**R:** Calculations  
**C:** Ratio and proportion. Direct proportion. Graphs  
**E:** Problems  

### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( \text{111} = 3 \times 37 \)  
  Factors: 1, 3, 37, 111
- \( \text{286} = 2 \times 11 \times 13 \)  
  Factors: 1, 2, 11, 13, 22, 26, 143, 286
- \( \text{461} \) is a prime number  
  Factors: 1, 461  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and \( 23^2 > 461 \))
- \( \text{1111} = 11 \times 101 \)  
  Factors: 1, 11, 101, 1111

---

### Activity 2

**Ratio and proportion 1**

a) Here are 8 blue marbles and 2 red marbles.

i) What is the ratio of the number of blue marbles to the number of red marbles?
   - Ps dictate what T should write on BB. Class agrees/disagrees.
   - BB: \( \text{blue} : \text{red} = 8 : 2 \)  
     (Read as 'the ratio of blue to red is equal to 8 to 2'.)
   - T: This ratio shows that the number of blue marbles is \( \text{BB: } 8 \div 2 = 4 \) times the number of red marbles.

ii) What is the ratio of the number of red marbles to the number of blue marbles?
   - Ps dictate what T should write on BB. Class agrees/disagrees.
   - BB: \( \text{red} : \text{blue} = 2 : 8 \)  
     (Read as 'the ratio of red to blue is equal to 2 to 8'.)
   - T: This ratio shows that the number of red marbles is \( \text{BB: } 2 \div 8 = \frac{1}{4} \) of the number of blue marbles.

iii) What equation could we write about the relationship between the blue and the red marbles? Ps dictate to T.
   - BB: \( \frac{B}{4} = R \) or \( R = \frac{1}{4} \times B \)  
     (\( = \frac{B}{4} \))

iv) What part of the marbles are blue? \( \frac{8}{10} = \frac{4}{5} \) are blue

v) What part of the marbles are red? \( \frac{2}{10} = \frac{1}{5} \) are red

b) Let’s draw diagrams to show these ratios. i) \( 3 : 5 \) ii) \( 4 : 1 \)
   - Ps come to BB to draw and explain. Class agrees/disagrees.
   - BB: e.g.
     - i) \( \square \square \square \square \square \square \square \) or \( \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup \bigtriangleup \)  
     \( \square \square \square \square \square \square \square \square \square \square \square \)  
     \( \text{3 : 5} \)

---

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Ratio and proportion 2

a) Let’s complete this table to show the mass of different lengths of wire if 1 metre of the wire weighs 60 g.

Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. What relationships do you notice in the table?

Ps come to BB to point and explain. (e.g. If the length increases by 2 times, the mass also increases by 2 times. etc.)

BB:

<table>
<thead>
<tr>
<th>Length (m)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>...</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (g)</td>
<td>0</td>
<td>60</td>
<td>120</td>
<td>180</td>
<td>240</td>
<td>300</td>
<td>360</td>
<td>420</td>
<td>480</td>
<td>540</td>
<td>600</td>
<td>...</td>
<td>60x</td>
</tr>
</tbody>
</table>

×4
×1.2
×1.2
× \( \frac{5}{4} \)
× \( \frac{5}{4} \)

T: We say that the length and the mass of the wire are in direct proportion to one another because as one increases or decreases by a certain number of times, so does the other.

We say that the ratio of the mass to the length of the wire is 60 g to 1 metre, or 60 g per metre. (\( M \neq 0 \))

b) If 3 loaves of bread weigh 1.5 kg, what do 8 loaves of bread weigh?

Ps suggest different ways to solve it. e.g.

BB: 3 loaves \( \rightarrow \) 1.5 kg

1 loaf \( \rightarrow \) 1.5 kg \( \div \) 3 = 0.5 kg = 0.5 kg \( \times \) 8

8 loaves \( \rightarrow \) 0.5 kg \( \times \) 8 = 4 kg = 4 kg

Ps (T) might suggest showing the masses of 1 to 8 loaves in a table.

BB: e.g.

<table>
<thead>
<tr>
<th>No. of loaves</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass (kg)</td>
<td>0.5</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>3.5</td>
<td>4</td>
</tr>
</tbody>
</table>

Answer: 8 loaves of bread weigh 4 kilograms.

What is the ratio of the mass and the number of loaves?

(The ratio is 0.5 kg per loaf of bread.)

c) The ratio or scale on a map is written like this. (BB)

What is the real distance between 2 villages, A and B, if they are 2.5 cm apart on the map?

Ps come to BB or dictate what T should write. Who agrees? Who thinks something else? etc. T directs Ps’ thinking if necessary.

BB: 1 cm \( \rightarrow \) 200 000 cm = 2000 m = 2 km

2.5 cm \( \rightarrow \) 2 km \( \times \) 2.5 = \( \frac{5}{2} \) km

Answer: The real distance between A and B is 5 km.
### Y6 Activity

**PbY6, page IIII, Q.1**

Deal with one part at a time. T (P) reads out the question, Ps calculate mentally or in Ex. Bks. and show ratios on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Class checks that they are correct. Mistakes corrected.

**Solution:**

a) i) **How many times 4 is 16?**  
   \((16 \div 4 = 4 \times 1)\)

ii) **Write their ratio.**  
   \((16 : 4 = 4 : 1)\)

b) i) **How many times 16 is 4?**  
   \((4 \div 16 = \frac{1}{4} = 4 \times \frac{1}{4})\)

ii) **Write their ratio.**  
   \((4 : 16 = 1 : 4)\)

c) i) **How many times \(\frac{1}{2}\) is \(\frac{2}{3}\)?**  
   \((\frac{2}{3} \div \frac{1}{2} = \frac{2}{3} \times \frac{2}{1} = \frac{4}{3} \times \frac{1}{2})\)

ii) **Write their ratio in whole numbers.**  
   \((\frac{2}{3} : \frac{1}{2} = \frac{4}{6} : \frac{3}{6} = 4 : 3)\)

d) i) **How many times \(\frac{3}{2}\) is \(\frac{1}{2}\)?**  
   \((\frac{1}{2} \div \frac{3}{2} = \frac{1}{2} \times \frac{2}{3} = \frac{3}{4} \times \frac{2}{4})\)

ii) **Write their ratio in whole numbers.**  
   \((\frac{1}{2} : \frac{3}{2} = \frac{3}{6} : \frac{4}{6} = 3 : 4)\)

e) i) **What part of 8 is 5?**  
   \((\frac{5}{8})\)

ii) **What part of 5 is 8?**  
   \((\frac{8}{5})\)

---

### Lesson Plan 111

**Notes**

Whole class activity but individual calculation

Responses shown in unison

Reasoning, agreement, checking, self-correction, praising

**Check:**  
\(4 \times 4 = 16\)

\(16 \times \frac{1}{4} = 4\)

\(\frac{1}{4} \times 3 = \frac{2}{3}\)

**Check:**  
\(\frac{5}{8} \times \frac{1}{8} = \frac{5}{8}\)

\(\frac{8}{5} \times \frac{1}{8} = \frac{8}{5}\)

---

**PbY6b, page IIII**

Q.2 Set a time limit. Ps read problem themselves, do necessary calculations and write answer as a sentence in Ex.Bks.

Review with whole class. T chooses Ps to read out each question, then Ps show answer on scrap paper or slates on command.
Ps with correct answers explain reasoning at BB. Who agrees? Who worked it out another way? Mistakes discussed/corrected.

**Solution:**  
*e.g.*

The ratio of boys to girls in a school is \(11 : 10\).

a) **How many girls are in the school if there are 220 boys?**

   \(B : G = 11 : 10 \Rightarrow 220 : 200\)  
   \(Answer: \) There are 200 girls in the school.

b) **What percentage of the number of girls is the number of boys?**

   \(\frac{B}{G} = \frac{11}{10} = \frac{110}{100} \Rightarrow 110\%\)
   
   \(Answer: \) The number of boys is 110% of the number of girls.

c) **What part of the number of pupils in the school are the boys?**

   \(B + G = 220 + 200 = 420; \ \frac{220}{420} = \frac{11}{21} (= 0.52)\)
   
   \(Answer: \) Eleven twenty-firsts of the pupils are boys.

---

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid method of solution but give extra praise for the methods shown below.

Feedback for T

or \(11 \div 10 = 1.1 \rightarrow 110\%\)

or \(B + G = 11 + 10 = 21; \ \frac{11}{21}\)

---

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### Activity 6

**PbY6b, page 111**

**Q.3** Read: Paul intends to plant 150 trees in his orchard. He has divided the orchard into two parts in the ratio 2 : 3. How many trees should he plant in:

- a) the smaller part of the orchard?
- b) the larger part of the orchard?

What does the diagram have to do with the problem? (Rectangle represents whole orchard, i.e. 150 trees; shaded units form the smaller part, unshaded units the larger part)

Set a time limit of 2 minutes. Ps write plans, do calculations and write answers as sentences in Ex. Bks.

Review with whole class. Ps show answers on scrap paper or slates on command. Ps with correct answers explain reasoning at BB, referring to diagram. Who agrees? Who did it another way? Mistakes discussed and corrected.

**Solution:** e.g.

- a) **Plan:** \(150 \div (2 + 3) = 150 \div 5 = 30; 2 \times 30 = 60\)
  
  or \(\frac{2}{5} \text{ of } 150 = \frac{2}{5} \times \frac{30}{150} = 60\)

  **Answer:** He should plant 60 trees in the smaller part.

- b) **Plan:** \(3 \times 30 = \frac{90}{1} \text{ or } 150 - 60 = 90\)
  
  or \(\frac{3}{5} \text{ of } 150 = \frac{3}{5} \times \frac{30}{150} = 90\)

  **Answer:** He should plant 90 trees in the larger part.

---

**Q.4** Read: From 1 kg of fresh apples you can get 150 g of dried apple. Why is there such a difference in mass? (Most of an apple is water and this water is lost through evaporation during the drying process.)

Set a time limit or deal with one part at a time. Ps work in Ex. Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps with correct answers explain to Ps who were wrong. Mistakes discussed and corrected.

**Solution:** e.g.

- a) i) **What part of the fresh apples is the dried apple?**
  
  \(\frac{150}{1000} = \frac{15}{100} = \frac{3}{20} = 0.15\)

  **Answer:** The dried apple is 0.15 of the fresh apple.

- ii) **What percentage of the fresh apples is the dried apple?**
  
  \(\frac{15}{100} \rightarrow 15\%\)

  **Answer:** The dried apple is 15% of the fresh apple.

---

### Notes

Individual work, monitored, (helped)

Grid drawn on BB:

![Grid Diagram]

Whole orchard: 150 trees

Remind Ps to check results.

Responses shown in unison.

Reasoning, agreement, checking, self-correction, praising

**Check:**

\[60 : 90 = 6 : 9 = 2 : 3 \checkmark\]

and \[60 + 90 = 150 \checkmark\]

---

Individual work, monitored, (helped)

Table drawn on BB or use enlarged copy master or OHP

Accept fractions or decimals.

T chooses Ps to say the answers in sentences.
b) i) What part of the mass of the fresh apples is lost in the drying process?

\[ 1000 \text{ g} - 150 \text{ g} = 850 \text{ g}; \quad \frac{850}{1000} = \frac{85}{100} = \frac{17}{20} = 0.85 \]

Answer: 0.85 of the mass of the fresh apples is lost.

ii) What percentage of the mass of the fresh apples is lost?

\[ \frac{85}{100} = 0.85 \rightarrow 85\% \text{ (or } 100\% - 15\% = 85\%) \]

Answer: 85% of the mass of the fresh apples is lost.

c) Complete the table.

<table>
<thead>
<tr>
<th>Mass of fresh apple (kg)</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>100</th>
<th>20</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass of dried apple (kg)</td>
<td>0</td>
<td>0.75</td>
<td>1.5</td>
<td>2.25</td>
<td>3</td>
<td>15</td>
<td>3</td>
<td>1.5</td>
<td>0.15</td>
</tr>
</tbody>
</table>

What is the rule for the table? Who can write it another way?

Rule: e.g. \( \frac{D}{F} = \frac{15}{100} = 0.15 \), \( \frac{F}{D} = \frac{100}{15} = \frac{20}{3} \)

\[ D = 0.15 \times F, \quad F = D \div 0.15, \text{ etc.} \]

T: We can say that the masses of fresh apples and of dried apples are in direct proportion to one another, because as one increases or decreases by a certain number of times, so does the other.

\[ 39 \text{ min} \]

8

PbY6b, page 111

Q.5 Read: Write different plans for each question. Use one of your plans to work out the answer.

Deal with one question at a time. Set a time limit. Ps write plans and do calculations in Ex. Bks.

Review with whole class. Ps come to BB or dictate plans to T. Class decides whether or not they are valid.

Ps shout out answer in unison.

Ps with wrong answers say which plan they used and try to explain what they did wrong. Mistakes corrected.

Solution: 3 lb of butter can be made from 25 litres of milk.

a) How much butter can be made from 48 litres of milk?

Plan: 25 litres → 3 lb

\[ 1 \text{ litre} \rightarrow 3 \text{ lb} \div 25 \left(= 12 \text{ lb} \div 100 = 0.12 \text{ lb} \right) \]

\[ 48 \text{ litres} \rightarrow 3 \text{ lb} \div 25 \times 48 \]

\(= 0.12 \text{ lb} \times 48 = 5.76 \text{ lb} \)

\(\text{or } 3 \text{ lb} \div 25 \times 48 \left(= 0.12 \text{ lb} \times 48 = 5.76 \text{ lb} \right) \)

\(\text{or } B : M = 3 : 25 = x : 48; \)

\[ x = \frac{3}{25} \times 48 = \frac{144}{25} = \frac{576}{100} = 5.76 \]

Answer: 5.76 lb of butter can be made from 48 litres of milk.
b) How much milk produces 17 lb of butter?

Plan: 3 lb → 25 litres

1 lb → 25 litres ÷ 3 (= \(8\frac{1}{3}\) litres)
17 lb → 25 litres ÷ 3 × 17

(= \(8\frac{1}{3}\) litres × 17 = 141 \(\frac{2}{3}\) litres)

or 25 litres ÷ 3 × 17 (= 141 \(\frac{2}{3}\) litres)

or \(M : B = 3 : 25 = 17 : y\)

\[y = \frac{25}{3} \times 17 = \frac{425}{3} = 141 \frac{2}{3}\)

or \(\times \frac{17}{3}\)

\[25 \text{ litres} \rightarrow 3 \text{ lb} \quad \frac{17}{3}\]

\[25 \text{ litres} \times \frac{17}{3} \rightarrow 17 \text{ lb} \quad \frac{17}{3}\]

= 141 \(\frac{2}{3}\) litres

Answer: 141 and 2 thirds litres of milk produces 17 lb of butter.

What can you say about the quantities of milk and butter?
(They are in direct proportion to one another, because if one quantity increases or decreases by a certain number of times, so does the other quantity.)
MEP: Primary Project

Y6

**Activity**

1. **Factorisation**
   - Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
   - Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
   - Elicit that:
     - $112 = 2 \times 2 \times 2 \times 7 = 2^4 \times 7$
     - Factors: 1, 2, 4, 7, 8, 14, 16, 28, 56, 112
     - $287 = 7 \times 41$
     - Factors: 1, 7, 41, 287
     - $462 = 2 \times 3 \times 7 \times 11$
     - Factors: 1, 2, 3, 6, 7, 11, 14, 21, 462, 231, 154, 77, 66, 42, 33, 22
     - $1112 = 2 \times 2 \times 2 \times 139 = 2^3 \times 139$
     - Factors: 1, 2, 4, 8, 139, 278, 556, 1112

   **8 min**

2. **Direct proportion**
   - Think about what happens when you turn on a tap and run water into a bucket. Do you think that the amount of water in the bucket is in direct proportion to the time it is running? Ask several Ps what they think.
   - Ps will probably think that it is, because if we increase the time by 2 (3, 4) times, the amount of water in the bath will also increase by 2, (3, 4) times – but one condition for this to be true is that the water runs from the tap at a constant rate. (Extra praise for Ps who think of this.) If no P mentions it, T brings it up (see opposite).
   - a) Suppose that we turn on a tap and water trickles into a measuring jug at a constant rate of 0.5 cl every second. BB: 0.5 cl per second
   - Let’s complete this table to show how much water will be in the jug after different lengths of time.
   - Ps come to BB or dictate to T, explaining reasoning. Class points out errors. What does the bottom row show? (The amount of water coming out of the tap every second – it is always the same.)
   - BB:
     | Time (t) | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
     |----------|---|---|---|---|---|---|---|---|---|---|----|
     | Volume (v) | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 | 4.5 | 5  |
     | v÷ t       | - | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |

   - What is the rule for the table? Ps come to BB or dictate to T.
   - Class checks that it is correct. Elicit different forms of the rule.
     - Rule: $v = 0.5 \times \ t \ , \ \ t = v ÷ 0.5, \ \ \frac{v}{t} = 0.5 \ (t \neq 0)$
   - b) Let’s show the data in a table. Ps come to BB to plot points on pre-prepared axes. Class points out errors. Is it correct to join up the points? (Yes, as the water was running continuously.)
     - What do you notice about the graph line? (It is a ray, starting at (0, 0) and slanting up to the right.)
     - We can say that the time and the volume of water are in direct proportion.

   **15 min**

Lesson Plan

**Notes**

Individual work, monitored (or whole class activity)
BB: 112, 287, 462, 1112
T decides whether Ps can use calculators.
Reasoning, agreement, self-correction, praising

Elicit that in the 1st column, dividing 0 by 0 makes no sense.

Whole class activity
(If possible, T demonstrates.)
Discussion involving several Ps. Praising only

If we shut down the tap to a trickle and then open it up to a gush, would the time and amount of water still be in direct proportion to one another?

(No, as time would continue the same but the bucket would fill at different rates.)

Drawn on BB or use enlarged copy master or OHP
Agreement, praising

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Lesson Plan 112

Individual or paired work, drawing (or cutting out), monitored.
Tables drawn in Ex. Bks.
At a good pace.
Extra praise for non-integer sides (or T could hint that the sides need not be whole cm).

At a good pace
Agreement, praising

What is the rule for the table?

At a good pace
Agreement, praising

Whole class discussion
Involve several Ps.
Agreement, praising

BB: inverse proportion

Individual work, monitored, helped
(or plotting the points done with the whole class)
Drawn/written on BB or use enlarged copy master or OHP
Differentiation by time limit and by extra task
(Quicker Ps could be asked to add extra columns to table.
  e.g. 10 seconds $\rightarrow$ 3.3 km
        15 seconds $\rightarrow$ 4.95 km)

Reasoning, agreement, self-correction, praising

Class checks rule with values from table.
Q.1  b) Distance (km)

\[ \text{Distance (km)} \]

\[ \text{Time (seconds)} \]

- The graph is a straight line (or ray).
- Distance and time are in direct proportion. (Because as time increases by a certain number of times, the distance also increases by that number of times.)

\[ 25 \text{ min} \]

Q.2  Read: Different vehicles travelled at different average speeds over a 40 km route.

- a) Complete the table to show the time taken at certain average speeds.
- b) Draw a graph in your exercise book to show the relationship between average speed (in km per hour) and time (in hours).
- c) Complete the sentence.

Deal with one part at a time or set a time limit. Ps do necessary calculations in Ex Bks. Ask Ps to write the rule for the table. Review the table with whole class and mistakes corrected before Ps draw the graph. (If necessary, draw the graph and plot points with the whole class, with Ps working on BB and rest of Ps working in Ex. Bks.) Otherwise, set a time limit, then review.

Ps come to BB to plot points, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Discuss whether Is it correct to join up the dots? (Yes, as time and speed are continuous.) Should we join them with a curved or a straight line? (Agree that a curved line fits the points better.)

Ps show missing word on slates or scrap paper on command. Ps answering correctly explain reasoning.

(The graph shows that as speed decreases by a certain number of times, time increases by that number of times.)

**Solution:**

- a)

<table>
<thead>
<tr>
<th>Speed (km/h)</th>
<th>40</th>
<th>30</th>
<th>20</th>
<th>16</th>
<th>10</th>
<th>8</th>
<th>5</th>
<th>4</th>
<th>80</th>
<th>120</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time (hours)</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>4</td>
<td>5</td>
<td>8</td>
<td>10</td>
<td>0.5</td>
<td>0.3</td>
<td>0.4</td>
</tr>
</tbody>
</table>

**Rule:** \( T = 40 \div S, \quad S = 40 \div T, \quad S \times T = 40 \)

- c) Speed and time are in inverse proportion.

**25 min**
**Y6**

**Activity 6**

**PbY6b, page 112**

Q.3 Set a time limit of 3 minutes. Ps read questions themselves and solve them in Ex. Bks, writing the answers in sentences. Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain reasoning. Mistakes discussed and corrected.

**Solution:**

a) 600 litres of juice are poured into bottles which have a capacity of 75 cl. How many bottles are needed?

*Plan:* 600 litres ÷ 75 cl = 6000 cl ÷ 75 cl = 800 (times)

*Answer:* 800 bottles are needed.

b) How many bottles would be needed if the bottles had a capacity of:

i) half a litre

\[ 600 \text{ } l \div 0.5 \text{ } l = 600 \text{ } l \div 5 \text{ } l = 1200 \text{ (bottles)} \]

ii) 1 litre

\[ 600 \text{ } l \div 1 \text{ } l = 600 \text{ (bottles)} \]

iii) 1.5 litres

\[ 600 \text{ } l \div 1.5 \text{ } l = 600 \text{ } l \div 15 \text{ } l = 400 \text{ (bottles)} \]

iv) 2 litres

\[ 600 \text{ } l \div 2 \text{ } l = 300 \text{ (bottles)} \]

c) Show the data in a table.

<table>
<thead>
<tr>
<th>Capacity (litres)</th>
<th>0.5</th>
<th>1</th>
<th>1.5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of bottles</td>
<td>1200</td>
<td>600</td>
<td>400</td>
<td>300</td>
</tr>
</tbody>
</table>

What kind of proportion do you notice?

Capacity and number of bottles are in inverse proportion.

37 min

**Lesson Plan 112**

**Notes**

Individual work, monitored, helped

Table drawn on on BB or SB or OHT

Differentiation by time limit Responses shown in unison.

Discussion, reasoning, agreement, self-correction, praising

---

**Y6**

**PbY6b page 112**

Q.4 Read: *The volume of a cuboid is 240 cm³.*

If \( a = 10 \text{ cm} \), complete the table for edges \( b \) and \( c \).

Set a time limit of 2 minutes. Ps calculate mentally or in Ex. Bks. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees or disagrees. Who wrote it a different way? Mistakes discussed and corrected. Elicit the rule.

**Solution:**

<table>
<thead>
<tr>
<th>( a )</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>10</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>2.4</td>
<td>4.8</td>
<td>6.6</td>
<td>7</td>
</tr>
<tr>
<td>( c )</td>
<td>24</td>
<td>8</td>
<td>4</td>
<td>10</td>
<td>5</td>
<td>3.6</td>
<td>3</td>
</tr>
</tbody>
</table>

\[ V = 240 \text{ cm}^3 \]

What is the rule for \( b \) and \( c \)? \( b = \frac{24}{c}, \ \text{ and } c = \frac{24}{b}, \ \text{ and } b \times c = 24 \)

Are \( b \) and \( c \) in proportion? (Yes, but in inverse proportion)

(As \( b \) increases by a certain number of times, \( c \) decreases by that number of times, and vice versa.)

41 min

---

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### Y6

<table>
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**PbY6b, page 112. Q.5**

Read: *Which of the formulae has elements which are in:*

- i) direct
- ii) inverse proportion?

First agree on actions to show what Ps think. (e.g. standing up for direct, remaining seated for inverse) Then T points to each formula in turn and Ps respond on command. Ps with different responses explain reasoning and class decides who is correct.

What do you notice about these 3 formulae? What could they be about?

**Solution:**

i) **Direct proportion:** \[ A = 8 \times b, \quad b = \frac{A}{8}, \quad 8 = \frac{A}{b} \]

They are all different forms of the same formula.

(A could be the area of a rectangle which has one side 8 units long and \(b\) could be the length of the other side.)

T: The numerator and denominator of fractions which are equal to 8 are always in direct proportion.

Ps check this on BB: \(8 = \frac{16}{2} = \frac{24}{3} = \ldots\)

ii) **Indirect proportion:** \[ 100 = e \times f, \quad e = \frac{100}{f}, \quad f = \frac{100}{e} \]

Elicit that they are also different forms of the same formula, but note that \(e\) and \(f\) cannot be equal to zero, as it is makes no sense to divide by zero!

(\(e\) and \(f\) could be the sides of a rectangle which has an area of 100 square units.)

T: The two factors of equal products are always in inverse proportion.

Extra praise if a P notices this without hint from T.

Ps check: e.g.

\[ 100 = 1 \times 100 = 2 \times 50 = 4 \times 25 = 5 \times 20 = \ldots \]

---

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## Activity 1

### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **113** is a prime number. Factors: 1, 113 (as not exactly divisible by 2, 3, 5, 7 and 11; 2² > 113)
- **288 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 = 2⁵ × 3²**
  Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 288, 144, 96, 72, 48, 36, 32, 24, 18

**[To Ts]**: We can tell how many factors there are from the powers.

- **463** is a prime number. Factors: 1, 463 (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and 23; 2³ > 463)
- **1113 = 3 × 7 × 53**
  Factors: 1, 3, 7, 21, 53, 159, 371, 1113

**[To Ts]**: 1113 has (1 + 1) × (1 + 1) × (1 + 1) = 2 × 2 × 2 = 8 factors

### Individual Work

- Individual work but class kept together on the steps.
- Agreement, praising (see below)

**Discussion, agreement, praising**
Involve several Ps.
T reminds Ps if necessary.

### Calculation

Elicit how to ascertain the size of the angles from the parts:

- **G**: \[ \frac{2}{3} \text{ of } 360° = \frac{2}{3} \times 360° = 240° \]
- **S**: \[ \frac{2}{9} \text{ of } 360° = \frac{2}{9} \times 360° = 80° \]
- **C**: \[ \frac{1}{9} \text{ of } 360° = \frac{1}{9} \times 360° = 40° \]

---

### Notes

- Individual work, monitored (or whole class activity)
- BB: 113, 288, 463, 1113
- T decides whether Ps can use calculators.
- Reasoning, agreement, self-correction, praising

**Example**

- **288**
  - Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 288, 144, 96, 72, 48, 36, 32, 24, 18
- **371**
  - Factors: 1, 371
- **159**
  - Factors: 1, 3, 53, 159
- **1113**
  - Factors: 1, 3, 7, 21, 53, 159, 371, 1113

**[Note that]**
- \[ 3 = 3¹, 7 = 7¹, 54 = 54¹ \]
Lesson Plan 113

Notes

T suggests a table if no P does and asks what Ps think.

Individual work, monitored, helped
Reasoning, agreement, self-correction, praising
Details: e.g.

100 kg ÷ 3 × 2
= 33 1/3 kg × 2 = 66 2/3 kg
etc.

Individual work, monitored helped

Initial clarification of context.

Ps could be allowed to use calculators.

Reasoning, agreement, self-correction, praising

Feedback for T

Extension (or homework task)

Ps draw a similar table in Ex. Bks. using their own handspan measurements.

(Allow Ps to use calculators.)

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### Activity

**4**  
*PbY6b, page 113*

**Q.3 Read:** Dianne measured the table with her hand and its length was 6 handspsans. Then she measured the length of the table in metres and it was \( \frac{6}{5} \) m.

- **a)** What is the length of Dianne's handspan in metres?
- **b)** Write the length of her handspan in centimetres and millimetres.

Set a time limit of 2 minutes.  
Review with whole class. Ps could show answers on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- **a)** Dianne's handspan: \( \frac{6}{5} \) m ÷ 6 = \( \frac{1}{5} \) m (= 0.2 m)
- **b)** \( \frac{1}{5} \) m = 20 cm = 200 mm

Compare Dianne's handspsan with the handspsan in Question 2 and with Ps' own handspsans. (Less than the handspsan in Q.2 but more than most pupils' handspsans, so Dianne is probably an adult.)

Discuss the merits of using standard units (they never change and are the same for everyone) and non-standard units (useful for estimating when rulers, etc. are not available).

---

**5**  
*PbY6b, page 113*

**Q.4 Read:** From 1 kg of fresh ham we can get about 625 g of smoked ham.

Why is there less ham when it is smoked? (The smoking process dries out the ham, so water is lost in evaporation.) Who likes ham? Who likes smoked ham better than non-smoked?

Set a time limit. Ps read questions themselves and solve them in Ex. Bks. under a time limit.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps responding correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

T chooses Ps to say the answers as sentences.

**Solution:**

- **a)** What percentage of the mass of the fresh ham is lost by smoking?
  
  Amount lost: 1000 g – 625 g = 375 g;
  
  Part lost: \( \frac{375}{1000} = \frac{37.5}{100} \) → 37.5%  
  
  **Answer:** 37.5% of the mass of fresh ham is lost by smoking.
b) How much smoked ham can we get from 6 kg of fresh ham?

1 kg of fresh ham → 625 g of smoked ham
6 kg of fresh ham → 625 g × 6 = 3750 g

Answer: We can get 3.75 kg of smoked ham from 6 kg of fresh ham.

c) How much fresh ham is needed to produce 6 kg of smoked ham?

0.625 kg of smoked ham → 1 kg of fresh ham
1 kg of smoked ham → 1 kg ÷ 0.625 = 1000 kg ÷ 625

Answer: We need 9.6 kg of fresh ham to produce 6 kg of smoked ham.

Q.5 Read: The areas of two rectangular gardens are equal.
The first garden is 64 m long and 30 m wide.
The length of the second garden is 120% of the length of the first garden.
a) How wide is the second garden?
b) What part of the width of the first garden is the width of the second garden?

Set a time limit or deal with one at a time.
Review with the whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly come to BB or dictate to T, explaining reasoning. Who did the same? Who worked it out in a different way? etc. Mistakes discussed and corrected. T chooses Ps to give the answers in sentences.

Solution:
a) Length of 2nd garden: 120% of 64 m = 64 m × 1.2 = 76.8 m

Area of each garden: 64 m × 30 m = 1920 m²

Width of 2nd garden: 1920 m² ÷ 76.8 m = 19 200 m ÷ 768

= 25 m

Answer: The second garden is 25 m wide.

b) Part of width of 1st garden: \( \frac{25}{30} = \frac{5}{6} \)

Answer: The width of the 2nd garden is 5 sixths of the width of the first garden.

Elicit/point out that the length and width of different rectangles which have equal areas are in inverse proportion to one another. (As one increases by a certain number of times, the other decreases by that same number of times, and vice versa.)

35 min
**Y6**

### Activity 7

**PbY6b, page 113. Q.6**

Read: Write different plans to answer each question.

Deal with one part at a time. Ps write plans in Ex. Bks, coming to BB or dictating to T when they think of a new one. Class agrees/disagrees. Ps choose one to work out the answer (using calculators) and dictate to T.

**Solution:**

a) What is 32% of £524.50?

*Plans:* e.g. £524.50 \times 0.32 (= £167.84)

or £524.50 \times \frac{32}{100} (= £524.50 \times \frac{8}{25})

or £524.59 ÷ 100 \times 32

b) What is 106% of £524.50?

*Plans:* e.g. £524.50 \times 1.06 (= £555.97)

or £524.50 \times \frac{106}{100} (= £524.50 \times \frac{53}{50})

or £524.59 ÷ 100 \times 106

c) What is p% of £524.50?

*Plans:* e.g. £524.50 \times \frac{p}{100}

or £524.59 ÷ 100 \times p = £5.2459 \times p

### Activity 8

**PbY6b, page 113. Q.7**

Read: Write different plans to answer each question.

Deal with one part at a time. Ps come to BB or dictate to T. Class agrees/disagrees. Highlight the preferred plans (see below) and Ps use one of them to work out the answer.

**Solution:**

a) 25% of which length is 72.5 cm?

*Plans:* e.g. \[ \frac{72.5 \text{ cm}}{0.25} (= 7250 \text{ cm} ÷ 25 = 290 \text{ cm}) \]

or \[ \frac{72.5 \text{ cm}}{\frac{25}{100}} (= 72.5 \text{ cm} ÷ \frac{1}{4} = 72.5 \text{ cm} \times 4) \]

or \[ \frac{72.5 \text{ cm}}{25 \times 100} \text{ or } 72.5 \text{ cm} \times \frac{100}{25} \]

b) 125% of which length is 72.5 cm?

*Plans:* e.g. \[ \frac{72.5 \text{ cm}}{1.25} (= 7250 \text{ cm} ÷ 125 = 58 \text{ cm}) \]

or \[ \frac{72.5 \text{ cm}}{\frac{125}{100}} (= 72.5 \text{ cm} ÷ \frac{5}{4} = 72.5 \text{ cm} \times \frac{4}{5}) \]

or \[ \frac{72.5 \text{ cm}}{125 \times 100} \text{ or } 72.5 \text{ cm} \times \frac{100}{125} \]

c) What is the whole length if p% of it is 72.5 cm?

*Plans:* e.g. \[ \frac{72.5 \text{ cm}}{\frac{p}{100}} \text{ or } \frac{72.5 \text{ cm}}{p \times 100} \]
Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:
1. \(114 = 2 \times 3 \times 19\)  
   Factors: 1, 2, 3, 6, 19, 38, 57, 114
2. \(289 = 17 \times 17\)  
   Factors: 1, 17, 289 (square number)
3. \(464 = 2 \times 2 \times 2 \times 29 = 2^4 \times 29\)  
   Factors: 1, 2, 4, 8, 16, 29, 58, 116, 232, 464
4. \(1114 = 2 \times 557\)  
   Factors: 1, 2, 557, 1114
   (557 is a prime number, as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, and \(27^2 > 557\))

Percentages 1
a) If the whole amount is 200 kg, what is:
   1% (50%, 10%, 75%, 20%, 150%, 1 quarter, 2 fifths, 7 quarters, etc.)?
   P: e.g. 1% of 200 kg = 1 hundredth of 200 kg = \(\frac{2}{100}\) kg, etc.

b) What is the whole amount if:
   10% is 21 m\(^2\) (40% is 8 litres, 75% is 60 kg, 200% is 40 minutes, etc.)?
   P: e.g. '10% means 1 tenth, so the whole amount is 21 m\(^2\) \times 10, which equals 210 m\(^2\).

c) What percentage of 600 m is:
   12 m (60 m, 30 m, 120 m, 1200 m, etc.)?
   P: e.g. 1% of 600 m is 6 m, so 12 m is 2% of 600 m,
   or \(\frac{12}{600} = \frac{2}{100}\) which is 2% of 600 m
Ps can suggest some percentages and quantities too if there is time.

Percentages 2
Let's review how to calculate a percentage value of a whole amount.
a) \textit{How can we work out 30\% of 410 kg?}
   Ps come to BB or dictate what T should write. Who agrees? Who would do it another way? Accept any valid method but highlight:
   BB: \(\frac{410 \text{ kg} \div 100 \times 30}{410 \text{ kg} \times 0.3} = 4.1 \text{ kg} \times 30 = 41 \text{ kg} \times 3 = 123 \text{ kg}\)
   or \(\frac{123 \text{ kg}}{410 \text{ kg} \times 0.3}\)
   We could show both methods in a diagram.
   T draws framework on BB (or on OHT) and Ps say what should be where.

Notes
Individual work, monitored (or whole class activity)
BB: 114, 289, 464, 1114
Ps can use calculators.
Reasoning, agreement, self-correction, praising
e.g. \(\frac{289}{17} = 17 \quad \frac{114}{2} = 17 \quad \frac{57}{3} = 17 \quad \frac{19}{17} = 289 \quad \frac{464}{2} = 2 \quad \frac{1114}{2} = 58 \quad \frac{557}{29} = 29 \quad \frac{1}{29} = 1\)

Whole class activity
T chooses Ps at random.
Accept any valid reasoning.
Class points out errors.
Ps may explain orally or write calculation on slates first or on BB or draw a diagram if necessary.
e.g. 40% is 8 litres

\begin{center}
\begin{tabular}{|l|l|l|}
\hline
\textbf{Whole quantity:} & \textbf{8 litres} & \textbf{20 litres} \\
\hline
8 litres \div 4 \times 10 = 20 & 10% is 2 litres, so 100% is 20 litres. & Reasoning, agreement, praising \\
\hline
\end{tabular}
\end{center}
(Continued)

Let's call the whole amount \( w \).

**What is 30\% of \( w \)?**

Ps come to BB or dictate to T. Class agrees/disagrees.

BB: \( 30\% \) of \( w = w \div 100 \times 30 \) or \( w \times 0.3 \)

Let's call the percentage \( p \) and the value of the part \( v \).

**What is \( p\% \) of \( w \)?** Ps come to BB or dictate to T.

BB: \[
\begin{align*}
\text{v} &= w \div 100 \times p \\
&\text{or } \begin{array}{c}
v = w \times \frac{p}{100}
\end{array}
\end{align*}
\]

Ps say the equations in unison. T asks individual Ps to say them too.

