### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- \(141 = 3 \times 47\) Factors: 1, 3, 47, 141
- \(316 = 2 \times 2 \times 79 = 2^2 \times 79\) Factors: 1, 2, 4, 79, 158, 316
- \(491\) is a prime number Factors: 1, 491 (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and 23^2 > 491)
- \(1141 = 7 \times 163\) Factors: 1, 7, 163, 1141

**Notes**

- Individual work, monitored (or whole class activity)
- BB: 141, 316, 491, 1141
- T decides whether Ps can use calculators.
- Reasoning, agreement, self-correction, praising

### Activity 2

**Sets**

a) Who can tell me a true statement about all the shapes in this base set?

BB: \(B = \{\begin{array}{c}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
\end{array}\}\)

e.g. (They are quadrilaterals.)

For which of the shapes in Set B is this statement true?

BB: *The quadrilaterals have at least one pair of perpendicular sides.*

T points to each shape in turn and Ps say 'True' or 'False'. Let's call this true subset T. Ps dictate what T should write.

BB: \(T = \{\begin{array}{c}
1 \\
3 \\
4 \\
\end{array}\}\)

b) Here is a base set of certain integers.

BB: \(B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3\}\)

For which of these numbers is this statement true?

BB: \(2 - x > 3\)

T points to each number in turn and Ps say whether or not it should be in the true subset and why. Class agrees/disagrees.

BB: \(T = \{-5, -4, -3, -2\}\)

c) This time, the base set is all the even numbers,

BB: \(B = \{\text{Even numbers}\}\)

and the true statement is: \(6 + x = 9\)

Elicit that no even number can make the statement true.

We say that the true set for this statement, based on the given base set of even numbers, is an *empty* set and write it like this.

BB: \(T = \{\emptyset\}\)

d) BB: \(B = \{\text{Positive numbers}\}\)

True statement: \(2 + y > 0\)

Elicit that all the numbers in the base set make the statement true.

The inequality is always true for all the numbers in the base set, so we can write: BB: \(T = B = \{\text{Positive numbers}\}\)

**Notes**

- Whole class activity
- Drawn (stuck) on BB or SB or OHT (or use enlarged copy master)
- Agreement, praising

Ps write details on BB. e.g.

\[
2 - (-5) = 2 + 5 = 7 > 3 \ (T)
\]

\[
2 - (-1) = 2 + 1 = 3 \n\not= 3 \ (F)
\]

Ask several Ps what they think. Extra praise if a P notices that \(x = 9 - 6 = 3\), but 3 is an odd number so is not in Set B.

BB: \(T = \{\emptyset\}\)

i.e. The true set *is* the base set.
## Y6

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### Notes

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### Q.1 Read:

The base set (B) of numbers for this question is:

\[ B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\} \]

Which of these numbers can be used instead of the letters to make the statements true?

Set a time limit. Ask Ps to write their answers in the form:

\[ T = \{ \ldots \} \]

Review with whole class. Ps come to BB to explain reasoning and to write the true set. Class agrees/disagrees. Mistakes discussed and corrected. T points out that the ‘true set’ is the solution to the equation or inequality.

**Solution:**

- **a) 5.4 + \_ = 3.4, \_ = 3.4 – 5.4 = –2 \ T = \{-2\}**

  If the base set was all the numbers you know, what would the true set be? (It would still be \( T = \{-2\} \). If the base set was only positive numbers, what would the true set be?

  Elicit that the true set would be empty: \( T = \{\emptyset\} \) as no positive number can make the statement true.

- **b) –3 + x = 4, x = 4 – (–3) = 4 + 3 = 7**

  but 7 is not an element of the base set, so the answer is:

  \( T = \{\emptyset\} \), i.e. it is an empty set.

- **c) \( y \text{ is divisible by 3} \) \ T = \{-3, 0, 3\} \ (i.e. multiples of 3)\**

- **d) 3 + z < 9, z < 9 – 3, z < 6, \ so \ T = B = \{-3, -2, -1, 0, 1, 2, 3, 4, 5\}**

- **e) \(-2.6 \times t \geq 2\)**

  Ps might try substituting each possible value:

  - \(-2.6 \times (-3) = 7.8 \) (T), \(-2.6 \times (-2) = 5.2 \) (T),
  - \(-2.6 \times (-1) = 2.6 \) (T), \(-2.6 \times 0 = 0 \) (F),
  - \(-2.6 \times 1 = -2.6 \) (F), \(-2.6 \times 2 = -5.2 \) (F),
  - \(-2.6 \times 3 = -7.8 \) (F), \(-2.6 \times 4 = -10.4 \) (F),
  - \(-2.6 \times 5 = -13 \) (F)

  so the true set is: \( T = \{-3, -2, -1\} \)

23 min
Y6

Activity

4  PbY6b, page 141

Q.2 Read: Which numbers in the given base set (B) can be used instead of the letters to make the equations true?

Set a time limit or deal with one or two at a time. Remind Ps to check their answers by substituting each value in the true set for the letter.

Review with whole class. Ps could show numbers on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) \( x - 9 = 3 \)  \( B = \{ \text{whole numbers} \} \)
   \( x = 3 + 9 = 12 \)
   \( T = \{ 12 \} \)

b) \( y + 8 = 7 \)  \( B = \{ 0, 1, 2, 3, 4 \} \)
   \( y = 7 - 8 = -1 \)
   Check: \(-1 + 8 = 7 \)  \( \checkmark \)
   but \(-1\) is not an element of \( B \), so \( T = \{ \varnothing \} \)

c) \( z - 6 = 6 - z \)  \( B = \{ 5 \frac{1}{3}, 5 \frac{2}{3}, 6, 6 \frac{1}{3}, 6 \frac{2}{3} \} \)
   \( z + z = 6 + 6 \)
   \( z = 6 \)
   Check: \( 6 - 6 = 6 - 6 \)  \( \checkmark \)
   \( T = \{ 6 \} \)

d) \( 3 + t \times 3 = (3 + t) \times 3 \)  \( B = \{ -2, -1, 0, 1, 2, 3, 4, 5 \} \)
   \( t \times 3 = 9 + t \times 3 - 3 \)
   \( t \times 3 = 6 + t \times 3 \)
   By substitution, none of the numbers in the base set makes the equation true, so \( T = \{ \varnothing \} \)

e) \( 3 \times t + 3 = (t + 1) \times 3 \)  \( B = \{ -7, -3 \frac{1}{5}, -0.21, 0, 0.375, 6 \frac{1}{7} \} \)
   \( 3 \times (t + 1) = (t + 1) \times 3 \)
   By substitution, all the numbers in set \( B \) make the equation true, so \( T = B = \{ -7, -3 \frac{1}{5}, -0.21, 0, 0.375, 6 \frac{1}{7} \} \)

f) \( 4 \times u - 2 = u + 10 \)  \( B = \{ -1, 4, 9, 14 \} \)
   \( 4 \times u = u + 12 \)
   \( 3 \times u = 12 \)
   \( u = 4 \)
   Check: \( 4 \times 4 - 2 = 14 = 4 + 10 \)  \( \checkmark \)
   \( T = \{ 4 \} \)

g) \( |v + 3| = v + 3 \)  \( B = \{ -5, -4, -3, -2, -1, 0, 1, 2, 3 \} \)
   Elicit or remind Ps that \(|v + 3|\) is read as ‘the absolute value of \( v + 3 \)’ and means its distance from zero, i.e. its numerical value disregarding whether it is positive or negative.
   Substitute each number in the base set for \( v \) to see whether it makes the equation true. \((-5 \text{ and } -4 \text{ do not make it true.}) \)
   \( T = \{ -3, -2, -1, 0, 1, 2, 3 \} \)

Notes

Individual work, monitored, helped
Discussion, reasoning, checking, agreement, self-correction, praising
Feedback for T

BB: absolute value
\( e.g. \) \(|-2| = 2, |+2| = 2 \)
\( -2 \)
\( 2 \)
\( 0 \)
\( 2 \)
\( -2 \)
\( 0 \)
\( 2 \)
\( 0 \)
\( -2 \)

\( e.g. \) \(|-5 + 3| = |-2| = 2 \)
\( \neq -5 + 3 = -2 \)
Activity

Lesson Plan 141

h) \((w + 1) \times (w - 2) = 0\) \(B = \{-3, -2, -1, 0, 1, 2, 3\}\)
By substituting each number in set \(B\) for \(w\) in the equation:
\((-3 + 1) \times (-3 - 2) = (-2) \times (-5) = 10 \neq 0\)
\((-2 + 1) \times (-2 - 2) = (-1) \times (-4) = 4 \neq 0\)
\((-1 + 1) \times (-1 - 2) = 0 \times (-3) = 0\)
\((0 + 1) \times (0 - 2) = 1 \times (-2) = -2 \neq 0\)
\((1 + 1) \times (1 - 2) = 2 \times (-1) = -2 \neq 0\)
\((2 + 1) \times (2-2) = 3 \times 0 = 0\)
\((3 + 1) \times (3 - 2) = 4 \times 1 = 4 \neq 0\)
So \(w = -1\) or \(w = 2\)
\(T = \{-1, 2\}\)

Y6

PbY6b, page 141

Q.3 Read: Which numbers can be written instead of the letters to make the statements true? Solve the equations and inequalities in your exercise book.

What is missing from this question? (There is no base set given.) \(T\) tells Ps that in such cases, they should take the base set is being all the numbers that they have learned. Elicit what they are: integers (positive and negative whole numbers and 0), positive and negative fractions and decimals.

\(T\): We call all these numbers the set of rational numbers (BB).

They are shown mathematically by using a capital 'Q'.

Set a time limit or deal with one at a time. Ps might use trial and error but encourage Ps to work it out logically where possible. Remind Ps to check their results by substitution.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

Solution:

a) \(a + b = b + a\) (The set of rational numbers)
\[\text{e.g. } -\frac{1}{3} + 4 = 4 +\left(-\frac{1}{3}\right) \quad T = Q\]

b) \(x - (-4) = 2.1\)
\[x = 2.1 + (-4) \quad \text{or } x + 4 = 2.1 \quad \text{or } x = 2.1 - 4 = -1.9\]

c) \(y \times (-5) = 30\)
\[y = 30 \div (-5) = -30 \div 5 = -6\]

d) \(2 \times (x + 1) = 2 \times x + 5\)
\[2x + 2 = 2x + 5\]
Impossible, as \(2x = 2x\) but \(2 \neq 5\), so \(T = \emptyset\)

e) \((u - 2) \times 3 = -6 + 3 \times u\)
\[u - 2 = -2 + u \quad \text{(The set of rational numbers)} \quad \text{or } u + (-2) = -2 + u\]
[Similar to part a): \(T = Q\)]
### Lesson Plan 141

#### Notes

**Check:**

If \( z = 3 \):
\[
44 \div 3 = 14 \frac{2}{3} < 11
\]

If \( z = 5 \):
\[
44 \div 5 = 8.8 < 11
\]

Show it on the number line.

**To T:**

Multiplication is commutative.

In this question, trial and error done in a logical way as opposite might be easier to understand.

Show what is happening on the number line as \( t \) becomes greater and smaller.

However, give extra praise if a P suggests taking \( t \) away from each side of the inequality, giving the result: \( -3 \geq t \)

Whole class activity

(or individual work if Ps wish, reviewed with whole class)

Discussion, reasoning, checking, agreement, praising

Allow Ps to write and explain, with help of class.

T interferes only if Ps are stuck.

**Check:**

If \( n = 5 \):
\[
(5 \times 4 + 28) \div 4 - 5 = 48 \div 4 - 5 = 12 - 5 = 7
\]

Ps check other numbers too.

---

### Activity

#### 5

(Continued)

\[ f) \quad 44 \div z < 11 \]

If \( 44 \div z = 11 \), \( z = 44 \div 11 = 4 \)

As the quotient is less than 11, \( z \) must be greater than 4.

i.e. \( z > 4 \).

\[ g) \quad u \times v = v \times u \quad (The \ set \ of \ rational \ numbers: \ T = Q) \]

(Any rational number can be substituted for \( u \) and \( v \) and the equation will be true, as the factors in a multiplication can be interchanged and the product will be the same.

**h) \quad t - 3 \geq 2 \times t**

If \( t = 1 \), \(-2 \geq 2 \)

\( t = 2 \), \(-1 \geq 4 \)

\( t = 10 \), \( 7 \geq 20 \)

\( t = 0 \), \(-3 \geq 0 \)

\( t = -1 \), \(-4 \geq -2 \)

\( t = -2 \), \(-5 \geq -4 \)

\( t = -3 \), \(-6 \geq -6 \)

\( t = -4 \), \(-7 \geq -8 \)

\( t = -10 \), \(-13 \geq -20 \)

so \( t \leq -3 \)

---

#### 6

**PbY6b, page 141. Q.4**

Read: Write an equation about the relationship between the given data. Solve the equation, then check your result in context.

Deal with one part at a time. T chooses a P to read out the question.

Who can write an equation about it? P comes to BB to write and say it. Class agrees/disagrees.

What should we do to solve this equation? Who agrees? Who thinks something else? P writes solution on BB, explaining reasoning. Class checks that the solution is correct by substitution.

**Solution:**

\[ a) \quad I \ think \ of \ a \ number. \ If \ I \ subtract \ 8 \ from \ 3 \ times \ my \ number, \ the \ result \ is \ 19. \ What \ is \ my \ number? \]

BB: \( 3 \times n - 8 = 19 \) (or \( 3n - 8 = 19 \))

\[ 3 \times n = 19 + 8 = 27 \]

\[ n = 27 \div 3 = 9 \]

**b) \quad I \ think \ of \ a \ number. \ If \ I \ divide \ my \ number \ by \ 5, \ then \ subtract \ 11 \ from \ the \ quotient, \ the \ result \ is \ 8. \ What \ is \ my \ number? \]

BB: \( n \div 5 - 11 = 8 \)

\[ n = 8 + 11 = 19 \]

\[ n = 19 \times 5 = 95 \quad n = 95 \]

**c) \quad I \ think \ of \ a \ number. \ I \ add \ 28 \ to \ 4 \ times \ my \ number, \ then \ divide \ the \ sum \ by \ 4. \ I \ subtract \ my \ number \ from \ the \ quotient \ and \ the \ difference \ is \ 7. \ What \ is \ my \ number? \]

BB: \( (n \times 4 + 28) \div 4 - n = 7 \)

\[ n + 7 - n = 7, \quad n + 7 = 7 + n \quad (\text{So } n \text{ is any rational number: } T = Q) \]

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<td><strong>Week 29</strong></td>
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<tr>
<td><strong>1 Factorisation</strong></td>
<td><strong>Notes</strong></td>
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<tr>
<td>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes. Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:</td>
<td>Individual work, monitored (or whole class activity)</td>
</tr>
<tr>
<td>• $142 = 2 \times 71$ Factors: 1, 2, 71, 142</td>
<td>BB: 142, 317, 492, 1142</td>
</tr>
<tr>
<td>• 317 is a prime number Factors: 1, 317 (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and 19 and $19^2 &gt; 317$)</td>
<td>T decides whether Ps can use calculators.</td>
</tr>
<tr>
<td>• $492 = 2 \times 3 \times 41 = 2^2 \times 3 \times 41$ Factors: 1, 2, 3, 4, 6, 12, 41, 82, 123, 164, 246, 492</td>
<td>Reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>• $1142 = 2 \times 571$ Factors: 1, 2, 571, 1142 (571 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23 and $29^2 &gt; 571$)</td>
<td>e.g.:</td>
</tr>
<tr>
<td><strong>8 min</strong></td>
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<tr>
<td><strong>2 Inequalities</strong></td>
<td>Whole class activity</td>
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<tr>
<td>Let’s show the answer to each question by writing an inequality. T asks the question and Ps come to BB to write and explain. Class agrees/disagrees. Ask Ps to show some of the inequalities on the number line. Class points out errors. e.g.</td>
<td>At a good pace</td>
</tr>
<tr>
<td>Which whole numbers are:</td>
<td>Agreement, praising</td>
</tr>
<tr>
<td>a) more than $– 5$ but less than $+ 3$ BB: $– 5 &lt; x &lt; 3$</td>
<td>Draw appropriate segments of number line on BB or show on class number line. e.g.</td>
</tr>
<tr>
<td>T: To show that $x$ is a whole number, we can also write: $x \in \mathbb{Z}$ ‘$\in$’ stands for the set of whole numbers or integers and ‘$\in$’ means ‘is an element of’</td>
<td>BB:</td>
</tr>
<tr>
<td>b) not less than $– 6$ but less than $+ 16$ BB: $– 6 \leq y &lt; 16$, $y \in \mathbb{Z}$</td>
<td>BB: <strong>Set of integers</strong> ($\mathbb{Z}$) (whole numbers)</td>
</tr>
<tr>
<td>c) not greater than $– 5$ and less than $– 2$ BB: $z \leq – 5$, $z \in \mathbb{Z}$ (If $z$ is not greater than $– 5$, it can be equal to $– 5$ or less than $– 5$, and numbers equal to or less than $– 5$ are also less than $– 2$)</td>
<td>BB:</td>
</tr>
<tr>
<td>d) at least $– 2$ and at most $+ 1$ BB: $– 2 \leq u \leq 1$, $u \in \mathbb{Z}$</td>
<td>BB:</td>
</tr>
<tr>
<td>e) positive but less than 4. BB: $0 &lt; v &lt; 4$, $v \in \mathbb{N}$ (If $v$ is a positive whole number, it is a natural number.)</td>
<td>BB:</td>
</tr>
<tr>
<td><strong>13 min</strong></td>
<td>Whole class activity</td>
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<tr>
<td><strong>3 Numbers written as operations</strong></td>
<td>At a fast pace</td>
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<tr>
<td>Answer each question by writing an operation. T asks the questions. Ps come to BB or dictate what T should write. Class agrees/disagrees. T shows the short form of notation where relevant.</td>
<td>Agreement, praising</td>
</tr>
<tr>
<td>Write the number which is:</td>
<td>Elicit or remind Ps that, e.g. $b \times c$ can be written as $bc$</td>
</tr>
<tr>
<td>a) i) $5$ more than $3$ $(3 + 5)$ ii) $x$ more than $3$ $(3 + x)$</td>
<td>Praising, encouragement only</td>
</tr>
<tr>
<td>ii) $3$ more than $y$ $(y + 3)$</td>
<td>Feedback for T</td>
</tr>
<tr>
<td>b) i) $7$ less than $x$ $(x – 7)$ ii) $u$ less than $7$ $(7 – u)$</td>
<td></td>
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</table>
c) i) 3 times \( t \) \( (3 \times t \text{ or } 3t) \) ii) \( s \) times \( 5 \) \( (s \times 5 \text{, or } 5s) \)
iii) \( b \) times \( c \) \( (b \times c, \text{ or } bc) \)

d) i) a quarter of \( x \) \( (x \div 4, \text{ or } \frac{x}{4}) \) ii) a third of \( y \) \( (y \div 3, \text{ or } \frac{y}{3}) \)

ii) 2 sevenths of \( z \) \( (z \div 7 \times 2, \text{ or } z \times \frac{2}{7}) \)

\[ 18 \text{ min} \]

Individual work, monitored, helped
Table (and axes for Extension) drawn on BB or use enlarged copy master or OHT

First elicit that:
• integers are whole numbers
• natural numbers are positive whole numbers
• rational numbers are positive and negative whole numbers, fractions, decimals and zero.

Reasoning, agreement, self-correction, praising
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<td><strong>5</strong></td>
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<td><strong>PbY6b, page 142</strong></td>
<td>Written on BB or use enlarged copy master or OHT</td>
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<tr>
<td><strong>Q.2</strong></td>
<td>Differentiation by time limit</td>
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<tr>
<td>Read: <em>Write a formula about the relationship between the data.</em></td>
<td>Responses shown in unison.</td>
</tr>
<tr>
<td>*Set a time limit. Ps write formulae in <em>Pbs.</em></td>
<td>Reasoning, agreement, self-correction, praising</td>
</tr>
<tr>
<td>Review with whole class. T chooses Ps to read out the relationship descriptions and Ps show formulae on scrap paper or slates on command. Class decides which formulae are correct. If there is disagreement, ask Ps to explain by drawing a diagram on BB. Mistakes discussed and corrected. [Elicit or show the short algebraic forms where relevant.]*</td>
<td>Feedback for T</td>
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<tr>
<td><strong>Solution:</strong></td>
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<tr>
<td>a) the area of a rectangle with sides <em>a</em> and <em>b</em></td>
<td></td>
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<tr>
<td>[ A = a \times b = ab ]</td>
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<tr>
<td>b) the perimeter of a rectangle with sides <em>e</em> and <em>f</em></td>
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<tr>
<td>[ P = 2 \times (e + f) = 2 \times e + 2 \times f \quad (= 2e + 2f) ]</td>
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<tr>
<td>c) the area of a square with side <em>c</em></td>
<td></td>
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<tr>
<td>[ A = c \times c = c^2 ]</td>
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<tr>
<td>d) the perimeter of a square with side <em>t</em></td>
<td></td>
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<tr>
<td>[ P = 4 \times t = 4t ]</td>
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<tr>
<td>e) the area of a square with diagonal <em>e</em></td>
<td></td>
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<tr>
<td>[ A = \frac{e \times e}{2} = \frac{e^2}{2} ]</td>
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<tr>
<td>f) the surface area of a cube with edge <em>c</em></td>
<td></td>
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<tr>
<td>[ A = 6 \times c \times c = 6c^3 ]</td>
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<tr>
<td>g) the volume of a cube with edge <em>a</em></td>
<td></td>
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<tr>
<td>[ V = a \times a \times a = a^3 ]</td>
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<tr>
<td>h) the volume of a cuboid with edges <em>a</em>, <em>b</em>, <em>c</em>.</td>
<td></td>
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<tr>
<td>[ V = a \times b \times c = abc ]</td>
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| 28 min |  |
| **** |  |
| **6** | Individual work, monitored, (helped) |
| **PbY6b, page 142** | Differentiation by time limit |
| **Q.3** | Responses shown in unison. |
| Read: *The difference between two numbers is 19.* | Reasoning, agreement, self-correction, praising |
| Ps read questions themselves and write the answers as operations involving *x* or *y*. | |
| Review with whole class. T chooses Ps to read out the questions and Ps show answers on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Mistakes discussed and corrected. | |
| **Solution:** | |
| a) What is the other number if: | |
| \[ \begin{align*} \text{i) the smaller number is } x & \quad \text{[}x + 19\text{]} \\
| \text{ii) the greater number is } y & \quad \text{[}y - 19\text{]} \end{align*} \] | |
| b) Write the sum of the two numbers using only one letter, *x*. | |
| \[ x + x + 19 = 2 \times x + 19 \quad (= 2x + 19) \] | |
| c) What are the two numbers if their sum is 40? | |
| \[ 2x + 19 = 40, \quad 2x = 40 - 19 = 21, \quad x = 10.5 \] | |
| The smaller number is **10.5** and the larger number is **10.5 + 19 = 29.5**. | |
| Check: **29.5 – 10.5 = 19** | |

| 32 min |  |
**Activity**

7  **PbY6b, page 142**

**Q.4**  Read: A natural number is 3 times another natural number.

What is a natural number? (A natural number is a positive whole number.) Ps read questions themselves and write the answers as operations involving y.

Review with whole class. T chooses Ps to read out the questions and Ps show answers on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) *If the smaller number is y, what is the greater number?*
   
   [The greater number is $3 \times y$ (or $3y$)]

b) *Write the sum of the two numbers.*
   
   [$y + 3 \times y = 4 \times y$ (or $y + 3y = 4y$)]

c) *Calculate the smaller number if the sum of the two numbers is 324.*
   
   $[4 \times y = 324, y = 324 \div 4 = 81]$
   
   The smaller number is 81.

---

8  **PbY6b, page 142, Q.5**

Read: In a box there are b apples. In a second box there are 7 apples more than b. In a third box there are 5 apples less than b.

T draws 3 'boxes' on BB.

a) *Read: How many apples are in each box?*
   
   Ps come to BB or dictate what T should write. Class agrees/disagrees.

b) *Read: How many apples are in the 3 boxes altogether?*
   
   T allows Ps half a minute to think about it and write in Ex. Bks.
   
   A, come and show us what you think. Who agrees? Who thinks it should be something else? Why?
   
   BB: Total number of apples: $b + b + 7 + b - 5 = 3 \times b + 2$

**c) How many apples are in the first box if there are 77 apples in all 3 boxes?**

I will give you a minute to work it out. Show me . . . now! (25)

P answering correctly explains reasoning at BB. Another P checks the answer. Mistakes corrected.

BB: $3b + 2 = 77$

$3b = 77 - 2 = 75$

$b = 75 \div 3 = 25$

**Answer:** There are 25 apples in the first box.

---

9  **Secret numbers**

a) I am thinking of a natural number. If I add 3 to 4 times my number, I will get 31. What is the number I am thinking of? (7)

b) I am thinking of another natural number. If I take it away from 7 times the number, I will get 102. What is my number? (17)

---

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**Lesson Plan**

**Week 29**

**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(143 = 11 \times 13\)
  - Factors: 1, 11, 13, 143

- \(318 = 2 \times 3 \times 53\)
  - Factors: 1, 2, 3, 6, 53, 106, 159, 318

- \(493 = 17 \times 29\)
  - Factors: 1, 17, 29, 493

- \(1143 = 3 \times 3 \times 127 = 3^2 \times 127\)
  - Factors: 1, 3, 9, 127, 381, 1143

Individual work, monitored (or whole class activity)

**Notes**

Individual work, monitored (or whole class activity)

BB: 143, 318, 493, 1143

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

**Activity 2**

**Equations and inequalities**

Let’s write a mathematical statement about each balance and solve it one step at a time. Unless we are told otherwise, we will assume that the base set of numbers to choose from is all the numbers we have learned.

Ps come to BB to say what they see in the diagram and write an equation or inequality. Elicit that when the two sides of the balance are level, it means that they are equal. If one side of the balance is lower than the other, it means that side is more than the other side. Class discusses how to work out the solution. Stress that what is done to one side of the balance must also be done to the other side to keep the balance in the same position. Ps say what they are doing at each step in a loud voice.

BB:

3 \times x + 5 = 12.5

Take 5 away from each side:

3 \times x = 7.5

Change to:

\(\frac{2}{1} + \frac{1}{2} + \frac{3}{1}\)

Divide each side by 3:

\(\frac{2}{1}\)

\(x = 2.5\)

\(2y + 30 = 3y + 10\)

[Take 2y from each side]

\(y + 10 = 30\)

[Take 10 from each side]

\(y = 20\)

Whole class activity

Drawn (stuck) on BB or use enlarged copy master or OHP (or use a real balance and equal bags/boxes and weights)

At a good pace

Agreement praising

T helps when necessary.

Ps could think of a context for each diagram.

(e.g. Bags of potatoes + kg weights. How many kg of potatoes are in each bag?)

\(3 \times 2.5 + 5\)

\(= 7.5 + 5 = 12.5\)

\(3 \times 20 + 10 = 70\)

RHS: 2 \times 20 + 30 = 70
Y6

**Activity**

2  
(Continued)

c) Elicit that LHS is more.

\[ 2t + 20 > t + 50 \]

[Take \( t \) from each side]

\[ t + 20 > 50 \]

[Take 20 from each side]

\[ t > 30 \]

\[ 20 \text{ min} \]

3  
*PbY6b, page 143*

Q.1 Read: Solve the equations and check your results.

Tell Ps that unless the base set is specified, they should take it as being all the numbers they have learned (the rational numbers).

Set a time limit or deal with one row at a time.

Review with whole class. Ps could show solutions on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Class checks their results mentally by substitution. Mistakes discussed and corrected.

Ask Ps to reason by generalising in words using the names of the components of the operations.

**Solution:**

a) \( x + 2.7 = 11 \), \( x = 11 - 2.7 = 8.3 \)

b) \(-6.2 + y = 3\), \( y = 3 - (-6.2) = 3 + 6.2 = 9.2 \)

To calculate the unknown term in a 2-term addition, subtract the known term from the sum.

c) \( z - (-3) = -2 \), \( z = -2 + (-3) = -5 \)

To calculate the reductant, add the subtrahend to the difference.

d) \( \frac{x}{4} \times 3 = \frac{9}{4} \), \( \frac{x}{4} = \frac{9}{4} \div 3 = \frac{3}{4} \), \( x = \frac{3}{4} \)

To calculate the multiplicand, divide the product by the multiplier.

e) \( u \div (-3) = 6 \), \( u = 6 \times (-3) = -18 \)

To calculate the dividend, multiply the quotient by the divisor.

f) \( (-42) \div v = 6 \), \( v = (-42) \div 6 = -7 \)

To calculate the divisor, divide the dividend by the quotient.

\[ 25 \text{ min} \]
**Y6**

**Activity**  
4  
*PbY6b, page 143*

Q.2 Read: *Solve the inequalities.*

Set a time limit or deal with one at a time. Ps can use any method they wish (including trial and error).

Review with whole class. Ps come to BB to explain reasoning. Who agrees with the answer? Who worked it out a different way? Give extra praise if Ps reason using the 'balance' model.

Ask Ps to show the solution to part a) on the number line.

**Solution:** e.g.

a) \( a + (-5) < -13 \)

\[ \text{e.g. If } a + (-5) = -13, \ a = -13 - (-5) = -13 + 5 = -8 \]

but \( a + (-5) < -13 \), so \( a < -8 \)

Check: If \( a = -9 \): \(-9 + (-5) = -14 < -13\)

or using the 'balance' model:

\[ a + (-5) < -13 \][Subtract (-5) from each side]

\[ a < -13 - (-5) = -13 + 5 \]

\[ a < -8 \]

BB:

<table>
<thead>
<tr>
<th>-12</th>
<th>-10</th>
<th>-8</th>
<th>-6</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b) \( b - \left( -\frac{7}{6} \right) \geq \frac{5}{9} \)

Check:

\[ b + \frac{7}{6} \geq \frac{5}{9} \]

\[ b \geq \frac{5}{9} - \frac{1}{6} = \frac{10 - 3}{18} = \frac{7}{18} - \left( -\frac{21}{18} \right) = \frac{28}{18} \]

\[ b \geq \frac{7}{18} \] [Check for \( b = \frac{6}{18} \) too.]

c) \( c - (+63) \leq -17 \)

\[ c \leq -17 + 63 \]

\[ c \leq 46 \] Check: If \( c = 47 \), \( 47 - 63 = -16 \)

\[ c \leq 46 \] Check: If \( c = 47 \), \( 47 - 63 = -16 \)

\[ c \leq 46 \]

Check:

\[ d + \frac{2}{3} > -2\frac{1}{4} \]

\[ d + \frac{8}{12} > -2\frac{3}{12} \] [Subtract \( \frac{8}{12} \) from each side.]

\[ d > -2\frac{11}{12} \]

**30 min**

---

**Lesson Plan 143**

**Notes**

Individual work, monitored, helped

Written on BB or SB or OHT

Differentiation by time limit.

Reasoning, agreement, self-correction, praising

Accept any valid reasoning.

**Extension**

T helps Ps to check the results. (See below)

If \( a = -7 \), \(-7 + (-5)\)

\[ = -12 \]

\[ -13 \]

\[ \checkmark \]

Elicit/remind Ps about the notation for showing inequalities on the number line:

- **black** (closed) circle above the number if it is included;
- **white** (open) circle above the number if it is not included.

If \( b = \frac{8}{18} \),

\[ \frac{8}{18} - \left( -\frac{21}{18} \right) = \frac{29}{18} \]

\[ = \frac{1}{11} > \frac{10}{18} \]

\[ \checkmark \]

If \( c = 46 \), \( 46 - 63 = -17 \)

\[ \checkmark \]

If \( c = 45 \), \( 45 - 63 = -18 \)

\[ \text{and} -18 < -17 \]

\[ \checkmark \]

Check:

If \( d = -2\frac{10}{12} \),

\[ -2\frac{10}{12} + \frac{8}{12} = -2\frac{2}{12} = -2\frac{1}{6} \]

\[ \text{and} -2\frac{1}{6} > -2\frac{1}{4} \]

\[ \checkmark \]
Lesson Plan 143

**Activity**

5  

*PbY6b, page 143*

Q.3  Read: Write an equation about the diagram. Solve the equation by changing the sides equally. Follow the steps.

What do you notice about this set of balances? (They are level, so the RHS = the LHS) Make sure that the RHS equals the LHS of your equations!

Set a time limit of 2 minutes. Ps write in *Pbs*.

Review with whole class. Ps come to BB to write equations and explain reasoning, saying what they have done to each side. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[ x + 17 = 2x + 1 \]

[Subtract \( x \) from each side.]

\[ 17 = x + 1 \]

[Subtract 1 from each side]

\[ 16 = x \]

or \( x = 16 \)

---

6  

*PbY6b, page 143*

Q.4  Read: Write an inequality about the diagram. Solve the inequality by changing the sides equally. Follow the steps.

What do you notice about this set of balances? (LHS is more than RHS) Make sure that the LHS is more than the RHS in your inequalities!

Set a time limit of 4 minutes. Ps write in *Pbs*.

Review with whole class. Ps come to BB to write inequalities and explain reasoning, saying what they have done to each side. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

\[ y + 7 > 4y + 1 \]

[\(-y\)]

\[ 7 > 3y + 1 \]

[\(-1\)]

\[ 6 > 3y \]

[\(\div 3\)]

\[ 2 > y \]

or \( y < 2 \)
**Y6**

**Activity**

**PbY6B, page 143, Q.5**

Read: *Draw diagrams to help you solve this equation.*

**BB:** \(3x - 2 = x + 6\)

Allow Ps a minute to think about it, discuss with their neighbours and try out ideas. T asks Ps for their suggestions.

If no P has a good idea (it is difficult because of the \(-2\)) T suggests a cash and debt model. e.g.

\[
\begin{array}{c}
\text{£1} \\
\text{-£1}
\end{array}
\]

means £1 in cash, i.e. +1, \(\begin{array}{c}
\text{-£1} \\
\end{array}\)

means £1 in debt, i.e. −1

Let’s show the equation using these symbols. Ps come to BB to draw symbols (or stick pre-prepared cards) on BB. Class points out errors. Ps suggest what to do to get the following line, and Ps come to BB in pairs to carry it out (redrawing or manipulating the symbols) while a 3rd P writes the matching equation. Check the solution.

**BB:** e.g.

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
</table>
| \(\begin{array}{c}
\text{£1} \\
\text{-£1}
\end{array}\) | \(\begin{array}{c}
\text{£1} \\
\text{£1} \\
\text{£1}
\end{array}\) |
| \(3x - 2 = x + 6\) | [+ 2] |

\[
\begin{array}{c}
\text{£1} \\
\text{£1} \\
\text{£1}
\end{array}
\]

\[
\begin{array}{c}
\text{£1} \\
\text{£1}
\end{array}
\]

\[
\begin{array}{c}
\text{£1}
\end{array}
\]

Check:

**LHS:** \(3 \times 4 - 2 = 10\)

**RHS:** \(4 + 6 = 10 \checkmark\)

**Extension**

Could we have done it this way? Is it correct? What do you think of it?

T shows prepared solution and asks several Ps what they think.

Elicit that it is correct but adding 7 to both sides does nothing to get closer to the solution. The solution above is better as every step makes progress (although the 1st and 2nd steps could be interchanged).

---

**Lesson Plan 143**

**Notes**

Whole class activity

(or individual trial first if Ps wish)

T has circles and rectangles already prepared (see diagram).

Discussion, agreement, praising

Extra praise if a P thinks of the cash and debt model.

At a good pace

In good humour.

Reasoning, agreement, praising

Ps could write the equations in *Ex. Bks.* too.

Elicit that:

\[
-2 + 2 = 0
\]

\[
x = 1 \times x
\]

\[
3x - x = 2 \times x
\]

\[
2x ÷ 2 = 1 \times x = x
\]

**Check:**

**LHS:** \(3 \times 4 - 2 = 10\)

**RHS:** \(4 + 6 = 10 \checkmark\)

**BB:** \(3x - 2 = x + 6\) [+ 7]

\[
3x + 5 = x + 13 \quad [-x]
\]

\[
2x + 5 = 13 \quad [÷ 2]
\]

\[
x + 2.5 = 6.5 \quad [-2.5]
\]

\[
x = 4
\]
R: Calculations
C: Equations and inequalities      Using the 'balance' method
E: Generalising relationships (Algebraic expressions)

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2 \quad (= \left (2^2 \times 3 \right )^2 = 12^2)$

  Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

  (Square number. No. of factors: \((4 + 1) \times (2 + 1) = 5 \times 3 = 15\))

- $319 = 11 \times 29$

  Factors: 1, 11, 29, 319

- $494 = 2 \times 13 \times 19$

  Factors: 1, 2, 13, 19, 26, 38, 247, 494

- $1144 = 2 \times 2 \times 2 \times 11 \times 13 = 2^3 \times 11 \times 13$

  Factors: 1, 2, 4, 8, 10, 11, 13, 22, 26, 44, 88, 143, 104, 572, 286, 144, 572, 286, 143, 104, 52, 44

  $\text{Factors: } 1, \ldots, 8 \text{ min}$

2

Solving equations and inequalities

Let's solve this equation and inequality, keeping in mind the two sides of a balance or set of scales.

