**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(81 = 3 \times 3 \times 3 \times 3 = 3^4\) (so a square number)
  
  Factors: 1, 3, 9, 27, 81

- \(256 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^8\)
  
  (= \(16 \times 16 = 16^2\), so it is a square number)
  
  Factors: 1, 2, 4, 8, 16, 32, 64, 128, 256

- \(431\) is a prime number
  
  Factors: 1, 431

(as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23 and \(29 \times 29 > 431\))

- \(1081 = 23 \times 47\)
  
  Factors: 1, 23, 47, 1081

**Notes**

Individual work, monitored (or whole class activity)

BB: 81, 256, 431, 1081

Calculators allowed.

Reasoning, agreement, self-correction, praising

- \(81\)
  
  Draw a line.

- \(256\)
  
  Draw a straight line.

- \(431\)
  
  a) i) Draw a curved line. BB: e.g.

  ii) Draw a straight line.

  b) Draw and label appropriately:

  i) a straight line

  ii) a half line or ray.

  iii) a line segment

  c) Draw a line, \(e\), and mark a point, \(A\), on it.

  Draw a line perpendicular to line \(e\) at point \(A\).

  How can we show that they are perpendicular? (Mark the angle with a square.)

  d) Draw and label 2 parallel lines 3 cm apart.

  How can we show that they are parallel? (Mark with single arrows.)

  e) i) Draw two lines, \(e\) and \(f\), which intersect one another.

  ii) Draw two line segments, \(AB\) and \(CD\), which intersect one another.

  What do you notice about the angles formed? (The opposite angles are equal.)

Quick individual activities but the whole class kept together T monitors, helps, corrects.

After each drawing, Ps say what they know, or T elicits:

- straight lines extend in both directions to infinity and the ends never meet.

- lines are usually labelled with small letters and points with capital letters.

- a ray extends from a point to infinity in one direction.

- line \(f\) extends beyond \(A\)

- each of the 4 angles formed is \(90^\circ\).

- the distance between two lines is the perpendicular distance between them; parallel lines never meet, however far they are extended.

- intersect means ‘cut or ‘cross’.

Ps mark the equal angles on BB and on own drawings.

(1 arc for 1st pair, 2 arcs for 2nd pair)
Activity 2
(Continued)
f) Draw: i) an \textbf{acute} angle  
\begin{center}
\begin{tikzpicture}
\draw[->] (0,0) -- (3,0);
\draw[->] (0,0) -- (0,3);
\draw[->] (3,0) -- (3,3);
\end{tikzpicture}
\end{center}
ii) a \textbf{right} angle,
iii) an \textbf{obtuse} angle
iv) a \textbf{reflex} angle.

T asks Ps for examples of sizes of angles for each type, then elicits their limits.

\begin{center}
BB:\begin{align*}
0^\circ &< \text{acute angle} < 90^\circ, \\
90^\circ &< \text{obtuse angle} < 180^\circ, \\
180^\circ &< \text{reflex angle} < 360^\circ
\end{align*}
\end{center}

Notes
Elicit that angles are usually labelled with Greek letters.
$\alpha$ (alpha), $\beta$ (beta), $\gamma$ (gamma), $\delta$ (delta)

Which types of angles are missing?
(null angle = $0^\circ$, straight angle = $180^\circ$, whole angle = $360^\circ$)

24 min

Reflection
In a PE lesson, Ps were put in pairs and told how to stand in a certain relationship with one another. What could the relationship be? Where should the missing pupil from each pair stand?

Ps study the diagram to find the 'rule' then come to BB to mark the missing points. Class agrees/disagrees. Elicit what the rule is and the main points about it. (T draws the dotted lines and squares on diagram.)

BB:
\begin{itemize}
  \item a) $A'\overline{A}$
  \item b) $E_1\overline{F_1}$
  \item c) $A'\overline{A}$
  \item d) $P'\overline{P}$
\end{itemize}

\textit{Reflection in the line $a$.}
A and its \textit{mirror image} $A'$ are the same perpendicular distance on line $e$, from line $a$.

\textit{Reflection in the point C.}
E and its mirror image, $E'$, are an \textit{equal} distance from point C on line $e$.

Translation by the same distance to the right and up (or by the same angle) each time.
The lines $PP'$, $QQ'$, $RR'$ and $SS'$ are all parallel.

24 min
**Activity 4**

*PbY6b page 81. Q.1*

Read: *What has been done to Triangle 1 to form the other shapes? Describe each transformation in your exercise book*

Ps come to BB to point out the relevant pairs of triangles and explain the transformations. Class agrees/disagrees. Ask Ps to draw the mirror lines or points of reflection, to say the scale of enlargement or reduction and to give the angle of rotation where relevant.

Agree on a good way to list the transformations in Ex. Bks.

T suggests the form below if no P has an idea. After agreement, Ps write the transformation in Ex. Bks.

**Extension**

Ask Ps to point out other relationships too (e.g. 7, 8, 9 and 10) and to show congruent and similar triangles.

**Solution:**

1 → 2: translation (4, 0)
1 → 3: reduction, reflection, translation
1 → 4: enlargement (2:1), translation
1 → 5: reflection in line e
1 → 6: reflection in point C
1 → 7: reflection in line f
1 → 8: reflection in horizontal axis, rotation by 90°, translation
1 → 9: reflection in vertical axis and rotation by –90° (i.e. clockwise)
1 → 10: reflection in line g, then translation by (16, 0)
1 → 11: stretch vertically (same width but 3 times as high)

**Lesson Plan 81**

**Notes**

Whole class activity

Draw on BB or use enlarged copy master or OHP

If possible, T has *Triangle 1* cut out to show the actual movements to the class.

At a good pace.

Discussion, reasoning, agreement, demonstration with model where possible

Elicit that in a reflection, the corresponding points on the original shape and its mirror image are the same perpendicular distance from the mirror line or axis.

Praising, encouragement only

Expect only the name of each transformation from less able Ps, but some details from the more able Ps. e.g.

1 → 3: we say that the ratio of reduction is 1 : 2 (the value of the image is given first); or we say that it is a reduction by scale factor 1 half, i.e. triangle 3 is half the size of triangle 1;
1 → 4: we say that the ratio of enlargement is 2:1, or it is enlargement by scale factor 2.

**Extension**

*PbY6b page 81. Q.2*

Read: *Draw the lines of symmetry and mark the centres of rotation.*

Set a time limit of 3 minutes. Expect only rough freehand drawing first and then discuss afterwards with the whole class how to draw the lines of symmetry and mark the centres of rotation accurately.

Review with whole class. Ps come to BB to draw, mark and label. Class agrees/disagrees. If no P shows the accurate method, demonstrate with BB compasses and ruler, involving Ps where possible.

**Solution:**

a) 

b) 

c) 

d) 

e) Not symmetrical!

f) 

e) The dog looks symmetrical at first sight but on closer inspection, the RHS of the dog is not exactly a mirror image of the LHS.

**Individual work, monitored, helped**

Use enlarged copy master or OHP

Discussion, agreement, self-correction, praising

Extra praise for Ps who remember how to construct the lines/points accurately.

(Instructions given on following page.)

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**Activity**

To Ts:

To draw a line of symmetry

Choose a point on the shape and its corresponding mirror image.

Set compasses to the distance between them. Put the compasses on one chosen point and draw arcs above and below it. Repeat with the other chosen point. Draw a straight line through the points where the arcs cross. This is the line of symmetry.

It is the perpendicular bisector of the line between the two chosen points (i.e. it is at right angles to it and cuts it in half).

To mark a centre of rotation

Choose a point on the shape and its corresponding mirror image.

Construct the perpendicular bisector of the line between them.

Repeat with another pair of corresponding points. (If necessary, extend the perpendicular bisectors so that they intersect.) The point of intersection is the centre of rotation.

It is an equal distance from both points in any corresponding pair.

---

**Notes**

Individual work, monitored, helped.

Grid drawn on BB or use enlarged copy master or OHP.

Ps could have copies of copy master grid too.

(Or T could have shape already prepared on OHT)

Discussion, agreement, self-correction, praising.

Images of the original shape are labelled appropriately.

e.g.

ABCDE
A'B'C'D'E'
A''B''C''D''E''
A*B*C*D*E*  

If preferred, in part b) Ps could use a different grid for each new image, so that the labelling is clearer.
(Continued)

b) Read: Change the coordinates of the points according to the instructions and draw the new shapes. Describe how the original pentagon's shape and size changes.

Deal with one part at a time. Set a short time limit. Ps first write the coordinates of the new points in Ex. Bks then draw the new shape and label its vertices appropriately.

T chooses Ps to work on grid on BB or OHT. Ps compare their drawing with the one on BB and any mistakes are discussed and corrected.

Elicit the type of transformation used.

Solution: (Diagrams shown on previous page)

i) Keep the x coordinate the same and multiply the y coordinate by (–1).
   
   \[ \begin{align*}
   A' & (–3, –2), \\
   B' & (0, –2), \\
   C' & (1, –3), \\
   D' & (1, –4), \\
   E' & (–3, –4)
   \end{align*} \]

   Transformation: Reflection in the x axis.

ii) Subtract 4 from both coordinates.

   \[ \begin{align*}
   A'' & (–7, –2), \\
   B'' & (–4, –2), \\
   C'' & (–3, –1), \\
   D'' & (–3, 0), \\
   E'' & (–7, 0)
   \end{align*} \]

   Transformation: Translation by (–4, –4).

iii) Multiply both coordinates by (–1).

   \[ \begin{align*}
   A''' & (3, –2), \\
   B''' & (0, –2), \\
   C''' & (–1, –3), \\
   D''' & (–1, –4), \\
   E''' & (3, –4)
   \end{align*} \]

   Transformation: Reflection in the origin, i.e. the point (0, 0).

iv) Multiply both coordinates by 2.

   \[ \begin{align*}
   A* & (–6, 4), \\
   B* & (0, 4), \\
   C* & (2, 6), \\
   D* & (2, 8), \\
   E* & (–6, 8)
   \end{align*} \]

   Transformation: Enlargement (2 : 1, or by scale factor 2)

v) Divide both coordinates by (–2).

   \[ \begin{align*}
   A* & (1.5, –1), \\
   B* & (0, –1), \\
   C* & (–0.5, –1.5), \\
   D* & (–0.5, –2), \\
   E* & (1.5, –2)
   \end{align*} \]

   Transformation: Reduction (1 : 2, or by scale factor 1 half)

c) Read: List the similar shapes.

   What are similar shapes? (They are the same shape but not necessarily the same size.) Elicit the symbol for 'similar to' (~) and agree that all the 6 shapes are similar to one another.

d) Read: List the congruent shapes.

   What are congruent shapes? (The same shape and size.) Elicit the symbol for 'congruent to' (≅). Ps come to BB or dictate to T. Class agrees/disagrees.
### Activity 7

**PbY6b, page 81**

Q.4 Read: *Draw the lines of symmetry and mark the centres of rotation.*

Set a time limit of 2 minutes. Ps can draw freehand or use rulers.

Review quickly with whole class. Ps come to BB to draw lines and mark dots. Class agrees/disagrees. T could have the shapes cut out for demonstration by folding or turning in case there is disagreement.

Which of the shapes are polygons? (a, b, c, d and h)

**Solution:**

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="shape-a" /></td>
<td><img src="image2" alt="shape-b" /></td>
<td><img src="image3" alt="shape-c" /></td>
<td><img src="image4" alt="shape-d" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>e)</th>
<th>f)</th>
<th>g)</th>
<th>h)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image5" alt="shape-e" /></td>
<td><img src="image6" alt="shape-f" /></td>
<td><img src="image7" alt="shape-g" /></td>
<td><img src="image8" alt="shape-h" /></td>
</tr>
</tbody>
</table>

45 min

### Notes

Individual work, monitored (or whole class activity if time is short)

Drawn (stuck on BB) or use enlarged copy master or OHP

Discussion, agreement, self-correction, praising

Elicit that the centre of rotation within a shape is the point where the lines of symmetry intersect.
Lesson Plan 82

Notes

Individual work, monitored (or whole class activity)
BB: 82, 257, 432, 1082
Calculators allowed.
Reasoning, agreement, self-correction, praising
e.g.

| 432 | 2 |
| 216 | 2 |
| 1082 | 2 |
| 541 | 541 |
| 27 | 3 |
| 3 | 3 |
| 1 |

6 min

2 Translation and rotation

a) Translation

1. Draw around the triangular shape on the left-hand side of one of the white sheets of paper and also on the tracing paper.
2. Place the tracing paper over the white sheet so that the two shapes line up exactly.
3. Slide the tracing paper to the right and up (about 8cm) over the white sheet but without turning it.
4. Pierce the vertices of the shifted triangle through the tracing paper, then draw its new position on the white sheet.
5. Very lightly draw the path of each vertex by joining up the corresponding vertices using a ruler.
6. Using the original card shape, show the movement again.

What kind of movement is it? (Translation)

What can you say about a translation? (e.g. the shape moves in the same plane; it moves in a straight line, it does not turn.)

Let’s label the vertices and sides of the two triangles on the white sheet. Ps dictate the labels to T and also label their own shapes.

(e.g. \(A'\) is the image of \(A\), \(B'\) is the image of \(B\), etc.)

BB: Translation

<table>
<thead>
<tr>
<th>A'</th>
<th>B'</th>
</tr>
</thead>
<tbody>
<tr>
<td>a'</td>
<td>b'</td>
</tr>
<tr>
<td>a</td>
<td>b</td>
</tr>
</tbody>
</table>

Who can tell me true statements about the translations?

How could we write it using mathematical notation?

(e.g. \(a = a'\), \(a' \parallel a\), \(\angle A = \angle A'\), \(\angle B'\), \(AA' = BB' = CC'\), \(ABC \cong A'B'C'\), etc.)

T: We say that \(AA'\), \(BB'\) and \(CC'\) are vectors because they show a movement by a certain distance in a certain direction.

Extension

Whole class activity but individual drawing and manipulating shapes
On desks, Ps have a right-angled triangle cut from thick coloured card, 2 sheets of plain white paper, a sheet of tracing paper, a ruler and a pair of compasses.

T gives instructions and monitors, helps, corrects.
Demonstrate on BB too.

Extra praise if a P can write it on BB.

Praising
(Continued)

b) **Rotation**

1. Draw around the coloured triangle on the other white sheet.
2. Place the tracing paper over the white sheet so that the shapes line up exactly.
3. Pierce the two sheets of paper with the pointed arm of your compasses at a point below the shapes and without letting go of your compasses, label the point O.
4. Turn just the tracing paper around point O by about $60^\circ$.
5. Pierce the vertices of the triangle on the tracing paper so that its new position is marked, then draw the triangle on the white sheet.
6. Check that you are correct by repeating the turn with the tracing paper.
7. Very lightly, using your compasses, draw the path of each vertex on the white sheet.
8. Using the original card shape, show the movement again.

What kind of movement is it? **(Rotation)**

What can you say about a rotation? (e.g. the shape turns around a certain point in a plane; corresponding vertices stay an equal distance from that point)

Let's label the vertices and sides of the two triangles on the white sheet. Ps dictate the labels to T and also label their own shapes.

d. e.g. $A \rightarrow A', B \rightarrow B'$, etc.)

**BB:** **Rotation**

Who can tell me true statements about the rotation? T writes them on BB using mathematical notation.

(e.g. $a = a'$, $\angle A = \angle A'$, $OA = OA'$, $ABC \cong A'B'C'$, etc.)

T shows translations and rotations with other shapes on BB or OHT and Ps say which type of transformation it is.

---

**Notes**

T gives instructions and monitors, helps, corrects.
Demonstrate on BB too using BB compasses and BB ruler.

Praising, encouragement only

**BB:** **Rotation**

motion in an arc around a central point

Involve several Ps.

Praising only

Class shouts out in unison.

---

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3  

**Activity**  

**PbY6b, page 82**  

Q.1 Read: *A boat sailed from one bank of the river to the opposite parallel bank, staying perpendicular to both banks during the crossing.*  

*This drawing shows the positions of the boat seen from above at equal intervals of time. The arrow shows the direction in which the river was flowing.*  

_Complete the drawing._  

Think about what we have been doing and what you could do first to help you draw the boats. I will give you 2 minutes.  

Review with whole class. P comes to BB to complete the drawing, explaining what he or she is doing. Who did the same? Who did it another way? Come and show us. Which method do you think is best? Why?  

Agree that it is easier and more accurate to draw the paths of the 3 vertices lightly in pencil first, then to draw the boats.  

What kind of transformation is it? *(Translation)*  

**Solution:**

![Diagram](image)

4  

**Activity**  

**PbY6b, page 82**  

Q.2 a) Read: *Rotate trapezium ABCD by 60° around the point O in a clockwise direction and show its route on the triangular grid.*  

First elicit that the grid lines form angles of 60° and in a rotation, the corresponding points on the shape and its image are the same distance from the centre of rotation.  

Set at time limit. Ps use protractors to measure the angle of turn and compasses to draw the rotation arcs. Ps label the image appropriately.  

Review with whole class. Ps come to BB or draw the rotation and label it. Class agrees/disagrees. Mistakes discussed and corrected.  

**Solution:**

![Diagram](image)
## Y6

### Activity
4

(Continued)

b) Read: *Complete the statements.*

Set a time limit of 1 minute. Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

Agree that in a rotation, the shape does not change – it moves in an arc in the plane around a centre point O.

**Solution:**

\[
\begin{align*}
A'B' &= AB \quad d' = a \\
B'C' &= BC \quad b' = b \\
C'D' &= CD \quad c' = c \\
D'A' &= DA \quad a' = d \\
\angle B' &= \angle B \\
\angle C' &= \angle C \\
B'D' &= BD \quad A'C' = AC \quad \text{(diagonals)} \\
A'B'C'D' &\cong ABCD \quad \text{(congruent)}
\end{align*}
\]

---

<table>
<thead>
<tr>
<th>5</th>
<th><strong>PbY6b, page 82</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.3 Read:</td>
<td></td>
</tr>
</tbody>
</table>

- **a)** Draw these rectangles in your exercise book.
- **b)** List the similar rectangles.
- **c)** List the congruent rectangles.

Set a time limit. Ps should use rulers to draw the rectangles. Ask quicker Ps to calculate the perimeter and area of each rectangle.

Review with whole class. T has rectangles already prepared or Ps finished early could have drawn them on squared BB or grid on OHT. Ps compare them with their own drawings and correct any mistakes.

Elicit the general formulas for perimeter and area then Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes corrected. Ps dictate the similar and congruent rectangles.

**Solution:** (actual size has been reduced)

\[
\begin{align*}
i) \quad P &= 7 \text{ cm} \\
& A = 3 \text{ cm}^2 \\
a &= 2 \text{ cm} \\
b &= 1.5 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
ii) \quad P &= 21 \text{ cm} \\
& A = 27 \text{ cm}^2 \\
a &= 6 \text{ cm} \\
b &= 4.5 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
iii) \quad P &= 15 \text{ cm} \\
& A = 14 \text{ cm}^2 \\
a &= 6 \text{ cm} \\
b &= 3.5 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
iv) \quad P &= 14 \text{ cm} \\
& A = 12 \text{ cm}^2 \\
a &= 6 \text{ cm} \\
b &= 4 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
v) \quad P &= 7 \text{ cm} \\
& A = 3 \text{ cm}^2 \\
a &= 2 \text{ cm} \\
b &= 1.5 \text{ cm}
\end{align*}
\]

\[
\begin{align*}
vii) \quad P &= 14 \text{ cm} \\
& A = 10 \text{ cm}^2 \\
a &= 5 \text{ cm} \\
b &= 2 \text{ cm}
\end{align*}
\]

---

<table>
<thead>
<tr>
<th>30 min</th>
<th>38 min</th>
</tr>
</thead>
</table>

---

**Notes**

Agreement, self-correction, praising

Ps point out the relevant components on the diagram on BB.

Ps might point out that the two shapes are also similar. 

A'B'C'D' ~ ABCD

---

Individual work, monitored, helped
Ps use squared Ex. Bks or squared 1 cm or 5 mm grid sheets.

Discussion, agreement, self-correction, praising

BB: \( P = 2 \times (a + b) \)

\[ A = a \times b \]

Similar rectangles:

i) ~ ii) ~ iv) ~ v)

Ask Ps for the ratio of the sides in pairs of similar shapes. e.g.

i) : ii) = 1 : 3

iv) : v) = 2 : 1

[Note that ii) and v) are similar even though the sides are not named respectively.]

Congruent rectangles:

i) \( \cong \) v)

[in v) the values of \( a \) and \( b \) have been exchanged, but the shape is still congruent to i)]

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Q.4 Read: A sprinkler was moved 60 m E from its 1st position to its 2nd position, then 30 m SW from its 2nd position to its 3rd position.

a) On the sketch, draw the direct route between its 1st and 3rd positions.

b) Measure this distance on the sketch and calculate its real length in metres.

T asks a P to come to BB to explain what has been done and what is required to be done in his or her own words, referring to the diagram as necessary.

What kind of transformation will the direct route be? (translation)

Set a time limit of 4 minutes. Ps use rulers, or rulers and compasses, to draw and measure.

Review with whole class. Ps show solutions on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected. Agree on the scale for the drawing, then Ps write it beside the diagram in Pbs.

Solution:

| Scale: 1 cm → 10 m |

```
1st position       2nd position
60 m       ↓
            3rd position
≈ 44 m
```

Length of direct route on diagram = 4.4 cm
Length of direct route in real life = 44 m

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**Y6**

<table>
<thead>
<tr>
<th>Activity 1</th>
</tr>
</thead>
</table>

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **83** is a prime number Factors: 1, 83
  (as not exactly divisible by 2, 3, 5, 7 and $11 \times 11 > 83$)
- **258** = $2 \times 3 \times 43$ Factors: 1, 2, 3, 6, 43, 86, 129, 258
- **433** is a prime number Factors: 1, 433
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and $23^2 > 433$)
- **1083** = $3 \times 19 \times 19 = 3 \times 19^2$
  Factors: 1, 3, 19, 57, 361, 1083

**Notes**

Individual work, monitored (or whole class activity)

<table>
<thead>
<tr>
<th>BB: 83, 258, 433, 1083</th>
</tr>
</thead>
</table>

Calculators allowed.

Reasoning, agreement, self-correction, praising

e.g.

```
258  2
129  3
43
43
```

```
1083  3
361  19
19
19
1
```

**Whole class activity,**

Initial discussion about the two shapes. Praising only

Mark positions beforehand.

Each solid is made from two cuboids stuck together (card, wood or multilink cubes).

Involve as many Ps as possible in the activity.

Agreement, praising

Ps might suggest measuring the distance between each solid and the mirror to check that they are the same.
### Activity 2 (Continued)

- **d)** T asks a P to hold his or her left hand in front of the mirror.
  - What do you notice? (It’s as if I am looking at my right hand.)
  - How could you see a left hand in the mirror? (Hold my right hand in front of the mirror.)
  - Could a right hand be the mirror image of a right hand? (No, that is impossible.)

- **e)** Ask Ps to position a pair of shoes (or other solids) so that they are mirror images of one other.

### Reflection 2

Ps have two congruent rectangles (one of white paper and one of tracing paper) on desks. T gives instructions and demonstrates with a large model and Ps follow.

1. Fold each rectangle in half and crease the fold.
2. Draw the letter 'L' on the RH side of the white paper and colour it. Label it A.
3. Lay the tracing paper exactly over the white paper, trace the 'L' and colour it.
4. Cut halfway along the fold from the top of the white paper down and from the bottom of the tracing paper up and fit them together. Label the fold line \( t \).
5. **Rotate** the tracing paper around the fold line \( t \) by 180°.
   - Pierce the vertices of the shape through the tracing paper onto the LHS of the white sheet and then draw the shape. Label it A'.

   What can you say about shape A and shape A'? e.g. (They are congruent shapes but facing in opposite directions. They are equal perpendicular distances from \( t \) but on opposite sides. The top and bottom horizontal sides are on the same line. The longest vertical sides are nearer the mirror line and the shortest further away. etc.)

T: We say that shape A' is the reflected image or the mirror image of shape A in the axis \( t \). Let’s check it with the mirror.

- How did we get from A to A'? (Rotation out of the plane around line \( t \) by 180°.)

Elicit that shape A cannot be moved onto shape A' within the plane of the sheet of paper. It has to be moved out of that plane and turned over. Demonstrate on BB with a cut-out shape A to prove it.

### Notes

Agreement, praising

Class agrees/disagrees before T checks with the mirror.

Individual or paired work monitored, helped

T has large versions for demonstration.

(or to save time, T could have the 2 sheets already prepared then Ps will only need to fix them together.)

Encourage Ps to work carefully and accurately.

Ps use a sharp pencil or the point of a pair of compasses.

Whole class discussion

Involve several Ps

Agreement, praising only

**BB:**

A' is the mirror image of A reflected in axis \( t \).

\[ A' \cong A \]
### Lesson Plan 83

#### Activity

4. **PbY6b, page 83**

**Q.1** Read:

- **a)** *Draw the letter P on a sheet of paper. Colour it green.*
- **b)** *Fold the sheet of paper along line t. Pierce the vertices of the shape, unfold the sheet then draw the mirror image of the shape on the other part of the sheet. Colour it red.*
- **c)** *Complete the sentences.*

Ps read the instructions themselves and carry them out. Make sure that Ps' drawings are correct before they do part c).


**Solution:**

a) and b)  

![](image)

- **i)** The red shape is the mirror image of the green shape.
- **ii)** The red shape and the green shape are congruent.
- **iii)** The red and green shapes are in symmetrical positions to axis t. (or line)

Elicit that the red shape is not a 'P' but a 'P' turned over.

What transformation have we done to get the red shape from the green shape? (Reflection in axis t, or rotation out of the plane around axis t by 180°.)

What can you tell me about the shape and its mirror image? (e.g. Any point and its mirror image are the same distance from t; Ps point out line segments on the shape and their mirror images which are perpendicular (parallel) to t, etc.)

---

#### Notes

**Ps have squared sheets of paper on desks.**

Individual work, monitored closely, helped

T has large sheet for demonstration if necessary.

Do one step at a time if class is not very able.

(or Ps could show missing words on scrap paper or slates on command)

Discussion, reasoning, agreement, self-correction, praising

Accept 'reflected image' too.

Sentences written on BB or SB or OH.

Whole class discussion

Involve several Ps.

Agreement, praising

Extra praise if a P mentions rotation. Demonstrate with a cut-out 'P'.

---

5. **PbY6b, page 83**

**Q.2** Read: *Reflect each shape in the given mirror line or axis.*

Use different colours.

Set a time limit. Ps use rulers to draw the mirror images and colour each mirror image a different colour.

Review with whole class. Ps come to BB or OHP to draw and label the mirror images, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

![](image)

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit

(or T has images already drawn and uncovers each one as it is dealt with)

Reasoning, agreement, self-correction, praising

Discuss the properties of the reflections. e.g.

- **a)** opposite orientation
- **b)** $AA' \perp t$, $CC' \perp t$, $AT = TA'$, $\angle A = \angle A'$, $ABC \cong A'B'C'$, etc.
- **c)** $B \equiv B'$ (identical to, i.e. exactly the same point)
### Activity

**5 (Continued)**

Elicit or point out the following if no P does so.

- The mirror image of a line which is parallel to \( t \) is a line parallel to \( t \) on the opposite side of \( t \);  
- The mirror image of a line which is perpendicular to \( t \) is the same line;  
- In f), the mirror image of the hexagon ABCDE is in the same place as ABCDE but corresponding points are on the opposite side of \( t \), except for the points on the axis, which are identical.  
- The shapes are labelled in an anti-clockwise direction, but the mirror images are labelled clockwise – the opposite direction.

**Notes**

Show on diagram.

---

**6**  
*PbY6b, page 83*

**Q.3** Read: Reflect each shape in the given axis. Use a different colour for each reflection.

Set a time limit or deal with one at a time. Remind Ps to make sure that the corresponding points on the image are the same perpendicular distance from the axis as the point on the shape. Review with whole class. Ps come to BB or OHP to draw and label the mirror images, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Ps say what they notice. (e.g. corresponding points, line segments and angles, lines which are parallel/perpendicular to \( t \), identical points, etc.)

**Solution:**

![Diagram of mirror images](https://via.placeholder.com/150)

**Notes**

Individual work, monitored, helped, corrected

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit

(Ps finished early could draw the mirror images hidden from the rest of the class, or T could have them already prepared and uncover each one as it is dealt with.)

Discussion, reasoning, agreement, self-correction, praising

Involve many Ps in noting the properties of the reflections.

T might point some out too and ask Ps if they are correct.

---

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Q.4 Read:

a) **Draw an axis (mirror line) in your exercise book and label it** \( t \).

b) **Place pairs of dried peas on the page so that they are mirror images of each other.**
   
   **Draw points to mark their positions and label the points.** (e.g. A and A’)
   
   c) **Do the same with pairs of matchsticks.**
   
   **Draw line segments to mark their positions.**

Set a time limit of 3 minutes. T chooses 3 or 4 Ps to show class what they did (by drawing on BB or OHT). Class agrees/disagrees on whether they are reflections.

Elicit that:

- in b), the 2 peas must be the same perpendicular distance from \( t \) but on opposite sides of \( t \); (unless they are actually on the line \( t \), when the two points are identical);

- in c), the corresponding end points of the two matchsticks must be the same perpendicular distance from \( t \), and many patterns are possible. (See below) T shows any of those below which are not shown by Ps.

**Solution:** e.g.

\[ \begin{align*}
\text{b)} & \quad \text{A} \hspace{1cm} \text{A'} \hspace{1cm} \text{P} \hspace{1cm} \text{P'} \hspace{1cm} \text{C} \equiv \text{C'} \\
\text{c)} & \quad \text{a} \hspace{1cm} \text{b} \hspace{1cm} \text{a'} \hspace{1cm} \text{b'} \hspace{1cm} \text{c} \equiv \text{c'}
\end{align*} \]

Elicit that if the lines representing the 2 matchsticks are extended they can:

- meet at axis \( t \), or
- be parallel to \( t \) and the same perpendicular distance from \( t \) but on opposite sides of \( t \), or
- be perpendicular to \( t \) and therefore are on the same line.

Individual work, monitored closely, helped, corrected

(or Ps stick magnetic dots and thin rectangular magnetic strips on BB)

Discussion, reasoning, agreement, praising

If there is disagreement on a pattern, check with a BB ruler, or BB compasses.)
**Lesson Plan**

**84**

**Notes**

Individual work, monitored
(or whole class activity)
BB: 84, 259, 434, 1084
Calculators allowed.
Reasoning, agreement, self-correction, praising

e.g.  
<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
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<tr>
<td>84</td>
<td>2</td>
<td>259</td>
<td>7</td>
</tr>
<tr>
<td>42</td>
<td>2</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>21</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>434</td>
<td>2</td>
<td>1084</td>
<td>2</td>
</tr>
<tr>
<td>217</td>
<td>7</td>
<td>542</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>31</td>
<td>271</td>
<td>271</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Whole class activity but individual trials of folding the paper
T monitors closely and notes which Ps are on the right track, helping, correcting
If a P does it, allow him/her to demonstrate/explain to class.

a) 

Elicit that:

\[
BB: \quad \text{A} \quad \text{B} \quad \text{T}
\]

b) 

Elicit that:

\[
\text{e} = f \\
\text{f} = f'
\]

Elicit (or point out) that:

- \(e = f'\) and \(f = e'\)
- the lines of symmetry **bisect** the central angles
Q.1 Read: Find points in the clearings which are an equal distance from:
   a) trees A and B  b) paths c and d  c) paths e and f.

Set a time limit of 3 minutes. Ps can draw freehand or accurately using rulers (measuring) or compasses (drawing arcs).

Review with whole class. T chooses Ps to show and explain what they did on BB or OHT. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

If no P used compasses, T demonstrates this method to the class, with Ps' help where possible.

Solution:

Q.2 Read: Draw the mirror image of each child's route.

Elicit that each capital letter stands for a child, m is the mirror line and the arrows show each child's path. (Ps could come to BB to trace the routes with their fingers.)

Set a time limit or deal with one part at a time. Encourage Ps to use a ruler and a pair of compasses.

Review with whole class. Ps come to BB or OHT to construct the routes, marking the important information. Class agrees/ disagrees. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

Who remembers the name we give to arrows which show direction and distance? (vectors)

Solution:

Ps say what they notice. e.g.
- Each line segment is the same length as its mirror image.
- Each angle is the same size as its mirror image.
- The mirror image of a turn anticlockwise is a turn clockwise (and vice versa). etc.

(Also elicit information about mirror images of line segments which are parallel or perpendicular to the axis.)
### Activity 5

**PbY6b, page 84, Q.3**

Read: *Reflect the point in the given axis. Construct and label its mirror image.*

Deal with one part at a time. First elicit the main features of the mirror image. (e.g. in a): A’ is a point which is the same perpendicular distance from m as A but on the opposite side of m

Then T leads a discussion on the steps needed to construct the mirror image. After agreement, carry out one step at a time, with T working on BB using BB instruments while Ps work in Pbs.

Who can say a true statement about the diagram? Who can write it using mathematical notation? Ps dictate to T or come to BB. Class agrees/disagrees.

a) Steps for construction of A'.
   1) Draw a perpendicular line from point A through m.
      (Lay a set square with its vertical edge on the axis and its bottom edge exactly on A. Place a ruler against the bottom of the set square, remove the set square and draw a line along the top of the ruler from A through the axis, extending to the other side of m.)
   2) Label the intersecting point M.
   3) Set a pair of compasses to the length AM, then measure this distance from M on the opposite side of M.
   4) Label this point A'.

b) Elicit the main features of, and the steps needed to construct, B'. Ps repeat the procedure, with a P working on BB with T's help. Then Ps write true mathematical statements about the diagram.

c) Ask one or two Ps where they think C’ should be. Elicit that the image of any point on the axis is that same point.

Who can write it mathematically? Who agrees? If necessary, T reminds Ps of the symbol which means 'identical to'. (≡)

Agree that the point C’ is the point C.

---

### Activity 6

**PbY6b, page 84**

Q.4 Read: *Reflect the line segment in the given axis. Construct and label its mirror image.*

We have been constructing the mirror image of a point but how can we construct the mirror image of a line segment? (Reflect the two end points, then join up their mirror images.)

Deal with one part at a time. Set a time limit. Ps use rulers, set squares and compasses. (If necessary, do part a) with the whole class first, with T (P) working on BB under Ps’ direction and P's working in Pbs.)

Review with whole class. Ps come to BB to show and explain their construction. Class agrees/disagrees. Mistakes discussed and corrected. Ps write true mathematical statements about each diagram. T could write some too and ask Ps if they are correct.

---

### Notes

Whole class activity but individual drawing, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Discussion, demonstration, agreement, praising

---

35 min

---

Individual work, monitored closely, helped, corrected

Drawn on BB or use enlarged copy master or OHP

(Less able Ps could have enlarged copies of the diagrams to make the construction and labelling easier.)