Let's show it in a diagram. T draws framework and Ps dictate what

BB:

\[
\begin{array}{c}
\text{whole amount} \\
(w)
\end{array} \quad \begin{array}{c}
\% \\
1\%
\end{array} \quad \begin{array}{c}
\text{value of part} \\
(v)
\end{array}
\]

\[
\times \frac{\text{percentage}(p)}{100}
\]

**b) I paid £120 in tax, which is 30\% of my gross income. How much is my gross income?**

Ps come to BB or dictate what T should write. Who agrees? Who would do it another way? Elicit that the operations are the reverse of a).

BB: \[
\begin{align*}
\text{£120} \div 30 \times 100 &= \text{£4} \times 100 = \text{£400} \\
or \quad \text{£120} \div 0.3 &= \text{£1200} \div 3 = \text{£400}
\end{align*}
\]

Let's show them in a diagram. T draws framework on BB (or on OHT) and Ps say what should be where.

Let's call the amount that I paid in tax \( v \) and my whole income \( w \).

If \( v \) is 30\% of \( w \), what is \( w \)?

Ps come to BB or dictate to T. Class agrees/disagrees.

BB: \( w = v \div 30 \times 100 \) or \( w = v \div 0.3 \)

**What is the whole amount if \( p\% \) of it is \( v \)?**

Ps come to BB or dictate to T.

BB: \[
\begin{align*}
\text{w} &= \text{v} \div p \times 100 \\
&\text{or } \begin{array}{c}
\text{w} = \text{v} \div \frac{p}{100}
\end{array}
\end{align*}
\]

Let's show them in a diagram. Ps come to BB or dictate to T.

BB:

\[
\begin{array}{c}
\text{whole amount} \\
(w)
\end{array} \quad \begin{array}{c}
\% \\
1\%
\end{array} \quad \begin{array}{c}
\text{value of part} \\
(v)
\end{array}
\]

\[
\div \frac{\text{percentage}(p)}{100}
\]

---

**Notes**

Reasoning, agreement, praising

Agreeent, praising

Ps write equations and draw the diagram in Ex. Bks.

T monitors, helps, corrects.

Elicit that gross income is the income before tax is deducted.

Accept any valid method but give extra praise for the two opposite.

or Ps draw the whole diagram on BB using the previous one to help them.

Elicit that to find the whole amount when we know the value of a part, divide the known value by the part.

Agreement, praising

Ps write equations and draw diagram in Ex. Bks. too.

T reviews and Ps repeat:

• To calculate a percentage of the whole amount, multiply the whole amount by the percentage.

• To calculate the whole amount from the value of a part, divide the value of the part by the percentage.
**Y6**

### Activity

**PbY6b, page 114**

**Q.1** Let's see how many of these you can do in 3 minutes.

Start...now!...Stop!

Review with whole class. T chooses a P to read each question and Ps show answers on scrap paper or slates on command.

Ps with correct answers explain reasoning on BB. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) What part is:

i) \( \frac{350}{400} \) of \( \frac{35}{40} = \frac{7}{8} = 0.875 \)

ii) \( \frac{350}{250} \) of \( \frac{35}{25} = \frac{7}{5} = \frac{14}{5} = 1.4 \)

b) What is the ratio between:

i) \( 350 \) and \( 400 \) \( [350 : 400 = 35 : 40 = \frac{7}{8}] \)

ii) \( 350 \) and \( 250 \) \( [350 : 250 = 35 : 25 = \frac{7}{5}] \)

c) What percentage is:

i) \( \frac{350}{400} \) \( \frac{350}{400} = \frac{87.5}{100} \rightarrow 87.5\% \)

or \( 400 \rightarrow 100\% \)

\( 4 \rightarrow 1\% \)

\( 350 \rightarrow \frac{350}{4} = 87.5\% \)

ii) \( \frac{350}{250} \) \( \frac{350}{250} = \frac{35}{25} = \frac{140}{100} \rightarrow 140\% \)

\( 25\ min \)

**Q.2** Read: *The ratio of the population of 3 cities (A, B and C) is 5 : 7 : 8.*

What part of the population of the 3 cities is the population of A (B, C)? \( (A: \frac{5}{20} = \frac{1}{4}; B: \frac{7}{20}; C: \frac{8}{20} = \frac{4}{5}) \)

Set a time limit (or deal with one part at a time). Ps read questions themselves and solve them. (Part a) in *Pbs*; b), c) and d) in *Ex. Bks.*

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it another way? etc. Deal with all methods used. Mistakes discussed and corrected.

T chooses Ps to say each answer in a sentence.

**Solution:**

a) Colour this strip in different colours to show the ratio.

BB: [Diagram of strip colored in different colors]

**Notes**

Individual work, monitored (helped)

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid method of calculation.

Elicit the decimal forms too.

---

**Lesson Plan 114**

Week 23
Lesson Plan 114

**Notes**

Accept any valid method of calculation with correct reasoning.

Extra praise for this.

Accept the ratios in any correct form but in the review make sure that all forms are dealt with.

Check:

\[25\% + 35\% + 40\% = 100\%\]

Individual work, monitored, helped

Briefly discuss gardens.

Who has a garden? What kind of flowers (fruit, vegetables) do you grow? What takes up most of your garden? What do you like best about it? Where is your favourite spot? etc.

Reasoning, agreement, self-correction, praising

Accept any valid method with correct reasoning.

Feedback for T
### Activity 6 (Continued)

b) i) Fl: $30\%$ of $1100\ m^2 = 1100\ m^2 \times 0.3 = 330\ m^2$

Answer: The area used to grow flowers is $330\ m^2$.

ii) Fr: $50\%$ of $1100\ m^2 = 1100\ m^2 \times 0.5 = 550\ m^2$

Answer: The area used to grow fruit is $550\ m^2$.

c) The ratio of vegetables to flowers to fruit is $20 : 30 : 50 = 2 : 3 : 5$

### Notes

**Extension (Homework)**
Ps could be asked to draw the garden to scale using appropriate dimensions and to colour the 3 different parts in different colours.

---

### Activity 7

**PbY6b, page 114, Q.4**

**Read:** Write a plan first, then calculate the result. Write the answer in a sentence.

Deal with one part at a time. T chooses a P to read out the question. Ps calculate mentally if they can or in Ex. Bks and show answer on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:** e.g.

a) *The price of an item was £438 but in the sale the price has been cut by 10%.*

i) *What is the sale price?*

**Plan:** £438 – £438 \times 0.1 = £438 – £43.80 = £394.20

**Answer:** The sale price is £394.20.

ii) *What percentage is the sale price of the original price?*

**Plan:** $100\% – 10\% = 90\%$

**Answer:** The sale price is $90\%$ of the original price.

b) *28\% of the inhabitants of a village live in blocks of flats.*

i) *How many people live in houses if 406 people live in flats?*

**Plan:** $28\% \rightarrow 406$

$1\% \rightarrow 406 \div 28 = 58 \div 4 = 14.5$

$72\% \rightarrow 14.5 \times 72 = 1044$

**Answer:** 1044 people live in houses.

ii) *How many people live in this village?*

**Plan:** $406 \div 28 \times 100 = 14.5 \times 100 = 1450$

**Answer:** In this village, there are 1450 people.

iii) *What percentage of the population of the village live in houses?*

**Plan:** $100\% – 28\% = 72\%$

**Answer:** 72\% of the population live in houses.

c) *The price of an item was cut by 10\% and it now costs £113.40.*

i) *What was the original price of the item?*

**Plan:** £113.40 \div 0.9 = £1134 \div 9 = £126

**Answer:** The original price was £126.

ii) *What percentage is the original price of the reduced price?*

**Plan:** $1 \div 0.9 = 10 \div 9 = 1.1 \rightarrow 111.1\%$

(or 0.9 is 9 tenths, so its reciprocal is 10 ninths, which is 1.1)

**Answer:** The original price is about 111.1\% of the reduced price.

---

Whole class activity but individual calculation (or individual work under a time limit, reviewed as usual)

Responses shown in unison. Reasoning, agreement, self-correction, praising

Accept any valid method of solution.

or £438 \times 0.9 = £394.20

or extra praise for:

406 + 1044 = 1450

d) *What percentage is 31.5 of 90?*

**Plan:**

\[
\frac{31.5}{90} = \frac{315}{900} = \frac{35}{100} \rightarrow 35\%
\]

or $31.5 \div 90 \times 100 = 3.15 \times 100 = 315 = 35\%$

**Answer:** 31.5 is 35\% of 90.
Factorising 115, 290, 465 and 1115. Revision, activities, consolidation

**PbY6b, page 115**

**Solutions:**

<table>
<thead>
<tr>
<th>Q.1</th>
<th>Q.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) 0.4 : 0.12 : 3.3 : 4.18 = 40 : 12 : 330 : 418 = 20 : 6 : 165 : 209</td>
<td>a) ( \frac{6}{24} = \frac{1}{4} ); of 36 lb = 9 lb</td>
</tr>
<tr>
<td>b) ( \frac{3}{5} : \frac{2}{3} : \frac{1}{6} : \frac{11}{15} = \frac{18}{30} : \frac{20}{30} : \frac{5}{30} : \frac{22}{30} = \frac{18}{20} : \frac{5}{22} )</td>
<td>G: ( \frac{7}{24} \times 36 \text{ lb} = \frac{21}{2} \text{ lb} = 10.5 \text{ lb} )</td>
</tr>
<tr>
<td>c) 12 ( \frac{1}{2} ) % : 42% : 64.5% : 11% = 25 : 84 : 129 : 22</td>
<td>L: ( \frac{5}{24} \times 36 \text{ lb} = \frac{15}{2} \text{ lb} = 7.5 \text{ lb} )</td>
</tr>
<tr>
<td>c) 12 ( \frac{1}{2} ) % : 42% : 64.5% : 11% = 25 : 84 : 129 : 22</td>
<td>R: ( \frac{4}{24} = \frac{1}{6} ); of 36 lb = 6 lb</td>
</tr>
<tr>
<td></td>
<td>S: ( \frac{2}{24} = \frac{1}{12} ); of 36 lb = 3 lb</td>
</tr>
<tr>
<td>b) i) Number of boys: ( \frac{11}{25} \times 1350 = 594 )</td>
<td>Number of girls: ( \frac{14}{25} \times 1350 = 540 + 216 = 756 )</td>
</tr>
<tr>
<td>ii) Number of teachers: ( 1350 \div 45 \times 2 )</td>
<td>( = 270 \div 9 \times 2 )</td>
</tr>
<tr>
<td>( = 30 \times 2 = 60 )</td>
<td>c) ( \frac{7}{37} ) ( \rightarrow ) 126 beads, so ( \frac{1}{37} ) ( \rightarrow ) 18 beads</td>
</tr>
<tr>
<td>B: ( \frac{13}{37} ) ( \rightarrow ) 18 ( \times ) 13 = 180 + 54 = 234 (beads)</td>
<td></td>
</tr>
<tr>
<td>G: ( \frac{17}{37} ) ( \rightarrow ) 18 ( \times ) 17 = 180 + 126 = 306 (beads)</td>
<td></td>
</tr>
</tbody>
</table>

Factors: 1, 5, 13, 115, 290 = 2 \( \times \) 5 \( \times \) 29, Factors: 1, 2, 5, 10, 29, 58, 145, 290, 465 = 3 \( \times \) 5 \( \times \) 31, Factors: 1, 3, 5, 11, 31, 93, 155, 465, 1115 = 5 \( \times \) 223 (or set factorising as homework at the end of Lesson 114 and review at the start of Lesson 115)
Q.3

a) Speed (km/hour)  | 30  | 6  | 20 | 10 | 12 | 3
Time (hours)       | 1   | 5  | 1.5| 3  | 2.5| 10

**Rule:** \( S \times T = 30 \), \( S = 30 \div T \), \( T = 30 \div S \)

b) They all travelled 30 km.

c) and d)

e) Speed and Time are in inverse proportion.

f) To cover the distance in 4 hours, you need to travel at a speed of 7.5 km per hour.

g) e.g. Data in table could refer to:
   - Column 1: cycling (or travelling on a bus which stops many times)
   - Column 2: walking quickly
   - Column 3: running quickly
   - Column 4: walking normally
   - Column 5: skateboarding, or using a non-motorised scooter
   - Column 6: jogging
   - Column 7: swimming

**Erratum**
In Pbs: 2nd 'e)' should be 'g)'
## Activity

### 1. Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 116 = 2 \times 2 \times 29 = 2^2 \times 29 \) Factors: 1, 2, 4, 29, 58, 116
- \( 291 = 3 \times 97 \) Factors: 1, 3, 97, 291
- \( 466 = 2 \times 233 \) Factors: 1, 2, 233, 466
- \( 1116 = 2 \times 2 \times 3 \times 3 \times 31 = 2^2 \times 3^2 \times 31 \) Factors: 1, 2, 3, 4, 6, 9, 12, 18, 233, 466, 1116, 558, 372, 279, 186, 124, 93, 62, 36

[No. of factors: \((2 + 1) \times (2 + 1) \times (1 + 1) = 18\)]

### 2. Percentage

Let's express these fractions, decimals, divisions and ratios as percentages.

Ps come to BB or dictate what T should write. Class agrees/disagrees.

**BB:** e.g.

- a) i) \( \frac{3}{4} = 0.75 \rightarrow 75\% \) ii) \( \frac{15}{10} = \frac{150}{100} \rightarrow 150\% \)
  
  iii) \( \frac{4}{9} = 0.4444 \ldots \rightarrow 44.4\% \)

- b) i) 0.27 \rightarrow 27\%  
  
  ii) 0.987 \rightarrow 98.7\%

- c) i) \( 1 \div 2 = 0.5 \rightarrow 50\% \) ii) \( 3 \div 100 = 0.03 \rightarrow 3\% \)

- d) i) \( 4 : 20 = \frac{2}{10} \rightarrow 20\% \) ii) \( 7 : 2 = \frac{7}{2} = 3.5 \rightarrow 350\% \)

### 3. Word problems

Deal with one at a time. Who can think of a word problem for this plan? Allow Ps a minute to think about it, then Ps suggest questions. Class decides whether they match the given plan and choose the one they like best. Ps work out the result and answer in context.

**BB:** e.g. [If 6 workmen can lay 216 m of pavement in a day, what length of pavement could 5 workmen lay in a day?]

- a) 216 \div 6 \times 5 = 36 \times 5 = 180

- b) (\[\begin{array}{c} 1.1 \times 0.1 \times 0.1 \\ 1.1 \times 1.1 \times 1.1 \end{array}\] = 720 \div 8 = 90

- c) 72 \div 0.8 = 4.341714
### Activity

**Q.1** Read: Two green marbles and one pink marble come out of a machine one after the other in a random order.

*Calculate the probability of each of these outcomes.*

Tell me what you know about probability. [It is the chance an event has of happening. It is measured on a scale of 0 (no chance) to 1 (certain). The probabilities in between can be given as fractions, or decimals, or percentages.]

What do we need to do first before we can answer the questions? (List all the possible outcomes.)

Encourage a *logical* listing in *Ex. Bks.* Set a time limit.

Review with whole class. Ps could show probabilities on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Mistakes discussed and corrected.

**Solution:**

6 possible outcomes:

<table>
<thead>
<tr>
<th>BB:</th>
</tr>
</thead>
<tbody>
<tr>
<td>G₁</td>
</tr>
<tr>
<td>G₂</td>
</tr>
<tr>
<td>P</td>
</tr>
<tr>
<td>G₁</td>
</tr>
<tr>
<td>P</td>
</tr>
<tr>
<td>G₂</td>
</tr>
</tbody>
</table>

a) *The first marble is pink.* \[
\frac{2}{6} = \frac{1}{3}
\]

b) *The first marble is green.* \[
\frac{4}{6} = \frac{2}{3}
\]

c) *The order of the three marbles is green, green, pink.* \[
\frac{2}{6} = \frac{1}{3}
\]

d) *The order of the marbles is green, pink, green.* \[
\frac{2}{6} = \frac{1}{3}
\]

**22 min**

**Q.2** Read: A computer program writes the letters A, B and C in a random order.

*What is the probability of each of these outcomes?*

What should you do first? (List all the possible outcomes.)

Encourage a *logical* listing in *Ex. Bks.* Set a time limit.

Review with whole class. Ps could show probabilities on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Mistakes discussed and corrected.

**Solution:**

6 possible outcomes: ABC BAC CAB ACB BCA CBA

a) *The first letter is A.* \[
\frac{2}{6} = \frac{1}{3}
\]

b) *The second letter is A.* \[
\frac{2}{6} = \frac{1}{3}
\]

c) *The third letter is C.* \[
\frac{2}{6} = \frac{1}{3}
\]

d) *The order is B, C, A.* \[
\frac{1}{6}
\]

**26 min**
### Activity 6

**PbY6b, page 116**

**Q.3** Read: *A computer program writes the digits 1, 2, 3 and 4 in a random order.*

**What is the probability of each of these outcomes?**

Set a short time limit for listing the possible outcomes in Ex. Bks. then review quickly. A, how many outcomes did you write? Who agrees? Who had more? Tell us what they are. Ps correct any mistakes/omissions before answering the questions.

Review probabilities. Ps could show them on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Mistakes discussed and corrected.

**Solution:**

24 possible outcomes:

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td>1234</td>
<td>2134</td>
<td>3124</td>
<td>4123</td>
<td></td>
</tr>
<tr>
<td>1243</td>
<td>2143</td>
<td>3142</td>
<td>4132</td>
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<tr>
<td>1324</td>
<td>2314</td>
<td>3214</td>
<td>4213</td>
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<td>1342</td>
<td>2341</td>
<td>3241</td>
<td>4231</td>
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<td>1423</td>
<td>2413</td>
<td>3412</td>
<td>4312</td>
<td></td>
</tr>
<tr>
<td>1432</td>
<td>2431</td>
<td>3421</td>
<td>4321</td>
<td></td>
</tr>
</tbody>
</table>

a) *The first digit is 3.*

\[
\frac{6}{24} = \frac{1}{4}
\]

b) *The first digit is 1.*

\[
\frac{6}{24} = \frac{1}{4}
\]

c) *The second digit is 3.*

\[
\frac{6}{24} = \frac{1}{4}
\]

d) *The second digit is 1.*

\[
\frac{6}{24} = \frac{1}{4}
\]

e) *The last digit is 2.*

\[
\frac{6}{24} = \frac{1}{4}
\]

f) *The last digit is 4.*

\[
\frac{6}{24} = \frac{1}{4}
\]

g) *The first two digits are 4, 3 in this order.*

\[
\frac{2}{24} = \frac{1}{12}
\]

h) *The order is 3, 1, 2, 4.*

\[
\frac{1}{24}
\]

32 min

### Activity 7

**PbY6b, page 116**

**Q.4** Read: *A computer program writes 2-digit, positive, whole numbers at random.*

**What is the probability of each of these outcomes?**

What are the possible outcomes? Ps come to BB or dictate to T. After 3 or 4 have been dictated, T asks if it is possible to work out the number of outcomes without listing them all. (For each of the 9 possible numbers (1 to 9) for the tens digit there are 10 possible numbers (0 to 9) for the units digit, so 90 possible outcomes.)

Agree that 0 cannot be used for the tens digit, as the number would then really be a 1-digit number, not a 2-digit number.

Set a short time limit for writing the probabilities beside the outcomes described in Pbs.

**Whole class activity to start**

Involve several Ps.

**BB:** 10, 11, 12, . . . 98, 99

\[
\begin{array}{c|c}
T & U \\
9 & (10) \\
9 \times 10 = 90
\end{array}
\]

Extra praise for a P who reasons like this without help from T.

Individual work, monitored, helped

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### Y6 Lesson Plan 116

#### Activity 7

**Notes**

Review probabilities with the whole class. Ps could show them on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Mistakes discussed and corrected.

**Solution:**

- **a)** The number is 37.  
  \[
  \frac{1}{90}
  \]

- **b)** The first digit is 8.  
  \[
  \frac{10}{90} = \frac{1}{9}
  \]

- **c)** The last digit is 5.  
  \[
  \frac{9}{90} = \frac{1}{10}
  \]

- **d)** The first digit is 0.  
  [0] (Impossible – 2-digit no.)

- **e)** The last digit is 0.  
  \[
  \frac{9}{90} = \frac{1}{10}
  \]

- **f)** The number is even.  
  \[
  \frac{45}{90} = \frac{1}{2}
  \]

---

### PbY6b, page 116

#### Erratum

In *Ps*, 2nd 'by' should be 'c'!

**Q.5 Read:** In a primary school, the number of girls is 176, which is 55% of the total number of pupils at the school.

- **a)** How many boys attend this school?
- **b)** How many pupils attend this school?
- **c)** If a computer program prints out the files of all the pupils in a random order, what is the probability of the computer selecting a file belonging to:
  - i) a girl
  - ii) a boy?

Deal with one at a time or set a time limit.

Review with whole class. Ps could show answers on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who worked it out another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

- **a)** G: 55%,  
  B: 100% – 55% = 45%
  
  Plan: 55% → 176  
  5% → 176 ÷ 11 = 16  
  45% → 16 × 9 = 144
  
  Answer: There are 144 boys at this school.

- **b)** Plan: G + B: 176 + 144 = 320  
  Answer: 320 pupils attend this school.

- **c)** i) \( p \) (a girl) is 176 out of 320, i.e. 55% (as given in question)  
  ii) \( p \) (a boy) is 144 out of 320, i.e. 45% (as calculated above)
  
  Answer: The probability of the computer printing a girl's file is 55% and of printing a boy's file is 45%.

---

Responses shown in unison.

Reasoning, agreement, self-correction, praising

**Extension**

Ps could think of other outcomes and ask Ps to give their probabilities, or give a probability and ask other Ps for a matching outcome.

10, 12, 14, 16, 18, . . .

For each of the 9 possible tens, there are 5 possible even digits.

Individual work, monitored, helped

Ps calculate in *Ex.Bks* and write the answers in sentences.

Responses shown in unison.

Reasoning agreement, self-correction, praising

Accept any valid method of solution with correct reasoning.  Deal with all methods used by Ps.

Accept fraction or decimal forms too – but unnecessary!

Elicit that a probability can be expressed as a ratio, a fraction, a decimal or a percentage.
Activity 9

PbY6b, page 116, Q.6

T puts 2 dark grey and 3 dark blue socks (real or cut-out, with the grey socks numbered 1 and 2, and the blue socks numbered 1, 2 and 3) in a bag or box.

X, come and take out 2 socks without looking at them. X does so and T notes the colours and numbers on the BB. What other possible combinations could X have taken out? Ps come to BB or dictate to T.

T helps Ps to form a logical listing.

BB: Possible outcomes:

\[
\begin{array}{ccccccc}
G_1 & G_2 & G_3 & B_1 & B_2 & B_3 \\
G_1 & G_2 & G_3 & B_1 & B_2 & B_3 \\
G_1 & G_2 & G_3 & B_1 & B_2 & B_3 \\
\end{array}
\]

T chooses a P to read out each outcome description and Ps show the probability of it happening on scrap paper or slates on command.

Ps with different answers explain reasoning. Class decides who is correct. Ps write agreed probabilities beside outcomes in Pbs.

**Solution:**

a) \( p \) (a pair of dark grey socks) = \( \frac{2}{20} = \frac{1}{10} \) (outcomes in shaded rectangle in diagram)

T also reasons that the chance of the first sock being grey is 2 out of 5, but the chance of the 2nd sock also being grey is 1 out of 4 (as a grey sock has already been taken out and only 4 socks are left in the bag, 1 grey and 3 blue).

So the probability of the 1st sock and the 2nd sock being grey is:

\[ p (G_1 \text{ and } G_2) = \frac{1}{5} \times \frac{1}{4} = \frac{1}{10} \rightarrow 10\% \]

b) \( p \) (a pair of dark blue socks) = \( \frac{6}{20} = \frac{3}{10} \) (outcomes in unshaded rectangle in diagram)

T (or a P) also reasons that the chance of the first sock being blue is 3 out of 5, but the chance of the 2nd sock also being blue is 2 out of 4 (as a blue sock has already been taken out, so only 4 socks are left in the bag, 2 blue and 2 grey).

So the probability of the 1st sock and the 2nd sock being blue is:

\[ p (B_1 \text{ and } B_2) = \frac{3}{5} \times \frac{2}{4} = \frac{3}{10} \rightarrow 30\% \]

c) \( p \) (2 socks the same colour) = \( \frac{8}{20} = \frac{2}{5} \) (outcomes in both rectangles in diagram)

or \( p \) (\( G_1 \text{ or } B_1 \text{ and } B_2 \)) = \( \frac{1}{10} \times \frac{4}{10} = \frac{4}{10} \rightarrow 40\% \)

**Extension**

What is the probability that the two socks are different colours?

---

**Notes**

Whole class activity

(or individual trial first if Ps wish)

Revision of how to determine the probability of certain outcomes, first through visualisation and listing then counting outcomes, then T leads Ps through the calculations of probability, involving Ps where possible.

Agreement, praising

Ps could write the outcomes in Ex. Bks too.

Responses shown in unison.

Discussion, reasoning, agreement, praising

T explains and Ps listen.

Stress that the probability of both socks being grey is less than that of one sock being grey.

If both conditions must be met, we multiply their probabilities.

Allow Ps to explain if they would like to try, with T’s help when necessary.

or 10% + 30% = 40%

If either condition can be met we add their probabilities.

BB: 100% - 40% = 60%
### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Lesson Plan</strong> 117</td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td><strong>R:</strong> Calculation</td>
<td>Individual work, monitored (or whole class activity)</td>
</tr>
<tr>
<td><strong>C:</strong> Simple probabilities</td>
<td>BB: 117, 292, 467, 1117</td>
</tr>
<tr>
<td><strong>E:</strong> Analysing games</td>
<td>Reasoning, agreement, self-correction, praising</td>
</tr>
</tbody>
</table>

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $117 = 3 \times 3 \times 13 = 3^2 \times 13$  
  Factors: 1, 3, 9, 13, 39, 117
- $292 = 2 \times 2 \times 73 = 2^2 \times 73$  
  Factors: 1, 2, 4, 73, 146, 292
- 467 is a prime number  
  Factors: 1, 467  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and $23^2 > 467$)
- 1117 is a prime number  
  Factors: 1, 467  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and $37^2 > 1117$)

#### Pby6b, page 117

Q.1 Read: A cash box contains gold and silver coins. The ratio of gold coins to silver coins is 47 to 53. The number of silver coins is 159.

a) How many:
   i) gold coins  
   ii) coins are in the cash box?

b) If you take out a coin with your eyes shut, what is the probability of the coin being gold?  
Give your answer as a percentage.

Set a time limit of 2 minutes. Ps work in Ex. Bks.

Review with whole class. Ps could show answers on scrap paper or slates on command. Ps who answer correctly explain reasoning at BB. Class agrees/disagrees. Mistakes discussed and corrected.

Solution: e.g.

a) i) Gold coins: $159 \div 53 \times 47 = 3 \times 47 = 141$  
   ii) Total coins: $141 + 159 = 300$

   Answer: In the cash box there are 141 gold coins. There are 300 coins altogether.

b) $p$ (a gold coin) = $\frac{141}{300} = \frac{47}{100} \rightarrow 47\%$

Individual work, monitored (helped)

Differentiation by time limit  
Responses shown in unison.  
Reasoning, agreement, self-correction, praising

or $G : S = 47 : 53 (\times 3)$  
$x : 159$  
$x = 47 \times 3 = 141$

What is the probability that the coin is silver? (53%)
**Activity 3**

**Roulette**

Who knows how to play the game of roulette? Allow Ps to explain if they can, otherwise T does so.

- Roulette consists of a wheel with the numbers 0 to 36 (not in order) in pockets around its edge and a roulette table with the numbers 0 to 36 in a grid. Some of the numbers are coloured red, some black.
- If each number has an equal chance, what is the probability of each of the numbers winning? (1 chance out of 37)
- You place your bet on a number on the grid. The teller spins the wheel in one direction and throws an ivory ball in the other direction. The number that the ball falls into when the wheel stops wins.
- If we bet on a single number and it wins, the bank pays 36 times the bet (including zero).
- If we bet on 2 adjacent numbers on the table and either of them wins, the bank pays 18 times our bet.

What do you think the bank will pay if we bet on 3 numbers and one of them wins? Ask several Ps what they think and why.

Elicit what we could win if we bet on 3, (4, 6, 12, 18) numbers.

**BB:**

- **3 numbers:**
  - e.g. $\begin{array}{c} 6 \\ 7 \\ 8 \end{array} \rightarrow 12 \times \text{bet} \\

- **4 numbers:**
  - e.g. $\begin{array}{c} 10 \\ 13 \\ 11 \\ 14 \end{array} \rightarrow 9 \times \text{bet} \\

- **6 numbers:**
  - e.g. $\begin{array}{c} 15 \\ 18 \\ 16 \\ 19 \\ 17 \\ 20 \end{array} \rightarrow 6 \times \text{bet} \\

- **12 numbers:**
  - e.g. 1 to 12, 13 to 24 or 25 to 36
  - or 1, 4, 7, 11, . . ., 36
  - or 2, 5, 8, 11, . . ., 36
  - or 3, 6, 9, 12, . . ., 36

- **18 numbers:**
  - $2 \times \text{bet} \\
  - (e.g. even or odd, red or black, high or low, etc.)

- If we bet on zero and it wins, all the other bets lose!

**Lesson Plan 117**

**Notes**

Whole class activity

If possible, T has a model roulette wheel and layout to demonstrate how they are used, otherwise show the wheel on the copy master.

Involves Ps when possible.

**BB:**

- e.g. $p (8) = \frac{1}{37}$

Ask a P to give an example.
- e.g. You bet £2 on number 10.
- If it wins the bank pays you £2 \times 36 = £72
- Do you think it is worth doing?
- Some Ps might think so, but extra praise if Ps point out that:
  - you have already paid £2, so you have won only £70;
  - the chance of winning is $\frac{1}{37}$
  - the chance of not winning (i.e. the bank wins) is $\frac{36}{37}$

This is why casinos make so much money!

**PbY6b, page 117**

**Q.2**

Read: In a game of Roulette, a wheel is spun and a ball comes to rest on one of the numbers 0 to 36.

*The even numbers from 2 to 36 are red numbers.*

*What is the probability of each of these outcomes?*

Set a time limit or deal with one at a time. Ps do necessary calculations and write results in Ex. Bks.

Review with whole class. Ps show probabilities on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

- a) 0 wins ($\frac{1}{37} \approx 2.7\%$)
- b) 21 wins ($\frac{1}{37} = 2.7\%$)
- c) 7 or 8 wins ($\frac{2}{37} \approx 5.4\%$)
- d) 31 or 34 wins ($\frac{2}{37} = 5.4\%$)

Individual work monitored, helped
(or whole class activity if Ps are unsure or not very able)

Differentiation by time limit
Responses shown in unison.
Discussion, reasoning, agreement, self-correction, praising
(More able Ps could be asked to give the probabilities as percentages, using a calculator to divide the numerator by the denominator and rounding appropriately.)
Lesson Plan 117

Notes

Week 24

<table>
<thead>
<tr>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 (Continued)</td>
</tr>
</tbody>
</table>

- **e)** 24 or 25 or 26 wins \( \left( \frac{1}{37} + \frac{1}{37} + \frac{1}{37} = \frac{3}{37} \approx 8.1\% \right) \)
- **f)** 12 ≤ n ≤ 17 wins. \( \left( \frac{6}{37} = 16.2\% \right) \) as 6 possible numbers
- **g)** 1 ≤ n ≤ 12 wins. \( \left( \frac{12}{37} \approx 32.4\% \right) \) as 12 possible numbers
- **h)** The winning number gives a remainder of 2 when divided by 3. \( \left( \frac{12}{37} = 32.4\% \right) \)
- **i)** A red number wins. \( \left( \frac{18}{37} = 48.6\% \right) \) as 18 possible numbers
- **j)** The numbers 25 to 36 do **not** win. \( \left( \frac{25}{37} = 67.6\% \right) \)

Which of the outcomes has most chance of happening? (j)

Do you think that this would be one of the bets we could make in a real game of roulette? (No, the bank always makes sure that it has more chance of winning than we have!)

---

| PbY6b, page 117 |

**Q.3** Read: *In a pack of 52 playing cards, there are 13 cards (Ace, 2, 3, 4, 5, 6, 7, 8, 9, 10, Jack, Queen, King) in each of 4 suites: Diamonds and Hearts (red), Clubs and Spades (black).*

If possible, T has a pack of large cards to show to class. Who has played cards? What games did you play? Did you win?

Set a time limit. Ps read question themselves and answer in Ex. Bks. Encourage Ps to write in this form: **BB:** \( p \) (an Ace) =

Review with whole class. T chooses Ps to give the answers and explain their reasoning. Who agrees? Who thinks something else? Why? etc. Mistakes discussed and corrected.

**Solution:**

*If you take a card from the pack at random, what is the probability that the card is:*

- **a)** an Ace \( \left( \frac{4}{52} = \frac{1}{13} \right) \)
- **b)** a 9 \( \left( \frac{4}{52} = \frac{1}{13} \right) \)
- **c)** a Club \( \left( \frac{13}{52} = \frac{1}{4} \right) \)
- **d)** a red card \( \left( \frac{26}{52} = \frac{1}{2} \right) \)
- **e)** a Queen of Diamonds \( \left( \frac{1}{52} \right) \)
- **f)** a Jack or a King of Spades \( \left( \frac{2}{52} = \frac{1}{26} \right) \)
- **g)** an Ace of Clubs or a King of Hearts \( \left( \frac{2}{52} = \frac{1}{26} \right) \)
- **h)** **not** an Ace? \( \left( \frac{48}{52} = \frac{12}{13} \right) \) or \( 1 - \frac{1}{13} = \frac{12}{13} \)

---

Individual work, monitored, helped

or T has packs of cards to hand round class, especially for Ps who have never played cards.

Reasoning, agreement, self-correction, praising

Involve several Ps.

At a good pace

Feedback for T

**Extension**

Ps think of other outcomes and choose Ps to say their probabilities.

T gives a probability and Ps think of an outcome to match it.
**Y6**

**Activity 6**

*PbY6b, page 117.*

Q.4  Read:  *These are the probabilities for certain outcomes when throwing a dice. Write a question to match each probability.*

Deal with one at a time (or one row at a time under a time limit)

Review with whole class.  T chooses Ps to read out their questions.

Class decides whether they are valid.  Ps who made a mistake or could not think of a question, write the one they like best from those offered.

**Solution:**  e.g. (but many others possible)

a)  \( \frac{1}{6} \) (throwing a 4)  

b) 0 (throwing a 7)  

c)  \( \frac{5}{6} \) (not throwing a 6)  

d) 1 (an odd or an even number)

e)  \( \frac{1}{3} \) (throwing a 1 or a 2)  

f)  \( \frac{1}{2} \) (throwing at least a 4)

g)  \( \frac{2}{3} \) (throwing an odd number or a 2)  

h)  \( \frac{33}{1} \) %  [as e)]

i) 50%  [as f]  

j) 100%  [as d)]

**Notes**

Lesson Plan 117

Individual work, monitored

Elicit that a dice is a cube with 6 faces, so the numbers 1 to 6 can be thrown.

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Give extra praise for creative answers (e.g. using 'or' or 'not')

Feedback for T

Whole class activity

Read:  *These are the probabilities for certain outcomes when 4 coins are tossed one after the other. Write an outcome to match each probability.*

What could the outcome be if one coin is tossed? (H or T)  What could the outcomes be if 4 coins are tossed?  Let's call the 4 coins A, B, C and D and write the possible outcomes in this table.

Ps come to BB or dictate to T.  Class points out errors.

**BB:**

<table>
<thead>
<tr>
<th>A</th>
<th>H</th>
<th>H</th>
<th>H</th>
<th>H</th>
<th>H</th>
<th>H</th>
<th>H</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>C</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>H</td>
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<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>D</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>T</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>T</td>
<td>H</td>
<td>H</td>
<td>T</td>
</tr>
</tbody>
</table>

Elicit that there are 16 possible outcomes (each shown in a column).

T states the probability and Ps suggest possible outcomes to match it.

Class decides whether or not they are valid.  Ps write a correct outcome beside the probability in Pbs.