What do you think the symbols mean?

$(-x)$ means £x in debt, $+x$ means £x in cash

a) Study this diagram. Who can tell me what it means in words?

(£14 in cash and £2x in debt is equal to £3x in cash and £1 in debt.)

Let's write it as a mathematical equation. Ps come to BB or dictate what T should write. Class agrees/disagrees.

Now let's solve it by changing each side equally. T or Ps suggest what to do at each step and Ps come to BB in pairs, one to draw (or stick on) symbols and the other to write the matching equation.

Class points out errors and checks the result.

BB:

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
<th>e.g.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-2$</td>
<td>$2$</td>
<td>$-2 \times x + 14 = 3 \times x - 1 \quad [+1]$</td>
</tr>
</tbody>
</table>

| $-10$ | $1$ | $-2 \times x + 15 = 3 \times x \quad [+2 \times x]$ |
| $-2$ | $1$ | $15 = 5 \times x \quad [\div 5]$ |

$3 = x \quad \text{or } \quad x = 3$

Check:

LHS: $-2 \times \frac{3}{2} + 14 = 8$

RHS: $3 \times \frac{3}{2} - 1 = 8$
b) Now study this diagram. Who can tell me what it means in words? (£3x in debt and £1 in cash is less than £x plus £10 in cash.)

Let’s write it as an inequality. Ps come to BB or dictate what T should write. Class agrees/disagrees.

Now let’s solve it by changing each side equally. T or Ps suggest what to do at each step and Ps come to BB in pairs.

Class points out errors. Check possible values for x.

BB:

<table>
<thead>
<tr>
<th>LHS</th>
<th>RHS</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-\frac{3}{4}x + 1)</td>
<td>(x + 10)</td>
</tr>
</tbody>
</table>

\[\frac{3}{4}x \leq 4\]

\[x \leq \frac{4}{3}\]

\[x \geq -\frac{2.25}{1}\]

-agree that if the context is money, the decimal form of the solution is better.

Discussion, reasoning, agreement, praising

At a good pace

Involve several Ps.
**Y6**

### Activity

**Letters in equations**

a) How could we write '17 apples plus 8 apples' in a shorter way?

BB: \(17a + 8a = 25a\)

We have written the first letter as an abbreviation of 'apple'.

What if the letter \(a\) meant the price of an apple? How would we write the equation then? Ps come to BB or dictate to T.

BB: \(17 \times a + 8 \times a = 25 \times a\)

but we could also write this too as:

\(17a + 8a = 25a\)

What else could \(a\) mean? (e.g. the mass of an apple)

b) How could we write mathematically '84 bananas divided by 12'?

BB: \(84b \div 12 = 7b\) (b is an abbreviation for banana)

What if the letter \(b\) means the mass of a banana? How would we write the division then?

BB: \(84 \times b \div 12 = 7 \times b\) or \(84b \div 12 = 7b\)

What else could \(b\) mean? (e.g. the price of a banana)

c) What could this mean if \(c\) is an abbreviation?

BB: \(30c \div 5c = 6\)

e.g. 30 chairs divided into groups of 5 chairs equals 6 (times).

T: We could also think of \(c\) as being the price of a chair. Then the division could mean the ratio of the chair prices.

BB: \(30 \times c \div 5 \times c = 6\)

\(\rightarrow 30 \times c : 5 \times c = 6 \rightarrow 30c : 5c = 6 \rightarrow 6c : c = 6\)

It means that the ratio of the price of 6 chairs to the price of 1 chair is 6, or that 6 chairs cost 6 times as much as 1 chair.

---

**Lesson Plan 144**

### Notes

Whole class activity

Ps come to BB or dictate what T should write. Class agrees/disagrees.

BB: abbreviation

(shortened form of a word)

Discussion, reasoning, agreement, praising

**To T:**

Note that in printed material (Pbs, LPs, etc.) letters which represent unknown amounts (variables) are shown in italic but this of course cannot be done on the BB or in Ex Bks.

T leads Ps to realise that in mathematics letters can be used to represent many things but they are always treated in the same way.

\((\rightarrow 6 : 1 = 6)\)

---

**PbY6b page 144**

Q.1 Read: Write each operation using abbreviations (e.g. 'a' instead of apricots) then do the operation.

Set a time limit of 3 minutes. Ps work in Ex. Bks.

Review quickly with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) 140 apricots + 27 apricots = 52 apricots

\([140a + 27a - 52a = 167a - 52a = 115a]\)

b) 150 apples \(\div 5\)

\([150a \div 5 = 30a]\)

c) 83 boxes \(\times 3\)

\([83b \times 3 = 249b]\)

d) 63 stamps \(\div 9\) stamps

\([63s \div 9s = 7\] times, not stamps!\)

e) \(\frac{4}{7}\) of 84 potatoes

\([\frac{4}{7} \text{ of } 84p = \frac{4}{7} \times 84p = 48p]\)

or \(\frac{4}{7}\) of 84p = 84p \(\div 7 \times 4 = 12p \times 4 = 48p\]

f) 4 apples + 10 apples + 5 bananas - 2 apples - 4 bananas

\([4a + 10a + 5b - 2a - 4b = 12a + 1b = 12a + b]\)

---

Individual trial, monitored, (helped)

Reasoning, agreement, self-correction, praising

63 stamps divided into groups of 9 stamps gives 7 groups

The apples and the bananas are calculated separately!
Lesson Plan 144

Notes

Individual work, monitored, helped
Drawn (stuck) on BB or use enlarged copy master or OHP
Accept any valid steps but tell Ps to make sure that, at each stage, both sides of the equation balance (are equal).

Responses shown in unison.
Reasoning, checking, agreement, self-correction, praising

Y6

Activity

5  PbY56, page 144

Q.2 Read:  Study the diagram to help you understand the equation. Solve the equation and check the result. (The first step is given.)

Elicit that the circles mean positive amounts and the rectangles mean negative amounts (e.g. a cash and debt model)

Set a time limit of 3 minutes. Ps copy what is given in Pbs in Ex. Bks and continue the steps. (If Ps can solve the equation without diagrams, they need not draw them.)

Review with whole class. Ps could show solution on scrap paper or slates on command. Ps with correct answer explain reasoning at BB. (T could have symbols already prepared to make this easier and quicker.) Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution:

\[
\begin{align*}
10 & \quad 0 & \quad 3 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
10 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 & \quad 0 \\
\end{align*}
\]

\[54 - 3 \times x = 6 \times x - 18 \quad [+ 18]\]

\[72 - 3 \times x = 6 \times x \quad [+ 3 \times x]\]

\[72 = 9 \times x \quad [\div 9]\]

\[\frac{72}{9} = 8 = x \quad \text{or} \quad x = 8\]

Check:

LHS: \[54 - 3 \times 8 = 54 - 24 = 30 \quad \checkmark\]

RHS: \[6 \times 8 - 18 = 48 - 18 = 30 \quad \checkmark\]

30 min

6  PbY6b, page 144

Q.3 Read:  Write an inequality about the diagram. Solve the inequality and check your result by filling in the table.

Ask Ps to write the inequality above the diagram in Pbs first, then review it and make sure that any mistakes are corrected before Ps solve it in Ex. Bks and check it in table in Pbs.

Set a time limit. Review with whole class. Ps show solution on scrap paper or slates on command. Ps with correct answer explain reasoning at BB. Who did the same? Who did it a different way? Ps come to BB to complete the table as a check. Class points out errors. Mistakes discussed and corrected.

Solution:

\[2y + 3 \geq 3y - 7 \quad [\text{or} \quad 2y + 3 \geq 3y + (-7)]\]

Check:

\[
\begin{array}{c|cccccccc}
& -1 & 0 & 3 & 7 & 9 & 10 & 11 & 16 \\
\hline
y \quad \text{LHS} & 1 & 3 & 9 & 17 & 21 & 23 & 25 & 35 \\
\text{RHS} & -10 & -7 & 2 & 14 & 20 & 23 & 26 & 41 \\
\hline
& \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark & \checkmark \\
\end{array}
\]

\[2y + 3 \geq 3y - 7 \quad [-2y]\]

\[3 \geq y - 7 \quad [+7]\]

\[10 \geq y \quad \text{or} \quad y \leq 10\]

Ask Ps to show the solution on the class number line.

35 min

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**Y6**

**Activity**

Q.4 Read: **Solve the equations and check the results in your exercise book.**

Set a time limit or deal with one part at a time. Ps may draw diagrams if they wish, but encourage Ps to try without them.

Review with whole class. Ps show solution on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected.

**Solution:**

- **a)** \(5 \times x = 2 \times x + 12\)  
  \(3 \times x = 12\)  
  \(x = 4\)

- **b)** \(5 \times y + 3 = 2 \times y + 9\)  
  \(5 \times y = 2 \times y + 6\)  
  \(3 \times y = 6\)  
  \(y = 2\)

- **c)** \(4 \times z - 5 = 10 + (-z)\)  
  \(4 \times z = 15 + (-z)\)  
  \(5 \times z = 15\)  
  \(z = 3\)

- **d)** \(45 - d = 25 - d\)  
  \(45 = 25\)  
  \([+ d]\)

  [but \(45 \neq 25\), so the equation is impossible! i.e. \(T = \emptyset\)]

- **e)** \(\frac{x}{4} + 5 = 8\)  
  \(\frac{x}{4} = 3\)  
  \(x = 12\)

**Check:**

- LHS: \(5 \times 4 = 20\)
  
  RHS: \(2 \times 4 + 12 = 20\)  
  ✔

- LHS: \(5 \times 2 + 3 = 13\)
  
  RHS: \(2 \times 2 + 9 = 13\)  
  ✔

- LHS: \(4 \times \frac{3}{2} - 5 = 7\)
  
  RHS: \(10 + (-3) = 7\)  
  ✔

Agree that there is no rational number which makes the equation true.

**Check:**

- \(\frac{12}{4} + 5 = 3 + 5 = 8\)  
  ✔

**Notes**

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

40 min
**Y6**

**Activity 8**

**PbY6b page 144, Q.5**

Read: We weighed out equal packs in kg. What can you write about the mass, m, of one pack?

Show the possible values for each pack on an appropriate segment of the number line.

T has inequalities and segment of number line already on BB or SB or OHT. Deal with one at a time. Class reads out the inequality in unison.

Ps come to BB to write each line of the solution, explaining reasoning. Class points out errors. T asks class for possible values of m as a check. After agreement, T chooses a P to show the solution on the number line.

**Solution:**

a) \(7 \times m + 1 \leq 22\) 
   \[7 \times m \leq 21\] 
   \[m \leq 3\]

but as the mass of a pack cannot be negative or zero (or there wouldn’t be a pack) the correct solution in context is:

\[0 < m \leq 3\]

b) \(4 \times m + 32 > 12 \times m\) 
   \[32 > 8 \times m\] 
   \[4 > m\]

but as the mass of a pack cannot be negative or zero, the solution in context is:

\[4 > m \geq 0\]

c) \(29.5 < 5 \times m + 2 < 32\) 
   \[27.5 < 5 \times m < 30\] 
   \[5.5 < m < 6\]

Ask Ps to say what each solution means in the context.

a) Each pack weighs more than 0 kg but less than, or equal to, 3 kg.

b) Each pack weighs more than 0 kg but less than 4 kg.

c) Each pack weighs more than 5.5 kg but less than 6 kg.

---

**Notes**

Whole class activity

(or individual trial first under a time limit if Ps wish and there is time)

(or if class is able, ask Ps to draw the appropriate number line segment on BB)

Discussion, reasoning, agreement, praising

Extra praise for Ps who realise that in the given context, \(m \leq 3\) is not the whole solution.

**BB:**

A white dot shows that the number is not included in the solution; a black dot shows that the number is included.

**BB:**

or

Each pack weighs between 5.5 kg and 6 kg.

---

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Factorising 145, 320, 495 and 1145. Revision and practice.  

PbY6b, page 145

Solutions:

Q.1 a) \( 5 \times n + 4 = 39 \)  
Check:  
\( 5 \times n = 35 \)  
\( 5 \times 7 + 4 = 39 \)  
\( n = 7 \)  
Answer: My number is 7.

b) \( \frac{n}{2} + 7 = 2 \times n \)  
\( \frac{\text{check}}{\text{check}} \)

Q.2 a) \( A_{\text{triangle}} = \frac{b \times h}{2} \)  
b) \( P_{\text{octagon}} = 8 \times a \)  
\( (= 8a) \)

c) \( A_{\text{cuboid}} = 2 \times (a \times b + b \times c + a \times c) \)  
\( = 2 \times (ab + bc + ac) \)

d) \( V_{\text{cuboid}} = a \times 2a \times 3a \)  
\( = a \times 2a \times a \times 3a \)  
\( = 6 \times a^3 \)  
\( = 6a^3 \)

e) \( A = 2 \times (a \times a + a \times 2a + a \times 2a) \)  
\( = 2 \times (a^2 + 2a^2 + 2a^2) \)  
\( = 2 \times 5a^2 \)  
\( = 10a^2 \)

Q.3 a) \( a - (-4) > -2 \)  
\( a > -2 + (-4) \)  
\( a > -6 \)

b) \( \frac{4}{5} + (-b) \leq \frac{3}{10} \)  
\( \text{Check: } b = \frac{5}{10} \)

Show on the number line why the sign needs to change!  
Also check that numbers less than a half do not make the inequality true.

Notes

145 = \( 5 \times 29 \)  
Factors: 1, 5, 29, 145

320 = \( 2^6 \times 5 \)  
Factors: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 64, 80, 160, 320

495 = \( 3^2 \times 5 \times 11 \)  
Factors: 1, 3, 5, 9, 11, 15, 33, 45, 55, 99, 165, 495

1145 = \( 5 \times 229 \)  
Factors: 1, 5, 229, 1145

(or set factorising as homework at the end of Lesson 144 and review at the start of Lesson 145.)
### Activity

**Solutions: (continued)**

**Q.3**

<table>
<thead>
<tr>
<th>c + (+ 4) ≥ 4</th>
<th>[– 4]</th>
</tr>
</thead>
<tbody>
<tr>
<td>c ≥ 0</td>
<td></td>
</tr>
</tbody>
</table>

**Check:**

| c = 0:       | 0 + (+ 4) = 4  ✔ |
| c = 3:       | 3 + (+ 4) = 7 > + 4  ✔ |
| c = –1:      | –1 + (+ 4) = 3  ≠  4  ✗ |

**Q.4**

<table>
<thead>
<tr>
<th>x + 6.2 = 9.3</th>
<th>[– 6.2]</th>
<th>Check: 3.1 + 6.2 = 9.3  ✔</th>
</tr>
</thead>
<tbody>
<tr>
<td>x = 3.1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>–3.7 + y = 5</th>
<th>[+ 3.7]</th>
<th>Check: –3.7 + 8.7 = 5  ✔</th>
</tr>
</thead>
<tbody>
<tr>
<td>y = 8.7</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>z × 2 = 1/4 [÷ 2]</th>
<th>Check: 1/8 × 2 = 2/8 = 1/4  ✔</th>
</tr>
</thead>
<tbody>
<tr>
<td>z = 1/8</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3 × a = a + 5 [– a]</th>
<th>Check: LHS: 3 × 2.5 = 7.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × a = 5</td>
<td>RHS: 2.5 + 5 = 7.5  ✔</td>
</tr>
<tr>
<td>a = 2.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5 × b + 2 = 3 × b – 8</th>
<th>[– 3 × b]</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 × b + 2 = –8</td>
<td>[– 2]</td>
</tr>
<tr>
<td>2 × b = –10</td>
<td>[÷ 2]</td>
</tr>
<tr>
<td>b = -5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>c/3 – 2 = 8 [+ 2]</th>
<th>Check: 30/3 – 2 = 10 – 2 = 8  ✔</th>
</tr>
</thead>
<tbody>
<tr>
<td>c/3 = 10</td>
<td>[× 3]</td>
</tr>
<tr>
<td>c = 30</td>
<td></td>
</tr>
</tbody>
</table>

**Q.5**

<table>
<thead>
<tr>
<th>U</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>35(\frac{1}{2})</td>
<td>36(\frac{3}{4})</td>
<td>38</td>
<td>39(\frac{1}{4})</td>
<td>40(\frac{1}{2})</td>
<td>41(\frac{3}{4})</td>
<td>43</td>
<td>44(\frac{1}{4})</td>
<td>45(\frac{3}{2})</td>
<td>46(\frac{3}{4})</td>
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</tbody>
</table>

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**Q.5**

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<td>44(\frac{1}{4})</td>
<td>45(\frac{3}{2})</td>
<td>46(\frac{3}{4})</td>
</tr>
</tbody>
</table>

| i) E = 40 → U = 6 \(\frac{1}{2}\) (between sizes 6 and 7) |
|----|------------------|
| ii) E = 38 → U = 5  |
| iii) E = 45 → U = 10 \(\frac{1}{2}\) (between sizes 10 and 11) |

(Ps read data from table – there is no need to do more calculations.)

**Check:**

<table>
<thead>
<tr>
<th>LHS: 5 × (–5) + 2 = –23</th>
</tr>
</thead>
<tbody>
<tr>
<td>RHS: 3 × (–5) – 8 = –23</td>
</tr>
</tbody>
</table>

**Details:** e.g. \( U = 3: \)

\[
E = \frac{5}{4} \times 3 + \frac{127}{4}
\]

\[
= \frac{15}{4} + \frac{127}{4} = \frac{142}{4}
\]

\[
= \frac{71}{2} = 35\frac{1}{2}
\]

After 3 or 4 columns have been completed, Ps might notice that the European sizes form a sequence with rule \(+ \frac{1}{4}\). Extra praise for this!
**Activity 1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- $146 = 2 \times 73$ Factors: 1, 2, 73, 146
- $321 = 3 \times 107$ Factors: 1, 3, 107, 321
- $496 = 2^4 \times 31$ Factors: 1, 2, 4, 8, 16, 31, 62, 124, 248, 496
- $1146 = 2 \times 3 \times 191$ Factors: 1, 2, 3, 6, 191, 382, 573, 1146

**Notes**

Individual work, monitored (or whole class activity)

BB: 146, 321, 496, 1146

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

- $146 \leq n \leq 2$  
  $73 \leq n \leq 107$  
  $248 \leq n \leq 496$  
  $1 \leq n \leq 1146$

- $3 \leq n \leq 107$  
  $62 \leq n \leq 321$  
  $124 \leq n \leq 496$  
  $1 \leq n \leq 1146$

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Discussion, agreement, praising

- $-2 \leq n \leq 5$, $n \in \mathbb{Z}$
- $-2 \leq n \leq 6$, $n \in \mathbb{Z}$

(i.e. $n$ is a member of the set of whole numbers or integers)

Two criteria are possible – both must be given.

- $-2 < n$, $n \in \mathbb{Z}$

(i.e. $n$ is a member of the set of all the numbers we know, or $n$ is a rational number)

or $n < -1$, $n \in \mathbb{Q}$

**Activity 2**

**Writing inequalities**

Let’s write an inequality about the set of numbers marked on these parts of the number line. Why are some shown by dots, some by lines?

Ps come to BB or dictate what T should write. Who agrees? Who can think of another way to write it? T gives hints where necessary. Class checks the inequality with some possible values.

BB:

<table>
<thead>
<tr>
<th>a)</th>
<th>$-5$</th>
<th>$0$</th>
<th>$5$</th>
<th>[–3 &lt; n &lt; 6, n \in \mathbb{Z}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>b)</td>
<td>$-5$</td>
<td>$0$</td>
<td>$5$</td>
<td>[-4 \leq n \leq -1\text{ OR } 4 \leq n \leq 5, n \in \mathbb{Z}]</td>
</tr>
<tr>
<td>c)</td>
<td>$-5$</td>
<td>$0$</td>
<td>$5$</td>
<td>[-1 \leq n, n \in \mathbb{Z}\text{ Elicit that ‘. . .’ means ‘and so on’.}]</td>
</tr>
<tr>
<td>d)</td>
<td>$-5$</td>
<td>$0$</td>
<td>$5$</td>
<td>[-1 \leq n &lt; 3, n \in \mathbb{Q}]</td>
</tr>
<tr>
<td>e)</td>
<td>$-5$</td>
<td>$0$</td>
<td>$5$</td>
<td>[n \leq -2, n \in \mathbb{Q}]</td>
</tr>
</tbody>
</table>

**Notes**

Whole class activity

Written on BB or SB or OHT

At a good pace

Involve as many Ps as possible.

Reasoning, agreement, checking, praising

**Check:**

LHS: $5 \times 2 + 3 = 13$

RHS: $2 + 11 = 13 \checkmark$

**Activity 3**

**Solving inequalities**

Let’s solve these equations and inequalities and show the solution on the number line. The base set is the set of rational numbers. ($n \in \mathbb{Z}$)

Ps come to BB to write each row of the solution, explaining reasoning. Class points out errors. When solution has been agreed and checked, Ps come to BB to draw appropriate section of number line (only a rough drawing is needed) and mark the solution.

BB:

| a) | $5 \times x + 3 = x + 11$  
  $4 \times x + 3 = 11$  
  $4 \times x = 8$  
  $x = 2$ |
|----|-------------------------------------------------|
| b) | \(-x\)  
  \([-3]\)  
  \([\div 4]\)  
  $x = 2$ |

**Notes**

Whole class activity
### Activity

(Continued)

b) \(5y - 2y \leq 10\)
\[3y \leq 10\] \[\div 3\]
\[y \leq 3\frac{1}{3}\]

![Graph](image)

\[3 \leq y \leq 3\frac{1}{3}\]

---

c) \(|e| < 3\)
What does this inequality mean?
(The absolute value of \(e\) is less than 3.)
So \(-3 < e < 3\)

![Graph](image)

---

d) \(|f| \geq 4\)
What does this inequality mean?
(The absolute value of \(f\) is greater than or equal to 4.)
\(f \leq -4\) OR \(f \geq 4\)

Check with various values for \(f\).
\[f = -6:\] \(|-6| = 6 \geq 4\] ✓
\[f = -4:\] \(|-4| = 4\] ✓
\[f = -3:\] \(|-3| = 3 \geq 4\] ✗
\[f = 5.5:\] \(|5.5| = 5.5 \geq 4\] ✓

---

### Notes

Check with some values, e.g.
\[y = 3\frac{1}{3}, y = 3, y = 4\]
Ps check in Ex. Bks or at side of BB.

Elicit that the absolute value of a number is how far it is from zero.
Check with some values for \(e\).

Two sets of numbers are possible (for negative \(t\) and for positive \(t\)) and both should be stated in the solution.

Individual work, monitored, helped
Written on BB or SB or OHT
Responses shown in unison.
Reasoning, checking, agreement, self-correction, praising

---

Check:
\[
\frac{1}{5} \times \frac{4}{9} = \frac{4}{5} \]
\[
\frac{9}{5} - 1 = \frac{9}{5} - \frac{5}{5} = \frac{4}{5} \]

---

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### Activity 4 (Continued)

| c) $35 \div c = 14$, $c = 35 \div 14 = 5 \div 2 = 2 \frac{1}{2}$ |
| d) $7 \times (d - 2) = d - 2$ or $[-(d-2)]$
| $7 \times d - 14 = d - 2$ $[-d]$ $6 \times (d - 2) = 0$
| $6 \times d - 14 = -2$ $[+14]$ $6 \times d - 12 = 0$ $[+12]$
| $6 \times d = 12$ $[\div 6]$ $6 \times d = 12$ $[\div 6]$
| $d = 2$

| e) $(4 - e) \times 5 = -5 \times e + 20$
| $20 - 5e = -5e + 20$ $[-20]$
| $-5e = -5e$
| So $e$ can be any rational number and the solution is: $e \in \mathbb{Q}$

| f) $6 \times f - 3 \times f = \frac{3}{8} + f \times 3$
| $3 \times f = \frac{3}{8} + f \times 3$ $[-3 \times f]$
| $0 = \frac{3}{8}$
| BUT $0 \neq \frac{3}{8}$, so the equation is impossible! $f = \emptyset$

### Notes

<table>
<thead>
<tr>
<th>Lesson Plan 146</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Check:</strong> $35 \div \frac{5}{2} = 7 \times \frac{2}{5} = 14 \checkmark$</td>
</tr>
</tbody>
</table>
| **Check:**
| LHS: $7 \times (2 - 2) = 7 \times 0 = 0$
| RHS: $2 - 2 = 0 \checkmark$ |

When the LHS of an equation is identical to the RHS, we say that it is an identity:
- e.g. $3a = 3a$, $-y = -y$, etc.
- and states what is obvious.

Extra praise if Ps did this correctly without T’s help.

Elicit the notation for an empty set.

### Whole class activity

(Or individual trial first if Ps wish, reviewed as usual)

Discussion, reasoning, agreement, (self-correction), praising

BB:

$6 < m < 8$

$m > 6$

$m < 8$

T asks a P to say the answer in a sentence.

---

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Q.3 Read: Show the relationship among the data by writing an equation. Solve the equation and check the result in the given context.

Deal with one at a time. Ps read question themselves and solve it in Ex. Bks. under a short time limit.

Review with whole class. Ps could show answer on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. T asks a P to say the answer in a sentence.

**Solutions:** e.g.

a) One fifth of a barrel is 20 litres less than the capacity of the whole barrel. What is the capacity of the barrel?

e.g. Let the capacity in litres be c.

\[ \frac{c}{5} = c - 20 \]
\[ \frac{c}{5} = 5c - 100 \]
\[ c + 100 = 5c \]
\[ 100 = 4c \]
\[ \frac{25}{c} = c = 25 \]

Answer: The capacity of the barrel is 25 litres.

b) On Monday, a shop sold x kg of honey. On Tuesday it sold 11 kg more than on Monday, and on Wednesday it sold 5 kg more than on Monday.

How much honey did the shop sell on each of these days if the total amount of honey sold was 220 kg?

Monday: x, Tuesday: x + 11, Wednesday: x + 5

Plan: \[ x + x + 11 + x + 5 = 220 \]
\[ 3x + 16 = 220 \]
\[ 3x = 204 \]
\[ x = 68 \]

Answer: The shop sold 68 kg of honey on Monday, 79 kg on Tuesday and 73 kg on Wednesday.

c) In one container there is twice as much water as there is in a second container.

If we took 30 litres of water out of the first container and 12 litres of water out of the second container, both containers would hold the same amount of water.

How much water is in each container?

Amount in 1st container: 2x, Amount in 2nd container: x

Plan: \[ 2x - 30 = x - 12 \]
\[ 2x - 18 = x \]
\[ 2x = x + 18 \]
\[ x = 18 \]

Answer: There are 36 litres of water in the first container and 18 litres of water in the second container.

**Notes**

Individual work, monitored, helped

Encourage Ps to draw digrams to help them understand the problem.

Responses shown in unison

Discussion, reasoning, agreement, self-correction, praising

Accept any valid method.

**Check: 68 + 79 + 73 = 220 ✔**

BB: [Diagram]

20 litres

or \( \frac{4}{5} \times c = 20 \)

\[ c = \frac{20 \times 5}{4} = \frac{100}{4} = 25 \] (litres)

Check: LHS: 25 ÷ 5 = 5

RHS: 25 – 20 = 5 ✔

Accept any valid steps towards the solution.

**Check: LHS: 36 – 30 = 6
RHS: 18 – 12 = 6 ✔**
d) Let \( \textcolor{red}{\text{£1}}, \textcolor{blue}{\text{£1}}, \textcolor{green}{\text{£x}} \) mean £1, \( \textcolor{red}{\text{£1}}, \textcolor{blue}{\text{£1}} \) mean – £1 and \( \textcolor{green}{\text{£1}} \) mean an £x banknote in an envelope.

Betty has: \( \textcolor{red}{\text{£1}}, \textcolor{blue}{\text{£1}}, \textcolor{blue}{\text{£1}}, \textcolor{green}{\text{£1}} \)

Larry has: \( \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{red}{\text{£1}}, \textcolor{blue}{\text{£1}} \)

If Betty has the same amount of money as Larry, what value of banknote is in each envelope?

**Plan:**

\[
\begin{align*}
1 + (-3) + 2x &= 9 + (-1) + x \\
-2 + x &= 8 \\
x &= 10
\end{align*}
\]

**Answer:** There is a £10 banknote in each envelope.

**Notes**

Symbols drawn (stuck) on BB.

BB: \( B = \textcolor{red}{\text{£1}}, \boxed{\textcolor{green}{\text{£1}}} = \)  
Ps could manipulate the symbols on BB to model what is being done to the equation.
<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| **Factorisation**<br>Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.<br>Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.<br>Elicit that:<br>• $147 = 3 \times 7 \times 7 = 3 \times 7^2$<br>Factors: 1, 3, 7, 21, 49, 147<br>• $322 = 2 \times 7 \times 23$<br>Factors: 1, 2, 7, 14, 23, 46, 161, 322<br>• $497 = 7 \times 71$<br>Factors: 1, 7, 71, 497<br>• $1147 = 31 \times 37$<br>Factors: 1, 31, 37, 1147<br>8 min | Individual work, monitored<br>(or whole class activity)<br>BB: 147, 322, 497, 1147<br>(Ps use calculators for 1147.)<br>Reasoning, agreement, self-correction, praising<br>e.g. $322 \quad 3161 \quad 7 \quad 71 \quad 71$
| **PbY6b, page 147, Q.1**<br>Read: The base set is the set of natural numbers. (N)<br>Write an equation or an inequality, solve it and check your result.<br>Which number am I thinking of?<br>Deal with one part at a time. T chooses a P to read out the description. Allow 1 minute for Ps to solve it in Ex. Bks, then Ps show the number on scrap paper or slates on command.<br>P answering correctly explains reasoning at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected.<br>Solution:<br>a) I add 5 to 3 times my number and the result is 53.<br>Plan: $3 \times n + 5 = 53$<br>Check: $3 \times n = 48$<br>$n = 16$
Answer: The number I am thinking of is 16.<br>b) I subtract 18 from 7 times my number and the result is 269.<br>Plan: $7 \times n - 18 = 269$<br>Check: $7 \times n = 287$<br>$n = 41$
Answer: The number I am thinking of is 41.<br>c) I subtract 4 times my number from 7 times my number and the result is 156.<br>Plan: $7 \times n - 4 \times n = 156$<br>Check: $3 \times n = 156$<br>$n = 52$
Answer: The number I am thinking of is 52.<br>d) I add 6 to 5 times my number and the result is less than 26.<br>Plan: $5 \times n + 6 < 26$<br>Check: $5 \times n = 20$<br>$n < 4$
Answer: The number I am thinking of could be 1, 2 or 3. | Whole class activity but individual calculation<br>Responses shown in unison.<br>In good humour.<br>Reasoning, checking, agreement, self-correction, praising<br>Feedback for T

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Lesson Plan 147

Activity 2 (Continued)

d) The difference between 7 times and 5 times my number is greater than 50.

Plan: \(7 \times n - 5 \times n > 50\)
\[
2 \times n > 50 \quad [\div 2]
\]
\(n > 25\)

Answer: The number I am thinking of could be any natural number greater than 25.

Notes

PbY6b, page 147

Q.2 Read: Solve each problem in two ways, with and without an equation.

How could you solve the problem if you don’t write an equation? (e.g. writing an operation, drawing a diagram, explaining in words)

Deal with one part at a time under a short time limit. Ps read problem themselves, solve it in two ways in Ex. Bks, check their result and write the answer in a sentence.

Review with whole class. T chooses a P to read out the question. A, read us your answer. Who agrees with A? Who has another answer? Who agrees with that? Ps come to BB to write the equation and to show other methods of solution. Class checks result and agrees on correct answer. Mistakes discussed and corrected. T asks some Ps which method they prefer and why.

Solution: e.g.

a) Daffy Duck is twice as old as Donald Duck. If the sum of their ages is 21 months, how old is Daffy and how old is Donald?

Donald: \(n \quad 2 \times n + n = 21\)

Daffy: \(2 \times n \quad 3 \times n = 21 \quad [\div 3]\)
\(n = 7\)

Check: \(2 \times 7 + 7 = 14 + 7 = 21\)

Answer: Daffy is 14 months old and Donald is 7 months old.

b) A 120 cm long stick is cut into two pieces so that one of the pieces is 30 cm longer than the other piece.

How long is each piece?

Short piece: \(x \quad x + x + 30 = 120 \quad [-30]\)

Long piece: \(x + 30 \quad 2 \times x = 90 \quad [\div 2]\)
\(x = 45\)

Check: \(45 + (45 + 30) = 45 + 75 = 120\)

Answer: The long piece is 75 cm and the short piece is 45 cm.

Notes

Individual work, monitored, helped

Discuss the meaning of 'equation' here. Elicit that it involves an unknown amount shown by a letter or symbol, whereas an 'operation' involves only numbers and operation signs.

Reasoning, checking, agreement, self-correction, praising

Extra praise if Ps draw diagrams, as shown below.

<table>
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<tr>
<th>21</th>
</tr>
</thead>
</table>

or \(21 \div 3 = 7\)
as the sum of their ages is divided into 3 equal parts: Donald’s age is 1 part, Daffy’s age is 2 parts.

or

If both pieces were the same length as the short piece:
\[(120 - 30) \div 2 = 90 \div 2 = 45\]
Long piece: \(45 + 30 = 75\)
or

If both pieces were the same length as the long piece:
\[(120 + 30) \div 2 = 150 \div 2 = 75\]
Short piece: \(75 - 30 = 45\)

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Activity 3 (Continued)

c) Liz has twice as many marbles as Julia. If Liz gave Julia 10 marbles they would both have the same amount. How many marbles do Liz and Julia each have?

Julia: \[ x + 10 = 2 \times x - 10 \] [+] 10
Liz: \[ 2 \times x \]
\[ 20 = x \]

Check: \[ 20 + 10 = 40 - 10, \text{ and } 2 \times 20 = 40 \]

Answer: Liz has 40 marbles and Julia has 20 marbles.

Notes

L

J 20

Liz must have \[ 2 \times 10 = 20 \] more marbles than Julia.

Answer:
Liz has 40 marbles and Julia has 20 marbles.

Individual work, monitored,
(helped)

Differentiation by time limit.

Responses shown in unison.

Reasoning, checking,
agreement, self-correction,
praising

Feedback for T

T asks several Ps to read out
their sentences. Ps say what
they think of them. If Ps hear
a better worded explanation
than their own, encourage them
to write it in Ex Bks. too.

Check:
\[ £37 + (– £12) = £37 − £12 = £25 \]

Check:
\[ £2.50 + £28.50 = £31 \]

Check:
\[ 3.35 + (– 1.15) = 2.20 \]

(or Colin is \( £1.15 \) in debt.)
### Activity 5

**PbY6b, page 147. Q.4**

Read:  *Solve the problems by writing equations.*

Deal with one part at a time. Ps decide what to do first and how to continue. T helps and guides where necessary by asking appropriate questions. Class points out any errors or suggests better steps.

[e.g. for a):]

**What is the unknown amount? What shall we call it? Who can write an amount for Alex (Ben)? Which of them has more? How many times more? Who can write an equation about the relationship between their money? How can we write it in a simpler way? What should we do first to solve the equation? What should we do next? How can we check the result? Who can say the answer in a sentence?]**

Ps could write the solution in Ex. Bks at the same time.

**Solution: e.g.**

a) Alex has £100 in cash, is £300 in debt and has two savings bonds of equal value. Ben is £100 in debt, has £400 in cash and has 3 such saving bonds.

*If Ben has 3 times as much money as Alex, how much is a savings bond worth?*

Let a savings bond be worth \( x \).

Then

\[
A = 100 + (-300) + 2 \times x,
\]

\[
3 \times A = B = -100 + 400 + 3 \times x
\]

\[
3 \times [100 + (-300) + 2 \times x] = -100 + 400 + 3 \times x
\]

\[
3 \times (-200 + 2 \times x) = 300 + 3 \times x
\]

\[
-600 + 6 \times x = 300 + 3 \times x \quad [+600]
\]

\[
6 \times x = 900 + 3 \times x \quad [-3 \times x]
\]

\[
3 \times x = 900 \quad [\div 3]
\]

\[
x = 300 \text{ (£)}
\]

**Answer:** A savings bond is worth £300.

b) Adam has £100 in cash, is £300 in debt and has a savings bond. Matthew has £500 in cash, is £100 in debt and also has a savings bond. Norah has £100 in cash, is £200 in debt and has two savings bonds. If the sum of the two boys’ money is greater than twice Norah’s money, how much can one of their savings bonds be worth?