Discussion, reasoning, agreement, self-correction, praising only
### Lesson Plan 84

#### Activity 6 (Continued)

**Solution:**

- **a)**
  - $a = a'$ or $AB = A'B'$
  - $AB \parallel m \parallel A'B'$
  - $AA' \parallel BB'$
  - $AA' \perp m$, $BB' \perp m$

- **b)**
  - $b = b'$ or $BC = B'C'$
  - $B \equiv B'$, $\angle \alpha = \angle \alpha'$
  - $CC' \perp m$

- **c)**
  - $c = c'$ or $CD = C'D'$
  - $c \perp m$, $c' \perp m$

- **d)**
  - $d = d'$ or $DE = D'E'$
  - $DD' \perp m$, $EE' \perp m$

- **e)**
  - $e = e'$ or $EF = E'F'$
  - $\angle \alpha = \angle \alpha'$
  - $EE' \perp m$, $FF' \perp m$

  *EF and E'F' intersect on the axis $m*.

- **f)**
  - $f = f'$ or $FG = F'G'$
  - $FG \perp m$, $FG' \perp m$

  *Line segments FG and F'G' lie on top of one another.*

- **g)**
  - $H \equiv H'$
  - $G \equiv G'$

  *GH \equiv G'H'*

---

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Factorising 85, 260, 435 and 1085. Revision, activities, consolidation

**PbY6b, page 85**

**Solutions:**

1. **Q.1**
   - 1 → 2: Enlargement (3 : 2), then translation (6, – 1)
   - 1 → 3: Reflection in axis \( m \)
   - 1 → 4: Reduction (1 : 2), then translation (20, 1)
   - 1 → 5: Rotation by 180° about point O, or reflection in point O
   - 1 → 6: Reflection in axis \( t \), then translation (6, 0)
   - 1 → 7: Stretch \( (\frac{3}{2}, 0) \), then translation (11.5, – 5)
   - 1 → 8: Reflection in axis \( s \), then translation (15, 0), then rotation about point A by 90° clockwise (i.e. by – 90°)

Expect more able Ps to give the complete answer, but accept only the names of the transformations from less able Ps.

2. **Q.2 a)**
   - Sketch
     - \( b = 3 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle a = 30° \)
     - \( \angle b = 30° \)
     - \( \angle c = 4 \text{ cm} \)
     - **Actual size**
     - \( b = 3 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle a = 30° \)
     - **Actual size**
     - \( b = 3 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle a = 30° \)
     - **Actual size**
     - \( b = 3 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle a = 30° \)

   (scalene triangle, obtuse-angled)

3. **Q.2 ii)**
   - Sketch
     - \( b = 20 \text{ mm} \)
     - \( c = 20 \text{ mm} \)
     - \( \angle a = 50° \)
     - **Actual size**
     - \( b = 2 \text{ cm} \)
     - \( c = 2 \text{ cm} \)
     - \( \angle a = 50° \)

   (isosceles triangle, acute-angled)

4. **Q.2 iii)**
   - Sketch
     - \( a = 4 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle b = 65° \)
     - \( \angle c = 65° \)
     - **Actual size**
     - \( a = 4 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle b = 65° \)
     - **Actual size**
     - \( a = 4 \text{ cm} \)
     - \( c = 4 \text{ cm} \)
     - \( \angle b = 65° \)

   (isosceles triangle, acute-angled)
Q.2  a) (Continued)

b) ii) ~ iii); iv) ~ v)

c) iv) \(\cong\) v)

Q.3

Q.4 Accept any shape with correct mirror images labelled appropriately (e.g. \(ABC \rightarrow A'B'C' \rightarrow A''B''C''\))
Y6  

**Activity**  

1  

**Factorisation**  

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.  

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.  

Elicit that:  

- \( 86 = 2 \times 43 \)  
  Factors: 1, 2, 43, 86  
- \( 261 = 3 \times 3 \times 29 = 3^2 \times 29 \)  
  Factors: 1, 3, 9, 29, 87, 261  
- \( 436 = 2 \times 2 \times 109 = 2^2 \times 109 \)  
  Factors: 1, 2, 4, 109, 218, 436  
- \( 1086 = 2 \times 3 \times 181 \)  
  Factors: 1, 2, 3, 6, 181, 362, 543, 1086  

**Rotation and reflection**  

T has a large model for demonstration. (If possible Ps have smaller versions to manipulate on desks too.)  

T demonstrates a rotation of plane P by \( 180^\circ \) around line \( m \). (Ps copy it if possible.)  

a) What transformation have we done? (rotation by \( 180^\circ \))  
   What other transformation could I have done instead? (Reflection in line \( m \) within the plane P.)  

b) Talk about the paths and mirror images of some points, line segments and parts of the shape (e.g. point T is shown in the diagram).  

c) Elicit the properties of reflection. e.g.  
   - any pair of corresponding points on the shape and its mirror image are the same perpendicular distance from the axis;  
   - any pair of corresponding line segments on the shape and its mirror image are the same perpendicular distance from the axis;  
   - the mirror image has the opposite orientation to the shape. etc.  

2

Whole class activity  

Use any simple shape, or use two copies of enlarged copy master (one on white paper and one on a transparency) fixed together as in LP 83/3.  

P could check using a mirror.  

Ps come to front of class to choose the points, line segments, etc. and to show and describe their paths.  

Discussion, reasoning, agreement, praising  

Involves several Ps.  

T reminds Ps of any they forget to mention.  

**PbY6b, page 86**  

Q.1 Read: **Complete the reflection of the clock in axis \( m \), then reflect its mirror image in axis \( n \).**  

Set a time limit. Ps use rulers and compasses to measure and draw.  

Review with whole class. T could have mirror images already prepared (or Ps finished early could draw them, hidden from the view of the rest of the class). Ps compare their own drawings with those on BB. Mistakes discussed and corrected.  

Label the clocks C, C’ and C” and elicit properties of the reflections.  

**Solution:**  

<table>
<thead>
<tr>
<th>C</th>
<th>C’</th>
<th>C”</th>
</tr>
</thead>
</table>

Individual work, monitored, helped  

Drawn (stuck) on BB or use enlarged copy master or OHP.  

Discussion, reasoning, agreement, self-correction, praising  

How can we get from C to C” with just one transformation? (Translation)  

Elicit or point out that the distance of the translation would be twice the distance between \( m \) and \( n \). Why?
**Lesson Plan 86**

**Notes**

Individual work, monitored, helped
drawn on BB or use enlarged copy master or OHP

T could have a cut out triangle for demonstration.

Differentiation by time limit

Discussion, reasoning, agreement, self-correction, praising

If necessary, revise how to reflect a point.

(Draw a perpendicular line from the point to the axis and extend it by the same distance on the other side of the axis.)

Whole class discussion

Involves several Ps.

Add extra labels (M and N) and the markings for parallel and perpendicular lines, etc. to the diagram as necessary.

Elicit or point out that the points on the original triangle and on the 2nd mirror image are labelled in an anti-clockwise direction (i.e. a positive turn), while the points on the 1st mirror image are labelled in a clockwise direction (i.e. a negative turn).

Feedback for T

---

**Activity 4**

*PbY6b, page 86*

Q.2 Read: Reflect triangle $ABC$ in axis $m$, then reflect $A'B'C'$ in axis $n$.

Label the vertices of the 2nd mirror image appropriately.

How can we reflect a triangle? (Reflect the 3 vertices, then join up their mirror images.)

Set a time limit. Ps use rulers, compasses and set squares to measure and draw. Ask Ps to label the sides with lower case letters too. ($a$ opposite $\angle A$, $b$ opposite $\angle B$, etc.)

Review with the whole class. Ps come to BB to demonstrate and explain their construction. Who agrees? Who did it another way? etc. Mistakes discussed and corrected.

**Solution:**

```
\begin{tikzpicture}
\coordinate (A) at (0,0);
\coordinate (B) at (1,0);
\coordinate (C) at (0,1);
\draw (A) -- (B) -- (C) -- cycle;
\draw (A) -- (M) -- (B);\draw (B) -- (N) -- (C);
\end{tikzpicture}
```

Tell me some properties of the reflections. (If Ps have no ideas, T suggests some and ask Ps if they are correct.) e.g.

$m \parallel n$, $\triangle ABD \cong \triangle A'B'C' \cong \triangle A''B''C''$

$a = a' = a''$, $b = b' = b''$, $c = c' = c''$

$\angle A = \angle A' = \angle A''$, $CC' \perp m$, $CC'' \perp n$, etc.

The orientation changes to its opposite, then to its opposite again, so the orientation of $\triangle ABC$ is the same as $\triangle A''B''C''$.

What single transformation could we have done instead? (Translation by 10 cm to the right, or by (10, 0), keeping perpendicular to the two axes.)

Who can explain why this is so?

e.g. Let's consider the motion of the point C:

$CM = MC'$, $CN = NC''$, $MC' + CN = 5$ cm (by measuring)

so $CM + MC' + CN + NC'' = 2 \times (MC' + CN) = 2 \times 5 \text{ cm} = 10 \text{ cm}$

---

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Q.3 Read: Reflect quadrilateral $ABCD$ in axis $m$, then reflect $A'B'C'D'$ in axis $n$.

Label the vertices of the 2nd mirror image appropriately.
(The 2 axes are perpendicular.)

What kind of shape is the quadrilateral? (Trapezium) We know that $\angle A$ is a right angle but what else can you tell me about the shape? ($DC \parallel AB$, $\angle D = 90^\circ$)

How can we reflect a quadrilateral in an axis? (Reflect each vertex, then join up the mirror images.)

Set a time limit. Ps use rulers, compasses and set squares to draw and measure, then label the vertices appropriately.

Review with whole class. (T could have mirror images already prepared and uncover each as it is dealt with, or Ps finished early could work on BB or OHT hidden from class.)

Ps compare their drawings with those on BB. Show each step if many Ps had difficulties. Mistakes discussed and corrected.

**Solution:**

![Diagram showing the reflection process](image)

Tell me some properties of the reflections. (If necessary, T could suggest some and ask Ps if they are correct.)

e.g. $ABCD \cong A'B'C'D' \cong A''B''C''D''$

$a = a' = a''$, $b = b' = b''$, $c = c' = c''$, $d = d = d''$

$\angle A = \angle A' = \angle A''$, etc.

$AB \parallel A''B''$, $BC \parallel B''C''$, $CD \parallel C''D''$, $DA \parallel D''A''$

How could we get from $ABCD$ to $A''B''C''D''$ in just one transformation?

(Rotation by $180^\circ$ around the point where $m$ and $n$ intersect.)

How can we prove that it is a rotation of $180^\circ$?

e.g. Let’s consider the rotation of point B.

BB: Angle of rotation from B to B': $\gamma + \gamma = 2\gamma$

Angle of rotation from B' to B'': $\varepsilon + \varepsilon = 2\varepsilon$, but $\gamma + \varepsilon = 90^\circ$, (as axes are perpendicular)

so angle of rotation from B to B'':

$$2 \times (\gamma + \varepsilon) = 2 \times 90^\circ = 180^\circ$$

Whole class discussion
 invols several Ps.

Extra praise if a P suggests labelling the sides of the trapezium $a, b, c, d$, otherwise T suggests it.

Ps might suggest $180^\circ$ because BB’ is a straight line. T demonstrates with cut-out shape.

Allow Ps a minute to think about it, then if no P has an idea, T leads Ps through the reasoning opposite.

Angles are usually labelled with Greek letters, e.g. $\gamma$ (gamma), $\varepsilon$ (epsilon)
**Activity**

**PbY6b, page 86**

Q.4 Read: **Reflect triangle ABC in axis m, then reflect A'B'C' in axis n.**

*Label the vertices of the 2nd mirror image appropriately.*

Set a time limit. Ps use rulers, compasses and set squares to draw and measure, then label the vertices.

Review with whole class. Ps come to BB to show and explain what they did. Class agrees/disagrees. Who did it a different way? Which way do you think is better? Mistakes discussed and corrected.

**Solution:**

Tell me some properties of the reflections. (If necessary, T suggests some and ask Ps if they are correct.)

e.g. ABC and A"B"C" have a positive orientation (i.e. labelled anti-clockwise) while A'B'C' has a negative orientation (i.e. labelled clockwise)

\[ \triangle ABC \cong \triangle A'B'C' \cong \triangle A''B''C'' \]

\[ a = a' = a'', \quad b = b' = b'', \quad c = c' = c'' \]

\[ \angle A = \angle A' = \angle A'', \quad \angle B = \angle B' = \angle B'', \quad \angle C = \angle C' = \angle C'' = 90^\circ \]

**Extension**

How could we get from triangle ABC to A"B"C" in just one transformation? (Rotation around the point where \( m \) and \( n \) intersect.)

What could we write about the angle of rotation? e.g. Let's consider the rotation of point A.

**BB:** Angle of rotation from A to A': \( \gamma + \gamma = 2\gamma \)

Angle of rotation from A' to A'': \( \varepsilon + \varepsilon = 2\varepsilon \)

Angle of rotation from A to A'': \( 2 \times (\gamma + \varepsilon) \)

(i.e. twice the angle between \( m \) and \( n \))
### Lesson Plan

**Week 18**

**R:** Properties of axial reflection (reflection in a line)  
**C:** Constructing axial reflections  
**E:** Problems

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td><strong>Lesson Plan</strong></td>
</tr>
<tr>
<td><strong>87</strong></td>
<td><strong>Notes</strong></td>
</tr>
</tbody>
</table>
| 1 | Individual work, monitored (or whole class activity)  
BB: 87, 262, 437, 1087  
Ps could try it without using calculators as division practice. Revise procedure for long division.  
Reasoning, agreement, self-correction, praising  
e.g.  
| 2 | Whole class activity  
Drawn on BB or use enlarged copy master or OHP  
Discussion, reasoning, agreement, praising  
| 3 | Individual work, monitored, helped  
Drawn on BB or use enlarged copy master or OHP  
Initial discussion on the constructions needed.  
Demonstrate on BB if necessary.  
Differentiation by time limit  
Discussion, reasoning, agreement, self-correction, praising  
Feedback for T  

---

**Factorisation**  
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.  
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.  
Elicit that:  
- \(87 = 3 \times 29\) Factors: 1, 3, 29, 87  
- \(262 = 2 \times 131\) Factors: 1, 2, 131, 262 e.g. \(\frac{23}{38}\)  
- \(437 = 19 \times 23\) Factors: 1, 19, 23, 437  
- \(1087\) is a prime number Factors: 1, 1087  
(as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and \(37 \times 37 > 1087\))  

| 8 min |

**Properties of reflection in a mirror line**  
Study this diagram. What does it show? (Reflection of a triangle in a mirror line. Elicit that two possible reflections are shown, \(\triangle AMB\) or \(\triangle AA'B\))  
Who can tell me true statements about the diagram? Ps dictate to T or come to BB to write mathematical statements. Class agrees/disagrees.  
BB:  
\[ \begin{align*}  
\triangle A'B' = \triangle AB, \quad B' &\equiv B, \quad (M' \equiv M),  
\triangle AMB = \triangle A'M'B', \quad AM = A'M'  
\triangle A'MB = \triangle A'M'B', \quad (AM' = A'M)  
\triangle AMB = \triangle A'MB' \equiv 90^\circ  
\triangle AMB \equiv \triangle A'MB'  
\text{The image of } \triangle AA'B \text{ is itself.}  
\end{align*} \]  

T: We say that triangle \(\triangle AA'B\) is a symmetrical triangle and its line of symmetry is line \(m\).  

| 13 min |

**Erratum**  
In Pbs, 2nd 'c') should be 'd')

**PbY6b, page 87**  
Q.1 Read: Construct the mirror image of each triangle. Colour the mirror image red and label its vertices appropriately.  
First revise how to reflect a point (draw a perpendicular line from the point to the mirror line then extend the line by the same distance on the other side of the mirror line) and how to reflect a triangle (reflect each vertex, then join up the mirror images). Ps explain in their own words.  
Set a time limit or deal with one part at a time.  
Ps use rulers, set squares and compasses to draw and measure, then label the images. (Ps finished early draw mirror images on BB hidden from class, or T has images already prepared and uncovers each one as it is dealt with.)  
Review with the whole class. Ps compare their drawings with those on BB and any mistakes are discussed and corrected. Ps point out the main features of the reflections. If necessary, T suggests missed features and asks Ps if they are correct.  

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### Y6

**Lesson Plan 87**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 (Continued)</td>
<td><strong>Solution:</strong></td>
</tr>
<tr>
<td><img src="" alt="Diagram" /></td>
<td><strong>Solution:</strong></td>
</tr>
<tr>
<td><img src="" alt="Diagram" /></td>
<td><strong>Solution:</strong></td>
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<td><img src="" alt="Diagram" /></td>
<td><strong>Solution:</strong></td>
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<tr>
<td>Solution:</td>
<td><strong>Solution:</strong></td>
</tr>
<tr>
<td>a) Read: Write the steps needed to reflect point A in axis m.</td>
<td>Elicit that in d), the triangle and its mirror image form a concave deltoid.</td>
</tr>
<tr>
<td>Study the diagram. It shows another way to reflect a point in a mirror line using only a pair of compasses. How do you think it was done? T asks several Ps for their ideas and class agrees on the steps of the construction. T repeats each step in a clear way and Ps write the steps in their Pbs (or Ex. Bks if they need more space).</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>BB: To reflect a point using compasses</td>
<td>Whole class activity</td>
</tr>
<tr>
<td>1. Mark 2 different points, P and Q, on axis m.</td>
<td>Drawn on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>2. Set the compasses to length PA and draw an arc around P.</td>
<td>Discussion, agreement, praising</td>
</tr>
<tr>
<td>3. Set the compasses to length QA and draw an arc around Q.</td>
<td>Involve several Ps.</td>
</tr>
<tr>
<td>4. The point of intersection of the two arcs is A'.</td>
<td>T writes steps on BB (or has them already prepared and uncovers each step as it is agreed on.)</td>
</tr>
<tr>
<td>b) Read: Carry out the construction on this diagram.</td>
<td>Individual work, monitored closely, helped, corrected</td>
</tr>
<tr>
<td>Set a short time limit. Ps finished quickly help slower Ps or repeat the construction in Ex. Bks, drawing their own points and axes.</td>
<td>Praising, encouragement only</td>
</tr>
<tr>
<td>30 min</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Individual trial, monitored (or whole class activity)</td>
</tr>
<tr>
<td><img src="" alt="Diagram" /></td>
<td>Discussion, reasoning, agreement, praising</td>
</tr>
<tr>
<td>PbY6b, page 87, Q.3</td>
<td>T could have some already prepared but use Ps’ diagrams where possible.</td>
</tr>
<tr>
<td>Deal with one part at a time. Allow Ps to think about it, discuss with their neighbours and try it in Ex. Bks first.</td>
<td></td>
</tr>
<tr>
<td>Review with whole class. Ps dictate the steps and demonstrate construction on BB. Class agrees/disagrees. T repeats the agreed steps in a clear way if necessary and Ps write them in Ex. Bks, if they have not already done so correctly.</td>
<td></td>
</tr>
<tr>
<td>Discuss and demonstrate different reflections for each type.</td>
<td></td>
</tr>
<tr>
<td>Ps say true statements about the shape and its mirror image.</td>
<td></td>
</tr>
</tbody>
</table>
Q.3 a) Read Write the steps needed to reflect any straight line in any axis. Draw an axis \( m \) and a straight line \( e \).

Reflect line \( e \) in \( m \).

Agree that in the diagram the line could cut the axis, or be parallel to the axis or neither cut it nor be parallel to it. Also elicit that the mirror image of a line which is perpendicular to the axis is that line.

Solution: e.g.

b) Read Write the steps needed to reflect any angle in any axis. Draw an axis \( m \) and an angle \( \alpha \).

Reflect angle \( \alpha \) in \( m \).

Three different types are possible: the angle does not touch the axis, crosses the axis, or touches the axis at its vertex.

Solution: e.g.

c) Read Write the steps needed to reflect any circle in any axis. Draw an axis \( m \) and a circle \( k \).

Reflect circle \( k \) in \( m \).

The axis does not touch the circle, or touches the circle at a point (i.e. the axis is a tangent to the circle and also to its mirror image), or lies on a chord of the circle, or lies on the diameter of the circle.

Solution e.g.

Elicit than when the axis lies on the diameter of the circle, the mirror image is the circle itself.

Extra praise if a P points out that a line which is neither parallel nor perpendicular to the axis will eventually cut the axis at some imagined point.

Steps to reflect a straight line
1. Mark any 2 points on the line (only 1 is needed if the line crosses the axis).
2. Reflect each point in the axis and label appropriately.
3. Draw a straight line through the 2 mirror images.

Steps to reflect an angle e.g.
1. Mark any point on each arm of the angle (only 1 arm is needed if the other arm crosses the axis).
2. Reflect each point and the point at the vertex in the axis. Label appropriately.
3. From the image of the point at the vertex draw rays through the images of the points on the arm(s).

Steps to reflect a circle e.g.
1. Mark the centre of the circle, \( O \), and any point, \( P \), on its circumference
2. Reflect the 2 points in the axis and label them.
3. Draw a circle with centre \( O' \) and radius \( O'P' \).

or

1. Mark the centre of the circle, \( O \).
2. Reflect \( O \) in the axis.
3. With compasses set to the radius of the original circle, draw a circle with centre \( O' \).
Q.3 a) Read Write the steps needed to reflect any straight line in any axis. Draw an axis $m$ and a straight line $e$.

Reflect line $e$ in $m$.

Agree that in the diagram the line could cut the axis, or be parallel to the axis or neither cut it nor be parallel to it. Also elicit that the mirror image of a line which is perpendicular to the axis is that line.

Solution: e.g.

(b) Read Write the steps needed to reflect any angle in any axis. Draw an axis $m$ and an angle $\alpha$.

Reflect angle $\alpha$ in $m$.

Many different types are possible, e.g. the arms do not touch the axis, or an arm crosses the axis, or vertex lies on axis, etc.

Solution: e.g.

(c) Read Write the steps needed to reflect any circle in any axis. Draw an axis $m$ and a circle $k$.

Reflect circle $k$ in $m$.

The axis does not touch the circle, or touches the circle at a point (i.e. the axis is a tangent to the circle and also to its mirror image), or lies on a chord of the circle, or lies on the diameter of the circle.

Solution e.g.

Elicit than when the axis lies on the diameter of the circle, the mirror image is the circle itself.

Extra praise if a P points out that a line which is neither parallel nor perpendicular to the axis will eventually cut the axis at some imagined point.

Steps to reflect a straight line
1. Mark any 2 points on the line (only 1 is needed if the line crosses the axis).
2. Reflect each point in the axis and label appropriately.
3. Draw a straight line through the 2 mirror images.

Steps to reflect an angle e.g.
1. Mark any point on each arm of the angle (only 1 arm is needed if the other arm crosses the axis).
2. Reflect each point and the point at the vertex in the axis. Label appropriately.
3. From the image of the point at the vertex draw rays through the images of the points on the arm(s).

Steps to reflect a circle e.g.
1. Mark the centre of the circle, $O$, and any point, $P$, on its circumference
2. Reflect the 2 points in the axis and label them.
3. Draw a circle with centre $O'$ and radius $O'P'$.

or
1. Mark the centre of the circle, $O$.
2. Reflect $O$ in the axis.
3. With compasses set to the radius of the original circle, draw a circle with centre $O'$.
**Lesson Plan**

**Y6**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>Factorisation</strong></td>
</tr>
<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:</td>
<td></td>
</tr>
<tr>
<td><strong>88</strong> = 2 × 2 × 2 × 11 = 2³ × 11</td>
<td></td>
</tr>
<tr>
<td>Factors: 1, 2, 4, 8, 11, 22, 44, 88</td>
<td></td>
</tr>
<tr>
<td><strong>263</strong> is a prime number</td>
<td></td>
</tr>
<tr>
<td>Factors: 1, 263</td>
<td></td>
</tr>
<tr>
<td>(as not divisible by 2, 3, 5, 7, 11, 13 and 17 × 17 &gt; 263)</td>
<td></td>
</tr>
<tr>
<td><strong>438</strong> = 2 × 3 × 73</td>
<td></td>
</tr>
<tr>
<td>Factors: 1, 2, 3, 6, 73, 146, 219, 438</td>
<td></td>
</tr>
<tr>
<td><strong>1088</strong> = 2 × 2 × 2 × 2 × 2 × 2 × 17 = 2⁶ × 17</td>
<td></td>
</tr>
<tr>
<td>Factors: 1, 2, 4, 8, 16, 17, 32, 34, 64, 68, 136, 272, 544, 1088</td>
<td></td>
</tr>
<tr>
<td><strong>8 min</strong></td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td><strong>Line symmetry</strong></td>
</tr>
<tr>
<td>a) Ps point out shapes in the classroom which have line symmetry. Class agrees/disagrees. Ps indicate where the lines of symmetry are. If disagreement, check with a mirror.</td>
<td></td>
</tr>
<tr>
<td>b) T has a collection of different shapes drawn (stuck) on BB. e.g. BB:</td>
<td></td>
</tr>
<tr>
<td>Which of these shapes have line symmetry? Ps come to BB to point to them and to draw their lines of symmetry. Class agrees/disagrees. (If the shapes are stuck on BB, Ps could check by folding them, so that one half covers the other half exactly.)</td>
<td></td>
</tr>
<tr>
<td>T: We say that a 2-dimensional shape has line symmetry if a line can be drawn which cuts the shape in half, so that one half can cover the other half exactly.</td>
<td></td>
</tr>
<tr>
<td>c) Ps have sheets of plain paper on desks and make symmetrical shapes by drawing, folding, tearing or cutting. T chooses Ps to show their shapes to the class and to point out the lines of symmetry. Class agrees/disagrees.</td>
<td></td>
</tr>
<tr>
<td><strong>18 min</strong></td>
<td></td>
</tr>
<tr>
<td><strong>3</strong></td>
<td><strong>PbY6b, page 88</strong></td>
</tr>
<tr>
<td>Q.1 Read: <em>Draw lines of symmetry on the shapes.</em></td>
<td></td>
</tr>
<tr>
<td>Set a time limit. Review with whole class. Ps come to BB to choose a shape, draw its lines of symmetry where possible, and explain why it is (or is not) symmetrical. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong> (A shape with no lines of symmetry is asymmetrical.)</td>
<td></td>
</tr>
<tr>
<td><strong>12 3 4 5 6 7</strong></td>
<td></td>
</tr>
<tr>
<td>Asymmetrical Asymmetrical</td>
<td></td>
</tr>
<tr>
<td><strong>23 min</strong></td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

Individual work, monitored (or whole class activity) BB: 88, 263, 438, 1088 (Ps could try it without using calculators as multiplication and division practice.) Reasoning, agreement, self-correction, praising

e.g. 1088 2
     88 2 544 2
     44 2 272 2
     22 2 438 2 136 2
     11 11 73 73 68 2
     1 1 34 2
     17 17 1

Whole class activity
At a good pace
Involve several Ps.
Agreement, praising
(Ps could have collected the shapes, or T has different shapes drawn or cut out, or use enlarged copy master.)

Reasoning, agreement, checking, praising

T asks one or two Ps to repeat the definition in their own words.

Individual work, monitored, helped, corrected
Extra praise for creativity.

Individual work, monitored
Drawn (stuck) on BB or use enlarged copy master or OHP (T could have cut-out versions for Ps to fold if necessary.)
Discussion, reasoning, agreement, self-correction, praising
Elicit that the circle on its own would have an infinite number of lines of symmetry.
### Y6

#### Activity

**PbY6b, page 88**

**Q.2** Read: *Construct the lines of symmetry.*

Set a time limit. Ps use rulers, compasses and set squares to construct the lines of symmetry. Ask Ps to label them too.

Review with whole class. Ps come to BB to explain their construction and to demonstrate on BB using BB instruments.

Class agrees/disagrees. Mistakes discussed and corrected. If no P drew 2 lines of symmetry in a) or c), T hints that there is another one and asks Ps to show where it is.

Ps point out the main features of the diagrams. (e.g. equal line segments, equal angles, perpendicular lines)

T asks one or two Ps to repeat the steps of construction for each diagram. Class points out errors or missed steps.

**Solution:**

- **a)**
  - 2 lines of symmetry:
  - \( m_1 \) is the perpendicular bisector of \( AB \).
  - \( m_2 \) lies on which \( A \) and \( B \).

- **b)**
  - 1 line of symmetry:
  - \( m \) is the bisector of \( \angle A \).

- **c)**
  - 2 lines of symmetry:
  - They cross at right angles, half way between \( A \) and \( B \).

28 min

#### Notes

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit

Discussion, reasoning, agreement, self-correction, praising

Involve several Ps. Praising

To bisect an angle: see *LP 84/3*

---

**Erratum**

In b): 'polyons' should be 'polygons'

**PbY6b, page 88**

**Q.3**

- **a)** Read: *Fold a rectangular sheet of paper along one of its diagonals and cut along the fold.*

  What shapes have you made? (2 triangles)

  What can you tell me about them? (right-angled, scalene, concave, congruent) Ps point out the equal sides and angles.

- **b)** Read: *Use the two pieces formed to make different polygons by placing equal sides together. Measure the sides and angles of these polygons and note the values.*

  Advise Ps to label equal sides with the same letters and mark the equal angles, so that they do not mix them up.

  Elicit that besides the original rectangle, we can make two different triangles and a deltoid. Ps show them by manipulating cut-out triangles on BB.

- **c)** Read: *In your exercise book, draw a sketch of each of the polygons you form and mark on the sketch the size of the angles and the lengths of the sides.*

  Set a time limit. Review with whole class. T chooses one or two Ps to draw their sketches on BB and write their measurements. Class points out errors.

  Discuss the main features of the two triangles and the deltoid.

  - Triangles: 2 equal sides, 2 equal angles, perpendicular bisector of the base meets the 3rd vertex
  - Elicit that they are symmetrical or isosceles triangles.

  - Deltoid: 2 pairs of adjacent equal sides, diagonals are perpendicular to each other.

36 min

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Q.4 Read: **Fill in the missing items.**

Set a time limit of 4 minutes. Ps fill in boxes in *Pbs.*

Review with whole class. Ps could show missing words or values on scrap paper or slates on command. Ps with different answers explain reasoning on diagram on BB. Class decides on the correct answer. Mistakes discussed and corrected.

**Solution:**

a) This **symmetrical** triangle has 2 equal sides and is called an **isosceles** triangle.

b) If a triangle has 2 equal sides, it is **symmetrical.**

c) \( AC = BC; \ angle A \neq angle B; \ angle ACD = angle BCD \)

d) The equal sides are called the **arms** of the triangle.

e) \( AB \) is the **base** of the triangle.

f) The line of symmetry **bisects** the base and is perpendicular to it.

g) \( AB \perp CD; \ AD \equiv DB \)

h) CD is the **height** of triangle ABC from its base.

**Extension**

What other statements could you write about the diagram? **41 min**

---

Q.5 Read: **If a triangle has 3 equal sides, it is called a regular or an equilateral triangle. Complete the statements.**

Ps come to BB to fill in the missing items on BB. Who agrees? Who thinks it should be something else? Why? Ps explain reasoning by referring to diagram. Class agrees on correct answer and Ps write it in *Pbs.* Ps think of other statements to make about the diagram.

**Solution:**

a) \( angle A = angle B = angle C; \ AB \perp CD; \ AD \equiv DB \)

b) Any equilateral triangle is an **isosceles** triangle.

c) An equilateral triangle has 3 lines of symmetry.

d) DC is the **height** of the equilateral triangle.

**Extension**

What other statements could you write about the diagram? (e.g. \( AM = BM = CM, \ angle ACD = angle BCD, \) etc.) **45 min**

---

**Notes**

Individual work, monitored, helped (or whole class activity)

Drawn/written on BB or use enlarged copy master or OHP

Responses shown in unison.

Agreement, self-correction, praising

**BB:**

---

**Whole class activity**

(or individual trial first if there is time)

Drawn (written) on BB or use enlarged copy master or OHP

or Ps show missing items on slates or scrap paper in unison

Reasoning, agreement, praising

**BB:**
### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **89** is a prime number  
  Factors: 1, 89  
  (as not exactly divisible by 2, 3, 5, 7 and 11 × 11 > 89)
- **264** = 2 × 2 × 2 × 3 × 11 = 2³ × 3 × 11  
  Factors: 1, 2, 3, 4, 6, 8, 11, 12, 264, 132, 88, 66, 44, 33, 24, 22
- **439** is a prime number  
  Factors: 1, 439  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and 23² > 439)
- **1089** = 3 × 3 × 11 × 11 = 3² × 11² = (3 × 11)² (square no.)  
  Factors: 1, 3, 9, 11, 33, 99, 121, 363, 1089

**Notes**

Individual work, monitored (or whole class activity)

BB: 89, 264, 439, 1089  
(Ps could try it without using calculators as multiplication and division practice.)

Reasoning, agreement, self-correction, praising  
(e.g.  
\[
\begin{align*}
264 &= 2 \times 2 \times 2 \times 3 \times 11 \\
&= 2^3 \times 3 \times 11 \\
89 &= \text{prime number} \\
439 &= \text{prime number} \\
1089 &= 3^2 \times 11^2 = (3 \times 11)^2 \\
\end{align*}
\]

**2**

**Isosceles triangles**

What is an isosceles triangle? (A triangle which has at least 2 equal sides) How could we create an isosceles triangle? Ps tell class what they already know or think of new ideas. T gives hints if necessary.

Ps demonstrate their different methods in front of class or on BB.

- **a)** Folding and cutting a rectangle along one of its diagonals and manipulating the 2 pieces:

- **b)** Fold a rectangle in half, fold the resulting rectangle diagonally, then open out the paper and cut along the diagonal folds.

- **c)** Construction using a ruler and set square:
  1) Draw line segment AB as the base.
  2) Measure and mark a point, M, halfway between A and B.
  3) Place the set square so that its bottom edge lies along AB and its vertical edge is on M.
  4) Draw a line, e, through M. (It is the perpendicular bisector of AB.)
  5) Mark any point C (which is not on AB) on line e. Join C to A and B.  
  Triangle ABC is an isosceles triangle.

- **d)** Construction using a ruler and compasses:
  1) Draw a base AB.
  2) Set compasses to more than half the distance between A and B.
  3) Draw arcs around A and B. Label their point of intersection C.
  4) Join A and B to C. C is the 3rd vertex of the isosceles triangle.

**Notes**

Whole class activity  
T has large rectangles of paper and BB ruler, compasses and set square at hand (or use OHP for Ps to demonstrate with own instruments)

T helps Ps to explain their ideas.

Discussion, demonstration, agreement, praising

Or set compasses and draw arcs around A and B  
Join up the points of intersection to draw the perpendicular bisector of AB, line e.
Q.1 Deal with one part at a time. Set a short time limit.

Review with whole class. Ps show results on scrap paper or slates where possible, or dictate to T, or come to BB to write missing values and explain reasoning. Mistakes discussed and corrected.

Part d) could be done with the whole class, with Ps suggesting ideas and explaining at BB. (T could have 2 cut-out triangles ready for demonstration.)