**Solution:**  e.g.

a) 0 (5 Tails)  

b) \( \frac{1}{16} \) (4 Heads)  

c) \( \frac{2}{16} = \frac{1}{8} \) (First 3 are Tails)

d) \( \frac{3}{16} \) (HHTH or HHTT or THHT, in order)

e) \( \frac{4}{16} = \frac{1}{4} \) (3 H + 1T, in any order)  

f) \( \frac{5}{16} \) (3 T + 1H or THHT)

g) \( \frac{6}{16} = \frac{3}{8} \)  (First 3 are Heads or 3T + 1H)
### Activity 7 (Continued)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>h)</td>
<td>$\frac{7}{16}$</td>
<td>(The first 2 are Heads or the first 3 are Tails or THTH in order)</td>
<td></td>
</tr>
<tr>
<td>i)</td>
<td>$\frac{8}{16} = \frac{1}{2}$</td>
<td>(The first is a Tail)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\frac{9}{16}$</td>
<td>(The first is a Head or TTTT)</td>
<td></td>
</tr>
<tr>
<td>k)</td>
<td>$\frac{10}{16} = \frac{5}{8}$</td>
<td>(The first is a Tail or the first 3 are Heads)</td>
<td></td>
</tr>
<tr>
<td>l)</td>
<td>$\frac{11}{16}$</td>
<td>(The first is a Tail or the first 3 are Heads or HTHH in order)</td>
<td></td>
</tr>
<tr>
<td>m)</td>
<td>$\frac{12}{16}$</td>
<td>(The first is a Head or the first 2 are Tails)</td>
<td></td>
</tr>
<tr>
<td>n)</td>
<td>$\frac{13}{16}$</td>
<td>(The first is a Head or the first 2 are Tails or THTH in order)</td>
<td></td>
</tr>
<tr>
<td>o)</td>
<td>$\frac{14}{16} = \frac{7}{8}$</td>
<td>(The first 3 are not all Tails)</td>
<td></td>
</tr>
<tr>
<td>p)</td>
<td>$\frac{15}{16}$</td>
<td>(Not HHHH)</td>
<td></td>
</tr>
<tr>
<td>q)</td>
<td>$\frac{16}{16} = 1$</td>
<td>(The first is a Head or a Tail)</td>
<td></td>
</tr>
<tr>
<td>r)</td>
<td>$50% = \frac{8}{16}$</td>
<td>(The first is not a Tail)</td>
<td></td>
</tr>
</tbody>
</table>

**45 min**
**Y6**

**R:** Natural numbers, fractions and decimals  
**C:** Revision: practising mental and written calculations  
**E:** Relationships among the components of operations. Word problems.

### Activity

#### 1: Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

**Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:**

<table>
<thead>
<tr>
<th>Number</th>
<th>Factorisation</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>118</td>
<td>$2 \times 59$</td>
<td>1, 2, 59, 118</td>
</tr>
<tr>
<td>293</td>
<td>is a prime number</td>
<td>1, 293</td>
</tr>
<tr>
<td>468</td>
<td>$2 \times 3 \times 3 \times 3 \times 13 = 2^2 \times 3^3 \times 13$</td>
<td>1, 2, 3, 4, 6, 9, 12, 13, 18</td>
</tr>
<tr>
<td>1118</td>
<td>$2 \times 13 \times 43$</td>
<td>1, 2, 13, 26, 43, 86, 559, 1118</td>
</tr>
</tbody>
</table>

#### 8 min

#### 2: Mental calculation

T says an operation and writes it on BB. Ps calculate mentally and stand up as soon as they have an answer. T chooses the quickest P to give the answer. Class agrees/disagrees. If correct, P explains how he or she calculated so quickly. If wrong, T chooses the next quickest P to answer and explain how they did it. Who did the same? Who worked it out a different way? etc.

**e.g.**

- a) $230 + 60 = 290$; $5230 + 60 = 5290$; $5230 + 460 = 5690$; $5680 + 320 = 6000$; $56 800 + 3200 = 60000$; etc.
- b) $6700 + 4900 = (e.g. 6600 + 5000 =) 11600$; $7500 – 3900 = (e.g. 7600 – 4000 =) 3600$; etc.
- c) $860 \times 40 = (e.g. 8600 \times 4 = 17200 \times 2 =) 34400$; $6600 \div 120 = (e.g. 660 \div 12 = 110 \div 2 =) 55$; etc.

#### 13 min

**Notes**

Individual work, monitored  
(or whole class activity)  
BB: 118, 293, 468, 1118  
Reasoning, agreement, self-correction, praising  
e.g.

<table>
<thead>
<tr>
<th>Number</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>468</td>
<td>2</td>
</tr>
<tr>
<td>234</td>
<td>2</td>
</tr>
<tr>
<td>117</td>
<td>3</td>
</tr>
<tr>
<td>39</td>
<td>3</td>
</tr>
<tr>
<td>13</td>
<td>13</td>
</tr>
<tr>
<td>559</td>
<td>13</td>
</tr>
<tr>
<td>43</td>
<td>43</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Whole class activity  
At a good pace  
In good humour!  
Encourage Ps to find easy, quick ways to do the calculations. Agreement, praising  
Ps can think of some operations too.  
Feedback for T
### Lesson Plan 118

**Notes**

Whole class activity  
Written on BB or use enlarged copy master or OHP  
(Ps could have copies on desks too.)  
At a good pace  
Reasoning, agreement, praising  
Remind Ps of names of components where necessary.

**Calculation:**

<table>
<thead>
<tr>
<th>Activity</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Missing numbers</strong></td>
<td></td>
</tr>
<tr>
<td>How can we work out the missing number? Ps come to BB to write an operation to calculate the missing number and to fill in the rectangle. Who agrees? Who would do it another way? etc. Elicit the general rule for such calculations (see boxes below).</td>
<td></td>
</tr>
<tr>
<td>BB: e.g.</td>
<td></td>
</tr>
<tr>
<td>Calculation:</td>
<td></td>
</tr>
<tr>
<td>a) i) $320 + \boxed{1240} = 1560$ as $1560 - 320 = 1240$</td>
<td></td>
</tr>
<tr>
<td>ii) $18 + 39 = 57$ as $57 - 39 = 18$</td>
<td></td>
</tr>
<tr>
<td>Missing term = Sum – known term</td>
<td></td>
</tr>
<tr>
<td>b) i) $317 - 81 = 236$ as $236 + 81 = 317$</td>
<td></td>
</tr>
<tr>
<td>ii) $11400 - 7800 = 3600$ as $3600 + 7800 = 11400$</td>
<td></td>
</tr>
<tr>
<td>Reductant = difference + subtrahend</td>
<td></td>
</tr>
<tr>
<td>c) i) $245 - \boxed{210} = 35$ as $245 - 35 = 210$</td>
<td></td>
</tr>
<tr>
<td>ii) $6170 - \boxed{620} = 5550$ as $6170 - 5550 = 620$</td>
<td></td>
</tr>
<tr>
<td>Subtrahend = reductant – difference</td>
<td></td>
</tr>
<tr>
<td>d) i) $11 \times \boxed{9} = 99$ as $99 \div 11 = 9$</td>
<td></td>
</tr>
<tr>
<td>ii) $302 \times 13 = 3926$ as $3926 \div 13 = 302$</td>
<td></td>
</tr>
<tr>
<td>Missing factor = product ÷ known factor</td>
<td></td>
</tr>
<tr>
<td>e) i) $85 \div \boxed{5} = 17$ as $85 \div 17 = 5$</td>
<td></td>
</tr>
<tr>
<td>ii) $264 \div \boxed{11} = 24$ as $264 \div 24 = 11$</td>
<td></td>
</tr>
<tr>
<td>Divisor = dividend ÷ quotient</td>
<td></td>
</tr>
<tr>
<td>f) i) $\frac{8}{9} \div 4 = \frac{2}{9}$ as $\frac{2}{9} \times 4 = \frac{8}{9}$</td>
<td></td>
</tr>
<tr>
<td>ii) $333 \div 10 = 33.3$ as $33.3 \times 10 = 333$</td>
<td></td>
</tr>
<tr>
<td>Dividend = quotient × divisor</td>
<td></td>
</tr>
</tbody>
</table>

**20 min**
**Y6**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 118</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td><strong>PbY6b, page 118</strong></td>
</tr>
</tbody>
</table>

**Q.1** Read: *Calculate the sums.*

Set a time limit of 3 minutes. Encourage Ps to estimate first and to check their results (against estimate and by adding in opposite direction for vertical addition).

Review with whole class. Ps come to BB to do calculations, explaining reasoning with place-value detail. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
a) & \quad \begin{array}{c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c} 8 & 7 & 0 & 4 \\ 1 & 6 & 0 & 7 & 1 \\ + & 8 & 0 & 3 & 2 & 6 \\ - & 9 & 5 & 6 & 5 & 11111 \\ \hline & 1 & 1 & 1 & 6 & 6 & 6 & 6 \\
\end{array} \\
b) & \quad \begin{array}{c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c} 4 & 3 & 0 & 2 \\ 7 & 2 & 1 & 4 & 5 \\ + & 9 & 0 & 8 & 3 & 11111 \\ \hline & 1 & 1 & 1 & 1 \\
\end{array} \\
c) & \quad \frac{4}{9} + \frac{3}{5} = 1 + \frac{20 + 27}{45} = 1 + \frac{47}{45} = \frac{2}{45} \\
d) & \quad 43.2 + 10 \frac{4}{5} = 43.2 + 10.8 = 54_{\text{25 min}}
\]

**5**

**PbY6b, page 118**

**Q.2** Read: *Calculate the differences.*

Set a time limit of 3 minutes. Encourage Ps to check their results (by addition, or subtracting difference from reductant).

Review with whole class. Ps come to BB to do calculations, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[
\begin{align*}
a) & \quad \begin{array}{c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c} 7 & 2 & 0 & 4 & 9 \\ - & 5 & 2 & 1 & 3 & 8 \\ \hline & 1 & 9 & 9 & 1 & 1 \\
\end{array} \\
b) & \quad \begin{array}{c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c@{\hspace{1em}}c} 6 & 3 & 1 & 1 & 0 & 1 & 0 & 8 \\ - & 1 & 6 & 1 & 3 & 4 & 1 & 1 \\ \hline & 4 & 6 & 7 & 5 & 9 & 8 \\
\end{array} \\
c) & \quad 2 \frac{1}{4} - 5 \frac{5}{6} = 2 + \frac{3 - 10}{12} = 2 - \frac{7}{12} = \frac{1}{12} \\
\quad \text{or} \quad 5 \frac{5}{4} - 5 \frac{5}{6} = 1 + \frac{15 - 10}{12} = \frac{5}{12} \\
d) & \quad 23 \frac{3}{4} - 15.05 = 23.75 - 15.05 = 8_{\text{29 min}}
\]
Y6

Activity
6  PbY6b, page 118

Q.3  Read: Calculate the products.
Set a time limit of 2 minutes. Encourage Ps to check their results (by division).

Solution:

a) \[
\begin{array}{c}
682 \\
\times 36
\end{array}
\]
\[
\begin{array}{c}
4092 \\
+ 2460
\end{array}
\]
\[
\begin{array}{c}
24552
\end{array}
\]

b) \[
\begin{array}{c}
415 \\
\times 0.71
\end{array}
\]
\[
\begin{array}{c}
415 \\
+ 29050
\end{array}
\]
\[
\begin{array}{c}
29465
\end{array}
\]

c) \[
\begin{array}{c}
425 \\
\times 37
\end{array}
\]
\[
\begin{array}{c}
225 \\
\times 3
\end{array}
\]
\[
\begin{array}{c}
66 \\
35
\end{array}
\]
\[
= 1 \frac{31}{35}
\]

or \[
5 \frac{2}{5} \times \frac{3}{7} = \frac{22}{5} \times \frac{3}{7} = \frac{66}{35} = 1 \frac{31}{35}
\]
\[
= 4 \times \frac{3}{7} + \frac{2}{5} \times \frac{3}{7} = \frac{12}{7} + \frac{6}{35}
\]
\[
= 1 \frac{5}{7} + \frac{6}{35} = 1 + \frac{25 + 6}{35} = 1 \frac{31}{35}
\]

33 min

7  PbY6b, page 118

Q.4  Read: Calculate the quotients.
Set a time limit of 2 minutes. Encourage Ps to check their results (using multiplication or dividing the dividend by the quotient).
Review with whole class. Ps come to BB to write calculations and explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) \[
\begin{array}{c}
5775 \\
\div 5
\end{array}
\]
\[
\begin{array}{c}
1183 \\
\div 6
\end{array}
\]

b) \[
\begin{array}{c}
6333 \\
\div 156
\end{array}
\]

1646.8 \div 26 = 63.3
(to nearest tenth, i.e to 1 d.p.)

In c), elicit that to divide by a fraction, multiply by its reciprocal value (i.e. the value which multiplies it to make 1)

\[
\begin{array}{c}
5 \frac{1}{3} \div 2 \frac{5}{3} = \frac{16}{5} \times \frac{5}{2} = \frac{40}{3} = 13 \frac{1}{3}
\end{array}
\]

37 min

Lesson Plan 118

Notes
Individual work, monitored, c) helped
Written on BB or SB or OHT
Reasoning, agreement, self-correction, praising
Discuss how the divisions would be answered if written horizontally and what to do about the remainders. (Write as a fraction or round to an appropriate number of decimal digits.)

Feedback for T
<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 118</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td></td>
</tr>
</tbody>
</table>

**PbY6b, page 118**

Q.5 **Deal with one part at a time (or set a time limit).**

Ps read questions themselves, write plans, estimate, do calculations, check them and write the answers as a sentence.

Review with whole class. T chooses a P to read out the question and Ps show answers on scrap paper or slates on command.

Ps with correct answers explain solution at BB. Who did the same? Who worked it out a different way? etc. Mistakes discussed and corrected.

**Solutions:**

a) **Six friends went on a day trip in a minibus. They spent £186.50 on petrol and £133.50 on food.** If they shared the costs equally, how much did they each have to pay?

**Plan:** £ \((186.50 + 133.50) ÷ 6 = 320 ÷ 6\)  
\[\approx £53.33\] (to nearest penny)

**Answer:** They each had to pay £53.33.

b) **Bob wanted to fill a 260 litre barrel from a 545 litre tank full of water.** He transferred the water from the tank to the barrel using two 5 litre buckets at a time. How many times did he need to fill the two 5 litre buckets?

**Plan:** 260 litres ÷ (2 × 5 litres) = 260 litres ÷ 10 litres  
\[= 26\] (times)

**Answer:** Bob needed to fill the two buckets 26 times.

c) **A beetle was 108 times as fast as a snail. If the beetle covered 54 cm in a certain time, how far could the snail move in the same time?**

**Plan:** 54 cm ÷ 108 = 540 mm ÷ 108 \(= 60 \text{ mm} ÷ 12\)  
\[= 5 \text{ mm}\]

**Answer:** The snail could move 5 mm (or 0.5 cm).

c) **The edges of a cuboid are 8 cm, 5.3 cm and 36 mm. What is its volume?**

**Plan:**  \(V = (8 \times 5.3 \times 3.6) \text{ cm}^3\)

\[= (42.4 \times 3.6) \text{ cm}^3\]

\[= 152.64 \text{ cm}^3\]

**Answer:** The volume of the cuboid is 152.64 cm\(^3\).

---

**Notes**

Individual work, monitored  
Do not allow calculators.  
Ps work in *Ex. Bks.*

Responses shown in unison.  
Reasoning, agreement, self-correction, praising  
Feedback for T

(or to 2 decimal places)

Note that the 545 litres in the water tank is irrelevant data!
### Activity

#### 1. Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **119** = \(7 \times 17\)  
  Factors: 1, 7, 17, 119

- **294** = \(2 \times 3 \times 7 \times 7 = 2 \times 3 \times 7^2\)  
  Factors: 1, 2, 3, 6, 7, 14, 21, 42, 49, 98, 147, 294

- **469** = \(7 \times 67\)  
  Factors: 1, 7, 67, 469

- **1119** = \(3 \times 373\)  
  Factors: 1, 3, 373, 1119

#### 2. Ladder Game

Let’s play a 3-rung ladder game!

**Rule:** There are 2 players, A and B. A says a natural number from 1 to 3. B adds 1, 2, or 3 to it and says the result. A adds 1, 2 or 3 to B’s number, and so on. The first player to say ‘20’ is the winner.

- **a)** T plays against the class, with T starting. e.g.
  
  **T:** 1, **P:** 4, **T:** 7, **P:** 8, **T:** 9, **P:** 11, **T:** 12, **P:** 14, **T:** 16, **P:** 19, **T:** **20**  
  T wins!

- **b)** T plays against the class, with Ps starting. e.g.
  
  **P:** 3, **T:** **4**, **P:** 5, **T:** 8, **P:** 10, **T:** 12, **P:** 15, **T:** **16**, **P:** 17, **T:** **20**  
  T wins!

- **c)** Ps play the game in pairs, taking turns to start. Ask the pairs to note who starts and who wins each time.

- **d)** Which player should always win? The player who starts or the player who goes second? What should you do to make sure that you win?
  
  If you want to finish on ‘20’, you must say ‘16’. To get to 16, you should say ‘12’. To get to 12 you should say ‘8’. To get to 8 you should say ‘4’. So the strategy is to aim for 4, 8, 12 and, most important of all, 16, as from there you cannot lose!
  
  i.e. i.e. 4 → 8 → 12 → 16 → **20**

### Notes

- **Individual work, monitored**  
  (or whole class activity)
  
  BB: 119, 294, 469, 1119

- **Reasoning, agreement, self-correction, praising**  
  
<table>
<thead>
<tr>
<th>119</th>
<th>294</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>119</td>
<td>7</td>
<td>17</td>
</tr>
<tr>
<td>7</td>
<td>147</td>
<td>3</td>
</tr>
<tr>
<td>17</td>
<td>49</td>
<td>7</td>
</tr>
<tr>
<td>1</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

- **Whole class activity, then paired work**  
  In good humour!

  T explains how to play it but gives no hints about how to ensure winning.

  [T’s strategy: Aim to say ‘4’, or ‘8’, or ‘12’, or ‘16’ and thus ‘20’.]

  After Ps have played it themselves, discuss with the whole class possible strategies. Allow Ps to make suggestions first, then T gives hints only if necessary.

  Elicit that:
  
  - if A starts, B should win!
  - A can win but only if B does not know the winning strategy or makes a mistake.
Here is another version of the ladder game but this time it is a 5-rung ladder.

Q.1 Read: A says a natural number from 1 to 5. B adds on a natural number 1 to 5. A adds a natural number, 1 to 5, to B's sum, and so on.

The winner is the player who reaches 40.

a) Play the game with a partner.
b) Work out a strategy for each player.
c) Which player can be sure of winning this game if he makes no mistakes?

Set a time limit. Ps play the game a few times, taking turns to start and noting who starts, which P says which number, and who wins. Pairs then analyse their findings, agree on possible winning strategies and test them out.

Review with whole class. Which player should not lose? (The player who starts.) What is the winning strategy? Who agrees? Who thinks something else? etc. Test them with the whole class if there is disagreement.

Solution:


i.e. 4 → 10 → 16 → 22 → 28 → 34 → 40

c) If A starts, then A should win.

B can only win if A does not know the strategy or makes a mistake.

Q.2 Read: The Ladder Game can be changed so that the player who says '40' is not the winner.

a) Play this version of the game with a partner.
b) Work out a strategy.
c) Which player can be sure of winning this game if he makes no mistakes?

Deal with this activity in the same way as Activity 3.

Solution:

b) If 40 does not win, then aim for 39. To reach 39, aim for 33. To reach 33, aim for 27. To reach 27, aim for 21. etc.

i.e. 3 → 9 → 5 → 21 → 27 → 33 → 39

c) Again, if A starts, then A should win.

B can only win if A does not know the strategy or makes a mistake.
**Y6**

**Activity**

> PbY6b, page 119

**Lesson Plan 119**

**Notes**

Individual work, monitored, in drawing the dots and writing the terms.

Dots drawn on BB or SB or OHT

or T could have all 10 terms prepared as dots on SB or OHT and uncover each as Ps dictate the number.

Whole class discussion on the rule or formula.

[As this formula is quite difficult to deduce, T gives it and asks Ps to check that it is true for all the terms.]

Elicit what \(a\) and \(n\) stand for:
- \(a\) (the value of the term)
- \(n\) (its position in the sequence)

Individual work in checking formula.

(T could allocate certain terms to certain Ps.)

Responses shown in unison.

Reasoning, agreement, praising

Individual work, monitored

Drawn on BB or SB or OHT

Reasoning, agreement, praising

BB: Square numbers

\[
\begin{align*}
1, & \quad 4, \quad 9, \\
16, & \quad 25, \quad 36, \quad 49, \quad 64, \quad 81, \quad 100 \\
\end{align*}
\]

\(a_n = n^2\) etc.

Individual work, monitored

Drawn on BB or SB or OHT

Reasoning, agreement, praising

BB: Square numbers

\[
\begin{align*}
1, & \quad 4, \quad 9, \\
16, & \quad 25, \quad 36, \quad 49, \quad 64, \quad 81, \quad 100 \\
\end{align*}
\]

\(a_n = n^2\) etc.

Q.3 a) Read: *Continue the pattern in your exercise book.*

Write the first 10 terms in this sequence of **triangular numbers**.

Set a time limit of 1 minute. Ps finished first come to BB to continue pattern on BB and to write the 10 numbers. Class agrees/disagrees. Mistakes, omissions corrected.

BB: 

\[
\begin{align*}
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
&\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \\
\end{align*}
\]

etc.

Why are the terms in the sequence called **triangular numbers**? (They can form triangles.)

We know the first 10 terms but how could we find out what the 25th or 100th terms in the sequence are without drawing dots for all the terms in between? (Use a rule or formula)

T suggests this if no P does so.

Let’s call the 1st term \(a_1\), the 2nd term \(a_2\), the 3rd term \(a_3\), and so on, and try out this formula:

\[
a_n = \frac{n(n + 1)}{2}, \quad (\text{where } n = 1, 2, 3, 4, \ldots)
\]

Check the first term, where \(n = 1\), on BB with Ps dictating to T, then Ps choose other terms (\(a_2\) to \(a_{10}\)) to check quickly in Ex. Bks. Has anyone found a term which does not match the formula? (No, they all match.)

BB: 

\[
\begin{align*}
a_1 &= \frac{1(1 + 1)}{2} = \frac{1 \times 2}{2} = \frac{2}{2} = 1 \\
a_2 &= \frac{2(2 + 1)}{2} = \frac{2 \times 3}{2} = \frac{6}{2} = 3 \\
&\quad \ldots \\
a_{10} &= \frac{10(10 + 1)}{2} = \frac{10 \times 11}{2} = \frac{110}{2} = 55
\end{align*}
\]

What is the 100th term? Ps calculate in Ex. Bks and show result on scrap paper on command. P answering correctly comes to BB to explain.

BB: 

\[
a_{100} = \frac{100(100 + 1)}{2} = \frac{100 \times 101}{2} = \frac{10100}{2} = 5050
\]

T: So from the general formula (T points to formula for \(a_n\)) for a sequence, we can work out any of its terms.

b) Read: *Continue the pattern in your exercise book.* Write the first 10 terms in this sequence of **square numbers**.

Set a time limit. (Ps need not draw dots for all the terms if they realise what the formula is and can calculate the terms.)

Again, Ps finished first come to BB to draw dots and/or write the terms. Class agrees/disagrees. Mistakes corrected.

What is the 15th term? (\(15 \times 15 = 15^2 = 225\))

What is the general formula? Ps dictate to T.

\(30\text{ min}\)
Activity 6

PbY6b, page 119

Q.4 Read: A family gathered 4 kg of cherries from the 1st tree in their orchard, 8 kg from the second tree and so on. They always gathered 4 kg more cherries from the next tree than from the one before it.

a) If there were 10 trees in the orchard, what mass of cherries was gathered altogether?

b) What mass of cherries would the family have collected if they had gathered 6 kg from the first tree and 4 kg more from one tree to the next?

Set a time limit or deal with one part at a time. P5s work in Ex. Bks.

Review with whole class. P5s could show results on scrap paper or slates on command. P5s with different answers explain reasoning at BB. Class points out errors and agrees on correct answer. Mistakes discussed and corrected.

Solution:

a) Plan: $4 + 8 + 12 + 16 + 20 + 24 + 28 + 32 + 36 + 40\]

= $220$ (kg)

How could we pair up the numbers to make the calculation easier? P5s make suggestions. T gives hint if necessary.

BB: $4 + 40 = 8 + 36 = 12 + 32 = 16 + 28 = 20 + 24$

So we could calculate the total like this:

BB: $(4 + 40) \times 5 = 44 \times 5 = 220$ (kg)

Answer: The mass of cherries gathered altogether was 220 kg.

b) Plan: $6 + 10 + 14 + 18 + 22 + 26 + 30 + 34 + 38 + 42$

= $240$ (kg)

or $220$ kg + $10 \times 2$ kg = $220$ kg + $20$ kg = $240$ kg

What other way could we do the calculation? P5s dictate to T.

BB: $6 + 42 = 10 + 38 = 14 + 34 = 18 + 30 = 22 + 26$

So we could calculate the total mass of cherries like this:

BB: $(6 + 42) \times 5 = 48 \times 5 = 240$ (kg)

T could also show: $\frac{6 + 42}{2} \times 10 = 24 \times 10 = 240$ (kg)

Answer: The mass of cherries gathered altogether was 240 kg.

35 min

Notes

Individual work, monitored (helped)

Responses shown in unison. Reasoning, agreement, self-correction, praising

Extra praise if P5s notice without a hint from T.

Ps say why it is correct.
### Lesson Plan 119

#### Activity 7

**PbY6b, page 119. Q.5**

a) Read: 1 + 2 + 3 and 2 + 3 + 4 and 3 + 4 + 5 are exactly divisible by 3.

*What can you say about the sum of three adjacent positive whole numbers?*

Give Ps a minute to consider it, then ask several what they think.

Ps might point out that:
- \(1 + 2 + 3 = 6, \ 2 + 3 + 4 = 9, \ 3 + 4 + 5 = 12, \) etc.
  
  The sums form a sequence: 6, 9, 12, 15, 18, 21, . . .
  
  in which each term is a multiple of 3.

or
- When divided by 3, one number has no remainder, one number gives a remainder of 1 and one number gives a remainder of 2, but \(0 + 1 + 2 = 3,\) so their sum is exactly divisible by 3.

or T might suggest, if no P does so:
- Let 1st number be \(n,\) then 2nd number is \(n + 1\) and 3rd number is \(n + 2.\)
  
  BB: \(n + (n+1) + (n+2) = 3n + 3, = 3(n + 1)\)
  
  which is exactly divisible by 3.

or
- Let \(n\) be the 2nd number, then \(n – 1\) is the 1st number and \(n + 1\) is the 3rd number.
  
  BB: \(n – 1 + n + n + 1 = 3n,\) which is a multiple of 3.

*Statement:*

The sum of any 3 adjacent positive whole numbers is a multiple of 3.

b) Read: 1 \(\times\) 2 \(\times\) 3 and 2 \(\times\) 3 \(\times\) 4 and 3 \(\times\) 4 \(\times\) 5 are exactly divisible by 6.

*What can you say about the product of three adjacent positive whole numbers?*

Ps suggest different ways to reason. e.g.
- \(1 \times 2 \times 3 = 6, \ 2 \times 3 \times 4 = 24, \ 3 \times 4 \times 5 = 60, . . .\)
  
  and 6, 24, 60, 120, 210, 336, . . . are all multiples of 6.

or
- In each product, one number is a multiple of 3 and at least one of the other numbers is even, so each product is exactly divisible by \(2 \times 3 = 6.\)

*Statement:*

The product of any 3 adjacent positive whole numbers is a multiple of 6.

---

**Notes**

- **Whole class activity**
  
  Involve several Ps
  
  Discussion, reasoning, agreement, praising
  
  Accept and praise the first reasoning given opposite but point out that we cannot write out every sum to check that it is a multiple of 3, so we must think of a more general reasoning which will apply to any 3 natural numbers (as in the next three examples)

  Elicit that \(n\) can be any positive, whole number, i.e. a natural number.

- **41 min**
  
  Ps say the statement in unison.

(but we cannot write out the product of every set of 3 adjacent natural numbers, so the 2nd reasoning is better)

Ps say the statement in unison.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 119</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Y6</strong></td>
<td></td>
</tr>
</tbody>
</table>

### Lesson Plan 119

#### Notes

- Whole class activity
- Allow time for Ps to think.
- Responses shown in unison.
- Reasoning, checking, agreement, praising
- If disagreement, check with actual values for \( x \) and \( n \).
- Feedback for T

### PbY6b, page 119, Q.6

T chooses a P to read out the question and Ps show answer on scrap paper or slates on command. Ps with correct answers explain reasoning, and check with the relevant operation.

**Solution:**

- **a)** *The difference between two numbers is 2.1. What is the larger number if the smaller number is \( x \)?*

  Larger number: \( 2.1 + x \)  
  Check: \( 2.1 + x - x = 2.1 \) ✓

- **b)** *Laura has \( n \) stamps. Laura and George have 125 stamps altogether. How many stamps does George have?*

  G: \( 125 - n \)  
  Check: \( n + 125 - n = 125 \) ✓

---

45 min
Y6

Activity

Factorising 120, 295, 470 and 1120. Revision, activities, consolidation

PbY6b, page 120

Solutions:

Q.1 a) i) \( p(\text{lemon}) = \frac{3}{15} = \frac{1}{5} \)  
ii) \( p(\text{strawberry}) = \frac{2}{15} \)

iii) \( p(\text{neither lemon nor strawberry}) = \frac{10}{15} = \frac{2}{3} \)

iv) \( p(\text{not blackcurrant}) = \frac{9}{15} = \frac{3}{5} \)

v) \( p(\text{orange or lemon}) = \frac{3}{15} + \frac{4}{15} = \frac{7}{15} \)

vi) \( p(\text{banana}) = 0 \)

b) 49

[No. of lemon jellies in bag: 1 fifth of 60 = 60 ÷ 5 = 12
No. of sweets which are not lemon jellies: 60 – 12 = 48
So the 1st 48 sweets taken out of the bag could be the blackcurrant, the orange and the strawberry jellies, but the 49th sweet must be a lemon jelly.]

Q.2 a) \( \frac{3}{7} + \frac{3}{4} = \frac{12 + 21}{28} = \frac{33}{28} \)

b) \( \frac{5}{8} + \frac{7}{8} = \frac{12}{8} = \frac{3}{2} = \frac{1}{2} \)

c) \( \frac{7}{11} + \frac{1}{2} = \frac{14 + 11}{22} = \frac{25}{22} = \frac{13}{22} \)

d) \( \frac{4}{9} + \frac{9}{13} = \frac{52 + 81}{117} = \frac{133}{117} = \frac{16}{117} \)

e) \( \frac{5}{6} \times \frac{1}{6} = \frac{5}{36} \)

f) \( \frac{1}{12} \times \frac{7}{11} = \frac{7}{12} \)

g) \( \frac{3}{15} \times \frac{6}{5} = \frac{3}{44} \)

h) \( \frac{11}{15} \times \frac{4}{13} = \frac{11}{169} \)

Q.3 a) i) \[
\begin{array}{ccc}
9 & 8 & 1 \\
2 & 7 & 0 \\
1 & 3 & 5 \\
\end{array}
\]

+ \[
\begin{array}{ccc}
1 & 5 & 2 \\
8 & 4 & 9 \\
9 & 7 & 8 \\
\end{array}
\]

b) i) \[
\begin{array}{ccc}
8 & 3 & 5 \\
6 & 3 & 0 \\
2 & 0 & 4 \\
\end{array}
\]

- \[
\begin{array}{ccc}
0 & 6 & 4 \\
4 & 1 & 9 \\
5 & 7 & 7 \\
\end{array}
\]

ii) \[
\begin{array}{ccc}
5 & 4 & 2 \\
2 & 7 & 8 \\
1 & 0 & 4 \\
\end{array}
\]

- \[
\begin{array}{ccc}
1 & 0 & 5 \\
1 & 7 & 6 \\
1 & 3 & 5 \\
\end{array}
\]

iii) \( \frac{5}{2} - 3.8 = 5.4 - 3.8 = 1.6 \)

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Q.3 c) iv) \[ 7 \times 0.25 = 1.75 \]
\[ \div 4 = 0.4375 \]

\[ iv) \quad 7 \times 0.25 = \frac{63}{8} \times \frac{1}{4} \]
\[ = \frac{63}{32} = \frac{1.96875}{32} \]

or

\[ d) \quad 6259 \div 23 = 272 \]
\[ \approx 272.13 \]

(to 2 d.p.)

Q.4 Many possibilities. e.g.

a) \( p\) (an Ace) = \( \frac{1}{13} \)

b) \( p\) (a Club) = \( \frac{1}{4} \)

c) \( p\) (a red card) = \( \frac{1}{2} \)

d) \( p\) (a black card) = \( \frac{13}{26} = \frac{1}{2} \)

e) \( p\) (a face card) = \( \frac{3}{13} \)

f) \( p\) (a Heart face card) = \( \frac{3}{52} \)

g) \( p\) (a red Queen) = \( \frac{1}{26} \)

h) \( p\) (a 2, 3, 4, 5 or 6) = \( \frac{5}{13} \)

i) \( p\) (not a 2, 3, 4, 5 or 6) = \( \frac{8}{13} \)

j) \( p\) (5 Kings) = 0

Q.5 a) e.g. Let smaller number be \( x \), then larger number is \( x + 1.1 \).

Sum: \( x + x + 1.1 = 8.3 \)
\[ 2x = 8.3 - 1.1 = 7.2 \]
\[ x = 7.2 \div 2 = 3.6 \]
\[ 3.6 + 1.1 = 4.7 \]

Answer: The smaller number is 3.6 and the larger number is 4.7.

b) \( 11^2 = 121 \)
\( 21^2 = 441 \)
\( 31^2 = 961 \)
(no more are possible, as 41 is a 4-digit number)

c) D: 8, B: 8 + 15 = 23, C: 23 - 8 = 15, A: 15 + 20 = 35
Lesson Plan

Week 25

Y6

R: Natural numbers
C: Multiples, factors. Calculations with remainders
E: Problems: reasoning and checking

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(121 = 11 \times 11 = 11^2\) Factors: 1, 11, 121
- \(296 = 2 \times 2 \times 2 \times 37 = 2^3 \times 37\) Factors: 1, 2, 4, 8, 37, 74, 148, 296
- \(471 = 3 \times 157\) Factors: 1, 3, 157, 471
- \(1121 = 19 \times 59\) Factors: 1, 19, 59, 1121

7 min

2

Factors and multiples

Study this diagram. What do the arrows mean? P comes to BB to explain, using one of the arrows as an example. Class agrees/disagrees.

BB:

What about the arrow pointing from a number to itself? (e.g. 7 is a multiple of 7 and 7 is also a factor of 7, as \(7 \times 1 = 7\), \(7 \div 7 = 1\))

10 min

3

PbY6b, page 121

Q.1 Read: The base set is the set of positive whole numbers.

a) Write 4 numbers which have exactly 2 factors.
b) Write a number which has exactly one factor.
c) Write 3 numbers which have exactly 3 factors.
d) Write 3 numbers which have exactly 4 factors.

Set a time limit. Ps check numbers on slates or in Ex. Bks before listing in Pbs.

Review with whole class. T asks some Ps for examples. Who had the same? Who had another number? Class decides which numbers are valid. Elicit what kind of numbers are in each category.

Solution:

a) e.g. 2, 11, 19, 157 (Prime numbers)

A prime number is exactly divisible only by itself and 1.

b) 1 (The unit number)

c) e.g. 4 (1, 2, 4), 9 (1, 3, 9), 25 (1, 5, 25)

These are the squares of prime numbers.

d) e.g. 6 (1, 2, 3, 6), 10 (1, 2, 5, 10), 15 (1, 3, 5, 15)

Each number is the product of 2 different prime numbers.

16 min

Notes

Individual work, monitored
(or whole class activity)
BB: 121 296, 471, 1121
Reasoning, agreement, self-correction, praising

e.g.  296 2

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>121</td>
<td>11</td>
<td>1148</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>74</td>
</tr>
<tr>
<td>1</td>
<td>37</td>
<td>37</td>
</tr>
<tr>
<td>471</td>
<td>3</td>
<td>1121</td>
</tr>
<tr>
<td>157</td>
<td>157</td>
<td>59</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Ps come to BB to point to each of the other arrows and explain in a similar way.

Agreement, praising

Individual work, monitored, helped

Differentiation by time limit

What do we call the set of positive whole numbers? (Natural numbers → \(\mathbb{N}\))

Discussion, reasoning, agreement, self-correction, praising

After agreeing on the type of numbers in a list, Ps could suggest a few more examples where possible.

\[ e.g. 6 = 2 \times 3, 10 = 2 \times 5, 15 = 3 \times 5, \text{ etc.} \]
### Activity

4  

**PbY6b, page 121**

Q.2  Read: **Simplify these fractions.**

What does simplify mean? (To change to its simplest form.)

How can we do that? (By dividing the numerator and denominator by their greatest common factor.)

Set a time limit. Ps can reduce the fractions in steps if necessary.

Review with whole class. Ps could show fractions on scrap paper or slates on command. Ps with different results explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Solution</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a) (\frac{42}{60} = \frac{7}{10})</td>
<td>b) (\frac{36}{48} = \frac{3}{4})</td>
</tr>
<tr>
<td>d) (\frac{140}{56} = \frac{20}{8} = \frac{5}{2} = 2\frac{1}{2})</td>
<td></td>
</tr>
</tbody>
</table>

T shows this method for simplifying large numerators and denominators in one step.

Factorise each number (as opposite) and circle their common factors. Their product is the highest common factor of the 2 numbers: \((2 \times 2 \times 7 = 28)\)

140 ÷ 28 = 5, 56 ÷ 28 = 2, i.e. the factors not circled

Peter worked out the greatest common factor of 140 and 56 like this. Who can explain it? Is his method correct?