Let a savings bond be worth \( x \).

**BB:**

\[
A = 100 + (-300) + x, \quad M = 500 + (-100) + x, \quad A + M > 2 \times N
\]

\[
100 + (-300) + x + 500 + (-100) + x > 2 \times [100 + (-200) + 2 \times x]
\]

\[
600 - 400 + 2 \times x > 200 - 400 + 4 \times x
\]

\[
200 + 2 \times x > -200 + 4 \times x \quad [+200]
\]

\[
400 + 2 \times x > 4 \times x \quad [-2 \times x]
\]

\[
400 > 2 \times x \quad [\div 2]
\]

\[
200 > x \quad \text{or} \quad x < 200
\]

**Answer:** A savings bond is worth less than £200.

---

**Notes**

Whole class activity
(or individual trial first if Ps wish and there is time, monitored, helped, corrected)

Involve as many Ps as possible.

At a reasonable pace, in good humour!

Discussion, reasoning, agreement, checking, (self-correction), praising

Feedback for T
Y6

**Activity**

**1**

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- \(148 = 2 \times 2 \times 37 = 2^2 \times 37\) Factors: 1, 2, 4, 37, 74, 148
- \(323 = 2 \times 3 \times 83\) Factors: 1, 2, 3, 6, 83, 166, 249, 498
- \(498 = 2 \times 3 \times 83\) Factors: 1, 2, 3, 6, 83, 166, 249, 498
- \(1148 = 2 \times 2 \times 7 \times 41 = 2^2 \times 7 \times 41\)

Factors: 1, 2, 4, 7, 14, 28, 41, 82, 164, 287, 574, 1148

8 min

**2**

**Problem**

Listen carefully to this problem and note down the important data.

*The perimeter of a quadrilateral is 117 cm. Side b is 4 cm shorter than side a. Side c is twice as long as side a and side d is 5 cm less than side c. What length is each side?*

a) Is there any missing or irrelevant data? (There is just enough data given to solve the problem and no irrelevant data.)

b) What should we do now? (Draw a diagram.)

P draws a quadrilateral on BB and labels its sides. Now what should we do? (Write down what we know.) Other Ps come to BB to write down the relationships (or dictate to T).

BB: e.g.

- \(P = a + b + c + d = 117\) cm
- \(b = a - 4\) cm, \(c = 2 \times a\)
- \(d = c - 5\) cm = \(2 \times a - 5\) cm

c) What should we do now? (Make a plan.) Let's write the plan as one equation. Ps come to BB or dictate to T. Class points out errors.

BB: Plan: \(a + (a - 4) + 2 \times a + (2 \times a - 5) = 117\)

Is this a good plan? (Yes) Can anyone think of a better one? (No)

d) What should we do before we solve our plan? (Estimate the solution.) How could we estimate the length of side a? (e.g. If the sides were equal, then the length of each side would be about 30 cm.)

e) Now let’s solve the equation. Ps come to BB to write and explain each step (or dictate steps to T). Class agrees/disagrees.

BB: \(a + (a - 4) + 2 \times a + (2 \times a - 5) = 117\)

\[6 \times a - 9 = 117\] [+9]

\[6 \times a = 126\] [÷ 6]

\(a = 21\) (cm)

Is this the answer to the question? (No, this is only side a – we need to work out the lengths of all the sides.) Ps dictate to T. (BB)

f) What should we do now? (Check the result.) How can we do it?

**Lesson Plan**

**148**

**Notes**

Individual work, monitored (or whole class activity)

BB: 148, 323, 498, 1148

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

| e.g. | 323 | 17 |
| 148 | 2 | 19 |
| 74 | 2 | 19 |
| 37 | 1 | 1148 |
| 498 | 2 | 19 |
| 249 | 3 | 74 |
| 83 | 3 | 74 |
| 1 | 1 |

Whole class activity

T could have problem written on BB or SB or OHT

Revision of the steps needed to solve word problems.

Involve as many Ps as possible. T leads or directs as necessary.

Discussion, reasoning, agreement, praising

Ps could write solution in Ex. Bks. at the same time.

Elicit that writing \(d\) as an expression of \(a\) will help in the solution – it will ensure that only one unknown amount needs to be dealt with.

If no P suggests this equation, T starts and Ps continue.

BB: \(E\):

- 117 cm ÷ 4 = 30 cm

So \(a = 21\) cm

\(b = 17\) cm

\(c = 42\) cm

\(d = 37\) cm

Check:

\(P = 21 + 17 + 42 + 37 = 117\) ✔
### Activity

(Continued)

h) What is the last thing we should do? (Write the answer in a sentence.)

Ps dictate what T should write.

**BB:** *Answer:* The lengths of the sides of the quadrilateral are 21 cm, 17 cm, 42 cm and 37 cm.

T asks Ps to think about other things too. e.g.

- Is the answer realistic?  (Yes, it is)
- Could we have solved the problem in a better way?

(Elicit/point out that we could have expressed each side in terms of \( b \) rather than \( a \) but it would have been more complicated, so we used the best method.)

15 min

### Notes

Let's summarise the steps needed to solve a word problem.

1. Note the relevant data.
2. Draw a diagram and/or write down what we know.
3. Look for relationships.
4. Write a plan.
5. Estimate the result.
6. Do the calculation.
7. Check the result in context.
8. Write the answer in a sentence.

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<thead>
<tr>
<th>Y6</th>
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</table>

| Lesson Plan 148 |

3  

**PbY6b, page 148**

Q.1 Read: Solve the equations and inequalities. Check your results.

Set a time limit or deal with one at a time. Ps work in Ex. Bks.

Review with whole class. Ps could show the solutions on scrap paper or slates on command. Ps responding correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:**

a)  
\[
\begin{align*}
2x - \frac{2}{5} &= \frac{7}{10} \\
\Rightarrow x &= \frac{11}{10} = 1 \frac{1}{10}
\end{align*}
\]

b)  
\[
\begin{align*}
y - \frac{3}{4} &> \frac{3}{4} - y \\
\Rightarrow 2y &> \frac{6}{4} + \frac{3}{4} \\
\Rightarrow y &> \frac{3}{4}
\end{align*}
\]

c)  
\[
\begin{align*}
\frac{4}{5} + u &= u + \frac{12}{15} \\
\Rightarrow \frac{4}{5} &= \frac{12}{15} \\
\text{[Simplify RHS]} &\text{This is an identity, so } u \text{ can be any rational number, i.e. } u \in \mathbb{Q}
\end{align*}
\]

d)  
\[
\begin{align*}
\frac{2}{3} \times t &= \frac{6}{30} \\
\Rightarrow t &= \frac{3}{2} \times \frac{1}{5} = \frac{3}{10} \times \frac{1}{2} \\
\Rightarrow t &= \frac{3}{10}
\end{align*}
\]

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, checking, agreement, self-correcting, praising

Feedback for T

**Checks:**

a)  
\[
\frac{11}{10} - \frac{4}{10} = \frac{7}{10} \checkmark
\]

b)  
\[
\begin{align*}
y &> \frac{3}{4}; \text{ e.g. } y = 1: \\
\text{LHS: } 1 - \frac{3}{4} &= \frac{1}{4} \\
\text{RHS } \frac{3}{4} - 1 &= -\frac{1}{4} \\
\text{so LHS} > \text{RHS } \checkmark
\end{align*}
\]

If \( y = \frac{3}{4} \),

LHS = 0 = RHS \( \times \)

If \( y < \frac{3}{4} \), e.g. 0:

LHS: \( -\frac{3}{4} \); RHS: \( \frac{3}{4} \)

so LHS < RHS \( \times \)

**Check:**

\[
\frac{2}{3} \times \frac{3}{10} = \frac{6}{30} \checkmark
\]
Lesson Plan 148

Notes

C:  $1.65 \div 1.5 = 16.5 \div 15$

$= 3.3 \div 3 = 1\frac{1}{3}$

Check for:

$v = 1.1, \ (✓)$

$v > 1.1, \ e.g. \ v = 2 \ (✓)$

$v < 1.1, \ e.g. \ v = 1 \ (✓)$

Check:

$-0.3 + 0.3 = 0 \ (✓)$

Activity

3 (Continued)

e) $0.2 \times v + 0.85 \leq 1.7 \times v - 0.8 \ [\pm 0.8]$

$0.2 \times v + 1.65 \leq 1.7 \times v \ [\pm 0.2 \times v]$

$1.65 \leq 1.5 \times v \ [\div 1.5]$

$1.1 \leq v \ \text{or} \ v \geq 1.1$

f) $w + 0.3 = 0 \ \text{[–0.3]}

$w = -0.3$

22 min

4 PbY6b, page 148

Q.2 Read: Write an equation for each question. Solve the equation and check your result.

Deal with one at a time or set a time limit. Ps read questions themselves and solve them in Ex. Bks.

Review with the whole class. Ps could show numbers on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected.

Solutions: e.g.

a) Brian said, "If I add my number to a quarter of my number I get $12\frac{1}{2}$. What is my number?"

Let Brian's number be $x$.

$\frac{x}{4} + \frac{x}{2} = 12\frac{1}{2} \ [\times 4]$

$4 \times x + x = 50$

$5 \times x = 50 \ [\div 5]$

$x = 10$

Answer: Brian's number is 10.

b) Tom said, "If I add a quarter of my number to half of my number I get the same result as if I had taken 2 away from 4 fifths of my number. What is my number?"

Let Tom's number be $x$.

$\frac{x}{2} + \frac{x}{4} = \frac{4}{5} \times x - 2 \ \text{[Convert to equivalent fractions with lowest common denominator: 20]}

\frac{10 \times x}{20} + \frac{5 \times x}{20} = \frac{16}{20} \times x - 2 \ [\times 20]$

$10 \times x + 5 \times x = 16 \times x - 40$

$15 \times x = 16 \times x - 40 \ [\pm 15 \times x]$

$0 = x - 40 \ [\pm 40]$

$40 = x$

Answer: Tom's number is 40.
### Y6

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<tr>
<td><strong>c)</strong> Two identical bottles contain 2.2 litres of squash altogether.</td>
<td></td>
</tr>
<tr>
<td>One bottle is ( \frac{2}{3} ) full and the other bottle is ( \frac{4}{5} ) full.</td>
<td></td>
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</tbody>
</table>

How much squash is there in a full bottle?

**Solution:**

Let the capacity of a bottle be \( x \).

\[
\frac{2}{3} \times x + \frac{4}{5} \times x = 2.2
\]

[Convert fractions to lowest common denominator: 15]

\[
\frac{10}{15} \times x + \frac{12}{15} \times x = 2.2
\]

\[
\frac{22}{15} \times x = 2.2 \quad \text{[× 15]}
\]

\[
22 \times x = 33 \quad \text{[÷ 22]}
\]

\[
x = 1.5 \text{ (litres)}
\]

**Answer:** There are 1.5 litres of squash in a full bottle.

### 5

**PbY6b, page 148**

Q.3 Read: Write an equation for each question. Solve the equation and check your result.

Deal with one at a time. Ps work in Ex. Bks.

Review with whole class. T asks a P to read out the question and Ps show answers on scrap paper or slates on command. Ps with correct answer explain reasoning at BB. Class agrees/disagrees and checks result. Who worked it out another way? Mistakes discussed and corrected.

**Solution:** e.g.

a) \( 130\% \) of a number is the same as adding 10.8 to 35% of the number. What is the number?

Let the number be \( x \).

\[
\frac{130}{100} \times x = \frac{35}{100} \times x + 10.8
\]

[× 100]

\[
95 \times x = 10.8
\]

[÷ 95]

\[
x = \frac{1080}{95} = \frac{216}{19} = 11 \frac{7}{19}
\]

**Check:**

\[
130\% - 35\% = 95\%
\]

\[
\frac{95}{100} \times 11 \frac{7}{19} = \frac{119}{20} \times \frac{216}{19} = \frac{54}{5} = 10 \frac{4}{5} = 10.8 \checkmark
\]

[N.B. Substituting \( 11 \frac{7}{19} \) for \( x \) and working out the LHS and RHS is very difficult. This is an easier way to check it.]

Extra praise if a P thinks of this or used this concept for the original equation:

\[95\% \text{ of } x = 10.8, \text{ etc.}\]

**Answer:** The number is \( 11 \frac{7}{19} \).
Activity 5

(Continued)

b) Lilly has £150 in her purse. This amount is £60 less than 1 sixth of all her money. How much money does Lilly have?

Let the amount of money be \( x \).

\[
\frac{x}{6} - 60 = 150 \quad [+ 60]
\]

\[
\frac{x}{6} = 210 \quad [\times 6]
\]

\[
x = 1260 \text{ (£)}
\]

Answer: Lilly has £1260.

c) I am thinking of a number. When I subtract 5 thirds of my number from the number itself, than add \(-\frac{1}{2}\) to the difference, the result is \(-\frac{7}{6}\). What is my number?

Let my number be \( x \).

\[
x - \frac{5}{3} \times x + (-\frac{1}{2}) = -\frac{7}{6} \quad [+ \frac{1}{2} = \frac{3}{6}]
\]

\[
\frac{3}{3} \times x - \frac{5}{3} \times x = -\frac{4}{6} \quad \times (-1)
\]

\[
-\frac{2}{3} \times x = -\frac{2}{3}
\]

\[
\frac{2}{3} \times x = \frac{2}{3} \quad [\div \frac{2}{3}, \text{i.e.} \times \frac{3}{2}]
\]

\[
x = 1
\]

Answer: My number is 1.

38 min
Lesson Plan 148

Notes

Whole class activity
(or individual trial first if Ps wish and there is time)
Involve as many Ps as possible.
At a good pace, in good humour.

Discussion, reasoning,
checking, agreement, (self-
correction), praising
T gives hints or directs Ps thinking if Ps have no ideas.
Accept any valid equation or other method of solution

Y6

Activity

PbY6b, page 148, Q.4

Read: Solve each problem with and without an equation.

Deal with one at a time. T chooses a P to read out the question and allows Ps a minute to think about how to solve it.

Who can solve it by writing an equation? P comes to BB to write equation. Class agrees/disagrees. Ps come to BB to solve it, doing one step each and explaining reasoning. Class points out errors.

T chooses a P to check the result. Class helps where necessary.

Who can think of a way to solve it without using a letter for the unknown amount? P comes to BB to show it. Class decides whether or not is valid. Who thought of another way? etc. Which method do you like best? Why? T chooses a P to say the answer in a sentence.

Solution: e.g.

a) Tim has covered \( \frac{3}{8} \) of his planned route plus an additional 2 km.

He still has 17 km to go. How long is Tim’s route?

Let the length of Tim’s route be \( x \).

\[
\frac{3}{8} \times x + 2 + 17 = x
\]

\[
19 = \frac{5}{8} \times x \quad \text{[} \div \frac{5}{8}, \text{i.e. } \times \frac{8}{5}\text{]}
\]

\[
19 \times \frac{8}{5} = x
\]

\[
\frac{152}{5} = x, \quad \text{or } x = 30.4 \text{ (km)}
\]

Answer: Tim’s route is 30.4 km.

b) Belinda spent half of her money plus another £40. Then she spent half of what was left plus £40. Her money has just run out.

How much money did Belinda have at first?

Let Belinda’s money be \( x \).

e.g. \[ \left( \frac{x}{2} - 40 \right) + 2 - 40 = 0 \quad [+40] \]

\[ \left( \frac{x}{2} - 40 \right) + 2 = 40 \quad [\times2] \]

\[ \frac{x}{2} - 40 = 80 \quad [+40] \]

\[ \frac{x}{2} = 120 \quad [\times2] \]

\[ x = 240 \text{ (£)} \]

Check: \[ £240 \div 2 = £120, \quad £120 - £40 = £80, \quad £80 \div 2 = £40, \quad £40 - £40 = £0 \]

Answer: Belinda had £240 at first.

45 min
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<td><strong>Lesson Plan 149</strong></td>
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<td><strong>C:</strong> Equations and inequalities. Word problems</td>
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<td>BB: 149, 324, 499, 1149</td>
</tr>
<tr>
<td><strong>E:</strong> Advanced problems</td>
<td></td>
<td>T decides whether Ps can use calculators.</td>
</tr>
</tbody>
</table>

### 1. Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **149** is a prime number
  - Factors: 1, 149
  - (as not exactly divisible by 2, 3, 5, 7, 11 and \(13^2 > 149\))
- **324** = \(2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^2 \times 3^4 \) \([= \left(2 \times 3^2\right)^2 = 18^2\] \)
  - Factors: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 324
- **499** is a prime number
  - Factors: 1, 499
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and \(23^2 > 499\))
- **1149** = \(3 \times 383\)
  - Factors: 1, 3, 383, 1149
  - (383 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, \(23^2 > 383\))

### 2. Inequality signs 1

Let’s write a suitable relationship sign between the two numbers in each pair to make a true statement. What are the relationship signs? Ps dictate and T writes on BB. (=, <, >)

Ps come to BB to write the signs, say the statements and check them on the class number line. Class agrees/disagrees.

BB:
- a) \(2 \leq 5\) but \(-2 > -5\)
- b) \(5.1 \geq -3\) but \(-5.1 < 3\)
- c) \(-2 \leq 4\) but \(2 > -4\)
- d) \(-2 \geq -1\) but \(2 > 1\)
- e) \(0 \leq 2\) but \(0 > -2\)
- f) \(0 \geq -2.5\) but \(0 < 2.5\)

### 3. Inequality signs 2

a) T write an inequality on BB. e.g. \(2.6 < 3.4\)

Follow my instructions for writing the next inequality. Ps come to BB or dictate what T should write. Class agrees/disagrees.

BB:
- \(2.6 < 3.4\) [Take the **opposite** value of each side.]
- \(-2.6 > -3.4\) [Multiply each side by \((-2)\)]
- \(5.2 < 6.8\) [Divide each side by \((-4)\)]
- \(-1.3 > -1.7\) [Divide each side by \((-1)\)]
- \(1.3 < 1.7\)

What do you notice about the signs? Elicit that when the numbers are divided or multiplied by a negative number, not only do the numbers change to their opposite value but the relationship sign changes to the opposite sign.

---

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Activity

(Continued)
b) Let’s solve this inequality. We can do it in two ways.

T (Ps) suggest what to do at each step and Ps come to BB to write
the instruction in square brackets and then to write and say the next
line. Class points out errors.

i) Avoiding multiplying or dividing by a negative number.

BB: \((-7) \times x > \frac{1}{3}\) \[+ 7 \times x\]

\[0 > 7 \times x + \frac{1}{3}\] \[-\frac{1}{3}\]

\[-\frac{1}{3} > 7 \times x\] \[\div \frac{1}{3}\]

\[-\frac{1}{21} > x\] or \(x < -\frac{1}{21}\)

ii) Dividing by a negative number.

BB: \((-7) \times x > \frac{1}{3}\) \[\div (-7)\]

\(x < -\frac{1}{21}\)

(The signs change to their opposites: ‘–’ to ‘+’, ‘+’ to ‘–’, ‘>’ to ‘<’)

c) Let’s think of different ways to solve this inequality.

Ps suggest what to do at each step, then come to BB or dictate
what T should write. Class agrees/disagrees.

BB: \(3 \times (-x) < 9\) \[\div 3\] \(0 < 9 + 3 \times x\) \[\div -3\]

\((-x) < 3\) \([\times (-1)]\) \(x > -3\)

\(x > -3\)

(Multiplying or dividing by a negative number changes the signs.)

Check:

ex.

\(x = -3:  \ 3 \times [-(-3)] = 3 \times 3 = 9 \ \checkmark\)

\(x = -2: \ 3 \times [-(-2)] = 3 \times 2 = 6 \ \checkmark\)

\(x = -4: \ 3 \times [-(-4)] = 3 \times 4 = 12 \ \times\)

\(20 \text{ min}\)

Notes

Elicit that to divide a fraction
by an integer, either divide the
numerator where possible or
multiply the denominator.

Check by substituting values
of \(x\) less than, equal to, and
greater than \(-\frac{1}{21}\). (Only
values of \(x\) less than \(-\frac{1}{21}\)
make the inequality true.)

or \(3 \times (-x) < 9\) \[+ 3 \times x\]

\(0 < 9 + 3 \times x\) \[\div -9\]

\(-9 < 3 \times x\) \[\div 3\]

\(-3 < x\) or \(x > -3\)

Individual work, monitored
helped

Written on BB or SB or OHT

Differentiation by time limit.

Discussion, reasoning,
agreement, self-correction,
praising

Extra praise for Ps who
realised that they need only
solve the first inequality in a).
The other two can be deduced
from the first solution!
(Continued)
a) ii) \(3.7 \times x - 2.4 = 4.9 \times x + 1.2\), \(x = -3\)

iii) \(3.7 \times x - 2.4 > 4.9 \times x + 1.2\), \(x < -3\)

b) i) \(\frac{3}{14} + x < -2\) \((-\frac{3}{14}\)]

\[x < -2 - \frac{3}{14}\]

ii) \(\frac{3}{14} + x = -2\), \(x = -2 - \frac{3}{14}\)

iii) \(\frac{3}{14} + x > -2\), \(x > -2 - \frac{3}{14}\)

c) i) \(3 \times x - (-10) < 20\) \([+ (-10)]\)

\[3 \times x < 10\] \([\div 3]\)

\[x < 3 - \frac{1}{3}\]

ii) \(3 \times x - (-10) = 20\): \(x = 3 + \frac{1}{3}\)

iii) \(3 \times x - (-10) > 20\): \(x > 3 + \frac{1}{3}\)

---

**PbY6b, page 149.**

Q. 2 Read: Solve the problem with or without an equation.

Set a time limit of 3 minutes. Ps read problem themselves and use any method they wish. Ps can work in pairs if they wish.

Review with whole class. Who managed to solve it? \(X\), come and show us what you did. Who agrees? Who did it a different way? Mistakes discussed and corrected. Ps who did not solve it, copy solution in Ex. Bks. If no P managed to solve it correctly, T draws diagram on BB and leads Ps through the solution, involving them where possible

**Solution:**

Two cities, \(A\) and \(B\), are 105 km apart.

A cyclist starts from \(A\) and cycles to \(B\) at a steady speed of 15 km per hour. At exactly the same time, another cyclist starts from \(B\) and cycles to \(A\) at a steady speed of 20 km per hour.

a) When will they meet each other?

BB: 

\[\begin{array}{ccc}
A & 105\text{ km} & B \\
\text{15 km/h} & & \text{20 km/h}
\end{array}\]

Let \(t\) be the number of hours after they started that they met.

**Plan:** \(15 \times t + 20 \times t = 105\)

\[35 \times t = 105\]

\[t = 105 \div 35 = 21 \div 7 = 3\text{ (hours)}\]

**Answer:** They will meet each other 3 hours after they started.
b) Where will the cyclists meet?
   From A: $3 \times 15 \text{ km} = 45 \text{ km}$, or
   From B: $3 \times 20 \text{ km} = 60 \text{ km}$ or $105 \text{ km} - 45 \text{ km} = 60 \text{ km}$
   
   Answer: The cyclists will meet at the point which is 45 km from A (and 60 km from B).

   c) When will they arrive at their destinations?
      
      A to B: $105 \div 15 = 7 \text{ (hours)}$
      B to A: $105 \div 20 = 5 \frac{1}{4} \text{ (hours)}$
      
      Answer: The cyclist going from A to B will arrive at B 7 hours after he or she started. The cyclist going from B to A will arrive at A 5 and a quarter hours after he or she started.

   BB: Meeting point
   
   A 45 km  60 km  B

   6  PBY6b, page 149. Q.3
   
   T chooses a P to read out the information given in the question.

   Read: Town A is 288 km from town B. Cindy leaves A at 08:00 and drives at a steady speed of 48 km per hour to B. Dan leaves B at 10:00 and drives at a steady speed of 80 km per hour to A.

   What can we do with this information to help us understand it better? (Draw a diagram.) P comes to BB to draw and label diagram, with prompts from class where necessary. Rest of Ps work in in Ex. Bks

   BB: e.g.  
   
   a) Read: When will they meet each other?
      
      A, how would you work it out? Who would do the same as A? Who can think of another way to do it? T gives hints if Ps have no ideas.
      
      Solution: e.g.
      
      Let $t$ be the number of hours from the time Cindy starts until they meet, so the number of hours from Dan's start time until they meet is $t - 2$.
      
      Then $48 \times t + 80 \times (t - 2) = 288$
      $48 \times t + 80 \times t - 160 = 288$  $[+ 160]$
      $128 \times t = 448$  $[\div 128]$
      $t = \frac{448}{128}$  $= \frac{112}{32}$  $= \frac{14}{4}$  $= \frac{7}{2}$  $= 3 \frac{1}{2} \text{ (h)}$
      
      C will meet D at: $8 \text{ h} + 3.5 \text{ h} = 11.5 \text{ h}$
      
      or  Let $t$ be the number of hours from the time Dan starts until they meet, so the number of hours from Cindy's start time until they meet is $t + 2$.
      
      Then $48 \times (t + 2) + 80 \times t = 288$
      $48 \times t + 96 + 80 \times t = 288$  $[-96]$
      $128 \times t = 192$  $[\div 128]$
      $t = \frac{192}{128}$  $= \frac{48}{32}$  $= \frac{12}{8}$  $= \frac{3}{2}$  $= 1 \frac{1}{2} \text{ (h)}$
      
      D will meet C at: $10 \text{ h} + 1.5 \text{ h} = 11.5 \text{ h}$
      
   Whole class activity
   (or individual trial first if Ps wish and there is time.)

   Agreement, praising
   
   Allow Ps time to think about the method of solution.
   Discussion, reasoning, agreement, praising
   Involve several Ps.

   or  C travels 96 km before D starts, so they approach each other over 288 – 96 = 192 (km) for $t$ hours.
      $48 \times t + 80 \times t = 192$
      $128 \times t = 192$
      $t = 192 \div 128 = 24 \div 16 = 3 \div 2 = 1 \frac{1}{2} \text{ (h)}$
      
      So D will travel for 1.5 hours and C will travel for 3.5 hours before they meet.

   Answer: Cindy and Dan will meet at 11:30.
### Activity 6 (Continued)

b) Where will they meet each other?
- From A: $48 \text{ km} \times 3.5 = 144 \text{ km} + 24 \text{ km} = 168 \text{ km}$, or
- From B: $80 \text{ km} \times 1.5 = 80 \text{ km} + 40 \text{ km} = 120 \text{ km}$

**Answer:** Cindy and Dan will meet each other at the point which is 168 km from A (and 120 km from B).

c) Read: **When will they reach their destinations?**
- Cindy: $288 \text{ km} \div 48 \text{ km} = 6 \text{ hours}$
- Dan: $288 \text{ km} \div 80 \text{ km} = 3.6 \text{ hours}$

**Answer:** Cindy will arrive at B at 14:00 (or 2.00 pm) and Dan will arrive at A at 13:36 (or 1.36 pm).

### PbY6b, Page 149

Q.4 Read: **Solve the problems by writing equations.**

Deal with one at a time. T chooses a P to read out the question. Allow Ps a minute to work it out in Ex. Bks, then Ps show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class points out errors and decides on the correct answer. Mistakes discussed/corrected.

T chooses a P to say the answer in a sentence.

**Solutions:** e.g.

a) **Piggy runs off at a speed of 5 metres per second. Two seconds later, Doggy chases Piggy at a speed of 7 metres per second.**

**When and where will Doggy catch up with Piggy?**

i) Let $t$ be the number of seconds that Piggy runs for.

Then the number of seconds that Doggy runs for is $t - 2$.

But the distance they each run is the same, so

Plan: $5 \times t = 7 \times (t - 2)$

$5 \times t = 7 \times t - 14$  
$0 = 2 \times t - 14$  
$14 = 2 \times t$  
$7 = t$, or $t = 7$ (seconds)

**Answer:** Doggy will catch up with Piggy 7 seconds after she started, and 35 metres from their starting point.

b) **The area of a rectangular garden is 150 m². If its length is 7.5 m, what is its width?**

Plan: $a \times 7.5 = 150$  

$\frac{a}{7.5} = \frac{150}{7.5} = \frac{2 \times 15 \times 10}{3 \times 5} = 20 \text{ m}$

**Answer:** The width of the garden is 20 m.

---

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7  (Continued)

c) The perimeter of a rectangular garden is 55 m and its length is 20 m. What is its width?

**Plan:** \(2 \times (20 + b) = 55\) \([\div 2]\)

\[
20 + b = 27.5 \quad [-20]
\]

\[
b = 7.5 \text{ (m)}
\]

**Answer:** The width of the garden is 7.5 m.

d) The area of a square garden is 196 m\(^2\). How long is each side?

\[
A = a \times a = 196 \text{ m}\(^2\)
\]

Factorise 196: \(196 = 2 \times 2 \times 7 \times 7 = (2 \times 7)^2 = 14^2\)

\[
a = \sqrt{196} = 14 \text{ m} \quad \text{(as } 14 \times 14 = 196)\]

**Answer:** Each side of the square is 14 metres long.

e) The perimeter of a square garden is \(60 \frac{4}{5}\) m.

How long is each side?

\[
P = 4 \times a = 60 \frac{4}{5} \text{ m}, \quad a = 60 \frac{4}{5} \div 4 = 15 \frac{1}{5} \text{ (m)}
\]

**Answer:** Each side is \(15 \frac{1}{5}\) metres long.

f) The surface area of a cuboid is 58 cm\(^2\). Its base edges are 4 cm and 2 cm. What is the height of the cuboid?

\[
A = 2 \times (a \times b + a \times c + b \times c) = 58 \text{ cm}\(^2\)
\]

\[
2 \times (4 \times 2 + 4 \times c + 2 \times c) = 58
\]

\[
2 \times (8 + 6 \times c) = 58 \quad [\div 2]
\]

\[
8 + 6 \times c = 29 \quad [-8]
\]

\[
6 \times c = 21 \quad [\div 6]
\]

\[
c = 3.5 \text{ (cm)}
\]

**Answer:** The height of the cuboid is 3.5 centimetres.

g) The volume of a cuboid is 28 cm\(^3\). Its base edges are 4 cm and 2 cm. What is its height?

\[
V = a \times b \times c = 28 \text{ cm}\(^3\)
\]

\[
4 \times 2 \times c = 28
\]

\[
8 \times c = 28 \quad [\div 8]
\]

\[
c = 3.5 \text{ (cm)}
\]

**Answer:** The height of the cuboid is 3.5 centimetres.
Factorising 150, 325, 500 and 1150. Revision and practice.

**PbY6b, page 150**

**Notes:**

**Lesson Plan 150**

150 = 2 × 3 × 5²
Factors: 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150

325 = 5³ × 13
Factors: 1, 5, 13, 25, 65, 325

500 = 2² × 5³
Factors: 1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500

1150 = 2 × 5² × 23
Factors: 1, 2, 5, 10, 23, 25, 46, 50, 115, 230, 575, 1150

(or set factorising as homework at the end of Lesson 149 and review at the start of Lesson 150.)
### Activity

**Solutions (continued)**

#### Q.3 a) Let $x$ be the length of the 3rd piece, so

- length of long piece: $x + 20$
- length of short piece: $x - 20$

$$x + x + 20 + x - 20 = 240$$

$$3 \times x = 240 \quad \text{[÷ 3]}$$

$$x = 80 \text{ (cm)}$$

*Answer:* The length of the 3rd piece of wire was 80 cm.

b) Let $x$ be the number of sweets that Louis has.

$$3 \times x - 8 = x + 8 \quad [-x]$$

$$2 \times x - 8 = 8 \quad [+ 8]$$

$$2 \times x = 16 \quad [÷ 2]$$

$$x = 8$$

*Answer:* Louis has 8 sweets and Sarah has 24 sweets.

c) Let the money that George had at first be $x$.

$$x - \frac{x}{2} - \frac{x}{4} - \frac{x}{8} = 2 \quad [\times 8]$$

$$8x - 4x - 2x - x = 16$$

$$x = 16 \text{ (£)}$$

*Answer:* George had £16 at first.

#### Q.4 a) $3a + 2a = 12$

$$5a = 12 \quad [+ 5]$$

$$a = 2.4$$

*Check:* $3 \times 2.4 + 2 \times 2.4 = 7.2 + 4.8 = 12$ ✓

b) $42 \div b = 3 \quad [\times b]$

$$42 = 3 \times b \quad [\div 3]$$

$$b = 14$$

*Check:* $42 \div 14 = 21 \div 7 = 3$ ✓

c) $2 \times (c + 2) = 3$

$$2 \times c + 4 = 3 \quad [-4]$$

$$2 \times c = -1 \quad [\div 2]$$

$$c = -0.5$$

*Check::*

$$2 \times (-0.5 + 2) = 2 \times 1.5 = 3$ ✓

d) $2d + 5d = 3d + \frac{1}{2}$

$$7d = 3d + \frac{1}{2} \quad [-3d]$$

$$4d = \frac{1}{2} \quad [\div 4]$$

$$d = \frac{1}{8}$$

*Check:* $LHS: 2 \times \frac{1}{8} + 5 \times \frac{1}{8} = \frac{7}{8}$

$$RHS: 3 \times \frac{1}{8} + \frac{1}{2} = \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ ✓
Solutions (continued)

e) \[ \frac{e}{9} - 2 \div 9 = e \div 15 \]

\[ e - 2 = \frac{9 \times e}{15} = \frac{3 \times e}{5} \] \[ \times 5 \]

\[ 5e - 10 = 3e \] \[ \div 3 \]

\[ 2e - 10 = 0 \] \[ \div 2 \]

\[ 2e = 10 \] \[ \times 5 \]

\[ e = 5 \]

f) \[ f^2 = f \times 111 \]

\[ f = 111 \]

Check:

LHS: \( \frac{5}{9} - 2 \div 9 = \frac{3}{9} = \frac{1}{3} \)

RHS: \( 5 \div 15 = 1 \div 3 = \frac{1}{3} \)

Q.5 Accept any valid rule with any numbers which make the rule correct but challenge the more able Ps to generalise, as below.

Rule: e.g. The sum of every two adjacent numbers is the number directly above them.

Let \( x \) be one of the terms of the addition which sums to 1000, then the other term is \( 1000 - x \), etc. Note that in the bottom row a second variable needs to be used.

C:

\[ 1000 - x - (x - 37.5) \]
\[ = 1000 - x - x + 37.5 \]
\[ = 1037.5 - 2x \]

\[ 1037.5 - 2x - (x + y - 75) \]
\[ = 1037.5 - 2x - x - y + 75 \]
\[ = 1112.5 - 3x - y \]

\[ x - 37.5 - (37.5 - y) \]
\[ = x - 37.5 - 37.5 + y \]
\[ = x + y - 75 \]
### Activity

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- 151 is a prime number  
  \( \text{Factors: } 1, 151 \)
  (as not exactly divisible by 2, 3, 5, 7, 11 and 13; \( 13^2 > 151 \))
- 326 = 2 \( \times \) 163  
  \( \text{Factors: } 1, 2, 163, 326 \)
- 501 = 3 \( \times \) 167  
  \( \text{Factors: } 1, 3, 167, 501 \)
- 1151 is a prime number  
  \( \text{Factors: } 1, 1151 \)
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and 37; \( 37^2 > 1151 \))

---

#### Properties of 3-D shapes

T has various solids on table at front of class. (e.g. sphere, cylinder, cone, prisms with various polygons as their base, cuboid, cube, other polyhedra) What do these shapes all have in common? (3-D shapes or solids) T holds up each in turn and Ps say its name if they know it.

Let's put them into sets. T specifies a set and Ps come to front of class to choose the appropriate solids and to say why they chose them. Class agrees/disagrees. Ps can choose the criteria for a set too.

- a) It has only plane faces (i.e. it is a polyhedron); it has no plane faces; it has some plane faces.
- b) It is a convex shape; it is a concave shape.

For the polyhedra only:
- c) Group according to the number of edges (faces, vertices)  
  T elicits or reminds Ps of the relationships in Euler's theorem.  
  (No. of faces + no. of vertices = no. of edges + 2)
- d) It has parallel faces; it has no parallel faces
- e) It has perpendicular faces; it has no perpendicular faces
- f) It has only regular faces; it has no regular faces.
  (Ps name and analyse the faces of the polyhedra: square, triangle, rectangle, parallelogram, trapezium, etc.)
### Activity 3

**PbY6b, page 151**

**Q.1** Read: *If the statement is true, write T in the box and if it is false, write F.*

I will give you 3 minutes to do it. Start...now!...Stop!