**Solution:**

a) *Measure the sides of this right-angled triangle.*
   
   \[
   a = 3 \text{ cm}, \quad b = 4 \text{ cm}, \quad c = 5 \text{ cm}
   \]

b) *Measure its angles.*

   \[
   \angle A = 37^\circ, \quad \angle B = 53^\circ, \quad \angle C = 90^\circ
   \]

   \[
   \angle C = 90^\circ
   \]

   \[
   \text{(Extra praise if a P remembers that the sum of the angles in any triangle is } 180^\circ\text{.)}
   \]

d) *Prove that } \angle A + \angle B = 90^\circ\text{.}
   
   By calculation:
   
   \[
   \angle A + \angle B = 180^\circ - \angle C = 180^\circ - 90^\circ = 90^\circ
   \]

   By demonstration:
   
   Show that 2 congruent right-angled triangles can be joined to form a rectangle. (Draw a diagram or use cut-out triangles.)

   

e) *Reflect triangle ABC in the line AC.*

   i) *What shape is formed from the triangle and its mirror image?*

   (ABB' is an isosceles triangle, or a symmetrical triangle)

   ii) *What is the sum of the angles of the new shape?*

   \[
   \angle B = \angle B' = 53^\circ, \quad \angle BAC = \angle B'AC = 37^\circ
   \]

   \[
   \angle A = \angle BAC + \angle B'AC = 2 \times 37^\circ
   \]

   So

   \[
   \angle A + \angle B + \angle B' = 2 \times (37^\circ + 53^\circ)
   \]

   \[
   = 2 \times 90^\circ = 180^\circ
   \]

   \[
   26 \text{ min}
   \]
### Activity 4

**PbY6b, page 89**

**Q.2**

a) Read: *Complete this sketch to show the construction of a triangle. (Step 1 is already given.)*

What is *Step 1*? Ps come to BB to point to it on diagram and explain what it is.

**Step 1**

Draw a ray from a point B. Set the compasses to length \(a\) and mark the point C.

What should be done next? Ps come to BB to explain each step and write its number on the diagram on BB. Class agrees/disagrees. Ps number the steps on own diagrams.

**Step 2**

Set the compasses to length \(b\), then draw an arc around C.

**Step 3**

Set the compasses to length \(c\) and draw an arc around B. Label the intersection of the 2 arcs A.

**Step 4**

Join up AB and AC.

What would be different when drawing an isosceles triangle? (\(c = b\), so in Step 3, we keep the compasses at width \(b\) when drawing an arc around B.)

b) Read: *In your exercise book, construct this isosceles triangle.*

**Base: \(a = 3.5\) cm, Arms: \(b = c = 5\) cm**

**Set a time limit.** Review with whole class. T asks Ps to tell class what they did at each step and demonstrate on BB or OHP. Who did the same? Who did it a different way? Mistakes corrected.

Agree that when constructing an isosceles triangle, it does not matter if *Step 2* and *Step 3* are interchanged.

What is the sum of the angles in your triangle? (180°)  
\[
\angle A + \angle B + \angle C = 180°
\]

**32 min**

### Activity 5

**PbY6b, page 89**

**Q.3**

Read: *In your exercise book, draw a sketch to show your construction plan, then construct these isosceles triangles accurately. Label them appropriately.*

First discuss the appropriate labelling of triangles. (Vertices labelled in an anticlockwise direction using capital letters; sides labelled with lower case letters, \(a\) opposite A, \(b\) opposite B, \(c\) opposite C, the perpendicular height is labelled \(h\).)

Deal with one triangle at a time. Ps first draw a construction plan (sketch) in Ex. Bks. Review the plan with the whole class and make sure that Ps correct any errors before they do the actual construction.

Ps finished early could demonstrate the construction on BB or help slower neighbours.

### Notes

Whole class discussion on the required steps.

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

T helps Ps to explain clearly.

**BB:**

![Completed sketch]

**Actual construction**

![Actual construction]

**\(\angle A + \angle B + \angle C = 180°\)**

Individual work, monitored closely, helped, corrected

Discussion, agreement, self-correction, praising

or T could have triangles already prepared and uncover each as it is dealt with.
Activity

5 (Continued)

**Solution:**

a) \( a = 6 \text{ cm} \)
\( h = 3 \text{ cm} \)
\( \angle B = \angle C = 60^\circ \)

b) \( a = 4 \text{ cm} \)
\( \angle B = \angle C = 60^\circ \)
\( b = 3 \text{ cm} \)

c) \( a = 4.5 \text{ cm} \)
\( b = 3 \text{ cm} \)

---

**Notes**

In a) elicit that the perpendicular bisector of BC should be constructed, but accept construction using ruler and set square, or compasses (as shown in diagram).

In b), after agreement on the plan, revise how to use a protractor accurately.

In c), \( b = c = 3 \text{ cm} \)

---

**Lesson Plan 89**

6 **PbY6b, page 89**

Q.4  

a) Read: Measure the angles of the isosceles triangles you drew in Questions 2 and 3. Write your results below these sketches.

Set a time limit. Ps use protractors, approximating to the nearest degree. Ps can extend the sides of the triangles to make the task easier where necessary.

Review with whole class. Ps could show the angles on scrap paper or slates on command. Ps with inaccurate results should measure again and amend their answers in \( Pbs \).

T asks Ps to describe each triangle.

**Solution:**

- \( A = 40^\circ \)
- \( \angle A = 90^\circ \)
- \( \angle A = 60^\circ \)

\( \angle B = \angle C = 70^\circ \)
\( \angle B = \angle C = 45^\circ \)
\( \angle B = \angle C = 60^\circ \)

It is an **acute-angled** isosceles triangle.  
It is a **right-angled** isosceles triangle.  
It is a regular or **equilateral** triangle.

b) Read: Calculate the area of the shaded triangle.

Set a time limit of 1 minute. Ps could show area on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Who did the same? Who did it a different way? T makes sure that both the methods below are discussed.

**Solution:**

By drawing and calculation:
\[ A = 3 \text{ cm} \times 3 \text{ cm} = 9 \text{ cm}^2 \]

By calculation:
\[ A = \frac{a \times h}{2} = \frac{6 \times 3}{2} \text{ cm}^2 = 9 \text{ cm}^2 \]
\( (h \text{ is the perpendicular height of the triangle from its base}) \)

---

**Notes**

Individual measuring, monitored (helped)

Drawn on BB or use enlarged copy master for recording only

Differentiation by time limit

Responses given in unison.

Agreement, self-correction, praising
Factorising 90, 265, 440 and 1090. Revision, activities, consolidation

**PbY6b, page 85**

**Solutions:**

**Q.1**

A single translation (9.9 cm, 0 cm) can replace the two reflections.

**Q.2**

a) e.g.  

b) e.g.  

c) e.g.  

d) e.g.  

e) To reflect any polygon in any axis, reflect each of its vertices then join up their mirror images.

**Q.3**

a) **Sketch**  

Acute-angled isosceles triangle

(Actual size reduced by 1 : 2 here to fit on the page.)
**Activity**

*Solutions (Continued)*

Q.3  

b) **Sketch**

![Equilateral triangle](image)

Equilateral triangle

\[ d = e = f = 7 \text{ cm} \]

Any triangle which has all 3 angles 60° is equilateral.

Whole class activity

Extra praise if Ps notice that two triangles are possible:

GHI₁ obtuse-angled triangle  
GHI₂ acute-angled triangle

Whole class activity

In c): accept *acute-angled*.

In d):

T has various triangles already prepared for discussion. Ps check the ratio of the sides and measure the angles.

Ps could try to draw a 3: 4 : 5 triangle which does **not** contain a right angle.  
(Impossible!)

Q.4  

a) An **equilateral** triangle has angles of 60° and has three equal sides.

b) An **isosceles** triangle has at least 2 equal sides. (or angles)

c) An **equilateral** triangle is also an **isosceles** triangle.

d) A triangle which has sides in the ratio of 3 : 4 : 5 is a **right-angled** triangle.

e) A triangle with 3 different sides is called a **scalene** triangle.

f) There is no triangle which has a reflex angle.  
(or straight, or whole, or null)

g) The sum of the angles of any triangle is 180°.
### Activity

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(91 = 7 \times 13\)  
  Factors: 1, 7, 13, 91
- \(266 = 2 \times 7 \times 19\)  
  Factors: 1, 2, 7, 14, 19, 38, 2, 266
- \(441 = 3 \times 3 \times 7 \times 7 = 3^2 \times 7^2\)  
  (square number)  
  Factors: 1, 3, 7, 9, 21, 49, 63, 147, 441
- \(1091\) is a prime number  
  Factors: 1, 1091
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37 \(\times 37 > 1091\))

**8 min**

#### Triangles and quadrilaterals

T has a collection of triangles and quadrilaterals (and other polygons if T wishes) drawn or stuck on BB.

BB: e.g.

```
1  2  3  4  5  6  7
8  9 10 11 12 13
14 15 16 17 18
```

a) T points to each shape in turn. Ps say what they know about it.

1. Obtuse-angled triangle, scalene, asymmetrical, convex
2. Right-angled triangle, scalene, asymmetrical, convex
3. Right-angled isosceles triangle, 1 line of symmetry, convex  
   angles: 90°, 45°, 45°
4. Acute-angled triangle, scalene, asymmetrical, convex
5. Acute-angled isosceles triangle, 1 line of symmetry, convex
6. Obtuse-angled isosceles triangle, 1 line of symmetry, convex
7. Equilateral triangle, 3 lines of symmetry, rotational symmetry, each angle is 60°, convex
8. Quadrilateral, asymmetrical, concave
9. Trapezium, 2 parallel sides, asymmetrical, convex
10. Deltoid, 2 pairs of adjacent equal sides, 2 equal angles, 1 line of symmetry, its 2 diagonals intersect at right angles, convex
11. Deltoid, concave, 1 line of symmetry, etc.
12. Parallelogram, opposite sides equal and parallel, opposite angles equal, rotational symmetry of 180°, no line symmetry, convex

b) Let's think of ways to group the shapes. T and Ps suggest criteria and choose Ps to list the numbers of the relevant shapes. Class agrees/disagrees.

T and Ps discuss properties such as:
- Symmetry
- Angles
- Sides
- Concave
- Convex
- Equilateral
- Isosceles

**20 min**

---

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## Lesson Plan 91

### Notes

- Individual work, monitored, helped
- Drawn on BB or use enlarged copy master or oHP
- Differentiation by time limit
- Discussion, reasoning, agreement, self-correction, praising
- T could have cut-out triangles for each part so that Ps can demonstrate the reflections as a check, especially if there is disagreement.

<table>
<thead>
<tr>
<th>Activity</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY6b, page 91</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Q.1 Read:</strong> Reflect the triangles in the side indicated. Write the name of the polygon formed by the original shape and its mirror image.</td>
<td></td>
</tr>
<tr>
<td>Set a time limit. Do not expect precise construction but encourage Ps to use rulers and to be reasonably accurate. Review with whole class. Ps come to BB to reflect the required point and complete the shape, saying what the name of the shape is and why they think so. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td><img src="images" alt="Images of polygons" /></td>
<td></td>
</tr>
<tr>
<td><strong>Extension</strong></td>
<td>Discuss the properties of the shapes (sides, angles, diagonals, lines of symmetry, etc.).</td>
</tr>
<tr>
<td><strong>30 min</strong></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Activity</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PbY6b, page 91</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Q.2 Read:</strong> To the left of AC construct an isosceles triangle which has 2 cm arms. To the right of AC construct another isosceles triangle which has 3 cm arms. We say that AC is the common base of the two triangles. What kind of polygon have you formed?</td>
<td></td>
</tr>
<tr>
<td>First elicit the steps needed in the construction. (Set compasses to 2 cm, then draw arcs on LHS of AC with pointed end of compasses on A, then on C. Label B the point where the 2 arcs intersect. Join A and C to B. Repeat on RHS of AC but with compasses set to 3 cm and the point of intersection labelled D.) Set a time limit. Ps draw the polygon and write its name. Review with whole class. Ps show name on scrap paper or slates on command. Ps with different names explain why they chose them. T shows prepared diagram for discussion. Class agrees on the correct name of the shape. (Deltoid)</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td><img src="images" alt="Images of polygons" /></td>
<td></td>
</tr>
<tr>
<td><strong>Extension</strong></td>
<td>Elicit some properties of the deltoid ABCD (see above)</td>
</tr>
<tr>
<td><strong>34 min</strong></td>
<td></td>
</tr>
</tbody>
</table>
### Lesson Plan 91

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td>Individual work, monitored, helped</td>
</tr>
</tbody>
</table>

#### PbY6b, page 91

**Q.3 Read:** Reflect:

- **a)** point B in line AC. Join B and B’ to A and C. What is ABCB’?
- **b)** point B in line AC. Join B and B’ to A and C. What is ABCB’?
- **c)** the linear shape in line EF. What is AA’D’D’?

Set a time limit. Ps use rulers, compasses and set squares to measure and draw, then write the name of the shape in Pbs.

Review with whole class. Ps could show names on scrap paper or slates on command. Ps with different names come to BB to complete the drawing and explain why they chose that name. Class agrees on correct answer. Mistakes discussed and corrected.

Discuss the main properties of each reflection.

**Solution:**

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="https://example.com/deltoid.png" alt="Deltoid" /></td>
<td><img src="https://example.com/deltoid.png" alt="Deltoid" /></td>
<td><img src="https://example.com/trapezium.png" alt="Trapezium" /></td>
</tr>
<tr>
<td>Deltoid (convex)</td>
<td>Deltoid (concave)</td>
<td>Trapezium (symmetrical)</td>
</tr>
</tbody>
</table>

---

**40 min**

---

**PbY6b, page 91. Q.4**

Read: Complete the sentences. Draw an example of each quadrilateral in your exercise book

T chooses a P to read out each sentence, saying 'something' instead of the missing word. (Ps can draw diagrams in Ex. Bks or on scrap paper to help them decide or to check.)

Ps write missing word on scrap paper or slates and show on command. Ps with different words explain their reasoning, drawing diagrams on BB, with T's help if necessary. Class decides if they are correct. Ps write agreed word(s) in Pbs.

**Solution:**

- **a)** A quadrilateral is called a **parallelogram** if its diagonals **bisect** each other.
- **b)** A quadrilateral with equal angles is called a **rectangle**.
- **c)** A quadrilateral with equal sides is called a **rhombus**.
- **d)** A **regular** quadrilateral is called a **square**.
- **e)** A quadrilateral is called a **deltoid** if one of its diagonals lies on a line of symmetry.
- **f)** Every deltoid has two pairs of adjacent **equal** sides.
- **g)** Every rectangle is a **trapezium**, (or parallelogram)
- **h)** Every rhombus is a **deltoid**, (or parallelogram, or trapezium)

---

**45 min**

---

**Erratum**

In Pbs:

the box in a) should be longer.
**MEP: Primary Demonstration Project**

**Week 19**

### Activity

#### 1 Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 92 = 2 \times 2 \times 23 = 2^2 \times 23 \) Factors: 1, 2, 4, 23, 46, 92
- \( 267 = 3 \times 89 \) Factors: 1, 3, 89, 267
- \( 442 = 2 \times 13 \times 17 \) Factors: 1, 2, 13, 17, 26, 34, 221, 442
- \( 1092 = 2 \times 2 \times 3 \times 7 \times 13 = 2^2 \times 3 \times 7 \times 13 \) Factors: 1, 2, 3, 4, 6, 7, 12, 13, 14, 21, 26, 28, 1092, 546, 364, 273, 182, 156, 91, 84, 78, 52, 42, 39

Individual work, monitored

BB: 92, 267, 442, 1092
Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c}
& 92 & 2 & 1092 & 2 & 546 & 2 & 273 & 3 & 91 & 7 & 13 & 13 \\
& 46 & 2 & 221 & 13 & 17 & 17 \\
& 23 & 1 & 17 & 17 \\
& 89 & 3 & 442 & 2 & 1 \\
\end{array}
\]

#### 2 Symmetrical shapes (trapeziums)

**a) Sheet with circle**

On your circle draw any chord and label it AB. (Elicit that a chord is a straight line segment which has its start and end points on the circumference of the circle.)

Draw another chord, CD, which is parallel to AB. Join AD and BC. What shape is ABCD? (trapezium) T shows one on BB. Who drew this type of trapezium? What do you notice about it? (symmetrical)

How many lines of symmetry does this trapezium have? (1)

Who can show us where it is? (By folding the paper so that C lies on D and B lies on A and creasing the fold; or by drawing the perpendicular bisector of the 2 parallel chords.)

Ps draw the lines of symmetry on own diagrams. Elicit that the lines of symmetry passes through the centre of the circle, point O.

T: We call such a symmetrical trapezium a chord trapezium, because each of its sides is a chord of the same circle.

We say that the trapezium is inscribed in the circle (i.e. each of its 4 vertices are points on the circumference of the circle.)

Let's collect the properties of the chord trapezium on the BB. Ps suggest some, in words, or using mathematical notation on BB, adding extra labelling to diagram as necessary. Class agrees/disagrees.

**Elicit that**

- a rectangle is a chord trapezium with equal angles
- a square is a regular chord trapezium (i.e. equal angles and equal sides)

Whole class activity

Ps have 3 sheets of paper on which are drawn a circle, an isosceles triangle and a deltoid. T has shapes drawn on BB too (or has large cut-out models).

BB: e.g.

\[
\begin{array}{c}
A \\
\hline
M \\
\hline
B \\
\end{array}
\]

Ps follow instructions individually but are involved in discussions with the whole class.

Discussion, reasoning, agreement, praising

Or T asks Ps who drew any of these types to show their diagrams on BB and explain their main features.

(e.g. 2nd from left: base side is the diameter of the circle)

Agree that all symmetrical (chord) trapeziums can be inscribed in a circle.
Activity 2 (Continued)

b) Sheet with isosceles triangle

On your isosceles triangle, draw any line which is parallel with the base. Label the points like this. (T shows on BB.)

What is the quadrilateral ABCD? (a symmetrical or chord trapezium)

Ps come to BB to point out its line of symmetry and equal angles and sides. Class agrees/disagrees.

i) How could we write the perimeter of ABCD? Ps dictate to T or come to BB. Who agrees? Who thinks something else?

ii) How could we write the area of ABCD? Ps make suggestions.

If necessary, T directs their thinking by drawing a diagram (or by manipulating two cut-out pieces of the trapezium) and eliciting the values of the sides.

BB:

\[ P = a + b + c + d, \]

but \( d = b \),

so \[ P = a + 2b + c \]

iii) What is the sum of the angles in ABCD? e.g.

\[ \angle A = \angle B = \alpha = \beta \text{ and } \hat{ADC} = \hat{BCD} = \gamma = \delta \]

but at vertex D, we can see that \( \alpha + \delta = 180^\circ \) (straight angle)

so the sum of the angles of ABCD is:

\[ \alpha + \beta + \gamma + \delta = 2\alpha + 2\beta = 2 \times (\alpha + \beta) = 2 \times 180^\circ = 360^\circ \]

c) Sheet with deltoid

What do you notice about this shape? (symmetrical) How many lines of symmetry does it have? (1) Ps show it by folding the paper or by drawing. Ps label the vertices appropriately.

Who can tell me some properties of the deltoid? Ps come to BB or dictate to T, adding extra labelling as necessary. Class agrees/disagrees. T suggests some if necessary.

(e.g. \( AC \perp BD \), \( DM = MB \), \( a = d \), \( b = c \), \( \hat{ADC} = \hat{ABC} \))

i) How can we write the perimeter of the deltoid?

BB:

\[ P = a + b + c + d = 2a + 2b = 2 \times (a + b) \]

ii) How can we write the area of the deltoid?

e.g. \( A = \frac{e}{2} \times f \) (where \( e \) and \( f \) are the lengths of the diagonals)

iii) What is the sum of the angles at the vertices of the deltoid?

\[ \hat{C} + \hat{D} = 90^\circ \quad [180^\circ - 90^\circ] \]

\[ \hat{C} + \hat{B} = 90^\circ \]

\[ \hat{A} + \hat{C} + \hat{D} = 4 \times 90^\circ = 360^\circ \]

Notes

BB:

\[ P = a + b + c + d, \]

but \( d = b \),

so \[ P = a + 2b + c \]

(or show that angles \( \alpha + \delta \) form a straight angle in the rectangle formed in ii).

BB:

\[ A = \frac{e}{2} \times f \] (mirror images of each other)

T draws a diagram or cuts the deltoid into 4 pieces and forms a rectangle.

(as the sum of the angles in a triangle is 180° and the 3rd angle in each triangle is 90° )
Q.1  a) Read: Construct this deltoid accurately using the data given in the sketch

Set a short time limit. Ask Ps to think about the order of the steps first before doing the actual construction.
Review quickly with whole class. T chooses Ps to tell the class how they drew their deltoid, referring to the diagram on BB. Who did the same? Who did it a different way? etc. Class agrees on a good method.

Solution:

b) Calculate the area of the deltoid. (Find right-angled triangles.)

Solution:  e.g.

\[ A = \frac{2.5 \times 2}{2} + \frac{2.5 \times 2}{2} + \frac{2.5 \times 3}{2} + \frac{2.5 \times 3}{2} \]

or

\[ A = \frac{1}{2} \times 2.5 \times 2 + \frac{1}{2} \times 2.5 \times 3 = 5 + 7.5 = 12.5 \text{ cm}^2 \]

\[ A = 5 \text{ cm} \times 2.5 \text{ cm} = 12.5 \text{ cm}^2 \]

c) Measure the angles of the deltoid and add them together.

Ps use protractors, extending the sides of the deltoid where necessary.

Solution:

\[ \angle A = 80^\circ, \quad \angle B = \angle D = 89^\circ, \quad \angle C = 102^\circ \]

\[ \sum \text{angles} = 80^\circ + 2 \times 89^\circ + 102^\circ = 182^\circ + 178^\circ = 360^\circ \]

d) Measure the sides of the deltoid and add their lengths together.

\[ AB = AD = 3.9 \text{ cm}, \quad CB = CD = 3.2 \text{ cm} \]

\[ P = (3.9 \text{ cm} + 3.2 \text{ cm}) \times 2 = 7.1 \text{ cm} \times 2 = 14.2 \text{ cm} \]
PbY6b, page 92

Q.2 Read:

a) Complete the drawing of a rhombus. Label its vertices.

b) Calculate the area of the rhombus.

c) Measure its angles and add them together.

d) Measure its sides and calculate its perimeter.

Deal with one part at a time under a time limit and review before continuing with the next part.

Ps come to BB to complete the shape on BB and explain their construction. Who did the same? Who did it another way? Elicit the main properties of the rhombus. (It is a special deltoid.)

Ps show area, sum of angles and perimeter on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Who did it another way? Mistakes discussed and corrected.

Solution:

a)

\[
\text{Actual size}
\]

\[
\begin{align*}
\text{Shaded triangle} & \quad \text{already given} \\
\end{align*}
\]

\[
\begin{align*}
\text{b) } A &= \frac{2}{4} \times \frac{2 \times 1.5}{2} = 2 \times 3 = 6 \text{ (cm}^2) \\
\text{or} & \\
A &= 2 \text{ cm} \times 3 \text{ cm} \\
&= 6 \text{ cm}^2 \\
c) & \quad \angle A = \angle C \approx 106^\circ, \quad \angle B = \angle C \approx 74^\circ \\
& \quad \sum \text{angles} = 2 \times (106^\circ + 74^\circ) = 2 \times 180^\circ = 360^\circ \\
d) & \quad AB = AD = CB = CD \approx 2.5 \text{ cm} \\
& \quad P = 2.5 \text{ cm} \times 4 = 10 \text{ cm} \\
\end{align*}
\]

31 min

Q.3 Read:

a) Construct a square which has sides 3.5 cm long.

b) Calculate its area.

c) Calculate its perimeter.

d) Calculate the sum of its angles.

e) Draw and measure its diagonals.

f) Measure the angles formed by the diagonals.

Set a time limit. Make sure that Ps' diagrams are correct before they do parts b) to f). Ps use rulers, set squares and compasses. Remind Ps to label their diagrams.

Review with whole class. Ps come to BB to explain reasoning. Class agree/disagrees. Mistakes discussed and corrected.

Elicit that a square is a regular deltoid and a regular rhombus.

Notes

Individual work, monitored, helped, a) corrected (or whole class activity for b), c) and d) if Ps are unsure)

Drawn on BB SB or OHT

Ps can do calculations and sketch any supplementary diagrams in Ex. Bks.

Discussion, reasoning, agreement, self-correction, praising

T could have the 4 separate right-angled triangles cut out so that Ps can manipulate them on the BB.

Ps can label the 2 diagonals e and f.
YPbY6b, page 92

Q.4 Read: a) Construct a rectangle which has sides 4 cm and 3 cm long.
   b) Calculate its area. c) Calculate its perimeter.
   d) Calculate the sum of its angles.
   e) Draw and measure its diagonals.
   f) Measure the angles formed by its diagonals.

As for Activity 5. Elicit that a rectangle is neither a deltoid nor a rhombus and that not all rectangles are squares but every square is a regular rectangle.

Solution:

\[
\begin{align*}
\text{a) Actual size} \\
A &= 4 \times 3 = 12 \text{ (cm}^2) \\
P &= 2 \times (4 + 3) = 14 \text{ (cm)} \\
\sum \text{angles} &= 4 \times 90^\circ = 360^\circ \\
e &= f = 5 \text{ cm} \\
f) \text{Angles at the intersection of the diagonals are approximately } 74^\circ \text{ and } 106^\circ .
\end{align*}
\]

(2 adjacent angles form a straight angle)
**Activity**

**Y6**

**Notes**

**PbY6b, page 92, Q.5**

Deal with one part at a time. Ps measure or calculate individually then dictate to T (or show results on scrap paper or slates on command). Ps explain reasoning where relevant and class agrees on correct answer.

In d), T has two parts of the trapezium cut out to show how they can form a rectangle to make the calculation of the area easier.

**Solution:**

a) What is the name of this shape? (chord trapezium)

b) Measure its diagonals. 

e = f ≈ 6.4 cm

c) Measure its sides. (a = 6 cm, b = d = 4.1 cm, c = 4 cm)

d) Calculate its perimeter. 

\[ P = 6 \text{ cm} + 4 \text{ cm} + 2 \times 4.1 \text{ cm} \]

\[ = 10 \text{ cm} + 8.2 \text{ cm} = 18.2 \text{ cm} \]

e) Measure its angles and add them together.

\[ \angle A = \angle B = 76^\circ, \angle C = \angle D = 104^\circ \]

\[ \angle A + \angle B + \angle C + \angle D = 2 \times (76^\circ + 104^\circ) = 2 \times 180^\circ \]

\[ = 360^\circ \]

e) Calculate its area. 

\[ A = \frac{a + c}{2} \times h = \frac{6 + 4}{2} \times 4 = \frac{10}{2} \times 4 \]

\[ = 5 \times 4 = 20 \text{ cm}^2 \]

45 min
R: Geometric definitions
C: Practice: Reflection in a line. Symmetry. Construction.
E: Problems

Lesson Plan

Notes

Individual work, monitored (or whole class activity)
BB: 93, 268, 443, 1093
Ps may use calculators.
Reasoning, agreement, self-correction, praising

e.g.  268 2
     134 2
     67 67
     1

Whole class activity

Lines of symmetry

T has diagrams on BB or OHT.
Let's draw lines of symmetry on these diagrams. Ps come to BB to use BB instruments (or to OHT using own instruments) to construct the lines of symmetry, explaining in a loud voice what they are doing. Class makes helpful suggestions or points out errors where necessary.

BB:

1. \(\text{a)}\)
   \[\begin{array}{c}
   A \\
   B \\
   \hline
   m_1 \\
   m_2 \\
   \end{array}\]

2. \(\text{b)}\)
   \[\begin{array}{c}
   A \\
   e \\
   \hline
   m \\
   \end{array}\]

3. \(\text{c)}\)
   \[\begin{array}{c}
   A \\
   k \\
   \hline
   O \\
   m \\
   \end{array}\]

4. \(\text{d)}\)
   \[\begin{array}{c}
   a \\
   b \\
   \hline
   m_1 \\
   m_2 \\
   \end{array}\]

5. \(\text{e)}\)
   \[\begin{array}{c}
   O \\
   \hline
   m \\
   \end{array}\]

6. \(\text{f)}\)
   \[\begin{array}{c}
   k_1 \\
   k_2 \\
   \hline
   O \\
   \end{array}\]

In \(\text{d)}, eliciting/point out that when two lines intersect:
- opposite angles are equal
- the lines of symmetry are the \text{bisectors} of the angles (i.e. they cut them in half).

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:

- \(93 = 3 \times 31\) Factors: 1, 3, 31, 93
- \(268 = 2 \times 2 \times 67 = 2^2 \times 67\) Factors: 1, 2, 4, 67, 134, 268
- \(443\) is a prime number Factors: 1, 443
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and \(23^2 > 443\))
- \(1093\) is a prime number Factors: 1, 1093
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31
  and \(37 \times 37 > 1093\))

8 min

15 min

Individual work, monitored

(If possible, P's have own version on desks too, so that they can check the lines of symmetry by folding the paper. Otherwise T has versions on sheets of paper.)
Reasoning, agreement, checking, praising

T makes sure that the main points of each construction are stressed.

In \(\text{e)}, eliciting or remind P's that the line \(t\) is a \text{tangent} to the circle (i.e. the line and the circle share one common point, T).
### Activity

<table>
<thead>
<tr>
<th>Lesson Plan 93</th>
</tr>
</thead>
</table>

#### Notes

- **Whole class activity**
- Discussion, reasoning, agreement, checking, praising
- Involve several Ps
- Extra praise if a P thinks of the method without help from T.

#### Constructing angles

**a)** How could we construct an angle of 60° using a ruler and compasses?

Ps make suggestions and come to BB to show their methods. T give hints if Ps have no ideas. [e.g. What shape do you know has 60° angles? (equilateral triangle) What is special about an equilateral triangle? (equal sides, as well as equal angles, so can be inscribed in a circle).]

If Ps still cannot think what to do, T leads Ps through the construction, involving them where possible, while rest of class construct the angle in *Ex. Bks.* too. Ps check that the angle is 60° with a protractor.

**Method**

1. Draw a ray, \( e \), from a point A.
2. Mark a point on \( e \) and label it B.
3. Set compasses to length AB, then draw an arc with radius AB around A.
4. Keeping compasses set to length AB, draw an arc with radius AB around B.
5. Draw a ray from A through the point where the two arcs intersect. Label it \( f \).

**b)** How could we draw an angle of 30° using a ruler and compasses?

(Draw an angle of 60°, then halve it.) What do we need to draw to halve the angle? (Construct the bisector of the angle.)

Ps come to BB to amend the angle on BB (or draw another 60° angle first) explaining what they are doing. Ps copy steps in *Ex. Bks*.

**Method:**

1. Construct an angle of 60° (as previously).
2. With compasses at the same width, mark points on \( e \) and \( f \) which are equal distances from A.
3. Draw arcs with equal radii around the marked points.
4. Draw a line from A through the point where the two arcs intersect.

Ps check that the angle is 30° using a protractor.

---

**PbY6a, page 93**

**Q.1 a)** Read: *Construct an equilateral triangle with 4 cm sides. Label its vertices.*

Set a short time limit. Review with whole class. Ps come to BB or OHP to show and explain the construction. Who did the same? Who did it a different way?

(e.g. Draw AB 4 cm long. Set compasses to 4 cm and draw arcs around A and B. Label C the point of intersection of the the two arcs. Join C to A and B.)

Ps compare their own drawings with that on BB and correct any mistakes.

What is the perimeter of triangle ABC? \( (P = 12 \text{ cm}) \)

What is the sum of its angles? \( (3 \times 60° = 180°) \)
Y6

Activity

(Continued)

b) Read: **Reflect it in the line BC. Label the mirror image of A with D. What kind of shape is ABDC?**

Allow Ps to use any method they wish but give extra praise to Ps who use only compasses to mark point D.

Review with whole class. Ps show name of shape on scrap paper or slates on command. (ABDC is a **rhombus**.)

Elicit the main features of rhombus ABCD. (4 equal sides, opposite sides parallel, diagonals meet at right angles, \(\angle A = \angle D = 60^\circ\), \(\angle B = \angle C = 2 \times 60^\circ = 120^\circ\))

c) Read: **Reflect the second triangle in line BD. Label the mirror image of C with E.**

Ps can use compasses to mark point E, or Ps might realize that they can extend AB by 4 cm. Accept both methods.

d) Read: **What shape do the three triangles form altogether? Measure or calculate its angles and add them together.**

Set a short time limit. Ps show name, then sum of angles on scrap paper or slates on command. Ps with different responses explain reasoning. Class agrees on correct answer.

**Solution:**

The 3 triangles form a symmetrical or chord **trapezium.**

Elicit that the line of symmetry is the **perpendicular bisector** of AE and CD (passing through point B)

---

Lesson Plan 93

Notes

T could have rhombus already constructed, or choose quicker Ps to demonstrate and explain construction on BB.

BB:

T has larger version already prepared for Ps to compare with their own drawings (or manipulates cut-out triangles)

Responses shown in unison.

Reasoning, agreement, self-correction, praising

\[\angle A + \angle E + \angle D + \angle C = 2 \times (60^\circ + 120^\circ)\]

\[= 2 \times 180^\circ = 360^\circ\]

What is its perimeter?

\[P = 5 \times 4 \text{ cm} = 20 \text{ cm}\]

---

5

**PbY6b, page 93**

Q.2 Read: **Calculate the missing angles in the table if AB = AC and the given angle is:**

\[a) \alpha = 40^\circ \quad b) \gamma = 65^\circ \quad c) \gamma^* = 120^\circ.\]

What kind of triangle is ABC? (isosceles triangle) What do you notice about angles alpha and alpha star, beta and beta star, gamma and gamma star? (Each pair forms a straight angle of 180°.)

Set a time limit or deal with one row at a time.

Review with whole class. Ps come to BB to explaining reasoning and referring to diagram. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[
\begin{array}{cccccccc}
\alpha & \beta & \gamma & \alpha^* & \beta^* & \gamma^* & \alpha + \beta + \gamma & \alpha^* + \beta^* + \gamma^* \\
\hline
a) & 40^\circ & 70^\circ & 70^\circ & 140^\circ & 110^\circ & 110^\circ & 180^\circ & 360^\circ \\
b) & 50^\circ & 65^\circ & 65^\circ & 130^\circ & 115^\circ & 115^\circ & 180^\circ & 360^\circ \\
c) & 60^\circ & 60^\circ & 60^\circ & 120^\circ & 120^\circ & 120^\circ & 180^\circ & 360^\circ \\
\end{array}
\]

---

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

BD:

Discussion, reasoning, agreement, self-correction, praising

**Bold** values are given.
Q.3 Read: Each of the angles below is 60°. Construct:
a) a 45° angle on this diagram
b) a 120° angle on this diagram
c) a 90° angle on this diagram.

Deal with one at a time. T points out that the angles could be measured with a protractor, but the word 'construct' does not mean measure. Ps should try it using only compasses and ruler, then check their angles with a protractor.

Set a short time limit. Ps who think they have done it come to BB to demonstrate their methods to class. Who did the same? Who used a different method? If no P thought of labelling the marked points, T could suggest it so that the method can be explained more easily, as below. Mistakes corrected.