BB: 140 ÷ 56 = 2, r 28  
56 ÷ 28 = 2

**Explanation:** e.g

If \(x\) is the greatest common factor of 140 and 56, then \(x\) is also a factor of 140 – 56 = 84.

So \(x\) is the greatest common factor of 84 and 56 and \(x\) is also a factor of 84 – 56 = 28.

So \(x\) is the greatest common factor of 56 and 28 and \(x\) is also a factor of 56 – 28 = 28.

So \(x\) is the greatest common factor of 28 and 28, which is 28.

i.e. \(x = 28\)

### Extension

**BB:**

<table>
<thead>
<tr>
<th>56</th>
<th>140</th>
<th>28</th>
<th>70</th>
<th>14</th>
<th>35</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3</td>
<td>2</td>
<td>7</td>
<td>2</td>
<td>7</td>
<td>1</td>
</tr>
</tbody>
</table>

So we only need to write the factorisation as above!

Whole class discussion. Involves several Ps.

T gives hint about differences if Ps do not think of it.

Extra praise if Ps think of it without help from T.

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### Lesson Plan 121

#### Activity 5

**PbY6b, page 121**

**Q.3** Read: *Decide whether the sum is exactly divisible by 3, then do the calculation.*

How can we decide? (If each term is divisible by 3 then the whole sum will be divisible by 3.)

Set a time limit. Ps check each term first, writing any remainder below the term before working out the result.

Review with whole class. Ps show by pre-agreed actions in unison whether they think each sum is divisible by 3, (e.g. standing up if Yes, remaining seated if No) Ps with different responses explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

**Solution:**

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>(36 + 18 + 27 + 45) ÷ 3</td>
<td>(Exactly divisible by 3, as each term is exactly divisible by 3)</td>
<td>12 + 6 + 9 + 15 = 42</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 12 + 6 + 9 + 15</td>
<td>= 42</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>(36 + 14 + 66 + 19) ÷ 3</td>
<td>(Exactly divisible by 3, as the sum of the remainders is 2 + 1 = 3, which is divisible by 3)</td>
<td>135 ÷ 3 = 45</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 2 0 1</td>
<td>= 45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>(45 + 73 + 46 + 90) ÷ 3</td>
<td>(Not exactly divisible by 3, as the sum of the remainders is 1 + 1 = 2, which is not divisible by 3)</td>
<td>254 ÷ 3 = 84, r 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 1 1 0</td>
<td>= 84, r 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

Extra praise if Ps realise that the remainders can also form groups of 3.

---

### Activity 6

**PbY6b, page 121**

**Q.4** Read: *Decide whether the sum is exactly divisible by 4, then do the calculation.*

Set a time limit of 3 minutes. Ps write remainders below terms first, then work out the result.

Review with whole class. Ps show by pre-agreed actions whether they think a sum is divisible by 4. Ps with different responses explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

**Solution:**

<p>| | | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>(33 + 41 + 62 + 240) ÷ 4</td>
<td>(Exactly divisible by 4, as the sum of the remainders is 1 + 1 + 2 = 4, which is divisible by 4)</td>
<td>11 + 15 + 5 + 3 = 34</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1 1 2 0</td>
<td>= 376 ÷ 4 = 94</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>(44 + 60 + 20 + 12) ÷ 4</td>
<td>(Exactly divisible by 4, as each term is divisible by 4)</td>
<td>33</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>0 0 0 0</td>
<td>= 11 + 15 + 5 + 3 = 34</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>(26 + 27 + 28 + 29) ÷ 4</td>
<td>(Not exactly divisible by 4, as the sum of the remainders is 2 + 3 + 1 = 6, which is not exactly divisible by 4)</td>
<td>110 ÷ 4 = 27, r 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2 3 0 1</td>
<td>= 110 ÷ 4 = 27, r 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes**

Extra praise if Ps reason that the remainders form another group of 4.

---

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### Activity 7

**PhbY6b, page 121**

**Q.5** Read: Decide whether the difference is exactly divisible by 5 then do the calculation.

Set a time limit of 3 minutes. Ps write remainders below reductant and subtrahend first, then work out the result.

Review with whole class. Ps show by pre-agreed actions whether they think a difference is divisible by 5. Ps with different responses explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

**Solution:**

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>(75 – 40) ÷ 5</td>
<td></td>
<td>(Exactly divisible by 5, as the reductant and subtrahend are exactly divisible by 5)</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 15 – 8 = 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>(78 – 43) ÷ 5</td>
<td></td>
<td>(Exactly divisible by 5, as the difference between the remainders: 3 – 3 = 0, is divisible by 5)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 35 ÷ 5 = 7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>c)</td>
<td>(82 – 35) ÷ 5</td>
<td></td>
<td>(Not exactly divisible by 5, as the difference between the remainders is 2 – 0 = 2, which is not exactly divisible by 5)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 47 ÷ 5 = 9, r 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d)</td>
<td>(36 – 14) ÷ 5</td>
<td></td>
<td>(Not exactly divisible by 5 as the difference between the remainders is 1 – 4 = −3, which is not divisible by 5)</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 22 ÷ 5 = 4, r 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>e)</td>
<td>(54 – 26) ÷ 5</td>
<td></td>
<td>(Difference between the remainders is 4 – 1 = 3, which is not exactly divisible by 5)</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 28 ÷ 5 = 5, r 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>f)</td>
<td>(90 – 36) ÷ 5</td>
<td></td>
<td>(Not exactly divisible by 5 as the difference between the remainders is 0 – 1 = −1, which is not divisible by 5)</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>= 54 ÷ 5 = 10, r 4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- **35 min**

**Q.6** Read: Which digit could be written in the box so that the sum inside the brackets:

- a) is exactly divisible by 7
- b) gives a remainder of 3 when divided by 7
- c) gives a remainder of 6 when divided by 7?

Set a time limit of 3 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class checks that they are correct. Mistakes corrected.

**Solution:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>= 2 or 9 (as 35 and 28 are multiples of 7, the middle term must also be a multiple of 7, i.e. 42 or 49)</td>
</tr>
<tr>
<td>b)</td>
<td>= 5 (missing term must be 3 more than a multiple of 7)</td>
</tr>
<tr>
<td>c)</td>
<td>= 8 or 1 (missing term must be 7 more than a multiple of 7)</td>
</tr>
</tbody>
</table>

- **40 min**

---

**Notes**

Individual work, monitored (helped)

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Extra praise if a P realises this.

Note that the remainder of the division is not −3. There are 7 groups of 5 in 36 and 2 groups of 5 in 14. 

\( 7 – 2 = 5, \) but one of these 5s must be combined with −3: 

\( −3 + 5 = 2, \) so the actual result of the division is 4, r 2.

[18 whole groups of 5 in 90, 7 whole groups of 5 in 35, and 18 – 7 = 11, but one of these 5s must be combined with the −1: \( −1 + 5 = 4, \) so the actual result of the division is 10, r 4.]

Individual work, monitored, helped

Written on BB or SB or OHT

BB: \((35 + 4\square + 28) ÷ 7\)

(or Ps could show middle term on slates in unison)

Reasoning, checking, agreement, self-correction, praising

**Check:**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>( 42 ÷ 7 = 6, 49 ÷ 7 = 7 )</td>
</tr>
<tr>
<td>b)</td>
<td>( 42 ÷ 7 = 6, r 3 ) (no others)</td>
</tr>
</tbody>
</table>
| c) | \( 48 ÷ 7 = 6, r 6 \)

\( 41 ÷ 7 = 5, r 6 \)

---

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**Lesson Plan 121**

**Notes**

Whole class activity
(or individual work if Ps wish, reviewed with whole class)

Written on BB or SB or OHT

Involve several Ps.

Reasoning, agreement, praising

Feedback for T

(or $4 \times \frac{3}{7} \times 8$, or $4 \times 6 \times 4$)

Stress that when the numerator is a sum, **each term** is divided by the denominator but when the numerator is a product, **only one** factor is divided by the denominator (but it can be any factor).

or $10 \times \frac{5}{7} \times 55$

or $10 \times 25 \times 11$

---

**Y6**

**Activity**

9

*PbY6b, page 121, Q.7*

Read: **Simplify the fractions in your exercise book.** Check that you are correct.

What does simplify mean? (Write each fraction in its simplest form.)

How can we do it? Ps come to BB or dictate what T should write, explaining reasoning. Who agrees? Who can think of another way to do it? (T gives hints if Ps have only one idea.)

Make sure that both the methods below are dealt with. Ask Ps which method they think is easier.

**Solution:**

a) \( \frac{4 + 6 + 8}{2} = \frac{18}{2} = 9 \) or \( \frac{4 + 6 + 8}{2} = \frac{2 + 3 + 4}{1} = 9 \)

b) \( \frac{4 \times 6 \times 8}{2} = \frac{192}{2} = 96 \) or \( \frac{4 \times 6 \times 8}{2} = 2 \times 6 \times 8 = 96 \)

c) \( \frac{10 + 25 + 55}{5} = \frac{90}{5} = 18 \)

or \( \frac{10 + 25 + 55}{5} = 2 + 5 + 11 = 18 \)

d) \( \frac{10 \times 25 \times 55}{5} = \frac{13750}{5} = 2750 \)

or \( \frac{2 \times 10 \times 25 \times 55}{5^2} = 2 \times 25 \times 55 = 50 \times 55 = 2750 \)

---

45 min
Y6

**Lesson Plan 122**

**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(122 = 2 \times 61\)  
  Factors: 1, 2, 61, 122

- \(297 = 3 \times 3 \times 3 \times 11 = 3^3 \times 11\)  
  Factors: 1, 3, 9, 11, 27, 33, 99, 297

- \(472 = 2 \times 2 \times 2 \times 59 = 2^3 \times 59\)  
  Factors: 1, 2, 4, 8, 59, 118, 236, 472

- \(1122 = 2 \times 3 \times 11 \times 17\)  
  Factors: 1, 2, 3, 6, 11, 17, 22, 33  
  1122, 561, 374, 187, 102, 66, 51, 34


**Notes**

Individual work, monitored  
(or whole class activity)

BB: 122 297 472 1122  
Reasoning, agreement, self-correction, praising

**PbY6b, page 122**

Q.1 Read: Write five 3-digit numbers which are exactly divisible by:

- a) 2
- b) 5
- c) 10.

Allow 3 minutes. Tell Ps to use the natural numbers as their base set. Ps write possible numbers in Ex. Bks.

Review with whole class. T asks a few Ps for their numbers. Class points out errors. Review the general 'tests' for divisibility by 2, 5 or 10.

**Solution:**

- a) e.g. 104, 236, 450, 788, 910  
  (any even number is divisible by 2)

- b) e.g. 105, 235, 450, 780, 915  
  (any number with units digit 0 or 5 is divisible by 5)

- c) e.g. 100, 130, 600, 800, 900  
  (any whole 10 is divisible by 10, i.e. any number which has zero in its units column)

**PbY6b, page 122**

Q.2 Read: Write five 4-digit numbers which are exactly divisible by:

- a) 4
- b) 25
- c) 100

Allow 4 minutes. Ps write possible numbers in Ex. Bks.

Review with whole class. T asks a few Ps for their numbers. Class points out errors. Review the general test for divisibility by 4, 25 or 100.

**Solution:**

- a) 4000, 7232, 3320, 8596, 2248
- b) 5500, 6925, 4850, 7175, 2025
- c) 6000, 9300, 5200, 8800, 1700

(To be divisible by 100, the tens and units digits should be 0.)

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**Lesson Plan 122**

### Y6

#### Activity

**PbY6b, page 122**

**Q.3** Read: Write four 5-digit numbers which are exactly divisible

- a) by 2 and by 5
- b) by 4 and by 25.

Set a time limit of 3 minutes. Again, Ps choose from the base set of natural numbers and write numbers in *Ex. Bks*.

Review with whole class. T asks a few Ps for their numbers. Class points out errors. Agree on the tests for divisibility.

**Solution:**

- a) e.g. 13 430, 76 000, 21 560, 55 550  
  (any number which is a multiple of 10, as \(2 \times 5 = 10\), and 2 and 5 are prime numbers)
- b) e.g. 26 400, 41 100, 70 900, 11 100  
  (any number which is a multiple of 100, as \(4 \times 25 = 100\), and 4 and 25 have no common factors apart from 1)

**Notes**

Individual work, monitored

Agreement, self-correction, praising

Elicit/remind Ps that the lowest common multiple of:
- 2 prime numbers, or
- 2 numbers with no common factors apart from 1
is their **product**.

---

**Q.4** Read: Decide on the remainder before doing the calculation by writing the remainder for each term below it.

Deal with one at a time or set a time limit.

Review with whole class. Ps could show remainders on scrap paper or slates on command. Ps answering correctly explain reasoning at BB, writing the result too. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:** e.g.

- a) \((45 + 63 + 18) \div 3\)
  
  \[
  \begin{array}{c}
  0 \ 0 \ 0 \\
  = 15 + 21 + 6 = \boxed{42}
  \end{array}
  \]
- b) \((41 + 72 + 81) \div 3\)
  
  \[
  \begin{array}{c}
  2 \ 0 \ 0 \\
  = 194 \div 3 = \boxed{64, r 2}
  \end{array}
  \]
- c) \((53 + 90 + 19) \div 3\)
  
  \[
  \begin{array}{c}
  2 \ 0 \ 1 \\
  = 162 \div 3 = \boxed{54}
  \end{array}
  \]
- d) \((1000 + 100 + 10 + 6) \div 3\)
  
  \[
  \begin{array}{c}
  1 \ 1 \ 1 \\
  = 1116 \div 3 = \boxed{372}
  \end{array}
  \]
- e) \((300 + 20 + 4) \div 3\)
  
  \[
  \begin{array}{c}
  0 \ 2 \ 1 \\
  = 324 \div 3 = \boxed{108}
  \end{array}
  \]
- f) \((4000 + 100 + 70 + 1) \div 3\)
  
  \[
  \begin{array}{c}
  1 \ 1 \ 1 \\
  = 4171 \div 3 = \boxed{1390, r 1}
  \end{array}
  \]

**Notes**

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Does anyone notice a quick way to determine the remainders?

Ps might remember from previous years or might just notice it now.

(\(\text{The remainder when a number is divided by 3 is the same as the remainder when the sum of its digits is divided by 3.}\))

If no P remembers or notices, T points it out.
Y6

**Activity**

6

*PbY6b, page 122*

Q.5 Read: Write the remainder after dividing each number by 9.

Think about what we have just said when doing this exercise!

Set a short time limit. Review with whole class. Ps come to BB to write remainders and explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) 100 [1]  

b) 200 [2]  

c) 800 [8]  

d) 900 [0]  

e) 1000 [1]  

f) 2000 [2]  

g) 6000 [6]  

h) 9000 [0]  

i) 819 [0]  

j) 7368 [6]  

k) 12 534 [6]  

l) 88 888 [4]

Elicit that the same strategy works for 9 as for 3: the remainder is the same as when the sum of the digits in the number is divided by 9. T (or Ps) might point out that if the sum of the remainders is a 2-digit number, then those 2 digits can be added together too.

Details: e.g.

819: 8 + 1 + 9 = 18, and 8 + 1 = 9 (so divisible by 9)

7368: 7 + 3 + 6 + 8 = 24, and 2 + 4 = 6, so remainder is 6

12 534: 1 + 2 + 5 + 3 + 4 = 15, and 1 + 5 = 6, so remainder is 6

88 888: 5 × 8 = 40, 4 + 0 = 4, so remainder is 4.

45 min

7

*PbY6b, page 122*

Q.6 Read: Decide on the remainder before doing the calculation by writing the remainder for each term below it.

Deal with one at a time or set a time limit.

Review with whole class. Ps could show remainders on scrap paper or slates on command. Ps answering correctly explain reasoning at BB, writing the result too. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:** e.g.

a) (45 + 63 + 18) ÷ 9  

0 0 0  

(no remainder, so divisible by 9)  

= 5 + 7 + 2 = 14

b) (41 + 72 + 81) ÷ 9  

5 0 0  

(remainder 5, so not divisible by 9)  

= 194 ÷ 9 = 21, r 5

c) (53 + 90 + 19) ÷ 9  

8 0 1  

(remainder 9, which is divisible by 9)  

= 162 ÷ 9 = 18

d) (1000 + 100 + 10 + 6) ÷ 9  

1 1 1 6  

(remainder 9, which is divisible by 9)  

= 1116 ÷ 9 = 124

e) (300 + 20 + 4) ÷ 9  

3 2 4  

(remainder 9, which is divisible by 9)  

= 324 ÷ 9 = 36

f) (4000 + 100 + 70 + 1) ÷ 9  

4 1 7 1  

(and 13 ÷ 9 = 1, r 4, so not divisible)  

= 4171 ÷ 9 = 463, r 4

40 min

---

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Lesson Plan 122

### Activity 8

**PbY6b, page 122**

**Q.7 Read:** Write the remainder after dividing each number by 3. Set a short time limit. Review with whole class. Ps come to BB to write remainders and explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>a) 100</th>
<th>b) 200</th>
<th>c) 800</th>
<th>d) 900</th>
<th>e) 1000</th>
<th>f) 2000</th>
<th>g) 6000</th>
<th>h) 9000</th>
<th>i) 819</th>
<th>j) 7368</th>
<th>k) 12,534</th>
<th>l) 88,888</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>[1]</td>
<td>[2]</td>
<td>[2]</td>
<td>[0]</td>
<td>[1]</td>
<td>[2]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
<td>[0]</td>
<td>[1]</td>
</tr>
</tbody>
</table>

Show details if there is disagreement. e.g.

- **819:**  $8 + 1 + 9 = 18$, and $8 + 1 = 9$ (so divisible by 3)
- **7368:**  $7 + 3 + 6 + 8 = 24$, and $2 + 4 = 6$ (so divisible by 3)
- **12,534:**  $1 + 2 + 5 + 3 + 4 = 15$, and $1 + 5 = 6$ (so divisible by 3)
- **88,888:**  $5 \times 8 = 40$, $4 + 0 = 4$, $4 \div 3 = 1$, $r 1$ (not divisible)

Ask individual Ps to explain in their own words how to calculate the remainder quickly when dividing by 2, 3, 4, 5, 9, 10, 25 or 100.

---

**Notes**

Individual work, monitored
(or if time is short, whole class activity with Ps coming to BB)

Written on BB or use enlarged copy master or OHP

Differentiation by time limit

Reasoning, agreement, self-correction, praising

T points out that instead of dividing by 3, we could subtract the nearest (smaller) multiple of 3, e.g. $4 - 3 = 1$, so remainder is 1.

Class agrees/disagrees.
Praising

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### Y6

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><strong>Factorisation</strong></td>
</tr>
<tr>
<td></td>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.</td>
</tr>
<tr>
<td></td>
<td>Elicit that:</td>
</tr>
<tr>
<td></td>
<td>• ( 123 = 3 \times 41 ) Factors: 1, 3, 41, 123</td>
</tr>
<tr>
<td></td>
<td>• ( 298 = 2 \times 149 ) Factors: 1, 2, 149, 298</td>
</tr>
<tr>
<td></td>
<td>• ( 473 = 11 \times 43 ) Factors: 1, 11, 43, 473</td>
</tr>
<tr>
<td></td>
<td>• ( 1123 ) is a prime number (as not exactly divisible by 2, 3, 5, 7, 11, 17, 19, 23, 29 and 31, and ( 37^2 &gt; 1123 ))</td>
</tr>
<tr>
<td></td>
<td><strong>8 min</strong></td>
</tr>
<tr>
<td>2</td>
<td><strong>Tests for divisibility</strong></td>
</tr>
<tr>
<td></td>
<td>Let’s revise how to work out whether a natural number is exactly divisible by a certain number, and if it is not, what the remainder will be. T says the divisor and chooses Ps to describe the test, say how to calculate the remainder and to write examples (divisible and not divisible) on BB. Class points out errors. e.g.</td>
</tr>
<tr>
<td></td>
<td>a) <strong>Divisor is 2</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 2 if its units digit is divisible by 2 (or even). If the number is not divisible by 2 (or odd), the remainder is 1.</td>
</tr>
<tr>
<td></td>
<td>b) <strong>Divisor is 3</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 3 if the sum of its digits is divisible by 3. If the number is not divisible by 3, the remainder is the same as when the sum of its digits is divided by 3.</td>
</tr>
<tr>
<td></td>
<td>c) <strong>Divisor is 4</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 4 if the last 2 digits are divisible by 4. If the number is not divisible by 4, the remainder is the same as when the last two digits are divided by 4.</td>
</tr>
<tr>
<td></td>
<td>d) <strong>Divisor is 5</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 5 if its units digit is 5 or 0. If the number is not divisible by 5, the remainder is the same as when the units digit is divided by 5.</td>
</tr>
<tr>
<td></td>
<td>e) <strong>Divisor is 9</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 9 if the sum of its digits is divisible by 9. If the number is not divisible by 9, the remainder is the same as when the sum of its digits is divided by 9.</td>
</tr>
<tr>
<td></td>
<td>f) <strong>Divisor is 10</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 10 if the units digit is 0. If the number is not divisible by 10, the remainder is the units digit.</td>
</tr>
<tr>
<td></td>
<td>g) <strong>Divisor is 25</strong></td>
</tr>
<tr>
<td></td>
<td>The number is divisible by 25 if the last 2 digits are divisible by 25. If the number is not divisible by 25, the remainder is the same as when the last two digits are divided by 25.</td>
</tr>
<tr>
<td></td>
<td><strong>15 min</strong></td>
</tr>
</tbody>
</table>

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### Activity

<table>
<thead>
<tr>
<th>3</th>
<th>PbY6b, page 123</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q.1</strong> Read:</td>
<td><strong>b)</strong> Increase the numbers so that when the new numbers are divided by 9:</td>
</tr>
<tr>
<td></td>
<td>i) there is a remainder of 1</td>
</tr>
<tr>
<td></td>
<td>ii) there is a remainder of 4.</td>
</tr>
<tr>
<td></td>
<td><strong>c)</strong> Decrease the original numbers so that when the new numbers are divided by 9 there is a remainder of 8.</td>
</tr>
<tr>
<td></td>
<td>Set a time limit. T monitors all Ps closely to check that they understand the task and to note Ps with different types of numbers. Review with whole class. T chooses a few Ps to come to BB to write their numbers and to explain how they adjusted them. Class checks that they are correct.</td>
</tr>
<tr>
<td><strong>Solution:</strong> e.g.</td>
<td>a) 11 115 46 728 70 002 99 999</td>
</tr>
<tr>
<td></td>
<td>b) i) 11 116 46 738 70 030 1 000 000</td>
</tr>
<tr>
<td></td>
<td>ii) 11 119 46 732 70 006 1 000 003</td>
</tr>
<tr>
<td></td>
<td>c) 11 114 46 727 70 001 99 998</td>
</tr>
</tbody>
</table>

### Notes

Individual work, monitored, helped, corrected

Reasoning, checking with the appropriate divisibility test, agreement, praising

(Most Ps might change only the units digits but show that other place-values can be changed too.)

<table>
<thead>
<tr>
<th>4</th>
<th>PbY6b, page 123</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q.2</strong> Read:</td>
<td><strong>b)</strong> Increase the numbers so that the new numbers are exactly divisible by 9.</td>
</tr>
<tr>
<td></td>
<td><strong>c)</strong> Increase the original numbers so that when the new numbers are divided by 3:</td>
</tr>
<tr>
<td></td>
<td>i) there is a remainder of 1</td>
</tr>
<tr>
<td></td>
<td>ii) there is a remainder of 2.</td>
</tr>
<tr>
<td></td>
<td>Set a time limit. Again, T monitors all Ps closely and note Ps with different types of numbers. Review with whole class. T chooses a few Ps to come to BB to write their numbers and to explain how they adjusted them. Class checks that they are correct.</td>
</tr>
<tr>
<td><strong>Solution:</strong> e.g.</td>
<td>a) 1110 6231 7866 3333</td>
</tr>
<tr>
<td></td>
<td>b) 1116 6237 7875 3339</td>
</tr>
<tr>
<td></td>
<td>c) i) 1111 6232 7867 3334</td>
</tr>
<tr>
<td></td>
<td>ii) 1112 6233 7868 3335</td>
</tr>
</tbody>
</table>

Individual work, monitored, helped, corrected

Reasoning, checking with the appropriate divisibility test, agreement, praising
**Notes**

Individual work, monitored

Written on BB or SB or OHT:

BB: 23 461 72 534 183 5606 444

Challenge the more able Ps to think of other ways to determine the remainders.

Responses shown in unison.

In good humour!

Praising

Discussion, reasoning, agreement, self-correction, praising

Extra praise if Ps thought of alternatives to division, otherwise T could show them and ask Ps if they are correct.

**Lesson Plan 123**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 123</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td><strong>PbY6b, page 123</strong></td>
</tr>
</tbody>
</table>
|          | Q.3  Read:  a) *Circle the numbers which are divisible by 2 and also by 3.*  
|          |       b) *Calculate the remainder when each number is divided by 6.*  
|          | Set a time limit of 3 minutes. Ps circle the numbers and write the remainders beneath in Pbs. (If necessary, Ps can do divisions in Ex. Bks.)
|          | Review with whole class. T points to each number in turn and Ps stand up if they think it is divisible by 2 and by 3 and/or show its remainder when divided by 6. Agree that numbers which are divisible by 2 and by 3 and are also exactly divisible by 6. T chooses Ps to explain their thinking or show their calculations. Who did the same? Who thought (calculated) in a different way? Mistakes discussed and corrected.
|          | Solution:
|          | a) 23 461 72 534 183 5606 444  
|          | (Odd numbers not possible. Add digits in other 3 numbers and divide their sum by 3.)
|          | b) \(23 461 \div 6 = 3910, r 1\)  
|          | (or 23 460 is even and the sum of its digits is divisible by 3 so it is also divisible by 6, so 23 461 gives a remainder of 1 when divided by 6)  
|          | \(72 534\): divisible by 2 and by 3, so divisible by 6, so \(r 0\)  
|          | \(183 \div 6 = 30, r 3\) or \(183 = 180 + 3\), so remainder is 3  
|          | \(5606 \div 6 = 934, r 2\) or \(5606 = 5400 + 180 + 24 + 2\), so \(r 2\)  
|          | (or 5604 is divisible by 2 and by 3, so also by 6, so 5606 when divided by 6 will give a remainder of 2.)  
|          | \(444\): divisible by 2 and by 3, so divisible by 6, so \(r 0\)  
|          | **30 min**

| 6        | **PbY6b, page 123. Q.4** |
|          | Read:  a) *Write the natural numbers from 150 to 170 in the Venn diagram.*  
|          | What is a natural number? (a positive whole number)  
|          | Ps come to BB one after the other to write the numbers from 150 to 170 in the correct place on diagram on BB, explaining reasoning. Rest of class write numbers in Pbs too and point out any errors.  
|          | BB:  
|          | ![Venn Diagram](image)  
|          | T: We call the part of the diagram where two sets overlap the *intersection* of the 2 sets. What do you notice about the numbers in this intersection? (T points) (They are multiples of 12.)  
|          | BB: *intersection*  
|          | Agreement, praising

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(Continued)

What about the other factors of 12? Is a number divisible by 12 if it is divisible by 2 and by 6? (No, e.g. 18 is divisible by 2 and by 6 but not by 12)

Read: Complete this sentence.
Ps read out the sentence, stressing the numbers to be written in the boxes. T fills them in on BB and Ps in Pbs.
BB: A natural number is divisible by 12 only if it is divisible by 4 and by 3.

<table>
<thead>
<tr>
<th>Multiple of 6</th>
<th>Not a multiple of 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>150, 160, 164</td>
<td>152, 160, 164</td>
</tr>
</tbody>
</table>

b) If a natural number is divisible by 4 and by 6, then it is also divisible by 12.

Individual work, monitored, helped
Drawn on BB or use enlarged copy master or OHP at a good pace.
Reasoning, agreement, self-corrections, praising
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Some Ps might have written '24' but 12 is divisible by 4 and 6 but not by 24. It is really the same concept as Q.4.

A natural number is divisible by 12 only if it is divisible by 4 and by 3.

**Solution:**

a) $150 \leq n \leq 170$

<table>
<thead>
<tr>
<th>Multiple of 4</th>
<th>Not a multiple of 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>150, 160, 164</td>
<td>150, 160, 164</td>
</tr>
</tbody>
</table>

b) If a natural number is divisible by 4 and by 6, then it is also divisible by 12.

Whole class activity
(or individual work if Ps wish)
Responses shown in unison.
Reasoning (with T's help if necessary), agreement, praising

BB: $2 \times 2 \times 3 = 12$

It's the same as:

$215 \times 1001$

= 215 000

+ 215

= 215 215
### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(124 = 2 \times 2 \times 31 = 2^2 \times 31\)  Factors: 1, 2, 4, 31, 62, 124
- \(299 = 13 \times 23\)  Factors: 1, 13, 23, 299
- \(474 = 2 \times 3 \times 79\)  Factors: 1, 2, 3, 6, 79, 158, 237, 474
- \(1124 = 2 \times 2 \times 281 = 2^2 \times 281\)  Factors: 1, 2, 4, 281, 562, 1124

### Activity 2

**Revision: Long division**

Let's do these divisions together and check the results. Ps come to BB or dictate each step, explaining with place-value detail. Ps write calculation in Ex. Bks too.

How can we check the result? (with multiplication) Again, Ps come to BB or dictate what T should write and Ps write it in Ex. Bks too.

**BB:**

- a)  
  \[
  \begin{array}{c}
  1462 \\
  \hline
  728 \\
  2912 \\
  -  \\
  4316 \\
  \hline
  1456 \\
  -  \\
  1456 \\
  \hline
  10
  \end{array}
  \]

  **Check:**
  \[
  \begin{array}{c}
  728 \\
  \times 4 \,
  \end{array}
  \]
  \[
  \begin{array}{c}
  1456 \\
  \end{array}
  \]
  \[
  \begin{array}{c}
  1456 \\
  \end{array}
  \]
  \[
  \begin{array}{c}
  10
  \end{array}
  \]

- b)  
  \[
  \begin{array}{c}
  248 \\
  \hline
  2791 \\
  \hline
  1412 \\
  \hline
  13824 \\
  \hline
  30720 \\
  \hline
  14736 \\
  \hline
  34177
  \end{array}
  \]

  **Check:**
  \[
  \begin{array}{c}
  2971 \\
  \times 4 \,+
  \end{array}
  \]
  \[
  \begin{array}{c}
  14736 \\
  \end{array}
  \]
  \[
  \begin{array}{c}
  34177
  \end{array}
  \]

Elicit different ways of writing the result in b). e.g.

**BB:** 448 \(\div\) 1536 = 291, r 1472

\[
\frac{1472}{1536} = \frac{291}{243} \]

or \(\approx\) 292 (to the nearest unit)

**Notes**

Individual work, monitored (or whole class activity)

BB: 124, 299, 474, 1124

T decides whether Ps may use calculators.

Reasoning, agreement, self-correction, praising

Written on BB or use enlarged copy master or OHP

At a good pace.

Class points out errors.

Reasoning, agreement, praising

Feedback for T

or Ps might suggest using a calculator to work out the result as a decimal.

(291.9583)
### Lesson Plan 124

#### Activity

3. **Solving equations**

Let’s work out what the letters stand for by solving these equations. Ps come to BB or dictate what T should write at each step, explaining reasoning. Ps write solutions in Ex. Bks. at the same time.

How can we check our result? (Substitute the value for the letter in the equation and check that the equation is true.)

**BB:**

- (\(a - \frac{2}{3}\) + 2) = 5
- 20 - (\(b + \frac{3}{4}\)) = 7

Then solve:

- \(a = 3 + 3\frac{2}{3}\)
- \(b = 13 - 2\frac{3}{4}\)

**Notes**

- Whole class activity
- Written on BB or SB or OHT
- At a good pace.
- Reasoning, checking, agreement, praising
- Feedback for T

**Checks:**

- (\(6\frac{2}{3} - 3\frac{2}{3}\) + 2
  \(= 3 + 2 = \frac{5}{2} \checkmark\))
- 20 - (10\(\frac{1}{4} + 2\frac{3}{4}\)
  \(= 20 - 13 = \frac{7}{2} \checkmark\))

4. **PbY6b, page 124**

**Q.1 Read:** Calculate:

- \(\frac{5}{14}\) times \(\frac{3}{7}\)
- One fifth of \(\frac{3}{7}\)
- Half of \(\frac{4}{5}\)

Set a time limit of 2 minutes. Ps write operations in Ex. Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Who did the same? Who did it another way?

**Solution:**

- \(5 \times \frac{3}{14} = 15 + \frac{5}{4} = 15 + 1\frac{1}{4} = 16\frac{1}{4}\)
  
  or \(5 \times \frac{13}{4} = \frac{65}{4} = 16\frac{1}{4}\)

- \(\frac{3}{7} \div 5 = \frac{3}{35}\) (or \(\frac{1}{5}\) of \(\frac{3}{7}\) = \(\frac{1}{5} \times \frac{3}{7} = \frac{3}{35}\))

- \(\frac{2}{4} \div 2 = \frac{1}{2}\) (or \(\frac{4}{5} + 2 = \frac{14}{5} \div 2 = \frac{7}{5} = \frac{2}{5}\))

**Notes**

- Individual work, monitored
- Written on BB or SB or OHT
- Responses shown in unison
- Reasoning, agreement, self-correction, praising
- Feedback for T

**What does \(\frac{3}{35}\) actually mean?**

(1 unit has been divided into 35 equal parts and we have taken 3 of the parts)

5. **PbY6b, Page 124**

**Q.2 Read:** Write these fractions in decreasing order in your exercise book.

What should you do first to make the comparison easier? (Write the fraction as equivalent fractions with a common denominator.)

Set a time limit of 3 minutes, then review with whole class.

Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

- \(\frac{8}{10} = \frac{4}{5}\) (Lowest common multiple of 2, 4, 5 and 20 is 20)
- \(\frac{4}{5}\) > \(\frac{3}{4}\) = \(\frac{75}{100}\) > \(\frac{11}{20}\) > \(\frac{3}{6}\)
**Y6**

**Activity**

**PbY6b, page 124**

**Q.3 Read:** Practise calculation.

Deal with one at a time under a short time limit. Ps calculate in **Pbs** (or in **Ex. Bks** if they need more space) then show result on scrap paper or slates on command. Ps with correct answer explain reasoning at BB to Ps who were wrong. Who did the same? Who did it a different way? Mistakes discussed and corrected. Accept any valid method of calculation.

**Solution:** e.g.

a) \[ 1 \frac{2}{5} + 2 \frac{2}{3} + 3 \frac{4}{5} - 4 \frac{1}{2} = 2 + \frac{6}{5} + \frac{2}{3} - \frac{1}{2} = 3 + \frac{1}{5} + \frac{2}{3} - \frac{1}{2} \]
\[ = 3 + \frac{6 + 20 - 15}{30} = \frac{11}{10} = 3 \frac{1}{10} \]

b) \[ 234 \times 0.34 = 79.56 \]

c) \[ \left( \frac{34}{5} - 12.4 \right) \times 5 = (34.6 - 12.4) \times 5 = 22.2 \times 5 = 111 \]

d) \[ \left( \frac{3}{4} + 2 \frac{1}{2} \right) \times \frac{2}{5} = \frac{5}{4} \times 2 = \frac{23}{2} \times 2 = \frac{23}{10} = 2 \frac{3}{10} \]

or \[ = (3.25 + 2.5) \times 0.4 = 5.75 \times 0.4 = 2.3 \]

e) \[ \left( \frac{7}{4} + \frac{9}{5} \right) \div \frac{3}{7} = (16 + \frac{15 + 16}{20}) \times \frac{7}{3} = 16 \frac{31}{20} \times \frac{7}{3} \]
\[ = \frac{235}{20} \times \frac{7}{3} = 819 \times \frac{20}{20} = 40 \frac{19}{20} \]

f) \[ 483 \div 1.5 = 483 \div 15 = 161 \div 5 = 32.2 \]

**Lesson Plan 124**

**Notes**

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison

Discussion, reasoning, agreement, self-correction, praising

Elicit that as 2, 3 and 5 are prime numbers, their lowest common multiple is their product

BB: \[ \begin{array}{c}
\frac{2}{13} \\
\times \frac{14}{14} \\
\frac{9}{3} \\
\frac{7}{15} \\
\frac{16}{16}
\end{array} \]

Elicit that:

- calculations in brackets should be done first;
- the product of long multiplication involving decimals should have the same number of decimal digits as the two factors combined;
- to divide by a fraction, multiply by its reciprocal value.

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Feedback for T

or \[ \frac{7}{40} \times 42 = \frac{147}{5} = 29 \frac{2}{5} \]

(On average!)
Activity

b) A group of students decided to walk a distance of 24 km over 4 days.

On the first day, they walked 6 and 2 fifths km, on the second day they walked 7 and 3 eighths km and on the third day they walked 5 and 3 quarter km.

What distance did they still have to walk on the 4th day?