Review with whole class. T chooses a P to read out the statement. Ps show T or F on scrap paper or slates (or use pre-agreed actions) on command. Ps with different answers explain reasoning by giving an example or counter-example. Class agrees on the correct response. Mistakes corrected.

**Solution:**

- a) A **cuboid** has 8 vertices, 6 faces and 10 edges. \[F\]
- b) Every **cube** has 6 faces, 8 vertices and 12 edges. \[T\]
- c) A **circle** is a 2-dimensional shape. \[T\]
- d) A **line segment** is a 2-D shape. \[F\]
- e) Every cuboid is a **prism**. \[T\]
  
  (Elicit that a prism is a polyhedron with at least one pair of parallel, congruent faces.)
- f) Any prism is a cuboid. \[F\]
  
  (If its base is neither a rectangle nor a square, the prism is not a cuboid.)
- g) If the diagonals of a quadrilateral are equal and **bisect** each other, the quadrilateral is a rectangle. \[T\]
- h) If a quadrilateral has 2 lines of symmetry it is a **rhombus**. \[F\]
  
  (Elicit that a rhombus is a quadrilateral with equal sides.)

---

### Activity 4

**PbY6b, page 151**

**Q.2** Read: *Construct an **isosceles** triangle which has a base side of 5 cm:*

- a) and its other two sides are 3 cm long
- b) and its height is 2.5 cm
- c) and the angles at its base are 75°
- d) and it is a regular triangle.

What is an isosceles triangle? (A triangle which has at least 2 equal sides) What instruments should you use to construct the triangles? (Ruler and compasses)

Deal with one at a time under a short time limit. Ps finished first construct the triangle on BB using BB instruments, explaining what they are doing at each step. Class points out errors. Ps’ own mistakes are discussed and corrected. What else do you notice about this isosceles triangle? (See below)

**Solution:**

- a) 

![Obtuse-angled isosceles triangle](image)

(A cuboid has 12 edges.)

(Has 1 dimension: length)

[It is a rectangular-based prism]

\[\text{e.g. Show a a triangular prism.}\]

\[\text{BB: e.g. } \square\]

\[\text{e.g. } \square \text{ is a rhombus} \]

\[\text{but } \bigtriangleup \text{ is not.}\]

---

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Elicit that the height of the triangle is the perpendicular distance from its base to the opposite vertex. Its line of symmetry bisects its base.

To construct a 75° angle, construct two 60° angles, bisect one of them to form two 30° angles, then bisect one of these to form a 15° angle: 15° + 60° = 75°.

Extend both arms until they intersect.

Elicit that its line of symmetry is the perpendicular bisector of its base.

Elicit that a regular triangle has equal sides and equal angles, so each angle is: 180° ÷ 3 = 60°

It has 3 lines of symmetry.
### Lesson Plan 151

#### Activity 5

*Pby6b, page 151, Q.3*

Read: *These triangles are made up of congruent triangles. The triangles in b), d) and e) are isosceles triangles.*

What are congruent triangles? (Triangles which are exactly the same size and shape.) What is an isosceles triangle? (It is a triangle which has at least two equal sides.) Is an equilateral Δ an isosceles Δ? (Yes)

Read: *Find relationships for each shape and write mathematical statements about them.*

What could we do to make it easier to write statements about the triangles? (Label the vertices.) T suggests it if Ps do not.

Deal with one triangle at a time. Ps come to BB or dictate what T should write. Class agrees/disagrees. T prompts if Ps run out of ideas or writes a statement and asks Ps if it is correct.

**Solution:**

<table>
<thead>
<tr>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
<th>e</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Diagram" /></td>
<td><img src="image2" alt="Diagram" /></td>
<td><img src="image3" alt="Diagram" /></td>
<td><img src="image4" alt="Diagram" /></td>
<td><img src="image5" alt="Diagram" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
<th>e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔABC ~ ΔFEC; Ratio of areas of ΔABC : ΔFEC = 4 : 1;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AF = FC = DE; AB = 2 × AD; FDEC is a rectangle;</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ADEF is a parallelogram; ( \angle A + \angle B = 90^\circ ); ( \text{EF} \parallel \text{AB} );</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DE ( \parallel ) CB; ( \frac{EF}{AB} = \frac{1}{2}; \ ΔADF \cong ΔDEF, \text{ etc.} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>f)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔCHG ~ ΔCIF ~ ΔCAB in the ratio 1 : 2 : 3;</td>
</tr>
<tr>
<td>Ratio of areas of ΔCHG : ΔCIF : ΔCAB = 1 : 4 : 9;</td>
</tr>
<tr>
<td>Extra praise if Ps notice that the ratio of the areas is the ratio of the sides squared. (1^2 = 1, 2^2 = 4, 3^2 = 9)</td>
</tr>
<tr>
<td>HG ( \parallel ) IF ( \parallel ) AB; ( \text{GH} = \frac{\text{IF}}{2} = \frac{\text{AB}}{4}; \text{ etc.} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole class activity</td>
</tr>
<tr>
<td>Drawn (stuck) on BB or use enlarged copy master or OHP</td>
</tr>
<tr>
<td>Elicit the sign which means 'congruent to'</td>
</tr>
<tr>
<td>BB: ( \cong )</td>
</tr>
<tr>
<td>Ps label triangles in Pb s too.</td>
</tr>
<tr>
<td>Involve as many Ps as possible.</td>
</tr>
<tr>
<td>At a good pace.</td>
</tr>
<tr>
<td>Agreement, praising</td>
</tr>
<tr>
<td>Extra praise for unexpected statements</td>
</tr>
<tr>
<td>Ps write a different type of statement below each triangle in Pb s.</td>
</tr>
</tbody>
</table>

Triangles b) to e) can also have similar statements made about them, except that the shape of FDEC is a square in b), a parallelogram in c) and a rhombus in d) and e);

In b): \( \angle A = \angle B = 45^\circ \), etc.

In f), as an extension, T could suggest marking the midpoint of AB (e.g. K) and elicit the ratios:

| CJ : JK = 2 : 1, |
| CJ : CK = 2 : 3, |
| KJ : KC = 1 : 3 |
Y6

**Activity 6**

*PbY6b, page 151*

**Notes**

- Read: *Mark the midpoints of the sides of each quadrilateral and join them up in order. Write the names of the polygons you have made.*

Set a time limit. Ps use rulers or compasses to mark the midpoints.

Review with whole class. T could have solution already prepared and uncover each shape as it is dealt with. Ps say the name of the original shape and the name of the shape they made by joining the midpoints of the sides. Mistakes corrected.

Which of the original shapes are symmetrical? Ps come to BB to draw the lines of symmetry. Class agrees/disagrees. Are these lines of symmetry the same for the new shapes? (Only in e) do they not match up: a rectangle has 2 lines of symmetry but the deltoid has only one)

**Solution:**

- (rectangle) \(\text{b)}\) (square) \(\text{c)}\) (parallelogram) \(\text{d)}\) (rhombus)

\(\text{a)}\) rhombus

\(\text{f)}\) parallelogram

\(\text{h)}\) (trapezium)

\(\text{g)}\) (quadrilateral)

**Extension**

How can we prove that, in g), EFGH is a parallelogram?

If Ps have no ideas, lead Ps through the proof for c) and g), involving Ps when possible but have no expectations of them understanding it.

E.g. in g): Join AC.

In triangle ACD, HG is parallel to AC and is half its length.

(as HG joins the midpoints of AD and DC)

In triangle ABC, EF is parallel to AC and is half its length.

So HG is parallel and equal to EF.

Therefore EFGH is a parallelogram.

or

Triangle DHG is similar to triangle DAC in the ratio of 1 : 2, so HG is parallel to AC and HG : AC is 1 : 2, i.e. HG is half of AC.

Triangle BFE is similar to triangle BCA in the ratio of 1 : 2, so EF is parallel to AC and EF : AC is 1 : 2, i.e. EF is half of AC.

So EF is equal and parallel to HG.

Therefore EFGH is a parallelogram.

**Individual work, monitored, helped**

**Discussion, reasoning, agreement, self-correction, praising**

**Whole class activity**

Review the properties of a parallelogram first.

(Quadrilateral with opposite sides equal and parallel)

Elicit that if one pair of opposite sides are equal and parallel, then the other pair of opposite sides must also be equal and parallel.

T starts to explain and Ps try to follow the reasoning.

Allow Ps who understood the first part to explain the 2nd part of the reasoning in their own words, with T’s help.

Similar reasoning can be applied to c).
R: Calculations
C: Review: Reflection in an axis and symmetry
E: Problems and challenges

**Y6**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
</table>
| 1 | **Factorisation**
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:
- **152** = \(2 \times 2 \times 2 \times 19\)
  Factors: 1, 2, 4, 8, 19, 38, 76, 152
- **327** = \(3 \times 109\)
  Factors: 1, 3, 109, 327
- **502** = \(2 \times 251\)
  Factors: 1, 2, 251, 502
- **1152** = \(2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3\) = \(2^7 \times 3^2\)
  Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 32, 576, 384, 288, 192, 144, 128, 72, 64, 48, 36
  [No. of factors: \((7 + 1) \times (2 + 1) = 8 \times 3 = 24\)]

8 min

| 2 | **Reflection in an axis**
A, come an mark a point on the BB and label it A. B, come and draw a mirror line and label it \(e\). How can we reflect point A in line \(e\)?
P comes to BB to draw the reflection, explaining what he/she is doing in a loud voice. Class agrees/disagrees. Who can think of another way to do it? Come and show us.
T chooses a P to summarise the steps needed (see below).
What are the main properties of the reflection? Ps come to BB or dictate what T should write. Class agrees/disagrees.
Repeat in a similar way for reflecting a line segment and a circle.

a) **Reflection of a point in an axis**
e.g.

\[
\begin{align*}
AT &= TA' \\
AA' &\perp e
\end{align*}
\]

1) Draw (using set square and ruler or compasses) a perpendicular line from A through \(e\). Label the point of intersection, e.g. T.
2) Using compasses (or ruler), measure the distance from A to T and then mark the same distance on the opposite side of T. Label the marked point A'. A' is the mirror image of A.

b) **Reflection of a line segment in an axis**
e.g.

\[
\begin{align*}
AB &= A'B', \quad AA' &\perp e, \quad BB' &\perp e. \\
AA' &\parallel BB', \quad AB' = A'B, \\
AB \text{ and } A'B' &\text{ intersect on line } e.
\end{align*}
\]

1) Reflect the points A and B in \(e\) and label them A' and B'.
2) Join A' to B'. A'B' is the mirror image of AB.

Individual work, monitored (or whole class activity)
BB: 152, 327, 502, 1152
T decides whether Ps can use calculators.
Reasoning, agreement, self-correction, praising

<table>
<thead>
<tr>
<th>No.</th>
<th>1152</th>
<th>576</th>
<th>327</th>
<th>288</th>
<th>109</th>
<th>144</th>
<th>72</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td>152</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>76</td>
<td>2</td>
<td>109</td>
<td>109</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>38</td>
<td>2</td>
<td></td>
<td>1</td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>

Whole class activity
T should have BB instruments available for Ps to use.
Involve several Ps.
Discussion, reasoning, agreement, praising
Ps could do drawings in Ex. Bks too.

Feedback for T

Elicit that a line segment has a start and end point whereas a line is never-ending (infinite) in both directions.
(Continued)

c) Reflection of a circle in an axis

e.g.  

1. Reflect the centre point, O, and a point, P, on the circumference, k. Label the mirror images O’ and P’.  
2. With compasses set to length OP, draw a circle around O’ passing through P’. Label the circumference k’.

15 min

3. **PbY6b, page 152**

Q.1 Read: **Reflect** each shape in the given mirror line. Use a ruler and a pair of compasses.

Set a time limit or deal with one at a time. Ps finished early can be asked to write mathematical statements about their reflections.

Review with whole class. T could have solution already prepared and ask Ps just to explain the steps to save time.

Class agrees/disagrees. Mistakes discussed and corrected.

Ask Ps to make true statements about the reflections.

**Solution:**

\[ \Delta ABC \cong \Delta A'B'C' \]

\[ AC = A'C', \ AC \parallel A'C' \parallel e \]

\[ \text{Line } BC \equiv \text{line } C'B' \perp e \]

\[ AA' \parallel BB', \ BA = B'A', \]

\[ \angle A = \angle A', \ \angle B = \angle B', \]

etc.

\[ \text{ABCD } \cong \text{A'B'C'D'} \] (deltoids)

\[ \text{AB } = \text{A'B'}, \text{ etc.} \]

\[ \angle A = \angle A', \text{ etc.} \]

\[ \text{AA' } \parallel \text{BB'} \parallel \text{CC'} \parallel \text{DD'} \]

\[ \text{AA' } \perp e, \text{ etc.} \]

The 2 circles are congruent.

\[ \text{OK } = \text{O'K'} = \text{ radius } (r) \]

\[ \text{OO' } \parallel KK', \]

\[ \text{OO'} \perp e, \text{ KK'} \perp e, \]

\[ \text{KK'O'O} \text{ is a trapezium.} \]

23 min
### Lesson Plan 152

#### Activity

<table>
<thead>
<tr>
<th>Y6</th>
<th>4</th>
<th><strong>PbY6b, page 152</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.2</td>
<td>Read: <em>Draw the mirror line in the correct place for each shape and its mirror image.</em></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Set a time limit. Ps use rulers and compasses.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps come to BB to draw the mirror lines, explaining what they are doing. Who did the same? Who did it a different way? etc. Class points out errors. Mistakes discussed and corrected.</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a)</td>
<td>The point ( A = A' ) is on the mirror line. The midpoints of ( BB' ) and ( CC' ) are on the mirror line. The perpendicular bisector of ( BB' ) (and of ( CC' )) is the mirror line.</td>
<td></td>
</tr>
<tr>
<td>b)</td>
<td>The points of intersection of ( CD' ) and ( CD ), and of ( AB ) and ( A'B' ) lie on the mirror line. Draw a straight line through these two intersections. This is the mirror line. Elicit that in each part the shape and its mirror image are congruent. The shapes in a) are right-angled triangles, the shapes in b) are rectangles, the shapes in c) are hexagons.</td>
<td></td>
</tr>
</tbody>
</table>

30 min

#### Activity

<table>
<thead>
<tr>
<th>5</th>
<th><strong>PbY6b, page 152</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Q.3</td>
<td>Read: <em>Which shapes are symmetrical? Draw the lines of symmetry where appropriate. Write the number of lines of symmetry below each shape.</em></td>
</tr>
<tr>
<td></td>
<td>Set a time limit of 3 minutes. Ps use rulers and set squares.</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. Ps come to BB to draw mirror lines, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Ask Ps to name the shapes if they can. Who had all 9 correct? Let’s give them a clap!</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
</tr>
<tr>
<td>a)</td>
<td>Deltoid</td>
</tr>
<tr>
<td>b)</td>
<td>Parallelogram</td>
</tr>
<tr>
<td>c)</td>
<td>Isosceles triangle</td>
</tr>
<tr>
<td>d)</td>
<td>Plane shape</td>
</tr>
<tr>
<td>e)</td>
<td>Circle</td>
</tr>
<tr>
<td>f)</td>
<td>Octagon</td>
</tr>
<tr>
<td>g)</td>
<td>Square</td>
</tr>
<tr>
<td>h)</td>
<td>Equilateral triangle</td>
</tr>
<tr>
<td>i)</td>
<td>Rhombus</td>
</tr>
<tr>
<td></td>
<td>Infinite ( \infty )</td>
</tr>
</tbody>
</table>

35 min

#### Notes

- Individual work, monitored, helped
- Drawn on BB or use enlarged copy master or OHP
- Reasoning, agreement, self-correction, praising
- (If majority of Ps are struggling, stop individual work and continue as a whole class activity, with Ps working in Ex. Bks. while a P works on BB.)

Individual work, monitored
- Drawn on BB or use enlarged copy master or OHP
- Differentiation by time limit.
- Reasoning, agreement, self-correction, praising
- Elicit or remind Ps of the symbol which means an infinite number of times (i.e. never-ending). BB: \( \infty \) means an infinite number (infinity)

Feedback for T
### Activity 6

*PbY6b, page 152, Q.4*

**Read:** Draw the path of the billiard ball after it has rebounded off the edge of the billiard table.

Who has played billiards? What are the rules? (T tells them if no P knows.) Who can explain what the diagrams mean? (The white circle is the ball, the shaded thick lines are parts of the surround of the billiard table, the arrow shows the path of the billiard ball after it has been hit by the billiard cue.)

1. **How can we tell at which point the ball will hit the edge of the table?**
   - (Extend the arrow line until it meets the shaded area.) P comes to BB to draw it. Rest of Ps draw it in *Pbs.*
   - Where do you think the ball will go after that? Ask several Ps what they think. T gives hint about reflection if Ps have no ideas. (The ball will rebound off the edge at the same angle as it hits it.)

2. **How can we draw these equal angles?**
   - (Draw a line perpendicular to the edge of the table at the point where the ball hits it, then either measure the angle made by the arrow line and the perpendicular and measure the same angle on the opposite side, or draw a mirror image of the arrow line.) Elicit that this time the arrowhead will point away from the edge of the table. T (P) works on BB and Ps work in *Pbs.*

3. **Once Ps have been given the idea, they might be able to do part b) more easily (but elicit that the procedure has to be done twice).**
   - T (or Ps) work on BB and Ps work in *Pbs.*
   - What do you notice about the path of the ball after it has rebounded the second time? (Its path is parallel to the path of the first hit but is moving in the opposite direction.)

   *If T has cue and ball and there is an expert snooker or billiard player in the class, the P could demonstrate the hit.*

4. **Solution:**
   - a) Measure $\angle \alpha$ and draw an equal angle on opposite side of perpendicular, or mark a point on original arrow, reflect it and join it to the rebound point.
   - b) Where do you think the ball will go after that? Ask several Ps what they think. T gives hint about reflection if Ps have no ideas. (The ball will rebound off the edge at the same angle as it hits it.)

5. **Individual (paired ) trial first, monitored**
   - (or whole class activity if time is short or Ps are not very able)
   - Drawn on BB or use enlarged copy master or OHT

   *If possible, T could have a real cue and balls to show.*

   **Discussion involving many Ps. T gives hints if necessary.**

   **Reasoning, agreement, praising**

   **Solution:**
   - a) $\alpha + \beta = 90^\circ$
   - b) Measure $\angle \alpha$ and draw an equal angle on opposite side of perpendicular, or mark a point on original arrow, reflect it and join it to the rebound point.

### Activity 7

*PbY6b, page 152*

**Q.5 Read:** We want the black billiard ball to hit the white ball after rebounding off the edge of the billiard table. Draw the path it should take. Explain why you drew it.

Set a time limit of 4 minutes. Ps can work in pairs if they wish and discuss with their neighbours. It is likely that most Ps will use trial and error and gradually get closer to the correct paths.

If no P is on the right track, T could give a hint about using the edge of the table as a mirror line for one of the balls (but which ball?)

If no P has solved it, either lead Ps through the solution or leave the problem open as homework.

**Solution:**

1. Reflect the white ball in the given table edge (by reflecting the centre point and drawing around it a circle of equal radius).
2. Join its centre to the centre of the black ball.
3. Join the point where it hits the edge of the table to the centre of the white ball and draw the appropriate arrowheads.

Ps could draw a perpendicular axis at the rebound point and check that the second path is a reflection of the first path.

---

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### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $153 = 3 \times 3 \times 17 = 3^2 \times 17$
  - Factors: 1, 3, 9, 17, 51, 153
- $328 = 2 \times 2 \times 2 \times 41 = 2^3 \times 41$
  - Factors: 1, 2, 4, 8, 41, 82, 164, 328
- $503$ is a prime number
  - Factors: 1, 503
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and $23^2 > 503$)
- $1153$ is a prime number
  - Factors: 1, 1153
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and $37^2 > 1153$)

---

### Activity 2

**PbY6b, page 153**

Q.1 Read:

a) Measure the length of segment $AC$ and mark $A'$ on the ray so that it is the reflection of $A$ in $C$.

b) Complete the statements.

What is a ray? (A straight line starting from a point and extending in only one direction.)

Set a time limit of 3 minutes. Advise Ps to use compasses rather than a ruler. (Set width of compasses to $AC$, then with point of compasses on $C$, draw an arc to cut the ray on the opposite side of $C$.)

Review with whole class. Ps come to BB to show and explain the construction (using BB compasses) and to fill in the missing items. Who agrees? Who wrote something else? Mistakes discussed and corrected.

**Solution:**

a) $\overrightarrow{AC}$

b) i) $AC \cong CA'$  
   ii) $C$ is the midpoint of $AA'$

T: We say that $A'$ is the reflection of $A$ in point $C$, or when $A$ is reflected in point $C$, its mirror image is $A'$.

---

### Notes

- Individual work, monitored (or whole class activity)
- BB: 153, 328, 503, 1153
- T decides whether Ps can use calculators.
- Reasoning, agreement, self-correction, praising

<table>
<thead>
<tr>
<th>153</th>
<th>328</th>
<th>503</th>
<th>1153</th>
</tr>
</thead>
<tbody>
<tr>
<td>153</td>
<td>3</td>
<td>328</td>
<td>2</td>
</tr>
<tr>
<td>51</td>
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</tr>
<tr>
<td>17</td>
<td>17</td>
<td>82</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>41</td>
<td>41</td>
<td></td>
</tr>
</tbody>
</table>

- Individual work, monitored, helped, construction corrected
- Drawn/written on BB or SB or OHT
- Agreement, praising

Using compasses is quicker and more accurate than measuring with a ruler.

Reasoning, agreement, self-correction, praising

Feedback for T
## Lesson Plan 153

### Activity

#### 3 Properties of reflection

Follow my instructions. T calls Ps to work on BB while rest of Ps work in *Ex. Bks* or on plain sheets of paper.

Mark 3 points, A, B and C, in your *Ex. Bks*. Arrange them like this. (BB)

Let's reflect point A in point C. How can we do it? Ps dictate the steps. (Join A to C and extend the line on the other side of C. Set width of compasses to AC then with point of compasses on C, draw an arc on the opposite side of C. Label the point A'.)

Now let's reflect point B in point C. (Ps again dictate the steps to get B'.)

Let's join A to B. Where is the mirror image of AB reflected in C? (A'B')

Who can tell me true statements about the diagram? Ps come to BB to write mathematical statements or to explain in words. Class agrees/disagrees. T prompts if Ps miss important properties. e.g. 

BB: \( AB = A'B' \), \( AC = CA' \), \( BC = CB' \), \( AB' = BA' \), 

\[ \Delta ABC \cong \Delta A'B'C', \quad \angle A = \angle A', \quad \angle B = \angle B', \quad B'C = B'C' \]

\( AB || A'B', \quad AB' || BA', \quad ABA'B' is a parallelogram, \)

Line segment AB can be rotated onto A'B' by 180° around C.

T might suggest that the direction of A'B' is the opposite of AB (draw appropriate arrowheads on B and B') and ask class if it is true. Elicit that the same could be said about AB' and A'B if we joined them up.

---

### Notes

Whole class activity but individual drawing, monitored

BB: \( A \)

Involve many Ps.

Have no expectations!

Reasoning, agreement, praising only

or e.g. \( AA' : AC = 2 : 1 \)

[Preparation for the concept of vectors.]

### Y6

#### 4 PbY6b, page 153

Q.2 Read: Reflect triangle \( ABC \) in point \( O \). Use a ruler and a pair of compasses.

Who can tell me how we will use the ruler and compasses? (Use a ruler to draw a ray from each vertex through point O and extend the ray on the other side of O. Use compasses to measure and mark the mirror images of the vertex. Use a ruler to join up the mirror images of the vertices to form triangle \( A'B'C' \)).

Set a time limit of 3 minutes. Review with whole class. Ps finished first show and explain construction on BB (or T has construction already prepared and Ps just explain what has been done to save time). Mistakes discussed and corrected

Let's think of true statements about the diagram. Ps tell class in words and/or come to BB to write it mathematically. Class agrees/disagrees. T suggests some too and asks if they are correct.

**Solution:** 

\( AB = A'B' \), etc.  

\( AO = OA' \), etc.  

\[ \Delta ABC \cong \Delta A'B'C' \]

\[ \Delta OAC \cong \Delta A'OC' \]

\( \angle A = \angle A' \), etc.  

\( AC || A'C', \quad AB || A'B' \)

\( AC' \cdot AC \) is a parallelogram, 

\( \Delta ABC \) can be rotated onto \( \Delta A'B'C' \) by 180° around O.

The mirror image of the mirror image of triangle \( ABC \) is itself.

---

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**Activity 5**

**PbY6b, page 153. Q.3**

Read: *Translate quadrilateral ABCD in the direction, and by the distance, shown by the arrow. Use a ruler and a pair of compasses.*

What does translate mean? (Move in the plane.) How can we translate the shape? (Translate each vertex, then join up the new points.)

How does the arrow help us to translate point A? Ps who have ideas come to BB to demonstrate. Who agrees? Who thinks we should do it another way? T gives hints if necessary. Elicit that the length of the arrow tells us how far the point should move and the direction in which the arrow is pointing shows us the angle it should be moved. Elicit that the path of point A will be equal and parallel to the arrow.

Revise how to draw parallel lines (using either ruler and set square or 2 rulers). T demonstrates if Ps have forgotten. Ps come to BB to draw parallel lines (with T's help), measure and mark the images of the points with compasses, label them, then join up the points to form A'B'C'D'. Rest of Ps follow in Pbs.

**Solution:**

Let's think of true statements about the diagram. Ps come to BB or dictate to T. Class agrees/disagrees. (See opposite.)

We could say that AD and A'D' are pointing in the same direction. T or P draws arrowheads at D and D'.

T: A line segment which has direction is called a **vector**. We write vectors with an arrow above them like this.

BB: $\overrightarrow{AD} = \overrightarrow{A'D'}$ and read it as, 'Vector AD equals vector A'D'.

Who can see another pair of vectors in the diagram? (Ps come to BB.)

**Extension**

29 min

**Notes**

Whole class activity

Drawn on BB or use enlarged copy master or OHP, for demonstration only

Discuss the steps required for the translation.

Involve several Ps.

T could have a template of the quadrilateral cut out and use it to demonstrate the translation.

Lay ruler (or set square) with one of its edges along the arrow. Place ruler beneath 1st ruler (or set square) and slide it along until it rests on A. Draw a ray from A along base of ruler.

Praising, encouragement only

**Assignment**

Statements: e.g.

BB: \[\begin{align*} \overrightarrow{ABCD} & \cong \overrightarrow{A'B'C'D'} \\ \overrightarrow{AD} & = \overrightarrow{A'D'}, \quad \overrightarrow{AD} \parallel \overrightarrow{A'D'}, \text{etc.} \\ \overrightarrow{AA'} & \parallel \overrightarrow{BB'} \parallel \overrightarrow{CC'} \parallel \overrightarrow{DD'} \\ \angle A & = \angle A', \text{etc.} \end{align*}\]

The labelling in both has the same orientation (anticlockwise)

Have no expectations!

Extra praise for unexpected criteria.

BB: **Vector**

a line segment with direction

**Activity 6**

**PbY6b, page 153**

Q.4 Read: *Rotate point A around centre O by 60° anticlockwise.*

First demonstrates the rotation with strips of card or straws. Discuss the steps needed to do draw it. (Join AO. Set compasses to width AO and with compass point on A, then on O, draw 2 arcs. Join O to the point of intersection, A'.)

This construction uses the concept of an equilateral triangle but Ps might also suggest using a protractor to measure the angle. Accept either method.

A P works on BB with T's help while rest of Ps work in Pbs.

**Solution:**

Individual work after initial discussion on method of construction, monitored, helped, corrected

Points drawn on BB or SB or OHT

Discussion, reasoning, agreement, praising

Elicit true statements about the diagram. e.g.

AO = A'O = AA'

AO and OA' are radii of a circle with centre O

\[\angle A = \angle A' = \angle O = 60^\circ\]
### Activity 6

Continued

T marks another point, B, below A. Ps mark it in \textit{Pbs}. too. Let's rotate B around O by 60°. P works on BB and rest of Ps work in \textit{Pbs}. Let's label the image B'. If we join up AB, where could we find its rotational image? (A'B')

What true statement could we write about the two line segments?

BB: \( AB = A'B' \) (the only one possible, as they are not parallel and are not facing in the same direction)

Who can tell us how to rotate any line segment or shape around a point? (Rotate each vertex, then join up the rotational images.)

35 min

---

### Activity 7

\textit{PbY6b}, page 153

Q.5 Read: \textit{Rotate the shape around centre O by 90° clockwise}.

\textit{Use a ruler and a pair of compasses (and a protractor if you wish).}

T has template already prepared. What kind of shape is it? (concave hexagon) Who can come and show us the rotation? Class decides whether it is correct. Elicit that a rotation clockwise means a rotation by a negative angle, i.e. by \(-90°\).

Do rotation of point A on BB with help of a P while rest of Ps follow in \textit{Pbs}. (First join O to A, then using the right angle on a set square or a corner of a ruler, or by construction of two 60° angles and bisecting one of them, or by using a protractor, draw a right angle, then set compasses to width OA and with point of compasses on O, mark point A' on the other arm of the angle.)

Set a time limit of 5 minutes for Ps to rotate the other vertices and join up the images.

Review with whole class. T could have construction already prepared (or Ps finished early complete the rotation on BB). Mistakes discussed and corrected.

Collect properties of the rotation (or if times runs out, set this task for homework and review before the start of \textit{Lesson 154}).

\textit{Solution:}

\[
\begin{align*}
\angle A &= \angle A' = \angle B = \angle B' = \angle C = \angle C' = \angle D = \angle D' = \angle E = \angle E' = 90°, \\
\angle F &= \angle F' = 270°, \\
& \text{etc.}
\end{align*}
\]

45 min

---
R: Calculations
C: Circle: names of its components, circumference
E: Introducing $\pi$

### Activity 1

#### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- $154 = 2 \times 7 \times 11$ Factors: 1, 2, 7, 11, 14, 22, 77, 154
- $329 = 7 \times 47$ Factors: 1, 7, 47, 329
- $504 = 2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7$
  Factors: 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 18, 21, 504, 252, 168, 84, 72, 63, 56, 42, 36, 28, 24
- $1154 = 2 \times 577$ Factors: 1, 577, 1154
  (577 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, and $29^2 > 577$)

### Activity 2

#### Components of a circle

Follow my instructions on drawing a circle. Ps draw in Ex. Bks. or on plain sheets of paper which they can stick in Ex Bks. T chooses different Ps to work on BB at each step.

a) Set your compasses to width 3 cm. Mark a point O and draw a circle around O. What do we call the line around the edge of a circle? (circumference)

b) Mark a point A on the circumference and draw OA. What part of the circle is OA? (radius) Write along the radius: $r = 3$ cm.

c) Draw a straight line through centre O and mark the points where it crosses the circumference as B and C. Label the line e. Line e is a line of symmetry of the circle because it divides it in half. How many lines of symmetry does a circle have? (an infinite number)

What part of the circle is line segment BC? (diameter)

What is the relationship between the length of the radius, $r$, and the length of the diameter? BB: $d = 2 \times r$

c) Draw a line, f, which crosses the circumference at 2 points, D and E, but does not pass through O. What do we call line f? (We say that line f is an intersector of the circle.)

What part of the circle is the line segment DE? (chord)

What name do we give to a part of the circumference, e.g. the part between A and B? (arc) What other arcs can you see?

Colour red the part of the plane inside the circle which is enclosed by the radii OA and OB and by the smaller arc AB. (It is like a slice of pizza.) What name to we call this part of the circle? (sector)

We say that $\angle$BOA in this sector is a central angle of the circle.

Colour blue the part of the plane inside the circle which is enclosed by the chord DE and by the smaller arc DE. What name to we give to this part of the circle? (segment)

Note that ECBD is also a segment of the circle.

### Notes

Individual work, monitored (or whole class activity)

BB: 154, 329, 504, 1154

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

Whole class activity but individual drawing, monitored closely, helped, corrected

T chooses a different P for each step shown on BB.

T could have flash cards of the underlined names prepared and stuck to side of BB. Ps choose the appropriate card at each step.

Allow Ps the opportunity to say the name if they know it.

BB: e.g.

**e)** Draw a line, t, which touches the circle at just one point, A. What do we call such lines? (We say that line t is a tangent to the circle at point A.)

A tangent is perpendicular to the radius which meets it at a common point.

BB: OA $\perp t$
### Activity 3

**PbY6b, page 154**

Q.1 Read: *Complete the statements about the diagram.*

T could leave the flash cards on show to help Ps to spell the missing words. Set a time limit of 4 minutes.

Review with whole class. T chooses a P to read out the sentence, saying 'something' instead of the box and another to identify the component on the diagram.

Show me the missing word...now! Ps with mistakes correct them and repeat the sentence again correctly.

**Solution:**

a) OT is a **radius** of the circle.

b) O is the **centre** of the circle.

c) AB is a **diameter** of the circle.

d) **Line segment** CD is a **chord** of the circle.

e) The smaller shape EOF is a **sector** of the circle.

f) The curve EF is an **arc** on the **circumference** of the circle.

g) \( \angle EOF = \alpha \) is the **central angle** of the smaller sector EOF.

h) Line CD is an **intersector** of the circle.

i) \( t \) is a **tangent** to the circle.

What other statements can you make about the diagram?

(e.g. \( OA = OE = OF = OB = OT = r \), \( OT \perp t \), Line \( t \) touches the circle at point T, or T is the common point of OT and t, etc.)

### Extension

**Ratio of the circumference and diameter of a circle**

Ps have cylinders (e.g. empty cans or wooden solids) and string or thick thread on desks. (Use at least 3 different sizes.)

a) What shape is the base of your cylinder? (a circle) Let's measure its circumference and then its diameter and compare them.

How can we measure its circumference? (T tells Ps what to do if no P has a suggestion.)

1. Coil the string tightly around the outside of the can close to the base, keeping it an equal distance from the base, and mark where the string meets itself.
2. Lay the string tightly along a ruler and note the marked length.

How can we measure the diameter of its base? Ps make suggestions.

   e.g. \( OA = OE = OF = OB = OT = r \). \( OT \perp t \).

   Line \( t \) touches the circle at point T, or T is the common point of OT and t, etc.)

b) Let's calculate the ratio of the lengths of the circumference and diameter. How should we do it? (Divide the length of the circumference by the length of the diameter.) Ps do calculations in Ex. Bks or on scrap paper and note the result.

### Notes

Individual work, monitored, helped
Drawn on BB or use enlarged copy master or OHP

**BB:**

Responses shown in unison.
Agreement, self-correction, praising

Feedback for T

Whole class activity
Praising only

---

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c) Let’s collect the different data and results in this table. A P from each pair dictates their results and the other P writes them in the table.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>c</td>
<td>18.8 cm</td>
<td>6.3 inches</td>
</tr>
<tr>
<td>d</td>
<td>6 cm</td>
<td>2 inches</td>
</tr>
<tr>
<td>( \frac{c}{d} )</td>
<td>3.13</td>
<td>3.15</td>
</tr>
<tr>
<td>( \frac{c}{d} )</td>
<td>3.125</td>
<td></td>
</tr>
</tbody>
</table>

BB: e.g.

d) What do you notice? (The ratio in the bottom row is between 3.1 and 3.2 each time, whatever the length of line or unit of measure.)

T: The ratio of the circumference of a circle and its diameter is a constant value (i.e. it does not change) and is about 3.14.

The exact value extends to an infinite number of decimal places. We call this value ‘pi’ and write it using a Greek letter.

BB: Ratio of the circumference of a circle to its diameter:

\[
\frac{\text{circumference}}{\text{diameter}} = \pi \quad (\text{pi}) \quad \approx \quad 3.14
\]

T chooses a P to write it on BB and class agrees/disagrees. Mistakes discussed and corrected.

Solution:

Length of \( A_1 A_3 A_5 A_7 A_9 \) \( \approx \) 15.2 cm

Length of \( A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 \) \( \approx \) 15.6 cm

b) Read: Write the lengths of the curved line, \( s \), and the two broken lines in increasing order.

Ps write it as an inequality inside semicircle in Pbs.

T quickly checks each P.

Q.2 Read: This semicircle has a radius of 5 cm. The length of its curved line is \( s \).

What is a semicircle? BB: (Half of a circle)

Who can come and show us \( s \) on the diagram?

What is \( s \)? (Half of the circumference of the whole circle)

What is \( d \) on the diagram? (The diameter of the whole circle.)