Solution: e.g.
a) Method
1. Set compasses and mark points on a and b which are an equal distance from A. Label the points B and C.
2. Draw arcs with radius AB around B and around C. Label D the point where they intersect.
3. From A, draw a ray, c, through point D. Ray c is the bisector of angle A.
4. Mark point E on ray c so that AE = AC.
5. Draw arcs with radius AC around C and around E. Label F the point where they intersect.
6. From A, draw a ray, d, through point F. Ray d is the bisector of angle CAD.
   The angle formed by rays a and d is 45°.

b) Method
1. Mark any point C on f.
2. Draw arcs with radius BC on the left of f around B and around C. Label D the point where the arcs intersect.
3. From B, draw a ray, g, through D. The angle formed by f and g is 60°. The angle formed by e and g is 120°.
(To explain the reasoning more easily, mark any point E on e.)

c) Method
1. Mark any point D on h.
2. Draw arcs with radius CD on the left of h around C and around D. Label E the point where the arcs intersect.
3. From C, draw a ray, i, through E.
   The angle formed by rays h and i is 60°.
4. Draw arcs with equal radius, between i and h, around D and around E. Label F the point where the arcs intersect.
5. From C, draw a ray through F and label it j. (j is the bisector of angle ECD.)
   The angle formed by rays g and j is 90°.
(To explain the reasoning more easily, mark any point G on g.)

37 min
PbY6b, page 93

Q.4 Read: Describe the steps needed to find the centre of the circle. A chord, AB, and its perpendicular bisector, line e, have been drawn.

Allow Ps a couple of minutes to write the steps and carry them out on the diagram as a check.

Review with whole class. T chooses Ps to read their descriptions while another P carries them out on diagram on BB or OHT. Who thought the same? Who thought of another way to do it? etc.

Class agrees on the correct form of words. Ps who were wrong or who could not write a description, copy the correct steps in Pbs, amending diagram appropriately.

Solution: e.g.

1. Mark another point, C, on the circumference and draw chord AC.
2. Construct the perpendicular bisector of AC. (Draw arcs of equal radius around A and around C. Draw a line through the 2 points of intersection). Label it f.
3. The point where e and f intersect is the centre of the circle. Label it O.

Elicit that OA = OB = OC, as they are radii of the circle.

41 min

PbY6b, page 93

Q.5 Read: Construct a trapezium which has these dimensions.

Base: a = 5.2 cm, Height: h = 3.4 cm, \( \angle \alpha = 60^\circ \)

Allow Ps a couple of minutes to think about it and try it out in Ex. Bks. Extra praise if Ps notice that there is not enough information given (as side b could be any length \( \geq 3.4 \) cm).

T accepts this, or suggests that Ps draw a chord (symmetrical) trapezium, so that both base angles are 60°.

Ps dictate the steps and T carries them out on a larger scale on BB using BB instruments. Class agrees/disagrees. Mistakes discussed and corrected.

Solution: e.g.

1. Draw base AB of length 5.2 cm.
2. Lay bottom edge of set square along AB and draw a line perpendicular to AB. Label it h.
3. Mark a point on h which 3.4 cm from AB.
4. Using a set square, draw a line through this marked point which is perpendicular to h (and also parallel to AB).
5. Construct an angle of 60° at A. (Mark a point E on AB. Draw arcs with radius AE around A and around E. Draw a line segment from A through the point where the arcs intersect to meet the line parallel to AB at D.)
6. Draw a line segment from B to meet the line parallel to AB at C. (Ps choose where they want C to be.)

45 min
### Lesson Plan

#### Activity

<table>
<thead>
<tr>
<th>Factorisation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:</td>
</tr>
<tr>
<td>• 94 = 2 × 47  Factors: 1, 2, 47, 94</td>
</tr>
<tr>
<td>• 269 is a prime number  Factors: 1, 269 (as not exactly divisible by 2, 3, 5, 7, 11, 13 and 17^2 &gt; 269)</td>
</tr>
<tr>
<td>• 444 = 2 × 2 × 3 × 3 × 37 = 2^2 × 3 × 37  Factors: 1, 2, 3, 4, 6, 12, 37, 74, 111, 148, 222, 444</td>
</tr>
<tr>
<td>• 1094 = 2 × 547  Factors: 1, 2, 547, 1094 (547 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, and 29 × 29 &gt; 547)</td>
</tr>
</tbody>
</table>

#### Notes

- Individual work, monitored (or whole class activity)
- BB: 94, 269, 444, 1094  
  T decides whether Ps can use calculators.
- Reasoning, agreement, self-correction, praising
- Whole class activity
  - Diagrams drawn by Ps on BB or OHT
  - (alternatively, T has points and lines already prepared)
  - At a good pace.
  - In good humour.
  - Involve majority of (all) Ps.
  - Class points out errors or missed steps.
  - Ps need only do freehand drawings as long as they give correct explanations and note important information on the diagrams.
  - Reasoning, agreement, praising

#### Reflection

- a) A, come and mark a point on the BB and label it P. B, come and draw a line on the BB and label it m. C, come and reflect P in m. C explains in a loud, clear voice. e.g. BB: 
  - 1. Draw a perpendicular line from P to m. Label M the point where the lines meet. 
  - 2. Extend the line by the same distance on the opposite side of m. 
  - 3. Label P' the mirror image of point P. (PM = PM', PP' ⊥ m, i.e. m is the perpendicular bisector of PP')

- b) D, come and draw a line segment and label it PQ. E, come and draw an axis and label it m. F, come and reflect PQ in m. F explains loudly and clearly. e.g. BB: 
  - 1. Reflect point P in m and label its mirror image P'. (F explains method as above.) 
  - 2. Reflect point Q in m and label its mirror image Q'. (As above) 
  - 3. Join P' to Q'. P'Q' is the mirror image of PQ. (PQ = P'Q', a = a', PE = EP', QF = FQ', m ⊥ PP', m ⊥ QQ')  
  What shape is PPQ'Q'? (symmetrical, or chord, trapezium)
2 (Continued)

(AB = A'B', AB and A'B' intersect on line m at point M, AM = A'M, BM = B'M, AA' \perp m, BB' \perp m, AA' \parallel BB')

Ps also give details of each reflection, as in b).

Ps label the shapes and mirror images appropriately. Praising, encouragement only. Feedback for T

3 Symmetrical solids

T has various solids on desk/table in front of class. Ps come to front of class one after the other to choose a solid and say what they know about it. (Name, number of faces, edges and vertices, type of faces, etc.) Class agrees/disagrees or points out any main feature missed.

Is this shape symmetrical? Stand up if you think it is. Ps standing show where the planes of symmetry are and class agrees on how many there are. e.g.

Whole class activity

Some shapes could be made from multilink cubes, or prepared so that they can be split along the relevant planes of symmetry and the parts shown as being congruent.

If possible, T has isometric diagrams of the shapes on BB too, as shown opposite. (At least one shape should not have planar symmetry.) At a good pace.

Reasoning, demonstration, agreement, praising Agree that a sphere or cylinder has an infinite number of planes of symmetry.
Q.1 Read: Divide the whole (360°) central angle of the circle into 3 equal parts.

Draw the radii and join up the 3 points where the radii meet the circumference.

What shape have you drawn?

What size will each angle be? (360° ÷ 3 = 120°)

Deal with one step at a time, with T demonstrating on BB and Ps working in Pbs.

Ps use protractors to measure 120° angles, mark points, then lay a ruler on each marked point and the centre point and draw straight lines from the centre of the circle to the circumference. Ps join up the 3 points on the circumference and label them appropriately.

What shape have we drawn? (equilateral triangle)

Let's collect its properties. Ps dictate to T and T writes on BB using mathematical notation where possible. T prompts where necessary (e.g. the angles at the base of the internal isosceles triangles and the number of lines of symmetry the shape has).

**Solution:**

BB: **Equilateral triangle**

- \( \angle A = \angle B = \angle C = 60° \)
- \( AB = BC = AC \)
- \( OA = OB = OC \) (radii of circle)
- \( OA \) bisects \( \angle A \), so \( \triangle OAB \cong \triangle OBC \cong \triangle OCA \) (isosceles triangles)

T: We can also call the shape a regular triangle.

If we rotate the triangle around O, after how many degrees will it line up with its original position?

(120°, 240°, 360°, 480°, 600°, . . . , i.e. multiples of 120°)

How many times does this happen in one complete turn? (3)

T: We say that the regular triangle has rotational symmetry of 120°, or has rotational symmetry of order 3, around O.

Ps shout out in unison.

Whole class discussion

Agreement, praising

T could have the shape cut out, so that it can be folded to check the number and positions of the lines of symmetry.

(3 lines of symmetry)

(as its angles and sides are equal)

T pins cut-out shape on top of the diagram to show the rotations.
### Q.2 Read:

*Divide the whole (360°) central angle of the circle into 4 equal parts.*

*Draw the radii and join up the 4 points where the radii meet the circumference.*

*What shape have you drawn?*

Set a time limit. Ps carry out construction as in Activity 4 and write the name of the shape in Pbs. Ps finished early write properties of the shape in Ex. Bks. in preparation for the discussion.

Review with whole class. Ps show name on scrap paper or slates on command. P with correct name explains to Ps who were wrong. Mistakes corrected.

T has shape already prepared on BB or OHT (or P finished first draws it, hidden from rest of class). Elicit its properties.

**Solution:**

**BB:** Square  

e.g.  \( \angle A = \angle B = \angle C = \angle D = 90^\circ \)  
\( AB = BC = CD = DA \)  
\( OA = OB = OC = OD \) (radii)  
\( OA \) bisects \( \angle A \), so  
\( \triangle BAO \cong \triangle DAO \cong \triangle OCD \cong \triangle ODA \)  
(right-angled, isosceles triangles)

T: We can also call the shape a regular quadrilateral.

If we rotate the square around O, after how many degrees will it line up with its original position?  
(90°, 180°, 270°, 360°, 450°, ... i.e. multiples of 90°)  
How many times does this happen in one complete turn? (4)

T: We say that the square has rotational symmetry of 90°, or has rotational symmetry of order 4, around O.  

### Notes

Individual work, monitored, helped, corrected  
(or whole class activity as in Activity 4 if Ps are unsure)

Circle drawn on BB or use enlarged copy master or OHP

Differentiation by time limit and by extra task

Name shown in unison.  
Agreement, self-correction, praising

Whole class activity  
Involve many Ps.

Praising, encouragement only  
T prompts where necessary.

(4 lines of symmetry)  
(a quadrilateral with equal sides and angles)

Demonstrate rotation with cut-out square pinned to diagram on BB.
**Activity 6**  
*PbY6b, page 94*

Q.3 Read: *Divide the whole (360°) central angle of the circle into 5 equal parts.*  
*Draw the radii and join up the 5 points where the radii meet the circumference.*  
*What shape have you drawn?*

Deal with this activity in a similar way to *Activity 5.*

**Solution:**

BB: **Pentagon**  
\[ \angle A = \angle B = \angle C = \angle D = \angle E = 108^\circ \]
\[ AB = BC = CD = DE = EA \]
\[ OA = OB = OC = OD = OE \]
OA bisects \( \angle A \), so  
\[ BAO = EAO = 54^\circ, \text{etc.} \]
\[ \triangle OAB \cong \triangle OBC \cong \triangle OCD \cong \triangle ODE \cong \triangle OEA \]  
(isosceles triangles)

T: We can also call the shape a **regular pentagon**.

If we rotate the pentagon around O, after how many degrees will it line up with its original position?

\((72^\circ, 144^\circ, 216^\circ, 288^\circ, 360^\circ, \ldots, \text{i.e. multiples of } 72^\circ)\)

How many times does this happen in one complete turn?  (5)

T: We say that the pentagon has rotational symmetry of \( 72^\circ \), or has rotational symmetry of **order 5**, around O.

**35 min**

---

**Activity 7**  
*PbY6b, page 94*

Q.4 Read: *Divide the whole (360°) central angle of the circle into 6 equal parts.*  
*Draw the radii and join up the 6 points where the radii meet the circumference. What shape have you drawn?*

Deal with this activity in a similar way to *Activity 5.*

**Solution:**

BB: **Regular Hexagon**  
\[ \angle A = \angle B = \ldots = \angle F = 120^\circ \]
\[ AB = BC = CD = DE = EF = FA \]
\[ OA = OB = OC = OD = OE = OF \]
OA bisects \( \angle A \), so  
\[ BAO = FAO = 60^\circ, \text{etc.} \]
\[ \triangle OAB \cong \triangle OBC \cong \triangle OCD \cong \triangle ODE \cong \triangle OEF \cong \triangle OFA \]  
(regular triangles)

If we rotate the hexagon around O, after how many degrees will it line up with its original position?

\((60^\circ, 120^\circ, 180^\circ, 240^\circ, 300^\circ, 360^\circ, \ldots, \text{i.e. multiples of } 60^\circ)\)

How many times does this happen in one complete turn?  (6)

T: We say that the pentagon has rotational symmetry of \( 60^\circ \), or has rotational symmetry of **order 6**, around O.

**40 min**

---

**Notes**

Individual work, monitored, helped, corrected under a time limit, then reviewed and properties discussed with the whole class, as in *Activity 5*

Ps measure \( \angle A \), etc. with protractors.

(5 lines of symmetry)

(as it is a pentagon with equal angles and equal sides)

Demonstrate rotation with cut-out pentagon pinned to diagram on BB.

---

**Lesson Plan 94**

Individual work, monitored, helped, corrected under a time limit, then reviewed and properties discussed with the whole class, as in *Activity 5*

Demonstrate rotation with cut-out hexagon pinned to diagram on BB.
**Activity 8**

PbY6b, page 94

Q.5 Read: Divide the whole (360°) central angle of the circle into 8 equal parts.

*Draw the radii and join up the 8 points where the radii meet the circumference.*

What shape have you drawn?

Deal with this activity in a similar way to Activity 5.

**Solution:**

BB: Regular Octagon

\[
\angle A = \angle B = \ldots = \angle H = 135°
\]

AB = BC = CD = DE = \ldots = HA

OA = OB = OC = OD = \ldots = OH

OA bisects \( \angle A \), so

\[
\text{BAO} \cong \text{HAO} = 67.5°, \text{etc.}
\]

\[
\Delta OAB \cong \Delta OBC \cong \Delta OCD \cong \Delta ODE \cong \Delta OEF \cong \Delta OFG \cong \Delta OGH \cong \Delta OHA
\]

(isosceles triangles)

If we rotate the octagon around O, after how many degrees will it line up with its original position? (45°, 90°, 135°, 180°, 225°, 270°, 315°, 360°, \ldots, i.e. multiples of 45°)

How many times does this happen in one complete turn? (8)

T: We say that the octagon has rotational symmetry of 45°, or has rotational symmetry of order 8, around O.

**Notes**

Individual work, monitored, helped, corrected under a time limit, then reviewed and properties discussed with the whole class, as in Activity 5

BB: \( 360° \div 8 = 45° \)

BB: \( 135° \div 2 = 67.5° \)

(8 lines of symmetry)

Demonstrate rotation with cut-out octagon pinned to diagram on BB.
Factorising 95, 270, 445 and 1095. Revision, activities, consolidation

**PbY6h, page 95**

**Solutions:**

Q.1

a) Line symmetry: 3, 4, 5, 7, 8, 10, 12, 13, 14
b) Rotational symmetry: 4, 7, 12, 13
c) Regular: 4, 7, 12
d) At least 1 obtuse angle: 1, 9, 10, 11, 14
e) Only acute angles: 4, 5, 6,
f) Trapezium: 9, 12, 13, 14
g) Deltoid: 8, 10, 12,
h) Rhombus: 12
i) Not a polygon: 7

Q.2

a) i) \[ a = 4.5 \text{ cm} \]

b) ii) \[ c = 3 \text{ cm} \]

\[ \begin{align*}
\angle A &= 35^\circ \\
\angle B &= 35^\circ \\
\angle C &= 180^\circ - 35^\circ - 35^\circ = 110^\circ \\
\end{align*} \]

\[ a = h = 10 \text{ cm} \]
Solutions (Continued)

Q.2  a) iv)  
\[ b = 37 \text{ mm} \]
\[ a = 25 \text{ mm} \]
\[ c = 43 \text{ mm} \]

Q.3  a) e.g.

b) Steps e.g.
1. Construct the perpendicular bisector of PQ. Label it \( e \).
2. Choose a point R on the circumference and draw the chord QR.
3. Construct the perpendicular bisector of QR. Label it \( f \).
4. Label 'O' the point where \( e \) and \( f \) intersect.
   O is the centre of the circle.

v)  e.g.
\[ d_2 = 6 \text{ cm} \]
\[ d_1 = 9.5 \text{ cm} \]

vi) Such a trapezium is impossible!

b) Accept any valid statement.

Q.4  
\[ 360^\circ \div 10 = 36^\circ \]
\[ \angle A = 144^\circ \]
\[ 144^\circ \div 2 = 72^\circ \]
**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 96 = 2 \times 2 \times 2 \times 2 \times 3 = 2^5 \times 3 \)
  
  Factors: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

- \( 271 \) is a prime number
  
  Factors: 1, 271
  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13 and \( 17^2 > 271 \))

- \( 446 = 2 \times 223 \)
  
  Factors: 1, 2, 223, 446

- \( 1096 = 2 \times 2 \times 2 \times 137 = 2^3 \times 137 \)
  
  Factors: 1, 2, 4, 8, 137, 274, 548, 1096

**Notes**

Individual work, monitored (or whole class activity)

BB: 96, 271, 446, 1096

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

E.g.

\[
\begin{array}{c|c|c|c|c|c}
96 & 2 & 446 & 2 & 271 & 223 \\
1096 & 2 & 24 & 2 & 1096 & 2 \\
446 & 2 & 12 & 2 & 548 & 2 \\
24 & 2 & 137 & 3 & 274 & 2 \\
271 & 1 & 137 & 1 & & \\
\end{array}
\]

**Extension**

Discuss whether each solid has planar and/or rotational symmetry.

**Activity 2**

**Properties of regular polyhedra**

T has various models on desk at front of class.

What do all these shapes have in common? (3-D solids, they have only plane faces) Who remembers the name of a 3-D shape which has many plane faces? (a **polyhedron**) T writes it on BB.

These solids are special kinds of polyhedra (plural of polyhedron).

T holds up each solid in turn. Elicit or tell Ps the name of each solid [Ps might guess the **hexahedron** (6 faces) and **octahedron** (8 faces)] then elicit its properties. T prompts only if necessary.

BB: Polyhedra

e.g. Tetrahedron  Hexahedron  Octahedron

\[
\begin{array}{c|c|c}
\text{Tetrahedron} & \text{Hexahedron} & \text{Octahedron} \\
6 \text{ edges} & 12 \text{ edges} & 12 \text{ edges (equal)} \\
4 \text{ faces: (congruent, regular triangles)} & 6 \text{ faces: (congruent squares)} & 8 \text{ faces: (congruent, regular triangles)} \\
4 \text{ vertices} & 8 \text{ vertices} & 6 \text{ vertices} \\
\end{array}
\]

T elicits/points out that in each case:

BB: edges + 2 = faces + vertices or \( e + 2 = f + v \)

T: Polyhedra which have congruent, regular faces (i.e. their faces have equal sides and equal angles) are called regular polyhedra.

Involve several Ps.

Reasoning/demonstrating, agreement, praising only

(Ps might remember this from Y5 as Euler's Formula)

Ps show the planes of symmetry and centres of rotation on the diagrams.
**Centre of rotation**

Study these shapes. Which of them has a point around which the shape can be rotated so that when it is turned, it covers itself exactly? (In addition to those below, T could also show shapes which do not have rotational symmetry.)

Where are these central points? Ps come to BB to point to them (or to construct them). Class agrees/disagrees. T helps if necessary. T checks that the points are in the correct position by pinning and rotating cut-out transparent shapes on top of the diagrams.

T: We call such a point the centre of rotation of the shape.

What is the smallest angle of rotation needed for the shapes to line up? T asks 2 or 3 Ps what they think and why. Class agrees/disagrees.

BB:

- It has rotational symmetry of 120°
- It has rotational symmetry of 90°
- It has rotational symmetry of 72°
- It has rotational symmetry of 45°
- It has rotational symmetry of 60°

**Q.1 Read: List the numbers of the shapes which match the descriptions.**

Set a time limit. Encourage Ps to answer by studying the diagrams, without using protractors to help them.

Review with whole class. Ps could show numbers on scrap paper or slates on command. Ps with different answers explain their reasoning at BB and class decides who is correct. Ps mark the lines of symmetry and centres of rotation on relevant diagrams on BB. Mistakes discussed and corrected.

**Solution:**

- a) It has line symmetry. [1, 3, 5]
- b) It has rotational symmetry. [1, 3, 4, 5, 6]
- c) It has rotational symmetry of 60°. [5]
- d) It has rotational symmetry of 120°. [3]
- e) It has rotational symmetry of 72°. [1, 6]
- f) It has rotational symmetry of 90°. [4]
- g) It has rotational symmetry of 180°. (None has smallest angle of rotation as 180°, but 4 and 5 can be rotated by this angle and cover themselves exactly.)
**Activity 5**

**PbY6b, page 96**

Q.2 Read: *Mark the centre of rotation. Write the smallest angle of rotation.*

First discuss how to find the centre of rotation. (Elicit that only 2 diagonals or perpendicular bisectors of sides are needed, as their point of intersection is the centre of the shape.)

Set a time limit. Ps use rulers and compasses to mark the points, then write an operation to calculate the angle of rotation.

Review with whole class. Ps could show the smallest angles of rotation on slates or scrap paper. Ps with correct answers explain reasoning at BB, showing how they constructed the centres of rotation. Who did the same? Who found it another way? etc.

Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
    360^\circ \div 5 &= 72^\circ \\
    360^\circ \div 8 &= 45^\circ \\
    360^\circ \div 3 &= 120^\circ \\
    360^\circ \div 6 &= 60^\circ 
\end{align*}
\]

Which shapes also have line symmetry? (a, b, c)

---

**Notes**

Individual work, monitored, helped, corrected

Drawn on BB or use enlarged copy master or OHP

Initial whole class discussion on method of construction.

Responses shown in unison.

Reasoning, agreement, self-correction, praising

T demonstrates constructions where necessary.

---

**Activity 6**

**PbY6b, page 96**

Q.3 Read:

- a) *Reflect points A and B in point O.*
- b) *Join up the points A, B', A', B and A in order. What shape have you formed?*
- c) *Join A to A' and B to B'. What do you notice?*

Set a time limit. Ps join A to O then extend the line by the same distance on the opposite side of O. Ps write the name of the shape below it in Pbs and write what they notice in Ex. Bks.

Review with whole class. Ps could show name on scrap paper or slates on command. Ps with correct answer come to BB demonstrate the construction, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Ps tell class what they noticed about the completed shape. (e.g. diagonals bisect each other.) Elicit other properties too.

**Solution:**

\[
\begin{align*}
    a) & \quad \text{Pallelogram} \\
    b) & \quad \text{e.g. Properties} \\
    & \quad AB = A'B', \ AB \parallel A'B' \\
    & \quad AB' = A'B, \ A'B \parallel B'A \\
    & \quad AO = OA', \ BO = B'O, \\
    & \quad \angle A = \angle A', \ \angle B = \angle B' \\
    c) & \quad \text{e.g. } ABA'B' \text{ has rotational symmetry of } 180^\circ \text{ (or of order 2) about the point } O; \ AO = OA', \ BO = OB'.
\end{align*}
\]

---

**Notes**

Individual work, monitored, helped

Points drawn on BB or SB or OHT

Discuss the procedure first.

Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

Praising, encouragement only

Feedback for T

‘Order 2’ means that twice during a whole turn, the shape covers its original position exactly. (at 180° and 360°).
Lesson Plan 96

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 7  | **PbY6b, page 96**

**Q.4** Read: *Draw any lines of symmetry and mark the centres of rotation.*

Set a time limit. Ask Ps to write the name of any shape they know. Review with whole class. Ps come to BB to draw and mark, explaining what they are doing. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

What are the properties of each shape?

**Solution:**

a) ![Diagram a](image1)

b) ![Diagram b](image2)

c) ![Diagram c](image3)

d) ![Diagram d](image4)

Elicit that:

a) is a hexagon

b) is a parallelogram.

**Extension**

What do we know about any regular polygon? (It has equal sides and equal angles; it can be inscribed in a circle.) We should keep this in mind when we make the polygon.

Ps come to BB to stick the triangles on BB to form a polygon. Class points out any errors. (e.g. 30° angles should be at the centre) Before the shape is completed, ask 2 or 3 Ps how many triangles they think will be needed and why. Let's see if you are correct! Ps complete the polygon and check that there are 12 triangles.

a) **How many vertices does the polygon have?** (12)

b) **How many sides does the polygon have?** (12)

Who remembers the name of a 12-sided polygon? (duodecagon)

**Extension**

**Q.5** Read: *Form a regular polygon with congruent triangles so that the line segments from the centre of the polygon to its vertices divide the whole central angle into angles of 30°.*

T has congruent isosceles triangles cut from coloured paper on desk. T holds one up. What kind of triangle is this? (isosceles) Which angle is 30°? P points to it. If this angle is 30°, what size are the other 2 angles? How could we calculate it? Ps come to BB or dictate what T should write. Class agrees/disagrees.

BB: \((180° - 30°) ÷ 2 = 150° ÷ 2 = 75°\)

Agree that each congruent triangle has angles of 30°, 75° and 75°.

What do we know about any regular polygon? (It has equal sides and equal angles; it can be inscribed in a circle.) We should keep this in mind when we make the polygon.

Ps come to BB to stick the triangles on BB to form a polygon. Class points out any errors. (e.g. 30° angles should be at the centre) Before the shape is completed, ask 2 or 3 Ps how many triangles they think will be needed and why. Let's see if you are correct! Ps complete the polygon and check that there are 12 triangles.

a) **How many vertices does the polygon have?** (12)

b) **How many sides does the polygon have?** (12)

Who remembers the name of a 12-sided polygon? (duodecagon)

b) **What size are its angles?** (150°) BB: 75° + 75° = 150°

c) **What is the sum of its angles?** (1800°) BB: 150° × 12 = 1800°

T labels vertices and midpoint and Ps say true statements about the polygon. Class agrees/disagrees. (sides, angles, symmetry, etc.)

Whole class activity

(If possible, Ps have triangles on desks too.)

Use copy master, enlarged and cut out, or use as one thick card template for Ps to draw around on BB or OHT.

Discussion, reasoning, agreement, checking, self-correcting, praising

Extra praise for unexpected properties (e.g. angle or order of rotation) or if Ps suggest labelling points in c) and d) to make explanation of the properties easier.

At a good pace. Involve several Ps.

BB: 360° ÷ 30° = 12 (times)

or 30° × 12 = 360°

BB:

12-sided polygon
Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **97** is a prime number
  - Factors: 1, 97
  - (as not exactly divisible by 2, 3, 5, 7 and 11² > 97)

- **272** = 2 × 2 × 2 × 2 × 17
  - Factors: 1, 2, 4, 8, 16, 17, 34, 68, 136, 272

- **447** = 3 × 149
  - Factors: 1, 3, 149, 447

- **1097** is a prime number
  - Factors: 1, 1097
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37² > 1097)

Revision of polygons
What can you tell me about the 2 triangles on your desk? (right-angled, congruent)

a) Form different polygons with the 2 triangles and draw the shape you have made in your Ex. Bks. Write the name of the shape below it. T monitors closely and chooses Ps to draw their different shapes on the BB (Ps could use a triangular template to help them). Elicit the name of the polygon and what was done to one triangle to get the position of the other triangle.

BB: e.g.

- **isosceles triangle**
  - (Reflection in AC)
  - (Reflection in BC)

- **deltoid**
  - (Reflection in AB)

- **parallelogram**
  - (Reflection in midpoint of BC)

- **rectangle**
  - (Rotation by 90° around C)

- **concave quadrilateral**
  - (Reflection in midpoint of AB)

b) Measure the acute angles of triangle ABC using a protractor.
  - (e.g. \( \angle A = 35°, \angle B = 55° \))
  - Calculate the sum of its angles in your Ex. Bks.

BB: \( \angle A + \angle B + \angle C = 35° + 55° + 90° = 180° \)

How could we prove it?

[The two triangles can form a rectangle, AC’BC: In the rectangle, \( \angle A + \angle B = 90°, \angle C = 90° \)
  - So \( \angle A + \angle B + \angle C = 90° + 90° = 180° \)]
**Activity**

(Continued)

c) Calculate the sum of the angles in this polygon. (T points to it.)
Ps do calculation in Ex. Bks then dictate to T. Class agrees/disagrees.

\[ \angle B = \angle B' = 55^\circ, \]

\[ B' \overrightarrow{AB} = \overrightarrow{CA} + \overrightarrow{B'C} = 2 \times 35^\circ \]

so \[ \sum \text{angles} = 2 \times (55^\circ + 35^\circ) = 2 \times 90^\circ = 180^\circ \]

d) Calculate the sum of the angles in this polygon. (T points to it.)
Ps do calculation in Ex. Bks then dictate to T. Class agrees/disagrees.

\[ \angle C = \angle C' = 90^\circ, \quad \angle B = \angle B' = 55^\circ, \]

\[ \overrightarrow{BA} = \overrightarrow{AB} + \overrightarrow{B'C} = 35^\circ \]

so \[ \sum \text{angles} = 2 \times (90^\circ + 55^\circ + 35^\circ) = 2 \times 180^\circ = 360^\circ \]

e) Calculate the sum of the angles in this polygon. (T points to it.)
Ps might realise that the calculation is the same as d):

\[ \sum \text{angles} = 2 \times (90^\circ + 55^\circ + 35^\circ) = 2 \times 180^\circ = 360^\circ \]

**Notes**

BB: [Diagram of a triangle]

(Ps might remember that the sum of the angles in any triangle is 180°.)

BB: [Diagram of a quadrilateral]

(Ps might remember that the sum of the angles in any quadrilateral is 360°.)

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Encourage Ps to mark the equal angles in the 4 triangles.

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Ps who were wrong, or think that their way of writing the calculations is not as good, copy those on BB.

Extension

What do you notice about the triangles in question iv)?

(Each small triangle is similar to the large triangle. The ratio of the large triangle to each small triangle is 2 : 1.)
**Activity**

4  *PbY5b, page 97*

**Q.2** Read: *The two triangles have been formed from congruent triangles.*

a) Measure the angles of the small internal triangles and of the large triangles.

b) **Prove** that the sum of the angles in each triangle is 180°.

Set a time limit. Ps use protractors to determine which angles are equal and mark them using the appropriate notation.

Review at BB with whole class. Ps come to BB or dictate the equal angles to T. (e.g. \( \angle AFD = \angle DEB = \angle FCE \))

Class agrees/disagrees. Mistakes corrected.

How can we **prove** that the sum of the angles is 180°? Ps tell their ideas. Who agrees? Who thinks something else? etc. Class agrees on the clearest way to write the 'proof'. Mistakes corrected.

**Solution:**

a)  

b) e.g. At D, the 3 angles form a **straight** angle, which is 180°

\[ \angle AFD + \angle DEB + \angle FCE = 180° \]

What else can you tell me about the triangles? e.g.

- The large triangle is **similar** to each of its small triangles.
- The ratio of the large triangle to a small triangle is 2 : 1.
- AC || DE, BC || DF, FE || AB
- D, E and F are the midpoints of the sides of the triangle.

**Extension**

**PbY6b, page 97**

**Q.3** Read:  

a) **Mark the centres of rotation.**

b) **By how many degrees has each shape been rotated?**

c) **Draw on the diagrams the paths taken by the vertices when they were rotated.**

Deal with one part at a time or set a time limit.

Elicit/remind Ps that the centre of rotation must be an **equal** distance from each point in a corresponding pair. Ask Ps to label the rotated triangle appropriately. Ps use compasses to draw the paths of rotation.

Review with whole class. Ps come to BB to mark points, write the angles of rotation and draw arcs, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Notes**

Individual work in measuring and marking, monitored, helped

Drawn on BB or use enlarged copy master or OHP

T tells Ps that they need not write the actual sizes of the angles on their diagrams – just mark equal angles with the same number of arcs.

Discussion, reasoning, agreement, self-correction, praising

Extra praise for Ps who noticed that the 3 different angles in both diagrams form a straight line.

(Or at E or at F)

Whole class activity

Praising, encouragement only

These apply to both diagrams.

**Individual work, monitored, helped**

(or whole class activity if Ps are unsure what to do)

Drawn on BB or use enlarged copy master or OHP

(In ii), Ps will need to construct the perpendicular bisectors of, e.g. AA' and BB'.

The point of intersection of the 2 bisectors is the centre of rotation.)

Reasoning, agreement, self-correction, praising
### Activity 5

**Solution:**

1. **i)**
   - Rotation by $90^\circ$ around $A$

2. **ii)**
   - Rotation by $90^\circ$ around $O$

3. **iii)**
   - Rotation by $90^\circ$ around $C$

---

### Lesson Plan 97

#### Notes

**Solution:**

**Construction**

1. Set compasses to width $AB$ and draw an arc around $A$.
   - The point where the arc cuts the horizontal line is $B'$.
2. Set compasses to width $BC$ and draw an arc around $B'$.
3. Set compasses to width $AC$ and draw an arc around $A$.
   - The point of intersection of the 2 arcs is $C'$.
4. Join $A$ and $B'$ to $C'$.

$\triangle AB'C'$ is the position of $\triangle ABC$ after the 1st turn onto $AB$.

Continue in a similar way for the next 2 turns onto $B'C'$ and $A'C''$, constructing the vertices in the appropriate order. Ps might notice that the 1st and 4th positions have the same orientation.

What do you notice about the angles of rotation? e.g.

1st rotation around $A$ is $180^\circ - \angle A$

2nd rotation around $B'$ is $180^\circ - \angle B'$

3rd rotation around $C''$ is $180^\circ - \angle C''$

and then these 3 steps are repeated.

---

**PbY6b, page 97**

Q.4 Read: Draw the paths of the vertices when the triangle is turned over along the straight line. (Use compasses.)

Set a time limit of 3 minutes. (Ps could also have a template of triangle $ABC$ to manipulate before construction and to check that they are correct afterwards.) Ask Ps to label the vertices of the triangles they draw.

Review with whole class. (T could have diagram already prepared or Ps finished early construct the solution on BB or OHT, as a model for slower Ps to follow.) Ps correct any mistakes.

Let's mark the equal angles. Ps come to BB, while rest of Ps mark the angles in $Pbs$.

**Solution:**

**Construction**

1. Set compasses to width $AB$ and draw an arc around $A$.
   - The point where the arc cuts the horizonal line is $B'$.
2. Set compasses to width $BC$ and draw an arc around $B'$.
3. Set compasses to width $AC$ and draw an arc around $A$.
   - The point of intersection of the 2 arcs is $C'$.
4. Join $A$ and $B'$ to $C'$.

$\triangle AB'C'$ is the position of $\triangle ABC$ after the 1st turn onto $AB$.

Continue in a similar way for the next 2 turns onto $B'C'$ and $A'C''$, constructing the vertices in the appropriate order. Ps might notice that the 1st and 4th positions have the same orientation.

What do you notice about the angles of rotation? e.g.

1st rotation around $A$ is $180^\circ - \angle A$

2nd rotation around $B'$ is $180^\circ - \angle B'$

3rd rotation around $C''$ is $180^\circ - \angle C''$

and then these 3 steps are repeated.

---

Individual work, monitored, helped, corrected

Drawn on BB or SB or OHT (or T has large cut-out triangle so that Ps can demonstrate the turns to the whole class)

Differentiation by time limit.