Plan: 24 km – (6 \( \frac{2}{5} \) + 7 \( \frac{3}{8} \) + 5 \( \frac{3}{4} \)) km

= 24 km – 18 km – \( \frac{2}{5} + \frac{3}{8} + \frac{3}{4} \) km

= 6 km – \( \frac{16 + 15 + 30}{40} \) km

= 6 km – \( \frac{61}{40} \) km = 6 km – \( \frac{21}{40} \) km = \( \frac{4}{19} \) km

Answer: On the 4th day they still had to walk \( \frac{4}{19} \) km.

c) The income of a group of 6 friends over a period of 3 weeks was £4500 in the first week, £3725.40 in the second week and £4105.50 in the third week.

What was the average income per person per week?

Plan: \( \frac{£4500 + £3725.40 + £4105.50}{3} \div 6 \)

= £12 330.90 \( \div 3 \div 6 \) = £4 110.30 \( \div 6 \) = £685.05

Answer: The average weekly income was £685.05 per person.

d) The Council has laid 12 \( \frac{1}{2} \) km of a cycle track, which is \( \frac{7}{8} \) of the planned length.

i) What length will the cycle track be when it is completed?

Plan: \( \frac{12 \frac{1}{2}}{2} \) km \( \times \frac{7}{8} \) = \( \frac{25}{2} \times \frac{4}{8} \) \( \frac{7}{7} \) km

= 14 \( \frac{2}{7} \) km

Answer: The completed cycle track will be 14 and 2 sevenths kilometres long.

ii) Next year, the Council plans to extend the cycle track by \( 2 \frac{1}{3} \) times the original length.

How long will the cycle track be then?

Plan: \( 14 \frac{2}{7} \) km \( \times 2 \frac{1}{3} \) = \( \frac{100}{7} \) km \( \times \frac{7}{3} \) = \( \frac{100}{3} \) km

= 33 \( \frac{1}{3} \) km

Answer: Next year, the cycle track will be 33 and a third kilometres long.
Factorising 125, 300, 475 and 1125. Revision, activities, consolidation

PbY6b, page 125

Solutions:

Q.1

a) \((177 - 42) \div 5\), or \((177 - 42) \div 5\)

b) \((84 + 56) \div 7\)

c) \((381 - 65) \div 4\), or \((383 - 75) \div 4\), or \((385 - 65) \div 4\), etc.

d) \((216 - 12) \div 6\), or \((216 - 18) \div 6\), or \((226 - 10) \div 6\), etc.

e) \((787 - 17) \div 10\), or \((787 - 27) \div 10\), or \((787 - 37) \div 10\), etc.

f) \((115 + 6) \div 11\), or \((125 + 7) \div 11\), or \((135 + 8) \div 11\), etc.

Q.2

a) \(1723 \ 3978 \ 1254 \ 8350 \ 1011\)

b) e.g. (but many others possible)

i) \(1724 \ 3988 \ 1244 \ 8352 \ 1012\) (divisible by 4)

ii) \(1720 \ 3975 \ 1255 \ 8450 \ 1010\) (divisible by 5)

iii) \(1722 \ 6978 \ 4254 \ 8340 \ 1014\) (divisible by 6)

iv) \(1722 \ 3976 \ 1224 \ 7350 \ 1211\) (divisible by 7)

v) \(1720 \ 3928 \ 1256 \ 8320 \ 1016\) (divisible by 8)

vi) \(1728 \ 3978 \ 1854 \ 8352 \ 1611\) (divisible by 9)

Q.3

a) \(20 \leq n \leq 60\)

b) A number which is divisible by 3 and by 4 and by 6 is also divisible by 12.

c) Ps could label each section with a letter or use different colours to identify them. (Many statements are possible.)

Q.4

a) \(4 \frac{1}{6} + \frac{1}{4} + 8 \frac{2}{3} - 11 \frac{1}{2} = 1 + \frac{2 + 3 + 8 - 6}{12} = 1 \frac{7}{12}\)

b) \(364 \times 4.36 = 1587.04\)
Q.4 c) \[ \left( \frac{3}{12} \times 12.5 \right) \times 6 = \left( \frac{5}{12} + \frac{1}{2} \right) \times 6 \]
\[ = (15 + \frac{5 + 6}{12}) \times 6 \]
\[ = 15 \frac{11}{12} \times 6 \]
\[ = 90 + \frac{66}{12} \]
\[ = 90 + 5 + \frac{6}{12} \]
\[ = 95 \frac{1}{2} \]

d) \[ \left( \frac{4}{4} + \frac{6}{3} \right) \div \frac{5}{6} = \left( \frac{10 + 9 + 4}{12} \right) \times \frac{6}{5} \]
\[ = 10 \frac{13}{12} \times \frac{6}{5} \]
\[ = 133 \frac{1}{12} \times \frac{6}{5} \]
\[ = 133 \frac{10}{10} \]
\[ = 13 \frac{3}{10} \text{ (} = 13.3) \]

e) \[ \left( \sqrt{16 \times 12} \right)^2 = \sqrt{16} \times 12 \times \sqrt{16} \times 12 \]
\[ = \sqrt{16} \times \sqrt{16} \times 12 \times 12 \]
\[ = 16 \times 144 \]
\[ = 1440 + 864 \]
\[ = 2304 \]

f) \[ 1864 + \left( \sqrt{100} \right)^2 = 1864 + 100 = 1964 \]

Q.5 a) No. of 2 litre tins needed: \[ 24 \div 2 \frac{1}{3} = 24 \div \frac{7}{3} \]
\[ = 24 \times \frac{3}{7} = \frac{72}{7} \]
\[ = 10 \frac{2}{7} \text{ (tins)} \]

Answer: Eleven 2 litre tins of paint had to be bought.

b) Tins left: \[ 11 - 10 \frac{2}{7} = \frac{5}{7}; \]

Amount left: \[ 2 \text{ litres} \times \frac{5}{7} = \frac{10}{7} \text{ litres} = 1 \frac{3}{7} \text{ litres} \]

Answer: There were 1 and 3 sevenths litres of paint left.
Lesson Plan

126

Notes

Individual work, monitored (or whole class activity)
BB: 126, 301, 476, 1126
T decides whether Ps may use calculators.
Reasoning, agreement, self-correction, praising

126 2 301 7
63 3 43 43
21 3 7 7
1 476 2 1126 2
238 2 563 563
119 7 1
17 17
1

Whole class activity
Involves several Ps.
Agreement, praising
T shows any of those opposite which Ps do not and asks class if it is correct.
Accept any valid meaning, e.g.

\[ \frac{3}{8} = \frac{1}{4} \times \frac{3}{2} \]
or \[ 1 \div 8 \times 3 \]

Extra praise for creativity!

Individual work, monitored,
helped
Some more difficult items could be done with the whole class.
Written on BB or SB or OHT
Reasoning, agreement, self-correction, praising

PY6b, page 126

Q.1 Read: Write the quotient as a fraction and as a decimal in your exercise book.
Deal with one row at a time under a time limit.
Review with whole class. Ps come to BB or dictate what T should write. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that to change a fraction to a decimal, if possible convert to an equivalent fraction with a denominator which is a whole 10 (or 100 or 1000) or divide the numerator by the denominator.

Fractions and decimals

a) Who can explain what \( \frac{3}{8} \) means?
Ps explain in different ways. Class agrees/disagrees. e.g.

\[ P_1: \text{Divide 1 unit into 8 equal parts and take 3 of the parts.} \]

\[ P_2: \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}; \quad P_3: \frac{1}{8} \times 3 = \frac{3}{8}; \quad P_4: \]

\[ P_5: \text{Divide each of 3 units into 8 equal parts and take 1 part from each unit.} \]

\[ P_6: \frac{1}{3} = \frac{3}{8}; \quad P_7: 3 \div 8 = \frac{3}{8}; \quad P_8: \frac{375}{1000}; \quad P_9: \]

\[ P_{10}: 3 \div 8 = 0.375; \quad P_{11}: \text{The ratio } 3:8 = \frac{3}{8}; \quad \text{etc.} \]

b) Who can explain what 0.81 means?
Ps explain in different ways. Class agrees/disagrees. e.g.

\[ P_{12}: 0.81 = \frac{81}{100}; \quad P_{13}: 0.81 \rightarrow 81%; \quad P_{14}: 0.81 = 81 \div 100 \]

\[ P_{15}: 0.81 = 81:100; \quad P_{16}: 0.81 = \frac{1}{100} \times 81; \quad P_{17}: 0.81 = 1 - 0.19 \]

Elicit that to change a fraction to a decimal, if possible convert to an equivalent fraction with a denominator which is a whole 10 (or 100 or 1000) or divide the numerator by the denominator.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 1 Factorisation | Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:

- \( 126 = 2 \times 3 \times 3 \times 7 = 2 \times 3^2 \times 7 \)
Factors: 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126
- \( 301 = 7 \times 43 \)
Factors: 1, 7, 43, 301
- \( 476 = 2 \times 2 \times 7 \times 17 = 2^2 \times 7 \times 17 \)
Factors: 1, 2, 4, 7, 14, 17, 28, 34, 68, 119, 238, 476
- \( 1126 = 2 \times 563 \)
Factors: 1, 2, 563, 1126

(563 is a prime number, as not divisible by 2, 3, 5, 7, 11, 13, 17, 19 and 23, and \( 29^2 > 563 \))

- 8 min |
| 2 Fractions and decimals | \( a) \) Who can explain what \( \frac{3}{8} \) means?
Ps explain in different ways. Class agrees/disagrees. e.g.

\[ P_1: \text{Divide 1 unit into 8 equal parts and take 3 of the parts.} \]

\[ P_2: \frac{1}{8} + \frac{1}{8} + \frac{1}{8} = \frac{3}{8}; \quad P_3: \frac{1}{8} \times 3 = \frac{3}{8}; \quad P_4: \]

\[ P_5: \text{Divide each of 3 units into 8 equal parts and take 1 part from each unit.} \]

\[ P_6: \frac{1}{3} = \frac{3}{8}; \quad P_7: 3 \div 8 = \frac{3}{8}; \quad P_8: \frac{375}{1000}; \quad P_9: \]

\[ P_{10}: 3 \div 8 = 0.375; \quad P_{11}: \text{The ratio } 3:8 = \frac{3}{8}; \quad \text{etc.} \]

\| | |
| b) Who can explain what 0.81 means?
Ps explain in different ways. Class agrees/disagrees. e.g.

\[ P_{12}: 0.81 = \frac{81}{100}; \quad P_{13}: 0.81 \rightarrow 81%; \quad P_{14}: 0.81 = 81 \div 100 \]

\[ P_{15}: 0.81 = 81:100; \quad P_{16}: 0.81 = \frac{1}{100} \times 81; \quad P_{17}: 0.81 = 1 - 0.19 \]

\| | |
| 3 PY6b, page 126 | White class activity
Involves several Ps.
Agreement, praising
T shows any of those opposite which Ps do not and asks class if it is correct.
Accept any valid meaning, e.g.

\[ \frac{3}{8} = \frac{1}{4} \times \frac{3}{2} \]
or \[ 1 \div 8 \times 3 \]

Extra praise for creativity!

Individual work, monitored,
helped
Some more difficult items could be done with the whole class.
Written on BB or SB or OHT
Reasoning, agreement, self-correction, praising

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Y6

Activity

3 (Continued)

Solution:

a)  
   i) \( 1 \div 2 = \frac{1}{2} = 0.5 \)
   ii) \( 3 \div 6 = \frac{3}{6} = \frac{1}{2} = 0.5 \)
   iii) \( 479 \div 958 = \frac{479}{958} = \frac{1}{2} = 0.5 \)

b)  
   i) \( 23 \div 4 = \frac{23}{4} = 5 \frac{3}{4} = 5.75 \)
   ii) \( 34.5 \div 6 = \frac{34.5}{6} = 5.75 = \frac{1}{2} = 0.5 \)
   iii) \( 479 \div 958 = \frac{479}{958} = \frac{1}{2} = 0.5 \)

c)  
   i) \( 2 \div 5 = \frac{2}{5} = 0.4 \)
   ii) \( 18 \div 5 = \frac{18}{5} = 3 \frac{3}{5} = 3.6 \)
   iii) \( 2.1 \div 5 = \frac{2.1}{5} = \frac{42}{100} = \frac{21}{50} \)

d)  
   i) \( 3 \div 16 = \frac{3}{16} = 0.1875 \)
   ii) \( 51 \div 20 = \frac{51}{20} = 2 \frac{11}{20} = 2.55 \)
   iii) \( 17 \div 80 = \frac{17}{80} = 0.2125 \)

e)  
   i) \( 2 \div 3 = \frac{2}{3} = 0.6 \) (recurring decimal)
   ii) \( 5 \div 7.5 = 10 \div 15 = \frac{10}{15} = \frac{2}{3} = 0.6 \)
   iii) \( 4 \div 9 = \frac{4}{9} = 0.4 \) (recurring decimal)

Ps can check the decimals with calculators.

Lesson Plan 126

Notes

Details: e.g.

b) ii) \( \frac{18}{4} = 4.5 \)

c) iii) \( \frac{0.4}{2} = 0.2 \)

d) i) \( \frac{1}{6} \)

Individual work, monitored, helped
Written on BB or SB or OHT
Reasoning, checking with calculators, agreement, self-correction, praising

BB: Recurring decimal
e.g. \( 0.46 = 0.4666 \ldots \)

Cyclic recurring decimal
\( 0.571428 \) or \( 0.57142857142857142 \ldots \)
e.g. \( 0.46 = 0.47 \) (to 2 d.p.)
**Solution:**

a) \( \frac{43}{64} = 0.671875 \)

b) \( \frac{89}{125} = 0.712 \)

c) \( \frac{74}{20} = \frac{36}{10} = 3.6 \)

d) \( \frac{5}{6} = 0.83 \)

e) \( \frac{14}{30} = 1.4 \div 3 = 0.46 \)

f) \( \frac{55}{36} = 1.527 \)

g) \( \frac{2}{7} = 0.285714 \) or \( 0.285714 \)

h) \( \frac{20}{35} = \frac{4}{7} = 0.571428 \) or \( 0.571428 \)

**Lesson Plan 126**

**Notes**

Q.3 Read: At the end of the Second World War in 1945, about \( \frac{11}{28} \) of the 3210 villages in Hungary had electricity.

By 1960, about 92.5% had electricity and by 1963, \( \frac{10}{10} \) had electricity.

a) How many villages had electricity in:
   i) 1945
   ii) 1960
   iii) 1963?

b) Express the numbers in 1945 and in 1963 as percentages of the total number of Hungarian villages.

First ask Ps what they know about the Second World War and about Hungary (T has some information prepared in case Ps know very little) and then ask a P to show where Hungary is on a world map.

Set a time limit. Ps write operations and do calculations in Ex. Bks.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected.

Individual work, monitored, helped

Initial whole-class brief discussion to clarify the context.

Involve as many Ps as possible.

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising
Solution:

a) i) Plan: \(3210 \times \frac{11}{28} = \frac{17655}{14} = 1261\)

Answer: In 1945, 1261 villages had electricity.

ii) Plan: \(3210 \times 0.925\)

\[ C: \begin{array}{r}
3210 \\
\times 0.925
\end{array} \]

\[ = \begin{array}{r}
2969.25
\end{array} \]

\[ \approx \begin{array}{r}
2969
\end{array} \]

Answer: In 1960, 2969 villages had electricity.

ii) Answer: In 1963, all 3210 villages had electricity.

b) In 1945: \(\frac{11}{28} \approx 0.39 \rightarrow 38\% \quad \text{In 1963:} \ \frac{10}{10} \rightarrow 100\%

\]

\[ 100\% \quad \rightarrow \quad 1800 \]

\[ \text{or} \quad 63 \div 0.035 \]

\[ = 63 \div 0.035 = \frac{63}{200} \times \frac{200}{7} = 1800 \]

Answer: That day, 1800 products were made.

b) Plan: \(1800 - 63 = 1737\)

Answer: 1737 products were not faulty.

c) In a year, the number of products will increase by about 250 times but so will the number of faulty products, so the % of faulty products will be the same.

Answer: I would expect 3.5% of products to be faulty in a year.
### Activity 7

**PbY6b, page 126, Q.5**

Read:  *The length of an aluminium cuboid is 150 cm, which is 150% of its width.*

*The height of the cuboid is \( \frac{3}{5} \) of its width.*

If the mass of 1 m\(^3\) of aluminium is 2700 kg, what is the mass of the cuboid?

Allow Ps a minute to think about it and discuss with their neighbours if they wish.

Ps who have ideas suggest what to do first and how to continue. Class agrees/disagrees or suggests better alternatives. T gives hints only if necessary (e.g. converting the dimensions to metres before doing the calculations for volume and mass).

**Solution:** e.g.

Length: 150 cm = 1.5 m, \( 150\% \rightarrow 150 \text{ cm} \)

\[ \frac{150}{100} \rightarrow 150 \text{ cm} = 1 \text{ m} \]

Height: \( \frac{3}{5} \) of 100 cm = \( \frac{3}{5} \times 100 \text{ cm} = 60 \text{ cm} = 0.6 \text{ m} \)

Volume: \( (1.5 \times 1 \times 0.6) \text{ m}^3 = (1.5 \times 0.6) \text{ m}^3 = 0.9 \text{ m}^3 \)

Mass: \( 2700 \text{ kg} \times 0.9 = 270 \text{ kg} \times 9 = 2430 \text{ kg} \)

**Answer:** The mass of the aluminium cuboid is 2430 kg.

**Notes**

Whole class activity
(or individual trial first if Ps wish and there is time)

Show a model or draw a diagram of a cuboid on BB.

Involve several Ps.
Discussion, reasoning, agreement, praising
Extra praise for Ps who suggest this without hint from T.

Class says the answer in a sentence in unison.

---

45 min
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1</strong></td>
<td><strong>Lesson Plan</strong></td>
</tr>
<tr>
<td><strong>Factorisation</strong></td>
<td><strong>127</strong></td>
</tr>
<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 5 minutes.</td>
<td></td>
</tr>
<tr>
<td>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td>Elicit that:</td>
<td></td>
</tr>
<tr>
<td>• 127 is a prime number Factors: 1, 127 (as not divisible by 2, 3, 5, 7, 11, and 13² &gt; 127)</td>
<td></td>
</tr>
<tr>
<td>• 302 = 2 × 151 Factors: 1, 2, 151, 302</td>
<td></td>
</tr>
<tr>
<td>• 477 = 3 × 3 × 53 = 3² × 53 Factors: 1, 3, 9, 53, 159, 477</td>
<td></td>
</tr>
<tr>
<td>• 1127 = 7 × 7 × 23 = 7² × 23 Factors: 1, 7, 23, 49, 161, 1127</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2</strong></td>
<td><strong>Solving inequalities</strong></td>
</tr>
<tr>
<td>Which side is greater? How much greater? Ps come to BB to fill in missing signs and explain reasoning. Who agrees? Who thinks something else? Ps give examples or counter examples to support what they think.</td>
<td></td>
</tr>
<tr>
<td>BB:</td>
<td></td>
</tr>
<tr>
<td>a) ( a + 3 \overset{&lt;}{\underset{+ 2}{\sim}} a + 5 ) (e.g. ( a = 10: \ 10 + 3 &lt; 10 + 5 ))</td>
<td></td>
</tr>
<tr>
<td>(Elicit that the sign is also correct if ( a = 0 ) and if ( a ) is negative.)</td>
<td></td>
</tr>
<tr>
<td>b) ( b \times 3 \overset{&lt;}{\underset{\text{if } b &gt; 0, \ i.e. \ b \text{ is positive}}{\sim}} b \times 5 )</td>
<td></td>
</tr>
<tr>
<td>e.g. if ( b = 2: \ 2 \times 3 &lt; 2 \times 5 )</td>
<td></td>
</tr>
<tr>
<td>or if ( b = \frac{2}{5}: \ \frac{2}{5} \times 3 &lt; \frac{2}{5} \times 5 )</td>
<td></td>
</tr>
<tr>
<td>(1 \frac{1}{5}) (2)</td>
<td></td>
</tr>
<tr>
<td>but if ( b ) is a negative number, ( b \times 3 \overset{&gt;}{\underset{\text{if } b &lt; 0}{\sim}} b \times 5 )</td>
<td></td>
</tr>
<tr>
<td>e.g. if ( b = (−4): \ (−4) \times 3 &gt; (−4) \times 5 )</td>
<td></td>
</tr>
<tr>
<td>(−12) (−20)</td>
<td></td>
</tr>
<tr>
<td>or if ( b = 0: \ b \times 3 \overset{=}{{\underset{(0)}{\sim}}} b \times 5 ) (0)</td>
<td></td>
</tr>
<tr>
<td>c) ( c \times 0 \overset{\text{Elicit that the missing sign could be } &lt;, =, \text{ or } &gt;.}{\underset{c}{\sim}} )</td>
<td></td>
</tr>
<tr>
<td>e.g ( c \times 0 \overset{&lt;}{\underset{\text{if } c \text{ is a positive number}}{\sim}} c )</td>
<td></td>
</tr>
<tr>
<td>or ( c \times 0 \overset{=}{{\underset{\text{if } c = 0}{\sim}}} c ) (if ( c = 0 ))</td>
<td></td>
</tr>
<tr>
<td>or ( c \times 0 \overset{&gt;}{\underset{\text{if } c \text{ is a negative number}}{\sim}} c ) (if ( c = 0 ))</td>
<td></td>
</tr>
<tr>
<td>e.g. if ( c = −2: \ (−2) \times 0 \overset{&gt;}{\underset{(0)}{\sim}} −2 )</td>
<td></td>
</tr>
</tbody>
</table>
**Activity**

3  
**PbY6b, page 127**

Q.1 Read: *Write the numbers in increasing order.*

Set a time limit of 2 minutes. Ps work in *Ex. Bks.*

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Ask Ps to show their approximate positions on a number line drawn on BB.

**Solution:**

a) \(0.8, \frac{2}{3}, -0.9, \frac{1}{2}, \frac{4}{5}, -\frac{3}{5}\)

\[
\begin{align*}
\frac{24}{30} &< \frac{27}{30} < \frac{15}{30} < \frac{24}{30} < \frac{18}{30} \\
-0.9 &< -\frac{3}{5} < \frac{1}{2} < \frac{2}{3} < 0.8 = \frac{4}{5}
\end{align*}
\]

b) \(2\frac{4}{5}, \frac{3}{4}, -\frac{1}{2}, \frac{4}{6}, -\frac{3}{2}\)

\[
\begin{align*}
-\frac{3}{2} &< -\frac{1}{2} < \frac{4}{6} < \frac{3}{4} < \frac{2}{5}
\end{align*}
\]

**Notes**

- Individual work, monitored (helped)
- Written on BB or SB or OHT
- Reasoning, agreement, self-correction, praising
- Feedback for T or accept comparison of one pair at a time, e.g.
  \(-0.9 = -\frac{9}{10} < -\frac{3}{5} = -\frac{6}{10}\)

Extra praise for Ps who realised that the ordering in b) can be done without changing all the fractions to equivalent fractions. We need only compare:

\[
\frac{3}{4} = \frac{9}{12} > \frac{4}{6} = \frac{8}{12}
\]

---

4  
**PbY6b, page 127**

Q.2 Read: a) *Round 7812 529 to the nearest:*

   i) 10  ii) 100  iii) 1000  iv) 1 000 000.

b) *Round 5.465 to the nearest:*

   i) unit  ii) tenth  iii) hundredth.

Set a short time limit. Ps work in *Ex. Bks.*

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) i) 7812 529 \(=\) 7812 530 (to the nearest 10)

ii) 7812 529 \(=\) 7812 500 (to the nearest 100)

iii) 7812 529 \(=\) 7813 000 (to the nearest 1000)

iv) 7812 529 \(=\) 8 000 000 (to the nearest 1 000 000)

b) i) 5.465 \(=\) 5 (to the nearest 1)

ii) 5.465 \(=\) 5.5 (to the nearest 10th, or to 1 d.p.)

iii) 5.465 \(=\) 5.47 (to the nearest 100th, or to 2 d.p.)

**Notes**

- Individual work, monitored, (helped)
- Numbers written on BB or SB or OHT
- Differentiation by time limit
- Reasoning, agreement, self-correction, praising
- Elicit/remind Ps that:
  - the digit ’5’ rounds up to the next greater place value
  - when rounding a number, round the whole number, not one digit at a time.

Feedback for T
Activity

5 PbY6b, page 127

Q.3 Read: Solve these equations.

Set a time limit or deal with one row at a time. Ps write operations in Ex. Bks and check results by substituting the value for the letter in each equation to see whether it is true.

Review with the whole class. Ps could show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class checks mentally and agrees on the correct answer. Mistakes discussed and corrected.

Solution:

a) \[2.75 + a = 7.1, \quad a = 7.1 - 2.75 = 4.35\]

b) \[b + \frac{2}{7} = 1 \frac{4}{5}, \quad b = 1 \frac{4}{5} - \frac{2}{7} = 1 + \frac{28 - 10}{35} = 1 \frac{18}{35}\]

c) \[c - 8.02 = 3.8, \quad c = 3.8 + 8.02 = 11.82\]

d) \[5 - d = 3 \frac{5}{8}, \quad d = 5 - 3 \frac{5}{8} = 2 - \frac{5}{8} = 1 \frac{3}{8}\]

e) \[7.2 \times e = 36, \quad e = 36 \div 7.2 = 360 \div 72 = 5\]

f) \[f \div 4.2 = 10.5, \quad f = 10.5 \times 4.2 = 42 + 2.1 = 44.1\]

g) \[\frac{4}{3} \div g = \frac{2}{5}, \quad g = \frac{4}{3} \div \frac{2}{5} = \frac{2}{3} \times \frac{5}{2} = \frac{10}{3} = 3 \frac{1}{3}\]

h) \[\frac{5}{6} \div h = 0, \quad \text{There is no possible value for } h.\]

i) \[\frac{72}{i} = 1.2, \quad i = 72 \div 1.2 = 720 \div 12 = 60\]

26 min

6 PbY6b, page 127

Q.4 Deal with one question at a time under a time limit.

Ps read questions themselves, write a plan, estimate, calculate and check the result and write the answer in a sentence in Ex. Bks.

Review with the whole class. T chooses a P to read out the question and Ps show results on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Who agrees? Who did it a different way? Mistakes discussed and corrected.

Solutions:

a) James had a 6.25 m length of wire. He used 125 cm one day, then he used 1.6 m on the next day, then \[2 \frac{1}{2} \text{ m on the day after that}. \quad \text{How much wire was left?}\]

Plan: \[6.25 \text{ m} - (1.25 + 1.6 + 2.5) \text{ m}\]

\[= 6.25 \text{ m} - 5.35 \text{ m} = 0.9 \text{ m} = 90 \text{ cm}\]

Answer: James had 90 cm of wire left.

Lesson Plan 127

Notes

Individual work, monitored, helped
Differentiation by time limit

Responses shown in unison.
Reasoning, agreement, self-correction, praising

Also ask Ps to give the general method for finding the missing component. T reminds Ps of the names of the components where necessary. e.g.

a) and b): to find the unknown term in a 2-term addition, subtract the known term from the sum.

c): to calculate the reductant, add the difference to the subtrahend etc.

(as there is no value which can multiply zero to make 5 sixths)
Activity 6  
(Continued)  
Solutions:  

b) The sides of a rectangular park are 800 m and $1 \frac{1}{4}$ km long.  
   What is: i) the perimeter of the park  
   ii) the area of the park?  
   i) Plan:  
   \[ P = 2 \times (800 + 1250) \text{ m} \]  
   \[ = 2 \times 2050 \text{ m} = 4100 \text{ m} = 4.1 \text{ km} \]  
   Answer: The perimeter of the park is 4.1 kilometres.  
   ii) Plan:  
   \[ A = (0.8 \times 1.25) \text{ km}^2 = 1 \text{ km}^2 \]  
   or  
   \[ A = \left( \frac{8}{10} \times \frac{5}{4} \right) \text{ km}^2 = \frac{2}{2} \text{ km}^2 = 1 \text{ km}^2 \]  
   Answer: The area of the park is one square kilometre.  

c) Calum has 45 stamps. Vanessa has $\frac{8}{9}$ of that number and George has 120% of that number.  
   How many stamps do Vanessa and George each have?  
   Plan:  
   \[ V: 45 \div \frac{8}{9} \times 8 = 5 \times 8 = 40 \text{ (stamps)} \]  
   G: \[ 45 \div 100 \times 120 = 4.5 \times 12 = 54 \text{ (stamps)} \]  
   Answer: Vanessa has 40 stamps and George has 54 stamps.

Notes  

BB:  

\[ \begin{array}{c} 800 \text{ m} \\ \frac{1}{4} \text{ km} = 1250 \text{ m} \end{array} \]  

or  

\[ P = 2 \times (0.8 + 1.25) \text{ km} \]  

\[ = 2 \times 2.05 \text{ km} \]  

\[ = 4.1 \text{ km} \]  

or  

\[ A = (800 \times 1250) \text{ m}^2 \]  

\[ = (8 \times 125 000) \text{ m}^2 \]  

\[ = 1 000 000 \text{ m}^2 \]  

\[ = 1 \text{ km}^2 \]  

BB:  

\[ \begin{array}{c} 800 \text{ m} \\ \frac{1}{4} \text{ km} = 1250 \text{ m} \end{array} \]  

or  

\[ P = 2 \times (0.8 + 1.25) \text{ km} \]  

\[ = 2 \times 2.05 \text{ km} \]  

\[ = 4.1 \text{ km} \]  

or  

\[ A = (800 \times 1250) \text{ m}^2 \]  

\[ = (8 \times 125 000) \text{ m}^2 \]  

\[ = 1 000 000 \text{ m}^2 \]  

\[ = 1 \text{ km}^2 \]  

or  

\[ V: 45 \div 9 \times 8 = 5 \times 8 = 40 \text{ (stamps)} \]  

G: \[ 45 \div 100 \times 120 \]  

\[ = 4.5 \times 12 = 54 \text{ (stamps)} \]  

Individual work, monitored, helped  

Responses shown in unison.  
Reasoning, agreement, self-correction, praising  
Accept any valid method of solution.  
Feedback for T  

or  

\[ 650 \div 2 - 6 = 325 - 6 \]  

\[ = 319 \]  

C:  

\[ \begin{array}{c} 4 \text{.}5 \\ \times 0 \text{.}8 \text{.}3 \\ \hline 1 \text{.}3 \text{.}5 \end{array} \]  

\[ + 3 \text{.}6 \text{.}0 \text{.}0 \]  

\[ \hline 3 \text{.}7 \text{.}3 \text{.}5 \]  

(kg)
Solutions:

c) The weight of 1 cm$^3$ of steel is 300% of the weight of 1 cm$^3$ of aluminium.

i) What is the ratio of the weight of a 25 cm$^3$ aluminium cuboid and that of a 25 cm$^3$ steel cuboid?

Plan: $a : s = 100 : 300 = 1 : 3$

Answer: The ratio of the weights of the aluminium and steel cuboids is 1 to 3.

ii) What is the mass of the aluminium cuboid if the steel cuboid's is 202.5 g?

Plan: $M_a = 202.5 \div 3 = 67.5$ g

Answer: The mass of the aluminium cuboid is 67.5 g.

iii) How many grams is 1 cm$^3$ of steel?

Plan: $25 \times 202.5 \div 25 = 8.1$ g

Answer: One cm$^3$ of steel weighs 8.1 g.

iv) How many grams is 1 cm$^3$ of aluminium?

Plan: $25 \times 67.5 \div 25 = 2.7$ g

Answer: One cm$^3$ of aluminium weighs 2.7 g.

T: We say that the density of steel is 8.1 g/cm$^3$ and the density of aluminium is 2.7 g/cm$^3$.

45 min
Y6

R:  Calculations
C:  Choosing and using appropriate operations to solve problems
E:  Problems involving proportion

Activity

1  

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^7\)
  Factors: 1, 2, 4, 8, 16, 32, 64, 128

- \(303 = 3 \times 101\) Factors: 1, 3, 101, 303

- \(478 = 2 \times 239\) Factors: 1, 2, 239, 478

- \(1128 = 2 \times 2 \times 2 \times 3 \times 47 = 2^3 \times 3 \times 47\)
  Factors: 1, 2, 3, 4, 6, 8, 12, 24, 1128, 564, 376, 282, 188, 141, 94, 47

[Number of factors: \(7 + 1 = 8\)]

2  

Operations

Let’s fill in the missing items in this table about the 4 operations.

Ps come to BB to write and say what is missing. Class agrees/disagrees. After each operation, T asks Ps to write examples using integers, fractions and decimals. Elicit that in addition and multiplication the order of the terms does not matter. (commutative)

BB:

<table>
<thead>
<tr>
<th>Operation</th>
<th>Name:</th>
<th>Rule:</th>
<th>Result:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a \times b = c)</td>
<td>Multiplication</td>
<td>missing factor = product ÷ known factor</td>
<td>Product</td>
</tr>
<tr>
<td>(a \div b = c)</td>
<td>Division</td>
<td>dividend ÷ divisor = quotient</td>
<td>Quotient</td>
</tr>
<tr>
<td>(a + b = c)</td>
<td>Addition</td>
<td>missing term = sum – known term</td>
<td>Sum</td>
</tr>
<tr>
<td>(a - b = c)</td>
<td>Subtraction</td>
<td>reductant = difference + subtrahend</td>
<td>Difference</td>
</tr>
</tbody>
</table>

Lesson Plan

128

Notes

Individual work, monitored (or whole class activity)

BB: 128, 303, 478, 1128

T decides whether Ps may use calculators.

Reasoning, agreement, self-correction, praising

e.g.

<table>
<thead>
<tr>
<th>128</th>
<th>2</th>
<th>3</th>
<th>64</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1128</td>
<td>32</td>
<td>2</td>
<td>1128</td>
<td>2</td>
</tr>
<tr>
<td>303</td>
<td>3</td>
<td>3</td>
<td>101</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>564</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>478</td>
<td>2</td>
<td>282</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>141</td>
<td>2</td>
<td>141</td>
<td>3</td>
<td></td>
</tr>
<tr>
<td>239</td>
<td>2</td>
<td>239</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>47</td>
<td>47</td>
<td></td>
</tr>
</tbody>
</table>

8 min

Whole class activity

Written on BB or use enlarged copy master or OHP

(Ps could have copies on desks too and stick in Ex. Bks when completed.)

Agreement, praising

Ps write examples of operations on BB and choose other Ps to do the calculation.

(T could write more difficult examples for very able Ps, e.g. using negative numbers)

At a good pace.

In good humour!

Praising, encouragement only

or use ‘multiplicand’ (a) and ‘multiplier’ (b)

Feedback for T
**PbY6b, page 128**

Q.1 Read: 84% of an apple is water.

a) How much water is in these quantities of apples?
   - i) 1 kg
   - ii) 2 kg
   - iii) 5 kg
   - iv) \(\frac{3}{2}\) kg
   - v) 0.4 kg

b) What amount of apples contains these quantities of water?
   - i) 420 g
   - ii) 2.52 kg

Set a time limit or deal with part a) then part b). Ps work in Ex. Bks.

Review with whole class. Ps come to BB or dictate what T should write. Who agrees? Who did it another way? Who made a mistake? What was your mistake? Make sure that you have corrected it.

**Solution:** e.g.

a) i) 1 kg of apples contains 0.84 kg of water
   - ii) 2 kg of apples \(\rightarrow 0.84 \times 2 = 1.68\) kg of water
   - iii) 5 kg of apples \(\rightarrow 0.84 \times 5 = 4.2\) kg of water
   - iv) \(\frac{3}{2}\) kg of apples \(\rightarrow 0.84 \times 3.5 = 2.94\) kg (of water)
   - v) 0.4 kg of apples \(\rightarrow 0.84 \times 0.4 = 0.336\) kg (= 336 g) of water

b) i) 420 g of water \(\rightarrow 420 \div 0.84 = 42000 \div 84 = 500\) g (of apples)
   - ii) 2.52 kg of water \(\rightarrow 2.52 \div 0.84 = 252 \div 84 = 3\) kg (of apples)

Q.2 Read: Two fifths of a garden had already been landscaped.

Five gardeners were employed to complete the job.

If they shared the remaining work equally, what part of the whole garden were they each responsible for?

I will give you 3 minutes to solve this problem. Start . . . now!

Stop! If you have an answer, show me it . . . now! \(\frac{3}{25}\)

A, come and tell us how you got your answer. Who did the same? Who did it another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

Part of garden still to be landscaped: \(1 - \frac{2}{5} = \frac{3}{5}\)

Part to be landscaped by each gardener: \(\frac{3}{5} \div 5 = \frac{3}{25}\)
Lesson Plan 128

Notes

Individual work, monitored, helped.
If you had 4 hours free time one afternoon, what would you like to do?
T asks several Ps.

Differentiation by time limit.
Responses shown in unison.
Reasoning, agreement, self-correction, praising.
Accept any valid method (e.g. working out how much time Charlie spent on each activity) but also show the solutions given opposite.

Activity

5  
PbY6b, page 128
Q.3 Read: Charlie spent his time between 2 o’clock and 6 o’clock in the afternoon doing different things.

He went shopping for \( \frac{2}{5} \) of the time, played with a friend for \( \frac{1}{4} \) of the time and read a book for \( \frac{1}{6} \) of the time.

a) What part of the time did Charlie spend doing other activities?

b) How many minutes did Charlie spend on other activities?