What are \( A_1, A_2 \), etc? (Points on the circumference) Let’s mark the points more clearly (with ‘ticks’). Ps do it in Pbs too.

a) Read: Measure the length of the two broken lines.

Set a short time limit. Ps use compasses and rulers to measure each part, find their sum and write lengths in Pbs.

Review with whole class. Ps could show lengths on scrap paper or slates on command. Accept slight inaccuracies but Ps who are very inaccurate should measure again (with the help of a more able P).

Solution:

Length of \( A_1 A_3 A_5 A_7 A_9 \) \( \approx \) 15.2 cm

Length of \( A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 \) \( \approx \) 15.6 cm

Ps might notice that the distances between each pair of points is about equal, measure one part and multiply by the number of parts.

Responses shown in unison.

Agreement, self-correction, praising

T explains, praising

Ps write the ratio on the blank page at the back of their Pbs.
c) Read: *If the ratio of the circumference to the diameter of any circle is about 3.14, what is the length of the curved line of the semi-circle?*

Allow Ps a minute to think about it and discuss with their neighbours if they wish. Who thinks they know what to do? Come and explain to us. Who agrees? Who would do it another way? etc. If no P is on the right track, T directs Ps’ thinking.

**Solution:**

\[ s = \text{half the length of the circumference of the whole circle} \]

\[ \frac{c}{d} \approx 3.14, \text{ so } c = 3.14 \times d = 3.14 \times 10 \text{ cm} = 31.4 \text{ cm} \]

\[ \text{and } s = 31.4 \text{ cm} \div 2 = 15.7 \text{ cm} \]

*Answer:* The length of the curved line of the semi-circle is about 15.7 cm.

d) Read: *Compare the lengths of the 3 lines and write their ratio.*

What can you tell me about the accuracy of the 3 lengths?

(The shorter broken line is a rough estimate of \( s \); the longer broken line is a better estimate of \( s \) and the calculated value is very close to the actual length.) Who can write the ratio of the 3 lengths?

**BB:** Shorter Broken Line : Longer BL : Calculated Value

\[ = 15.2 : 15.6 : 15.7 = 152 : 156 : 157 \]

37 min

Q.3 Read: *If the circumference of a circle with diameter 1 unit is about 3.14 units, calculate the circumference of a circle which has these lengths.*

What is the relationship between circumference and diameter?

**BB:** \( c = 3.14 \times d \)

Set a time limit of 4 minutes. Ps write answers in *Ex. Bks.*

Review orally with whole class. T chooses Ps to give their answers and explain their reasoning. Class agrees/disagrees. Mistakes discussed and corrected. If there is disagreement, ask Ps to show details of the calculations on BB.

**Solution:**

a) a 1 cm diameter: \( c = 3.14 \times 1 \text{ cm} = 3.14 \text{ cm} \)

b) a 7 cm diameter: \( c = 3.14 \times 7 \text{ cm} = 21.98 \text{ cm} \)

c) a 1 m diameter: \( c = 3.14 \times 1 \text{ m} = 3.14 \text{ m} \)

d) a 5 m diameter: \( c = 3.14 \times 5 \text{ m} = 15.7 \text{ m} \)

e) a 1 cm radius: \( c = 3.14 \times 2 \text{ cm} = 6.28 \text{ cm} \)

f) a 3 cm radius: \( c = 3.14 \times 6 \text{ cm} = 18.84 \text{ cm} \)

g) a 1 m radius: \( c = 3.14 \times 2 \text{ m} = 6.28 \text{ m} \)

h) a 2 m radius: \( c = 3.14 \times 4 \text{ m} = 12.56 \text{ m} \)

42 min
### Activity 7

**PbY6b, page 154, Q.4**

T reads out the question, Ps calculate mentally or in Ex. Bks. and stand up when they have an answer. T goes to them and Ps whisper answer in T’s ear. T tells them whether they are correct or not. If wrong they must sit down and calculate again.

When a few Ps are standing, T asks P who stood up first to explain how he/she got the answer so quickly. Ps correct their mistakes.

**Solution:**

If $\pi \approx 3.14$, calculate the circumference of a circle which has a:

- **a) 10 cm diameter**
  
  \[ c = 3.14 \times 10 \text{ cm} = 31.4 \text{ cm} \]

- **b) 8 m radius**
  
  \[ c = 3.14 \times 2 \times 8 \text{ m} = 6.28 \times 8 \text{ m} = 50.24 \text{ m} \]

- **c) 4 m radius**
  
  \[ c = 3.14 \times 2 \times 4 \text{ m} = 3.14 \times 8 \text{ m} = 25.12 \text{ m} \]

- **d) radius $r$**
  
  \[ c = 2 \times \pi \times r (= 6.28r) \]

### Homework

Set Question 5 for homework and review before the start of Lesson 155.

**PbY6b, page 154, Q.5**

- **a) $c = \pi \times 22 \text{ cm} (= 22\pi \text{ cm})$**
- **b) $c = \pi \times 2.5 \text{ m} (= 2.5\pi \text{ m})$**
- **c) $c = \pi \times d (= \pi d)$**
- **d) $c = 2 \times \pi \times r (= 2\pi r)$**

### Notes

Whole class activity but individual calculation

In good humour!

Praising, encouragement only

Class applauds quickest P(s).
Factorising 155, 330, 505 and 1155. Revision and practice. 

**PbY6b, page 155**

**Solutions:**

Q.1  

a) *Every isosceles triangle has angles of 60°.*  

(An isosceles triangle can be acute-angled, right-angled or obtuse-angled.)

b) *No rectangle has adjacent equal sides.*  

(A square has adjacent equal sides and is a rectangle.)

c) *The diameter of a circle is twice the length of its radius.*

(T)

d) The *circumference* of a circle is its radius multiplied by \( \pi \).  

(The circumference of a circle is its diameter multiplied by \( \pi \).)

e) *There is a prism which has congruent faces.*  

(e.g. A cube has congruent faces and is a prism.)

f) *If the diagonals of a quadrilateral bisect each other at right angles, the quadrilateral is a rhombus.*

(A square fits this description but it too is a rhombus.)

h) *A tangent to a circle can touch the circle at more than 1 point.*  

(A tangent touches the circle at only one point.)

Q.2  

a), d) and e): e.g.

(b) AC \( \approx 7.1 \) cm, DB \( \approx 2.3 \) cm

c) e.g. the length of one of its diagonals and the length of its long and short sides, but other answers are possible.

d) ABCD could have been transformed to A"B"C"D" by a single rotation.

N.B. A convex deltoid is shown but a concave deltoid could be constructed with the same dimensions.


Y6

Activity

Solutions (continued)

Q.3  a) i) \( c = 1.5 \text{ cm} \times 3.14 = 4.71 \text{ cm} \)
ii) \( c = 3.5 \text{ cm} \times 3.14 = 10.99 \text{ cm} \)
iii) \( c = 2 \times 3 \times \pi \times \pi = 6 \times \pi^2 = 6\pi^2 \text{ units} \)
   or \( \approx 2 \times 3 \times \pi \times 3.14 = 18.84\pi \text{ units} \)
   or \( \approx 2 \times 3 \times 3.14 \times 3.14 \approx 59.16 \text{ units} \) (to 2 d.p.)
b) i) \( c = 8.9\pi \text{ cm} \) ii) \( c = 5.24\pi \text{ cm} \) iii) \( c = \pi \text{ cm} \)

Q.4  a) The largest area is enclosed by the most regular shape, so the pen should be a square and the length of each side will be:
\( P = 4a = 200 \text{ m} \), so \( a = 200 \text{ m} \div 4 = 50 \text{ m} \)
b) This is probably best done by trial and error using a table to show different possible values for \( a \) and \( b \).
Elicit that: \( a + 2b = 200 \), so \( b = \left(\frac{200 - a}{2}\right) \)

<table>
<thead>
<tr>
<th>( a )</th>
<th>10</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>100</th>
<th>110</th>
<th>104</th>
<th>101</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b )</td>
<td>95</td>
<td>90</td>
<td>80</td>
<td>70</td>
<td>60</td>
<td>50</td>
<td>45</td>
<td>48</td>
<td>49.5</td>
</tr>
<tr>
<td>( A )</td>
<td>950</td>
<td>1800</td>
<td>3200</td>
<td>4200</td>
<td>4800</td>
<td>5000</td>
<td>4950</td>
<td>4992</td>
<td>4999.5</td>
</tr>
</tbody>
</table>

The greatest area is enclosed when the longer side is 100 m and the two shorter sides are 50 m.
c) \( A_1 = 50 \times 50 = 2500 \text{ (m}^2) \), \( A_2 = 50 \times 100 = 5000 \text{ (m}^2) \)
   \( \frac{A_2}{A_1} = \frac{5000}{2500} = \frac{2}{1} \rightarrow 200\% \)
   or \( \frac{A_1}{A_2} = \frac{2500}{5000} = \frac{1}{2} \rightarrow 50\% \)

Q.5  Accept any valid net. Ps use rulers and compasses. Ps check their nets by cutting out and folding.

Notes

Accept any of these answers.
Activity 1
Factorisation
Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:
- \[ 156 = 2 \times 2 \times 39 = 2^2 \times 39 \] Factors: 1, 2, 4, 39, 78, 156
- \[ 331 \] is a prime number Factors: 1, 331
(as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and 19 \( > 331 \))
- \[ 506 = 2 \times 11 \times 23 \] Factors: 1, 2, 11, 22, 23, 46, 253, 506
- \[ 1156 = 2 \times 2 \times 17 \times 17 = 2^2 \times 17^2 \] \[ = (2 \times 17)^2 = 34^2 \] Factors: 1, 2, 4, 17, 34, 68, 289, 578, 1156 (square number)

8 min

Activity 2
Perimeter and area
a) Let's calculate the perimeter and area of these polygons.

i) (rectangle) \[ b = 2 \text{ cm} \]
\[ a = 5 \text{ cm} \]
\[ P = 2 \times (5 + 2) \text{ cm} = 14 \text{ cm} \]
\[ A = (5 \times 2) \text{ cm}^2 = 10 \text{ cm}^2 \]

ii) (square) \[ a = 1.1 \text{ m} \]
\[ P = 4 \times 1.1 \text{ m} = 4.4 \text{ m} \]
\[ A = (1.1 \times 1.1) \text{ m}^2 = 1.21 \text{ m}^2 \]

iii) (right-angled triangle) \[ a = 2 \text{ km} \]
\[ b = 4 \text{ km} \]
\[ c = 4472 \text{ m} \]
\[ P = (2 + 4 + 4.472) \text{ km} = 10.472 \text{ km} \]
\[ A = \frac{1}{2} \times 2 \text{ km}^2 = 4 \text{ km}^2 \]

b) Let's see if you are clever enough to do it using letters instead of numbers! Elicit that each letter could stand for any value so the resulting equations are general formulae for perimeter and area.

i) \[ a \]
\[ b \]
\[ P = 2 \times (a + b) \]
\[ A = a \times b = ab \]

ii) \[ a \]
\[ P = 4 \times a = 4a \]
\[ A = a \times a = a^2 \]

iii) \[ a \]
\[ b \]
\[ c \]
\[ P = a + b + c \]
\[ A = \frac{a \times b}{2} \]

iv) (rhombus) \[ P = 4 \times a = 4a \]
\[ A = \frac{a \times f}{2} \]

v) (isosceles triangle) \[ P = a + b + b = a + 2b \]
\[ A = \frac{a \times h}{2} \]

If necessary, remind Ps how to calculate the area of a rhombus by drawing the surrounding rectangle. e.g.

Elicit that the diagonals of a rhombus bisect each other at right angles.
### Activity 2 (Continued)

(c) Let’s write the surface area and volume of each of these polyhedra.  
What is a polyhedron? (3-D shape with many plane faces)

<table>
<thead>
<tr>
<th>Polyhedron</th>
<th>Surface Area</th>
<th>Volume</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Cuboid</td>
<td>$A = 2 \times (4 \times 2 + 4 \times 3 + 2 \times 3) \text{ cm}^2 = 2 \times (8 + 12 + 6) \text{ cm}^2$</td>
<td>$V = (4 \times 2 \times 3) \text{ cm}^3 = 24 \text{ cm}^3$</td>
</tr>
<tr>
<td>ii) Cube</td>
<td>$A = 6 \times (1.5 \times 1.5) \text{ m}^2 = 6 \times 2.25 \text{ m}^2 = 13.5 \text{ m}^2$</td>
<td>$V = (1.5 \times 1.5 \times 1.5) \text{ m}^3 = 1.5 \times 2.25 \text{ m}^3 = 3.375 \text{ m}^3$</td>
</tr>
<tr>
<td>iii) Isosceles Triangle</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iv) Square</td>
<td>$A = 6 \times (a \times a) = 6 \times a^2 \quad [= 6a^2]$</td>
<td>$V = a \times a \times a = a^3$</td>
</tr>
</tbody>
</table>

### Notes

**Lesson Plan 156**

- **Y6**
- **Activity 2**

Q1 Read: Write below each polygon its perimeter and area.

What is a polygon? (a plane shape with many straight sides)

Set a time limit of 3 minutes. Ps calculate mentally (or in Ex. Bks) and write results in Pbs.

Review with whole class. First elicit the name of the shape then Ps show results on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>a) Rectangle</td>
<td>$P = 2 \times (12 + 8) \text{ cm} = 2 \times 20 \text{ cm} = 40 \text{ cm}$</td>
<td>$A = (12 \times 8) \text{ cm}^2 = 96 \text{ cm}^2$</td>
</tr>
<tr>
<td>b) Rhombus</td>
<td>$P = 4 \times a \quad (= 4a); \quad A = \frac{x \times y}{2} \quad (= \frac{xy}{2})$</td>
<td></td>
</tr>
<tr>
<td>c) Isosceles Triangle</td>
<td>$P = 8 \text{ cm} + 2 \times 5 \text{ cm} = 18 \text{ cm}; \quad A = \frac{3 \times 8^4}{2^1} \text{ cm}^2 = 12 \text{ cm}^2$</td>
<td></td>
</tr>
<tr>
<td>d) Square</td>
<td>$P = 4 \times u \quad (= 4u); \quad A = u \times u = u^2$</td>
<td></td>
</tr>
</tbody>
</table>

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**Activity 4**

PbY6b, page 156

Q.2 Read: Write below each polyhedron its surface area and volume.

What is a polyhedron? [a solid (or 3-D shape) which has many plane faces (or only faces which are polygons)]

Set a time limit of 4 minutes. Ps do necessary calculations in Ex. Bks and write results in Pbs.

Review with whole class. First elicit the name of the shape then Ps show results on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

**Solution:**

a) 

A = \(2 \times (5 \times 3 + 5 \times 1 + 3 \times 1) \text{ cm}^2\) \\
= \(2 \times 23 \text{ cm}^2 = 46 \text{ cm}^2\) \\
V = \((5 \times 3 \times 1) \text{ cm}^3 = 15 \text{ cm}^3\)

b) 

A = \(2 \times (a \times b + a \times c + b \times c) = 2(ab + ac + bc)\) \\
V = \(a \times b \times c = abc\)

d) 

A = \(6 \times x \times x = 6x^2\) \\
V = \(x \times x \times x = x^3\)

**Extension**

What are the general formulae for the surface area and volume of a *square-based* cuboid?

A = \(4 \times a \times b + 2 \times a \times a\) \\
= \(4 \times a \times b + 2a^2\) \\
V = \(a \times b \times a = a^2b\)

**Activity 5**

PbY6b, page 156

Q.3 Read: In the diagram, the points on the two sides are midpoints. What part of the square has been shaded?

What shape has been shaded? (a concave deltoid)

Allow Ps a couple of minutes to think and try to solve it.

(If no P is on the right track, T gives hint about labelling the vertices and calculating the area of each unshaded part.)

If you have an answer, show me it . . . now! \(\left(\frac{1}{4}\right)\)

P with correct answer explains reasoning at BB to class. Class agrees/disagrees.

If no P has the correct answer, T leads Ps through the solution, involving them once they understand what to do.

Ps write solution in Ex. Bks too.

**Solution:** e.g.

Let each side of the square be 1 unit. Then:

Area of AEGF = \(\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\) (square unit)

Area of \(\Delta EBC = \frac{1}{2} \times 1 = \frac{1}{2}\) \\

Area of \(\Delta CDF = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}\) (sq. unit)

So Area of ECFG = \(1 - \frac{1}{4} - \frac{1}{4} - \frac{1}{4} = \frac{1}{4}\) (sq. unit)

**Answer:** One quarter of the square has been shaded.

**Notes**

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP (or show actual models)

Responses shown in unison.

Reasoning, agreement, self-correcting, praising

Extra praise if Ps give the short forms too.

Feedback for T

**Extension**

What are the general formulae for the surface area and volume of a *square-based* cuboid?

A = \(4 \times a \times b + 2 \times a \times a\) \\
= \(4 \times a \times b + 2a^2\) \\
V = \(a \times b \times a = a^2b\)
<table>
<thead>
<tr>
<th>Activity</th>
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<tr>
<td><strong>Y6</strong></td>
<td></td>
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</tbody>
</table>

**PbY6b, page 156**

**Q.4 Read:** In the diagram, the sides of the large square are 3 units long. The sides of the large square have been divided into 3 equal parts and some of the dividing points have been joined up.

What is the area of the shaded square?

Use the same idea to solve this question as we used in Q.3. Set a time limit of 3 minutes.

Review with whole class. Ps with answers show results on scrap paper or slates on command. P with correct answer explains reasoning at BB. Who agrees? Who did it another way? etc.

(If no P has the correct answer, T directs Ps’ thinking.)

Ps correct their mistakes or if they could not solve it, write the solution in Ex. Bks.

**Solution:**

Area of $\Delta$ AEH = Area of $\Delta$ EBF = Area of $\Delta$ FCG

$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$ (square unit)

Area of ABCD = $3 \times 3 = 9$ (square units)

Area of EFGH = $9 - 4 \times 1 = 9 - 4 = 5$ (square units)

**Answer:** The area of the shaded square is 5 square units.

---

**Notes**

Individual work, monitored, helped

(Revert to a whole class activity if no P has an idea)

Drawn on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, self-correction, praising

---

**PbY6b, page 156, Q.5**

Read: A wooden cube was cut by some planes. The cuts were parallel to two opposite faces. The sum of the surface area of the pieces formed is 3 times the surface area of the cube. How many planes cut the cube?

Allow 2 minutes for Ps to think and discuss with their neighbours if they wish. Who thinks they know what to do? Come and explain it to us. Who agrees? Who thinks something else? If no P has an idea, T directs Ps’ thinking and class solves it together.

**Solution:**

Let the length of an edge of the cube be $a$, then

area of each face = $a^2$

surface area of the cube = $6 \times a^2$

When we make one cut along a suitable plane, the 2 pieces formed have 2 extra faces, i.e. their total surface area increases by $2 \times a^2$.

If we let the number of cuts be $n$, then

BB: $6 \times a^2 + n \times 2 \times a^2 = 18 \times a^2 \quad [- (6 \times a^2)]$

$n \times 2 \times a^2 = 12 \times a^2 \quad [\div (2 \times a^2)]$

$n = 6$

**Answer:** Six planes cut the cube.

---

**Whole class activity**

(or individual trial first if Ps wish)

Discussion, reasoning, agreement, (self-correction) praising

Extra praise if a P has a good idea of what to do.

T involves different Ps where possible.

Ps could copy the solution in Ex. Bks. too

Show the cuts in a diagram (or demonstrate on a model made from inter-locking plastic cubes)
**Y6**

**Activity**

8

**HMC:** Hungarian Mathematics Competition 1996 Age 11

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**Lesson Plan 156**

**Notes**

Whole class activity
(or individual or paired trial if Ps wish)

If the class is not very able, T (or Ps) could build the model first but if Ps are able, let them imagine it first, using enlarged copy master or OHP, then confirm with a model.

Discussion, reasoning, agreement, praising

Involve several Ps.

**BB:**

Extra praise for Ps who work out the solution without help.

N.B. If done as an individual challenge, leave the problem open as homework if no P can solve it in the time remaining.

---

**PbY6b, page 156, Q.6**

Read: Imagine a cube built from 27 small 1 cm cubes.

The middle cube in each face is removed and so is the small cube at the centre of the large cube.

What is:

a) the surface area of the remaining solid

b) the volume of the remaining solid?

Allow Ps a minute to think about it and discuss with their neighbours.

Who thinks they know what to do? Come and explain it to us. Who agrees? Who thinks something else? If no P has an idea, T directs Ps’ thinking and class solves it together.

**Solution:** e.g.

a) Original volume is 27 cm$^3$, and $27 = 3 \times 3 \times 3$, so the length of each edge is 3 cm.

Area of each face of the cube = $(3 \times 3)$ cm$^2$ = 9 cm$^2$

Surface area of the original cube = $6 \times 9$ cm$^2$ = 54 cm$^2$

After removing the 7 unit cubes:

Area lost = $6 \times 1$ cm$^2$ = 6 cm$^2$

Area gained = $6 \times 4$ cm$^2$ = 24 cm$^2$

So surface area of solid = $54 - 6 + 24 = 72$ (cm$^2$)

b) After removing 7 unit cubes, the volume of the remaining solid is $27$ cm$^3$ – $7$ cm$^3$ = 20 cm$^3$

45 min
### Lesson Plan

#### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **157** is a prime number
  - Factors: 1, 157
  - (as not exactly divisible by 2, 3, 5, 7, 11, and 13: 13² > 157)
- **332** = \(2 \times 2 \times 83\)
  - Factors: 1, 2, 4, 83, 166, 332
- **507** = \(3 \times 13 \times 13\)
  - Factors: 1, 3, 13, 39, 169, 507
- **1157** = \(13 \times 89\)
  - Factors: 1, 13, 89, 1157

#### Activity 2

**Angles**

a) Which of these angles do you think are equal? Ps study them carefully then come to BB or dictate to T. Class agrees/disagrees. How did you decide whether they were equal or not? (Ps will probably mention imagining them being moved together or turned around in their heads, or noticing parallel or perpendicular arms, etc.)

BB:

\[
\angle A = \angle B = \angle D = \angle C; \quad \angle E = \angle F
\]

b) Which pairs of angles do you think make an angle of 180°?

Ps come to BB or dictate to T. Class agrees/disagrees. How did you decide? (By imagining, e.g. \(\angle A\) and \(\angle F\) being moved together so that their vertices are at one point, then of course, each of the 2 angles can be replaced by an angle which is equal to it.)

BB:

\[
\angle A + \angle F = 180°, \quad \angle A + \angle E = 180°, \quad \angle B + \angle F = 180°, \\
\angle B + \angle E = 180°, \quad \angle C + \angle F = 180°, \quad \angle C + \angle E = 180°, \\
\angle D + \angle F = 180°, \quad \angle D + \angle E = 180°
\]

#### Activity 3

**Circumference**

What length is the circumference of each of these circles? Ps come to BB or dictate to T. Class agrees/disagrees. T helps Ps to express the exact length using \(\pi\).

BB:

\[
\text{a) } \quad P \approx 3.14 \text{ units} \\
\text{b) } \quad P \approx 6.28 \text{ units}
\]
### Activity

**Lesson Plan 157**

<table>
<thead>
<tr>
<th>Y6</th>
<th>Notes</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Stress that when we use ( \pi ), the value is exact but when we use 3.14, the value is only approximate.</td>
</tr>
</tbody>
</table>

#### Activity 3 (Continued)

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>c)</td>
<td>![Diagram](r = 1)</td>
</tr>
<tr>
<td>d)</td>
<td>![Diagram](r = 2)</td>
</tr>
</tbody>
</table>

\[
P \approx 6.28 \text{ units} \quad \quad \quad P \approx 4 \times 3.14 = 12.56 \text{ (units)}
\]

*PbY6b, page 157, Q.1*

Read: *The circumference of a circle with radius 1 unit is*

\[2 \times \pi \approx 6.28 \text{ units}.*

*Colour an arc of the given length on the circumference of the circle.*

a) If the whole circumference (T draws finger around it) is \(2 \times \pi\), what part of the circumference is \(\pi\) (half)? A, come and show us the arc with your finger. Who agrees? Who can show us it another way? Elicit that the arc is a semicircle and there are lots of ways to show it but agree that it is easiest to start from the point on the circumference where the radius meets it on the diagram (although the semicircle could be shown above or below the radius)

T colours the agreed arc on BB and Ps colour it in Phs too.

Continue in this way, dealing with one part at a time but if T thinks that most Ps understand what to do, change to individual work under a time limit and review as usual with the whole class.

What size of angle is formed by the two radii at the endpoints of each arc? Ps come to BB or dictate what T should write. Elicit that the whole central angle of a circle is \(360^\circ\), so, e.g. in a) the angle formed is half of \(360^\circ\), which is \(180^\circ\). etc.

*Solution:*

\[
a) \pi \\
b) \frac{\pi}{2} \\
c) \frac{\pi}{3} \\
d) \frac{\pi}{4} \\
e) \frac{\pi}{6} \\
f) \frac{3}{4}\pi \\
g) \frac{3}{2}\pi
\]

\[
180^\circ \\
90^\circ \\
60^\circ \\
45^\circ \\
30^\circ \\
135^\circ \\
270^\circ
\]

T: Mathematicians used to give the size of an angle using \( \pi \) in exactly this way before the unit we use now for measuring angles (degrees) had been thought of.

\[
23 \text{ min}
\]
Pby6b, page 157

Q.2 a) Read: *Is there a square in which the numerical value of its perimeter length (in cm) is equal to the numerical value of its area (in cm²)?*

Allow Ps 2 minutes to think about it and discuss it with their neighbours if they wish.

Stand up if you think there is. Show me the length of one of its sides . . . now! (4 cm)

P with correct answer explains reasoning to class. Who agrees? Who thought in another way? etc. If no P was correct, T leads Ps through the reasoning below. Ps who were wrong write the correct solution in Ex. Bks.

**Solution:**

Let the length of a side of the square be $a$.

$$P = 4 \times a \text{ and } A = a \times a$$

$$4 \times a = a \times a \quad [\div a]$$

$$4 = a$$

**Answer:** Yes, there is such a square and it has sides of 4 cm.

b) Read: *Is there a cube in which the numerical value of its surface area (in cm²) is equal to the numerical value of its volume (in cm³)?*

Again, set a time limit of 2 minutes. Continue in a similar way to a) but more Ps might be able to solve it this time.

**Solution:**

Let the length of an edge of the cube be $a$.

$$A = 6 \times a \times a \text{ and } V = a \times a \times a$$

$$6 \times a \times a = a \times a \times a \quad [\div (a \times a)]$$

$$6 = a$$

**Answer:** Yes, there is such a cube and it has edges of 6 cm.

---

HMC: Hungarian Mathematics Competition
1992
Age 11

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Pby6b, page 157

Q.3 Read: *You have 60 congruent small cubes and you want to build different kinds of cuboids. You must use all the unit cubes to build each cuboid. How many different sized cuboids could be built?*

Set a time limit of 3 minutes. Ps work in Ex. Bks. (If Ps are struggling, T gives hints about factorising and drawing a table.)

Review with whole class. Ps who have an answer show result on scrap paper or slates on command. P answering correctly explains at BB to Ps who were wrong. Who agrees? Who did it a different way? Mistakes discussed and corrected.

**Solution:** e.g. Let the edges of the cuboid be $a$, $b$ and $c$.

<table>
<thead>
<tr>
<th>$a$</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$c$</td>
<td>60</td>
<td>30</td>
<td>20</td>
<td>15</td>
<td>12</td>
<td>10</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>$V$</td>
<td>60</td>
<td>30</td>
<td>20</td>
<td>15</td>
<td>12</td>
<td>10</td>
<td>15</td>
<td>10</td>
</tr>
</tbody>
</table>

**Answer:** Ten different sized cuboids could be built.

---

Individual work, monitored, helped

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Ps who could not do it, copy solution in Ex. Bks.

Factors of 60: 60 2 1, 2, 3, 4, 5, 15 3 6, 10, 12, 15, 5 5 20, 30, 60 1
### Activity

#### PbY6b, page 157, Q.4

**Read:** The two regular octagons are congruent. 
Show that the two shaded areas are equal.

What is a regular octagon? (Polygon with 8 equal sides and angles)

Allow Ps a minute to think about the problem and discuss with their neighbours if they wish. Who has an idea? Who agrees? Who thinks something else? etc. If no P is on the right track T hints about dividing up the two shaded areas into equal parts. How could we do it? Ps come to BB to show it. Class agrees/disagrees. (Elicit that the shaded areas can be divided into congruent right-angled triangles: each triangle has base length half the side of the octagon, height from the centre of the octagon perpendicular to a side, and hypotenuse from the centre of the octagon to a vertex.) Ps do the same in Pbs too.

**Solution:**

To find the centre of each octagon:
- **LHS:** draw diagonals of rectangle
- **RHS:** join up another pair of opposite vertices

**Answer:** Each of the two shaded areas contains 8 congruent right-angled triangles which form half of the octagon, so the shaded areas are equal.

---

### Notes

**Whole class activity**

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

Extra praise for Ps who think of this without help.

Remind Ps of the name of the side opposite the right angle in a right-angled triangle

**BB:** hypotenuse

Class agrees on a form of words for the answer and Ps write it in Pbs.

---

#### PbY6b, page 157, Q.5

**Read:** The shorter side of a rectangle is 2 units and each of its diagonals is 4 units.

a) What size are the angles formed by the diagonals?

b) What size are the angles formed by the diagonals and the sides?

What should we do first? (Draw a diagram and label it.)

What do we know about a diagonal of a rectangle? (They are equal and bisect one other.)

Set a time limit of 3 minutes. Ps work in Ex. Bks.

Review with whole class. Ps show angles on scrap paper or slates on command. Ps answering correctly explain reasoning at BB, with T’s help if necessary. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

a) \( AD = BC = 2, \ AC = BD = 4 \)

In \( \triangle AOD \):

\[
 AD = 2, \quad \text{and} \quad AO = DO = \frac{4}{2} = 2
\]

\( \triangle AOD \) is equilateral so each of its angles is: 
\[
 \frac{180^\circ}{3} = 60^\circ.
\]

\( \hat{AOD} = \hat{CÔB} = 60^\circ \), (opposite angles)

so \( \hat{AÔB} = \hat{CÔD} = 180^\circ - 60^\circ = 120^\circ \)

(as BD is a straight line)

**Answer:** The smaller angles formed by the diagonals are 60° and the larger angles are 120°.

---

**Individual trial first, monitored, helped**

[If Ps are struggling, stop individual work and continue as a whole class activity, with T working on BB with help of class and Ps working in Ex. Bks.]

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Extra praise for Ps who realised that two different sized angles are formed by the diagonals.

**BB:**

\[
 \hat{DAB} = 90^\circ, \quad \hat{CAB} = 90^\circ - 60^\circ = 30^\circ
\]

**Answer:** The angles formed by the diagonals with the vertical sides are 60° and by the diagonals with the horizontal sides are 30°.
**Activity**

9  
HMC: Hungarian Mathematics Competition 1994  
Age 12

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**PbY6b, page 157, Q.6**

Read: *In this right-angled triangle, the lines DA and EA divide the right angle CAB into 3 equal parts.*

*DA is perpendicular to the hypotenuse BC. E is the midpoint of BC.*

What size are the acute angles of triangle ABC?

Allow Ps a minute to think about it and discuss with their neighbours if they wish. Who thinks they know what to do? Come and show us. Who agrees? Who would do it another way? If no P has an idea, T gives hint about the sum of the angles in a triangle. If still no P can do it, T leads Ps through the solution below, involving Ps once they understand. Ps write the solution in Ex Bks.

BB: e.g.

\[
\begin{align*}
\angle CDA &= \angle DAE = \angle EAB = \frac{90^\circ}{3} = 30^\circ \\
\text{In } \triangle ADC, \quad \angle DCA &= 180^\circ - 90^\circ - 30^\circ \\
&= 90^\circ - 30^\circ = 60^\circ \\
\text{In } \triangle ABC, \quad \angle A + \angle B + \angle C &= 180^\circ \\
\text{so } \angle ABC &= 180^\circ - 90^\circ - 60^\circ = 30^\circ 
\end{align*}
\]

Answer: In triangle ABC, angle C is 60° and angle B is 30°.

---

**Notes**

Whole class activity  
Drawn on BB or use enlarged copy master or OHP  
Ps decide what to do first and how to continue.  
Discussion, reasoning, agreement, praising  
Feedback for T

[Other methods are possible, e.g. \( \triangle ABE \) is an isosceles \( \triangle \), with base AB and EA = EB, so \( \angle EAB = \angle EBA = \frac{30^\circ}{2} \) or obtain angle B from \( \triangle ABD \), etc.]
**Y6**

**Activity**

### Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

<table>
<thead>
<tr>
<th>Number</th>
<th>Factorisation</th>
<th>Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>158</td>
<td>$2 \times 79$</td>
<td>1, 2, 79, 158</td>
</tr>
<tr>
<td>333</td>
<td>$3 \times 3 \times 37 = 3^2 \times 37$</td>
<td>1, 3, 9, 37, 111, 333</td>
</tr>
<tr>
<td>508</td>
<td>$2 \times 2 \times 127 = 2^2 \times 127$</td>
<td>1, 2, 4, 127, 254, 508</td>
</tr>
<tr>
<td>1158</td>
<td>$2 \times 3 \times 193$</td>
<td>1, 2, 3, 6, 193, 386, 579, 1158</td>
</tr>
</tbody>
</table>

8 min

### Different number systems

a) Do you remember in Year 3 learning about number systems which were not based on 10? We called them Numberlands. We imagined creatures called Dizzy Dombles who lived in Threeland. They used the number 3 as their base number instead of 10, and this is one of their numbers. Who can read it out? What does it mean? Ps come to BB or dictate to T. Class agrees/disagrees.

BB: $2102_3$ (read as 'two one zero two, in base 3')

Means in base 10: $2 \times (3 \times 3 \times 3) + 1 \times (3 \times 3) + 0 \times 3 + 2 \times 1$

$= 2 \times 27 + 1 \times 9 + 0 \times 3 + 2 \times 1$

$= 54 + 9 + 0 + 2 = 65$

('sixty five' in the base 10 number system: $6 \times 10 + 5 \times 1$)

Who can read out this number? (one zero two two, in base 3)

What does it mean? Ps come to BB or dictate to T. Class agrees/disagrees.

BB: $1022_3$

$= 1 \times 3^3 + 0 \times 3^2 + 2 \times 3 + 2 \times 1$

$= 1 \times 27 + 0 \times 9 + 2 \times 3 + 2 \times 1$

$= 27 + 6 + 2 = 35$ (base 10)

Let's show the numbers from 1 to 10 in base 3 in a table.

### Notes

Individual work, monitored (or whole class activity)

BB: 158, 333, 508, 1158

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

<table>
<thead>
<tr>
<th>Number</th>
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</thead>
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<tr>
<td>158</td>
<td>2, 333, 508, 1158</td>
</tr>
<tr>
<td>79</td>
<td>1, 37</td>
</tr>
<tr>
<td>111</td>
<td>3, 37</td>
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<td>333</td>
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<td>1, 2, 579</td>
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<td>127</td>
<td>1, 127</td>
</tr>
<tr>
<td>193</td>
<td>1, 193</td>
</tr>
</tbody>
</table>

Whole class activity

Written on BB.

Discussion, reasoning, agreement, praising

Involve as many Ps as possible.

[Tell Ps that when a base is not specified, we assume that the number is in base 10]

i.e. 'thirty-five'

$3 \times 10 + 5 \times 1$

Table already prepared and Ps fill in the numbers.

In unison. Praising
Activity

2

(Continued)

b) Who can say these numbers? What do they mean? Ps come to BB or dictate to T. Class agrees/disagrees.

BB:  
- i) \[413 = 4 \times 7^2 + 1 \times 7 + 3 \times 1\]
  \[= 4 \times 49 + 1 \times 7 + 3 \times 1 = 196 + 7 + 3 = 206\] (base 10)
- ii) \[101101 = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1\]
  \[= 1 \times 32 + 1 \times 8 + 1 \times 4 + 1 = 45\] (base 10)

c) Let’s code the number 74 in the base 5 number system. What would the value of the place-value columns be? Ps come to BB or dictate to T.

BB: \[
\ldots 5^3 5^2 5^1 5^0
\]
i.e. \(\ldots 125 25 5 1\)

How can we do it? (Divide 74 by 25, then divide the remainder by 5, then the remainder after that is the number of ‘ones’.) Ps come to BB or dictate what T should write.

BB: \[
74 \div 25 = 2, \text{r} 24
\]
\[
24 \div 5 = 4, \text{r} 4
\]
\[
20 \text{ min}
\]

Ps say in unison: ‘Seventy-four is two four four in base 5.’