Discussion, demonstration, agreement, self-correction, praising

Elicit the steps from Ps if possible.

T points this out if no P notices it.
**Activity 7**

PbY6b, page 97

Q.5 Read: Construct:  
  a) a 90° angle  
  b) a 45° angle  
  c) a 240° angle.

Deal with one at a time. The more able Ps could be asked to construct each angle in 2 different ways. Ps should use only compasses and rulers (‘construct’ does not mean ‘measure’).

T monitors closely, choosing Ps to demonstrate and explain their constructions to the class. Who did the same? Who did it a different way? Come and show us.

Solution: e.g.

\[ \begin{align*}  
\text{a)} & \quad \text{perpendicular bisector of AB} \\
& \quad \begin{align*} & 60^\circ + \frac{60^\circ}{2} \\
& = 90^\circ \quad \text{or} \\
& 180^\circ + \frac{60^\circ}{2} = 150^\circ \quad \text{or} \\
& \frac{180^\circ}{2} \text{ or } \frac{60^\circ}{2} \quad \text{or} \\
& \frac{30^\circ + 30^\circ}{2} \end{align*} \\
\text{b)} & \quad \begin{align*} & 45^\circ \quad \text{or} \\
& \left( \frac{60^\circ + 60^\circ}{2} \right) \div 2 \quad \text{or} \\
& \frac{180^\circ}{2} \div 2 \quad \text{or} \\
& 30^\circ + 30^\circ \div 2 \quad \text{or} \\
& \frac{180^\circ}{2} \div 2 \end{align*} \\
\text{c)} & \quad \begin{align*} & 240^\circ \quad \text{or} \\
& 60^\circ \times 4 \quad \text{or} \\
& \frac{180^\circ}{2} \quad \text{or} \\
& \frac{180^\circ + 60^\circ}{2} \end{align*} \\
& \quad \text{or} \\
& 45 \text{ min} 
\end{align*} \]

**Lesson Plan 97**

- Individual work, monitored, helped, corrected
- If necessary, quickly revise how to construct a 60° angle (equilateral triangle) and how to bisect an angle.
- Reasoning, demonstration, (self-correction), praising
- Accept any valid method.
- Feedback for T

T could demonstrate the methods not used by Ps and ask if they are correct.

If time runs out, ask Ps to complete the activity at home.

Review before the start of Lesson 98.

[Ps should have their own sets of mathematical instruments.]
Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- \(98 = 2 \times 7 \times 7 = 2 \times 7^2\)  
  Factors: 1, 2, 7, 14, 49, 98
- \(273 = 3 \times 7 \times 13\)  
  Factors: 1, 3, 7, 13, 21, 39, 91, 273
- \(448 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 7 = 2^6 \times 7\)  
  Factors: 1, 2, 4, 7, 8, 14, 16, 28, 32, 56, 64, 112, 224, 448
- \(1098 = 2 \times 3 \times 3 \times 61 = 2 \times 3^2 \times 61\)  
  Factors: 1, 2, 3, 6, 9, 18, 61, 122, 183, 366, 549, 1098

8 min

Angles 1

Each pair of Ps has angle models cut from coloured paper. T has larger versions stuck on BB.

BB:

a) Find equal angles.
Ps can use protractors to measure the angles (or extra praise for Ps who put one angle on top of the other with the vertices lined up to see if the arms of the angles coincide). Ps dictate the equal angles and T writes on BB. Ask for the type of angle too. Class agrees/disagrees. Agree that the length of the arms does not determine the angle size! (T extends the arms of an angle on BB to show this.)

b) Put the angles in increasing order. (Equal angles can be laid one on top of the other or one below the other.) Ps dictate to T and T writes on BB. Class agrees/disagrees. (See below)

c) What size are these angles? (Ps give accurate measurements if they used protractors or estimates if they did not.)

BB: (If using copy master)
\[
\angle B = \angle E < \angle A = \angle D < \angle C = \angle J < \angle F = \angle G < \angle H = \angle I
\]
\[
(30^\circ) \quad (90^\circ) \quad (120^\circ) \quad (230^\circ) \quad (320^\circ)
\]

15 min
**Lesson Plan 98**

### Activity

**Angles 2**

Let's revise what we know about angles.

What is an angle? (It is a turn around a point.)

What unit do we use to measure angles? (Degrees, angle minutes, angle seconds) What is the relationship between them?

BB: (1 degree) $1^\circ = 60'$ (60 angle minutes),

$1' = 60''$ (angle seconds)

What tool do we use to measure angles? (protractor) T shows it.

What types of angles are there? Let's start from the smallest. Ps dictate the names and size ranges. Class points out errors.

BB:  Null angle $= 0^\circ$ (no turn)

$0^\circ < \text{Acute angle} < 90^\circ$

Right angle $= 90^\circ$ (arms perpendicular, quarter of a turn)

$90^\circ < \text{Obtuse angle} < 180^\circ$

$180^\circ = \text{Straight angle}$ (straight line, half a turn)

$180^\circ < \text{Reflex angle} < 360^\circ$

Whole angle $= 360^\circ$ (a complete turn)

a) T (P) says an angle type and Ps show examples on slates or scrap paper on command. T (P) points out errors.

b) T (P) draws an angle on BB and Ps say its name.

c) Everyone stand up! I will say an angle size and you show me what you think it roughly looks like using 2 pencils (or 2 rulers). Point to the angle you want me to look at.

T: e.g. $60^\circ$, $150^\circ$, $270^\circ$, $100^\circ$, $340^\circ$, etc.

20 min

### Notes

Whole class activity
At a good pace
In good humour.
Involves all Ps.
Elicit that angle minutes and angle seconds are so small that only computers or scientists use them in calculations.

Ps could come to BB to draw an example for each type. Class agrees/disagrees.
Elicit that we show the angle by drawing an arc between its 2 arms.

Responses shown in unison.
Agreement, praising
T quickly checks all Ps with a large protractor, praising or correcting where necessary.

Individual work, monitored, helped in measuring
Drawn on BB or use enlarged copy master or OHP
Differentiation by time limit.
Responses shown in unison.
Demonstration, agreement, self-correction, praising

Show the 2 methods of measuring reflex angles: e.g.

$\angle C = 180^\circ + 90^\circ = 270^\circ$

or

$\angle C = 360^\circ - 90^\circ = 270^\circ$

$\angle E = 180^\circ + 120^\circ = 300^\circ$

or

$\angle E = 360^\circ - 60^\circ = 300^\circ$
### Activity

#### PbY6b, page 98

**Q.2 Read:** Work in your exercise book.

*Draw two parallel lines, then draw a line which crosses both of them. Label the angles as shown in the sketch.*

T monitors closely to make sure that P’s diagrams are correctly drawn and labelled. Ps use 2 rulers (or ruler and set square) to draw the parallel lines. Then Ps mark them with arrows. (The arcs indicating the angles can be drawn freehand.)

**Read:**

- a) Measure the angles formed and write down the values.
- b) List the angles which are equal.
- c) Find other relationships among the angles.

Set a time limit. Ps measure angles and write statements. Review with whole class. T chooses 3 or 4 Ps to come to BB or dictate their angles and findings. Although we have different angle sizes depending on where we drew line w, what general statements can we make about the angles? Ps come to BB or dictate to T. Ps check that it is also true for their angles. (Ask Ps who disagree to measure again!)

**Solution:**

- **a)** e.g. Let’s mark the equal angles on a general diagram so that we can see the pattern more clearly.

- **b)** $\angle A = \angle D = \angle E = \angle H$

- $\angle B = \angle C = \angle F = \angle G$

- **c)** e.g. There are 4 equal acute angles and 4 equal obtuse angles. (Altogether, they form 8 right or 4 straight angles.)

- $\angle A + \angle B = \angle C + \angle D = \angle A + \angle C = \angle B + \angle D = 180^\circ$

- $\angle E + \angle F = \angle G + \angle H = \angle E + \angle G = \angle F + \angle H = 180^\circ$

- $\angle A + \angle B + \angle C + \angle D = \angle E + \angle F + \angle G + \angle H = 360^\circ$

**Extension**

T: Angles such as A and D, formed when 2 lines intersect are called **opposite** angles. **Opposite** angles are always equal. Angles such as A and E, formed when a line crosses 2 parallel lines, are in corresponding positions and are called **corresponding** angles. **Corresponding** angles are always equal. Angles such as C and F, on alternate sides of the line which intersects the parallel lines, are called **alternate** angles. **Alternate** angles are always equal. Angles such as A and B, or C and E, together form a straight angle. They are called **complementary** angles. **Complementary** angles always sum to $180^\circ$.

---

### Lesson Plan 98

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Make sure that P’s have a correct diagram before they attempt the questions.

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Agree that the sum of any two angles which form a straight angle (i.e. a straight line) must be $180^\circ$.

**BB: General diagram**

Extra praise if a P reasons like this:

$\angle A + \angle B = 180^\circ$, but $\angle A + \angle C = 180^\circ$ so $\angle B = \angle C$

T shows it if no P does so.

Ps point out other pairs of:

**opposite** angles: B and C, E and H, F and G

**corresponding** angles: B and F, C and G, D and H

**alternate** angles: D and E

**complementary** angles: C and D, E and F, G and H, D and F, A and C, etc.
**Activity**

6  

*PbY6b, page 98*

Q.3 Read: *Calculate the sizes of the unknown angles.*  
First ask Ps to say the Greek letters labelling the angles.  
T reminds Ps if they have forgotten. (alpha, beta, gamma, delta)  
Deal with one part at a time or set a time limit. Tell Ps that their general diagram for Q.2 will help them in parts a) and c).  
Review with the whole class. Ps come to BB or dictate to T, explaining reasoning. T repeats Ps’ reasoning in a clearer way if necessary. Who agrees? Who worked it out another way? etc. Mistakes discussed and corrected.

**Solution:**  
e.g.  
\[ \alpha = 58^\circ \text{ (opposite angles)} \]
\[ \beta = 180^\circ - 58^\circ = 122^\circ \]
\[ \gamma = 58^\circ \text{ (corresponding angles)} \]
\[ \delta = 180^\circ - 58^\circ = 122^\circ \]

b)  
\[ \beta = 180^\circ - 90^\circ - 58^\circ = 90^\circ - 58^\circ = 32^\circ \]  
(\( \Sigma \) angles in any triangle is 180\(^\circ \))

c)  
e.g. \( \Delta ABD = \Delta ABC \)  
(equal base and height)

so \( \alpha = \angle BCA = 180^\circ - 90^\circ - 67^\circ = 90^\circ - 67^\circ = 23^\circ \)
\[ \gamma = \angle CAB = 67^\circ \text{ (alternate angles)} \]

In \( \Delta AMD, \angle DAM = 90^\circ - 67^\circ = 23^\circ \),  
so \( \delta = 180^\circ - 23^\circ - \alpha = 157^\circ - 23^\circ = 134^\circ \)
\[ \beta + \delta = 180^\circ, \text{ so } \beta = 180^\circ - \delta = 180^\circ - 134^\circ = 46^\circ \]

d)  
\[ \gamma = 180^\circ - 71^\circ - 36^\circ = 109^\circ - 36^\circ = 73^\circ \]  
(\( \Sigma \) angles in any triangle is 180\(^\circ \))

35 min
PbY6b, page 98

Q.4 a) Read: Construct these angles in your exercise book and write their names below them.

Deal with one at a time or set a time limit. Ps use only compasses and rulers.

T monitors closely, choosing Ps to demonstrate and explain their constructions to the class. Who did the same? Who did it a different way? Come and show us. Mistakes corrected, or construction done again.

Solution: e.g.

\[
\begin{align*}
60^\circ + 60^\circ & \div 2 = 75^\circ \\
180^\circ + 60^\circ & \div 2 = 105^\circ \\
\end{align*}
\]

\(\text{acute angle}\)

b) Read: Draw an angle of 40°.

As we are not asked to construct this angle, how can we draw it? (Use a protractor)

Ps suggest what to do first and how to continue. T (P) works on BB or OHT while rest of class work in Ex. Bks. e.g.

1. Draw a straight line, \(e\), and mark on it a point, A.
2. Place the protractor so that its zero line lies along \(e\) and its 90 degree line is on A.
3. Make a mark at the 40° line on the right of A.
4. Remove protractor and draw a ray from A through the marked point to form the 2nd arm of the angle. Label it \(f\).
5. Draw an arc around A from \(e\) to \(f\) to show the angle and write 40° inside it.

Solution:

\[
\begin{align*}
45^\circ & + 90^\circ = 135^\circ \\
\end{align*}
\]

\(\text{obtuse angle}\)

Whole class activity but individual drawing

Revision of how to draw an angle using a protractor.

Discussion, agreement on the steps.

T monitors, helps, corrects.

Stress the difference between drawing an angle using a protractor and constructing an angle using only compasses and ruler.

Which method do you think is more accurate? T asks several Ps what they think and why.

(Ps will probably think that using a protractor is more accurate, as there are more chances of inaccuracies in a construction.)
### Activity 8

**PbY6b, page 98**

Q.5 Read: *Calculate with angles. (1° = 60')*

What other unit of measuring angles do we know and what is its relation to those given? Ps dictate to T. (BB)

Set a time limit of 3 minutes. Ps can work in *Pbs* or in *Ex. Bks* if they need more space.

Review with whole class. Ps who have an answer show results on scrap paper or slates on command. Ps answering correctly come to BB to explain reasoning to Ps who were wrong. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>22° 20'</td>
<td>179° 60'</td>
<td>71° 103'</td>
</tr>
<tr>
<td></td>
<td>38° 30'</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+</td>
<td>75° 75'</td>
<td>137° 05'</td>
<td></td>
</tr>
<tr>
<td></td>
<td>111° 28'</td>
<td></td>
<td>43° 52'</td>
</tr>
<tr>
<td></td>
<td>2°</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(120' = 2°)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **d)** $16° 42' \times 5 = 80° + 210' = 80° + 3° + 30' = 83° 30'$
- **e)** $13° 24' \div 6 = 12° 84' \div 6 = 2° 14'$
- **f)** $173° 15' \div 10 = 17.3° + 1.5' = 17° + 0.3 \times 60' + 1' + 30''$
  $= 17° + 18' + 1' + 30''$
  $= 17° 19' 30''$

or

$173° 15' \div 10 = 170° 195' \div 10 = 17° 19.5' = 17° 19' 30''$

### Notes

Individual work, monitored, less able Ps helped

Written on BB or SB or OHT

BB: 1' = 60"

(1 angle minute = 60 angle seconds)

Differentiation by time limit.

Discussion, reasoning, agreement, self-correction, praising

Accept any valid method of calculation.

Ps say what type of angle each result is (and estimate what it looks like using 2 pencils)

Feedback for T

(or $\frac{3}{10}$ of $1° = \frac{3}{10}$ of $60'$

= 18')
R: Calculations
C: Measures: time; money, converting £s to other currencies
E: Percentage problems

**Y6**

**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- \( 99 = 3 \times 3 \times 11 = 3^2 \times 11 \)
  
  Factors: 1, 3, 9, 11, 33, 99
- \( 274 = 2 \times 137 \)
  
  Factors: 1, 2, 137, 274
- \( 449 \) is a prime number
  
  (not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and \( 23^2 > 449 \))
- \( 1099 = 7 \times 157 \)
  
  Factors: 1, 7, 157, 1099

**Notes**

Individual work, monitored (or whole class activity)

BB: 99, 274, 449, 1099

Ps could practise calculation without calculators.

Reasoning, agreement, self-correction, praising

\[ \begin{align*}
99 & = 3 \times 3 \times 11 = 3^2 \times 11 \\
274 & = 2 \times 137 \\
449 & \text{ is a prime number} \\
1099 & = 7 \times 157
\end{align*} \]

**Activity 2**

**Measures: revision**

a) **Mass**

T has various objects on desk at front of class and various types of weighing equipment. [e.g. a balance with different weights (1 kg, 500 g, 200 g, 100 g, 50 g, 20 g, 10 g, 5 g, 2 g, and 1 kg), scales with metric and imperial units and metric digital scales]

Let's measure the mass of each of these objects. What is mass?

Ps come to front of class to choose an object and weigh it using one type of equipment, then another P weighs it on a different type. Ps write the values on BB. Class discusses the different forms of the same mass and the pros and cons of each type of weighing machine.

Repeat for different objects. (Pupils could be weighed too!)

Let's list the metric units of mass. Ps dictate to T in unison.

BB: 1 gram (g) < 1 kilogram (kg) < 1 tonne (t) \times 1000 \times 1000

Are there any smaller or greater units? Ps say what they know, otherwise T tells them and writes them on BB.

BB: Smaller: 1 milligram (1 mg) = 0.001 g

Greater: 1 kilogramme (kg) = 1000 kg

1 Megatonne (Mt) = 1 million tonnes = 1 billion kg

b) **Capacity**

T has various spoons, cups, glasses, bottles, measuring jugs and cylinders and a bucket of water at front of class.

What is capacity? (How much liquid a container can hold)

Let's measure the capacity of each of these containers.

Ps come to front of class to choose a measuring container and to use it to measure the capacity of other containers.

Elicit or remind Ps that 'milli' means \( \frac{1}{1000} \)

'kilo' means 1000 times

Whole class activity

Have bathroom scales too.

At a good pace.

In good humour.

Involve several Ps.

Discussion, agreement, praising

Agreement, praising

Ps could write the units of mass on the back page of Pbs as a reminder.

Elicit or remind Ps that

Ps explain to class what they are doing and write any calculations on BB.

Reasoning, agreement, praising

\[ \begin{align*}
1 \text{ litre} & \times 1000 = 1000 \text{ cl} \\
1 \text{ cl} & \times 1000 = 1 \text{ litre}
\end{align*} \]
Activity

2 (Continued)

Let’s list the units of capacity. Ps dictate to T or come to BB.

BB: 1 millilitre (1 ml) < 1 centilitre (1 cl) < 1 litre (1 l)

\times 10 \times 100

These units measure capacity as how much liquid a container can hold but we can also think of capacity as the volume of space inside a container. Let’s list the corresponding values of capacity and volume. Ps dictate to T if they know it or can work it out.

BB:
1 ml → 1 cm³ (1 ml of pure water can fill a 1 cm cube)
1 cl → 10 cm³ (1 cl . . . . . . . . . . . . 10 cm cubes)
1 litre → 1000 cm³ (1 litre . . . . . . . . . . . . a 10 cm cube)
1000 litres → 1 m³ (1000 litres . . . . . . a 1 m cube)

Who remembers a connection between length, capacity and mass? Allow Ps to explain if they can, otherwise T reminds them.

Using pure water at sea level and at a temperature of 4°C:

BB: 1 litre → 1000 cm³ → 1 kg (Ps write it in Pbs.)

Notes

Agreement, praising
Ps could write the units on back page of Pbs as a reminder.

Discussion, reasoning, agreement, praising
If possible, T has 1 cm and 10 cm open cubes for demonstration.
Ps write corresponding values on back page of Pbs.

How can we check it?
(Weigh an empty 1 litre jug, then fill it with water and weigh it again. The difference is the mass of 1 litre of water.)

3 PbY6b, page 99

Q.1 Read: The temperature was 16°C at 07:00.
   a) By 12:00 the temperature had risen by 60%. What was the temperature at 12:00?
   b) By 18:00, the mid-day temperature had fallen by 60%. What was the temperature at 18:00?

Set a time limit of 3 minutes. Ps write a plan, do the calculation, check the result and write the answer as a sentence.

Review with whole class. Ps could show answer on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who worked it out a different way? Come and show us what you did. Mistakes discussed and corrected.

Solution: e.g.

a) Plan: 16 + 60% of 16 = 16 + 0.6 × 16 = 16 + 9.6

= 25.6 (°C)

or 160% of 16 = 16 ÷ 100 × 160

= 0.16 × 160 = 1.6 × 16 = 25.6 (°C)

Answer: At 12:00 the temperature was 25.6 degrees Celsius.

b) Plan: 25.6 – 60% of 25.6 = 25.6 – 0.6 × 25.6

= 25.6 – 15.36 = 10.24 (°C)

or 40% of 25.6 = 25.6 × 0.4 = 2.56 × 4 = 10.24 (°C)

Answer: At 18:00 the temperature was 10.24 degrees Celsius.

Lesson Plan 99

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising
Deal with all methods used by Ps and accept any correct method of solution.

Feedback for T

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Q.2 Read: On 20 November 2003, 1 EUR (Euro) was worth 0.7021 GBP (£).

a) Calculate the value of 1 GBP in Euros on that day.

b) i) If 1 GBP = 1.42 EUR, what is the Euro equivalent of 532 GBP?

ii) What percentage of 1 Euro is 1 GBP?

c) i) If 1 EUR = 0.7 GBP, what is the GBP equivalent of 532 Euros?

ii) What percentage of 1 GBP is 1 Euro?

Why is a certain date given in the question? (The exchange rate of currencies change from day to day. These values were correct on that day but might not be correct now!)

Deal with one part at a time. Ps suggest what to do first and how to continue, with T prompting where necessary. Ps come to BB or dictate to T. Class agrees/disagrees. Ps work in Ex. Bks. at the same time. Ps do any calculations on BB without a calculator as revision, then use a calculator to check the result.

Elicit/remind Ps that to find the value of a whole unit from a known part, divide the whole by the known part.

**Solution:**

ea) £0.7021 = 1 Euro

\[ 1 \div 0.7021 = 1.42429853 \]

\( \approx 1.4243 \) Euros

T shows Ps how to check with a calculator. Ps follow.

\[ 1 \div 0.7021 = 1.42429853, \text{ which is 1.4243 correct to 4 d.p.} \]

Answer: On 20 November 2003, £1 was worth 1.4243 Euros.

b) i) £1 = 1.42 Euros

\[ 532 \times 1.42 = 755.44 \text{ Euros} \]

Answer: 755.44 Euros is the equivalent of 532 GBP.

ii) £1 = 1.42 of 1 Euro = 142% of 1 Euro

Answer: One GBP is 142 percent of one Euro.

c) i) 1 Euro = £0.7

\[ 532 \times 0.7 = 372.40 \text{ GBP} \]

Answer: 372.40 GBP is the equivalent of 532 Euros.

ii) 1 Euro = 0.7 of £1 = 70% of £1

Answer: One Euro is 70 percent of one GBP.

**Notes**

Whole class activity

If possible, T has a few of the relevant coins and notes to pass round class. Ps tell class if they know about any of them (e.g. Ps might have seen or used them on holiday)

Discuss the countries in which the currencies are used and ask Ps to point them out on a world map.

[Ps could be asked to find out what today's exchange rates are for homework (from newspapers or the internet) ]

Discussion, reasoning, agreement, checking, praising

At a good pace.

Involve many Ps.

T tells Ps that it is usual to give the answer correct to the same number of decimal places as the value in the question, unless stated otherwise.

So to find a result correct to 4 decimal places, we need to calculate to 5 decimal places so that we can round correctly.

BB: 1.42429 \( \approx 1.4243 \) (to 4 d.p.)

\[ \begin{array}{cccccc}
0 & 1 & 4 & 2 & 4 & 9 \\
\end{array} \]

\[ \begin{array}{cccccc}
1 & 4 & 2 & 4 & 9 & 3 \\
\end{array} \]

\[ \begin{array}{cccccc}
1 & 4 & 2 & 4 & 9 & 8 \\
\end{array} \]

\[ \begin{array}{cccccc}
1 & 4 & 2 & 4 & 9 & 6 \\
\end{array} \]

\[ \begin{array}{cccccc}
1 & 4 & 2 & 4 & 9 & 5 \\
\end{array} \]

\[ \begin{array}{cccccc}
1 & 4 & 2 & 4 & 9 & 4 \\
\end{array} \]
### Activity

**PbY6b, page 99**

Q.3 Read: On 20 November 2003, 1 GBP was worth 1.6998 USD ($).

- **a)** If 1 GBP = 1.7 USD, how many £s can you get for 1 USD?
- **b)** i) If 1 GBP = 1.7 USD, what is the USD equivalent of 532 GBP?
   
   ii) What percentage of 1 USD is 1 GBP?
- **c)** i) If 1 USD = 0.59 GBP, what is the GBP equivalent of 532 USD?
   
   ii) What percentage of 1 GBP is 1 USD?

#### Solution: e.g.

- **a)** 1.7 USD = £1
  
  \[1 \text{ USD} = \frac{\text{£1}}{1.7} = \frac{10}{17} = \text{£0.5882} \text{ (to 4 d.p.)} \]
  
  **Answer:** You can get 0.5882 GB Pounds for one US Dollar.

- **b)** i) \£1 = 1.7 USD
  
  \[532 = 1.7 \text{ USD} \times 532 = 904.40 \text{ USD} \]
  
  **Answer:** 904.40 US Dollars is the equivalent of 532.
  
  ii) \£1 = 1.7 of 1 USD = 170% of 1 USD
  
  **Answer:** One pound is 170 percent of one US Dollar.

- **c)** i) 1 USD = £0.59
  
  \[532 \text{ USD} = 0.59 \times 532 = £313.88 \]
  
  **Answer:** £313.88 is the equivalent of 532 US Dollars.
  
  ii) 1 USD = 0.59 of £1 = 59% of £1
  
  **Answer:** One USD is 59 percent of one GBP.

---

### Notes

- Individual work, monitored, helped
- Elicit that 1 USD means 1 United States Dollar ($)
  
  (T has dollars to show to class if possible.)
  
  Ps say what they know about the USA.

- (or Ps do calculations in Ex. Bks and use a calculator to check their results,)
  
  Responses shown in unison.
  
  Reasoning, agreement, self-correction, praising
  
  Allow any form of each currency (e.g. £ or GBP or pounds)

- Extra praise for Ps who used the 1/x button.
Q.4 Read: On 20 November 2003, 1 GBP was worth 185.11 JPY.
   a) If 1 GBP = 185 JPY, how many £s can you get for 1 Japanese Yen?
   b) i) If 1 GBP = 185 JPY, what is the JPY equivalent of 532 GBP?
      ii) What percentage of 1 JPY is 1 GBP?
   c) i) If 1 JPY = 0.0054 GBP, what is the GBP equivalent of 532 JPY?
      ii) How much more or less than 1% of £1 is 1 Japanese Yen?

Deal with this in a similar way to Activity 5.

Solution: e.g.
   a) 185 JPY = £1
      1 JPY = £1 ÷ 185 = £0.0054054 = £0.0054 (to 4 d.p.)
      Answer: You can get 0.0054 pounds for one Japanese Yen.

   b) i) £1 = 185 JPY
      £532 = 185 JPY × 532 = 98420 JPY
      Answer: 98420 JPY is the equivalent of £532.
      ii) £1 = 185 JPY = 185% of 1 JPY
      Answer: One pound is 185 percent of one Japanese Yen.

   c) i) 1 JPY = £0.0054
      532 JPY = £0.0054 × 532 = £2.8728 ≈ £2.87
      Answer: £2.87 is the equivalent of 532 Japanese Yen.
      ii) 1 JPY = 0.0054 of £1, which is 0.54% of £1
      Answer: One JPY is about half a percent less than 1% of £1.

40 min

Q.5 Who can explain what a gross price and a net price are?
   (Gross price is the full price, including VAT. Net price is the price before VAT has been added on.)
   What is VAT? (The tax put on certain goods by the government to collect extra money for its treasury.)
   Let's see how many of these you can solve in 4 minutes!
   Start...now! Ps work in Ex. Bks and may use calculators.
   Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain reasoning to class. Who agrees? Who did it another way? etc. Mistakes discussed and corrected. T choooses Ps to say the anwer as a sentence.
**Y6**

**Activity**

7  
(Continued)

**Solution:**

a) *The price of a bicycle is £60 + VAT. Calculate its gross price if the Value Added Tax (VAT) is 15% of the net price.*

*Plan:* £60 + 15% of £60 = £60 × 1.15 = £6 × 11.5 = £69

*Answer:* The gross price of the bicycle is £69.

b) *The gross price of a computer is £450, including VAT. Calculate the net price if the VAT is 12.5% of the net price.*

*Plan:* £450 ÷ 1.125 = £450 000 ÷ 1125 = £400

or 112.5% → £450

225% → £900

1% → £900 ÷ 225 = £4

100% → £4 × 100 = £400

*Answer:* The net price of the computer is £400.

c) *How much is the VAT on a product which can be bought for £37.50 but its net price is £30?*

*Plan:* VAT = £37.50 – £30 = £7.50

% rate of VAT: \( \frac{7.5}{30} = \frac{75}{300} = \frac{25}{100} \rightarrow 25\% \)

or 37.50 ÷ 30 = 3.75 ÷ 3 = 1.25

Gross price is 1.25 or 125% of the net price, so VAT is 25% of the net price.

*Answer:* The VAT on the product is £7.50, which is 25% of the net price.

---

**Lesson Plan 99**

**Notes**

Deal with all methods used by Ps.
Accept any valid method of solution but also show the simplest calculation if no P used it.

Any questions not done in the time in class could be completed for homework and reviewed before the start of *Lesson 100.*
Activity

Factorising 100, 275, 450 and 1100. Revision, activities, consolidation

\[100 = 2^2 \times 5^2 \quad (= 10^2)\]
Factors: 1, 2, 4, 5, 10, 20, 25, 50, 100

\[275 = 5^2 \times 11\]
Factors: 1, 5, 11, 25, 55, 275

\[450 = 2 \times 3^2 \times 5^2\]
Factors: 1, 2, 3, 5, 6, 9, 10, 15, 18, 25, 30, 45, 50, 75, 90, 150, 225, 450

\[1100 = 2^2 \times 5^2 \times 11\]
Factors: 1, 2, 4, 5, 10, 11, 20, 22, 25, 44, 50, 55, 100, 110, 220, 275, 550, 1100
(or set factorising as homework at the end of Lesson 99 and review at the start of Lesson 100

Notes

100 = 2^2 \times 5^2 \quad (= 10^2)
Factors: 1, 2, 4, 5, 10, 20, 25, 50, 100

275 = 5^2 \times 11
Factors: 1, 5, 11, 25, 55, 275

450 = 2 \times 3^2 \times 5^2
Factors: 1, 2, 3, 5, 6, 9, 10, 15, 18, 25, 30, 45, 50, 75, 90, 150, 225, 450

1100 = 2^2 \times 5^2 \times 11
Factors: 1, 2, 4, 5, 10, 11, 20, 22, 25, 44, 50, 55, 100, 110, 220, 275, 550, 1100
(or set factorising as homework at the end of Lesson 99 and review at the start of Lesson 100

Q.1

\[360^\circ \div 3 = 120^\circ\]

\[360^\circ \div 8 = 45^\circ\]

\[360^\circ \div 2 = 180^\circ\]

\[360^\circ \div 9 = 40^\circ\]

Q.2

\[\angle A = 45^\circ\]

\[\angle B = 130^\circ\]

\[\angle C = 95^\circ\]

Q.3

\[\angle A = \angle B = 37^\circ\]

\[x = 180^\circ - 2 \times 37^\circ = 180^\circ - 74^\circ = 106^\circ\]

b) \[\angle A = \angle C = x\]

\[x = (180^\circ - 68^\circ) \div 2 = 112^\circ \div 2 = 56^\circ\]
Solutions (Continued)

Q.4 

c) In $\triangle{AMD}$,
\[\angle{AMD} = 180^\circ - 98^\circ = 82^\circ\]
AM = DM (as diagonals in a rectangle are equal and bisect one another)
so $\angle{DAM} = \angle{MDA} = x$ (base angles in an isosceles triangle)
\[2 \times x = 180^\circ - 82^\circ = 98^\circ\]
so $x = 98^\circ \div 2 = 49^\circ$
d) $180^\circ - 90^\circ - 47^\circ = 90^\circ - 47^\circ = 43^\circ$

Q.5 a) Plan: $12^\circ \text{C} - (-6^\circ \text{C}) = 12^\circ \text{C} + 6^\circ \text{C} = 18^\circ \text{C}$
Answer: The temperature rose by 18 degrees Celsius.

b) Plan: $117.5\%$ of £240 = £240 $\times$ $1.175$
\[= £24 \times 11.75 = £264 + £18 = £282\]
Answer: We will have to pay £282 for the gate.

c) Sum of the angles in a quadrilateral is $360^\circ$.
Plan: $360^\circ - (41^\circ 56' + 63^\circ 45' + 122^\circ 8')$
\[= 360^\circ - 227^\circ 49' = 132^\circ 11'\]
Answer: The size of the 4th angle is $132^\circ 11'$.

d) Plan: $360^\circ \div 18^\circ = 20$ (times)
Answer: There are 20 spokes on the wheel.

e) Plan: $15\% \rightarrow 60\$
\[1\% \rightarrow 60\$ \div 15 = 4\$
\[100\% \rightarrow 4\$ \times 100 = 400\$
\[1.6 \text{ USD} = £1\]
\[400 \text{ USD} = £1 \div 1.6 \times 400 = £400 \div 1.6\]
\[= £400 \div 16\]
\[= £1000 \div 4\]
\[= £250\]
Answer: Molly changed £250 to US Dollars for her holiday.
**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 101 is a prime number  \[\text{Factors: 1, 101}\]
  (not exactly divisible by 2, 3, 5, 7 and 11 \[11 \times 11 > 101\])

- 276 = \[2 \times 2 \times 3 \times 23 = 2^2 \times 3 \times 23\]
  \[\text{Factors: 1, 2, 3, 4, 6, 12, 23, 46, 69, 92, 138, 276}\]

- 451 = \[11 \times 41\]
  \[\text{Factors: 1, 11, 41, 451}\]

- 1101 = \[3 \times 367\]
  \[\text{Factors: 1, 3, 367, 1101}\]
  (367 is a prime number, as not exactly divisible by 2, 3, 5, 7, 11, 13, 19 and 23 \[23 \times 23 > 367\])

---

**2 Length: revision**

a) Stand up and hold your hands about 1 m apart. (When you were in Year 1 you needed to stretch out your arms to show it but now they need not be so far apart!) T quickly checks all Ps with a metre rule, praising or correcting.

Repeat for 10 cm, 25 cm, 1 cm, 500 mm, 200 mm, 10 mm, etc.

b) Let's list the units we use to measure length. Ps dictate to T.

BB:  \[1 \text{ mm} < 1 \text{ cm} < 1 \text{ m} < 1 \text{ km}\]
\[\times 10 \quad \times 100 \quad \times 1000\]

T might mention smaller and greater units and ask Ps if they have heard of them and who might use them.

BB: Smaller units: 1 micrometre (\(\mu\)m) = 0.001 mm
(used by scientists using microscopes)

1 angstrom (A) = 0.000 000 1 mm
(used to measure wave lengths of radiation)

Greater units: (used by astronomers, space scientists)

1 Astronomical Unit (AU) = \[1.495 \times 10^8\] km
(mean distance of the Earth from the Sun)

1 light-year = \[9.46 \times 10^{12}\] km = 63 275 AU
\[= 9 460 000 000 000 \text{ km}\]

1 parsec (pc) = \[3.08 \times 10^{13}\] km = 3.26 light-years

---

**Notes**

Individual work, monitored (or whole class activity)

BB: 101, 276, 451, 1101

Ps may use calculators.