Set a time limit of 4 minutes. Ps work in Ex. Bks.
Review with whole class. Ps show results on scrap paper or slates on command. Ps answering correctly come to BB to explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Solution: e.g.

\( a) \) Plan: \( 1 - \left( \frac{2}{5} + \frac{1}{4} + \frac{1}{6} \right) = 1 - \frac{24 + 15 + 10}{60} = 1 - \frac{49}{60} = \frac{11}{60} \)

\( \text{Answer:}\) Charlie spent 11 sixtieths of the time doing other activities.

\( b) \) Plan: \( 6 \text{ h} - 2 \text{ h} = 4 \text{ h}; \)

\( \frac{11}{60} \text{ of } 4 \text{ h} = \frac{11}{60} \times \frac{4}{1} \times 240 \text{ min} = 44 \text{ min} \)

\( \text{Answer:}\) Charlie spent 44 minutes on other activities.

29 min

6  
PbY6b, page 128. Q.4
Read: When experiments in television broadcasting first began in 1923, scientists could only transmit images across a distance of 2.5 metres.

Which two things in the classroom are 2.5 m apart? Ps make suggestions then other Ps check with a tape measure. Class applauds the nearest estimate.

Read: Some years later, a Hungarian engineer, Denes Mihaly, who was working in Berlin in Germany, managed to transmit images across a distance of 1000 m.

Which places are about 1000 m from the school? Ps suggest some and class agrees/disagrees. Elicit that 1000 m = 1 km.
T chooses Ps to read one question at a time. Allow Ps time to think and calculate, then Ps show answers on command. Ps with different answers explain reasoning at BB. Class agrees on the correct answer. Ps write agreed answers beside questions in Pbs.

Solution:

\( a) \) How many times more is 1000 m than 2.5 m? (400)

\( b) \) What percentage is 1000 m of 2.5 m? (40 000%)

\( c) \) Write their ratio with whole numbers. (1000 : 2.5 = 400 : 1)

Whole class activity

or T has a 2.5 metre length of string prepared to quickly check Ps’ estimates.

T should have some places already in mind in case Ps’ have no idea or are very inaccurate.

Responses shown in unison.
Reasoning, agreement, praising.

\( a) \) \( 1000 \div 2.5 = 2000 \div 5 = 400 \)

\( b) \) \( 400 \times 100\% = 40 000\% \)
PbY6b, page 128

Q.5 Read: Emma bought shares in the stock market for £100 000 but very soon their value began to fall. To avoid losing too much money, she sold half of her shares at a 15% loss.

Two weeks later, the value of her shares rose again and reached a level which was 20% more than the amount she had paid for them. She then sold the rest of her shares.

How much profit or loss did she make on the shares?

First talk about the stock market to clarify the context. Ps say what they know and T has information prepared in case Ps know very little.

Set a time limit. Ps solve problem in Ex. Bks and write the answer in a sentence.

Review with whole class. Ps show results on scrap paper or slates on command. Ps with different answers explain reasoning. Class points our errors and agrees on correct answer. Who had the correct answer but did it a different way? Mistakes discussed and corrected.

Solution: e.g.

Value of shares bought: £100 000

Loss made on 1st sale: £50 000 × 0.15 = £7 500

Profit made on 2nd sale: £50 000 × 0.2 = £10 000

Difference: £10 000 – £7500 = £2500

Answer: Overall, Emma made a profit of £2500.

40 min

PbY6b, page 128, Q.6

Read: \( \frac{2}{5} \) of Tom's money is the same as \( \frac{3}{4} \) of Frank's money.

a) If Frank has £220, how much does Tom have?

b) What ratio is:

i) Tom's to Frank's money

ii) Frank's to Tom's money?

Allow Ps a minute to think about it and discuss with their neighbours.

Ps suggest what to do first and how to continue. Class agrees/disagrees or makes alternative suggestions. T helps only if necessary.

Ps could write a solution in Ex. Bks too.

Solution: e.g.

a) \( \frac{2}{5} \) of \( T = \frac{3}{4} \) of £220 = £220 ÷ 4 × 3 = £55 × 3 = £165

Tom has: £165 ÷ 2 × 5 = £82.50 × 5 = £412.50

b) i) \( T : F = 412.5 : 220 \) (= 15 : 8)

[T shows: \( T \times \frac{2}{5} = F \times \frac{3}{4} \),

\( \frac{T}{F} = \frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \frac{5}{2} = \frac{15}{8} \)]

ii) \( F : T = 220 : 412.5 \) (= 8 : 15)

So \( T : F = 15 : 8, \ F : T = 8 : 15 \]

45 min

40 min

Individual work, monitored, helped

Initial whole-class brief discussion on the stock market.

Some Ps or T might have their own shares or T could show share values in a newspaper or on the internet.

Differentiation by time limit.

Responses shown in unison.

Reasoning, agreement, self-correction, praising or

Income from the 2 sales:

£50 000 × 0.85 = £42 500

£50 000 × 1.2 = £60 000

£42 500 + £60 000 = £102 500

Profit: £102 500 – £100 000 = £2500

Whole class activity

(or individual trial first if Ps wish and there is time)

Drawn on BB or SB or OHT

BB: Tom

\[ \begin{align*}
\text{Frank} &= \frac{3}{4} \\
\text{Tom} &= \frac{5}{4} \\
\text{T} &= \frac{200}{412.5} \\
\end{align*} \]

Discussion, reasoning, agreement, praising or

Involve several Ps.

Extra praise for a plan in one line for a):

\( \frac{55}{412.5} \times \frac{3}{4} \div \frac{2}{5} = \frac{5}{2} \times \frac{825}{2} \)

Check: \( = \frac{412.50}{2} \)

412.5 : 220 = 825 : 440

= 165 : 88 = 15 : 8

45 min

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### Lesson Plan

#### Y6

**R:** Calculations

**C:** Revision: fractions, decimals and percentages

**E:** Problems

#### Activity

**1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(129 = 3 \times 43\)
  - Factors: 1, 3, 43, 129
- \(304 = 2 \times 2 \times 2 \times 2 \times 2 \times 19 = 2^4 \times 19\)
  - Factors: 1, 2, 4, 8, 16, 19, 38, 76, 42, 304
- \(479\) is a prime number
  - Factors: 1, 479
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and 23^2 > 479)
- \(1129\) is a prime number
  - Factors: 1, 1129
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and 37^2 > 1129)

**2**

**Review of addition**

T writes an addition on B and Ps dictate the sum. e.g. \(12.3 + 4.5 = 16.8\)

Let's use this result to help us calculate the sum if we:

- increase one term
  - e.g. \((12.3 + 2) + 4.5 = 16.8 + 2 = 18.8\)
  - or \((12.3 + 4.5 + 0.2) = 16.8 + 0.2 = 17\)
  - or \((12.3 + a) + 4.5 = 12.3 + (4.5 + a) = 16.8 + a\)
- increase both terms
  - e.g. \((12.3 + 1) + (4.5 + 2) = 16.8 + 1 + 2 = 19.8\)
  - or \((12.3 + a) + (4.5 + b) = 16.8 + a + b\)
- decrease one term
  - e.g. \((12.3 - 0.3) + 4.5 = 16.8 - 0.3 = 16.5\)
  - or \((12.3 + 4.5 - 4) = 16.8 - 4 = 12.8\)
  - or \((12.3 - a) + 4.5 = 12.3 + (4.5 - a) = 16.8 - a\)
- decrease both terms
  - e.g. \((12.3 - 1.3) + (4.5 - 1) = 16.8 - (1.3 + 1) = 16.8 - 2.3 = 14.5\)
  - or \((12.3 - a) + (4.5 - b) = 16.8 - (a + b)\) or \(16.8 - a - b\)
- increase one term and decrease the other term
  - e.g. \((12.3 + 3) + (12.3 - 3) = 16.8 + 3 - 3 = 16.8\)
  - or \((12.3 - 1) + (4.5 + 0.5) = 16.8 - 1 + 0.5 = 16.3\)
  - or \((12.3 + a) + (12.3 - b) = 16.8 + a - b\)

### Notes

Individual work, monitored (or whole class activity)

BB: 129, 304, 479, 1129

T decides whether Ps may use calculators.

Reasoning, agreement, self-correction, praising

\[
\begin{array}{c|c|c|c|c}
129 & 3 & 304 & 2 \\
43 & 43 & 152 & 2 \\
19 & 19 & 38 & 2 \\
1 & 1 & 76 & 2 \\
\end{array}
\]

Whole class activity

Written on BB or SB or OHT

Ps come to BB or dictate what T should write.

Class points out errors.

At a good pace

Agreement, praising

Elicit generalisations after each type. Ps dictate what T should write. Class agrees/disagrees.

Ps point out what they have noticed, e.g.

If we increase (decrease) one of the terms in a 2-term addition by a certain amount, the sum also increases (decreases) by that amount.

If we increase one term and decrease the other term in a 2-term addition by the same amount, the sum does not change.

Etc.
### Activity

#### PbY6b, page 129

**Q.1 Read:** Do the multiplications.

- Set a time limit. Ask Ps to give the results as decimals too. Allow Ps to use calculators.
- Review with whole class. Ps come to BB or dictate what T should write. Class agrees/disagrees. Mistakes corrected.

**Solution:**

- \( a) \quad \frac{1}{7} \times \frac{2}{7} \times \frac{3}{7} \times \frac{4}{7} \times \frac{5}{7} = \frac{720}{117649} = 0.00612 \)
- \( b) \quad \frac{1}{2} \times \frac{1}{3} \times \frac{4}{5} \times \frac{1}{7} = \frac{1}{7} = 0.142857 = 0.143 \)
- \( c) \quad \frac{1}{9} \times \frac{7}{8} \times \frac{3}{5} \times \left( \frac{1}{6} \right) \times \left( \frac{5}{16} \right) = \frac{-1 \times 1 \times 1 \times 1 \times 1}{3 \times 2 \times 1 \times 1 \times 1} = -\frac{1}{6} = 0.16 (= 0.167) \)

Ask Ps to say to how many decimal digits (or places) they have rounded. 

20 min

---

#### PbY6b page 129

**Q.2 Read:** Solve the equation then check your result.

- How can you check your result? (Substitute the result for the letter in the equation and check that the equation is true.)
- Set a time limit. Ps work in Pbs (or in Ex. Bks. if they need more space).
- Review with whole class. Ps could show results on scrap paper or slates on command. Ps answering correctly explain at BB to Ps who were wrong. Class checks that they are correct. Mistakes discussed and corrected.

**Solution:**

- \( a) \quad \left( x + \frac{1}{5} \right) + 6 = 10, \quad x = 10 - 6 - \frac{4}{5} = 4 - \frac{4}{5} = 2 \frac{1}{5} \)
  
  \( \text{Check: } 2 \frac{1}{5} + \frac{4}{5} + 6 = 4 + 6 = 10 \checkmark \)

- \( b) \quad \frac{3}{5} \times y = \frac{13}{7}, \quad y = \frac{13}{7} \times \frac{5}{3} = \frac{13}{7} \times \frac{5}{3} = \frac{65}{21} = \frac{5}{21} \)

  \( \text{Check: } 2 \frac{3}{5} \times \frac{5}{7} = \frac{13}{5} \times \frac{5}{7} = \frac{13}{7} \checkmark \)

- \( e) \quad z \div 4 = \frac{1}{4}, \quad z = 3 \frac{1}{4} \times 4 = \frac{13}{4} \times 4 = 13 \)

  \( \text{Check: } 13 \div 4 = \frac{13}{4} = 3 \frac{1}{4} \checkmark \)

25 min
**Activity**

5

*PbY6b, page 129*

Q.3 Read: *Calculate in your exercise book.*

Set a time limit. Ps can use any method of calculation.

Review with whole class. T asks 3 or 4 Ps for their answers, then chooses Ps to show their calculations on BB. Class agrees or disagrees. Who calculated in a different way? Come and show us. Mistakes discussed and corrected.

**Solution:**

a) 0.7 of 415: \( 0.7 \times 415 = 290.5 \)

b) 1.43 of 19: \( 1.43 \times 19 = 27.17 \)

c) 3% of 34.2: \( 34.2 \times 0.03 = 1.026 \)

d) 69% of 5500: \( 5500 \times 0.69 = 3795 \)

e) 210% of 46.1: \( 46.1 \times 2.1 = 99.81 \)

**Notes**

Individual work, monitored, (helped)

Reasoning, agreement, self-correction, praising

Elicit that 'of' means multiply.

Feedback for T

30 min

---

6

*PbY6b, page 129*

Q.4 Ps read question themselves and do calculations in *Ex. Bks.* under a time limit.

Review with whole class. T asks 3 or 4 Ps for their answers, then chooses Ps to show their calculations on BB. Class agrees or disagrees. Who calculated in a different way? Come and show us. Mistakes discussed and corrected.

**Solution:**

What is the number if:

a) \( \frac{3}{10} \) of it is 28.5: \( 28.5 \div 0.3 = 285 \div 3 = 95 \)

b) 2.5 of it is 8260: \( 8260 \div 2.5 = 82600 \div 25 = 3304 \)

c) 12% of it is 58.2: \( 58.2 \div 0.12 = 5820 \div 12 = 485 \)

d) 99% of it is 346.5: \( 346.5 \div 0.99 = 34650 \div 99 = 350 \)

e) 250% of it is 8260? \( 8260 \div 2.5 = 3304 \) [same as b]

**Notes**

Individual work, monitored

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Accept any valid method of calculation. e.g.

a) \( 28 \frac{1}{2} \div \frac{3}{10} = \frac{57}{2} \times \frac{5}{3} = 95 \)

or \( 28.5 \div 3 \times 10 = 9.5 \times 10 = 95 \)

Feedback for T

Extra praise if Ps noticed this.
### Activity

**PbY6b, page 129**

**Q.5 Read:** The lengths of the sides of a rectangle are 40 cm and 60 cm. One of the sides of a second rectangle is 110% of one of the sides of the first rectangle. The adjacent side of the second rectangle is 1.1 times as long as the adjacent side of the first rectangle.

What percentage of the area of the first rectangle is the area of the second rectangle?

Allow Ps half a minute to think about it and discuss it with their neighbours if they wish.

What should we do first? (Calculate the sides of the second rectangle.) Then what should we do? (Calculate the areas of the two rectangles, then write the ratio of the 2nd area to the 1st area as a fraction, then as a percentage.)

Ps carry out each step of the solution in Ex. Bks., drawing diagrams to help them and writing the answer as a sentence.

Review with whole class. Ps could show result on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Mistakes discussed and corrected.

**Solution:** e.g.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>a = 40 cm</td>
<td>c = 110% of a</td>
</tr>
<tr>
<td>b = 60 cm</td>
<td>d = 1.1 \times b</td>
</tr>
</tbody>
</table>

Rectangle 2: 
\[
\begin{align*}
    c &= 40\text{ cm} \times 1.1 = 44\text{ cm} \\
    d &= 60\text{ cm} \times 1.1 = 66\text{ cm}
\end{align*}
\]

\[
A_2 = (44 \times 66) \text{ cm}^2 = 2904 \text{ cm}^2
\]

\[
A_2 = \frac{2904}{2400} = 2904 \div 2400 = 29.04 \div 24 = 2.42 \div 2 = 1.21 \rightarrow 121\%
\]

**Answer:** The area of the second rectangle is 121% of the area of the first rectangle.

---

**Notes**

Individual work, monitored, helped

Initial whole-class discussion and agreement on the steps needed to solve the problem.

T gives hints if necessary.

Agreement, praising

Responses shown n unison.

Reasoning, agreement, self-correction, praising

or sides can be labelled a' and b'

\[
\begin{align*}
    c &= 40\text{ cm} \times 1.1 = 44\text{ cm} \\
    d &= 60\text{ cm} \times 1.1 = 66\text{ cm}
\end{align*}
\]

or \[
A_2 : A_1 = 2904 : 2400 \quad \frac{2904}{2400} = 1.21
\]
Q.6 Read: The perpendicular sides of a right-angled triangle are:
\[ a = 10 \text{ cm}, \quad b = 6.2 \text{ cm} \]

If we cut 20% off side a and shorten side b to \( \frac{4}{5} \) of its length a second triangle is formed.

a) Calculate the area of both triangles.

b) What percentage of the area of the 1st triangle is the area of the 2nd triangle?

c) What percentage smaller than the 1st triangle is the area of the 2nd triangle?

This is the same type of question as Q.5 except that it is about areas of triangles rather than rectangles.

How do you calculate the area of a right-angled triangle? (Half the length of its base multiplied by its height)

T chooses Ps to work on BB while rest of Ps work in Ex. Bks.

T monitors everyone closely, especially Ps at BB, helping, correcting. Class also points out any errors they see.

Review quickly with whole class. Ps at BB explain what they have done and class agrees/disagrees. Mistakes corrected.

**Solution:**

**BB:**

\[ b = 6.2 \text{ cm} \]

\[ a = 10 \text{ cm} \]

\[ b' = 6.2 \text{ cm} \times 0.8 = 4.96 \text{ cm} \]

\[ a' = 10 \text{ cm} \times 0.8 = 8 \text{ cm} \]

\[ A_1 = \frac{6.2 \times 5}{2} \times \frac{4}{2} = 31 \text{ cm}^2 \]

\[ A_2 = \frac{4.96 \times 8}{2} \times \frac{4}{2} = 19.84 \text{ cm}^2 \]

b) \[ \frac{A_2}{A_1} = \frac{19.84}{31} = 0.64 \rightarrow 64\% \]

**Answer:** The area of the 2nd triangle is 64% of the area of the 1st triangle.

c) \[ A_1 - A_2 = 100\% - 64\% = 36\% \]

or \( A_2 \) is \( 0.8 \times 0.8 = 0.64 \) of \( A_1 \), so is 0.36 (i.e. 36%) less

**Answer:** The area of the 2nd triangle is 36% smaller than the area of the 1st triangle.

45 min
Factorising 130, 305, 480 and 1130. Revision, activities, consolidation  
*PbY6b, page 130*

**Solutions:**

**Q.1**

a) \(0.75 = \frac{75}{100} = \frac{3}{4} = 3 \div 4\)

b) \(1.6 = \frac{16}{10} = \frac{8}{5} = 8 \div 5\)

c) \(0.1 = \frac{1}{9} = 1 \div 9\)

d) \(1.8 = \frac{18}{10} = \frac{9}{5} = 9 \div 5\)

e) \(0.6 = \frac{6}{9} = \frac{2}{3} = 2 \div 3\)

f) \(0.625 = \frac{5}{8} = 5 \div 8\)

g) \(2.5 = \frac{25}{10} = \frac{5}{2} = 5 \div 2\)

h) \(1.125 = \frac{1}{8} = \frac{9}{8} = 9 \div 8\)

i) \(0.375 = \frac{3}{8} = 3 \div 8\)

j) \(0.16 = \frac{1}{6} = 1 \div 6\)

**Q.2**

As colour cannot be shown, one set of equal numbers is shaded and the numbers in each of the other sets are joined together.

**Q.3**

a) i) \(13.64 = \frac{1364}{100} = 13 \frac{16}{25}\)

ii) \(9.015 = \frac{9015}{1000} = 9 \frac{3}{200}\)

iii) \(0.875 = \frac{7}{8}\) (as 0.875 is 7 \(\times\) 0.125)

iv) \(0.7 = \frac{7}{9}\) (as 0.7 is 7 \(\times\) 0.1)

v) \(5.55 = \frac{555}{100} = 5 \frac{11}{20}\)

b) i) \(\frac{11}{25} = \frac{44}{100} = 0.44\)

ii) \(\frac{5}{8} = 0.625\)

iii) \(\frac{19}{20} = \frac{95}{100} = 0.95\)

iv) \(\frac{1}{6} = 0.16\)

v) \(\frac{3}{11} = 0.272727\ldots = 0.\overline{27}\) (or 0.\(\overline{27}\))
Activity

Solutions: (Continued)

Q.4  a) \(a + 3.26 = 8.2\), \(a = 8.2 - 3.26 = 4.94\)

b) \(b - \frac{3}{5} = 4\frac{6}{7}\), \(b = 4\frac{6}{7} + \frac{3}{5} = 4 + \frac{30 + 21}{35} = \frac{51}{35} = 1\frac{16}{35}\)

c) \(0.91 - c = 1\), \(c = 0.91 - 1 = -0.09\)

d) \(\frac{2}{9} \times d = \frac{1}{27}\), \(d = \frac{1}{27} \div \frac{2}{9} = \frac{1}{27} \times \frac{2}{9} = \frac{1}{6}\)

e) \(\frac{3}{4}\) of \(e = e - 25\), \(\frac{1}{4}\) of \(e = 25\), so \(e = 25 \times 4 = 100\)

f) \(f \times 2.7 = \frac{27}{100}\), \(f = \frac{27}{100} \div \frac{27}{10} = \frac{27}{100} \times \frac{27}{27} = \frac{1}{10}\)

or \(f = 0.27 \div 2.7 = 0.1\)

(7² = 49, so \(\sqrt{49} = 7\))

Q.5  a) \(26\text{ km} 350\text{ m} \div 5 \times 8 = 26.35\text{ km} \div 5 \times 8 = 5.27\text{ km} \times 8 = 42.16\text{ km}\)

b) \(6.78\text{ litres} \div 4 \times 15 = 1.695\text{ litres} \times 15 = 25.425\text{ litres}\)

c) \(\£65.23 \div 11 \times 13 = \£5.93 \times 13 = \£77.09\)

d) \(4\text{ kg} 308\text{ g} \div 0.75 = 4308\text{ g} \div \frac{3}{4} = 4308\text{ g} \div 3 \times 4 = 1436\text{ g} \times 4 = 5744\text{ g} = 5\text{ kg} 744\text{ g}\)

e) \(7\text{ h} 4\text{ min} \div 1.06 = 424\text{ min} \div 1.06 = 42400\text{ min} \div 106 = 400\text{ min} = 6\text{ h} 40\text{ min}\)

f) Whole amount: \(110 \times 110 = 1100 \times 11 = 12100\)

Q.6  a) \(A_1 = (3 \times 3)\text{ cm}^2 = 9\text{ cm}^2 = 25\%\text{ of } A_2\)

\(A_2 = 9\text{ cm}^2 \times 4 = 36\text{ cm}^2\)

\(a_2 = \sqrt{36} \text{ cm} = 6\text{ cm}\) (because \(6^2 = 36\))

b) i) \(A_2 = (4.5 \times 3.2)\text{ cm}^2 \div 2 = 14.4\text{ cm}^2 \div 2 = 7.2\text{ cm}^2\)

ii) \(A_1 = \frac{2}{5}\) of \(A_2 = 0.4 \times 7.2\text{ cm}^2 = 2.88\text{ cm}^2\)

\(b_1 = (2.88 \div h_1 \times 2)\text{ cm} = (2.88 \div 8 \times 2)\text{ cm} = (0.36 \times 2)\text{ cm} = 0.72\text{ cm}\)

Answer: The length of each side of the second square is 6 cm.

Answer: The area of the second triangle is 7.2 cm².

Answer: The length of the base of the second triangle is 0.72 cm.
### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- **131** is a prime number  
  Factors: 1, 131  
  (as not exactly divisible by 2, 3, 5, 7, 11, and 13^2 > 131)
- **306** = \(2 \times 3 \times 3 \times 17 = 2 \times 3^2 \times 17\)  
  Factors: 1, 2, 3, 6, 9, 17, 18, 34, 51, 102, 153, 306
- **481** = 13 \(\times\) 37  
  Factors: 1, 13, 37, 481
- **1131** = \(3 \times 13 \times 29\)  
  Factors: 1, 3, 13, 29, 39, 87, 377, 1131

**8 min**

### Review of subtraction

T writes a subtraction on BB and Ps dictate the difference.

**e.g.** BB: \(125.7 - 35.2 = 90.5\)

Let's use this result to help us calculate the difference if we make changes.

- **a)** Increase the reductant.
  - e.g. \((125.7 + 2.5) - 35.2 = 90.5 + 2.5 = 93\)
  - or \((125.7 + a) - 35.2 = 90.5 + a\)
- **b)** Increase the subtrahend.
  - e.g. \(125.7 - (35.2 + 1) = 90.5 - 1 = 89.5\)
  - or \(125.7 - (35.2 + a) = 90.5 - a\)
- **c)** Decrease the reductant.
  - e.g. \((125.7 - 4.5) - 35.2 = 90.5 - 4.5 = 86\)
  - or \((125.7 - a) - 35.2 = 90.5 - a\)
- **d)** Decrease the subtrahend.
  - e.g. \(125.7 - (35.2 - 2) = 90.5 + 2 = 92.5\)
  - or \(125.7 - (35.2 - a) = 90.5 + a\)
- **e)** Increase the reductant and decrease the subtrahend by the same amount.
  - e.g. \((125.7 + 1) - (35.2 - 1) = 90.5 + 1 - 1 = 92.5\)
  - or \((125.7 + a) - (35.2 - a) = 90.5 + a + a = 90.5 + 2a\)
- **f)** Decrease the reductant and increase the subtrahend by the same amount.
  - e.g. \((125.7 - 1) - (35.2 + 1) = 90.5 - 1 - 1 = 88.5\)
  - or \((125.7 - a) - (35.2 + a) = 90.5 - a - a = 90.5 - 2a\)
- **g)** Increase the reductant and increase the subtrahend by the same amount.
  - e.g. \((125.7 + 10) - (35.2 + 10) = 90.5 + 10 - 10 = 90.5\)
  - or \((125.7 + a) - (35.2 + a) = 90.5 + a - a = 90.5\)  
    (no change)
- **h)** Decrease the reductant and decrease the subtrahend by the same amount.
  - e.g. \((125.7 - 5) - (35.2 - 5) = 90.5 - 5 - 5 = 90.5\)
  - or \((125.7 - a) - (35.2 - a) = 90.5 - a - a = 90.5\)  
    (no change)

**Whole class activity**

Written on BB or SB or OHT

Ps come to BB or dictate what T should write.

Class points out errors.

At a good pace

Agreement, praising

Elicit a generalisation after each type. Ps dictate what T should write. Class agrees/disagrees.

Ps point out what they have noticed, e.g.

If we increase the reductant (or decrease the subtrahend) by a certain amount, the difference increases by that amount.

If we decrease the reductant (or increase the subtrahend) by a certain amount, the difference decreases by that amount.

If we increase or decrease the reductant and the subtrahend by the same amount, the difference does not change.

etc.
Activity

2

(Continued)

i) How could we write the calculation if we increase or decrease the reductant and subtrahend by different amounts, e.g. $a$ and $b$?

BB: $(125.7 + a) - (35.2 + b) = 90.5 + a - b$

Ask for actual examples where $a$ and $b$ are positive or negative whole numbers, fractions or decimals, or zero.

Agree that the equation is true for all these values of $a$ and $b$.

Notes

Ps dictate what T should write. Class agrees/disagrees and checks with different values for $a$ and $b$.

Praising only

$
\begin{align*}
\text{Q.1} & \quad \text{Calculate:} \\
\text{a) half of } \frac{4}{5} & \quad [\frac{4}{5} \div 2 = \frac{19}{5} + 2 = \frac{19}{10} = 1 \frac{9}{10} (= 1.9)] \\
\text{b) one fifth of } \frac{7}{8} & \quad [\frac{7}{8} \div 5 = \frac{7}{40}] \\
\text{c) seven times } \frac{3}{5} & \quad [\frac{3}{5} \times 7 = 14 + \frac{21}{5} = 18 \frac{1}{5} (= 18.2)]
\end{align*}
$

$
\begin{align*}
\text{Q.2} & \quad \text{A 1 metre metal tube weighs } \frac{9}{20} \text{ kg. What is the mass of four similar 7 metre tubes?} \\
\text{Plan:} & \quad \frac{9}{20} \times 4 \times 7 = \frac{63}{5} \text{ kg} = 12 \frac{3}{5} \text{ kg} \\
\text{Answer:} & \quad \text{The mass of four similar 7 metre tubes is } 12 \frac{3}{5} \text{ kg.}
\end{align*}
$

$\begin{align*}
\text{Q.3} & \quad \text{a) Convert these fractions to thirtieths:} \\
\frac{5}{6} & = \frac{25}{30}; \quad \frac{4}{5} = \frac{24}{30}; \quad \frac{7}{10} = \frac{21}{30}; \quad \frac{2}{3} = \frac{20}{30} \\
\text{b) Write the fractions in increasing order:} \\
\frac{20}{30}, \frac{21}{30}, \frac{24}{30}, \frac{25}{30} \\
\text{c) What is the sum of the fractions?} \\
\frac{20}{30} + \frac{21}{30} + \frac{24}{30} + \frac{25}{30} = \frac{90}{30} = 3
\end{align*}$

$\begin{align*}
\text{Q.4} & \quad \text{a) Draw a rectangle which has sides 7 cm and 4 cm long.} \\
\text{b) i) Draw its lines of symmetry. (2 lines of symmetry)} \\
\text{ii) Which plane shapes did you form by drawing these lines of symmetry? (4 congruent small rectangles)} \\
\text{c) How many times larger than the perimeter of one of the smaller shapes is the perimeter of the original rectangle? (× 2)} \\
\text{d) How many times larger than the area of one of the smaller shapes is the area of the original rectangle? (× 4)}
\end{align*}$

Notes

This $Pb$ page could be used as a diagnostic test in 2 parts:

Part A: Q. 1–4
Part B: Q. 5–8

Allow 15 minutes working and 5 minutes review for each part.

Review Part A interactively with the whole class and make sure that mistakes are corrected before continuing with Part B.

If there is no time for the two parts during a single lesson, Part B could be set as homework and reviewed interactively before the start of Lesson 132.

If not done as a test, but as practice, deal with the questions one at a time, reviewing interactively with the whole class after each question as usual.

Elicit that 30 is the smallest common multiple of 3, 5, 6 and 10.
Q.5 Write:
   a) two 4-digit natural numbers which are divisible by 2, 5 and 6.
      (Lowest common multiple of 2, 5 and 6 is 30, so numbers
      must be multiples of 30. e.g. 3330, 4590)
   b) two 5-digit natural numbers which are divisible by 3, 4 and 25.
      Lowest common multiple of 3, 4 and 25 is 300, so numbers
      must be multiples of 300. e.g. 96300, 51900)

Q.6 List these fractions in increasing order:
   \[
   \frac{3}{5}, \frac{7}{10}, \frac{1}{2}, \frac{60}{100}, \frac{13}{20}, \frac{14}{20}
   \]
   Solution:
   Write the fractions as equivalent fractions with a common
denominator. (20)
   \[
   \frac{3}{5} = \frac{12}{20}; \quad \frac{7}{10} = \frac{14}{20}; \quad \frac{1}{2} = \frac{10}{20}; \quad \frac{60}{100} = \frac{12}{20};
   \frac{10}{20} < \frac{12}{20} < \frac{13}{20} < \frac{14}{20} = \frac{14}{20}
   \]

Q.7 72 radishes are tied in equal bundles, with no radishes left
over. How many radishes could be in each bundle?
   \[
   72 = (72 \times 1) = \frac{36}{2} \times 2 = \frac{24}{3} \times 3 = \frac{18}{4} \times 4
   = \frac{12}{6} \times 2 = \frac{9}{3} \times 8 = \frac{8}{4} \times 9 = \frac{6}{3} \times 12 = \frac{4}{2} \times 18
   = \frac{3}{24} \times 2 = \frac{1}{36} \times (\frac{1}{1} \times 72)
   \]
   or Ps might show the result in a table:
   BB:
   | No. of bundles | 1 | 2 | 3 | 4 | 6 | 9 | 12 | 18 | 24 | 36 |
   | No. of radishes in each bundle | 72 | 36 | 24 | 18 | 12 | 9 | 8 | 6 | 4 | 3 |

Q.8 a) Draw a point, then draw two 3 cm segments from the point so
     that the angle they form is 60°.
     Ps use ruler and compasses, or a protractor, to draw the angle.

b) If each of the two segments is half of a diagonal of the same
   rectangle, construct the rectangle.

c) Measure the necessary dimensions, then calculate:
   i) the perimeter of the rectangle
      \[
      P = (3 \text{ cm} + 5.2 \text{ cm}) \times 2 = 8.2 \text{ cm} \times 2 = 16.4 \text{ cm}
      \]
   ii) the area of the rectangle.
      \[
      A = (3 \times 5.2) \text{ cm}^2 = \frac{15.6}{55} \text{ cm}^2
      \]

Class applauds Ps who have all questions correct (or fewest errors)

Feedback for T
Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(132 = 2 \times 2 \times 3 \times 11 = 2^2 \times 3 \times 11\)
  - Factors: 1, 2, 3, 4, 6, 11, 12, 22, 33, 44, 66, 132
- \(307\) is a prime number
  - Factors: 1, 307
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17 and 19: \(2 \geq 307\))
- \(482 = 2 \times 241\)
  - Factors: 1, 2, 241, 482
  (241 is not exactly divisible by 2, 3, 5, 7, 11, and 17: \(2 \geq 241\))
- \(1132 = 2 \times 2 \times 283 = 2^2 \times 283\)
  - Factors: 1, 2, 4, 283, 566, 1132

8 min

Review of multiplication

T writes a multiplication on BB and Ps dictate the product.

- e.g. BB: \(436 \times 2.8\)
- Let’s use this result to help us calculate the product if we make changes.

  a) Increase the multiplicand by a certain number of times.
     - e.g. \((436 \times 2) \times 2.8 = 1220.8 \times 2 = 2441.6\)
     - or \((436 \times a) \times 2.8 = 1220.8 \times a\)
  
  b) Increase the multiplier by a certain number of times.
     - e.g. \(436 \times (2.8 \times 2) = 1220.8 \times 2 = 2441.6\)
     - or \(436 \times (2.8 \times a) = 1220.8 \times a\)  [same result as a]
  
  c) Decrease the multiplicand by a certain number of times.
     - e.g. \((436 \div 4) \times 2.8 = 1220.8 \div 4 = 305.2\)
     - or \((436 \div a) \times 2.8 = 1220.8 \div a\)
  
  d) Decrease the multiplier by a certain number of times.
     - e.g. \(436 \times (2.8 \div 7) = 1220.8 \div 7 = 174.4\)
     - or \(436 \times (2.8 \div a) = 1220.8 \div a\)  [same result as c]
  
  e) Increase both factors by different numbers of times.
     - e.g. \((436 \times 0.1) \times (2.8 \times 2) = 1220.8 \times 0.1 \times 2 = 244.16\)
     - or \((436 \times a) \times (2.8 \times b) = 1220.8 \times a \times b\)  [= 1220.8 \times ab]
  
  f) Decrease both factors by different numbers of times.
     - e.g. \((436 \div 4) \times (2.8 \div 2) = 1220.8 \div 4 \times 2 = 1220.8 \div 8 = 152.6\)
     - or \((436 \div a) \times (2.8 \div b) = 1220.8 \div (a \times b)\)  [= 1220.8 \div ab]
  
  g) Multiply one factor and divide the other factor by the same number.
     - e.g. \((436 \times 4) \times (2.8 \div 4) = 1220.8 \times 4 \div 4 = 1220.8\)
     - or \((436 \times a) \times (2.8 \div a) = 1220.8 \times a \div a = 1220.8\)

15 min

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**MEP: Primary Project**

**Y6**

**Activity 3**

<table>
<thead>
<tr>
<th>TEST 2, Part A</th>
</tr>
</thead>
</table>

**PbY6b, page 132**

| Q.1 | 3 \( \frac{2}{3} + 2 \frac{4}{5} + 1 \frac{1}{2} - 4 \frac{3}{4} \) = | 6 \(-4 + \frac{40 + 48 + 30 - 45}{60}\) |
|     | = 2 + \frac{118 - 45}{60} = 2 + \frac{73}{60} = 3 \frac{13}{60} |

| Q.2 | On the 1st day of a 4-day walking holiday, we walked 7 \( \frac{1}{4} \) km. |
|     | On the 2nd day we walked 6 \( \frac{3}{5} \) km and on the 3rd day we |
|     | walked 5 \( \frac{2}{8} \) km. If we walked 25 km altogether, how far did |
|     | we walk on the 4th day? |
| Plan: | e.g. \( 25 \text{ km} - (7 \frac{1}{4} + 6 \frac{3}{5} + 5 \frac{7}{8}) \text{ km} \) |
|     | = 25 \text{ km} - 18 \text{ km} - (\frac{10 + 24 + 35}{40}) \text{ km} |
|     | = 7 \text{ km} - \frac{69}{40} \text{ km} = 7 \text{ km} - 1 \frac{29}{40} \text{ km} |
|     | = 6 \text{ km} - \frac{29}{40} \text{ km} = 5 \frac{11}{40} \text{ km} |

| Answer: On the 4th day we walked 5 \( \frac{11}{40} \) kilometres. |

| Q.3 a) | Construct an isosceles triangle with base 3 cm long and |
|        | arms 5 cm long. |
|        | Elicit that an isosceles triangle has at least 2 equal sides (angles). |
| b) i) | Draw its lines of symmetry. (1) |
|       | ii) Which plane shapes did you |
|       | form by drawing these lines of |
|       | symmetry? (2 congruent |
|       | right-angled triangles) |
| c) | Calculate the area of: |
|    | i) one of the smaller shapes |
|    | ii) the original triangle. |
| First measure the perpendicular |
| height of ABC: \( h = 4.8 \text{ cm} \) |
| i) Area of smaller triangle: |
| \( A = (4.8 \times 1.5 \div 2) \text{ cm}^2 = (2.4 \times 1.5) \text{ cm}^2 = 3.6 \text{ cm}^2 \) |
| ii) Area of triangle ABC: |
| \( A = (4.8 \times 3 \div 2) \text{ cm}^2 = (2.4 \times 3) \text{ cm}^2 = 7.2 \text{ cm}^2 \) |
| or \( A = 3.6 \text{ cm}^2 \times 2 = 7.2 \text{ cm}^2 \) |

| Construction: e.g. |
| 1. Draw the 3 cm base and |
| label it AB. |
| 2. Set compasses to 5 cm |
| and draw arcs around A and |
| around B. |
| 3. Label C the point where |
| the arcs intersect. |
| 4. Join A and B to C. |
| Triangle ABC is an isosceles |
| triangle. |

| Elicit that |
| • the line of symmetry in an |
| isosceles triangle is the |
| perpendicular bisector of |
| its base; |
| • the area of a triangle is |
| half its base \( \times \) its height. |

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**Q.4** A group of 8 people in an office earned these amounts over a period of 4 weeks.
1st week: £3684, 2nd week: £3341, 3rd week: £3435.40, 4th week: £3256.80

How much did each person earn on average over the 4-week period?