Notes

‘four one three, in base 7’

‘one zero one one zero one, in base 2’

Whole class activity
(or individual trial if Ps wish)

Written on BB

BB: 1 2 3

Discussion, reasoning, agreement, praising
(e.g. If \(\square = 3\), number is \[1 \times 9 + 2 \times 3 + 3 = 18\])
Extra praise if Ps think of this, otherwise T gives hints or suggests it and asks Ps if the reasoning is correct
Reasoning, agreement, praising
Ps write solution in Ex. Bks.

Answer:
The base number is 4 or 6.
### Lesson Plan 158

**Notes**

Individual work, monitored, (helped), one part at a time

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>BB:</strong></td>
<td><strong>2004 Ordinal Values</strong></td>
</tr>
<tr>
<td>1 Jan</td>
<td>→ 1 or ( n )</td>
</tr>
<tr>
<td>1 Feb</td>
<td>→ 32 or ( 31 + n )</td>
</tr>
<tr>
<td>1 Mar</td>
<td>→ 61 or ( 60 + n )</td>
</tr>
<tr>
<td>1 Apr</td>
<td>→ 92 or ( 91 + n )</td>
</tr>
<tr>
<td>1 May</td>
<td>→ 122 or ( 121 + n )</td>
</tr>
<tr>
<td>1 Jun</td>
<td>→ 153 or ( 152 + n )</td>
</tr>
<tr>
<td>1 Jul</td>
<td>→ 183 or ( 182 + n )</td>
</tr>
<tr>
<td>1 Aug</td>
<td>→ 214 or ( 213 + n )</td>
</tr>
<tr>
<td>1 Sep</td>
<td>→ 245 or ( 244 + n )</td>
</tr>
<tr>
<td>1 Oct</td>
<td>→ 275 or ( 274 + n )</td>
</tr>
<tr>
<td>1 Nov</td>
<td>→ 306 or ( 305 + n )</td>
</tr>
<tr>
<td>1 Dec</td>
<td>→ 336 or ( 335 + n )</td>
</tr>
</tbody>
</table>

<p>| | |</p>
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<tr>
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<tbody>
<tr>
<td><strong>BB:</strong></td>
<td><strong>2004 Product Values</strong></td>
</tr>
<tr>
<td>1 Jan</td>
<td>→ ( 1 \times n = n )</td>
</tr>
<tr>
<td>1 Feb</td>
<td>→ ( 2 \times n = 2n )</td>
</tr>
<tr>
<td>1 Mar</td>
<td>→ ( 3 \times n = 3n )</td>
</tr>
<tr>
<td>1 Apr</td>
<td>→ ( 4 \times n = 4n )</td>
</tr>
<tr>
<td>1 May</td>
<td>→ ( 5 \times n = 5n )</td>
</tr>
<tr>
<td>1 Jun</td>
<td>→ ( 6 \times n = 6n )</td>
</tr>
<tr>
<td>1 Jul</td>
<td>→ ( 7 \times n = 7n )</td>
</tr>
<tr>
<td>1 Aug</td>
<td>→ ( 8 \times n = 8n )</td>
</tr>
<tr>
<td>1 Sep</td>
<td>→ ( 9 \times n = 9n )</td>
</tr>
<tr>
<td>1 Oct</td>
<td>→ ( 10 \times n = 10n )</td>
</tr>
<tr>
<td>1 Nov</td>
<td>→ ( 11 \times n = 11n )</td>
</tr>
<tr>
<td>1 Dec</td>
<td>→ ( 12 \times n = 12n )</td>
</tr>
</tbody>
</table>

**Solution:**

\( n = n \) (Identity – any day)

Jan: 31 such days in January

Feb: 31 + \( n \) = 2\( n \) \(- n\)

Mar: 60 + \( n \) = 3\( n \) \(- n\)

Dec: 335 + \( n \) = 12\( n \) \(- n\)

but there is no 31st of February!

so 1 day in March (30th)

\( 30 \div 11 = n \) (impossible)

so no such days in December.

---

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### Activity

**PbY6b, page 158**

**Q.2 a)** Read: *Number the days of 2004 in order. *This is their ordinal value.*

Example: 1st of January is 1; 5th of February is \( 31 + 5 = 36 \), etc.

What is special about the year 2004? [It was a leap year (divisible by 4) so there were 29 days in February.]

Quickly revise the number of days in the other months.

Set a short time limit. Ask Ps to list just the first day of each month and its ordinal value (it would take too long to include every day) in Ex. Bks. (Less able Ps could have a 2004 calendar on desks.)

Review with whole class. T has days already listed and Ps dictate to T and check any mistakes.

If we let any date in a month be \( n \), who can think of another way to write the ordinal values? If necessary, T does the first one, then Ps dictate the others.

**b)** Read *Multiply each date in every month by the ordinal value of the month. This is their product value.*

Example: 11 April: \( 11 \times 4 = 44 \); 31 October: \( 31 \times 10 = 310 \), etc.

Do we need to write out the product values of every day in a month? Elicit that, again, we could use \( n \) for any date in a month. Ps make another column in Ex. Bks and list the product values for the months (as opposite).

Review quickly with whole class. Ps dictate to T and check and correct their own lists.

**c)** Read: *How many days were there in the year 2004 when the ordinal value and the product value were equal?*

How do you think we can do this without having to compare the two values for all 366 days? (If the two expressions involving \( n \) are equal, \( n \) should work out as a whole day.) T gives hint if Ps cannot think of it. Do a random example on BB, with Ps dictating what T should write, e.g.

BB: Jul: \( 182 + n = 7n \) \(- n\)

\[ 182 = 6n \ [\div 6] \]

\[ 30 \frac{2}{6} = n \] (impossible, so none in July)

Ps do the rest in Ex. Bks. under a time limit.

How many such days did you find? Show me . . . now! (32)

P answering correctly explains reasoning. Who agrees? Who found another day? Come and show us. Class points out errors. Mistakes corrected. T chooses a P to say the answer in a sentence.

**Answer:** In 2004, there were 32 days when the ordinal value and product value were equal.

[N.B. Ps could, of course, list ordinal and product values for all 366 days and then count those which are equal – a lot of work!]

---

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N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done in class time as individual challenges or as whole class activities, with any remaining questions set as optional homework and reviewed interactively before the start of Lesson 159. Or T could divide the class into teams of roughly equal ability and set the remaining questions as a ‘maths challenge’ competition.

**Q.3 Read:** I cut a rectangle into two parts by drawing a straight line. Then I cut one of the two parts into two polygons by drawing another straight line. Then I cut one of the two polygons by drawing another straight line, and so on. After I had drawn 100 dividing lines, I counted the vertices of all the polygons I had formed. I counted 300 vertices. Is this possible? Give a reason for your answer.

**Solution:** e.g.

After 100 cuts, we would have 101 polygons. Even if they were all triangles (the polygon with the least number of vertices), the number of vertices would be

\[ 101 \times 3 = 303 \text{ and } 303 > 300 \]

**Answer:** No, it is not possible, as the number of vertices must be at least 303.

**Q.4 Read:** Prove that if all the natural numbers from 1 up to and including a number which has units digit 5 (in a base 10 number system), the sum will be divisible by 5.

**Solution:** e.g.

\[ 1 + 2 + 3 + 4 + 5 = 15, \text{ and } 15 \text{ is divisible by 5} \]
\[ 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 16 \times 7 + 8 = 112 + 8 = 120, \text{ and } 120 \text{ is divisible by 5, etc.} \]

or ‘the sum of any 5 consecutive natural numbers is a multiple of 5 because the remainders will be 1, 2, 3, 4 and 0, which sum to a multiple of 5, so any number of such groups will also sum to a multiple of 5’.

**Q.5 Read:** Liz started to write the whole numbers from 1 and now she is writing the 2893rd digit. Which whole number is she now writing?

**Solution:**

There are 9 1-digit, 90 2-digit and 900 3-digit numbers.

The number of digits is:

\[ 1 \times 9 + 2 \times 90 + 3 \times 900 = 9 + 180 + 2700 = 2889 \]

Liz is writing the 2893rd digit: \( 2893 - 2889 = 4 \)

So Liz is writing the 4th digit after 999, i.e. the first 4-digit number, 1000, and she has reached the last ‘0’ of that number.
**Q.6** Read: *Four equilateral triangles have been drawn, one inside the other. The area of the innermost, smallest triangle is 1 square unit.*

*What is the sum of the areas of the 4 triangles?*

**Solution:** e.g.

\[
\begin{align*}
A \text{ of } \nabla & = 1 \\
A \text{ of } \triangle & = 4 \\
A \text{ of } \triangledown & = 16 \\
A \text{ of } \bigtriangleup & = 64
\end{align*}
\]

**Sum of the 4 areas:**

\[
1 + 4 + 16 + 64 = 85 \text{ (square units)}
\]
Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \( 159 = 3 \times 53 \) Factors: 1, 3, 53, 159
- \( 334 = 2 \times 167 \) Factors: 1, 2, 167, 334
- \( 509 \) is a prime number Factors: 1, 509
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and \( 23^2 > 509 \))
- \( 1159 = 19 \times 61 \) Factors: 1, 19, 61, 1159

8 min

**Lesson Plan 159**

Notes

Individual work, monitored (or whole class activity)

BB: 159, 334, 509, 1159

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

| 159 | 3 | 1159 | 19 |
| 53 | 53 | 61 | 61 |
| 334 | 2 | 167 | 167 |
| 1 |

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Check with a calculator.

**Check:**

\[
1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \\
\times 8 \times 9 \times 10 = 3,628,800
\]

20 min

Activity 2

**PbY6b, page 159**

Q.1 Read: *The first 10 positive integers are multiplied together.*

*How many zeros are at the right-hand side of the product?*

Set a time limit of 4 minutes. Ps work in Ex. Bks.

Review with whole class. Ps could show number of zeros on scrap paper or slates on command. P answering correctly explains reasoning. Who thought the same? Who did it a different way? etc. Mistakes discussed and corrected.

**Solution:**

\[
1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10 \\
= 1 \times 2 \times 3 \times 2^2 \times 5 \times (2 \times 3) \times 7 \times 2^3 \times 3^2 \times (2 \times 5)
\]

This product has two prime factors which are 5 and more than two prime factors which are 2, so there are two factors which are \( (2 \times 5 = 10) \).

So the product is divisible by \( 10 \times 10 = 100 \),

but as there are no more factors of 10, it is not divisible by 1000.

**Answer:** There are 2 zeros at the right-hand side of the product.

14 min

**Notes**

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Check with a calculator.

**Check:**

\[
1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \\
\times 8 \times 9 \times 10 = 3,628,800
\]

20 min

Activity 3

**PbY6b, page 159. Q.2**

Read: *The first 100 positive whole numbers are multiplied together. In the product, which digit is in the place value 24th from the right?*

How can we show the multiplication without writing all the factors?

BB: \( 1 \times 2 \times 3 \times 4 \times 5 \times \ldots \times 98 \times 99 \times 100 \)

Which digits do we know will be 1st and 2nd from the right? (00, as one of the factors is 100). What other factor will produce a zero in the product? (10) Let’s see if we can make 100s or 10s from the factors.

How many factors of \( 5 \times 5 = 25 \) are in the multiplication? (4)

Elicit that there are more than 4 factors of 4 in the multiplication, so there are 4 factors involving \( 25 \times 4 = 100 \).

How many factors of 5 are in the multiplication other than those we have used for the 25s? (16) Elicit that there are more than 16 factors containing 2, so there are 16 factors of \( 5 \times 2 = 10 \) in the multiplication.

This means that the 4 factors of 100 and 16 factors of 10 will give at least \( 4 \times 2 + 16 = 8 + 16 = 24 \) zeros on the RHS of the product.

20 min
<table>
<thead>
<tr>
<th>Y6</th>
<th>Activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>N.B. We have not prescribed timings for the following questions as it is important that P's have time to think and try out their ideas. The questions can be done in class time as individual challenges or as whole class activities, with any remaining questions set as optional homework and reviewed interactively before the start of Lesson 160. Or T could divide the class into teams of roughly equal ability and set the remaining questions as a 'maths challenge' competition.</td>
</tr>
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</table>

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<thead>
<tr>
<th></th>
<th>Notes</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Make sure that the questions are reviewed by the whole class, whether P's attempted them or not. T could have a prize for the team which solves most questions correctly.</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>4</th>
<th>PBY6b, page 159</th>
</tr>
</thead>
</table>
| Q.3 | Read: *Imagine that this fraction is simplified as far as possible.*  
\[
\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{2^{10}}
\]  
*\[2^{10}\text{ means the product of 10 factors and each factor is 2.}\]*  
Which number will be the denominator of the simplified fraction? |

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<tbody>
<tr>
<td></td>
<td>Set a time limit. P's work in <em>Ex. Bks</em> and discuss with neighbours if they wish.</td>
</tr>
<tr>
<td></td>
<td>Review with whole class. P's show number on scrap paper or slates on command. P answering correctly explains reasoning to class. Who agrees? Who thought in a different way? etc. Mistakes discussed and corrected.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Solution: e.g</th>
</tr>
</thead>
</table>
|    | \[
\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{2^{10}}
\]  
\[
= \frac{1 \times 3 \times 5 \times 3 \times 7 \times 9 \times 5}{2 \times 2} = \frac{3 \times 5 \times 3 \times 7 \times 9 \times 5}{4}
\]  
and it cannot be simplified further. |
|    | or |
|    | \[
\frac{1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10}{2^{10}}
\]  
\[
= \frac{1 \times 2 \times 3 \times (2^3) \times 5 \times (2 \times 3) \times 7 \times (2^3) \times 9 \times (2 \times 5)}{2^{10}}
\]  
\[
= \frac{2^8 \times (3 \times 5 \times 3 \times 7 \times 9 \times 5)}{2^{10}}
\]  
\[
= \frac{3 \times 5 \times 3 \times 7 \times 9 \times 5}{2^2} = \frac{3 \times 5 \times 3 \times 7 \times 9 \times 5}{4}
\]  
*Answer:* The denominator of the simplified fraction will be 4. |

<table>
<thead>
<tr>
<th></th>
<th>Individual trial first</th>
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<tbody>
<tr>
<td></td>
<td>Written on SB or OHT</td>
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</table>

<table>
<thead>
<tr>
<th></th>
<th>Also accept an explanation in words: e.g.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>There are 5 even factors in the numerator, so 5 '2's can be cancelled out in the numerator and denominator.</td>
</tr>
<tr>
<td></td>
<td>There are 2 factors which are divisible by 4, so another two '2's can be cancelled out in the numerator and denominator.</td>
</tr>
<tr>
<td></td>
<td>There is one factor which is divisible by 8, so another '2' in the numerator and denominator can be cancelled out.</td>
</tr>
<tr>
<td></td>
<td>So 8 '2's can be cancelled out in both the numerator and the denominator, leaving 2 '2's in the denominator, so the denominator is 4.</td>
</tr>
</tbody>
</table>
### Y6

**Activity**

- **PbY6b, page 159**
  - **Q.4** Read: *Imagine that this fraction is simplified as far as possible.*
    
    $$\frac{1 \times 2 \times 3 \times 4 \times 5 \times \ldots \times 98 \times 99 \times 100}{2^{100}}$$
    
    *[2^{100} means the product of 100 factors and each factor is 2.]*
    
    Which number will be the denominator of the simplified fraction?
    
    **Solution:**
    
    - $100 \div 2 = 50$ (no. of even factors)
    - $100 \div 4 = 25$ (no. of factors which are divisible by $2^2$)
    - $100 \div 8 = 12\ldots$ (no. of factors which are divisible by $2^3$)
    - $100 \div 16 = 6\ldots$ (no. of factors which are divisible by $2^4$)
    - $100 \div 32 = 3\ldots$ (no. of factors which are divisible by $2^5$)
    - $100 \div 64 = 1\ldots$ (no. of factors which are divisible by $2^6$)
    
    So no. of ‘2’s in numerator which can cancel ‘2’s in denominator:
    
    BB: $50 + 25 + 12 + 6 + 3 + 1 = 97$
    
    $$\frac{2^{100}}{2^{97}} = 2^3 = 2 \times 2 \times 2 = 8$$
    
    **Answer:** The denominator of the simplified fraction will be 8.

### Lesson Plan 159

**Notes**

- Whole class activity
- Written on BB or SB or OHT

Elicit that the ellipsis stands for the numbers not shown.

Agree that it would take too long to write out every number in the numerator, so ask Ps to think of another way of solving the problem.

T directs Ps thinking if necessary.

- [We are assuming that the base set of numbers is the set of natural numbers!]

---

**PbY6b, page 159, Q.5**

Read: *Some consecutive whole numbers, from 1 to a positive whole number which is greater than 1, are added together.*

Which digit can be in the units place value in the sum?

(Give a reason for your answer.)

What does consecutive mean? (one following the other in order)

- e.g. $1 + 2 + 3 + \ldots$ Ps dictate the sums for 2, 3, 4, etc. numbers.

**BB:**

- $1 + 2 = 3$
- $1 + 2 + 3 = 6$
- $1 + 2 + 3 + 4 = 10$
- $1 + 2 + 3 + 4 + 5 = 15$
- $1 + 2 + 3 + 4 + 5 + 6 = 21$
- $1 + 2 + 3 + 4 + 5 + 6 + 7 = 28$
- $1 + 2 + \ldots + 8 = 36$
- $1 + 2 + 3 + \ldots + 9 = 45$
- $1 + 2 + 3 + \ldots + 10 = 55$

So it seems as if the units digit in the sum can be 0, 1, 3, 5, 6 or 8.

[Or T suggests calling the number of terms (i.e. the greatest term) $n$.]

**BB:**

- $1 + 2 + 3 + \ldots + (n - 3) + (n - 2) + (n - 1) + n = \frac{1 + n \times n}{2}$

Substituting numbers for $n$:

- $\frac{1 \times 2}{2} = 1$
- $\frac{2 \times 3}{2} = 3$
- $\frac{3 \times 4}{2} = 6$
- $\frac{4 \times 5}{2} \rightarrow 10$
- $\frac{5 \times 6}{2} \rightarrow 15$
- $\frac{6 \times 7}{2} \rightarrow 21$
- $\frac{7 \times 8}{2} \rightarrow 28$
- $\frac{8 \times 9}{2} \rightarrow 36$
- $\frac{9 \times 10}{2} \rightarrow 45$

**Answer:** Any of the digits 0, 1, 3, 5, 6 or 8 can be in the units place-value column in the sum.
### Activity

**PbY6b, page 159**

**Q.6** Read: A new volume in a series of books is published every 7 years. When the 7th volume was published, the sum of all the year numbers in which a book was published was 13727.

In which year was the first volume in the series published?

**Solution:** e.g.

Let the 1st year of publication be $n$. Then

$$n + (n + 7) + (n + 14) + (n + 21) + (n + 28) + (n + 35) + (n + 42) = 13727$$

$$7 \times n + 147 = 13727 \quad \rightarrow \quad 7 \times n = 13580 \quad \div 7$$

$$n = 1940$$

**Answer:** The first volume was published in 1940.

---

**Notes**

Individual work

- **Check:**
  - 1940
  - 1947
  - 1954
  - 1961
  - 1968
  - 1975
  - 1982
  - 13727

---

**Whole class activity**

**PbY6b, page 159, Q.7**

Read: The whole numbers from 1 to 1999 are added together. Is the sum the square of a natural number? Give a reason for your answer.

Ps decide what to do first and how to continue. T gives hint or directs Ps' thinking only if necessary. Class agrees/disagrees. Ps write solution in Ex. Bks.

**Solution:** e.g.

$$1 + 2 + 3 + \ldots + 1997 + 1998 + 1999 = \frac{1 + 1999}{2} \times 1999$$

$$= 1000 \times 1999 = 1999000$$

1999 000 cannot be a square number, as it is divisible by $10^2$ but not by $(10^2)^2 = 10^4$.

**Check:**

- 1999 000
- 999 500
- 499 750
- 249 875
- 49 975
- 9 995
- 1 999

Elicit or point out that the prime factors of a square number have even powers.

and 1999 is a prime number (as it is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and $47^2 > 1999$)
Y6

Activity

Factorising 160, 335, 510 and 1160. Revision and practice.  
*PbY6b, page 160*

Solutions:

Q.1

- **a)**  
  \[ P = 2 \times (4 + 3) \text{ cm} = 14 \text{ cm}; \quad A = 4 \times 3 = 12 (\text{cm}^2) \]

- **b)**  
  \[ P = 2 \times (a + b) = 2 (a + b); \quad A = a \times b = ab \]

- **c)**  
  \[ P = 6 \ \text{cm} + (2 \times 5) \ \text{cm} = 16 \ \text{cm}; \quad A = \frac{6 \times 4^2}{2} \ \text{cm} = 12 \ \text{cm} \]

- **d)**  
  \[ P = a + 2 \times b = a + 2b; \quad A = \frac{a \times h}{2} = \frac{ah}{2} \]

Q.2

- **a)**  
  \[ A = 2 \times \frac{6 \times 4^2}{2} \ \text{cm}^2 + 2 \times (5 \times 10) \ \text{cm}^2 + (6 \times 10) \ \text{cm}^2 \]
  \[ = 2 \times 12 \ \text{cm}^2 + 2 \times 50 \ \text{cm}^2 + 60 \ \text{cm}^2 \]
  \[ = 24 \ \text{cm}^2 + 100 \ \text{cm}^2 + 60 \ \text{cm}^2 \]
  \[ = 184 \ \text{cm}^2 \]
  \[ V = \frac{6 \times 4^2}{2} \times 10 = 12 \times 10 = 120 (\text{cm}^3) \]

- **b)**  
  \[ A = \frac{1}{2} \times \frac{a \times h}{2} + 2 \times (b \times c) + a \times c = \frac{ah}{2} + 2bc + ac \]
  \[ V = \frac{a \times h}{2} \times c = \frac{ahc}{2} \]

- **c)**  
  \[ A = 2 \times a \times a + 4 \times (a \times b) = 2a^2 + 4ab \]
  \[ V = a \times a \times b = a^2b \]

Q.3

First factorise 40, then show the possibilities in a table.

<table>
<thead>
<tr>
<th>40</th>
<th>2</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>10</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

**Factors:** 1, 2, 4, 5, 8, 10, 20, 40

**Answer:** Six different cuboids could be built.
### Activity

#### Solutions (continued)

#### Q.4

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
<th>d)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2\pi$</td>
<td>$\frac{5\pi}{4}$</td>
<td>$\frac{2\pi}{3}$</td>
<td>$\frac{5\pi}{6}$</td>
</tr>
</tbody>
</table>

Elicit that for circles with radius 1 unit:

- \(\pi = \frac{1}{2}\) of the circumference,
- \(\frac{\pi}{4} = \frac{1}{8}\) of the circumference, so \(\frac{5\pi}{4} = \frac{5}{8}\) of the circumference
- \(\frac{\pi}{3} = \frac{1}{6}\) of the circumference, so \(\frac{2\pi}{3} = \frac{1}{3}\) of the circumference
- \(\frac{\pi}{6} = \frac{1}{12}\) of the circumference, so \(\frac{5\pi}{6} = \frac{5}{12}\) of the circumference

#### Q.5

<table>
<thead>
<tr>
<th>a)</th>
<th>b)</th>
<th>c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 sides</td>
<td>5 sides</td>
<td>6 sides</td>
</tr>
<tr>
<td>d)</td>
<td>e)</td>
<td>f)</td>
</tr>
</tbody>
</table>
| 7 sides | 8 sides | Impossible!

#### Q.6

First calculate the RH square in top row:

1. \(42 - (10 + 26) = 42 - 36 = 6\)
2. Let the middle unknown square be \(a\), then the other unknown squares can be expressed in terms of \(a\). e.g.
   - \(36 - a\)
   - \(16 - a\)
   - \(32 - a\)
   - \(4 + a\)
3. \(42 - (32 - a) = 4 + a\)
4. \(42 - a - (4 + a) = 38 - 2a\)

As any column, row or diagonal sums to 42, then, e.g. taking the LH column:

- \(10 + (38 - 2a) + (36 - a) = 42\)
- \(84 - 3a = 42\) [+ 3a]
- \(84 - 3a + 2a = 42\) [- 42]
- \(42 = 3a\) [+ 3]
- \(14 = a\)

Now the values of all the unknown squares can be calculated, as opposite.
Check that each row, column and diagonal sums to 42.

<table>
<thead>
<tr>
<th>10</th>
<th>26</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>22</td>
<td>2</td>
<td>18</td>
</tr>
</tbody>
</table>

Bold numbers are given.

- \(36 - 14 = \frac{22}{2}\)
- \(16 - 14 = 2\)
- \(32 - 14 = 18\)
- \(4 + 14 = 18\)
- \(38 - 2 \times 14 = 38 - 28 = 10\)
Lesson Plan

Y6

Activity

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(161 = 7 \times 23\) Factors:  1, 7,  23,  161
- \(336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7\)
  Factors: 1, 2, 3, 4, 6, 7, 8, 12, 14, 16, 24, 336, 168, 112, 84, 48, 42, 28, 24, 21
- \(511 = 7 \times 73\) Factors:  1, 7,  73,  511
- \(1161 = 3 \times 3 \times 3 \times 43 = 3^3 \times 43\)
  Factors: 1, 3, 9, 27, 43, 129, 387, 1161

Notes

Individual work, monitored (or whole class activity)

BB: 161, 336, 511, 1161

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

\[
\begin{array}{cccc}
161 & 7 & 336 & 2 \\
23 & 23 & 168 & 2 \\
1 & 84 & 1 & 2 \\
1161 & 3 & 42 & 2 \\
511 & 7 & 387 & 3 \\
73 & 3 & 129 & 7 \\
1 & 43 & 43 & 1 \\
\end{array}
\]

\[
\begin{array}{ccc}
8 \text{ min} & & \\
\end{array}
\]

PbY6b, page 161

Q.1 Read: Decide whether each statement is true or false and write T or F in the box.

Set a time limit of 4 minutes. Remind Ps to think of examples or counter examples to support their answers.

Review with whole class. T chooses a P to read out each statement and Ps show 'T' or 'F' on slates or scrap paper (or by pre-agreed actions) on command. Ps with different responses explain their reasoning, drawing diagrams where necessary.

Class decides who is correct. Mistakes discussed and corrected.

Solution:

a) The product of two numbers can be less than each of the two numbers.  \(\text[T]\)

b) The arithmetic mean of two negative numbers can be positive.  \(\text[F]\)

(The sum of two negative numbers is always negative, so half that sum is also negative.)

c) There is an isosceles triangle which has two right angles.  \(\text[F]\)

(e.g. \(\alpha + \alpha + \beta = 180^\circ\)

If \(\alpha = 90^\circ\), then \(\beta = 0^\circ\), which is impossible.)

d) There is a positive fraction less than 1 which is equal to its reciprocal.  \(\text[F]\)

(Impossible – if the reciprocal of \(a\) is \(b\), then \(a \times b = 1\).)

When \(0 > a < 1\), then \(b = \frac{1}{a} > 1\))

e) If a product is zero, at least one of its factors is zero.  \(\text[T]\)

(If \(a \neq 0\) and \(b \neq 0\), then \(a \times b \neq 0\))

f) If the areas of two triangles are equal, the triangles are congruent.  \(\text[F]\)

(A right-angled triangle and an isosceles triangle can have the same area but they are not congruent.)

\[
\begin{array}{ccc}
1 & 2 & 4 \\
3 & 10 & 10 \\
4 & 10 & 10 \\
\end{array}
\]

\[
\begin{array}{ccc}
2 & 2 & 2 \\
2 & 2 & 2 \\
\end{array}
\]

\(A = \frac{2 \times 2 \times 1}{2 \times 2} = 2\) (sq. units)
Activity

2

(Continued)

9) There is a quadrilateral which is both a deltoid and a parallelogram but is not a square. [T]

(A rhombus which does not have right angles is a deltoid and also a parallelogram.)

N.B. We do not wish to prescribe timings for the following questions as it is important that P's have time to think and try out their ideas. The questions can be done in class time as individual challenges or as whole class activities, with any remaining questions set as optional homework and reviewed interactively before the start of Lesson 162, or T could divide the class into teams of roughly equal ability and set the remaining questions as a 'maths challenge' competition.

Notes

14 min

Individual trial or whole class activity
(or whole class activity to start with, then individual completion)

PbY6b, page 161

Q.2 Read: Frank, Charlie, Johnny and George went to visit their friend. The surnames of the 4 boys, in no particular order, are Little, Grant, Tailor and Miller.

Miller arrived first, then Johnny, then Little. George arrived last.

Each boy gave his friend a present. Miller gave a magic cube, Frank gave a pen, George gave a bar of chocolate and Tailor gave a book.

What is the full name of each boy?

Allow P's time to think about it and try to solve it. If P's are struggling, stop individual work and suggest showing the information in a table. P's dictate what T should write.

Deal with the information in the 2nd paragraph with the whole class, then either P's complete the table as individual work if they wish, or the class completes it together, with T's help.

Solution:
From 2nd paragraph:

<table>
<thead>
<tr>
<th></th>
<th>Little</th>
<th>Grant</th>
<th>Tailor</th>
<th>Miller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frank</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Charlie</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Johnny</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td></td>
</tr>
<tr>
<td>George</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td></td>
</tr>
</tbody>
</table>

Miller is not Johnny.
Miller is not George.
Little is not Johnny.
Little is not George.

From 3rd paragraph:

<table>
<thead>
<tr>
<th></th>
<th>Little</th>
<th>Grant</th>
<th>Tailor</th>
<th>Miller</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frank</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Charlie</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>Johnny</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
<tr>
<td>George</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
<td>✔</td>
</tr>
</tbody>
</table>

Miller is not Frank, so Miller must be Charlie.
Therefore Charlie is not Little, Grant or Tailor.
Frank must be Little, so Frank is not Grant or Tailor.

Tailor is not George, so Tailor must be Johnny, so Grant is George.

Answer: The four boys are called Frank Little, Charlie Miller, Johnny Tailor and George Little.
### Activity

#### PbY6b, page 161

**Q.3** Read: *In a Canadian city, 80% of the population speaks English and 70% speaks French. Every inhabitant can speak either French or English. What percentage of the population can speak both languages?*

If Ps are struggling, suggest that they show the information in a Venn diagram, as below.

**Solution:**

Reasoning: e.g. 
\[80\% + 70\% = 150\%\], which is 50% more than 100% so 50% speak both languages.

**Check:** 
\[30\% + 50\% + 20\% = 100\% \checkmark\]

**Answer:** 50% of the population can speak English and French.

#### PbY6b, page 161

**Q.4** Read: *Ten pupils took part in a mathematics competition in which 5 problems were set. Thirty-five answers were handed in. We know that there was a pupil who handed in only 1 answer, a pupil who handed in 2 answers and a pupil who handed in 3 answers. Show that there must be a pupil who answered all five problems.*

**Solution:** e.g.

Let's suppose that exactly 1 pupil solved only 1 problem, exactly 1 pupil solved 2 problems and exactly 1 pupil solved 3 problems.

Then 7 pupils would have handed in 29 answers, but 
\[29 \div 7 = 4, \frac{1}{7}\], so at least one of the pupils must have answered all 5 problems.

or

Suppose that the other 7 pupils handed in 4 answers each, then in total there would be 
\[1 + 2 + 3 + 7 \times 6 + 28 = 34\] answers but as there are 35 answers, at least one pupil must have done all five problems.

---

**Notes**

Individual trial

[If the number of pupils who solved 1 (2, 3) problems was more than 1, the problem would be easier to solve.]
**Activity 6**

**PbY6b, page 161. Q.5**

Read: A year 6 class of 42 pupils took part in a special Physical Education lesson. The pupils could choose from basketball, swimming and gymnastics.

We know that 20 of them did swimming, 19 did gymnastics and 18 played basketball. We also know that 7 pupils swam and played basketball, 8 pupils swam and did gymnastics and 6 pupils did gymnastics and played basketball.

How many pupils took part in all 3 sports?

Allow Ps time to think about it for a few minutes and discuss with their neighbours. If any Ps have good ideas, T helps them to develop the solution, involving other Ps where possible. If no P has an idea, T suggests drawing a Venn diagram and calling the number of pupils who took part in all 3 sports $n$. Then Ps might be able to proceed from there, otherwise T directs Ps thinking and class solves the problem together.

**Solution:** e.g.

BB: 20 + 19 + 18 − (8 + 7 + 6) + $n$ = 42

57 − 21 + $n$ = 42

36 + $n$ = 42

$[−36]$ 

$n$ = 6

**Answer:** Six pupils took part in all three sports.

---

**Notes**

Whole class activity
(or individual trial first if Ps wish)

(or alternative equation to the one given in the solution:
20 + (18 − 7) + (19 − 6 − 8 + $n$)  = 42

20 + 11 + 5 + $n$ = 42

36 + $n$ = 42

$n$ = 6

**Check:**

11 + 1 + 11 + 11 + 6 + 2 = 42

$\emptyset$ means ‘empty set’

**To T:** $\cap$ means ‘intersection’,

$\cup$ means ‘union’

A $\cup$ B $\cup$ C = A + B + C − (A $\cap$ B + A $\cap$ C + B $\cap$ C) + A $\cap$ B $\cap$ C

---

**Activity 7**

**PbY6b, page 161. Q.6**

Read: A shooting practice target is shaped like an equilateral triangle and each of its sides is 1 metre long. If 10 shots hit the target, show that two of the shots must be less than 34 cm apart.

Allow Ps time to think about it and discuss with their neighbours. Ps who have ideas develop them with help of T and class. If no P is on the right track, T gives hints or directs Ps' thinking, involving Ps where possible.

**Solution:** e.g.

First draw an equilateral triangle. As 34 cm = \( \frac{1}{3} \) m, divide each side of the triangle into thirds to form 9 smaller, congruent, equilateral triangles. Any point in one of these small triangles is at most 33 and 1 third cm away from another point on that triangle.

The worst possible scenario is that the first 9 shots hit different small triangles.

However, the 10th shot must hit one of these 9 triangles. As the distance between any 2 points on a triangle is at most 33 and 1 third cm apart, then at least two of the shots must be less than 34 cm apart.
### Lesson Plan 162

**Y6**

**R:** Calculations  
**C:** Revision: arithmetic, algebra  
**E:** Problems and challenges

#### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **162**  
  - Factors: 1, 2, 3, 6, 9, 18, 27, 54, 81, 162

- **337**  
  - Factors: 1, 337  
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and 19² > 337)

- **512**  
  - Factors: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 (cubic number)

- **1162**  
  - Factors: 1, 2, 7, 14, 83, 166, 581, 1162

**8 min**

#### Activity 2

**Arithmetic laws**

Let's complete these equations using the letters, then check that they are true using numbers.

Ps come to BB or dictate what T should write. Class agrees/disagrees. After each equation has been checked with numbers, ask Ps to explain the general rule or law in words using the components of the 4 operations.

**BB:** e.g.

a) \((a + b) \times c = [a \times c + b \times c]\)

LHS: \((2 + \frac{3}{4}) \times 5 = 2 \times \frac{3}{4} \times 5 + \frac{11}{4} \times 5 = \frac{55}{4} = 13 \frac{3}{4}\)

RHS: \(2 \times 5 + \frac{3}{4} \times 5 = 10 + \frac{15}{4} = 10 + \frac{3}{4} = 13 \frac{3}{4}\) ✔

[Multiplying a sum by a number has the same result as multiplying each term of the sum by the number and adding the products.]

b) \(a + \left(\frac{-b}{c}\right)\)

\(\frac{-b}{c}\)

[Adding a negative number has the same result as subtracting the opposite positive number.]

c) \(a - \left(\frac{b}{c}\right)\)

\(\frac{b}{c}\)

[Subtracting a negative number has the same result as adding the opposite positive number.]

d) \(\left(\frac{a}{c}\right) - \left(\frac{b}{c}\right)\)

\(\left(\frac{a}{c}\right) + \left(\frac{-b}{c}\right)\)

LHS: \(\frac{15}{3} - \frac{6}{3} = \frac{9}{3} = 3\)

RHS: \(\frac{15}{3} - \frac{6}{3} = 5 - 2 = 3\) ✔

[Dividing a difference by a number has the same result as dividing the reductant and the subtrahend by that number and subtracting the two quotients.]
Y6

Activity

2 (Continued)

e) \[ \frac{a}{b} \times c = \left[ \frac{a \times c}{b} \right] \] e.g. \[ \frac{5}{8} \times 3 = \frac{5 \times 3}{8} = \frac{15}{8} = 1 \frac{7}{8} \]

[To multiply a fraction by a whole number, multiply the numerator by that number.]

f) \[ \frac{a}{b} + \frac{c}{d} = \left[ \frac{a \times d + c \times b}{b \times d} \right] \] e.g. LHS: \[ \frac{4}{7} + \frac{2}{7} = \frac{6}{7} \] RHS: \[ \frac{4 + 2}{7} = \frac{6}{7} \]

[To add two fractions which have the same denominator, add the numerators and keep the same denominator.]

g) \[ \frac{a}{b} - \frac{c}{d} = \left[ \frac{a \times d - b \times c}{b \times d} \right] \] e.g. \[ \frac{2}{3} + \frac{5}{8} = \frac{2 \times 8}{3 \times 8} + \frac{5 \times 3}{3 \times 8} = \frac{2 \times 8 + 5 \times 3}{3 \times 8} = \frac{16 + 15}{24} = \frac{31}{24} = 1 \frac{7}{34} \]

[To add two fractions with different denominators, multiply each numerator by the denominator of the other fraction, add the two products, then divide by the product of the two denominators.]