Reasoning, agreement, self-correction, praising

\[\begin{array}{cccc}
276 & 2 & 451 & 11 \\
138 & 2 & 41 & 41 \\
69 & 3 & 1 & 1 \\
23 & 23 & 1101 & 3 \\
367 & 367 & 1 & 1 \\
\end{array}\]

Whole class activity

At a good pace
In good humour!

Praising, encouragement only

Agreement, praising

Ps could write units on blank page in Pb too.

T might also mention that in some countries they use a unit of length called a decimetre (1 tenth of a metre)

BB: 1 decimetre (dm) = 10 cm = 0.1 m

Why is it a mean distance?
( Distance between Earth and Sun varies according to the time of year)

A light year is the distance that light travels in 1 year.
(about 6 million million miles)

Individual estimation and measurement then whole class check and discussion

Involve several Ps.

Agreement, evaluation, praising

---

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### Activity

**PbY6b, page 101**

**Q.1** Read: *Colour the equal values in the same colour.*

Set a time limit. Ps calculate mentally (or in *Ex. Bks.*), write values above or below each ellipse then colour appropriately.

Review with whole class. Which operation does not have a matching value? \((8 \times 0.7)\) Who can think of a partner for it?

Ps come to BB to write results, explaining reasoning, and to colour appropriately. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>480</th>
<th>56</th>
<th>480</th>
</tr>
</thead>
<tbody>
<tr>
<td>((400 + 100) \times 12%)</td>
<td>56</td>
<td>(400 \times 12%)</td>
</tr>
<tr>
<td>(400 \times 20%)</td>
<td>56</td>
<td>(400 + 20%)</td>
</tr>
</tbody>
</table>

\[8 \times 0.7 = 56\]

\[70\%\] of 80 = \[80 \times 0.7 = 56\]

\[120\%\] of 400 = \[400 \times 1.2 = 480\]

**20 min**

**Q.2** Read: *Convert the quantities.*

Deal with one part at a time or set a time limit.

Review with whole class. Ps come to BB to fill in missing values or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) \(45.8\, \text{kg} = \frac{45\,800}{1000} = 45.8\, \text{g}; 718\, \text{g} = 0.718\, \text{kg}; 5.1\, \text{t} = 5100\, \text{kg}\)

b) \(3.4\, \text{litres} = \frac{340}{100} = 340\, \text{ml}; 216\, \text{cl} = \frac{216}{100} = 2.16\, \text{litres}; 470\, \text{ml} = 0.47\, \text{litres}\)

c) \(2.9\, \text{km} = \frac{2900}{1000} = 2.9\, \text{m}; 53\, \text{cm} = \frac{53}{100} = 0.53\, \text{m}; 4280\, \text{mm} = 4.28\, \text{m}\)

d) \(233\, \text{min} = \frac{233}{60} = 3\frac{53}{60}\, \text{hr}; 10.4\, \text{hr} = 624\, \text{min}; 45\, \text{sec} = \frac{45}{4}\, \text{min}\)

\[\frac{53}{60}\, \text{hr} = 25\, \text{min}\]

**Q.3** Read: *If 1 EUR (Euro) = 7.4 DK (Danish Kroner) and £1 = 1.4 EUR:*

i) how many Danish Kroner is £1 worth

ii) how many £s is 1 DK worth?

b) Calculate 18% of 360 DK and give your answer in £s.

Deal with one part at a time or set a time limit. Ps write operations, do calculations and write the answers as sentences in *Ex. Bks.* Ps may use calculators for a) ii) and b).

Review with whole class. Ps could show answers on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Who did the same? Who worked it out in a different way? etc. Mistakes discussed and corrected.

**Notes**

Individual work, monitored helped

Drawn (stuck) on BB or use enlarged copy master or OHP

Ps make suggestions. Class checks that they are valid.

Reasoning, agreement, self-correction, praising

Ps show details of calculations on BB if there is disagreement.

\[70\%\] of 80 = \[80 \times 0.7 = 56\]

\[120\%\] of 400 = \[400 \times 1.2 = 480\]

\[5.6\]

\[480\]

\[233\, \text{min} = \frac{233}{60}\, \text{hr}; 10.4\, \text{hr} = 624\, \text{min}; 45\, \text{sec} = \frac{3}{4}\, \text{min}\]

\[\frac{53}{60}\, \text{hr} = 25\, \text{min}\]

\[\frac{480}{400} + (20\%\) of 400) \]

\[56\]

\[480\]

\[480\]

\[0.75\, \text{min}\]

\[\text{or } \frac{0.75}{1}\]

\[\text{min}\]

\[\frac{56}{480}\]

\[56\]

\[480\]

\[40\]

\[12\]

\[400\]

\[12\]

\[480\]

\[400\]

\[12\]

\[480\]

\[40\]

\[12\]

\[480\]

\[400\]

\[12\]

\[480\]

\[40\]

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\[480\]

\[40\]

\[12\]

\[480\]

\[400\]

\[12\]

\[480\]

\[40\]

\[12\]

\[480\]

\[400\]

\[12\]

\[480\]
Activity 5  
(Continued)

Solution: e.g.

a) i) £1 = 1.4 EUR = 1.4 × 7.4 DK = 10.36 DK
   Answer: One pound is worth 10.36 Danish Kroner.

ii) 1 DK = £ (1 ÷ 10.36) ≈ £0.0965 ≈ £0.10 (i.e. 10 p)
   Answer: One Danish Kroner is worth about £0.10.

b) 18% of 360 DK = 360 DK × 0.18 = 64.80 DK
   Answer: 18% of 360 Danish Kroner is worth about £6.25.

Lesson Plan 101

Notes

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>7.4</td>
</tr>
<tr>
<td></td>
<td>×1.4</td>
</tr>
<tr>
<td></td>
<td>2 \ 9 \ 6</td>
</tr>
<tr>
<td></td>
<td>+ 7 \ 4 \ 0</td>
</tr>
<tr>
<td></td>
<td>1 \ 0 \ 3 \ 6</td>
</tr>
</tbody>
</table>

or 360 × 0.0965 ÷ 100 × 18 
≈ 6.25 ( £ )

6  
PbY6b, page 101

Q.4 Read: On 1 January, Martin put £3600 into an account which had an interest rate of 4% per year.

a) Calculate the yearly interest for Martin’s account.

b) If Martin did not touch his account, how much money would be in his account:
   i) 1 year later
   ii) 2 years later?

c) What percentage of his starting amount would be in his account:
   i) 1 year later
   ii) 2 years later?

Set a time limit or deal with one part at a time.

Review with whole class. Ps show answers on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution:

a) 4% of £3600 = £3600 ÷ 100 × 4 = £36 × 4 = £144
   or = £3600 × 0.04 = £36 × 4 = £144
   Answer: The yearly interest for Martin’s account was £144.

b) i) £3600 + £144 = £3744
   Answer: After 1 year, there would be £3744 in his account.

ii) £3744 + £3744 × 0.04 = £3744 + £149.76
    = £3893.76
   Answer: After 2 years, there would be £3893.76 in Martin’s account.

c) i) 100% + 4% = 104%
   Answer: After one year, there would be 104% of Martin’s starting amount in his account.

ii) 3893.76 ÷ 3600 × 100 = 1.0816 × 100 = 108.16%
   Answer: After 2 years, there would be 108.16% of Martin’s starting amount in his account.

Point out that the calculation is not 4% + 4% = 8%, as Martin had more money in his account during the 2nd year so he received more interest than in the 1st year.

Individual work, monitored, helped
Ps who have a bank account tell class about it. (e.g. name of bank, where it is, how long they have had it, how often they put money in/take money out, etc.) Otherwise T talks about his/her account.

Differentiation by time limit.
Calculators can be used where necessary.

Answers shown in unison.
Reasoning, agreement, self-correction, Praising

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### Activity 7

**PbY6b, page 101, Q.5**

Read: *Mr. Yamamoto is a very clever businessman. His software company has made a profit of 262 million JPY this year. The company's value is now 140% of what it was last year.*

a) **By what percentage** has his company's value increased?
   - Show me . . . now! (40%)

b) **What was the value of the company at the end of last year?**
   - Allow Ps to think about it or discuss with their neighbours, then Ps come to BB to write an operation and do the calculation, explaining reasoning. Who thought of doing the same? Who thought of a different way? If Ps have no ideas, T gives hints or directs Ps' thinking.
   - T chooses a P to say the answer in a sentence.
   - **Solution:** e.g.
     - 40% → 262 million JPY (or 262 ÷ 40 × 100)
     - 10% → 65.5 million JPY (or 262 ÷ 0.4 = 2620 ÷ 4)
     - 100% → 655 million JPY
   - **Answer:** At the end of last year, the value of the company was 655 million Japanese Yen.

c) **What is the value of the company now?**
   - Allow Ps to think about it and discuss.
   - Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Who thought of another way to do it? Come and show us. Which do you think is best? Why?
   - **Solution:** e.g.
     - 140% of 655 million JPY = (655 million + 262 million) JPY
     = 917 million JPY
   - **Answer:** The company is now worth 917 million Japanese Yen.

---

### Notes

Whole class activity

- T chooses Ps to read out the questions.
- Ps show answer on scrap paper or slates in unison.

Discussion, reasoning, agreement, praising

- Involves several Ps.

Ps write the method they like best in *Ex. Bks.*

Discussion, reasoning, agreement, praising

- Ps write the method they like best in *Ex. Bks.*
  - 655 million × 1.4
  - 655 million ÷ 100 × 140
  - 655 million ÷ 10 × 14
Activity 8

PbY6b, page 101

Q.6 Read: Calculate the whole quantity if:

a) \( \frac{3}{8} \) of it is 210 kg  

b) 35% of it is £1812.30

c) \( \frac{1}{2} \) of it is \( 11 \frac{2}{3} \) m²  

d) 130% of it is 32.5 miles.

Set a time limit of 3 minutes. Ps work in Ex. Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly show their solution on the BB. Who did the same? Who did it another way? Mistakes discussed and corrected.

Solution:

a) \( 210 \text{ kg} \div \frac{3}{8} \times 8 = 70 \text{ kg} \times 8 = \frac{560}{3} \text{ kg} \) (or 210 kg ÷ 0.375)

b) £1812.30 ÷ 35 × 100 = £362.46 ÷ 7 × 100
   = £51.78 × 100 = £5178

or £181230 ÷ 0.35
   = £181230 ÷ 0.35 = £5178

Elicit or remind Ps that to divide by a fraction, multiply by its reciprocal value (i.e. the value which multiplies the original fraction to make 1)

Elicit or remind Ps that to divide by a fraction, multiply by its reciprocal value (i.e. the value which multiplies the original fraction to make 1)
Lesson Plan 102

Notes

Individual work, monitored (or whole class activity)
BB: 102, 277, 452, 1102
Ps could practise calculation without using calculators.
Reasoning, agreement, self-correction, praising
e.g.

<table>
<thead>
<tr>
<th>102</th>
<th>277</th>
<th>452</th>
<th>1102</th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>2</td>
<td>51</td>
<td>29</td>
</tr>
<tr>
<td>51</td>
<td>3</td>
<td>113</td>
<td>113</td>
</tr>
<tr>
<td>1102</td>
<td>2</td>
<td>551</td>
<td>19</td>
</tr>
<tr>
<td>551</td>
<td>19</td>
<td>29</td>
<td>29</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

7 min

2 Units of measure: conversion practice (using calculators)
T gives the metric → Imperial rate of conversion. Ps use calculators to work out the Imperial → metric conversion rate.
All Ps do the calculation, then T chooses a P to dictate the reverse rate (rounding to an agreed number of decimal places: the more decimal places, the closer to the actual value) and class agrees or disagrees. T writes agreed value in a table on BB (or on a prepared wall chart) and Ps write it on their own Units of Measure sheet.

[If possible, in the case of disagreement, use a calculator on a computer projected onto a screen so that the whole class can see the buttons pressed and the result generated before agreement on an appropriate rounding.] Ps stick completed sheet in the back of their Pbs/Ex. Bks. for reference.

Rates given by T: Calculated by Ps: e.g.

<table>
<thead>
<tr>
<th>Units of Measure</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td></td>
</tr>
<tr>
<td>1 inch (1”) = 25.4 mm (= 2.54 cm)</td>
<td>1 cm = 0.3937” (= 0.4”)</td>
</tr>
<tr>
<td>1 foot (1’) = 12” = 30.48 cm (= 0.3 m)</td>
<td>1 m = 3.281’ (= 3.3’)</td>
</tr>
<tr>
<td>1 yard (1 yd) = 3’ = 914.4 mm (= 0.9144 m)</td>
<td>1 m = 1.0936 yd (= 1.1 yd)</td>
</tr>
<tr>
<td>1 mile = 1.609 km</td>
<td>1 km = 0.6215 mile (= 0.6 mile)</td>
</tr>
<tr>
<td>1 Nautical mile = 1.852 km</td>
<td>1 km = 0.53996 Naut. mile (= 0.54 N. mile)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Area</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1 square inch ≈ 6.54 cm²</td>
<td>1 cm² ≈ 0.153 square inches</td>
</tr>
<tr>
<td>1 square foot ≈ 929 cm²</td>
<td>1 m² = 10.76 square feet</td>
</tr>
<tr>
<td>1 square yard ≈ 0.836 m²</td>
<td>1 m² ≈ 1.196 square yards</td>
</tr>
<tr>
<td>1 acre = 0.4047 hectares (ha)</td>
<td>1 ha = 2.471 acres</td>
</tr>
<tr>
<td>1 hectare = 10 000 m²</td>
<td>1 km² = 1 000 000 m² = 100 hectares</td>
</tr>
</tbody>
</table>

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Lesson Plan 102

Notes

T gives Ps an idea of what each measure is.
(e.g. a grain of rice, a pint of milk, an ounce of sugar, 1 lb of potatoes, 1 yard marked on classroom wall, a metre rule, places which are 1 km and 1 mile from school, etc.to make the measures more relevant)

Ps suggest values for \( x \) and use the formulae to convert degrees Fahrenheit to degrees Celsius and vice versa.
Discuss temperature related to daily life (weather, ovens, body, washing machines, etc.)

Individual work, monitored, helped

T reminds Ps that:
BB: 1 foot = 1 ft = 1'

Responses shown in unison. Reasoning, agreement, self-correction, praising
Ask Ps to demonstrate the heights, first without a tape measure then with one to see how close their estimate was.
In good humour! Praising

Feedback for T

---

**Activity**

(Continued)

<table>
<thead>
<tr>
<th>Mass (weight)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 grain (1 gr) = 0.06481 g</td>
</tr>
<tr>
<td>1 ounce (1 oz) = 28.33 g</td>
</tr>
<tr>
<td>1 pound (1 lb) = 0.4535 kg</td>
</tr>
<tr>
<td>1 hundredweight (1 cwt) = 50.792 kg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pint (1 pt) = 0.5682 litre</td>
</tr>
<tr>
<td>1 gallon (1 gal) = 4.5455 litres</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cubic inch = 16.387 cm³</td>
</tr>
<tr>
<td>1 cubic foot = 0.02832 m³</td>
</tr>
<tr>
<td>1 cubic yard = 0.764 m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fahrenheit → Celsius</td>
</tr>
<tr>
<td>( x \times F = (x - 32) \times \frac{5}{9} \text{ (°C)} )</td>
</tr>
<tr>
<td>Celsius → Fahrenheit</td>
</tr>
<tr>
<td>( x \times C = \frac{9}{5} \times x + 32 \text{ (°F)} )</td>
</tr>
</tbody>
</table>

---

**Notes**

Lesson Plan 102

Notes

T gives Ps an idea of what each measure is.
(e.g. a grain of rice, a pint of milk, an ounce of sugar, 1 lb of potatoes, 1 yard marked on classroom wall, a metre rule, places which are 1 km and 1 mile from school, etc.to make the measures more relevant)

Ps suggest values for \( x \) and use the formulae to convert degrees Fahrenheit to degrees Celsius and vice versa.
Discuss temperature related to daily life (weather, ovens, body, washing machines, etc.)

Individual work, monitored, helped

T reminds Ps that:
BB: 1 foot = 1 ft = 1'

Responses shown in unison. Reasoning, agreement, self-correction, praising
Ask Ps to demonstrate the heights, first without a tape measure then with one to see how close their estimate was.
In good humour! Praising

Feedback for T
Lesson Plan 102

Notes

Individual work, monitored, helped
Allow Ps to use calculators but when reviewing, go through the vertical division below with Ps help, as revision practice.

Responses shown in unison.
Discussion, reasoning, agreement, self-correction, praising

Extension

What percentage of a normal (or statutory) mile is a Nautical mile?

1.85 × 16 × 100

= 116 (%)
### Activity 5

(Continued)

b) Michael Schumacher, the German racing driver, did a road test on his car and said that he had covered a distance of 410 km.

If David Coulthard, the Scottish racing driver, had done the same road test, what distance would he say that he had covered?

Plan: 410 ÷ 1.6 = 4100 ÷ 16 = 256.25 = 256 (miles)

Answer: He would have said that he had covered a distance of about 256 miles (or 256 and a quarter miles).

### Notes

If possible, T has pictures of the racing drivers and asks Ps what they know about them. (T should have information already prepared in case Ps know nothing.)

or 256.25 miles = 256 \( \frac{1}{4} \) miles

---

<table>
<thead>
<tr>
<th>6</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Erratum</strong></td>
<td><strong>Lesson Plan 102</strong></td>
</tr>
</tbody>
</table>

In Pbs: 'c)' should be 'b')

PbY6b, page 102

Q.4 Read: One acre is approximately equal to 0.4 of a hectare.

- Lazlo, a Hungarian farmer, has a farm covering 120 hectares. Ian, a British farmer, has a farm covering 375 acres.
  - a) What is the ratio of: i) Ian’s land to Lazlo’s land
    - ii) Lazlo’s land to Ian’s land?
  
  b) By what percentage is Ian’s land greater than Lazlo’s land?

Set a time limit or deal with one part at a time. Allow calculators. Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who worked it out a different way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

a) i) 1 : 375 acres = 375 × 0.4 hectares = 150 hectares

   Ratio of 1 : L = 150 : 120 = 5 : 4

   ii) Ratio of L : 1 = 120 : 150 = 4 : 5

   Answer: The ratio of Ian’s land to Lazlo’s land is 5 to 4.

   The ratio of Lazlo’s land to Ian’s land is 4 to 5.

b) 150 – 120 = 30 (ha)

   \[
   \frac{30}{120} = \frac{1}{4} \rightarrow 25\%
   \]

   or \[
   \frac{5 - 4}{4} \times 100 = \frac{1}{4} \times 100 = 25\% \]

   Answer: Ian’s land is 25\% greater than Lazlo’s land.

---

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PbY6b, page 102, Q.5

Read: 1 kilogram is approximately equal to 2.2 pounds (lb).

Sarah bought \( \frac{1}{2} \) lb of meat for £12 in a butcher’s shop.

Olga bought 500 g of the same kind of meat for £7 in the supermarket.

a) Who had the better bargain?

Allow Ps 2 minutes to think about it. If you think Sarah had the better bargain, stand up . . . now!

T chooses a P standing and a P sitting to explain their reasoning at BB. Class decides who is correct.

Solution: e.g.

S: \( £12 \div 1.5 = £24 \div 3 = £8 \) (per lb)

O: 500 g = 0.5 kg, 1 kg \( \approx \) 2.2 lb, so 0.5 kg \( \approx \) 1.1 lb

\( £7 \div 1.1 = £70 \div 11 = £6.36 \) (per lb)

Answer: Olga had the better bargain.

b) What would 1 kg of the meat cost in each shop?

Show me what it would cost in the supermarket . . . now! (£14)

A, tell us how you worked it out. (\( £7 \times 2 = £14 \))

How can we work out what it would cost in the butcher's shop?

B, come and show us. Who agrees? Who would do it another way? etc. Class helps if necessary.

e.g. \( £12 \div 1.5 \times 2.2 = £8 \times 2.2 = £17.60 \)

T chooses a P to say the answer in a sentence.

Answer: One kilogram of the meat would cost £14 in the supermarket and £17.60 in the butcher’s shop.

\[ 45 \text{ min} \]
<table>
<thead>
<tr>
<th>Y6</th>
<th><strong>Lesson Plan</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>103</td>
</tr>
</tbody>
</table>

**Activity**

1 **Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- 103 is a prime number  
  Factors: 1, 103
  (as not exactly divisible by 2, 3, 5, 7 and 11 × 11 > 103)
- 278 = 2 × 139  
  Factors: 1, 2, 139, 278
- 453 = 3 × 151  
  Factors: 1, 3, 151, 453
- 1103 is a prime number  
  Factors: 1, 1103
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 19, 23, 29, 31 and 37² > 1103)

   6 min

2 **Problem 1**

How many times do the two hands on a clock cover each other exactly from 12 noon to 12 midnight? T asks several Ps what they think. Let's check it. T has large traditional analogue clock at a height that Ps can reach and class can see easily. (The hands must turn together.)

Let's list the times when they cover each other exactly. Agree that the first time is 12 noon. Then Ps say what they think the next time will be and a P turns the hands to check. T writes checked times on BB.

BB: At 12:00 (noon), after 13:05, after 14:10; after 15:15, after 16:20, after 17:25, after 18:30, after 19:35, after 20:40, after 21:45, after 22:50, at 12:00 (midnight)

Agree that they cover each other exactly 12 times (not 13).

   10 min

3 **Problem 2**

Listen carefully and think about how you would work out the answer.

a) How much time has passed from

BB: 6 h 14′ 25″ to 13 h 08′ 43″?

What kind of operation do we need to do? (subtraction) How could we do it? Ps come to BB to write it vertically and to do the calculation, explaining reasoning in detail. Who can think of another way to do the subtraction? T shows it if no P does.

BB:

\[
\begin{array}{c}
6 \text{ h} 14′ 25″ \\
13 \text{ h} 08′ 43″
\end{array}
- \begin{array}{c}
6 \text{ h} 14′ 25″ \\
13 \text{ h} 08′ 43″
\end{array}
= \begin{array}{c}
6 \text{ h} 54′ 18″
\end{array}
\]

Here are some calculations about time but they are written in a different form. Who can explain them? (No units are given so, e.g., 11: 43 could mean 11 h 43 minutes or 11 minutes 43 seconds)

Let's work out the result and then think of a word problem about it. Ps come to BB, explaining reasoning. Class points out errors. Ps suggest contexts and class decides whether they are valid.

   6 min

**Notes**

Individual work, monitored (or whole class activity)

BB: 103, 278, 453, 1103
Ps could practise calculation without a calculator.

Reasoning, agreement, self-correction, praising

E.g.

\[
\begin{array}{cccc}
278 & 139 & 453 & 3 \\
139 & 151 & 151 & 1
\end{array}
\]

and 139 and 151 are prime numbers

Whole class activity

If possible, Ps have own clocks too.

At a good pace

In good humour.

Agreement, praising

Class applauds Ps who were correct at the beginning.

Whole class activity

Elicit that 1 hour = 60 min (60’)
1 ′ = 60 sec (60 “)

Discussion, reasoning, agreement, praising

Methods:

- Borrow 1 hour (= 60’) from the hours column, then pay it back, or
- Change 1 hour in the hours column to 60 minutes and add it to minutes column.

Written on BB or SB or OHT

Elicit that the result does not depend on whether they are hours or minutes, as there are 60 minutes in 1 hour and also 60 seconds in 1 minute

© CIMT, University of Exeter
Q.1 Read: One foot is approximately equal to 30.5 cm and 1 yard is approximately equal to 91.5 cm.

The members of a school’s athletics team were training for a competition and their coach noted how far they could run in a set time.

a) Leslie ran 610 yards 2 feet. Cora ran 90% of Leslie’s distance in the same time.

How many metres did Cora run?

b) Jane ran 502 m 88 cm. Adam ran 120% of Jane’s distance in the same time.

How many yards did Adam run?

Set a time limit or deal with one at a time. Ps write plans, do calculations (with calculators) and write answers in sentences in Ex.Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution:

a) L: 610 yards 2 ft = (610 × 91.5 + 2 × 30.5) cm
   = 55 815 cm + 61 cm
   = 55 876 cm = 558.76 m
C: 90% of 558.76 m = 558.76 m × 0.9 = 502.884 m
Answer: Cora ran 502.884 metres.

b) J: 502 m 88 cm = 50 288 cm = (50 288 ÷ 91.5) yd
   = (502 880 ÷ 915) yd
   = 549.60 yd
A: 120% of 549.60 yd = 549.60 yd × 1.2 = 659.52 yd
Answer: Adam ran 659.52 yards.

Extension (for quicker Ps)
Put the children in order of speed.
1st: Adam (659.52 yd)
2nd: Leslie: (610 yd 2 ft)
3rd equal: Cora and Jane (549.6 yd)
Who can explain what the formulae in the box mean? Allow Ps to explain if they can, otherwise T reminds class.

Degrees Celsius is a metric unit of measure and degrees Fahrenheit is an Imperial unit of measure.

• To convert degrees Celsius \(x\) to degrees Fahrenheit:
  multiply the value \(x\) by \(\frac{9}{5}\) and add 32.

• To convert degrees Fahrenheit \(x\) to degrees Celsius:
  subtract 32 from the value \(x\) and multiply the difference by \(\frac{5}{9}\).

a) Read: "It’s 32° here and I’m cold!" said Kate on the phone in London.
   "It’s 32° here and I’m hot!" Lucia answered from Sao Paolo in Brazil.
   Who is correct? Give a reason for your answer.
   Allow Ps a minute to think about it. Stand up if you think that Kate is correct . . . now!
   Raise your hand if you think that Lucia is correct . . . now!
   Ps who did both explain to class. Both are correct, as Kate could have meant 32 degrees Fahrenheit which is very cold and Lucia could have meant 32 degrees Celsius, which is hot.

   Let’s work out what the actual temperatures are so that we can compare them. Ps come to BB or dictate what T should write, substituting 32 for \(x\) in each formula.

   BB:
   K: 32°F = \(\frac{9}{5} \times (32 - 32) = \frac{9}{5} \times 0\) °C = 0 °C
   L: 32°C = \(\frac{9}{5} \times 32 + 32 = \frac{288}{5} + 32 = 57.6 + 32\) °F

   Let’s see if you can do parts b) and c) on your own. I will give you 3 minutes! Start . . . now! . . . . . Stop!
   Review with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.
   Ask Ps to say whether they think the temperatures are hot or cold. Solution:
   a) Convert to degrees Celsius:
      i) 0°F = \(\frac{5}{9} \times (0 - 32) = \frac{5}{9} \times (-32) = -\frac{160}{9}\) °C
         = -17.7 °C ≈ -17.8 °C
      ii) 50°F = \(\frac{5}{9} \times (50 - 32) = \frac{5}{9} \times \frac{2}{1}\) °C = 10 °C
      iii) 104°F = \(\frac{5}{9} \times (104 - 32) = \frac{5}{9} \times \frac{8}{1}\) °C = 40 °C

Elicit/tell that water freezes at 0°C and boils at 100°C.
**Activity 5** (Continued)

**c) Convert to degrees Fahrenheit:**

i) \(100^\circ C = \left(\frac{9}{5} \times 100 + 32\right) ^\circ F = 212 ^\circ F\)

ii) \(30^\circ C = \left(\frac{9}{5} \times 30 + 32\right) ^\circ F = 86 ^\circ F\)

iii) \(-10^\circ C = \left[\frac{9}{5} \times (-10) + 32\right] ^\circ F = 14 ^\circ F\)

---

**Lesson Plan 103**

**Notes**

Individual work, monitored, less able Ps helped
Written on BB or SB or OHT
Differentiation by time limit
Reasoning agreement, self-correction, praising
Accept any valid reasoning.
Feedback for T

---

**Activity 6**

**PbY6b, page 103**

**Q.3** What are these calculations about? (time) What units are used? [hours (h), minutes (min or '), seconds (sec or "')] Let's see if you can do them in 3 minutes! Start...now!...Stop! Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

Ps finished early could think of a word problem for each one.

**Solution:** e.g.

a) 4 h 16 min 37 sec + 5 h 57 min 43 sec

\[\begin{array}{c}
4 h 16 min 37 sec \\
+ 5 h 57 min 43 sec \\
\hline
10 h 14 min 20 sec
\end{array}\]

b) 17 h 31' 18"

\[\begin{array}{c}
17 h 31' 18" \\
- 6 h 50' 32" \\
\hline
10 h 40' 46"
\end{array}\]

---

**Activity 7**

**Pb6b, page 103**

**Q.4** Read: *Calculate the arrival time if a plane took off at:*

a) 3.24 pm and the flight lasted 9 hours 44 minutes
b) 11.45 am and the flight lasted 3 hours 16 minutes
c) 21:18 and the flight lasted 5 hours 33 minutes.

Set a time limit or deal with one at a time. Ps write plans, do calculations and write the answer in a sentence in Ex. Bks.

Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain at BB. Who did the same? Who did it another way? etc. Mistakes discussed and corrected. T chooses Ps to say the answer in a sentence.

**Solution:**

a) Plan: 3 h 24 min + 9 h 44 min

\[\begin{array}{c}
3 h 24' \\
+ 9 h 44' \\
\hline
12 h 08'
\end{array}\]

**C:**

\[\begin{array}{c}
12 h 08' \\
+ 9 h 44' \\
\hline
21 h 52'
\end{array}\]

31 min

**Answer:** The arrival time was 01:08 the next day.

---

Responses shown in unison.
Reasoning, agreement, self-correction, praising
Accept any valid method of calculation with correct reasoning.

Extra praise for Ps who realised that:

\[25 h 8' = 24 h + 1 h + 8' = 1 \text{ day } + 1 h + 8'\]

(or 1.08 am)
Y6

Activity
(Continued)

7

b) Plan: 11 h 45 min + 3 h 16 min  C: + 3 h 16 min

15 h 01 min

Answer: The plane arrived at 15:01 (or 3:01 pm, or 1 minute past 3 in the afternoon).

c) Plan: 21 h 18 min + 5 h 33 min  C: 21 h 18 min + 5 h 33 min

26 h 51 min

Answer: The arrival time was 2:51 am (or 02:51) the next day.

Lesson Plan 103

Notes

Elicit different ways to express the answer.

Individual work, monitored, helped

Responses shown in unison.
Reasoning, agreement, self-correction, praising
Accept any valid method of calculation with correct reasoning.
Feedback for T

Extra praise for this!

PbY6b, page103

Q.5 Read: Calculate tour journey time if we left at:

a) 9:35 am and arrived at 11.56 am
b) 9.35 am and arrived at 13:25
c) 09:35 and arrived at 4.10 pm
d) 09:35 and arrived at 07:25 the next day.

Set a time limit or deal with one at a time. Ps write plans, do calculations and write the answer in a sentence in Ex. Bks.

Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly explain at BB. Who did the same? Who did it another way? etc. Mistakes discussed and corrected. T chooses Ps to say the answer in a sentence.

Solution:

a) Plan: 11 h 56 min – 9 h 35 min  C: 11 h 56 min – 9 h 35 min

Answer: Our journey time was

2 hours 21 minutes.

b) Plan: 13 h 25 min – 9 h 35 min  C: 13 h 25 min – 9 h 35 min

Answer: Our journey time was

3 hours 50 minutes.

c) Plan: 16 h 10 min – 9 h 35 min  C: 16 h 10 min – 9 h 35 min

Answer: Our journey time was

6 hours 35 minutes.

d) Plan: 24 h – 9 h 35 min + 7 h 25 min = 14 h 25 min + 7 h 25 m = 21 h 50 min

or 09:35 one day to 09:35 the next day is 24 hours

Answer: Our journey time was 21 hours 50 minutes.
Lesson Plan 103

**Notes**

Whole class activity
(or individual work if there is time, reviewed as usual with whole class)

Quick revision of time zones
(T could use Y6 CM LP 47/2)
Involve several Ps.

Answers shown in unison.
Reasoning, agreement, praising
Accept any valid method of solution.

<table>
<thead>
<tr>
<th>C:</th>
<th>14 h 60'</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.g.</td>
<td>7 h 30'</td>
</tr>
<tr>
<td>---</td>
<td>6 h 45'</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C:</th>
<th>15 h 60'</th>
</tr>
</thead>
<tbody>
<tr>
<td>e.g.</td>
<td>8 h 30'</td>
</tr>
<tr>
<td>---</td>
<td>6 h 45'</td>
</tr>
</tbody>
</table>

Optional
Review before the start of Lesson 104.

Ask Ps to find Vietnam on a map of the world and Hanoi on a map of Vietnam.
Ps say what they know about Vietnam. (T could have information already prepared.)

---

**Activity 9**

**PbY6b, page 103, Q.6**

Read: *When the time is 09:00 in Exeter in the UK, it is 10:00 in Kassel in Germany.*

T (P) shows Exeter and Kassel on a map of Europe. Why is the time different in the two countries? (Talk about the Earth turning on its axis so the sunrise and sunset is at different times around the world. To make life easier, countries have been put into different agreed time zones, measured from the Meridien line in Greenwich, London.)

Deal with one at a time. T chooses a P to read the sentence. Ps calculate mentally or in Ex. Bks. and show results on scrap paper or slates on command. Ps with different answers explain reasoning. Class decides who is correct. T chooses a P to say the answer in a sentence.

*Solution:* e.g.

a) *David left Exeter at 7.30 am and arrived in Kassel at 15:15. How long did his journey take?*

When David arrived in Kassel the time would be 14:15 in Exeter.

*Plan:* 14:15 – 07:30 = 6 h 45'

*Answer:* David's journey took 6 and 3 quarter hours.

b) *A month later, Werner left Kassel at 08:30 and arrived in Exeter at 14:15. How long did his journey take?*

When Werner arrived in Exeter the time would be 15:15 in Kassel.

*Plan:* 15:15 – 08:30 = 6 h 45'

*Answer:* Werner's journey also took 6 and 3 quarter hours.

---

**Homework**

When the time is 10:00 in London, it is 17:00 in Hanoi.
A plane leaves London at 12:40 and 13 hours later it lands in Hanoi.
What is the time in Hanoi when the plane lands?

*Solution:* e.g.

12 h 40' + 13 h = 25 h 40' = 24 h + 1 h + 40' = 1 day + 1 h + 40'
so the time in London when the plane lands is 01:40 the next morning.

But Hanoi time is 7 hours ahead of London time, so the time in Hanoi when the plane lands is 01:40 + 07:00 = 08:40
(or 12:40 + 13:00 + 7:00 = 12:40 + 20:00 = 08:40)
R: Calculations
C: Perimeter and area. Area of squares, rectangles and triangles
E: Problems

**Lesson Plan**

**104**

**Notes**

Individual work, monitored (or whole class activity)

BB: 104, 279, 454, 1104

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

Whole class activity but individual (or paired) work in measuring, monitored, less able Ps helped

Use copy master and 1 cm measuring grids from Y6 LP 46/2 copied onto OHT and cut out.

Reasoning, agreement, self-correction, praising

Discussion and agreement on the general formulae.

(Only one example for each type is shown here.)

