \times 0.5 = 2.35 \)

e) \( 2 \frac{1}{4} \div t = 6 \frac{3}{4} \), \( t = 2 \frac{1}{4} \div \frac{3}{4} = \frac{9}{4} \times \frac{4}{1} \times \frac{1}{3} = \frac{1}{3} \)

f) \( 3 \times z + 0.5 \times z = 1.4 \), \( 3.5 \times z = 1.4 \), \( \frac{z}{3.5} = 1.4 \), \( z = 1.4 \div 3.5 \) = \( 2 \div 5 = 0.4 \) (or = \( \frac{2}{5} \))

Q.6  GONE TO WATCH GLADIATORS. BACK AT SEVEN.

To get the original message, each letter in the code needs to be replaced by the letter which is 3 places before it.


The original message was encoded using a shift code of + 3.

Ps could write the letters of the alphabet on two strips of card and make up their own codes using different 'shifts', then ask other Ps to decode them.

Check by substituting solution for letter in the equation to see whether it makes the equation true.