Notes

Lesson Plan 162

3 PbY6b, page 162

Q.1 Read: Complete the arithmetic laws. Try them with numbers if necessary.

Set a time limit of 5 minutes. Ps check mentally or in Ex. Bks. (The more difficult equations can be done with the whole class.

Review with whole class. Ps come to BB to complete the equations, explaining reasoning. Who agrees? Who wrote something else? etc. Mistakes discussed and corrected.

Ask Ps to explain the laws in words where appropriate.

Solution:
a) \( a + (–b) – (c) – (d) = [a – b – c + d] \)
b) \((a – b) \times c = [a \times c – b \times c] \) \( (= ac – bc) \)
c) \( x \times y \times x \times z = [x \times (y + z)] \) \( (= x(y + z)) \)
d) \((a – b) \div c = [a \div c – b \div c] \)
e) \( u \div w + v \div w = [(u + v) \div w] \)
f) \(2 \times f + 3 \times f – 4 \times f = [2(3 – 4) \times f = 1 \times f = f] \)
g) \(6t – 4t – 9t = [-7t] \)
h) \[ \frac{a \times c}{b \times c} = \left[ \frac{a}{b} \right] \] i) \[ \frac{a + b}{c} = \left[ \frac{a}{c} + \frac{b}{c} \right] \]

j) \[ \frac{a}{b} – \frac{c}{d} = \left[ \frac{a \times d – b \times c}{b \times d} \right] = \frac{ad – bc}{bd} \]

k) \[ \frac{a \times n}{n} = [a] \]

l) \[ \frac{a \times b}{b} = [a] \] m) \[ \frac{a}{b} \div c = \left[ \frac{a \div c}{b \times c} \right] \]

Elicit that, e.g. \( 6t = 6 \times t \)

n) \[ \frac{a \times c}{b \times d} = \left[ \frac{a \times c}{b \times d} \right] \]

o) \( \frac{a}{b} \div \frac{c}{d} = \left[ \frac{a}{b} \div \frac{c}{d} \right] \)

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

(If class is not very able, deal with one or two at a time.)

Reasoning, checking with actual values, agreement, self-correction, praising

Feedback for T
## Lesson Plan 162

### Activity

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

### Notes

Review the questions with the whole class, whether Ps attempted them or not.

---

### PbY6b, page 162

#### Q.2

Read: *How is it possible to share 7 equal sized loaves of bread among 12 hungry people without cutting each loaf into 12 pieces? Try to solve it using as few cuts as possible.*

**Solution:** e.g.

\[
7 = 3 + 4 = 12 \times \frac{1}{4} + 12 \times \frac{1}{3},
\]

so the loaves could be cut like this, using 14 cuts:

BB:

- - - - - - - - - - - - - - -

Each person would get \( \frac{1}{4} + \frac{1}{3} = \frac{3 + 4}{12} = \frac{7}{12} \) of the bread.

---

#### Q.3

Read: *What is the smallest positive whole number which has units digit 6 and if we move this digit from the right-hand side to the left-hand side but leave the other digits unchanged, we get 4 times the original number?*

**Solution:** e.g.

Let's write the multiplication using the digits we know about and using an *ellipsis* (dots) for those we don't.

BB:

\[
\begin{array}{c}
\ldots 6 \times 4 \\
\ldots 46 \times 4 \\
\ldots 846 \times 4 \\
\ldots 3846 \times 4 \\
\ldots 53846 \times 4 \\
\ldots 153846 \times 4
\end{array}
\]

(6 × 4 = 24, so 4 must be the 10s digit, and 2 is carried over to the next greater place-value column)

(4 × 4 = 16, 16 + 2 = 18, so 8 must be the 100s digit, and 1 is carried over to the next greater place-value column)

(8 × 4 = 32, 32 + 1 = 33, so 3 must be the 1000s digit, and 3 is carried over to the next greater place-value column)

(3 × 4 = 12, 12 + 3 = 15, so 5 must be the 10 000s digit, and 1 is carried over to the next greater place-value column)

(5 × 4 = 20, 20 + 1 = 21, so 1 must be the 100 000s digit, and 2 is carried over to the next greater place-value column)

(1 × 4 = 4, 4 + 2 = 6, there is nothing to be carried over and we have reached digit 6 in the product)

**Answer:** The number is 153 846.

---

### Individual trial

Allow Ps a minute or two to think and discuss with their neighbours. Ps who have ideas develop them with the whole class.

If no P has an idea, T starts solution and involves Ps when they understand what is happening.

**Check:**

\[
\begin{array}{ccccccc}
1 & 5 & 3 & 8 & 4 & 6 & \times 4 \\
\hline
6 & 1 & 5 & 3 & 8 & 4 & \checkmark
\end{array}
\]
**Y6**

**Activity**

6

HMC: Hungarian Mathematics Competition 1980

Age 12

**PbY6b, page 162**

Q.4  Read: We multiply the digits of a 3-digit whole number, then multiply the digits of the product. We can represent the number and the two products in this way: The same shape means the same digit.

What was the original number? Explain your reasoning.

**Solution:** e.g.

We can write the products as multiplications:

BB: \[ \triangle \times \bigcirc \times \bigcirc = \triangle \bigcirc ; \triangle \times \bigcirc = \bigcirc \]

If \( \triangle = 1 \), then \( \bigcirc \neq 0, 1, 2 \) or 3 (as product is 1-digit)

if \( \bigcirc = 4 \rightarrow 144, 16, \) and \( \bigcirc = 6 \)

so \( 144 \) is o.k.

also, \( \bigcirc \neq 5, 6, 7, 8 \) or 9 (as tens digit in product is not 1)

If \( \triangle = 2 \), then \( \bigcirc \neq 0, 1, 2 \) (as product is 1-digit)

\( \bigcirc \neq 3, 4, 5, 6, 7, 8, 9 \)

(as digits in products do not match the shapes)

Also \( \bigcirc \neq 3, 4, 5, 6, 7, 8 \) or 9,

(as digits in product do not match the shapes)

The only possible answer is: \( \triangle = 1, \bigcirc = 4, \bigcirc = 6 \)

**Notes**

Individual trial first

Drawn on BB or SB or OHT

BB:

\[ \triangle \bigcirc \bigcirc ; \triangle \bigcirc \bigcirc \]

The easiest method is to use trial and error, done in a logical way.

**Check:**

\[ 144: 1 \times 4 \times 4 = 16, \]

\[ 1 \times 6 = 6 \]

e.g. \( 155: 1 \times 5 \times 5 = 25 \)

e.g. \( 233: 2 \times 3 \times 3 = 18 \)

e.g. \( 244: 5 \times 4 \times 4 = 80 \)

**Lesson Plan 162**

**Week 33**

---

**7**

HMC: Hungarian Mathematics Competition 1992

Age 12

**PbY6b, page 162, Q.5**

Read: We put £255 into 8 envelopes, seal the envelopes and write on each how much money it contains. There is a different amount in each envelope. Without opening any of the envelopes we can pay any whole amount from £1 to £255. How much money is in each envelope?

Ps make suggestions and class tries them out. If necessary, T suggests starting at £1 and seeing what notes are needed.

BB:

\[ £1 \rightarrow £1, \ £2 \rightarrow £2, \ £3 \rightarrow £2 + £1, \ £4 \rightarrow £4, \ £5 \rightarrow £4 + £1, \ £6 \rightarrow £4 + £2, \ £7 \rightarrow £4 + £2 + £1, \ £8 \rightarrow £8, \ £9 \rightarrow £8 + £1, \ £10 \rightarrow £8 + £2, \ etc. \]

After a while, Ps might realise that what are needed are the powers of 2 (\( 2^0 \) to \( 2^7 \)), i.e. the place values in the base 2 number system.

**Solution:**

The 8 envelopes contain, in increasing order:

BB:

\[ £1, \ £2, \ £4, \ £8, \ £16, \ £32, \ £64, \ £128 \]

\( (2^0), (2^1), (2^3), (2^4), (2^5), (2^6), (2^7) \)

[N.B. \( 2^8 = 2 \times 128 = 256 > 255, \) so is not needed.]

Whole class trials and solution (or individual challenge if Ps wish, left open as homework if no P can solve it during the lesson)

There is no need to check every number to 255.

Once Ps have realised what is needed, ask Ps to suggest some larger numbers to check.

e.g. \( 213 = 128 + 64 + 16 + 4 + 1 \)

\[ 213 \div 2 = 106, \ r \frac{1}{2} \] (13)

\[ 106 \div 2 = 53, \ r \frac{1}{2} \] (53)

\[ 53 \div 2 = 26, \ r \frac{1}{2} \] (26)

\[ 26 \div 2 = 13, \ r \frac{1}{2} \] (13)

\[ 13 \div 2 = 6, \ r \frac{1}{2} \] (6)

\[ 6 \div 2 = 3, \ r \frac{1}{2} \] (3)

\[ 3 \div 2 = 1, \ r \frac{1}{2} \] (1)

\[ 1 \div 2 = 0, \ r \frac{1}{2} \] (0)

so \( 213 = 11010101_2 \) (128's)
Activity 1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **163**: is a prime number  
  Factors: 1, 163 
  (as not exactly divisible by 2, 3, 5, 7, 11, and 13² > 163)

- **338**:  
  Factors: 1, 2, 13, 26, 169, 338
  \(338 = 2 \times 13 \times 13 = 2 \times 13^2\)

- **513**:  
  Factors: 1, 3, 9, 19, 27, 57, 171, 513
  \(513 = 3 \times 3 \times 3 \times 19 = 3^3 \times 19\)

- **1163**: is a prime number  
  Factors: 1, 1163  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31, and 37² > 1163)

---

2

\[8\text{ min}\]

**PbY6b, page 163**

Q.1 Read: Solve the equations and inequalities. Check your results. 

(The base set is in brackets.)

What kind of numbers are in the given base sets?

- **Z:** the set of integers (whole numbers)
- **Q:** the set of rational number, i.e. all the numbers we have learned about (positive and negative integers, fractions and decimals)
- **N:** the set of natural numbers (positive whole numbers)

Deal with one part at a short time under a time limit.

Review with whole class. Ps show solutions on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong. Mistakes discussed and corrected.

**Solution:**

a) \(x - 5 > -5\)  
   \(x > 0\), \((x \in \mathbb{Z})\) \([\leq 5\text{ means 'is a member of the set']}

b) \((-3) \times (4 \times y) + 8 \leq 2 \times (5 \times y + 6)\)  
   \(-12 \times y + 8 \leq 10 \times y + 12\)  
   \([-8]\)

   \(-12 \times y \leq 10 \times y + 4\)  
   \([-12 \times y]\)

   \(0 \leq 22 \times y + 4\)  
   \([-4]\)

   \(-4 \leq 22 \times y\)  
   \([\div 22]\)

   \(-\frac{4}{22} \leq y\),  
   or \(y \geq -\frac{2}{11}\)  
   \((y \in \mathbb{Q})\)

**Check:** e.g.

\[\begin{align*}
   y &= -\frac{4}{11} \quad \text{LHS:} & \frac{48}{11} + 8 &= 4 \frac{4}{11} + 8 &= 12 \frac{4}{11}; \\
   \text{RHS:} & -\frac{40}{11} + 12 &= -3 \frac{7}{11} + 12 &= 8 \frac{4}{11} \times \end{align*}\]

\[\begin{align*}
   y &= -\frac{2}{11} \quad \text{LHS:} & \frac{24}{11} + 8 &= 2 \frac{2}{11} + 8 &= 10 \frac{2}{11}; \\
   \text{RHS:} & -\frac{20}{11} + 12 &= -1 \frac{9}{11} + 12 &= 10 \frac{2}{11} \checkmark \end{align*}\]
**Activity 2**

(Continued)

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<thead>
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<tbody>
<tr>
<td><strong>Y6</strong></td>
<td><strong>Lesson Plan 163</strong></td>
</tr>
</tbody>
</table>

**Y6**

**Activity**

2

(Continued)

c) \[
\frac{3}{8} x - 2 + \frac{2}{3} t = -\frac{5}{6} + \frac{5}{12} t \quad \text{[Convert to 24ths]}
\]

\[
\frac{9}{24} x - \frac{48}{24} + \frac{16}{24} t = -\frac{20}{24} + \frac{10}{24} t \quad \times 24
\]

\[
9 x - 48 + 16 t = -20 + 10 t \quad [\times 48]
\]

\[
25 t = 28 + 10 t \quad [-10 \times t]
\]

\[
15 t = 28 \quad [\div 15]
\]

\[
t = \frac{28}{15} = \frac{13}{15}
\]

**Check:**

**RHS:**

\[
-\frac{5}{6} + \frac{15 \times 28}{3 \times 15} = -\frac{5}{6} + \frac{7}{9} = -\frac{15}{18} + \frac{14}{18} = -\frac{1}{18} \checkmark
\]

d) \[
\frac{3}{2} v + \frac{5}{-2} = -4 \quad [\times (-2)]
\]

\[
3 \times v + 5 = 8 \quad [-5]
\]

\[
3 \times v = 3 \quad [\div 3]
\]

\[v = 1\]

**15 min**

**PbY6b, page 163, Q.2**

Read: *I have 18 coins (2 p and 5 p pieces) in my pocket.*

*If I had as many 5 p coins as I have 2 p coins and as many 2 p coins as I have 5 p coins, I would have twice as much money as I have now. How much money do I have?*

Allow Ps a couple of minutes to think about how to solve it. Ps who have ideas explain them to the class. Class decides whether they are valid. If no P has a good idea, T gives hint about using a letter for the number of one type of coin and helps class to form an equation. Then Ps come to BB or dictate what T should write to solve it, check the solution and agree on a form of words for the answer.

**Solution:** e.g.

Let \( x \) be the number of 2 p coins, then the number of 5 p coins is \( 18 - x \) and the amount of money in my pocket is

**BB:**

\( 2 \times x + 5 \times (18 - x) \)

If I did what is suggested, then the amount in my pocket would be:

\( 5 \times x + 2 \times (18 - x) \)

and it would be twice as much as I have now. So now we can write:

**BB:**

\[ [2 \times x + 5 \times (18 - x)] \times 2 = 5 \times x + 2 \times (18 - x) \]

\((-3 \times x + 90) \times 2 = 3 \times x + 36 \]

\(-6 \times x + 180 = 3 \times x + 36 \quad [+ 6 \times x]
\]

\(180 = 9 \times x + 36 \quad [-36]
\]

\(144 = 9 \times x \quad [\div 9]
\]

\(16 = x \)

**Answer:** I have 16 2 p coins and two 5 p coins, so I have 42 p altogether.

**21 min**

**Notes**

**Check:**

**LHS:**

\[
\frac{3 \times 10}{8 \times 15} - 2 + \frac{2 \times 28}{3 \times 15}
\]

\[
= \frac{7}{10} - 2 + \frac{56}{45}
\]

\[
= -1 \frac{3}{10} + \frac{11}{45}
\]

\[
= -\frac{27}{90} + \frac{22}{90}
\]

\[
= -\frac{5}{90} - \frac{1}{18}
\]

**Check:**

\[
\frac{3}{2} \times 1 + \frac{5}{2} = 8 \quad 2
\]

\[= -4 \checkmark
\]

**Whole class activity**

(or individual trial first if Ps wish)

Discussion, reasoning, agreement, checking, (self-correction), praising

Involve many Ps.

Ps could write solution in Ex. Bks. too.

**Check:**

\[
5 \times 16 + 2 \times (18 - 16)
\]

\[
= 80 + 2 \times 2
\]

\[= 80 + 4 p
\]

\[= 84 p
\]

\[= 2 \times 42 p \checkmark
\]
### Y6

#### Activity

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize) Any questions not done in class could be set as voluntary homework.

#### Notes

Review the questions with the whole class, whether Ps attempted them or not.)

### 4

*Hungarian Mathematics Competition 1981 Age 11*

**PbY6b, page 163**

Q.3 Read: *Steve and a dog can be balanced on a seesaw by 5 equal-sized boxes. The dog and 2 cats can be balanced on the seesaw by 3 of the boxes and the dog can be balanced by 4 cats. How many cats are needed to balance Steve?*

**Solution:** e.g.

Let Steve's weight be S, a dog's weight be D, a cat's weight be C and a box's weight be B. Then

\[ S + D = 5 \times B, \quad D + 2 \times C = 3 \times B, \quad D = 4 \times C \]

Writing \( 4 \times C \) for D in the first two equations:

\[ S + 4 \times C = 5 \times B \quad \ldots \ldots \quad (1) \]

and

\[ 4 \times C + 2 \times C = 3 \times B \]

\[ 6 \times C = 3 \times B \quad [\div 3] \]

\[ 2 \times C = B \]

Writing \( 2 \times C \) for B in the first substituted equation (1),

\[ S + 4 \times C = 5 \times (2 \times C) \]

\[ S + 4 \times C = 10 \times C \quad [-4 \times C] \]

\[ S = 6 \times C \]

**Answer:** Six cats are needed to balance Steve.

### 5

*Hungarian Mathematics Competition 1991 Age 11*

**PbY6b, page 163**

Q.4 Read: *The average age of the 11 members of a football team is 22 years. When one member of the team was sent off because of a bad tackle, the average age of the rest of the team was 21 years. How old is the player who was sent off?*

**Solution:** e.g.

Total age of the 11 players: \( 11 \times 22 \) years = 242 years

Total age of 10 players: \( 10 \times 21 \) years = 210 years

So age of the 11th player: \( 242 - 210 = 32 \) (years)

**Answer:** The player who was sent off was 32 years old.

Individual trial

What does average age mean?

(As if the older players cancelled out the younger players and they were all the same age.

or

If all their ages were added together and then divided by the number of players the result would be their average age.)
### Lesson Plan 163

#### Activity

**PbY6b, page 163**

Q.5 Read: If I had four times as much money as I have now, my money would be as much over £1000 as the amount I have now is less than £1000. How much money do I have?

**Solution:**

Let the amount of money I have now be £x, then:

\[
\begin{align*}
4 \times x - 1000 &= 1000 - x \\
5 \times x &= 2000 \\
x &= 400
\end{align*}
\]

**Answer:** I have £400 now.

**Check:**

\[
\begin{align*}
\text{LHS: } 4 \times £400 - £1000 &= £1600 - £1000 = £600 \\
\text{RHS: } £1000 - £400 &= £600
\end{align*}
\]

Whole class activity

(or individual or paired trial if Ps wish)

#### Notes

- Individual work, monitored

---

**Y6**

<table>
<thead>
<tr>
<th>Activity</th>
<th>Notes</th>
</tr>
</thead>
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<tr>
<td><strong>PbY6b, page 163</strong></td>
<td>Individual work, monitored</td>
</tr>
<tr>
<td>Q.5 Read: If I had four times as much money as I have now, my money would be as much over £1000 as the amount I have now is less than £1000. How much money do I have?</td>
<td></td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
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<tr>
<td>Let the amount of money I have now be £x, then:</td>
<td></td>
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</tbody>
</table>
| \[
\begin{align*}
4 \times x - 1000 &= 1000 - x \\
5 \times x &= 2000 \\
x &= 400
\end{align*}
\] | |
| **Answer:** I have £400 now. | |
| **Check:** | |
| LHS: \(4 \times £400 - £1000\) & \(£1600 - £1000 = £600\) | |
| RHS: \(£1000 - £400\) & \(£600\) | |

---

**PbY6b, page 163, Q.6**

Read: A lorry and a car started from two cities at the same time and travelled towards each other at steady speeds. The lorry took 6 hours to cover the distance between the two cities and the car took 4 hours.

After what amount of time did they pass each other?

What should we do first? (Draw a diagram) Ps come to BB to draw and explain. Class agrees/disagrees.

**BB:**

![Diagram of a lorry and a car moving towards each other](image)

What part of the distance did they each cover every hour?

(Lorry: \(\frac{1}{6}\) of the distance; Car: \(\frac{1}{4}\) of the distance)

So every hour they approach each other by \((\frac{1}{6} + \frac{1}{4})\) of the distance.

If they meet in \(x\) hours and we think of the whole distance as 1 unit, then

\[
\begin{align*}
\frac{1}{6} + \frac{1}{4} &= \frac{1}{x} \\
2x + 3x &= 12 \\
5x &= 12 \\
x &= 2.4 \text{ (hours)}
\end{align*}
\]

**Answer:** They passed each other after 2 hours 24 minutes.
Q.7 Read: A matchbox contains some matches. If we double the number of matches then take away 8, then double the number of matches left and take away 8 again, then do the same for a third time, the box will be empty. How many matches are in the matchbox?

Solution: e.g.
Let the number of matches in the matchbox be \( n \).

Then number in box after:
Action 1: \( n \times 2 – 8 \)
Action 2: \( (n \times 2 – 8) \times 2 – 8 \)
Action 3: \[ (n \times 2 – 8) \times 2 – 8 \] \times 2 – 8

But \[ (n \times 2 – 8) \times 2 – 8 \] \times 2 – 8 = 0 [+ 8]

so \[ (n \times 2 – 8) \times 2 – 8 \] = 8 [+ 2]

\( n \times 2 – 8 = 6 \) [+ 8]

\( n \times 2 = 14 \) [÷ 2]

\( n = 7 \)

or \[ (n \times 2 – 8) \times 2 – 8 \] = 0

\( n \times 4 – 16 – 8 \) \times 2 – 8 = 0

\( n \times 4 – 24 \) \times 2 – 8 = 0

\( n \times 8 – 48 – 8 \) = 0

\( n \times 8 – 56 \) = 0 [+ 56]

\( n \times 8 = 56 \) [÷ 8]

\( n = 7 \)

Answer: There are 7 matches in the matchbox.
Y6

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $164 = 2 \times 2 \times 41 = 2^2 \times 41$
  Factors: 1, 2, 4, 41, 82, 164

- $339 = 3 \times 113$
  Factors: 1, 3, 113, 339

- $514 = 2 \times 257$
  Factors: 1, 2, 257, 514
  (257 is not exactly divisible by 2, 3, 5, 7, 11, 13 and $17^2 > 257$)

- $1164 = 2 \times 2 \times 3 \times 97 = 2^2 \times 3 \times 97$
  Factors: 1, 2, 3, 4, 6, 12, 97, 194, 291, 388, 582, 1164

2

PhY6b, page 164

Q.1 Read: Find a relationship between the corresponding values and complete the table.

Show the data in a graph in your exercise book.

Deal with one table at a time. Class agrees on one form of the rule then Ps complete the table and write the rule in different ways.

Why must the rule in the form $y = \frac{1}{x}$ have the extra condition that $x$ cannot be equal to zero? (Because it is nonsense to divide by 0.)

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed/corrected.

Ps write different forms of the rule and choose other Ps to check them using values from the table.

Draw the graph with the whole class. Ps work on BB and rest of class work in Ex. Bks. First agree on the range of values needed for the two axes, then Ps draw the axes and label them, Ps come to BB one after the other to choose a column in the table and plot that point. Class points out errors. Ps do the same in Ex Bks.

In b) plot extra points (e.g. the white dots shown) to confirm where the curve should lie.

Solution:

a) $x \quad 0 \quad -1 \quad \frac{1}{2} \quad 2 \quad \frac{2}{3} \quad -\frac{1}{6} \quad 2.5 \quad -4 \quad \frac{1}{6} \quad 0.2 \quad -\frac{1}{2}$

$y \quad 0 \quad -3 \quad \frac{3}{2} \quad 6 \quad 2 \quad -\frac{1}{2} \quad 7.5 \quad -12 \quad \frac{1}{2} \quad 0.6 \quad -\frac{3}{2}$

Rule: $y = 3 \times x$, $x = y \div 3 = \frac{1}{3} \times y$, $\frac{y}{x} = 3 \ (x \neq 0)$

Who can think of another form of the rule? [$\frac{x}{y} = \frac{1}{3} \ (y \neq 0)$]

What does the graph show us about the relationship between $x$ and $y$? ($x$ and $y$ are in direct proportion to one another.)
**Activity**

2 (Continued)

**Solution**:

b)  

<p>| | | | | | | | | | | | | |</p>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>u</td>
<td>1</td>
<td>( \frac{1}{2} )</td>
<td>2</td>
<td>( \frac{1}{3} )</td>
<td>-4</td>
<td>( \frac{1}{2} )</td>
<td>8</td>
<td>( \frac{2}{3} )</td>
<td>-3</td>
<td>-12</td>
<td>-4</td>
<td></td>
</tr>
<tr>
<td>v</td>
<td>4</td>
<td>-8</td>
<td>2</td>
<td>12</td>
<td>-1</td>
<td>8</td>
<td>( \frac{1}{2} )</td>
<td>6</td>
<td>4</td>
<td>( \frac{3}{2} )</td>
<td>0</td>
<td>-1</td>
</tr>
</tbody>
</table>

**Rule:**  

\[ v = \frac{4}{u} \quad (u \neq 0), \quad u = \frac{4}{v} \quad (v \neq 0), \quad u \times v = 4 \]

What does the graph show us about the relationship between \( u \) and \( v \)? (\( u \) and \( v \) are in inverse proportion to one another.)

**Notes**

- Note that neither \( u \) nor \( v \) can be zero.
- (As one value increases by a certain amount, the other value decreases by that amount, and vice versa.)

---

**Y6**

**Lesson Plan 164**

**Notes**

- Individual work, monitored, helped
- Differentiation by time limit
- Responses shown in unison
- Reasoning, agreement, self-correction, praising
- Feedback for T

**Q.2 Read:** Solve the problems. Think about the ratio between the quantities.

Deal with one at a time. Ps read the problem themselves and solve it in Ex. Bks. under a time limit.

Review with whole class. Ps show results on scrap paper or slates on command. P answering correctly explains reasoning at BB. Who did the same? Who solved it in a different way? etc.

Mistakes discussed and corrected.

Elicit whether the quantities are in direct or inverse proportion.

**Solution**:

a) If \( \frac{4}{5} \) kg of apples cost £2.40, what is the price of \( \frac{2}{3} \) kg of apples?

\[
\text{Plan: } \frac{2.40}{\frac{4}{5}} \times \frac{2}{3} = \frac{2.40 \times 5 \times 2}{3} = \frac{2.40}{\frac{4}{5} \times \frac{2}{3}} = \frac{2.40 \times 5 \times \frac{2}{3}}{4} = \frac{2.40 \times 5}{3} \times \frac{2}{3} = \frac{2.40 \times 5}{3} \\
= \frac{2.40 \times 5}{3} \\
= \frac{2.40}{\frac{4}{5}} \times \frac{2}{3} = \frac{2.40 \times 5 \times \frac{2}{3}}{4} \\
\]

or \[ \frac{2.40 \times 5}{3} \times \frac{2}{3} = \frac{2.40 \times 5}{3} \times \frac{2}{3} = \frac{2.40 \times 5}{3} \times \frac{2}{3} = \frac{2.40 \times 5}{3} \]

**Answer:** The price of \( \frac{2}{3} \) kg of apples is £2.

[Price and quantity are in direct proportion to one another.]

---

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b) \( \frac{1}{5} \) kg of strawberries costs £1.50, including the cost of the punnet. If all punnets cost 10 p, what would you pay for 400 g of strawberries in a punnet?

**Plan:**
\[
\frac{1}{5} \text{ kg} = 200 \text{ g} \rightarrow £1.40 + £0.10 = £1.50
\]
\[
400 \text{ g} \rightarrow £1.40 \times 2 + £0.10 = £2.90
\]
**Answer:** You would pay £2.90 for 400 g of strawberries in a punnet.

c) When our car used \( \frac{5}{3} \) litres of petrol every 100 km, a full tank lasted for 864 km. If our car had used \( \frac{7}{5} \) litres of petrol every 100 km, how far could we have driven with a full tank?

**Plan:**
\[
100 \text{ km} \rightarrow \frac{5}{3} \text{ litres}
\]
\[
864 \text{ km} \rightarrow \frac{5}{3} \text{ litres} \times 8.64 = 43.20 + 2.88 \text{ (litres)}
\]
\[= 46.08 \text{ litres}
\]
(This is the capacity of the tank.)
\[
46.08 \text{ litres} \div \frac{7}{5} \text{ litres} \times 100 = 4608 \div 7.2 = 640 \text{ (km)}
\]
or
\[
\text{Consumption} \times \text{Distance covered}
\]
\[
\frac{7}{5} \times \frac{5}{3} \text{ litres (per 100 km)} \rightarrow 864 \text{ km} \times \frac{5}{3} \text{ litres (per 100 km)} \rightarrow \ ? \text{ km} \times \frac{7}{5}
\]
\[C: \ 864 \times \frac{16}{3 \times 7.2} = 864 \times \frac{16}{21.6} = 40 \times 16 = 640 \text{ (km)}
\]
**Answer:** We could have driven 640 km with a full tank.

---

The price and quantity are not in direct proportion and not in inverse proportion – they are not in proportion at all.

(As the mass increases by a certain number of times, the price does not increase or decrease by that same number of times.)

Amount of petrol in tank and the distance covered (for the same consumption) are in direct proportion. (The more petrol there is in the tank, the further the distance which can be covered.)

Consumption and distance covered (with the same amount of petrol in the tank at the start) are in inverse proportion.

(The higher the consumption of the engine, the shorter the distance covered.)

T could show this method of multiplying by the reciprocal value if no P used it and ask Ps if it is correct.

\[
(864 \div 21.6 = 108 \div 2.7 = 12 \div 0.3 = 120 \div 3 = 40)
\]
N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

**Activity**

**PbY6b, page 164. Q.3**

Read: *The big hand and the little hand of a clock coincide at 12 o’clock. When will the two hands of the clock next be in a straight line?*

Allow Ps a minute or two to think and discuss with their neighbours. Ps who have ideas come to BB to explain and develop them, with help of T and class. If no Ps have ideas, T gives hint about the angle covered by each hand in 1 minute. If Ps still cannot suggest what to do, T directs Ps thinking and helps class to solve it together, involving Ps where possible.

**Solution:** e.g.

When the hands are next in a straight line, the angle between the two hands will have increased from $0^\circ$ to $180^\circ$.

**Angle speed of the hands:**

- Hour hand (small): 60 minutes $\rightarrow 30^\circ$
  1 minute $\rightarrow 30^\circ \div 60 = 0.5^\circ$
- Minute hand (big): 60 minutes $\rightarrow 360^\circ$
  1 minute $\rightarrow 360^\circ \div 60 = 6^\circ$

So every minute, the angle between the 2 hands increases by $6^\circ - 0.5^\circ = 5.5^\circ$ but the angle has to increase by $180^\circ$, so the time needed is:

$$180^\circ \div 5.5^\circ = \frac{180}{5.5} = \frac{360}{11} = 32 \frac{8}{11} \text{ (minutes)}$$

**Answer:** The two hands will next form a straight line at 12 h $32 \frac{8}{11}$ min.

**Notes**

Review the questions with the whole class, whether Ps attempted them or not.

**Lesson Plan 164**

Whole class activity (or individual or paired trial first if Ps wish)

Use real or model clock or draw diagram on BB or use enlarged copy master or OHP.

BB:

Discussion, reasoning, agreement, praising Ps could write agreed solution in Ex. Bks.

Check solution by showing the position of the hands on a real clock or on the diagram.

BB:

**Individual work, monitored**

Diagram drawn on BB or SB or OHT

Alternative method:
Let the time taken together be $x$, then

<table>
<thead>
<tr>
<th></th>
<th>Time needed</th>
<th>Part dug per hour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dad</td>
<td>2 h</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Benny</td>
<td>3 h</td>
<td>$\frac{1}{3}$</td>
</tr>
<tr>
<td>Charlie</td>
<td>6 h</td>
<td>$\frac{1}{6}$</td>
</tr>
<tr>
<td>Together</td>
<td>$x$</td>
<td>$\frac{1}{x}$</td>
</tr>
</tbody>
</table>

$$\frac{1}{x} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{6}{6} = 1$$

$x = 1$ (hour)
Q.5 Read: The ratio of the lengths of the sides of a right-angled triangle is $3 : 4 : 5$.

If the area of the triangle is $24 \text{ cm}^2$, what is the length of each of its sides?

Advise Ps to draw a diagram first.

Solution: e.g.

$$A = \frac{a \times b}{2} = 24 \text{ cm}^2 \quad [\times 2]$$

$$a \times b = 48 \text{ cm}^2$$

If $a:b:c = 3 \text{ cm} : 4 \text{ cm} : 5 \text{ cm}$, $a \times b = 12 \text{ cm}^2 \times$

If $a:b:c = 6 \text{ cm} : 8 \text{ cm} : 10 \text{ cm}$, $a \times b = 48 \text{ cm}^2 \checkmark$

Answer: The length of the sides of the triangle are 6 cm, 8 cm and 10 cm.

Individual work, monitored or

Let $x$ be the scale factor required, then

$$A = \frac{3x \times 4x}{2} = 24 \quad [\times 2]$$

$$3x \times 4x = 48$$

$$12 \times x^2 = 48 \quad [\div 12]$$

$$x^2 = 4$$

$$x = 2$$

So the lengths of the sides are 6 cm, 8 cm and 10 cm.
Factorising 165, 340, 515 and 1165. Revision and practice.

**PbY6b, page 165**

Solutions:

Q.1  

a) *If the areas of two rectangles are equal, the rectangles are congruent.*  
(e.g. $A = 12$ cm$^2$: sides could be 3 cm and 4 cm, or 2 cm and 6 cm, or 1 cm and 12 cm)

b) *All equilateral triangles are similar.*  
(All equilateral triangles have angles of 60°, so they are the same shape, but not necessarily the same size.)

c) *The arithmetic mean of two numbers is always positive.*  
(e.g. the mean of −4 and −8 is −6)

d) *There is an isosceles triangle which has three equal angles.*  
(An isosceles triangle has at least 2 equal sides and angles, so an equilateral triangle is also an isosceles triangle.)

e) *The diagonals of a parallelogram intersect at right angles.*  
(Only if the parallelogram is a rhombus, i.e. it has equal sides)

f) *If the areas of two squares are equal, the squares are congruent.*  
(All squares are similar, so if two squares have the same area, they must also be exactly the same size.)

Q.2  

a) $(a + b) \times c = a \times c + b \times c = ac + bc$

b) $(a + b) \div c = a \div c + b \div c = \frac{a}{c} + \frac{b}{c} = \frac{a + b}{c}$

c) $\frac{a}{b} + \frac{c}{d} = \frac{a \times d + c \times b}{b \times d} = \frac{ad + cb}{bd}$

d) $\frac{a}{b} \times \frac{b}{a} = 1$ (reciprocal values)

e) $\frac{a}{b} \div a = \frac{1}{b}$

f) $\frac{a - b}{c} = \frac{a}{c} - \frac{b}{c}$

Q.3  

a) $\frac{1}{x} + \frac{1}{y} = \frac{1}{8} + \frac{1}{24} = \frac{3 + 1}{24} = \frac{4}{24} = \frac{1}{6}$

b) $x = 7, y = 42; x = 9, y = 18; x = 10, y = 15$; and $x = 12, y = 12$

Q.4  

a) $V = 24$ unit cubes

b) $A = 2 \times 4 + 4 \times 12 = 8 + 48 = 56$ (unit squares)

c) i) Cuboid with **smallest** possible surface area has dimensions 2 units $\times$ 3 units $\times$ 4 units  
(i.e. the most symmetrical shape)

$A = 2 \times (6 + 8 + 12) = 2 \times 26 = 52$ (sq. units)
### Activity

*(Solutions: Q.4 continued)*

- **c) ii)** Cuboid with the greatest possible surface area has dimensions 24 units × 1 unit × 1 unit  
  (i.e. the most **asymmetrical** shape)

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<tr>
<td>a)</td>
<td>e.g.</td>
<td><img src="image1.png" alt="Cube" /></td>
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<tr>
<td>b)</td>
<td>e.g.</td>
<td><img src="image2.png" alt="Cube" /></td>
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<tr>
<td>c)</td>
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</table>

Shortest path from S to F is **3 units**.