**1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(104 = 2 \times 2 \times 2 \times 13 = 2^3 \times 13\)
  - Factors: 1, 2, 4, 8, 13, 26, 52, 104
- \(279 = 3 \times 3 \times 31 = 3^2 \times 31\)
  - Factors: 1, 3, 9, 31, 93, 279
- \(454 = 2 \times 227\) (227 is a prime number)
  - Factors: 1, 2, 227, 454
- \(1104 = 2 \times 2 \times 2 \times 2 \times 3 \times 23 = 2^4 \times 3 \times 23\)
  - Factors: 1, 2, 3, 4, 6, 8, 12, 16, 23, 24, 1104, 552, 368, 276, 184, 138, 92, 69, 48, 46

**2**

**Perimeter and area**

Ps each have a sheet of squares, rectangles and triangles and a transparent 10 cm by 10 cm measuring grid.

Use your grids to measure or calculate the area and the perimeter of the rectangles and only the area of the triangles. Write your results in your Ex. Bks. Deal with one set of polygons at a time.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes corrected.

Elicit the general formula for area and perimeter of rectangles and for area of triangles. e.g.

**Square** e.g.

- \(P = 12 \text{ cm} \) (or \(3 \times 4 = 12 \text{ cm}\))
- \(A = 9 \text{ cm}^2 \) (or \(3 \times 3 = 9 \text{ cm}^2\))
- \(P(\text{square}) = 4a, \ A(\text{square}) = a \times a = a^2\)

**Rectangle** e.g.

- \(P = 11 \text{ cm} \) \([2 \times (3 + 2 \frac{1}{2}) = 11 \text{ cm}]\)
- \(A = 7 \frac{1}{2} \text{ cm}^2 \) \([3 \times 2 \frac{1}{2} = 7 \frac{1}{2} \text{ cm}^2]\)
- \(P(\text{rectangle}) = 2 \times (a + b), \ A(\text{rectangle}) = a \times b\)

**Right-angled triangles** e.g.

- \(A = 1 + \frac{1}{2} + \frac{1}{2} = 2 \text{ (cm}^2\) \)
  - (or as half the square: \(\frac{2 \times 2}{2} = 2 \text{ cm}^2\))
- \(A(\text{right-angled } \Delta) = \frac{a \times b}{2} \) (or \(\frac{a \times a}{2}\) if also isosceles)
  - \(\frac{\text{base} \times \text{height}}{2}\)

\[20 \text{ min}\]
### Activity 3  
**PbY6b, page 104**

**Q.1** Read: *Measure the data needed to calculate the perimeter and area of the rectangles. Write the perimeter and area inside each rectangle.*

Set a time limit. Ask Ps to measure with rulers as accurately as they can (to the nearest mm). Ps do necessary calculations in Ex. Bks.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

1. **a)**
   - \( b = 3 \text{ cm} \)
   - \( a = 5.3 \text{ cm} \)
   - \( P = 2 \times (5.3 + 3) \)
   - \( A = 5.3 \times 3 = 15.9 \text{ cm}^2 \)

2. **b)**
   - \( d = 4.7 \text{ cm} \)
   - \( c = 3.4 \text{ cm} \)
   - \( P = 2 \times (3.4 + 4.7) \)
   - \( A = 3.4 \times 4.7 = 15.98 \text{ cm}^2 \)

3. **c)**
   - \( a = 2.5 \text{ cm} \)
   - \( P = 4 \times 2.5 \text{ cm} = 10 \text{ cm} \)
   - \( A = 2.5 \text{ cm} \times 2.5 \text{ cm} = 6.25 \text{ cm}^2 \)

4. **d)**
   - \( b = 2.1 \text{ cm} \)
   - \( A = 2.1 \text{ cm} \times 2.1 \text{ cm} = 4.41 \text{ cm}^2 \)

**Notes**

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP (for demonstration only)

Ps can estimate first using 1 cm grids, then calculate exactly.

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Feedback for T

---

### Activity 4  
**PbY6b, page 104**

**Q.2** Read: *Measure the necessary data, then calculate the area and perimeter as required.*

What kind of triangles are they? [a] and [b] are right-angled, [c] is isosceles and [d] is scalene.

Deal with one triangle at a time. Set a short time limit. Ps do necessary calculations in Ex. Bks.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

1. **a)**
   - \( b = 3 \text{ cm} \)
   - \( a = 4 \text{ cm} \)
   - \( c = 5 \text{ cm} \)
   - \( A = \frac{3 \times 4}{2} = \frac{12}{2} = 6 \text{ cm}^2 \)
   - \( P = 3 + 4 + 5 = 12 \text{ cm} \)

**Extension**

What other measurement could we have used to find the area of a square?

(Measure a diagonal.) e.g.

**d)**

\( d = 3 \text{ cm} \)

\( A = \frac{3 \times 3}{2} \)

\( = \frac{9}{2} \text{ cm}^2 = 4.5 \text{ cm}^2 \)

Agree that this method is not as accurate as measuring a side – but acceptable.
Y6  

**Activity**  

4  

(Continued)

b) \[ b = 3.5 \text{ cm} \]  
\[ a = 4.9 \text{ cm} \]  

\[ A = \frac{3.5 \times 3.5}{2} = \frac{12.25}{2} = 6.125 \text{ (cm}^2) \]  

\[ P = 2 \times 3.5 + 4.9 = 7 + 4.9 = 11.9 \text{ (cm)} \]  

\[ c) \[ a = 3 \text{ cm} \]  
\[ m = 3.5 \text{ cm} \]  
\[ b = 3.8 \text{ cm} \]  

\[ A = \frac{3 \times 3.5}{2} = \frac{10.5}{2} = 5.25 \text{ (cm}^2) \]  

\[ P = 2 \times 3.8 + 3 = 7.6 + 3 = 10.6 \text{ (cm)} \]  

\[ d) \[ a = 4.3 \text{ cm} \]  
\[ h = 2.2 \text{ cm} \]  

\[ A = \frac{4.3 \times 2.2}{2} = \frac{9.46}{2} = 4.73 \text{ (cm}^2) \]  

5  

*PbY6b, page 104, Q.3*  

Deal with one question at a time. T chooses a P to read out the question, Ps calculate mentally or on scrap paper or slates and show result on command. Ps with different answers explain reasoning. Class points out any errors and decides on correct answer. Who did the same? Who worked out the correct answer another way? etc. T chooses a P to say the answer in a sentence.  

**Solution:**  

a) *The landing strip at an airport is 4 km long and 200 m wide.*  

*What is the area of the landing strip?*  

**Plan:**  

\[ A = 4 \text{ km} \times 200 \text{ m} = 4 \text{ km} \times 0.2 \text{ km} = 0.8 \text{ km}^2 \]  

or  

\[ A = 4000 \text{ m} \times 200 \text{ m} = 800 000 \text{ m}^2 \]  

**Answer:** The area of the landing strip is 0.8 km².  

b) *A park is square-shaped and its sides are 3.1 km long.*  

i) *How much fencing is needed to enclose it?*  

**Plan:**  

\[ P = 3.1 \text{ km} \times 4 = 12.4 \text{ km} \]  

or  

\[ P = 3100 \text{ m} \times 4 = 12 400 \text{ m} = 12 \text{ km 400 m} \]  

**Answer:** It would need 12.4 km of fencing to enclose the park.  

ii) *What is the area of the park?*  

**Plan:**  

\[ A = 3.1 \text{ km} \times 3.1 \text{ km} = 9.61 \text{ km}^2 \]  

or  

\[ A = 3100 \text{ m} \times 3100 \text{ m} = 9 610 000 \text{ m}^2 (= 961 \text{ hectares}) \]  

**Answer:** The area of the park is 9.61 km².  

Lesson Plan 104

**Notes**

or  

C) \[ a_1 = 1.5 \text{ cm} \]  

\[ A = 1.5 \times 3.5 = 5.25 \text{ (cm}^2) \]  

or  

\[ d) \]  

\[ A = \frac{a_1 \times h}{2} + \frac{a_2 \times h}{2} \]  

Whole class activity but individual calculation.  

(or individual work under a time limit, reviewed with whole class as usual)  

Responses shown in unison.  

Reasoning, agreement, self-correction, praising  

Accept the answers in any correct form.  

(or = 80 hectares, as 1 hectare = 10 000 m²)  

Extra praise if Ps point out that the actual amount needed would be less than this, as there needs to be space for a gate!  

Feedback for T
PbY5b, page 104, Q.4

Read: The length of one side of a triangular park is 2.6 km and the opposite corner is 2.1 km from this side.

Calculate the area of the park.

What should we do first? (Draw a diagram.) A, come and draw the diagram. Is A correct? Could it be drawn another way? Come and show us. Who can think of yet another way?

Now what should we do? (Write a plan.) Ps come to BB to write operation and do the calculation, explaining reasoning. Class agrees/disagrees. Could we write the answer in a different form? (m² or hectares) Class says the answer in a sentence.

Solution:

Diagram:

Plan: $A = \frac{2.6 \times 2.1}{2} = \frac{5.46}{2} = 2.73 \text{ (km}^2\text{)}$

($= 2730000 \text{ m}^2 = 273 \text{ hectares}$)

Answer: The area of the park is 2.73 square kilometres.

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Factorising 105, 280, 455 and 1105. Revision, activities, consolidation

**PbY6b, page 105**

**Solutions:**

Q.1  
   a) If £1 = 1.43 Euros, 1 Euro = £1 ÷ 1.43 = £0.70
   
   b) If 1 EUR = 7.47 DK, 1 DK = 1 EUR ÷ 7.47 = 0.13 EUR
   
   c) If 1 USD = 0.62 GBP, 1 GBP = 1 USD ÷ 0.62 = 1.61 USD
   
   d) If £1 = 183.2 JPY, 1 JPY = £1 ÷ 183.2 = £0.00546
      (→ 0.55 p)

Q.2  
   a) Interest after 1 year: £397.50 – £375 = £22.50
      
      Rate of interest: \[
      \frac{22.50}{375} \times 100\% = 6\% \]
      
      Answer: The interest rate on Jenny’s account was 6%.
   
   b) £397.50 × 106% = £397.50 × 1.06 = £421.35
      
      Answer: Jenny would have £421.35 in her account at the end of the 2nd year.

Q.3  
   a) i) 312 ft = (312 × 0.3) m = 93.6 m
      
      ii) 11 m = (11 ÷ 0.3 = 110 ÷ 3) ft = 36.67 ft
      
   b) i) 36.4 cm = (36.4 ÷ 2.54 = 3640 ÷ 254) inches
      
      = 14.33 inches
      
      ii) 13 inch = (13 × 25.4) m = 330.2 mm
      
   c) i) 580 lb = (580 ÷ 2.2 = 5800 ÷ 22) kg = 263.64 kg
      
      ii) 37 kg = (37 × 2.2) lb = 81.4 lb
      
   d) i) 22°C = (\frac{9}{5} × 22 + 32 = \frac{198}{5} + 32 = 39.6 + 32) °F
      
      = 71.6 °F
      
      ii) 28°F = [\frac{5}{9} × (28 – 32) = \frac{5}{9} × (-4) = -\frac{20}{9} °C
      
      = -2.2 °C

Q.4  
   a) 14 h 10 min – 8 h 35 min = 13 h 70 min – 8 h 35 min
      
      = 5 h 35 min
      
   b) 27 h 22 in – 17 h 55 min = 26 h 82 min – 17 h 55 min
      
      = 9 h 27 min
      
   c) 24 h 24 min – 10 h 15 min = 14 h 9 min
      
   d) 18 h 52 min – 18 h 35 min = 17 min

105 = 3 × 5 × 7
Factors: 1, 3, 5, 7, 15, 21, 35, 105

280 = 2³ × 5 × 7
Factors: 1, 2, 4, 5, 7, 8, 10, 14, 20, 28, 35, 40, 56, 70, 140, 280

455 = 5 × 7 × 13
Factors: 1, 5, 7, 13, 35, 65, 91, 455

1105 = 5 × 13 × 17
Factors: 1, 5, 13, 17, 65, 85, 221, 1105

(or set factorising as homework at the end of Lesson 104 and review at the start of Lesson 105)
Activity

Solutions (Continued)

Q.5  a) \( a \times b = 16 \text{ cm}^2 \) and \( 16 = 1 \times 16 = 2 \times 8 = 4 \times 4 \)
    
    If \( a = 1, b = 16 \): \( P = 2 \times (1 + 16) = 2 \times 17 = 34 \neq 16 \)
    
    If \( a = 2, b = 8 \): \( P = 2 \times (2 + 8) = 2 \times 10 = 20 \neq 16 \)
    
    If \( a = 4, b = 4 \): \( P = 2 \times (4 + 4) = 2 \times 8 = 16 \checkmark \)
    
    The rectangle is a square with side 4 cm.

    b) \( a \times b = 24 \text{ cm}^2 \)
    
    and \( 24 = 1 \times 24 = 2 \times 12 = 3 \times 8 = 4 \times 6 \)
    
    If \( a = 1, b = 24 \): \( P = 2 \times (1 + 24) = 2 \times 25 = 50 \neq 28 \)
    
    If \( a = 2, b = 12 \): \( P = 2 \times (2 + 12) = 2 \times 14 = 28 \checkmark \)
    
    If \( a = 3, b = 8 \): \( P = 2 \times (3 + 8) = 2 \times 11 = 22 \neq 28 \)
    
    If \( a = 4, b = 6 \): \( P = 2 \times (4 + 6) = 2 \times 10 = 20 \neq 28 \)
    
    The rectangle has shorter side 2 cm and longer side 12 cm.

    c) \( a \times b = 72 \text{ cm}^2 \)
    
    and \( 72 = 1 \times 72 = 2 \times 36 = 3 \times 24 = 4 \times 18 = 6 \times 12 = 8 \times 9 \)
    
    If \( a = 1, b = 72 \): \( P = 2 \times (1 + 72) = 2 \times 73 = 146 \neq 34 \)
    
    If \( a = 2, b = 36 \): \( P = 2 \times (2 + 36) = 2 \times 38 = 76 \neq 34 \)
    
    If \( a = 3, b = 24 \): \( P = 2 \times (3 + 24) = 2 \times 27 = 54 \neq 34 \)
    
    If \( a = 4, b = 18 \): \( P = 2 \times (4 + 18) = 2 \times 22 = 44 \neq 34 \)
    
    If \( a = 6, b = 12 \): \( P = 2 \times (6 + 12) = 2 \times 18 = 36 \neq 34 \)
    
    If \( a = 8, b = 9 \): \( P = 2 \times (8 + 9) = 2 \times 17 = 34 \checkmark \)
    
    The rectangle has shorter side 8 cm and longer side 9 cm.

Q.6  a) \( b = 54 \times 2 \div 9 = 108 \div 9 = 12 \text{ (cm)} \)
    
    b) \( h = 42 \times 2 \div 12 = 84 \div 12 = 7 \text{ (cm)} \)
    
    c) \( A = 3.8 \times 2.2 = 8.36 \text{ (cm}^2) \)
    
    d) \( a = 37.1 \div 5.3 \times 2 = 371 \div 53 \times 2 = 7 \times 2 = 14 \text{ (cm)} \)

Notes

- Not to scale
- e.g.

© CIMT, University of Exeter
<table>
<thead>
<tr>
<th>Y6</th>
<th>R: Calculations</th>
<th>C: Area. Squares and square roots</th>
<th>E: Problems</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td><strong>Factorisation</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Elicit that:</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $106 = 2 \times 53$ Factors: 1, 2, 53, 106</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• 281 is a prime number Factors: 1, 281</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(as not exactly divisible by 2, 3, 5, 7, 11, 13, and $17^2 &gt; 281$)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $456 = 2 \times 2 \times 2 \times 3 \times 19 = 2^3 \times 19$ Factors: 1, 2, 3, 4, 6, 8, 12, 19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>456, 228, 152, 114, 76, 57, 38, 24</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>• $1106 = 2 \times 7 \times 79$ Factors: 1, 2, 7, 14, 79, 158, 553, 1106</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>7 min</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td><strong>Square numbers</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>a) Let's find squares which have sides of 1, 2, 3, 4, 5 and 6 units on this diagram. Ps come to BB to point out each square, then T asks them to write a calculation for its area. Class agrees/disagrees.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB: 1, 4, 9, 16, 25, 36, ... [Square numbers]</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>These are special numbers. Who knows what they are called? (square numbers) T adds name to BB.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>T: A square number is the product of a number multiplied by itself. We can write square numbers like this using a 'power' symbol.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB: $1^2 = 1$, $2^2 = 4$, $3^2 = 9$, $4^2 = 16$, $5^2 = 25$, $6^2 = 36$, ... Who can tell me the next square numbers in the sequence?</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ps dictate to T. ($7^2 = 49$, $8^2 = 64$, $9^2 = 81$, etc.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Let's find isosceles right-angled triangles in the diagram. Ps come to BB to point them out. T asks Ps to write calculations for their areas (or T starts and Ps continue). Class points out errors.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB: $1 \times 1$, $2 \times 2$, $3 \times 3$, etc. (i.e. half the area of the squares)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>c) Let's calculate the area of a square which has sides of length 11 cm (41 cm). Ps come to BB or dictate what T should write. Class agrees/disagrees.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>BB: $A = 11 \text{ cm} \times 11 \text{ cm} = 11^2 \text{ cm}^2 = 121 \text{ cm}^2$.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>15 min</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Notes</td>
<td>Individual work, monitored (or whole class activity) BB: 106, 281, 456, 1106 Ps can use calculators. Reasoning, agreement, self-correction, praising e.g.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>456 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>228 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>114 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>57 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>19 19</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1106 2 553 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>79</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>7 min</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td><strong>15 min</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Activity 3**

*PbY6b, page 106*

Q.1 Read: *Calculate the area of these squares.*

What do \(a\) and \(e\) stand for on the diagram? (\(a\) is the length of each side and \(e\) is the length of each diagonal)

Set a time limit of 4 minutes. Ps do calculations in *Ex. Bks.*

Review with whole class. Ps come to BB, draw a square and write on it the given data before doing the calculation. Class agrees/disagrees. Mistakes discussed and corrected.

*Solution:*

- a) \(a = 27\) cm:
  
  \[ A = 27 \, \text{cm} \times 27 \, \text{cm} = 27^2 \, \text{cm}^2 = 729 \, \text{cm}^2 \]

- b) \(a = 365\) mm:
  
  \[ A = 365 \, \text{mm} \times 365 \, \text{mm} = 365^2 \, \text{cm}^2 = 133225 \, \text{mm}^2 \]

- c) \(a = 2.3\) m:
  
  \[ A = 2.3 \, \text{m} \times 2.3 \, \text{m} = 2.3^2 \, \text{m}^2 = 5.29 \, \text{m}^2 \]

- d) \(e = 15\) cm:
  
  \[ A = \frac{15 \times 15}{2} \, \text{cm}^2 = \frac{15^2}{2} \, \text{cm}^2 = \frac{225}{2} \, \text{cm}^2 = 112.5 \, \text{cm}^2 \]

- e) \(e = 72\) mm:
  
  \[ A = \frac{72 \times 72}{2} \, \text{mm}^2 = \frac{72^2}{2} \, \text{mm}^2 = \frac{5184}{2} \, \text{mm}^2 = 2592 \, \text{mm}^2 = 25.92 \, \text{cm}^2 \]

*21 min*

**Notes**

Individual work, monitored, d) and e) helped or done with whole class

BB:

Reasoning, agreement, self-correction, checking with calculators, praising

\(= 1332.25 \, \text{cm}^2 \)

BB:

Elicit/show that to find the area of a square from its diagonal length, calculate the area of a square with side length \(e\), then halve it.

**Activity 4**

*PbY6b, page 106*

Q.2 Read: *Fill in the missing numbers if \(A = a^2\).*

Set a time limit of 2 minutes. Review with whole class.

Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

*Solution:*

<table>
<thead>
<tr>
<th>(a)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>(A)</td>
<td>0</td>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>36</td>
<td>49</td>
<td>64</td>
<td>81</td>
<td>100</td>
<td>121</td>
<td>144</td>
<td>169</td>
<td>196</td>
<td>225</td>
</tr>
</tbody>
</table>

What kind of numbers are in the bottom row of the table? (square numbers) Ps turn over *Pbs* and T covers up table on BB.

Let’s see if you can say the square numbers from 1 to 225 in increasing (decreasing) order. Encourage Ps to learn them by heart.

*25 min*
Activity

5

PbY6b, page 106, Q.3

Read: The area of a square is 1156 cm$^2$. Follow these methods to find the length of its sides.

T draws a square on BB and writes the area inside it.

a) Read: Between which two whole tens is the length of each side?

Ps make suggestions, saying why they chose those tens. Who agrees? Who thinks something else? Class decides what T should write on BB and Ps write the same tens in Pbs too.

BB:

\[ 30 < a^2 < 40 \]

as \[ 900 < a^2 < 1600 \]

Now let’s find \( a \) by trial and error. Ps suggest numbers to try and Ps check their squares on BB (or with calculators).

BB:

\[
35^2 = 35 \times 35 \\
34^2 = 34 \times 34 \\
= 175 + 1050 \\
= 136 + 1020 \\
= 1225 \text{ (too big)} = 1156 \checkmark
\]

So \( a^2 = 1156 = 34^2 \), and \( a = 34 \)

b) Read: First factorise 1156, then work out the value of \( a \).

Ps factorise 1156 in Ex. Bks then dictate what T should write on BB.

BB:

\[
\begin{array}{c|c}
\text{1156} & 2 \\
\text{578} & 2 \\
\text{289} & 17 \\
\text{17} & 17 \\
\end{array}
\]

So \( a = 34 \)

T: We say that the square root of 1156 is 34, because 34 squared is 1156. We can write it mathematically like this.

Let’s read the equation together. ‘The square root of 1156 equals 34.’

30 min

6

PbY6b, page 106

Q.4 Read: Fill in the missing numbers if \( A = \sqrt{A} \) (or \( a^2 = A \)).

Set a time limit of 2 minutes. Ps complete table in Pbs, factorising in Ex. Bks where necessary.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. e.g. ‘The square root of 4 is 2, as 2 squared is equal to 4.’ Class points out errors. Mistakes discussed and corrected. Show details of factorisation if there is disagreement.

What could \( A \) and \( a \) stand for? (e.g. \( A \) could be the area of a square and \( a \) could be the length of a side.)

Solution:

\[
\begin{array}{cccccccccccccccc}
A & 1 & 4 & 9 & 16 & 25 & 36 & 49 & 64 & 81 & 100 & 121 & 144 & 169 & 196 & 225 \\
\hline
a & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 \\
\end{array}
\]

Who can think of another way to write the rule for the table?

(e.g. \( A \div a = a \), or \( \frac{A}{a} = a \), or \( a \times a = A \))

Ps suggest values for extra columns in the table.

34 min

Notes

Whole class activity

Written on BB or SB or OHT

BB:

\[
A = 1156 \text{ cm}^2
\]

Discussion, reasoning, agreement, praising

Extra praise if Ps suggest 35 first as it is halfway between 30 and 40.

35 is too big, so try the next smaller integer.

Agreement, praising

T intervenes only if necessary.

Allow Ps to work it out if they can.

BB: Square root

\[
\sqrt{1156} = 34 \]

(as \( a^2 = 1156 \))

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Extra praise for clever Ps who realised that this table is the reverse of the table in Q.2, so the missing values could be copied from the Q. 2 table!

Agreement, praising

Class checks that they are correct.
\[ \text{Solution:} \]

a) i) \( A = 25 \text{ cm}^2 \) = 5 cm \times 5 cm, so \( a = 5 \text{ cm} \)
ii) \( A = 250 \text{ cm}^2 \): 15 \times 15 = 225 (too small)
   16 \times 16 = 256 (too big but quite close)
   15.9 \times 15.9 = 252.81 (still too big but closer)
   15.8 \times 15.8 = 249.64 (very close)
   so \( a = 15.8 \text{ cm} \)
iii) \( A = 2500 \text{ cm}^2 \): 2500 = 25 \times 100
   = 5 \times 5 \times 10 \times 10 = (5 \times 10)^2 = 50^2
   so \( a = 50 \text{ cm} \)

b) i) \( A = 64 \text{ cm}^2 \) = 8 cm \times 8 cm, so \( a = 8 \text{ cm} \)
ii) \( A = 6.4 \text{ cm}^2 \): 2 \times 2 = 4 (too small)
   3 \times 3 = 9 (too big)
   2.5 \times 2.5 = 6.25 (still too small but closer)
   2.6 \times 2.6 = 6.76 (too big)
   2.52 \times 2.52 = 6.3504 (slightly too small)
   2.53 \times 2.53 = 6.4009 (very close)
   so \( a = 2.53 \text{ cm} \)
iii) \( A = 0.64 \text{ cm}^2 \) = 0.8 cm \times 0.8 cm, so \( a = 0.8 \text{ cm} \)
   (or 0.64 cm\(^2\) = 64 mm\(^2\) = 8 mm \times 8 mm = 0.8 cm \times 0.8 cm)

We have seen how difficult it is to work out square roots which are not whole numbers. We can do it easily using a calculator. T shows Ps how to use the \( \sqrt{ } \) button and the \( x^2 \) button as a check. Ps copy T.

\[ 40 \text{ min} \]
**Activity 8**

PbY6b, page 106

Q.6 Read: *Work out the square roots. Use a calculator where necessary.*

Set a time limit or deal with one row at a time. Encourage Ps to check their results by squaring the values.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Who worked it out another way? Come and show us. Mistakes discussed and corrected. If no P thinks of it, T shows a method using fractions:

BB:  

b) ii) \( \sqrt{2.56} = \sqrt{\frac{256}{100}} = \frac{\sqrt{256}}{\sqrt{100}} = \frac{16}{10} = 1.6 \)

c) i) \( \sqrt{0.25} = \sqrt{\frac{25}{100}} = \frac{\sqrt{25}}{\sqrt{100}} = \frac{5}{10} = 0.5 \)

d) i) \( \sqrt{1.96} = \sqrt{\frac{196}{100}} = \frac{\sqrt{196}}{\sqrt{100}} = \frac{14}{10} = 1.4 \)

**Solution:**

a) i) \( \sqrt{100} = 10 \)  ii) \( \sqrt{10000} = 100 \)

iii) \( \sqrt{1000000} = 1000 \)

b) i) \( \sqrt{256} = 16 \)  ii) \( \sqrt{2.56} = 1.6 \)

iii) \( \sqrt{25600} = 160 \)

c) i) \( \sqrt{0.25} = 0.5 \)  ii) \( \sqrt{25} = 5 \)

iii) \( \sqrt{2500} = 50 \)

d) i) \( \sqrt{1.96} = 1.4 \)  ii) \( \sqrt{196} = 14 \)

iii) \( \sqrt{19.6} = 4.43 \)

45 min

**Notes**

Individual work, monitored, helped.

(or whole class activity if time is short or Ps are unsure)

Written on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising.

Note that d) iii) is the only calculation which requires a calculator, although it could be done using approximation if Ps do not have a square root button on their calculators:

e.g.

\( 4 \times 4 = 16 \) (too small)

\( 5 \times 5 = 25 \) (too big)

\( 4.5 \times 4.5 = 20.25 \) (too big)

\( 4.4 \times 4.4 = 19.36 \) (too small)

\( 4.42 \times 4.42 = 19.5364 \) (too small)

\( 4.43 \times 4.43 = 19.6249 \) (very close)

\( 4.44 \times 4.44 = 19.7136 \) (too big)
Y6

**Activity**

1. **Factorisation**

   Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

   - **107** is a prime number
     - Factors: 1, 107
     - (as not exactly divisible by 2, 3, 5, 7 and 11² > 107)
   - **282** = \(2 \times 3 \times 47\)
     - Factors: 1, 2, 3, 6, 47, 94, 141, 282
   - **457** is a prime number
     - Factors: 1, 457
     - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and 23² > 457)
   - **1107** = \(3 \times 3 \times 3 \times 41\) = \(3³ \times 41\)
     - Factors: 1, 3, 9, 27, 41, 123, 369, 1107

   **6 min**

2. **Area and volume of cuboids**

   - **T** has 3 large cuboids on desk and if possible Ps have a smaller set on desks also. T's set could have edges which are 10 times greater: e.g. \(2 \times 2 \times 2\) (cube), \(4 \times 3 \times 2\) (cuboid), \(2 \times 2 \times 4\) (square-based cuboid), each a different colour.
   - T should also have a diagram drawn on BB or OHT for each type.
   - What name can you give to all these shapes? (cuboids)
   - Who can tell me properties of a cuboid? (e.g. 3-D, 6 rectangular faces, 90° angles at the 8 vertices, 12 straight edges; Ps point out congruent, parallel/perpendicular faces and edges, equal sides, etc.)

   a) Let's measure the edges of this cuboid. (T holds up one cuboid at a time and Ps measure it (with rulers or 1 cm square grids). T writes measurements on BB.

   - **R**
     - \(a = 2\, cm\)
     - \(b = 3\, cm\)
     - \(c = 4\, cm\)
   - **G**
     - \(a = 4\, cm\)
     - \(b = 2\, cm\)
     - \(c = 2\, cm\)
   - **Y**
     - \(a = 2\, cm\)
     - \(b = 2\, cm\)
     - \(c = 3\, cm\)

   b) What is the area of each cuboid? Ps dictate to T. Class agrees/disagrees.

   - **R**: \(A = 6 \times 2 \times 2\) cm² = \(24\, cm²\)
   - **G**: \(A = 2 \times (4 \times 2 \times 2 + 4 \times 3 \times 2 + 2 \times 3 \times 2)\)
     - = \(2 \times (8 \, cm² + 12 \, cm² + 6 \, cm²)\)
     - = \(2 \times 26 \, cm²\) = \(52\, cm²\)
   - **Y**: \(A = 2 \times 2 \times 2 + 4 \times 2 \times 2 \times 2\)
     - = \(2 \times 4 \, cm² + 4 \times 8 \, cm²\)
     - = \(8 \, cm² + 32 \, cm²\) = \(40\, cm²\)

   What is the **general** formula for the area of any cuboid? Ps dictate to T. Class agrees/disagrees. T shows a short notation without ‘×’.

   - **BB**: \(A\) (cuboid) = \(2 \times a \times b + 2 \times a \times c + 2 \times b \times c\)
     - = \(2 \times (a \times b + a \times c + b \times c)\) = \(2(ab + ac + bc)\)

   **Whole class activity**
   [It would be useful if the T’s cuboids were transparent plastic containers with 1 unit grids on the faces and a lid so that unit cubes could be placed inside when dealing with volume.]

   - Involve several Ps.
   - Praising, encouragement only

   Ps measure own cuboids if they have them or come to front of class to measure T’s.

   - Agree on a consistent method of listing, e.g.
     - \(a = \text{width}\)
     - \(b = \text{depth}\)
     - \(c = \text{height}\)
   - At a good pace
   - Agreement, praising

   - Who can tell me other units of area? (e.g. \(\text{mm}², \text{m}², \text{km}², \text{acres, hectares}\))

   - Also elicit the general formula for the surface area of a cube and a square-based cuboid.
   - **BB**: \(A\) (cube) = \(6 \times a² = 6a²\)
   - \(A\) (s. b. cuboid) = \(2a² + 4ab\)

   Ps write formulae in back of Pb.
(Continued)

**c)** How many unit cubes would we need to build each cuboid?  
(How many would be needed along the front edge?  How many in each layer?  How many layers?)  Elicit that this value is the **volume** of the cuboid, i.e. how much space it takes up.

Let's write a calculation for each volume.  Ps come to BB or dictate what T should write.

- **BB:**  
  \[
  V = 2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm} = 8 \text{ cm}^3 
  \]
  \[
  V = 2 \text{ cm} \times 2 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^3 
  \]
  \[
  V = 4 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 24 \text{ cm}^3 
  \]

Who can tell me the **general** formula for the volume of any cuboid?  
Ps dictate to T.  Class agrees/disagrees.  T shows a shortened notation.  Also elicit general formulae for the volume of a cube and a square-based cuboid.

- **BB:**  
  \[
  V(\text{cuboid}) = a \times b \times c = abc 
  \]
  \[
  V(\text{cube}) = a \times a \times a = a^3 
  \]
  \[
  V(\text{square-based cuboid}) = a \times a \times b = a^2 b 
  \]

<table>
<thead>
<tr>
<th>Along an edge</th>
<th>In a layer</th>
<th>Total number of cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>5 \times 3 = 15</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>4 \times 2 = 8</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>3 \times 3 = 9</td>
</tr>
</tbody>
</table>

Who can tell me other units of volume?  (e.g. mm$^3$, m$^3$)

Ps say what the lengths of each edge would be in these cases.  
(1 mm, 1 m)

Ps write formulae (including short forms) in back of *Pbs* or Ex. Bks.

**Lesson Plan 107**

- **Notes**

  If possible, Ps check by placing unit cubes in the transparent plastic cubes.
  
  At a good pace.
  
  Agreement, praising
  
  Who can tell me other units of volume?  (e.g. mm$^3$, m$^3$)
  
  Ps say what the lengths of each edge would be in these cases.  
  (1 mm, 1 m)

  Ps write formulae (including short forms) in back of *Pbs* or Ex. Bks.

  Individual work, monitored, helped
  
  Drawn on BB or use enlarged copy master or OHP
  
  (if possible, T has large transparent boxes and unit cubes for demonstration)

  Differentiation by time limit
  
  In unison
  
  Reasoning, agreement, self-correction, praising
  
  Feedback for T
Activity

2 (Continued)

- How many unit cubes would we need to build each cuboid? (How many would be needed along the front edge? How many in each layer? How many layers?) Elicit that this value is the volume of the cuboid, i.e. how much space it takes up.

Let's write a calculation for each volume. Ps come to BB or dictate what T should write.

BB: R: \( V = 2 \text{ cm} \times 2 \text{ cm} \times 2 \text{ cm} = 8 \text{ cm}^3 \)
G: \( V = 2 \text{ cm} \times 2 \text{ cm} \times 4 \text{ cm} = 16 \text{ cm}^3 \)
Y: \( V = 4 \text{ cm} \times 3 \text{ cm} \times 2 \text{ cm} = 24 \text{ cm}^3 \)

Who can tell me the general formula for the volume of any cuboid? Ps dictate to T. Class agrees/disagrees. T shows a shortened notation. Also elicit general formulae for the volume of a cube and BB: 

- \( V(\text{cuboid}) = a \times b \times c = abc \) a square-based cuboid.
- \( V(\text{cube}) = a \times a \times a = a^3 \)
- \( V(\text{square-based cuboid}) = a \times a \times b = a^2b \)

20 min

Q.1 Read: These are 3 different boxes for storing unit cubes.

BB: 

- What shape are they? (All are cuboids, B is a square-based cuboid, although it is not standing on its square face; C is a cube.)

Set a time limit. Ps read questions themselves, write answers to a) and b) in Ex Bks (Ps might find it helpful to label the edges in diagrams with letters) then complete the table in Pbs.

Review with whole class. Ps show answers to a) and b) on scrap paper or slates on command, then come to BB to write in table, referring to diagrams. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

- How many cubes will fit along the front edge of the bottom layer in each box?
  A: \( a = 5 \); B: \( a = 4 \); C: \( a = 3 \)

- How many: i) rows ii) cubes can be put in each bottom layer?
  i) A: \( b = 3 \); B: \( b = 2 \); C: \( b = 3 \)
  ii) A: \( ab = 15 \); B: \( ab = 8 \); C: \( ab = 9 \)

- Fill in the table.