**Plan:**
\[
\frac{(3684 + 3341 + 3435.40 + 3256.80)}{8} = \frac{13717.20}{8} = \mathbf{1714.65}
\]

**Answer:** Each person earned on average £1714.65 over the 4-week period.

**Q.5** In a recipe for making bread, 1 kg of flour produces 1.8 kg of dough. After the dough has been kneaded and proved, it is put into the oven to bake.

During baking, the dough loses \(\frac{1}{5}\) of its mass.

How much bread can be made from 2 kg of flour using this recipe?

**Plan:**
\[
1.8 \times 2 \times \frac{4}{5} = 3.6 \times 0.8 = 2.88 \text{ kg}
\]
or ratio of flour : dough : bread = 1 : 1.8 : 1.8 \(\times 0.8 = 1.44\)
\[
= 2 : 3.6 : 2.88 \text{ (\(\times 2\))}
\]

**Answer:** 2.88 kg of bread can be made from 2 kg of flour.

**Q.6** Dad cut these lengths from a 2.5 m plank of wood:
\(\frac{4}{5}\) m, \(\frac{3}{4}\) m and \(\frac{5}{8}\) m. What length of plank was left?

**Plan:**
\[
2.5 - \left(\frac{4}{5} + \frac{3}{4} + \frac{5}{8}\right) \text{ m}
\]
\[
= 2 \frac{1}{2} m - \left(\frac{32 + 30 + 25}{40}\right) m
\]
\[
= 2 \frac{1}{2} m - \frac{87}{40} m = 2 \frac{20}{40} m - 2 \frac{7}{40} m = \frac{13}{40} \text{ m}
\]

**Answer:** There was \(\frac{13}{40}\) (or 0.325) of a metre of plank left.
### Activity

**Q.7 a)** Construct an angle of $45^\circ$.

**b)** Mark a point 4 cm from the vertex on one of the arms of the angle.

**c)** Draw a line which is perpendicular to the arm at this point and extend it to cut the other arm, forming a triangle.

**d)** Measure the sides and angles of this triangle.

N.B. Pupils need not label points M to R. They are labelled here to make the explanation of the construction easier (as given opposite).

**e)** What kind of triangle have you drawn?

(Right-angled isosceles triangle)

**f)** Calculate its area and perimeter.

\[
A = \frac{4 \times 4}{2} \text{ cm}^2 = 8 \text{ cm}^2, \quad P = (4 + 4 + 5.7) \text{ cm} = 13.7 \text{ cm}
\]

**Notes**

**Construction e.g.**

1. Construct an angle of $90^\circ$.
   - Set compasses to an appropriate width and keep that width throughout.
   - Mark a point A and draw a ray.
   - With compass point on A, draw an arc around A to cut the ray at M.
   - With compass point on M, draw an arc to cut the 1st arc at N.
   - With compass point on N, then on O, draw 2 arcs which intersect at P.
   - Draw a ray from A through P. $\angle PAM = 90^\circ$.

2. Construct the bisector of $\angle PAM$ to form a $45^\circ$ angle.
   - With compass point on M, then on Q (the intersection of the 1st arc and AP) draw 2 arcs which intersect at R.
   - Draw a ray from A through R. $\angle RAM = 45^\circ$.

Follow the rest of the instructions as given, extending the arms (rays) if necessary.

Class applauds Ps who have all questions correct (or the fewest errors).
## Activity 1

### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit a generalisation after each type. Ps dictate what T should write. Class agrees/disagrees.

Ps point out what they have noticed, e.g.

<table>
<thead>
<tr>
<th>Number</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>133</td>
<td>1, 7, 19, 133</td>
</tr>
<tr>
<td>308</td>
<td>1, 2, 4, 7, 11, 14, 22, 28, 44, 77, 154, 308</td>
</tr>
<tr>
<td>483</td>
<td>1, 3, 7, 21, 23, 69, 161, 483</td>
</tr>
<tr>
<td>1133</td>
<td>1, 11, 103, 1133</td>
</tr>
</tbody>
</table>

Let’s use this result to help us calculate the quotient if we make changes.

<table>
<thead>
<tr>
<th>Change Description</th>
<th>Example Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Increase dividend by a certain number of times.</td>
<td>$15.6 \div 3 = \frac{3}{2}$ (no change)</td>
</tr>
<tr>
<td>b) Decrease the divisor by a certain number of times.</td>
<td>$15.6 \div 5.2 = \frac{3}{2}$ (no change)</td>
</tr>
<tr>
<td>c) Decrease the dividend by a certain number of times.</td>
<td>$15.6 \div 3 = \frac{1}{a}$</td>
</tr>
<tr>
<td>d) Decrease the divisor by a certain number of times.</td>
<td>$15.6 \div 5.2 = \frac{3}{2}$ (no change)</td>
</tr>
<tr>
<td>e) Increase the dividend and divisor by the same number of times.</td>
<td>$15.6 \times 2 \div (5.2 \times 2) = \frac{3}{2}$ (no change)</td>
</tr>
<tr>
<td>f) Decrease the dividend and divisor by the same number of times.</td>
<td>$15.6 \div 5.2 = \frac{3}{2}$ (no change)</td>
</tr>
<tr>
<td>g) Increase the dividend and decrease the divisor by the same number of times.</td>
<td>$15.6 \div 5.2 = \frac{3}{2}$ (no change)</td>
</tr>
<tr>
<td>h) Decrease the dividend and increase the divisor by the same number of times.</td>
<td>$15.6 \div 5.2 = \frac{3}{2}$ (no change)</td>
</tr>
</tbody>
</table>

### Review of division

T writes a division on BB and Ps dictate the quotient.

- e.g. BB: $15.6 \div 5.2 = \frac{3}{2}$ (because $\frac{3}{2} \times 5.2 = 15.6$)

Let’s use this result to help us calculate the quotient if we make changes.

- a) Increase the dividend by a certain number of times.
  - $15.6 \times 2 \div (5.2 \times 2) = \frac{3}{2}$ (no change)
  - or $15.6 \times a \div (5.2 \times a) = \frac{3}{a}$ (same as b)

- b) Decrease the divisor by a certain number of times.
  - $15.6 \div 5.2 = \frac{3}{2}$ (no change)

- c) Decrease the dividend by a certain number of times.
  - $15.6 \div 3 = \frac{3}{a}$ (same as b)

- d) Decrease the divisor by a certain number of times.
  - $15.6 \div 5.2 = \frac{3}{2}$ (no change)

- e) Increase the dividend and divisor by the same number of times.
  - $15.6 \times a \div (5.2 \times a) = \frac{3}{a}$ (same as a)

- f) Decrease the dividend and divisor by the same number of times.
  - $15.6 \div 5.2 = \frac{3}{2}$ (no change)

- g) Increase the dividend and decrease the divisor by the same number of times.
  - $15.6 \times 2 \div 5.2 = \frac{3}{a}$ (same as a)

- h) Decrease the dividend and increase the divisor by the same number of times.
  - $15.6 \div a \div (5.2 \times a) = \frac{3}{a^2}$ (same as a)

Ps come to BB or dictate what T should write.
Class points out errors.
At a good pace
Agreement, praising

If we increase the dividend or decrease the divisor by a certain number of times, the quotient increases by that number of times.

If we decrease the dividend or increase the divisor by a certain number of times, the quotient decreases by that number of times.

If we increase or decrease the dividend and divisor by the same number of times, the quotient stays the same.

etc.

T could show the forms in square brackets to familiarise Ps with algebraic notation.
Activity 2 (Continued)

i) Increase the dividend and divisor by a different number of times, e.g. the dividend by \( a \) and the divisor by \( b \).

BB: \((15.6 \times a) \div (5.2 \times b) = 3 \times a \div b = \frac{3a}{b}\)

Ask Ps to suggest values for \( a \) and \( b \) and elicit that \( a \) can be any positive or negative whole number or fraction or decimal.

Notes

Ps come to BB or dictate to T. Agreement, praising

3

TEST 3, Part A

PbY6b, page 133

Q.1 a) \(\frac{3}{4} \times \frac{5}{7} = \frac{15}{28}\)  
b) \(\frac{2}{15} \times \frac{1}{6} = \frac{2}{15}\)

c) \(1\frac{3}{5} \times \frac{5}{8} = \frac{8}{51} \times \frac{5}{8} = \frac{1}{1}\)

d) \(2\frac{1}{3} \times 3\frac{1}{4} = \frac{7}{3} \times \frac{13}{4} = \frac{91}{12} = 7\frac{7}{12}\)

Q.2 Write each percentage as a fraction and as a decimal.

a) \(43\% \rightarrow \frac{43}{100} = 0.43\)

b) \(206\% \rightarrow \frac{206}{100} = 2\frac{6}{50} = 2.06\)

Q.3 What are these parts of 838 km?

a) 0.67 of 838 km = 838 km \(\times 0.67\)  
\[= 561.46\text{ km}\]

b) 838 km \(\times 4\frac{1}{3}\) = 838 km \(\times 4 + 838\text{ km} \div 3\)
\[= 3352\text{ km} + 279.3\text{ km} = 3631.3\text{ km}\]

c) 86% of 838 km \(\rightarrow 838\text{ km} \times 0.86 = 720.68\text{ km}\)

Q.4 A container was \(\frac{4}{5}\) full of honey. Then 2 thirds of this honey was sold.

a) What part of the container still contains honey?

If 2 thirds were sold, then 1 third is left.

Plan: \(\frac{1}{3}\) of \(\frac{4}{5}\) = \(\frac{4}{5} \div 3 = \frac{4}{15}\)

or \(\frac{1}{3}\) of \(\frac{4}{5}\) = \(\frac{1}{3} \times \frac{4}{5} = \frac{4}{15}\)

Answer: Four fifteenths of the container still contains honey.
(Test 3, Part A continued)

b) If the container has a capacity of 50 litres:

i) how much honey was sold

Plan: \(\frac{2}{3}\) of \(\frac{4}{5}\) of 50 litres

\[
= \frac{2}{3} \times \frac{4}{5} \times 50 \text{ litres} = \frac{80}{3} \text{ litres} = 26 \frac{2}{3} \text{ litres}
\]

Answer: Twenty-six and two thirds litres of honey were sold.

ii) how much honey was left?

Plan: \(\frac{1}{3}\) of \(\frac{4}{5}\) of 50 litres

\[
= \frac{1}{3} \times \frac{4}{5} \times 50 \text{ litres} = \frac{40}{3} \text{ litres} = 13 \frac{1}{3} \text{ litres}
\]

or \(\frac{2}{3}\) \(\rightarrow\) 26 \(\frac{2}{3}\) litres,

\[
\frac{1}{3} \rightarrow 26 \frac{2}{3} \text{ litres} ÷ 2 = 13 \frac{1}{3} \text{ litres}
\]

Answer: Thirteen and one third litres of honey were left.

Q.5 A jewellery firm bought 3.6 m\(^2\) of gold leaf. First 15% of the gold leaf was used, then \(\frac{2}{9}\) of it, then 0.4 of it.

a) How much gold leaf was used altogether?

Plan: \((3.6 \times 0.15) + (3.6 \times \frac{2}{9}) + (3.6 \times 0.4) \text{ m}^2\)

\[
= (0.54 + 0.8 + 1.44) \text{ m}^2 = 2.78 \text{ m}^2
\]

Answer: Altogether, 2.78 m\(^2\) of gold leaf was used.

b) If the firm employed 10 craftsmen, how much gold leaf did each craftsman use on average?

Plan: 2.78 m\(^2\) ÷ 10 = 0.278 m\(^2\)

Answer: Each craftsman used 0.278 m\(^2\) of gold leaf on average.

or

Amount in container at start:

\[
\frac{4}{5} \text{ of 50 litres} = 40 \text{ litres}
\]

Amount sold:

\[
\frac{2}{3} \times 40 \text{ litres} = \frac{80}{3} \text{ litres}
\]

Amount left:

\[
(40 - 26 \frac{2}{3}) \text{ litres} = 13 \frac{1}{3} \text{ litres}
\]

N.B. This is easier than adding the three parts together first, as in fraction form, the lowest common multiple of 100, 9 and 10 is 900, and in decimal form,

\[
\frac{2}{9} = 0.\overline{2}, \text{ a recurring decimal.}
\]
**Activity 4**

**TEST 3, Part B**

**PbY6b, page 133**

**Q.6 a)**

\[
\left( \frac{3 \frac{1}{2} + 2 \frac{1}{4}}{5} \right) \times \frac{3}{5} = \frac{5 \frac{3}{4} \times \frac{3}{5}}{23 \frac{3}{4} \times \frac{3}{5}} = \frac{69}{20} = 3 \frac{9}{20}
\]

**b)**

\[
\left( \frac{8 \frac{1}{5} - 2 \frac{3}{4}}{5} \right) \times \frac{2}{3} = \left( \frac{7 \frac{6}{5} - 2 \frac{3}{4}}{5} \right) \times \frac{2}{3}
\]

\[
= (5 + \frac{24 - 15}{20}) \times \frac{2}{3}
\]

\[
= 5 \frac{9}{20} \times \frac{2}{3} = \frac{109}{20} \times \frac{1}{3} = \frac{109}{30}
\]

\[
= 3 \frac{19}{30}
\]

**Q.7** What quantity is:

- **a)** \( \frac{2}{3} \) of 543 m: \( \frac{181}{543} \) m \( \times \frac{2}{3} = 362 \) m

- **b)** \( \frac{3}{4} \) of 615 kg: \( \frac{4305}{615} \) kg = 1076 \( \frac{1}{4} \) kg

- **c)** \( \frac{2}{5} \) of 15 \( \frac{2}{5} \) km = \( \frac{15}{2} \) km = 38 \( \frac{1}{2} \) km

- **d)** 1.17 of 63.3 m\(^2\): 63.3 m\(^2\) \( \times \) 1.17 = 74.061 m\(^2\)

or (63 + 63.3 \( \times \) 0.17) m\(^2\)

**Q.8** In 2003, a firm planned for an income of £25.7 million. They exceeded this plan by 20%. How much income did the firm actually achieve?

**Plan:** £25.7 million \( \times \) 1.2 = £30.84 million

or £ (25.7 + 25.7 \div 5) million

\[= £ (25.7 + 5.14) million\]

\[= £30.84 million\]

**Answer:** The firm achieved an income of £30.84 million.

---

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**Lesson Plan 133**

---

### Activity

**4 (Test 3, Part B continued)**

<table>
<thead>
<tr>
<th>Q.9</th>
<th>During a sale, the price of a £185 suit was reduced by 13%, then reduced again by 15%.</th>
</tr>
</thead>
<tbody>
<tr>
<td>a)</td>
<td>By how many £s was the price reduced?</td>
</tr>
<tr>
<td></td>
<td>Original price: £185</td>
</tr>
<tr>
<td></td>
<td>1st reduction: £185 × 0.13 = £24.05</td>
</tr>
<tr>
<td></td>
<td>Price after 1st reduction: £185 – £24.05 = £160.95</td>
</tr>
<tr>
<td></td>
<td>2nd reduction: £160.95 × 0.15 = £24.14</td>
</tr>
<tr>
<td></td>
<td>Total reductions: £24.05 + £24.14 = £48.19</td>
</tr>
<tr>
<td></td>
<td><strong>Answer:</strong> The price was reduced by £48.19.</td>
</tr>
<tr>
<td>b)</td>
<td>What was the new price?</td>
</tr>
<tr>
<td></td>
<td><strong>Plan:</strong> £185 – £48.19 = £136.81</td>
</tr>
<tr>
<td></td>
<td><strong>Answer:</strong> The new price was £136.81.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Q.10</th>
<th>Construct a rhombus which has an angle of 60° and a longer diagonal of length 7 cm.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Measure the necessary data then calculate the perimeter and area of the rhombus.</strong></td>
</tr>
<tr>
<td></td>
<td>Elicit that a rhombus is a parallelogram which has equal sides. Its diagonals cross at right angles and bisect each other.</td>
</tr>
<tr>
<td></td>
<td><strong>BB:</strong></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td></td>
<td>(DAB = ABD = BDA = 60°, ABC = ADCA = 120°, but the size of the angles is not required for area and perimeter)</td>
</tr>
<tr>
<td></td>
<td><strong>P</strong> = 4 cm × 4 = 16 cm</td>
</tr>
<tr>
<td></td>
<td><strong>A</strong> = BD × AC / 2 = 4 × 7 / 2 = 28 / 2 = 14 cm²</td>
</tr>
<tr>
<td></td>
<td><strong>65 min</strong></td>
</tr>
</tbody>
</table>

---

**Notes**

<table>
<thead>
<tr>
<th>Construction</th>
<th>e.g.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Mark a point A and draw the longer diagonal, AC, 7 cm long.</td>
</tr>
<tr>
<td>2.</td>
<td>Construct angles of 30° above and below AC at A. ∠ A = 60°</td>
</tr>
<tr>
<td>3.</td>
<td>Construct the perpendicular bisector of AC and label B and D the points where the bisector cuts the arms of angle A. BD is the other diagonal of the rhombus.</td>
</tr>
<tr>
<td>4.</td>
<td>Join B and D to C. ABCD is a rhombus.</td>
</tr>
</tbody>
</table>

Triangles ABD and BCD are congruent equilateral triangles.

The area of the rhombus ABCD is half the area of the dotted rectangle shown opposite, i.e. half of BD × AC.

---

**Feedback for T**

Class applauds Ps who have all questions correct (or the fewest errors) and also the most improved score from Test 2.
**Lesson Plan**

### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(134 = 2 \times 67\) Factors: 1, 2, 67, 134
- \(309 = 3 \times 103\) Factors: 1, 3, 103, 309
- \(484 = 2 \times 2 \times 11 \times 11 = 2^2 \times 11^2 = (2 \times 11)^2\) (square number) Factors: 1, 2, 4, 11, 22, 44, 121, 242, 484
- \(1134 = 2 \times 3 \times 3 \times 3 \times 3 \times 7 = 2 \times 3^4 \times 7\) Factors: 1, 2, 3, 6, 7, 9, 14, 18, 21, 27, 42, 63, 81, 162, 189, 378, 567, 1134

[No. of factors: \( (1 + 1) \times (4 + 1) \times (1 + 1) = 2 \times 5 \times 2 = 20\) ]

### Activity 2

**Generalisation in addition**

T writes an addition on BB: \(a + b = c\)

Let’s see how the result changes if we increase or decrease the terms.

Ps suggest changes (using positive and negative whole numbers, fractions and decimals) and choose other Ps to dictate the result. e.g.

a) Increase \(a\) by 1.2: BB: \((a + 1.2) + b = c + 1.2\)

b) Increase \(b\) by \(-\frac{3}{4}\): \(a + [b + \left(-\frac{3}{4}\right)] = c - \frac{3}{4}\)

c) Decrease \(a\) by \(-\frac{2}{3}\): \([a - \left(-\frac{2}{3}\right)] + b = c + \frac{2}{3}\)

d) Increase \(a\) and \(b\) by different amounts: \((a + 0.1) + (b + 0.7) = c + 0.8\)

e) Decrease \(a\) and increase \(b\) by the same amount: \((a - 5) + (b + 5) = c\)

f) Increase \(a\) by 3 times: \((a \times 3) + b = a + b + (a \times 2) = c + a \times 2\)

T: We could write it like this:

BB: \(c + 2a\), as \(a \times 2 = a + a = 2a\)

What is the result of this addition? What does it mean?

BB: \(2a + 5a = (7a)\)

\((= 2 \times a + 5 \times a = 7 \times a)\)

Whole class activity

At a good pace

In good humour

Class points out errors.

Agreement, praising

If there is disagreement, ask Ps to check result by using actual values for \(a\) and \(b\).

Ask Ps to explain the generalisations in words too. e.g.

If we increase or decrease either term in a 2-term addition by a certain amount, the result also increases or decreases by that amount.

If we increase one term and decrease the other term by the same amount in a 2-term addition, the result does not change.
**PbY6b, page 134**

**Q.1** An observatory on a mountain in Scotland measured the temperature at 6 am each day during the second half of February. This table shows the data collected.

<table>
<thead>
<tr>
<th>Day</th>
<th>15th</th>
<th>16th</th>
<th>17th</th>
<th>18th</th>
<th>19th</th>
<th>20th</th>
<th>21st</th>
<th>22nd</th>
<th>23rd</th>
<th>24th</th>
<th>25th</th>
<th>26th</th>
<th>27th</th>
<th>28th</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>0</td>
<td>-4</td>
<td>-5</td>
<td>-2</td>
<td>-3</td>
<td>+1</td>
<td>+2</td>
<td>0</td>
<td>-1</td>
<td>+1</td>
<td>+2</td>
<td>+2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a) **Draw a graph to show how the temperature changed.**

(Accept any correct form: bars, lines, dots, crosses)

Less able Ps could have prepared grids (see copy master).

b) **Calculate the mean temperature.**

\[
\left( [0 + (-1) + (-4) + (-5) + (-2) + (-3) + (-2) + 1 + 2 + 0 + (-1) + 1 + 2 + 2] \right) \div 14 = \left[ -18 + 8 \right] \div 14 = -\frac{10}{14} = -\frac{5}{7} \approx -0.71 \, (^\circ \text{C})
\]

**Q.2**

a) \(5 \div \frac{2}{3} = 5 \times \frac{3}{2} = \frac{15}{2} = \frac{7 \frac{1}{2}}{2}\)

b) \(16 \div 4 \frac{1}{2} = 16 \div \frac{9}{2} = 16 \times \frac{2}{9} = \frac{32}{9} = 3 \frac{5}{9}\)

c) \(54 \div 5 \frac{1}{5} = 54 \div \frac{26}{5} = \frac{54}{\frac{26}{5}} = \frac{54 \times 5}{26} = \frac{135}{13} = 10 \frac{5}{13}\)

d) \(100 \div \left( \frac{8}{4} - 7 \frac{1}{2} \right) = 100 \div \left( \frac{5}{4} - \frac{15}{2} \right) = 100 \div \frac{3}{4} = 100 \times \frac{4}{3} = \frac{400}{3} = 133 \frac{1}{3}\)

**Q.3** What is the whole quantity if:

a) \(\frac{1}{4}\) of it is 28 kg: \([28 \, \text{kg} \times 4 = 112 \, \text{kg}]\)

b) \(\frac{2}{3}\) of it is 28 litres: \([28 \, \text{litres} \div 2 \times 3 = 14 \, \text{litres} \times 3 = 42 \, \text{litres}]\)

or \(28 \div \frac{2}{3} = \frac{28 \times 3}{2} = 42 \, \text{litres}\)

Accept any correct method but elicit that to calculate the whole amount when we know the value of part of it, we can divide the known value by the part.

**Notes**

This Pb page could be used as a diagnostic test in 2 parts:

- **Part A:** Q. 1–4
- **Part B:** Q. 5–8

Allow 20 minutes for each part (working and review).

Review **Part A** interactively with the whole class before continuing with **Part B**.

If there is no time for the two parts during a single lesson, **Part B** could be set as homework and reviewed interactively before the start of **Lesson 135**.

If done as practice, deal with one question at a time and review interactively after each question as usual, with any mistakes discussed and corrected.

**Extension**

Also elicit the mode of the data (most common): \(+2^\circ \text{C}\) and median (middle in ordered set of data): \(-1 + 0 = -\frac{1}{2} = -0.5 \, (^\circ \text{C})\)

As there is an even number of data values, the median is the mean of the 2 middle values.

Elicit that to divide by a fraction, multiply by its **reciprocal** value, i.e. the value which multiplies it to result in 1, or the fraction which has the numerator and denominator values exchanged.

or \(28 \div \frac{1}{4} = 28 \times \frac{4}{1} = 112 \, (\text{kg})\)
**Activity**

3

(**Test 4, Part A continued**)

c) \(2 \frac{3}{4}\) of it is 121 m:
\[
121 \text{ m} \div 2 \frac{3}{4} = 121 \text{ m} \div \frac{11}{4}
\]
\[
= 11 \text{ m} \times \frac{4}{11} = 44 \text{ m}
\]
d) \(1 \frac{4}{5}\) of it is 189 cm:
\[
189 \text{ cm} \div 1 \frac{4}{5} = 189 \text{ cm} \div \frac{9}{5}
\]
\[
= 189 \text{ cm} \times \frac{5}{9} = 105 \text{ cm}
\]
e) 0.17 of it is 61.2 g:
\[
61.2 \text{ g} \div 0.17 = 360 \text{ g}
\]

Q.4 What is the whole quantity if:

a) 1% is £4.25:
\[
£4.25 \times 100 = £425
\]
b) 1% is 0.7 m:
\[
0.7 \text{ m} \times 100 = 70 \text{ m}
\]
c) 25% is 32.6 kg:
\[
32.6 \text{ kg} \times 4 = 130.4 \text{ kg}
\]
(or 32.6 kg \(\div 25 \times 100\))
d) 10% is 43.75 km:
\[
43.75 \text{ km} \times 10 = 437.5 \text{ km}
\]
e) 50% is £159.80?
\[
£159.80 \times 2 = £319.60
\]

---

**Notes**

or 121 m \(\div 11 \times 4 = 44 \text{ m}

etc.

Q.5 a) \((6.2 + 5.8) \div \frac{2}{3} = \frac{6}{12} \times \frac{3}{2} = 18\)

b) \(\left(5 \frac{1}{4} - \frac{3}{5}\right) \div 1 \frac{1}{2} = (2 + \frac{5}{4} - \frac{4}{20}) \div \frac{3}{2}\)
\[
= 2 \frac{1}{20} \times \frac{2}{3}
\]
\[
= \frac{41}{20} \times \frac{2}{3} = \frac{41}{30} = \frac{11}{30}
\]

Q.6 What is the whole quantity if:

a) \(\frac{7}{8}\) of it is 315 cm:
\[
315 \text{ cm} \div \frac{7}{8} = 315 \text{ cm} \times \frac{8}{7} = 360 \text{ cm}
\]
b) \(\frac{4}{3}\) of it is 611 m:
\[
611 \text{ m} \div \frac{13}{3} = \frac{47}{13} \text{ m} \times \frac{3}{13} = 141 \text{ m}
\]
c) 65% of it is 20.28 kg?
\[
20.28 \text{ kg} \div 0.65 = 31.2 \text{ kg}
\]
(or 20.28 kg \(\div 65 \times 100\))

---

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**Y6**

**Activity**

4

(Test 4, Part B continued)

Q.7  A country bought 1 199 300 tonnes of oil, which was 33.5% of its imports that year. What mass of goods did the country import that year?

Plan: 1 199 300 t ÷ 0.335

= 1 199 300 thousand tonnes ÷ 335

= 3 580 thousand tonnes

= 3 580 000 tonnes

*Answer:* The country imported 3 million 580 thousand tonnes of goods that year.

Q.8  The length of a cuboid-shaped iron block is 140 cm.

Its width is 0.7 of its length and \( \frac{5}{9} \) of its height.

L: 140 cm

W: 140 cm \times 0.7 = 98 cm

H: 98 cm \div \frac{5}{9} = 98 cm \div \frac{14}{9} = 98 cm \times \frac{9}{14} = 63 cm

a) Calculate:

i) its surface area:

\[ A = 2 \times (140 \times 98 + 140 \times 63 + 98 \times 63) \text{ cm}^2 \]

\[ = 2 \times (13 720 + 8820 + 6174) \text{ cm}^2 \]

\[ = 2 \times 28 714 \text{ cm}^2 \]

\[ = 57 428 \text{ cm}^2 \]

ii) its volume:

\[ V = (140 \times 98 \times 63) \text{ cm}^3 \]

\[ = 864 360 \text{ cm}^3 \ ( = 0.86436 \text{ m}^3) \]

b) How much does the block weigh if 1 cm³ of iron weighs 7.6 g?

Mass: 864 360 \times 7.6 g = 6 569 136 g

\[ = 6 156 9 \text{ kg} \ ( = 6.6 \text{ t}) \]

*Answer:* The iron block weighs 6 tonnes, 569 kilograms and 136 grams (or about 6.6 tonnes).

---

**Notes**

Ps could be allowed to use calculators for this question, but calculation details could be shown in the review as practice in long division.

BB:

Class applauds Ps with all correct (or the fewest errors) and also the most improved score from Test 3.
### Activity

Factorising 135, 310, 485 and 1135. Revision and practice.  
*PbY6b, page 135*

#### Solutions:

**Q.1**

| a) | 236.8 - 46.3 = 190.5 |
| b) | 236.8 - (46.3 + 2) = 190.5 - 2 = 188.5 |
| c) | (236.8 - 5.6) - 46.3 = 190.5 - 5.6 = 184.9 |
| d) | 236.8 - (46.3 - 3) = 190.5 + 3 = 193.5 |
| e) | (236.8 + 2) - (46.3 - 1) = 190.5 + 2 + 1 = 193.5 |
| f) | (236.8 - 1) - (46.3 + 1) = 190.5 - 2 = 188.5 |
| g) | (236.8 + 10) - (46.3 - 10) = 190.5 + 10 + 10 = 210.5 |
| h) | (236.8 - 6) - (46.3 - 6) = 190.5 - 6 + 6 - 190.5 |
| i) | (236.8 + a) - (46.3 + b) = 190.5 + a - b |
| j) | (236.8 - 3c) - (46.3 - 5c) = 190.5 - 3c + 5c = 190.5 + 2c |

**Q.2**

| a) | 325 × 1.5 = 487.5 |
| b) | (325 × 3) × 1.5 = 487.5 × 3 = 1462.5 |
| c) | 325 × (1.5 × 3) = 487.5 × 3 = 1462.5 |
| d) | (325 ÷ 5) × 1.5 = 487.5 ÷ 5 = 97.5 |
| e) | 325 × (1.5 ÷ 3) = 487.5 ÷ 3 = 162.5 |
| f) | (325 × 0.2) × (1.5 × 4) = 487.5 × 0.2 × 4 = 487.5 × 0.8 |
| g) | (325 ÷ 4) × (1.5 ÷ 3) = 487.5 ÷ 4 ÷ 3 = 487.5 ÷ 12 |
| h) | (325 × 11) × (1.5 ÷ 11) = 487.5 × 11 ÷ 11 = 487.5 |
| i) | (325 ÷ a) × (1.5 ÷ b) = 487.5 ÷ a ÷ b = 487.5 ÷ ab |
| j) | (325 × a) ÷ (1.5 ÷ b) = 487.5 ÷ a × b = 487.5 ÷ b |

**Q.3**

| a) | (x + 2.3) + y = z + 2.3 |
| b) | x + [y + (\(\frac{-4}{5}\))] = z - \(\frac{4}{5}\) |
| c) | [x - (\(\frac{-3}{4}\))] + y = z + \(\frac{3}{4}\) |
| d) | (x + 1.2) + (y + 1.6) = z + 1.2 + 1.6 = z + 2.8 |
| e) | (x - 7) + (y + 7) = z - 7 + 7 = z |
| f) | (x × 4) + y = (x × 3) + x + y = 3x + z |

---

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By measuring: $BC \approx 3.9$ cm, $DC \approx 2.2$ cm, $AD \approx 3.1$ cm

a) i) $P \approx (5.5 + 3.9 + 2.2 + 3.1) \text{ cm} = 14.7 \text{ cm}$

ii) Draw lines to form 3 rectangles (as shown by dotted lines) and measure the unknown lengths, $DE$ and $CF$.

\[
A = (0.8 \times 3 + 2.5 \times 3) \div 2 + (2.2 \times 3) \text{ cm}^2
= (2.4 + 7.5) \div 2 + 6.6 \text{ cm}^2
= 9.9 \div 2 + 6.6 \text{ cm}^2
= 4.95 + 6.6 \text{ cm}^2
= 11.55 \text{ cm}^2
\]

b) Accept any correct reflection with correct labelling. e.g.

Q.5 The 4-digit number could be:
1671 or 1761; 2562 or 2652; 3453 or 3543

No more are possible, as if we use the next digit, 4, for the outside digits, then

$4 + 4 = 8, 15 - 8 = 7, \text{ and } 7 = 3 + 4,$

but neither 3 nor 4 are greater than 4!

Q.6 Son: $\frac{1}{6}$ of $M$, Daughter: $\frac{1}{10}$ of $M$, Dad: $M + 2$

\[
M + (\frac{1}{6} \div \frac{1}{10}) M + M + 2 = 70
\]

\[
2M + \frac{5 + 3}{30} M = 68, \quad 2 \cdot \frac{8}{30} M = 68, \quad \frac{68}{30} M = 68, \quad M = 30
\]

\[
\text{Mum's age: } 30
\]

\[
\text{Dad's age: } 30 + 2 = 32
\]

\[
\text{Son's age: } 30 \div 6 = \frac{5}{2}
\]

\[
\text{Daughter's age: } 30 \div 10 = \frac{3}{2}
\]

Check:

$30 + 32 + 5 + 3 = 70 \checkmark$

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**Y6**

R: Handling data  
C: Review and practice: diagnostic test  
E:

### Activity 1

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(136 = 2 \times 2 \times 2 \times 17 = 2^3 \times 17\)
  - Factors: 1, 2, 4, 8, 17, 34, 68, 136

- \(311\) is a prime number
  - Factors: 1, 311
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and 19^2 > 311)

- \(486 = 2 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^5\)
  - Factors: 1, 2, 3, 6, 9, 18, 27, 54, 81, 162, 243, 486

- \(1136 = 2 \times 2 \times 2 \times 2 \times 71 = 2^4 \times 71\)
  - Factors: 1, 2, 4, 8, 16, 71, 142, 284, 568, 1136

#### Extension

Let’s think of the factors of 486 as a sample set of data. What is the:

a) **range** of the data \((486 - 1 = 485)\)

  Elicit that the range of a set of data is the difference between the smallest and greatest values.

b) **mean** of the data

  \[
  \frac{1 + 2 + 3 + 6 + 9 + 18 + 27 + 54 + 81 + 162 + 243 + 486}{12} = \frac{1092}{12} = 91
  \]

  Elicit that the mean is the average value, i.e. the sum of all the values divided by the number of data values in the set.

c) **median** of the data \(\frac{18 + 27}{2} = \frac{45}{2} = 22.5\)

  Elicit that the median is the middle value in an ordered data set

  (if there is an even number of values, it is the mean of the two middle values)

d) **mode** of the data? (all of them)

  Elicit that the mode is the value which occurs most often, i.e. the most common value.

**10 min**

### Activity 2

#### Revision: Handling data

What is the range, median, mode and mean of these sets of data? Ps come to BB to write and explain. Class points out errors.

a) BB: \{-5.2, 3, 0.7, \(-\frac{1}{5}\), -2, \(\frac{15}{5}\), 4.2\}

i) **range**: \(4.2 - (-5.2) = 9.4\) (middle)

ii) **median**: ordered set: \(-5.2, -2, -1.2, 0.7, 3, 3, 4.2\)

iii) **mode**: 3, as there are two values equal to 3 \((3 \text{ and } \frac{15}{5})\)

iv) **mean**: \(\frac{0.7 + 3 + 3 + 4.2 - (5.2 + 2 + 1.2)}{7}\)

\[
= \frac{(10.9 - 8.4)}{7} = \frac{2.5}{7} = 0.357
\]

or \(\frac{2.5}{7} = \frac{5}{14}\)

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Activity 2

b) BB: \{0.001, 0.01, 0.1, 1\}
   i) range: \(1 - 0.001 = 0.999\)
   ii) median: (set is already in order, with an even no. of values)
      \[BB: (0.01 + 0.1) \div 2 = 0.11 \div 2 = 0.055\]
   iii) mode: All the data values
   iv) mean: \[0.001 + 0.01 + 0.1 + 1 \div 4 = 1.111 \div 4 = 0.27775\]

Lesson Plan 136

Notes

Q.1 A man walks at an average speed of \(4\ \frac{2}{3}\) km/hour.

How far does he walk in \(2\ \frac{2}{3}\) hours?

Solution:

Plan: \(4\ \frac{2}{3}\ \text{km} \times 2\ \frac{2}{3} = \frac{14}{3} \times \frac{8}{3} = \frac{112}{9} \text{ km}\)

Answer: The man walks 11 and 11 fifteenths kilometres in 2 and 2 thirds hours.

Q.2 What is the whole quantity if:

a) \(\frac{6}{7}\) of it is 60 kg: \(60 \text{ kg} \div \frac{6}{7} = 60 \text{ kg} \times \frac{7}{6} = 70 \text{ kg}\)

b) 55\% of it is £273.02: £273.02 \div 0.55 = £27302 \div 55 = £496.40

c) \(\frac{1}{3}\) of it is 14\frac{2}{5} litres: 14.4 litres \div 1.6 = 144 litres \div 16 = 72 litres \div 8 = 9 litres

Q.3 If \(a = 12 \div 3\ \frac{1}{3}\) and \(b = 12 \div 2\ \frac{3}{4}\), what is the value of:

a) \(a = 12 \div 3\ \frac{1}{3} = 12 \div \frac{10}{3} = \frac{36}{3} \times \frac{3}{10} = \frac{18}{5} = \frac{3}{5}\)

b) \(b = 12 \div 2\ \frac{3}{4} = 12 \div \frac{11}{4} = 12 \times \frac{4}{11} = \frac{48}{11} = \frac{4}{11}\)

c) \(a + b = \frac{3}{5} + \frac{4}{11} = 7 + \frac{33 + 20}{55} = \frac{73 + 20}{55} = \frac{93}{55} = \frac{93}{55}\)

f) \(b \div a = \frac{48}{11} \div \frac{18}{5} = \frac{48}{11} \times \frac{5}{18} = \frac{20}{3} = \frac{20}{3} = \frac{7}{33}\)

This Pb page could be used as a diagnostic test in 2 parts:

Part A: Q. 1–5
Part B: Q. 6–8

Allow 20 minutes for each part (working and review).

Review Part A interactively with the whole class before continuing with Part B.

If there is no time for the two parts during a single lesson, Part B could be set as homework and reviewed interactively before the start of Lesson 137.

If done as practice, deal with one question at a time and review interactively after each question as usual, with any mistakes discussed and corrected.

or \(14\ \frac{2}{5} \div \frac{3}{5} = \frac{72}{5} \div \frac{8}{5} = \frac{9}{\frac{72}{5}} \times \frac{\frac{8}{5}}{1} = 9\) (litres)

or \(= 12 \times \frac{3}{10} = \frac{36}{10} = \frac{36}{10}\)

but it is best to stay in fraction form for the remaining calculations

\(= \frac{33}{40}, \text{ so } \frac{a}{b} = \frac{40}{33}\) (as \(\frac{b}{a}\) is the reciprocal of \(\frac{a}{b}\))
Q.4 If $\frac{1}{2} \cdot \frac{2}{5}$ of a number is $\frac{2}{3}$, what is $\frac{3}{2} \cdot \frac{2}{5}$ of the same number?

\[\frac{1}{2} \cdot \frac{2}{5} = \frac{7}{5} \Rightarrow \frac{2}{3} \div \frac{7}{5} = \frac{26}{3} \div \frac{7}{5} = \frac{26}{21} = \frac{1}{21}\]

\[\frac{5}{5} \Rightarrow \frac{1}{5} \times 5 = \frac{25}{21} = \frac{4}{21} \text{ (This is the number.)}\]

3 of \[\frac{6}{21} = \frac{17}{3} \times \frac{430}{21} = \frac{442}{21} = 21 \times \frac{1}{21}\]

Answer: Three and 2 fifths of the same number is $21\frac{1}{21}$.

Q.5 Here is some information about the dimensions of an aluminium cuboid:

\[a = 38.5 \text{ cm}, \ b = 80\% \text{ of } a, \ b = 1 \frac{2}{3} \text{ of } c.\]

Dimensions: \[b = 38.5 \text{ cm} \times 0.8 = 30.8 \text{ cm}\]

\[c = 30.8 \text{ cm} \div 1 \frac{2}{3} = 30.8 \text{ cm} \div \frac{5}{3} = 6.16 \text{ cm} \times \frac{3}{\frac{5}{1}} = 18.48 \text{ cm}\]

a) Calculate the volume of the cuboid.

\[V = a \times c \times b = 38.5 \times 30.8 \times 18.48 \text{ (cm$^3$)}\]

\[\approx 21913.584 \text{ cm}^3\]

b) Calculate the mass of the solid if 1 cm$^3$ of aluminium weighs 2.7 g.

\[M = 21913.584 \times 2.7 = 59166.6768 \text{ g}\]

\[\approx 59.167 \text{ kg}\]

Q.6 a) Draw a square and label its vertices, sides and diagonals.

b) Write true statements about the square, using words or mathematical notation.

\[e \parallel f; \ e \perp f; \ \angle A = \angle B = \angle C = \angle D = 90^\circ\]

DA \perp AB, BC \perp AB, AB \parallel CD, etc.

A square is a regular rectangle.

A square has 4 lines of symmetry and rotational symmetry. etc.

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Y6

Activity

4

(Test 6, Part B, continued)

Q.7  a) Draw a rectangle and label its vertices A, B, C and D.

b) Mark the mid-points of the sides and label them E, F, G and H.

c) Draw the line of symmetry through E and G and mark the midpoint of line segment EG. Label the midpoint O.

d) What are the mirror images of points F, D and O?

F' = H, D' = C, O' = O

e) What are the mirror images of triangles AEG, GCB and AOB?

Δ (AEG)' = Δ BEG, Δ (GCB)' = Δ GDA,
Δ (AOB)' ≡ Δ BOA

Q.8  Reflect triangle ABC in line AB.

Using ruler and set square

1. Lay base of set square along AB, with perpendicular edge against point C, and draw a line from C to AB.

2. Measure this perpendicular line and extend it on the opposite side of AB by the same distance.

3. Label its end point C'.

4. Join A and B to C'.

or

Using ruler and compasses

1. Set the width of the compasses to length AC.

2. With point of compasses on A, draw an arc on the opposite side of AB from C.

3. Set the width of the compasses to length BC.

4. With point of compasses on B, draw an arc on the opposite side of AB from C.

5. Label the intersection of the 2 arcs C'.

6. Join A and B to C'.

55 min

Class applauds Ps with all correct (or the fewest errors) and also Ps with the most improved scores from Test 5.

Notes

BB:

Δ ≡ \text{means 'identical to'}

Accept either method.

T should have BB instruments for Ps to demonstrate their construction to class.

Feedback for T
Week 28

R: Calculations
C: Review and practice: diagnostic test (Geometry)

Notes

Individual work, monitored (or whole class activity)
BB: 137, 312, 487, 1137
T decides whether Ps may use calculators.
Reasoning, agreement, self-correction, praising
e.g. 312

\[ 312 = 2 \times 2 \times 2 \times 3 \times 13 = 2^3 \times 3 \times 13 \]
Factors: 1, 2, 3, 4, 6, 8, 12, 13, 312, 156, 104, 78, 52, 39, 26, 24

8 min

This Pb page could be used as a diagnostic test in 2 parts:

Part A: Q. 1–4
Part B: Q. 5–7

Allow 25 minutes for each part (working and review).
Review Part A interactively with the whole class before continuing with Part B.
If there is no time for the two parts during a single lesson, Part B could be set as homework and reviewed interactively before the start of Lesson 138.
If done as practice, deal with one question at a time and review interactively after each question as usual, with any mistakes discussed and corrected.

or

a) construct two 60° angles and bisect the 2nd angle to form the 90° angle, rather than bisecting a 180° angle, or

construct a 60° angle, bisect it to form two 30° angles and bisect one of the 30° angles.
Q.3  
(a) Draw an **equilateral triangle** which has sides of length 2 cm.
(b) Draw a triangle which has sides 3 times longer than those in the 1st triangle.

![Diagram of two triangles: Triangle 1 and Triangle 2.](image)

**Triangle 1**
- \( a = 2 \text{ cm} \)
- \( h = 1.7 \text{ cm} \)

**Triangle 2**
- \( b = 6 \text{ cm} \)
- \( h = 5.2 \text{ cm} \)

(c) How many times more than the area of the 1st triangle is the area of the 2nd triangle?

Measure the perpendicular height of each triangle (see diagram).

\[
A_1 = \frac{1}{2} \times 2 \times 1.7 = 1.7 \text{ cm}^2
\]
\[
A_2 = \frac{3}{2} \times 6 \times 5.2 = 15.6 \text{ cm}^2
\]
\[
\frac{A_2}{A_1} = \frac{15.6}{1.7} = \frac{156}{17} = 9.2
\]

Answer: The area of the 2nd triangle is about 9 times more than the area of the 1st triangle.

(c) How many times more than the perimeter of the 1st triangle is the perimeter of the 2nd triangle?

\[
P_1 = 3 \times 2 = 6 \text{ cm}
\]
\[
P_2 \approx 3 \times 6 = 18 \text{ cm}
\]
\[
\frac{P_2}{P_1} = \frac{18}{6} = 3
\]

Answer: The perimeter of the 2nd triangle is 3 times more than the perimeter of the 1st triangle.
Q.4  

a) Construct an isosceles triangle which has a side of length 4 cm as its base and angles of 75° at its baseline.

b) Measure the necessary data then calculate the perimeter of the triangle.

\[ AC = BC = 7.7 \text{ cm} \]
\[ P \approx 4 \text{ cm} + (2 \times 7.7 \text{ cm}) = 4 \text{ cm} + 15.4 \text{ cm} = 19.4 \text{ cm} \]

c) Calculate the area of the triangle.

Measure the perpendicular height. (\( h \approx 7.5 \text{ cm} \))
\[ A \approx \frac{2 \times 7.5}{2} \text{ cm}^2 = 15 \text{ cm}^2 \]
**Activity** 3

PbY6b, page 137

Q.5 a) Construct a deltoid which has sides of length 4 cm and 6 cm and the length of the diagonal which lies on its line of symmetry is 8 cm.

b) Calculate its perimeter.

\[ P = 2 \times (4 + 6) \text{ cm} = 2 \times 10 \text{ cm} = 20 \text{ cm} \]

c) Measure the necessary data, then calculate its area.

Measure the other diagonal: \( BD \approx 5.8 \text{ cm} \)

\[ A = \frac{AC \times BD}{2} = \frac{4.8 \times 5.8}{2} \text{ cm}^2 = 23.2 \text{ cm}^2 \]

Q.6 Two opposite angles of a deltoid are 50° and 110°. Calculate the size of the other two angles.

The sum of the angles in any quadrilateral is 360°.

\[ \angle B = \angle D = \frac{360° - (110° + 50°)}{2} \]
\[ = \frac{360° - 160°}{2} = \frac{200°}{2} = 100° \]

*Answer:* The other two angles are each 100°.
### Activity

<table>
<thead>
<tr>
<th>Test 7, Part B, continued</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Q.7</strong></td>
</tr>
<tr>
<td>a) <strong>Construct a rhombus</strong> which has diagonals 8 cm and 5 cm long.</td>
</tr>
<tr>
<td>b) <strong>Measure the distance between two opposite sides.</strong></td>
</tr>
<tr>
<td>(e.g. the perpendicular distance between AB and DC, as shown in diagram, or BC and AD:  ( h \approx 4.2 \text{ cm} ))</td>
</tr>
</tbody>
</table>

**Diagram:**

- **Construction**
  1. Mark a point A and draw a ray.
  2. Set compasses to 8 cm and with point of compasses on A, mark point C on the ray.
  3. Set compasses to an appropriate width and draw 2 arcs around A and around C above and below AC.
  4. Draw a line through the 2 points of intersection. This is the perpendicular bisector of AC.
  5. Set compasses to 2.5 cm. With point of compasses on point of intersection of the 2 diagonals, mark points B and D on the perpendicular bisector of AC.
  6. Join B and D to A and C. ABCD is the required rhombus.

**Diagram:**

- c) **Measure the angles and add them together.**
  \[ \angle A = \angle C \approx 64^\circ, \quad \angle B = \angle D \approx 116^\circ \]
  \[ \sum \text{angles} = 2 \times (64^\circ + 116^\circ) = 2 \times 180^\circ = 360^\circ \]
  (Extra praise for Ps who realised that they did not need to calculate, as the sum of the angles in any quadrilateral is 360°.)

- d) **Calculate the perimeter of the rhombus.**
  Measure the length of a side:  \( a \approx 4.7 \text{ cm} \)
  \[ P = 4 \times 4.7 \text{ cm} = 18.8 \text{ cm} \]

- e) **Calculate the area of the rhombus.**
  \[ A = \frac{AC \times BD}{2} = \frac{8 \times 5}{2} \text{ cm}^2 = 20 \text{ cm}^2 \]

**Extension**

| Class applauds Ps with all correct (or the fewest errors) and also the Ps chosen by the T as having the neatest drawings. |

### Notes

**Construction**

1. Mark a point A and draw a ray.
2. Set compasses to 8 cm and with point of compasses on A, mark point C on the ray.
3. Set compasses to an appropriate width and draw 2 arcs around A and around C above and below AC.
4. Draw a line through the 2 points of intersection. This is the perpendicular bisector of AC.
5. Set compasses to 2.5 cm. With point of compasses on point of intersection of the 2 diagonals, mark points B and D on the perpendicular bisector of AC.
6. Join B and D to A and C. ABCD is the required rhombus.

(\( \sum \) means 'sum of')

Elicit that a rhombus is a deltoid which has equal sides, and its area is half the product of its diagonals (i.e. half the area of the dotted rectangle shown in the diagram).

(See dashed rectangle in diagram.)

**Feedback for T**
## Activity 1

### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **138**  =  $2 \times 3 \times 23$  
  Factors: 1, 2, 3, 6, 23, 46, 69, 138

- **313** is a prime number  
  Factors: 1, 313 
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and 19; $19^2 > 313$)

- **488**  =  $2 \times 2 \times 2 \times 61$  
  Factors: 1, 2, 4, 8, 61, 122, 244, 488

- **1138**  =  $2 \times 569$  
  Factors: 1, 2, 569, 1138 
  (569 is not divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, and $29^2 > 569$)

---

### Solving equations

Let’s solve these equations. What does ‘solve’ mean? (Find out what number could be written instead of the letter to make the equation true.)

Ps come to BB or dictate to T, explaining reasoning. Class points out errors and checks that the solution is correct. T asks Ps to think of a word problem for which the equation is the solution. e.g.

a) $\frac{d}{2} = 60$  
   $d = 60 \times 2 = 120$  
   e.g. If a car travelled at a steady speed of 60 miles/hour for 2 hours, what distance would it have covered? (120 miles)

b) $100 \times t = 1500$  
   $t = \frac{1500}{100} = 15$  
   e.g. How long would it take a train travelling at an average speed of 100 km/hour to cover a distance of 1500 km? (15 hours)

c) $s \times 4 = 80$  
   $s = \frac{80}{4} = 20$  
   e.g. If a cyclist covered a distance of 80 km in 4 hours, what was his average speed? (20 km/hour)

d) $30 = 2 \times b \times 5$  
   $b = \frac{30}{2 \times 5} = \frac{30}{10} = 3$  
   e.g. What is the width of a cuboid which has length 2 cm, height 5 cm and volume 30 cm$^3$?

etc. Ps could suggest other equations if there is time.

---

### Notes

- Individual work, monitored (or whole class activity)
- BB: 138, 313, 488, 1138
- T decides whether Ps may use calculators.
- Reasoning, agreement, self-correction, praising

---

### Lesson Plan

- **Y6**
  - **R:** Calculations
  - **C:** Review and practice: diagnostic test
  - **E:** Formulae

---

### Extra praise for unexpected contexts.
Q.1 The cross-section of a 3.5 m long pine beam is a 16 cm square. If 1 m³ of pinewood weighs 500 kg, what is the mass of the beam?

\[
V = (0.16 \times 0.16 \times 3.5) \text{ m}^3 = (0.0256 \times 3.5) \text{ m}^3 = 0.0896 \text{ m}^3
\]

\[
M = 500 \text{ kg} \times 0.0896 = 5 \text{ kg} \times 8.96 = 44.8 \text{ kg}
\]

Answer: The mass of the beam is 44.8 kg.

Q.2 A container shaped like a 35 cm cube was filled with water. We ladled out half of the water, then ladled out \(\frac{2}{5}\) of the water. How much water was left in the container?

Give your answer in litres.

\[
V = (35 \times 35 \times 35) \text{ cm}^3 = (1225 \times 35) \text{ cm}^3 = 42875 \text{ cm}^3
\]

So 42875 cm³ → 42875 ml

Amount of water remaining:

\[
42875 \text{ litres} \times \frac{1}{2} \times \frac{3}{5} = 42875 \text{ litres} \times \frac{3}{10} = 128625 \text{ litres}
\]

Answer: The amount of water left in the container was about 12.9 litres.

Q.3 The spire of a church is shaped like a pyramid. The edges of its square base are 3.5 m long and each of its side faces is 5.2 m high. How many m² of tin plate are needed to cover the spire?

\[
A = \frac{2 \times 3.5 \times 5.2}{2} \text{ m}^2 = 2 \times 18.2 \text{ m}^2 = 36.4 \text{ m}^2
\]

Answer: To cover the spire, 36.4 m² of tin is needed.
Activity

3

(Test 8, Part A, continued)

Q.4 The volume of a square-based pyramid can be calculated using this formula:

\[ V = \frac{A \times h}{3} \]

where \( A \) is the area of the base and \( h \) is the height of the pyramid.

How high is the pyramid if its base edge is 36 cm and its volume is 17289 cm\(^3\)?

Plan:

\[ h = \frac{V \times 3}{A} = \frac{17289 \times \frac{1}{3}}{36 \times \frac{36}{12}} \quad \text{cm} = \frac{1921}{48} \text{ cm} = 40 \text{ cm} \]

Answer: The pyramid is about 40 cm high.

Lesson Plan 138

Notes

or Ps might do 3 separate calculations: work out the area of the base first, then multiply the volume by 3, then divide this product by the area of the base.

<table>
<thead>
<tr>
<th>4</th>
<th>8</th>
<th>1</th>
<th>9</th>
<th>2</th>
<th>1</th>
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<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Tell Ps not to be dismayed by this calculation but to work through it carefully doing one step at a time.

Class applauds Ps who did it correctly but also praise Ps who made a good attempt.

b) or \( \frac{5}{6} \) of \( \frac{5}{7} \) kg

\[ = \frac{26}{7} \text{ kg} \div 6 \times 5 \]

\[ = \frac{26}{42} \text{ kg} \times 5 \]

\[ = \frac{13}{21} \text{ kg} \times 5 = \frac{65}{21} \text{ kg} \]

or \( \frac{7}{8} \text{ m} \div \frac{7}{2} \)

\[ = \frac{25}{84} \text{ m} \times \frac{7}{7} \]

\[ = \frac{25}{4} \text{ m} \]

\[ = 6 \frac{1}{4} \text{ m} \]
### Q.6
A tailor bought 35 rolls of a certain material. Each roll originally contained 26.5 m of material but the tailor has already used 19 and 3 quarter rolls.

**How many men’s suits can he make from the remaining material if each suit needs on average 3.1 m of material?**

In steps:

- **Rolls left:** $35 - 19 \frac{3}{4} = 15 \frac{1}{4}$
- **Material left:** $26.5 \times 15 \frac{1}{4} = 404.125$ m
- **No: of suits:** $\frac{404.125}{3.1} \approx 130.36$

**Answer:** The tailor could make 130 suits from the remaining material. [Ext: 1.125 m would be left over.]

### Q.7
I spent 9.5% of my money and had £304.08 left. How much money did I have at first?

- **Spent:** 9.5%     **Had left:** 100% – 9.5% = 90.5% → £304.08
- **Plan:** £304.08 ÷ 0.905 = £336

**Answer:** I had £336 at first.

### Q.8
52% of the 350 pupils in a school are girls. How many girls and how many boys attend this school?

- **G:** 52% of 350 = $350 \times 0.52 = 182$
- **B:** 350 – 182 = 168

**Check:** 182 + 168 = 350 ✓

**Answer:** 182 girls and 168 boys attend this school.

### Q.9
The edge of a container shaped like a cube is 24 cm. A second container shaped like a cuboid holds the same amount of liquid. If the base edges of the second container are 36 cm and 24 cm, how high is it?

- **$V_1$** = $(24 \times 24 \times 24)$ cm$^3$ = **$V_2$** = $(24 \times 36 \times h)$ cm$^3$
- **So** $24 \times 24 = 36 \times h$

**Answer:** The height of the second container is 16 cm.

---

**Erratum**

In P3: 'taylor' should be 'tailor'

---

**Notes**

- **Lesson Plan 138**

**Class applauds Ps with all correct (or the fewest errors) and also the Ps who have made most progress from Test 7.**

---

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### Activity

#### 1 Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(139\) is a prime number

Factors: \(1, 139\)

(as not exactly divisible by \(2, 3, 5, 7, 11\) and \(13^2 > 139\))

- \(314 = 2 \times 157\)

Factors: \(1, 2, 157, 314\)

- \(489 = 3 \times 163\)

Factors: \(1, 3, 163, 489\)

- \(1139 = 17 \times 67\)

Factors: \(1, 17, 67, 1139\)

**8 min**

#### 2 Sequences

Let’s write the first 5 terms of these sequences if \(n = 1, 2, 3, 4,\) etc.

Who can explain what we should do? (Substitute 1 for \(n\) to get the 1st term, 2 for \(n\) to get the 2nd term, 3 for \(n\) to get the 3rd term, etc.)

Ps calculate mentally or on scrap paper or slates, then come to BB or dictate what T should write. Class points out errors and agrees on another form of the rule (where possible).

**BB:**

a) \(a_n = \frac{2}{5} n - 1:\) \((-\frac{3}{5}, -\frac{1}{5}, \frac{1}{5}, \frac{3}{5}, 1, \ldots\) )

*Rule:* Increasing by 2 fifths from \(-3\) fifths \([\text{or } + \frac{2}{5}\] )

b) \(b_n = 14.2 - 6.5n:\) \((7.7, 1.2, –5.3, –11.8, –18.3, \ldots\) )

*Rule:* Decreasing by 6.5 from 7.7 \([\text{or } –6.5]\]

c) \(c_n = \frac{n \times n - 2n + 1}{3} :\)

\((0, \frac{1}{3}, \frac{4}{3}, 3, \frac{16}{3}, \ldots)\)

or \((0, \frac{1}{3}, \frac{1}{3}, 3, \frac{5}{3}, \ldots)\)

or \((\frac{0^2}{3}, \frac{1^2}{3}, \frac{2^2}{3}, \frac{3^2}{3}, \frac{4^2}{3}, \ldots)\)

d) \(d_n = \frac{n - 2}{n} :\)

\((-1, 0, \frac{1}{3}, \frac{1}{2}, \frac{3}{5}, \ldots)\)

or \((-\frac{1}{1}, \frac{0}{2}, \frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \ldots)\)

**15 min**

### Lesson Plan

#### Notes

Individual work, monitored (or whole class activity)

BB: \(139, 314, 489, 1139\)

T decides whether Ps may use calculators.

Reasoning, agreement, self-correction, praising

e.g. \[
\begin{array}{c|c|c|c|c}
3 & 489 & \text{3} \\
163 & 314 & \text{163} \\
157 & \text{2} & \text{1} \\
67 & \text{1139} & \text{17} \\
67 & \text{1} & \text{1} \\
\end{array}
\]

Whole class activity

Written on BB or SB or OHT

At a good pace

Involve several Ps.

Reasoning, agreement,

praising

Elicit that \(\frac{2}{5}n\) means \(\frac{2}{5} \times n\)

Point out that:

\[
\frac{n \times n - 2n + 1}{3}
\]

can be written as

\[
\frac{n^2 - 2n + 1}{3}
\]

Agree that in c) and d) the rules are best described by the given formulae, as it is difficult to explain them in words.

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Q.1 Deal with a), b) and c) one at a time. Ps work in Ex. Bks under a time limit. T could suggest that Ps draw a tree diagram for part c).

Review with whole class. T chooses a P to read out each question and Ps show answers on scrap paper or slates on command. Ps answering correctly come to BB to explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) i) In how many ways can you put a blue and a white counter in order?
   
   Possible orders: b w, w b (2 ways)

   ii) If it is done randomly, what is the probability that the order will be blue, white?

   1 chance out of 2, so \( p(b \text{ w}) = \frac{1}{2} \)

b) i) How many ways are there of putting in order a blue, a white and a red counter?

   Possible orders: b w r, b r w, w b r, r b w, r w b (6 ways)

   ii) If the orders happen at random, what is the probability that the order will be red, blue, white?

   1 chance out of 6, so \( p(r \text{ b w}) = \frac{1}{6} \)

c) i) How many ways are there of putting in order a blue, a white and a red and a green counter?

   Possible orders:
   
   24 ways

   ii) If the orders happen at random, what is the probability that the order will be white, red, green, blue?

   1 chance out of 24, so \( p(w \text{ r g b}) = \frac{1}{24} \)

Let’s show the number of orders which are possible for different numbers of colours in a table. Ps come to BB or dictate what T should write for the results above, explaining reasoning in words. What about the numbers that we don’t know? (Circled in table below)

How can we work out what they should be? Ps make suggestions.

<table>
<thead>
<tr>
<th>BB: Number of different colours</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of possible orders</td>
<td>1</td>
<td>2</td>
<td>6</td>
<td>24</td>
<td>120</td>
<td>720</td>
<td>5040</td>
<td>40 320</td>
</tr>
</tbody>
</table>

4 colours: \( 4 \times 3 \times 2 \times 1 = 24 \) (ways)

5 colours: \( 5 \times 4 \times 3 \times 2 \times 1 = 5 \times 24 = 120 \) (ways), etc.
Lesson Plan 139

Notes

Individual work, monitored, helped (or c) done with whole class)

(T could have labelled cut-out horses stuck on BB.)

Responses shown in unison.

Reasoning, agreement, self-correction, praising

or by calculation:

\[3 \times 2 \times 1 = 6\]

[For each of the 3 possible horses for 1st place, there are 2 possible horses for 2nd place and for each of these there is 1 possible horse for 3rd place.]

or by calculation:

\[4 \times 3 \times 2 = 24\]

[For each of the 4 possible horses for 1st place, there are 3 possible horses for 2nd place and for each of these there are 2 possible horses for 3rd place.]

Ps come to BB or dictate to T, explaining in words too.

[For each of the 5 possible horses for 1st place there are 4 possible horses for 2nd place, and for each of these there are 3 possible horses for 3rd place.]

Whole class activity

Ps come to BB or dictate to T.

Class agrees/disagrees.

Praising

If no P has an idea, T gives hint or writes the operation and asks Ps if it is correct.

Agreement, praising

---

PbY6b, page 139

Q.2 Deal with a), b) and c) one at a time as in Q.1. Ps work in Ex. Bks under a time limit.

Review with whole class. T chooses a P to read out each question and Ps show answers on scrap paper or slates on command. Ps answering correctly come to BB to explain reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) i) Three horses, A, B and C, are running in a race. How many orders are possible for 1st, 2nd and 3rd places?

Possible orders:

ABC, ACB, BAC, BCA, CAB, CBA  \((6 \text{ orders})\)

ii) If each of the different orders has an equal chance of happening, what is the probability of the order C, A, B?

\(1\) chance out of \(6\), so \(p (\text{CAB}) = \frac{1}{6}\)

b) i) Four horses, A, B, C and D, are running in another race. How many orders are possible for 1st, 2nd and 3rd places?

\(24\) orders

\[\begin{array}{c|c|c|c}
\text{1st} & \text{2nd} & \text{3rd} \\
\hline
(5) & (4) & (3) \\
\end{array}\]

No. of possible orders:

\(5 \times 4 \times 3 = 60\)

\(p (\text{CAB}) = \frac{1}{24}\)  \(\text{(as 1 chance out of 24)}\)

ii)

c) i) Five horses, A, B, C, D and E, are running in a 3rd race. How many orders are possible for 1st, 2nd and 3rd places?

This time, let’s show the possible orders in a table. How many possibilities for 1st place (2nd place, 3rd place)?

BB:

\[\begin{array}{c|c|c|c}
\text{1st} & \text{2nd} & \text{3rd} \\
\hline
(5) & (4) & (3) \\
\end{array}\]

No. of possible orders:

\(5 \times 4 \times 3 = 60\)

\(p (\text{CAB}) = \frac{1}{60}\)  \(\text{(as 1 chance out of 60)}\)

Extension

If there were 17 (40) horses in a race, how could we calculate how many possible orders there would be for 1st, 2nd and 3rd places?

BB: \(17 \times 16 \times 15\)  \((40 \times 39 \times 38)\)

What if we did not know the number of horses in the race and called the number \(n\)? How could we calculate the number of orders?

BB: \(n \times (n - 1) \times (n - 2)\)  \(\text{(where } n \leq 3, \text{ and a natural number)}\)

Elicit that \(n\) must be a positive whole number, as we cannot have parts of a horse or a negative horse, and if \(n\) was less than 3, there could not be a 3rd place!
**Activity 5**

**PbY6b, page 139**

Q.3 Read: Two white marbles and one red marble are in a bag. If you take out a marble with your eyes shut, what is the probability of each of these outcomes?

Deal with one part at a time. Encourage Ps to picture what is happening in their heads. Ask Ps to write the answer using probability notation in Ex. Bks. e.g. BB: $p \text{(red)} = ?$

Review with whole class. T chooses Ps to read out the actions. Ps show probabilities on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class points out errors and agrees on correct answer. Mistakes discussed and corrected

**Solution:**

a) You take out the red marble. $p \text{(red)} = \frac{1}{3}$

b) You take out a white marble. $p \text{(white)} = \frac{2}{3}$

c) You take out the red marble, replace it, then take out the red marble again.

$p \text{(red, red)} = \frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$

d) You take out a white marble, then take out the other white marble.

$p \text{(white, white)} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

e) You take out a white marble, replace it then take out a white marble again.

$p \text{(white, white)} = \frac{2}{3} \times \frac{2}{3} = \frac{4}{9}$

f) You take out a white marble, replace it then take out the red marble.

$p \text{(white, red)} = \frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$

g) You take out the red marble, replace it then take out a white marble.

$p \text{(red, white)} = \frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

**Notes**

Individual work, monitored, helped

T could have real marbles, or coloured discs, in a bag for demonstration in case there is disagreement.

Responses shown in unison. Reasoning, agreement, self-correction, praising

38 min

The probability that the marble is red the first time is 1 chance out of 3, but if it is replaced, the probability that red is drawn the 2nd time is also 1 chance out of 3.

If it is not replaced, there are only 2 marbles left in the bag, so the chance of drawing the 2nd white marble is 1 out of 2.

T reminds Ps that when there are two conditions to be met we multiply the 2 probabilities.

(There is less chance of both conditions happening than of only one happening.)
**Lesson Plan 139**

**Notes**

Whole class activity
(or individual trial first if Ps wish, reviewed with whole class)

In good humour!
Praising

Ps make suggestions. T shows diagram if Ps do not think of it. (see below)

---

**Activity**

PbY6b, page 139, Q.4

Read: If each member of a group shakes hands with each of the others, how many handshakes occur if there are:

- a) 2 members in the group
- b) 3 members in the group
- c) 4 members in the group
- d) 5 members in the group
- e) 11 members in the group
- f) n members in the group?

Ask Ps to picture the handshakes in their heads, then T asks several Ps what they think. Demonstrate with Ps shaking hands at front of class and class keeping count of the number of handshakes.

How could we show it mathematically? (e.g. use letters or numbers for the people in the group, or show in a diagram using dots for people)

[Agree that each member of the group shakes hands with each of the other members but that, e.g. A–B and B–A is the same handshake! e.g. in a group of 3, each of the 3 people shakes hands with 2 others (3 × 2) but each handshake involves 2 people, so the actual number of handshakes is \( \frac{3 \times 2}{2} = 3 \)]

**Solution:**

a) 2 members: A–B (1) (B–A is the same handshake as A–B)

b) 3 members: A–B, A–C, B–C (3 different handshakes)

c) 4 members: 1–2, 1–3, 1–4, 2–3, 2–4, 3–4 (6 handshakes)

or \( \frac{2 \times 4 \times 3}{2} = 6 \)

d) 5 members: 1–2, 1–3, 1–4, 1–5, 2–3, 2–4, 2–5, 3–4, 3–5, 4–5

or \( \frac{5 \times 4 \times 3}{2} = 10 \)

e) 11 members: (Too many to write out or draw, so let’s just calculate.)

BB: \( \frac{11 \times 10}{2} \) = 55

f) n members: BB: \( \frac{n \times (n - 1)}{2} \) (Have no expectations for this!)

If Ps cannot do it, T writes it and asks Ps if it is correct and why.

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Activity

Factorising 140, 315, 490 and 1140. Revision and practice.

PbY6b, page 140

Solutions:
Q.1 a) and b)

3 lines of symmetry

AB = BC = CA, \( \angle A = \angle B = \angle C = 60^\circ \).
CM \perp AB, \( \angle A + \angle B + \angle C = 180^\circ \).
\( \Delta AMC \cong \Delta BMC \), etc. \( MC\parallel AB \), \( \angle A + \angle B + \angle C = 180^\circ \), etc.
The point of intersection of the lines of symmetry is the centre of the triangle.

d) Yes, all equilaterals are similar because the size of their angles does not change, so they are always the same shape.

Q.2 a) Length = Volume ÷ area of triangular face
\[ = \frac{720}{(6 \times 8)} \text{ cm} = \left( \frac{720}{48} \right) \text{ cm} = \left( \frac{60}{4} \right) \text{ cm} = 15 \text{ cm} \]
b) \( A = (2 \times 48 \text{ cm}^2) + 3 \times (10 \times 15) \text{ cm}^2 \)
\[ = 96 \text{ cm}^2 + 3 \times 150 \text{ cm}^2 = 96 \text{ cm}^2 + 450 \text{ cm}^2 = 546 \text{ cm}^2 \]

Q.3 a) Whole quantity: \( \£60 \div \frac{5}{7} = \frac{\£60 \times 7}{5} = \£84 \)
b) Whole quantity: \( \£272.8 \div 0.11 = \£2728 \div 11 = \£248 \)
c) Whole quantity: \( 12 \frac{3}{5} \text{ litres} \div 2 \frac{1}{3} = \frac{63}{5} \div \frac{7}{3} \) (litres)
\[= \frac{63}{5} \times \frac{3}{7} \text{ litres} = \frac{27}{5} \text{ litres} = 5 \frac{2}{5} \text{ litres} \]

Q.4 \( V_{\text{cube}} = (8 \times 8 \times 8) \text{ cm}^3 = (64 \times 8) \text{ cm}^3 = 512 \text{ cm}^3 \)
Height of water level
in cuboid: \( 512 \div (4 \times 4) \text{ cm} = 512 \div 16 \text{ (cm)} = 128 \div 4 \text{ (cm)} = 32 \text{ cm} \)

Answer: The water level will reach a height of 32 cm.

Notes

140 = \( 2^2 \times 5 \times 7 \)
Factors: 1, 2, 4, 5, 7, 10, 14, 20, 28, 35, 70, 140

315 = \( 3^2 \times 5 \times 7 \)
Factors: 1, 3, 5, 7, 9, 15, 21, 35, 45, 63, 105, 315

490 = \( 2 \times 5 \times 7^2 \)
Factors: 1, 2, 5, 7, 10, 14, 35, 49, 70, 98, 245, 490

1140 = \( 2^2 \times 3 \times 5 \times 19 \)
Factors: 1, 2, 3, 4, 5, 6, 10, 12, 15, 19, 20, 30, 38, 57, 60, 76, 95, 114, 190, 228, 285, 380, 570, 1140
(or set factorising as homework at the end of Lesson 139 and review at the start of Lesson 140.)
Activity

Solutions (continued)

Q.5 100% – 7.5% = 92.5%  
94.35 kg ÷ 0.925 = 94350 kg ÷ 925 = 102 kg  
Answer: I weighed 102 kg before the race.

Q.6  
a) \( p\) (red) = \( \frac{2}{6} = \frac{1}{3} \)  
b) \( p\) (blue) = \( \frac{3}{6} = \frac{1}{2} \)  

c) \( p\) (white, white) = \( \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \)  
d) \( p\) (red, red) = \( \frac{1}{3} \times \frac{1}{5} = \frac{1}{15} \)  
e) \( p\) (blue, blue) = \( \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} \)  
f) i) \( p\) (blue, blue, blue) = \( \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \)  
ii) \( p\) (2 different colours) = 1 - \( \frac{1}{8} \) = \( \frac{7}{8} \)  
(i.e. you do not take out 3 blue marbles)

Q.7 Accept any valid contexts but stress that the different outcomes must have an equal chance of happening.

Notes

Lesson Plan 140

If you take out a red marble and do not replace it, there will be 5 marbles left in the bag and only 1 of them red.