<table>
<thead>
<tr>
<th></th>
<th>Along an edge</th>
<th>In a layer</th>
<th>Total number of cubes</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5</td>
<td>( 5 \times 3 = 15 )</td>
<td>( 5 \times 3 \times 4 = 60 )</td>
</tr>
<tr>
<td>B</td>
<td>4</td>
<td>( 4 \times 2 = 8 )</td>
<td>( 4 \times 2 \times 4 = 32 )</td>
</tr>
<tr>
<td>C</td>
<td>3</td>
<td>( 3 \times 3 = 9 )</td>
<td>( 3 \times 3 \times 3 = 27 )</td>
</tr>
</tbody>
</table>

26 min

Notes

If possible, Ps check by placing unit cubes in the transparent plastic cubes.

At a good pace.

Agreement, praising

Who can tell me other units of volume? (e.g. \( \text{mm}^3, \text{m}^3 \))

Ps say what the lengths of each edge would be in these cases. (1 mm, 1 m)

Ps write formulae (including short forms) in back of Pbs or Ex. Bks.

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

(If possible, T has large transparent boxes and unit cubes for demonstration)

Differentiation by time limit

In unison

Reasoning, agreement, self-correction, praising

Feedback for T

Does it matter in which order we multiply the lengths?

No, as the factors in a multiplication can be interchanged without affecting the result.
Activity

7

PbY6b, page107

Q.5 T chooses a P to read out the question. Ps calculate in Ex. Bks and show answers on command. T chooses Ps with different answers to explain their reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Use a model if necessary.

Solution:

a) Calculate the volume of a cuboid which has edges 3 cm, 4 cm and 5 cm long.
\[ V = 3 \text{ cm} \times 4 \text{ cm} \times 5 \text{ cm} = 60 \text{ cm}^3 \]

b) What is the volume of a cuboid with edges a, b and c?
\[ V = a \times b \times c = abc \]

39 min

8

PbY6b, page107

Q.6 Read:

a) The surface area of each face of an ice cube is 49 cm². Calculate:
   i) the volume of the ice cube
   ii) its mass, if 1 cm³ of ice weighs 0.91 g.

b) The surface area of a square-based prism is 64 cm² and its base edge is 2 cm. What is the volume of the prism?

Set a time limit or deal with one part at a time. Ps draw a diagram, write an operation, do the calculation and write the answer in a sentence in Ex. Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

Solution:

a) i) \[ A \text{ (face)} = a \times a = 49 \text{ cm}^2 \]

BB:

\[
\begin{array}{c}
\text{a} \\
\text{a}
\end{array}
\]

\[ a = \sqrt{49} \text{ (cm)} = 7 \text{ cm} \]

\[ V = a \times a \times a = 7 \text{ cm} \times 7 \text{ cm} \times 7 \text{ cm} = 343 \text{ cm}^3 \]

ii) \[ M = 0.91 \text{ g} \times 343 = 312.13 \text{ g} \approx 0.312 \text{ kg} \]

Answer: The mass of the ice cube is 312.13 grams.

b) \[ A \text{ (square-based prism)}: 2 \times 2 \times 2 + 4 \times 2 \times h = 64 \text{ (cm}^2\)\]

BB:

\[
\begin{array}{c}
2 \\
\text{h}
\end{array}
\]

\[ 2 \times 2 \times 2 + 4 \times 2 \times h = 64 \text{ (cm}^2\)

\[ 8 + 8 \times h = 64 \text{ (cm}^2\)

\[ 8 \times h = 64 - 8 = 56 \text{ (cm}^2\)

\[ h = 56 \div 8 = 7 \text{ (cm)} \]

\[ V \text{ (square-based prism)}: a \times a \times h = 2 \text{ cm} \times 2 \text{ cm} \times 7 \text{ cm} = 4 \text{ cm}^2 \times 7 \text{ cm} = 28 \text{ cm}^3 \]

Answer: The volume of the prism is 28 cm³.

45 min

Lesson Plan 107

Notes

Individual work, monitored
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Who can put into words the rule for finding the volume of a cuboid? T might suggest: ‘The volume of a cuboid is the product of the lengths of the 3 edges which meet at a vertex.’ and ask Ps if it is correct. Class repeats it in unison.

Individual work, monitored, helped
Ps can discuss the method of solution with their neighbours.
(If majority of Ps are struggling, stop individual work and continue as a whole class activity.)

Differentiation by time limit

Responses shown in unison.
Discussion reasoning, agreement, self-correction, praising

Feedback for T

(If possible, T has a 7 cm cube weighing approximately 312 g to pass round class.)

Class applauds any Ps who worked out the answer without help.
### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3\)
  - Factors: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
- \(283\) is a prime number
  - Factors: 1, 283
    - (as not exactly divisible by 2, 3, 5, 7, 11, 13, and 17 \(2^7 > 283\))
- \(458 = 2 \times 229\)
  - Factors: 1, 2, 229, 458
- \(1108 = 2 \times 2 \times 277 = 2^2 \times 277\) (and 277 is a prime number)
  - Factors: 1, 2, 4, 277, 554, 1108

**Notes**

Individual work, monitored (or whole class activity)

BB: 108, 283, 458, 1108

Ps could practise without using calculators.

Reasoning, agreement, self-correction, praising

### Activity 2

**Area and volume**

Let’s calculate the values asked for. First elicit the name of the shape and the formula to use (in words). Then Ps come to BB to write operations and do calculations, explaining reasoning. Class helps and points out errors. T gives hints where necessary and helps Ps to write the units of measure in the correct place [within the operation or at the end with the operation in brackets: \(7 \text{ cm} \times 7 \text{ cm} = 49 \text{ cm}^2\) or \((7 \times 7) \text{ cm}^2 = 49 \text{ cm}^2\)]

BB:

#### a)

\[
\begin{align*}
A &= \left(\frac{5 \times 3}{2}\right) \text{ cm}^2 = \frac{15}{2} \text{ cm}^2 = 7.5 \text{ cm}^2 \\
A &= \left(\frac{3.2 \times 4}{2}\right) \text{ cm}^2 = \frac{12.8}{2} \text{ cm}^2 = 6.4 \text{ cm}^2
\end{align*}
\]

#### b)

\[
\begin{align*}
A &= \left(\frac{60 \times 40}{2}\right) \text{ mm}^2 = \frac{2400}{2} \text{ mm}^2 = 1200 \text{ mm}^2 \\
A &= \left(1.6 \times 1.6\right) \text{ m}^2 = 5.6 \text{ m}^2 \\
A &= \sqrt{81} \text{ m}^2 = 9 \text{ m}^2
\end{align*}
\]

#### c)

\[
\begin{align*}
A &= \left(\frac{60 \times 40}{2}\right) \text{ mm}^2 = 1200 \text{ mm}^2 \\
A &= \left(\frac{6 \times 7}{2}\right) \text{ cm}^2 = \frac{42}{2} \text{ cm}^2 = 21 \text{ cm}^2
\end{align*}
\]

#### d)

\[
\begin{align*}
A &= \left(\frac{6 \times 7}{2}\right) \text{ cm}^2 = \frac{42}{2} \text{ cm}^2 = 21 \text{ cm}^2 \\
A &= 6 \times 49 \text{ cm}^2 = 294 \text{ cm}^2 \\
V &= 7 \times 7 \times 7 \text{ cm}^3 = 49 \times 7 \text{ cm}^3 = 343 \text{ cm}^3
\end{align*}
\]

#### e)

\[
\begin{align*}
A &= \left(\frac{6 \times 7}{2}\right) \text{ cm}^2 = \frac{42}{2} \text{ cm}^2 = 21 \text{ cm}^2 \\
A &= 6 \times 7 \times 7 \text{ cm}^2 = 6 \times 49 \text{ cm}^2 = 294 \text{ cm}^2 \\
V &= 7 \times 7 \times 7 \text{ cm}^3 = 49 \times 7 \text{ cm}^3 = 343 \text{ cm}^3
\end{align*}
\]

#### f)

\[
\begin{align*}
A &= \left(\frac{6 \times 7}{2}\right) \text{ cm}^2 = \frac{42}{2} \text{ cm}^2 = 21 \text{ cm}^2 \\
A &= 6 \times 49 \text{ cm}^2 = 294 \text{ cm}^2 \\
V &= 7 \times 7 \times 7 \text{ cm}^3 = 49 \times 7 \text{ cm}^3 = 343 \text{ cm}^3
\end{align*}
\]

#### g)

\[
\begin{align*}
A &= \left(\frac{6 \times 7}{2}\right) \text{ cm}^2 = \frac{42}{2} \text{ cm}^2 = 21 \text{ cm}^2 \\
A &= 6 \times 49 \text{ cm}^2 = 294 \text{ cm}^2 \\
V &= 7 \times 7 \times 7 \text{ cm}^3 = 49 \times 7 \text{ cm}^3 = 343 \text{ cm}^3
\end{align*}
\]

**Notes**

Whole class activity

Drawn on BB or use enlarged copy master or OHT

Ps could have copy on desks too.

At a good pace

Involve majority of class

Reasoning, agreement, praising

Feedback for T

Elicit that:

- Area of a triangle = half its base \(\times\) its height
- Area of a rhombus = half the product of its diagonals
- Area of a square = the length of a side squared
- Length of the side of a square = the square root of its area
- Surface area of a cube = 6 \(\times\) the length of an edge squared
- Volume of a cube is the length of an edge cubed etc.
Q.1 Read: Write the areas and volumes below the diagrams, as required.

Set a time limit or deal with one at a time. Ps do calculations in Ex. Bks, then write only the results in Pbs.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

Solution: e.g.

a) \[ A = 5 \times 2.5 + \frac{5 \times 2}{2} = 12.5 + \frac{10}{2} = 12.5 + 5 = 17.5 \text{ (m}^2) \]

b) \[ A = \frac{4 \times 4}{2} + \frac{4 \times 2}{2} = \frac{16}{2} + \frac{8}{2} = 8 + 4 = 12 \text{ (cm}^2) \]

c) \[ A = 6 \times 2.3 \times 2.3 = 6 \times 5.29 = 31.74 \text{ (m}^2) \]
\[ V = 2.3 \times 2.3 \times 2.3 = 5.29 \times 2.3 = 12.167 \text{ (m}^3) \]

d) \[ A = 2 \times (3 \times 3) + 4 \times (5 \times 3) = 2 \times 9 + 4 \times 15 = 18 + 60 = 78 \text{ (mm}^2) \]
\[ V = 5 \times 3 \times 3 = 45 \text{ (mm}^3) \]

e) \[ A = 2 \times (e \times f + e \times g + f \times g) = 2 (ef + eg + fg) \]
\[ V = e \times f \times g = efg \]

f) \[ A = 6 \times s \times s = 6 \times s^2 = 6s^2 \]
\[ V = s \times s \times s = s^3 \]

Extra praise if Ps can write the short forms of the equations but if no P does so, T shows them.

---

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**Y6**

**Activity**

4  *PbY6b, page 108, Q.2*

Read: *A cuboid was cut into two equal pieces. This is the net of one of the halves.*

![Net of a cuboid](image)

T has 2 enlarged copies of the nets cut out and pre-folded. (If possible, Ps have actual scale versions on desks too.) Who can show us what the whole cuboid looks like? Ps fold the two nets and hold them together to show T (or class if only using T’s models). Now we have an idea of where this part of the cuboid fits.

Read: *Calculate the surface area and the volume of this prism.*

Why is half of the cuboid called a prism and what kind of prism is it? (It is a prism because it is a polyhedron with 2 of it faces: base and top, equal and parallel. It is a triangular-based prism.)

How can we calculate its area and volume? Ps come to BB or dictate what T should write on BB. Class agrees/disagrees or suggests other calculations. T gives help only if necessary. Ps work in *Ex. Bks* too.

**Solution:**

\[
A = (5 \times 3.5 + 4 \times 3.5 + 3 \times 3.5 + 2 \times \frac{4 \times 3}{2}) \text{ cm}^2 = (17.5 + 14 + 10.5 + 12) \text{ cm}^2 = 54 \text{ cm}^2
\]

or \( A = 12 \text{ cm} \times 3.5 \text{ cm} + 4 \text{ cm} \times 3 \text{ cm} = 42 \text{ cm}^2 + 12 \text{ cm}^2 = 54 \text{ cm}^2 \)

\[
V = \frac{4 \times 3 \times 3.5}{2} \text{ cm}^3 = \frac{42}{2} \text{ cm}^3 = 21 \text{ cm}^3
\]

33 min

5  *PbY6b, page 108*

Q.3  Read: *What is the volume of a cube if its edge is 1, 2, 3, 4, 5, 6 or 7 units? Fill in the table to show the volumes for different edge lengths.*

Set a time limit of 2 minutes. Encourage mental calculation where possible, otherwise Ps calculate in *Ex. Bks*. Ask for the rule too.

Review with whole class. Ps dictate to T or come to BB, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. If disagreement, demonstrate with unit cubes.

**Solution:**

<table>
<thead>
<tr>
<th>(a) (units)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V) (unit cubes)</td>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>125</td>
<td>216</td>
<td>343</td>
<td>512</td>
<td>729</td>
<td>1000</td>
</tr>
</tbody>
</table>

**Rule:** \(V = a \times a \times a = a^3\)

The numbers in the bottom row are called *cubic numbers*. Let’s say them together. Now let’s try to say them with our eyes closed!

Another form of the rule is to say that \(a\) is the *cubic root* of \(V\). We write it mathematically like this. Ps read out the equation, with T pointing to the relevant components.

38 min

---

**Notes**

Whole class activity  
(or individual trial first if Ps wish, using the net models afterwards to confirm or dispute their results)  
Drawn on BB or use enlarged copy master or OHP  
Differentiation by time limit

Discussion, reasoning, agreement, praising

Extra praise for Ps who realise that:
- 3 of the faces form a wide rectangle 12 cm by 3.5 cm and the 2 triangular faces form another rectangle 3 cm by 4 cm;
- the volume of the prism is half the volume of the whole cuboid.

Individual work, monitored, helped  
Drawn on BB or use enlarged copy master or OHP  
Differentiation by time limit  
Reasoning, agreement, self-correction, praising

In unison. In good humour. Encourage Ps to learn them.

BB: **Rule:** \(a = \frac{1}{3} \sqrt[3]{V}\)  
(as \(a^3 = V\))
Q.4 Deal with one question at a time. T chooses a P to read out the question, and asks Ps to picture the shape in their heads first. Ps solve problem in Ex. Bks. under a short time limit, then show result on scrap paper or slates on command. Ps with different answers explain reasoning at BB (with T's help if needed). Class points out errors and agrees on correct answer. Who worked it out another way? Mistakes discussed and corrected. T chooses a P to say the answer in a sentence.

Solution:

a) An empty cubic box contains 8000 cm$^3$ of air.
   How long is its edge?
   \[ V = a \times a \times a = 8000 \text{ cm}^3 \]
   but \[ 20 \text{ cm} \times 20 \text{ cm} \times 20 \text{ cm} = 8000 \text{ cm}^3, \] so \[ a = 20 \text{ cm} \]
   or \[ a = \frac{1}{3} \sqrt[3]{8000} = 20 \text{ cm} \]
   [as \[ 8000 = 8 \times 1000 = 2^3 \times 10^3 = (2 \times 10)^3 = 20^3 \]
   Answer: The length of each edge of the box is 20 cm.

b) i) How many metres long is the edge of a 1 km$^3$ cube?
   \[ V = a \times a \times a = a^3 = 1 \text{ km}^3 \]
   but \[ 1 \text{ km} \times 1 \text{ km} \times 1 \text{ km} = 1 \text{ km}^3, \] so \[ a = 1 \text{ km} = 1000 \text{ m} \]
   or \[ a = \frac{1}{3} \sqrt[3]{1 \text{ km}^3} = 1 \text{ km} = 1000 \text{ m} \]
   T: We say that the cubic root of 1 is 1, as \[ 1^3 = 1 \]
   Answer: The edge of a 1 km$^3$ cube is 1000 m long.

   ii) What is the surface area of the cube?
   \[ A = 6 \times 1 \text{ km} \times 1 \text{ km} = 6 \times 1 \text{ km}^2 = 6 \text{ km}^2 \]
   Answer: The surface area of the cube is 6 km$^2$.

c) i) How many centimetres long is the edge of a 1 m$^3$ cube?
   \[ V = a \times a \times a = a^3 = 1 \text{ m}^3 \]
   but \[ 1 \text{ m} \times 1 \text{ m} \times 1 \text{ m} = 1 \text{ m}^3, \] so \[ a = 1 \text{ m} = 100 \text{ cm} \]
   or \[ a = \frac{1}{3} \sqrt[3]{1 \text{ m}^3} = 1 \text{ m} = 100 \text{ cm} \]
   Answer: The edge of a 1 m$^3$ cube is 100 cm long.

   ii) What is the surface area of the cube?
   \[ A = 6 \times 1 \text{ m} \times 1 \text{ m} = 6 \times 1 \text{ m}^2 = 6 \text{ m}^2 \]
   Answer: The surface area of the cube is 6 m$^2$.

d) How many mm long is the edge of a 729 000 cm$^3$ cube?
   Use the table in Question 3 to help you.
   \[ V = a \times a \times a = 729 \text{ 000 cm}^3 \]
   but \[ 90 \text{ cm} \times 90 \text{ cm} \times 90 \text{ cm} = 729 \text{ 000 cm}^3, \] so \[ a = 90 \text{ cm} \]
   or \[ a = \frac{1}{3} \sqrt[3]{729 \text{ 000 cm}^3} = 90 \text{ cm} \]
   [as \[ 729 \text{ 000} = 729 \times 1000 = 9^3 \times 10^3 = (9 \times 10)^3 = 90^3 \]
   Answer: The length of each edge of the cube is 90 cm.

Notes

Individual work, monitored, helped, but class kept together Difference by time limit.
Advise Ps to use the results in the table in Q.2 to help them.
Responses shown in unison.
Discussion, reasoning, agreement, self-correction, praising
Feedback for T

T: We say that the cubic root of 8000 is 20, as \[ 20^3 = 8000 \]

or \[ V = 1 \text{ km}^3 \]
   \[ = 1000 \text{ m} \times 1000 \text{ m} \]
   \[ \times 1000 \text{ m} \]
   so \[ a = 1000 \text{ m} \]

or \[ V = 1 \text{ m}^3 \]
   \[ = 100 \text{ cm} \times 100 \text{ cm} \]
   \[ \times 100 \text{ cm} \]
   so \[ a = 100 \text{ cm} \]

Finish lesson with mental practice at speed round class.
e.g. a miscellany of:
• T saying a number and Ps saying its square (cubic).
• T saying a square (cubic) number and Ps saying its square (cubic) root.
Lesson Plan

109

Notes

Individual work, monitored (or whole class activity)
BB: 109, 284, 459, 1109
T decides whether Ps can use calculators.
Reasoning, agreement, self-correction, praising
e.g.

<table>
<thead>
<tr>
<th>284</th>
<th>459</th>
</tr>
</thead>
<tbody>
<tr>
<td>142</td>
<td>153</td>
</tr>
<tr>
<td>71</td>
<td>51</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
</tr>
</tbody>
</table>

Whole class activity
Written on BB or SB or OHT
At a good speed
Involve several Ps.
Discussion, agreement, praising

(1) It is possible for a book to be written by more than one person, but make sure that only single authored books are shown here.)

Drawn on BB or SB or OHT
Ps come to BB to draw the joining arrows.

What if we drew the arrows in the opposite direction?

Elicit that the rule would be:

BB: \( a = \frac{-2}{3} \times b \) or \( a = b \div (-1.5) \) which is also unique.

---

### Activity

#### 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **109** is a prime number
  - Factors: 1, 109
  - (as not exactly divisible by 2, 3, 5, 7, and \( 11^2 > 109 \))
- **284** = \( 2 \times 2 \times 71 = 2^2 \times 71 \)
  - Factors: 1, 2, 4, 71, 142, 284
- **459** = \( 3 \times 3 \times 3 \times 17 = 3^3 \times 17 \)
  - Factors: 1, 3, 9, 17, 27, 51, 153, 459
- **1109** is a prime number
  - Factors: 1, 1109
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and \( 37^2 > 1109 \))

---

### Connections

Let’s join up the elements in one set to the matching elements in the other set. e.g.

a) **Variable connections** (Ps could suggest their favourite books and authors or T has sets already prepared, using books in class.)

Let’s join up the authors with their books. Ps come to BB to draw arrows. Class agrees/disagrees.

**BB:** e.g.

```
Authors
J. K. Rowling
Roald Dahl
Jacqueline Wilson

Books
Harry Potter and the Philosopher's Stone
Harry Potter and the Prisoner of Azkeban
The BFG
The Witches
Double Act
```

What do you notice?

(An author can be joined to more than one book.)

If we drew arrows in the opposite direction, what do you notice?

(Each book can be joined to only one author.)

Agree that although some authors only ever write one book, generally they write several books, especially if they sell lots of copies of their first book.

b) **Unique connections**

What is the connection between these 2 sets? (Each element, \( a \), in Set A has been multiplied by \(-1.5\), giving a number, \( b \), in Set B.)

**BB:**

```
A \[\ldots -3 -2 -1 0 1 2 3 4 5 \ldots\]
\times -1.5

B \[\ldots 4.5 3 1.5 0 -1.5 -3 -4.5 -6 -7.5 \ldots\]
```

Could an element in A be connected to more than 1 element in B?

(No) T: We say that the relationship between A and B is **unique**, i.e. each value in Set A can be connected to only **one** in Set B.
Y6

Activity

2

(Continued)

c) What can you tell me about the relationship between the rows in this table and is it unique?

\[
\begin{array}{c|c|c|c|c|c|c|c}
\text{x} & -4.1 & 2 & -\frac{3}{4} & 0 & -0.7 & -11 & -0.93 & 2 \\
\text{|x|} & 4.1 & 2 & \frac{3}{4} & 0 & 0.7 & 11 & 0.93 & 2 \\
\end{array}
\]

First elicit or remind Ps that \(|x|\) means the absolute value of \(x\), i.e. its distance from zero. Then ask several Ps what they think about the connections from top to bottom row and bottom to top row.

Agree that:
- \(x \rightarrow |x|\) is unique,
- \(|x| \rightarrow x\) is not unique (as an absolute value of, e.g. 2, can be connected to +2 or to –2)

---

3

PbY6b, page 109,

Q.1 Read: Let \(y\) be 60% of \(x\).

What part of \(x\) is 60%? (60 hundredths, 6 tenths, 3 fifths, 0.6)

a) Read: Complete the table.

Set a short time limit. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes corrected.

\[
\begin{array}{c|c|c|c|c|c|c|c}
\text{x} & 1 & -1 & 4 & 0 & 2.5 & -2 & 5 \\
\text{y} & 0.6 & -0.6 & 2.4 & 0 & 1.5 & -12 & 3 \\
\end{array}
\]

b) Read: Represent the pairs of values as dots in the coordinate grid. Join up the points with a line.

What can you tell me about the grid? (e.g. The \(x\) axis ranges from –2 to 3 and the \(y\) axis from –2 to 5; there is a dotted grid line at every 0.2 of a unit and a solid grid line at every unit.)

Ps come to BB to mark the points on BB, explaining exactly what they are doing, while rest of Ps work in Pbs.

Ps join up the dots with rulers.

---

Lesson Plan 109

Notes

Drawn on BB or use enlarged copy master or OHP

Extra praise for Ps who remember about absolute value.

Discussion, reasoning, agreement, praising

---

Individual work to start, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP

Ps do necessary calculations in Ex Bks or on scrap paper.

Reasoning, agreement, self-correction, praising

Whole class activity but Ps working in Pbs at same time.

Discussion, reasoning, agreement, praising

Elicit that the coordinates of the first point in the table are written as (1, 0.6), with the \(x\) coordinate given first.

P at BB points to 1 on \(x\) axis with right hand and 0.6 on \(y\) axis with left hand, moves his or her fingers along the grid lines until they meet, then marks that point.

Agreement, praising

T monitors individual work as well as keeping an eye on Ps working on BB.

Are we correct to join up the dots? (Yes, as \(x\) and \(y\) could be any value between those given.)

Praising
Who can tell me the coordinates of points on the graph line which are not given in the table? [e.g. (3, 1.8), (2, 1.2), etc.]

If we increase the value of \(x\) by 2 (3) times, what happens to the \(y\) value? (It will also increase by 2 (3) times.)

Let's write the rule for the table in different ways. Ps come to BB or dictate to T. Class agrees/disagrees.

\[y = 0.6 \times x, \quad y = x ÷ 100 \times 60, \quad y = \frac{3}{5} \text{ of } x, \text{ etc.}\]

What about defining \(x\) rather than \(y\)? Ps dictate. e.g.

\[x = y \times \frac{5}{3}, \quad \text{or } x = y ÷ 0.6, \quad \text{or } x = \frac{10}{6} \times y, \text{ etc.}\]

What about defining \(x\) rather than \(y\)? Ps dictate. e.g.

\[x = y \times \frac{5}{3}, \quad \text{or } x = y ÷ 0.6, \quad \text{or } x = \frac{10}{6} \times y, \text{ etc.}\]

Solution:

\(a\)

\[A = a \times a = a^2, \quad \text{or } a = \sqrt{A} \quad (A \geq 0, \quad a \geq 0)\]

\(c\)

\(a\) could be the length of a side of a square and \(A\) could be its area.

What do you notice about this graph? (It is curved, not straight) T tells Ps that if there is a square number in the rule, the graph is always curved.
**PbY6b, page 109**

**Activity 5**

**Q.3 Read:** Complete the table so that \( a \) is the edge of a cube and \( A \) is its surface area. Write the rule in different ways.

Agree on one form of the rule in words. (e.g. the surface area of a cube is equal to 6 times the square of the length of a side)

Set a time limit or deal with one column at a time. The more difficult columns could be done with the whole class. Ps do any necessary calculations in Ex. Bks.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Agree on different forms of the rule.

**Solution:**

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
a & 0.1 & 0.9 & 2 & \frac{3}{4} & 1.6 & 2.5 \times 5 \times 10 \times 1 \\
\hline
A & 0.06 & 4.86 & 24 & \frac{3}{8} & \frac{1}{6} & 37.5 & 150 & 600 & 6 \times 6 \\
\hline
\end{array}
\]

Rule: \( A = 6 \times a \times a = 6 \times a^2 = 6a^2 \),

or \( a^2 = \frac{A}{6} \), or \( a = \sqrt{\frac{A}{6}} \)

34 min

**Activity 6**

**PbY6b, page 109**

**Q.4 Read:** The area of a rectangle is 5 cm\(^2\).

a) How long is side \( b \) if side \( a \) is: i) 1 cm ii) 0.5 cm

iii) 2 \( \frac{1}{2} \) cm iv) 5 cm v) 3 cm?

b) Show the data in a table in your exercise book.

c) Represent the pairs of values on the coordinate grid. Join up the dots.

What equation could we write about the area of the rectangle?

What operation could we use to calculate \( b \)?

Deal with part a) first under a short time limit, then review and make sure that mistakes are corrected before Ps do parts b) and c).

Extra praise for Ps who notice that the \( b \) axis is not long enough to show 10 cm (ii). Ps could extend the \( b \) axis by another 5 units and mark the point or leave it out.

Is it correct to join up the dots? (Yes, because \( a \) and \( b \) could be any value between the given points – length is continuous.)

Should we join the dots with a straight or curved line?

**Solution:**

a) i) \( 5 \text{ cm}^2 \div 1 \text{ cm} = 5 \text{ cm} \)

ii) \( 5 \text{ cm}^2 \div 0.5 \text{ cm} = 50 \text{ cm}^2 \div 5 \text{ cm} = 10 \text{ cm} \)

iii) \( 5 \text{ cm}^2 \div 2.5 \text{ cm} = 50 \text{ cm}^2 \div 25 \text{ cm} = 2 \text{ cm} \)

iv) \( 5 \text{ cm}^2 \div 5 \text{ cm} = 1 \text{ cm} \)

v) \( 5 \text{ cm}^2 \div 3 \text{ cm} = \frac{5}{3} \text{ cm} = 1 \frac{2}{3} \text{ cm} \)

52 min
Activity 6

(Continued)

When Ps have said what kind of joining line they think should be drawn, check by choosing values for \(a\) which are not in the table, working out what the \(b\) value is and marking the points on the graph (e.g. white dots opposite). Agree that a curved line fits the extra points better than straight line segments.

Notes

Ps could suggest extra values for \(a\) and T helps them to calculate the corresponding values for \(b\).

Is there a squared value in the rule for the graph? (Yes, \(a \times b = 5\) square cm)

Lesson Plan 109

7 PbY6b, page 109

Q.5 Read: Fill in the missing values if \(a\) is the edge of a cube and \(V\) is the volume of the cube.

Individual work, monitored, helped (or do more difficult columns with the whole class)

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Accept any valid method of calculation – give extra praise for creativity (see details).

T could show how to use the \(\sqrt[3]{\phantom{0}}\) button on a calculator if Ps have them and Ps use it to check the data in the table.

T could also demonstrate using a scientific calculator to work out cubic roots. e.g. $8 \sqrt[3]{\phantom{0}} \approx 2$

or

- $0.1 \times 0.1 = 0.01$
- $0.01 \times 0.1 = 0.001$
- $0.9 \times 0.9 = 0.81$
- $0.81 \times 0.9 = 0.729$

Individual work, monitored, helped (or do more difficult columns with the whole class)

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Accept any valid method of calculation – give extra praise for creativity (see details).

T could show how to use the \(\sqrt[3]{\phantom{0}}\) button on a calculator if Ps have them and Ps use it to check the data in the table.

T could also demonstrate using a scientific calculator to work out cubic roots. e.g. $8 \sqrt[3]{\phantom{0}} \approx 2$

or

- $0.1 \times 0.1 = 0.01$
- $0.01 \times 0.1 = 0.001$
- $0.9 \times 0.9 = 0.81$
- $0.81 \times 0.9 = 0.729$
Y6

Activity

Factorising 110, 285, 460 and 1110. Revision, activities, consolidation

PbY6b, page 110

Solutions:

Q.1  a) i) $A = 25 \text{ cm}^2$ ii) $A = 3.61 \text{ cm}^2$
   iii) $A = 529 \text{ mm}^2 \approx 529 \text{ cm}^2$ iv) $A = 22.09 \text{ km}^2$
   v) $A = 0.01 \text{ m}^2 \approx 100 \text{ cm}^2$

   b) i) $a = 4 \text{ cm}$ ii) $a = 10 \text{ m}$ iii) $a = 13 \text{ m}$ iv) $a = 16 \text{ m}$ v) $a = 35 \text{ m}$

$110 = 2 \times 5 \times 11$
Factors: 1, 2, 5, 10, 11, 22, 55, 110

$285 = 3 \times 5 \times 19$
Factors: 1, 3, 5, 15, 19, 57, 95, 285

$460 = 2^2 \times 5 \times 23$
Factors: 1, 2, 4, 5, 10, 20, 23, 46, 92, 115, 230, 460

$1110 = 2 \times 3 \times 5 \times 37$
Factors: 1, 2, 3, 5, 6, 10, 15, 30, 37, 74, 111, 185, 222, 370, 555, 1110

(or set factorising as homework at the end of Lesson 109 and review at the start of Lesson 110)

<table>
<thead>
<tr>
<th>$a$ (cm)</th>
<th>1</th>
<th>0.2</th>
<th>5</th>
<th>6</th>
<th>12</th>
<th>0.1</th>
<th>3.7</th>
<th>4</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V$ (cm$^3$)</td>
<td>1</td>
<td>0.008</td>
<td>125</td>
<td>216</td>
<td>1728</td>
<td>0.001</td>
<td>50.653</td>
<td>64</td>
<td>1000</td>
<td>1331</td>
</tr>
<tr>
<td>$A$ (cm$^2$)</td>
<td>6</td>
<td>0.24</td>
<td>150</td>
<td>216</td>
<td>864</td>
<td>0.06</td>
<td>82.14</td>
<td>96</td>
<td>600</td>
<td>726</td>
</tr>
</tbody>
</table>

Q.2  a) i) $V = (13 \times 13 \times 13) \text{ cm}^3 = 2197 \text{ cm}^3$
   ii) $A = 6 \times (13 \times 13) \text{ cm}^2 = 6 \times 169 \text{ cm}^2 = 1014 \text{ cm}^2$

   b) i) $a = \sqrt{\frac{486}{6}} = \sqrt{81} = 9 \text{ (cm)}$
   ii) $V = 9 \text{ cm} \times 9 \text{ cm} \times 9 \text{ cm} = 81 \text{ cm}^2 \times 9 \text{ cm} = 729 \text{ cm}^3$

   c) i) $a = \sqrt{\frac{100}{4}} = \sqrt{25} = \frac{10}{2} = 5 \text{ (cm)}$
   ii) $A = (2 \times 25 + 4 \times 20) \text{ cm}^2 = (50 + 80) \text{ cm}^2 = 130 \text{ cm}^2$

Q.3

<table>
<thead>
<tr>
<th>$a$ (cm)</th>
<th>1</th>
<th>0.2</th>
<th>5</th>
<th>6</th>
<th>12</th>
<th>0.1</th>
<th>3.7</th>
<th>4</th>
<th>10</th>
<th>11</th>
</tr>
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<td>1000</td>
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<tr>
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<td>6</td>
<td>0.24</td>
<td>150</td>
<td>216</td>
<td>864</td>
<td>0.06</td>
<td>82.14</td>
<td>96</td>
<td>600</td>
<td>726</td>
</tr>
</tbody>
</table>

Q.4  a) i) $\sqrt{81} = 9$ ii) $\sqrt{8100} = 90$ iii) $\sqrt{0.81} = 0.9$
   b) i) $\sqrt{169} = 13$ ii) $\sqrt{1.69} = 1.3$ iii) $\sqrt{16.900} = 130$
   c) i) $\sqrt{144} = 12$ ii) $\sqrt{1440000} = 1200$

Q.5

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>0.8</th>
<th>2</th>
<th>3.2</th>
<th>4</th>
<th>-0.8</th>
<th>-1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>0</td>
<td>0.6</td>
<td>1.5</td>
<td>2.4</td>
<td>3</td>
<td>-0.6</td>
<td>-1.2</td>
</tr>
</tbody>
</table>

b) Rule: $y = \frac{3}{4} \times x$, or $y = 0.75x$, or $y = \frac{3x}{4}$.

\[ x = \frac{4}{3} \times y, \quad \text{or} \quad x = \frac{4y}{3}, \quad \text{or} \quad x = y + 3 \times 4 \]

c) $x$ and $y$ could be the sides of a rectangle, or Euros and £s, etc.

Notes

110 = $2 \times 5 \times 11$
Factors: 1, 2, 5, 10, 11, 22, 55, 110

285 = $3 \times 5 \times 19$
Factors: 1, 3, 5, 15, 19, 57, 95, 285

460 = $2^2 \times 5 \times 23$
Factors: 1, 2, 4, 5, 10, 20, 23, 46, 92, 115, 230, 460

1110 = $2 \times 3 \times 5 \times 37$
Factors: 1, 2, 3, 5, 6, 10, 15, 30, 37, 74, 111, 185, 222, 370, 555, 1110

(or set factorising as homework at the end of Lesson 109 and review at the start of Lesson 110)