Longest path from S to F is **7 units**.

\[ 2 + 4 + 4 + 4 = 14 \text{ different paths are possible.} \]

### Notes

\[ A = 2 \times 1 + 4 \times 24 \]
\[ = 2 + 96 = 98 \text{ (sq. units)} \]
Lesson Plan

166

Notes

Individual work, monitored (or whole class activity)
BB: 166, 341, 516, 1166
Ps try it without calculators.
Reasoning, agreement, self-correction, praising

Individual work, monitored, helped
Written on BB or SB or OHT
Responses shown in unison.
Reasoning, agreement, self-correction, praising
Accept any valid form of calculation.
Feedback for T

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.
Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.
Elicit that:

- \(166 = 2 \times 83\)
  Factors: 1, 2, 83, 166
- \(341 = 11 \times 31\)
  Factors: 1, 11, 31, 341
- \(516 = 2 \times 2 \times 3 \times 43 = 2^2 \times 3 \times 43\)
  Factors: 1, 2, 3, 4, 6, 12, 43, 86, 129, 172, 258, 516
- \(1166 = 2 \times 11 \times 53\)
  Factors: 1, 2, 11, 22, 53, 106, 583, 1166

8 min

2

PbY6b, page 166

Q.1 Read: Which is more, A or B? Circle the appropriate letter.
Deal with one at a time, or set a time limit. Ps calculate mentally or write calculations in Ex. Bks.
Review with whole class. Ps could show their answer with an equation or inequality on scrap paper or slates on command.
Ps with different answers explain reasoning. Class points out errors and agrees on correct answer. Who worked it out in the same way? Who thought in a different way, etc. Mistakes discussed and corrected.

Solution: e.g.

a) \(0.15 \text{ times } A\) is \(3300\text{ kg}. \ 0.25 \text{ times } B\) is \(4000\text{ kg}. \ [A \ > \ B]\)
\[
A = 3300 \text{ kg} \div 0.15 = 330000 \text{ kg} \div 15 = 22000 \text{ kg}
\]
\[
B = 4000 \text{ kg} \div 0.25 = 400000 \text{ kg} \div 25 = 80000 \text{ kg} + 5
\]
\[
= 16000 \text{ kg}
\]

b) \(\frac{47}{100} \) of \(A\) is \(564\) litres. \(\frac{55}{100}\) of \(B\) is \(605\) litres. \([A \ > \ B]\)
\[
A = 564 \text{ litres} \div 0.47 = 56400 \text{ litres} \div 47 = 1200 \text{ litres}
\]
\[
B = 605 \text{ litres} \div 0.55 = 60500 \text{ litres} \div 55 = 1100 \text{ litres}
\]

c) \(A\) is \(75\%\) of \(900\) m. \(B\) is \(120\%\) of \(562.5\) m. \([A = B]\)
\[
A = \frac{900 \text{ m}}{1.2} = 675 \text{ m}
\]
\[
B = 562.5 \text{ m} \times 1.2 = 675 \text{ m}
\]

d) \(A\) is \(30\%\) more than \(\£5000.\ \ 80\%\) of \(B\) is \(\£5000.\ \ [A \ > \ B]\)
\[
A = 130\% \text{ of } \£5000 = \£5000 \times 1.3 = \£6500
\]
\[
B = \£5000 \div 0.8 = \£50000 \div 8 = \£6250
\]

22 min
Lesson Plan 166

**Notes**

Individual work, monitored, helped

Responses shown in unison. Reasoning, agreement, self-correction, praising
Accept and praise any valid method of solution.
Feedback for T

_E: a little more than £10 000_ or _Plan:_

\[ £10000 \times \frac{1000}{1050} = £10650 \]

_E: a little less than £21 300_

_C: 21300 ÷ 1.065_

\[ = 2130000 \div 1065 \]

\[ = 4260000 \div 213 \]

\[ = 20000 \]

_E: 1200 – 800 = 400_

\[ \frac{400}{1200} = \frac{1}{3} = 33\% \]

or _\[ \frac{65}{1200} \times 100\% = 65\% \_

Lost: 100\% – 65\% = 35\%

Elicit that if a side is reduced by 40\%, it is 60\% or 6 tenths or 0.6 of the original length.

If no P used this method of solution, T could lead Ps through it and ask what they think of it.

\[ b' = b \div 12 \times \frac{10}{6} = b \times \frac{5}{3} \rightarrow 166.6\% \text{ of } b \]

[i.e. \( b' = \frac{2}{3} \text{ of } b \)]

---

32 min
\[ Y6 \]

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**PbY6b, page 166**

Q.2 Read: Solve the problems and check your results in context.

Deal with one at a time under a short time limit. Ps read problem themselves, solve it and check the result, then write the answer in a sentence.

Review with whole class. T chooses a P to read out the question and Ps show results on scrap paper or slates on command. P with correct answer explains reasoning at BB. Class agrees/disagrees. Who did the same? Who worked it out in another way? etc. Mistakes discussed and corrected.

T chooses a P to say the answer in a sentence.

**Solutions:** e.g.

a) The sum of two numbers is 76.8 and their ratio is 2 : 3. What are the two numbers?

Let the two numbers be \( a \) and \( b \).

Then \( a + b = 76.8 \), \( a : b = 2 : 3 \)

\[
\begin{align*}
a &= \frac{2}{5} \text{ of } 76.8 = 0.4 \times 76.8 = 30.72 \\
b &= \frac{3}{5} \text{ of } 76.8 = 0.6 \times 76.8 = 46.08 \\
or b &= 76.8 - 30.72 = 46.08
\end{align*}
\]

**Answer:** The two numbers are 30.72 and 46.08.

b) The difference between two positive numbers is 37.6 and their ratio is 4 : 3. What are the two numbers?

Let the two numbers be \( a \) and \( b \).

Then \( a - b = 37.6 \), \( a : b = 4 : 3 \)

\[
\begin{align*}
a &= 4 \times 37.6 = 150.4 \\
b &= 3 \times 37.6 = 112.8 \\
or b &= 150.4 - 37.6 = 112.8
\end{align*}
\]

**Answer:** The two numbers are 150.4 and 112.8.

c) The ratio of two angles in a quadrilateral is 2 : 7. The third angle is \( 40^\circ \) less, and the 4th angle is \( 60^\circ \) less, than the largest angle. What sizes are the angles in the quadrilateral?

The sum of the angles in any quadrilateral is \( 360^\circ \), so

\[ \angle A + \angle B + \angle C + \angle D = 360^\circ \]

\[ \angle A : \angle B = 2 : 7, \]

so if we let the scale factor be \( \alpha \),

\[ \angle A = 2\alpha \], \( \angle B = 7\alpha \), \( \angle C = 7\alpha - 40^\circ \), \( \angle D = 7\alpha - 60^\circ \)

Adding the angles:

\[
\begin{align*}
2\alpha + 7\alpha + 7\alpha - 40^\circ + 7\alpha - 60^\circ &= 360^\circ \\
23\alpha - 100^\circ &= 360^\circ \quad [\div 100^\circ] \\
23\alpha &= 460^\circ \quad [\div 23] \\
\alpha &= 20^\circ
\end{align*}
\]

**Answer:** The angles in the quadrilateral are \( 40^\circ \), \( 80^\circ \), \( 100^\circ \) and \( 140^\circ \).
d) The ratio of the two shorter sides of a right-angled triangle is $7:5$ and its area is $8470 \text{ cm}^2$.

How long are these two sides?

Let the two side lengths be $a$ and $b$.

Then $a:b = 7:5$, $A = \frac{ab}{2}$

If we let the scale factor be $x$, then

$a = 7x$, $b = 5x$, $A = \frac{7x \times 5x}{2} = 8470 \text{ cm}^2$

Solve: $\frac{7x \times 5x}{2} = 8470 \quad [\times 2]$

$35 \times x^2 = 16940 \quad [\div 35]$

$x^2 = 484$

$x \times x = 4 \times 121 = (2 \times 11) \times (2 \times 11)$

$x = 2 \times 11 = 22$

Now we can work out the values of $a$ and $b$.

$a = 7 \times 22 \text{ (cm)} = 154 \text{ cm}$, $b = 5 \times 22 \text{ (cm)} = 110 \text{ cm}$

**Answer:** The lengths of the two shorter sides of the triangle are 154 cm and 110 cm.

e) The ratio of the lengths of 3 edges meeting at a vertex of a cuboid is $2:4:5$. The volume of the cuboid is $320 \text{ cm}^3$.

What lengths are the edges of the cuboid?

Let the 3 edge lengths be $a$, $b$ and $c$.

Then $a:b:c = 2:4:5$, $V = a \times b \times c$

If we let the scale factor be $x$, then

$a = 2x$, $b = 4x$, $c = 5x$, $V = 2x \times 4x \times 5x$

Solve: $2x \times 4x \times 5x = 320$

$40 \times x^3 = 320 \quad [\div 40]$

$x^3 = 8$

$x = 2$

Now we can work out the values of $a$, $b$ and $c$.

$a = 2 \times 2 \text{ (cm)} = 4 \text{ cm}$, $b = 4 \times 2 \text{ (cm)} = 8 \text{ cm}$,

$c = 5 \times 2 \text{ (cm)} = 10 \text{ cm}$

**Answer:** The lengths of the 3 edges of the cuboid are 4 cm, 8 cm and 10 cm.
### Activity

**Y6**

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**PbY6b, page 166**

Q.4 Read: *The perimeter of an irregular pentagon is 54 cm. The length of its sides are in the ratio 1 : 2 : 5 : 6 : 7 : 9.*

*Calculate the length of each side.*

Set a time limit of 3 minutes. Ps work individually or in pairs. Review with whole class. T asks a P for the lengths of the sides. Who agrees with A? Who has different lengths? Who agrees with B? Ps with different lengths explain reasoning at BB. Class agrees on the correct answer. Who had the correct answer but worked it out in another way? etc. Mistakes discussed and corrected.

**Solution:** e.g.

Let the scale factor be \(x\), then

\[
BB: \quad x + 2x + 5x + 6x + 7x + 9x = 54
\]

\[
30x = 54 \quad \left[ \div 30 \right]
\]

\[
x = 1.8
\]

*Check:* \(1.8 + 3.6 + 9 + 10.8 + 12.6 + 16.2 = 54 \checkmark\)

*Answer:* The lengths of the sides are 1.8 cm, 3.6 cm, 9 cm, 10.8 cm, 12.6 cm and 16.2 cm.

---

**Lesson Plan 166**

**Notes**

Individual (paired) work, monitored, helped

Differentiation by time limit.

Reasoning, agreement, checking, self-correction, praising

[If no P had the correct answer, the question could be left open for voluntary homework and reviewed before the start of Lesson 167.]

or \(1 + 2 + 5 + 6 + 7 + 9 = 30\)

\[
\frac{1}{30}\text{ of }54 \text{ cm} = 1.8 \text{ cm},
\]

\[
\frac{2}{30}: \quad 3.6 \text{ cm}, \quad \frac{5}{30}: \quad 9 \text{ cm},
\]

\[
\frac{6}{30}: \quad 10.8 \text{ cm}, \quad \frac{7}{30}: \quad 12.6 \text{ cm}
\]

\[
\frac{9}{30}: \quad 16.2 \text{ cm}
\]
**Activity**

### 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **167** is a prime number  
  Factors: 1, 167  
  (as not exactly divisible by 2, 3, 5, 7, 11 and 13² > 167)

- **342** = 2 × 3 × 3 × 19 = 2 × 3² × 19  
  Factors: 1, 2, 3, 6, 9, 18, 19, 38, 57, 114, 171, 342

- **517** = 11 × 47  
  Factors: 1, 11, 47, 517

- **1167** = 3 × 389  
  Factors: 1, 3, 389, 1167  
  (389 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and 23² > 389)

**8 min**

### 2

**PbY6b, page 167, Q.1**

a) Read: **Write five numbers using non-zero digits so that their ratio is 1 : 2 : 3 : 4 : 5. Use each digit only once.**

What is the obvious solution? (1, 2, 3, 4, 5) Who can think of another solution? Is it correct? What scale factor has been used? Is there another solution?

Elicit that using scale factor:

- **2**: 2, 4, 6, 8, 10  
  (not valid, as zero used)

- **3**: 3, 6, 9, 12, 15  
  (not valid, as two ’1’s used)

- **4, 5, 6, 7, 8**:  
  (not valid either)

- **9**: 9, 18, 27, 36, 45

What do you notice about the solution using scale factor 9? (Every non-zero digit has been used once.)

If we wanted only this answer, how should we have worded the question? (Write five numbers using all the non-zero digits . . .)

b) Read: **Use all possible digits once each to make five numbers in the ratio 1 : 2 : 3 : 4 : 5.**

What is different about this question? (We must use zero.)

Give Ps a minute or two to try out some numbers, then Ps come to BB or dictate them to teacher. Class agrees/disagrees.

**BB**: 18, 36, 54, 72, 90  
(scale factor 18, i.e. 2 × 9)

Who found a different solution? (No other solution is possible.)

**T**: When there is only one solution to a problem, we say that the solution is **unique**.

**14 min**

---

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### Activity

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

### Notes

Review the questions interactively with the whole class, whether Ps attempted them or not.

#### Individual trial

Accept any valid method of solution.

#### Check:

- $72 : 48 = 36 : 24 = 3 : 2$ ✓
- $72 \div 6 = 12$, $48 \div 6 = 8$ and $12 = 8 + \frac{4}{3}$ ✓

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<td><strong>Notes</strong></td>
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### Activity

#### HMC: Hungarian Mathematics Competition 1987

**Age 11**

#### Lesson Plan 167

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<td><strong>HMC:</strong> Hungarian Mathematics Competition 1987</td>
<td>(Revert to a whole class activity if Ps are struggling)</td>
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**PbY6b, page 167**

#### Q4

**Read:** In a mathematics competition, 9 pupils got through to the final round. In the final round, 6 tenths of the girls solved at least two problems correctly.

How many boys and how many girls reached the final of the competition?

**Solution:**

Let the number of girls be \( g \) and the number of boys be \( b \).

Then \( b + g = 9 \)

\[
\frac{6}{10} = \frac{3}{5}, \text{ so } \frac{3}{5} \text{ of } g \text{ must be a natural number, and } 0 < g \leq 9
\]

\[
\frac{3}{5} \text{ of } 5 \text{ (} = 3\text{) gives the only whole number among } \frac{3}{5} \text{ of '1 to 9'}
\]

So \( g = 5 \) and \( b = 9 - 5 = 4 \)

**Answer:** Four boys and five girls reached the final.

---

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<tr>
<td><strong>HMC:</strong> Hungarian Mathematics Competition 1990</td>
<td>We cannot have part of a girl!</td>
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**Erratum**

In *PbY6b* the question should be: 'How much money do I have in each pocket?'

---

**PbY6b, page 167**

#### Q5

**Read:** I have £2 in my two pockets altogether.

If I transfer a quarter of the money that I have in one pocket plus an additional 20 p from the same pocket to the other pocket, I would have an equal amount of money in each pocket. How much money do I have in each pocket?

**Solution:**

Let the amount of money in one pocket be \( a \) (in pence), and in the other pocket be \( b \) (in pence).

Then \( a + b = 200, \quad b = 200 - a \)

\[
a - \frac{a}{4} - 20 = 200 - a + \frac{a}{4} + 20 \quad [\times 4]
\]

\[
4a - a - 80 = 800 - 4a + a + 80
\]

\[
3a - 80 = 880 - 3a
\]

\[
6a - 80 = 880 \quad [+ 80]
\]

\[
6a = 960 \quad [\div 6]
\]

\[
a = 160 \text{ (p)}
\]

So \( b = 200 - 160 = 40 \text{ (p)} \)

**Answer:** I have £1.60 in one pocket and 40 p in the other pocket.

---

**Check:**

LHS: \( 160 - (40 + 20) = 100 \)

RHS: \( 40 + (40 + 20) = 100 \)
**Lesson Plan 167**

**Notes**

Whole class activity
(or individual trial if Ps wish)

Allow Ps a minute or two to think about it and discuss with their neighbours.
P.s develop their ideas with the help of T and class.
If no P is on the right track, T gives hints or directs Ps thinking, involving them as much as possible.

---

**Y6**

**Activity**

**PbY6b, page 167, Q.6**

Read: Sally owns a hotel. She has seen some material which matches the colour scheme in her public rooms exactly. She needs 51 m\(^2\) of material to make cushions and drapes.

However, Sally has been told that when the material is washed, it shrinks by \(\frac{1}{16}\) of its length and by \(\frac{1}{18}\) of its width, so she intends to wash the material before she uses it.

How many square metres of unshrunk material should she buy?

**Solution:** e.g.

Let the amount of material to be bought be \(x\) (in square metres).

If the material shrinks by \(\frac{1}{16}\) of its length and by \(\frac{1}{18}\) of its width then the shrunken material will be \(\frac{15}{16}\) of the original length and \(\frac{17}{18}\) of the original width.

BB: \(x \times \frac{15}{16} \times \frac{17}{18} = 51\) m\(^2\) \[\times 16 \times 18\]

\(x \times 15 \times 17 = 51 \times 16 \times 18\) (m\(^2\)) \[\div (15 \times 17)\]

\(x = \frac{51 \times 16 \times 18}{15 \times 17} \) m\(^2\)

\(= \frac{3 \times 96}{5} \) m\(^2\) = \(\frac{288}{5} \) m\(^2\) = 57.6 m\(^2\)

**Answer:** Sally should buy 57.6 m\(^2\) of unshrunk material.

---

**PbY6b page 167, Q.7**

Read: Which is more: \(\frac{3}{4}\) or \(\frac{3000001}{4000001}\)?

**Solution:** e.g.

Let’s think of 3 quarters as its equivalent fraction \(\frac{3000000}{4000000}\)

BB: If \(\frac{3000000}{4000000} > \frac{3000001}{4000001}\)

\(3 \times 4000000 \times 4000001 > 3 \times 4000000 \times 4000000\) [\(\times 4 000 000 \times 4 000 001\)]

\(3 000 000 \times 4 000 000 + 3 000 001 > 3 000 000 \times 4 000 000 + 4 000 000\)

\(3 000 000 > 4 000 000\) ✗ Not true!

or If \(\frac{3000000}{4000001} < \frac{3000001}{4000001}\)

\(3 000 000 \times 4 000 000 < 3 000 001 \times 4 000 000\)

\(3 000 000 \times 4 000 000 + 3 000 000 < 3 000 000 \times 4 000 000 + 4 000 000\)

\(3 000 000 < 4 000 000\) ✓ True!

**Answer:** \(\frac{3000001}{4000001}\) is more than \(\frac{3}{4}\).
### Activity

#### 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **168** = \(2 \times 2 \times 2 \times 3 \times 7 = 2^3 \times 3 \times 7\)
  - Factors: \(1, 2, 3, 4, 6, 7, 8, 12, 168, 84, 56, 42, 28, 24, 21, 14\)  
  - 8 min

- **343** = \(7 \times 7 \times 7 = 7^3\) (cubic number)
  - Factors: \(1, 7, 49, 343\)

- **518** = \(2 \times 7 \times 37\)
  - Factors: \(1, 2, 7, 14, 37, 74, 259, 518\)

- **1168** = \(2 \times 2 \times 2 \times 2 \times 73 = 2^4 \times 73\)
  - Factors: \(1, 2, 4, 8, 16, 73, 146, 292, 584, 1168\)

#### 2

*PbY6b, page 168, Q.1*

Read: *Freddie Fox decided that in future he would tell lies on Mondays, Wednesdays and Fridays but he would always tell the truth on the other days of the week. One day, Freddie said, "Tomorrow I will tell the truth." On what day of the week could he have said it?*

Allow Ps a minute or two to think about it and discuss with their neighbours if they wish. Who thinks that they know the answer? Why do you think so? Who agrees? Who thinks something else? etc. If no P has the correct explanation, try each day in turn.

Could he have said it on a Monday (Tuesday, etc.)? Why not?

**Solution:**

Freddie could not have said it on:

- Monday, Wednesday or Friday, as he tells lies on these days and as on the days following them he tells the truth, he would not be telling a lie.
- Tuesday, Thursday or Sunday, as he tells the truth on these days and as on the days following them he tells lies, he would not be telling the truth.

The only possible day is a **Saturday**, as he tells the truth on Saturdays and also tells the truth on Sundays, so his statement is true.

13 min
<table>
<thead>
<tr>
<th><strong>Y6</strong></th>
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</thead>
<tbody>
<tr>
<td><strong>Activity</strong></td>
<td><strong>Notes</strong></td>
</tr>
<tr>
<td>N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.</td>
<td>Review the questions interactively with the whole class, whether Ps attempted them or not.</td>
</tr>
<tr>
<td>3</td>
<td><strong>PbY6b, page 168</strong></td>
</tr>
<tr>
<td>Q.2 <strong>Read:</strong> Is it possible for four whole numbers to have an odd number as their sum and an odd number as their product? If so, write the numbers. If not, say why not.</td>
<td>Individual trial first</td>
</tr>
<tr>
<td><strong>Solution:</strong> To get an odd sum, there must be an odd number of odd terms. e.g. 4 + 8 + 10 + 17 is odd (1 odd number) but 4 × 8 × 10 × 17 is even (units digit 0) 20 + 3 + 41 + 15 is odd (3 odd numbers) but 20 × 3 × 41 × 15 is even (units digit zero) It is impossible for the sum and product of 4 whole numbers to be odd. To get an odd sum there must be an odd number of odd terms and to get an odd product, there must be an odd number of factors which are all odd numbers.</td>
<td>Ps try out different numbers in Ex. Bks.</td>
</tr>
<tr>
<td>4</td>
<td><strong>PbY6b, page 168</strong></td>
</tr>
<tr>
<td>Q.3 <strong>Read:</strong> Five empty glasses and five glasses full of grape juice are standing in a row. <strong>BB:</strong></td>
<td>Then whole class discussion and agreement on the correct form of words for the answer.</td>
</tr>
<tr>
<td><strong>Solution</strong> Pour the contents of glasses 7 and 9 into empty glasses 2 and 4. <strong>BB:</strong></td>
<td></td>
</tr>
<tr>
<td>They cannot exchange places as that would involve touching 4 glasses!</td>
<td></td>
</tr>
</tbody>
</table>
### Activity

**5**  

**PbY6b, page 168**

*Q.4* Read: *On an old smudged sheet of paper we can see this writing:*

- BB: 72 barrels: £37.8

...but the digits marked are illegible. What could the price of a barrel have been?

**Solution:** e.g.

Get rid of the decimal by changing the amount to pence.

BB: 72 barrels: £378

If £378 is exactly divisible by 72, it must also be exactly divisible by 8 and by 9, as $8 \times 9 = 72$.

- £378 is divisible by 8 only if 78 is divisible by 8.
- 78 is divisible by 8 only if \( \frac{78}{8} = 9.75 \), so $\Box378 = \Box378\,\text{p}$

If £3784 is exactly divisible by 9, its digits must add up to a multiple of 9.

BB: \(4 + 8 + 7 + 3 = 22\), 22 + 5 = 27 (the only digit possible)

so $\Box3784 = \Box3784\,\text{p}$

BB: 72 barrels: £537.84

1 barrel: £537.84 $\div 8 = £67.23$  $\div 9 = £7.47$

**Answer:** The price of a barrel was £7.47.

---

**Notes**

Individual trial first

Change to a whole class activity if Ps are struggling.

T directs’ Ps thinking.

(as whole thousands are exactly divisible by 8)

Extra praise for Ps who worked out the answer without help.

---

**6**  

**PbY6b, page 168. Q.5**

Read: *Ben had to make a 4-digit number, choosing from the digits 1, 2, 3, 4, 5 and 6. He was allowed to use a digit more than once. Ben wrote his number on a piece of paper and put it in his pocket. The rest of the class had to guess Ben’s number. The first suggestion was 4215. Ben said that two digits were correct but only one of them was in the correct place-value column. The second suggestion was 2365. Ben said that again two digits were correct but only one of them was in the correct place-value column. The third suggestion was 5525. This time Ben said that no digits were correct. What do you think Ben’s number could be?*

**Solution:**

- 3rd clue: 5525 (No digits correct, so number does not contain 2 or 5)
- 2nd clue: 2365 (3 and 6 correct, so number has 3H or 6T)
- 1st clue: 4215 (4 and 1 correct, so number has 4Th or 1T)

If 4Th is correct, then 1T is not correct, so 6T must be correct.

If 6T is correct, then 3H is not correct, so number is 4163.

If 4Th is not correct, then 1T is correct, so 6T is not correct.

If 6T is not correct, then 3H is correct, so number is 6314.

**Answer:** Ben’s number could be 4163 or 6314.

---

**Th H T U**

BB: $\begin{array}{c}
4 \\
6 \\
6 \\
3 \\
4 \\
1 \\
6 \\
3
\end{array}$

or $\begin{array}{c}
\square \\
\square \\
\square \\
\square \\
3 \\
1 \\
6 \\
3
\end{array}$
Q.6 Read: We know this information about a certain square and a certain rectangle.

• Their areas are equal.
• The perimeter of the square is 4 fifths of the perimeter of the rectangle.
• The long side of the rectangle is 4 times the length of its short side.
• The perimeters, areas and sides of the 2 shapes are whole numbers less than 100.

What could be the lengths of the sides of the square and the rectangle?

Solution: e.g.
BB:

\[ a \times a = b \times c \] (1)

Clue 2: \[ 4 \times a = \frac{4}{5} \times 2 \times (b + c) \] (2)

Clue 3: \[ b = 4 \times c \] (3)

Clue 4: \[ a, b, c, 4a, 2 \times (b + c), a^2, b \times c < 100 \] and whole numbers

Substitute \( 4 \times c \) for \( b \) in equation (2):

\[ 4 \times a = \frac{4}{5} \times 2 \times (4 \times c + c) \]

\[ 4 \times a = \frac{4}{5} \times 2 \times 5 \times c \] [÷ 4]

\[ a = \frac{1}{5} \times 10 \times c \]

\[ a = 2 \times c \] (4)

Substitute \( 2 \times c \) for \( a \) in equation (1):

\[ 2 \times c \times 2 \times c = b \times c \]

\[ 4 \times c \times c = b \times c \]

but \( b \times c < 100 \) and is a whole number,

so \( 4 \times c \times c < 100 \), and is a whole number \[ [ ÷ 4] \]

\[ c \times c < 25 \), and is a whole number

If \( c = 1 \), then from (3) \( b = 4 \). and from (4) \( a = 2 \)
If \( c = 2 \), \( b = 8 \). \( a = 4 \)
If \( c = 3 \), \( b = 12 \). \( a = 6 \)
If \( c = 4 \), \( b = 16 \). \( a = 8 \)

Answer: The sides of the square could be 2, 4, 6 or 8 units. The sides of the rectangle could be 1 unit by 4 units, or 2 units by 8 units, or 3 units by 12 units, or 4 units by 16 units.

Individual or paired trial first (left open as voluntary homework if Ps have not time to solve it during the lesson)

If all Ps are struggling, stop individual work and continue as a whole class activity, with T directing Ps’ thinking and involving them whenever possible.

Ps could write agreed solution in Ex. Bks.
**Lesson Plan**

169

### Notes

- Individual work, monitored (or whole class activity)
- BB: 169, 344, 519, 1169
- T decides whether Ps can use a calculator.
- Reversal will be whole class activity if Ps are struggling.
- Reasoning, agreement, self-correction, praising

- Review the questions interactively with the whole class, whether Ps attempted them or not.

---

**Y6**

<table>
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<tr>
<td><strong>E:</strong> Challenges</td>
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#### 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **169** = 13 × 13 = 13²
  - Factors: 1, 13, 169 (square number)
- **344** = 2 × 2 × 2 × 43 = 2³ × 43
  - Factors: 1, 2, 4, 8, 43, 86, 172, 344
- **519** = 3 × 173
  - Factors: 1, 3, 173, 519
- **1169** = 7 × 167
  - Factors: 1, 7, 167, 1169

- **8 min**

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

---

#### 2

**PbY6b, page 169**

Q.1 Read: *Can you pay a bill of £500 using exactly 12 notes which are £5, £20, or £50 notes? Give a reason for your answer.*

Solution: e.g.

Let the number of £5 notes be \( a \), and \( a \) is a whole number and the number of £20 notes be \( b \), and \( b \) is a whole number, then the number of £50 notes is \( 12 - a - b \).

\[
BB: 5 \times a + 20 \times b + 50 \times (12 - a - b) = 500
\]

\[
5a + 20b + 600 - 50a - 50b = 500 \quad [-500]
\]

\[
100 - 45a - 30b = 0 \quad [+45a + 30b]
\]

\[
20 = 9a + 6b \ldots . (1)
\]

Substitute possible values for \( a \) in equation (1).

If \( a = 0 \), \( 6b = 20 \), so \( b = \frac{20}{6} = 3 \frac{1}{3} \) Impossible!

If \( a = 1 \), \( 6b = 20 - 9 = 11 \), so \( b = \frac{11}{6} = 1 \frac{5}{6} \) Impossible!

If \( a = 2 \), \( 6b = 20 - 18 = 2 \), so \( b = \frac{2}{6} = \frac{1}{3} \) Impossible!

\( a \) cannot be greater than 2, as \( 9 \times 3 = 27 \), which is more than 20, so there is no solution to the equation.

**Answer:** It is impossible to pay a bill of £500 using exactly 12 notes which are £5, £20 or £50 notes, because there is no combination of 12 such notes which can make £500.
**Activity**

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</table>

**Q.2** Read: The square base of a solid wooden cuboid has 4 cm edges. The height of the cuboid is 3 cm. The outside of the cuboid is painted red. If the cuboid is cut into 1 cm cubes, how many of the unit cubes will have:

- a) 3 red faces
- b) 2 red faces
- c) 1 red face
- d) no red faces?

**Solution:**

The cuboid is made from $4 \times 4 \times 3 = 48$ cm cubes

- a) 3 red faces: 8 cubes at each vertex
- b) 2 red faces: 20 cubes remaining cubes on edges
- c) 1 red face: 16 cubes middle cubes on each face
- d) no red face: 4 cubes at centre of cuboid

**Check:** 48 cubes

---

| 4   | **PbY6b, page 169** |

**Q.3** Read: We marked the midpoints of the edges of a cube. Then we joined up each point to the next with straight lines and cut the corners off the cube along these lines. The surface of the remaining solid is made up of triangular and square faces.

- a) How many triangles and how many squares make up its surface?
- b) How many vertices and edges does this solid have?
- c) Draw this solid.

**Solution:**

- a) 8 triangles and 6 squares make up its surface.
- b) It has 12 vertices and 24 edges.
- c) [Diagram]

**Extension**

Who remembers the relationship between the number of faces, vertices and edges of a polyhedron? T reminds Ps if necessary and class checks that it is true for this polyhedron.

**BB:** $f + v = e + 2$ [Euler's theorem]

LHS: $(8 + 6) + 12 = 26$

RHS: $24 + 2 = 26$ ✔

---

**Notes**

Individual or paired trial

Less able Ps could have unit cubes on desk to build the solid and/or T has model prepared from unit cubes to confirm Ps' solution

![Diagram](image)

(8 × 2 + 4 × 1)

(4 × 2 + 2 × 4)

---

Individual trial

T could show a model at the start if Ps need some help in imagining what the solid would look like.

(or Ps could make their own rough models from plasticine).

What is this solid called?

(polygon: a 3-D shape with many plane faces)

Extra praise if a P remembers.

Ps could write the theorem in the blank page at back of Pb to help them remember it.
### Activity

**PbY6b, page 169**

Q.4 Read: At Primary school, Peter was asked for a clue about his age. This is what he said.

'The current age of my father can be written with two digits and his age when I was born could be written with the same two digits.'

How old is Peter?

**Solution:**

Let the 2 digits in Peter's father's age be $a$ and $b$.

$ab - ba < 12$ (as Peter is in Primary school)

so $10a + b - (10b + a) < 12$

$9a - 9b < 12$

so $a - b = 1$ (they must be whole numbers)

and $9(a - b) = 9$

**Answer:** Peter is 9 years old.

---

Q.5 Read: There were 25 cars in a car park. There were 3 times as many Renaults as Hondas and twice as many Peugeots as Fords. The Hondas were not the same colour.

How many of each type of car was in the car park?

**Solution:**

Data: 25 cars, $R = 3 \times H$, $P = 2 \times F$

$H \neq 1$ (as there was more than one colour)

If $H = 2$, then $R = 6$, and $F + 2 \times F = 25 - (6 + 2)$

$3 \times F = 17$

So $H \neq 2$

If $H = 3$, then $R = 9$, and $F + 2 \times F = 25 - (9 + 3)$

$3 \times F = 13$

So $H \neq 3$

If $H = 4$, then $R = 12$, and $F + 2 \times F = 25 - (12 + 4)$

$3 \times F = 9$

So $H = 4$, $R = 12$, $P = 6$

$F = 3$, ✓

If $H = 5$, then $R = 15$, and $F + 2 \times F = 25 - (15 + 5)$

$3 \times F = 5$

So $H \neq 5$

If $H = 6$, then $R = 18$, and $F + 2 \times F = 25 - (18 + 6)$

$3 \times F = 1$ Impossible!

$H < 7$, as if $R = 21$, $21 + 7 = 28$ and $28 > 25$

**Answer:** There were 4 Hondas, 12 Renaults, 6 Peugeots and 3 Fords in the car park.

---

**Lesson Plan 169**

**Notes**

Individual trial first

(T gives hint about naming the 2 digits or reverts to a whole class activity if Ps are struggling.)

Individual or paired trial first

H, R, P and F must be whole numbers, as there could not be part of a car.

If Ps are struggling, T leads them through the reasoning for $H = 2$, then Ps try the other possible values by themselves.

Check: $4 + 12 + 6 + 3 = 25$ ✓

All the conditions are fulfilled.

So $H = 4$ is the only possible solution.
**Y6**

**Activity**

7

HMC: Hungarian Mathematics Competition 1981 Age 12

_PbY6b, page 169. Q.6_

Read: Six bars of plain milk chocolate cost the same as 4 bars of fruit and nut chocolate or 5 bars of dark chocolate.

If we buy two bars of each type of chocolate, we are given change from £1.

What is the price of each type of chocolate bar?

Ps decide what to do first and how to continue. T directs Ps' thinking if Ps have no ideas.

**Solution:** e.g.

Let the price of a bar of: plain milk chocolate be \( m \) (pence)

fruit and nut chocolate be \( f \) (pence)

dark chocolate be \( d \) (pence)

\[
6 \times m = 4 \times f, \quad 6 \times m = 5 \times d
\]

so \( f = \frac{6}{4} \times m = \frac{3}{2} \times m \) and \( d = \frac{6}{5} \times m \)

Ratio of \( m : f : d = 1 : \frac{3}{2} : \frac{6}{5} = 10 : 15 : 12 \)

But \( 2 \times m + 2 \times f + 2 \times d < 100 \) \([\div 2]\)

\( m + f + d < 50 \)

So the only possible solution is: \( m = 10, \quad f = 15, \quad d = 12 \)

**Answer:** The price of a plain milk chocolate bar is 10 p, the price of a fruit and nut chocolate bar is 15 p and the price of a dark chocolate bar is 12 p.

**Check:**

\[
6 \times 10 \text{ p} = 60 \text{ p}
\]

\( 4 \times 15 \text{ p} = 60 \text{ p} \)

\( 5 \times 12 \text{ p} = 60 \text{ p} \)

\[
2 \times 10 \text{ p} + 2 \times 15 \text{ p} + 2 \times 12 \text{ p} = 20 \text{ p} + 30 \text{ p} + 24 \text{ p} = 74 \text{ p} < £1 \quad \checkmark
\]

(i.e. the scale factor can be only 1, otherwise the sum would be more than 50)

**Notes**

Whole class activity

(or individual or paired trial first if Ps wish)

**Lesson Plan 169**

**PbY6b, page 169**

Q.7 Read: Draw a square, ABCD, with 2 cm sides.

Draw a point P in the plane of the square so that these isosceles triangles are formed.

\( \text{ABP, BCP, CDP, DAP} \)

Find more than one solution!

Suggest that Ps label the possible points \( P_1, P_2, P_3 \) etc and hint that Ps should use compasses to find them.

When a P has found a point, ask him or her to show it on a digram on BB or SB or OHT and elicit the types of triangles formed.

**Solution:**

\[
\begin{array}{c}
\text{BB:} \\
\text{P}_3 \\
\text{ } \\
\text{ } \\
\text{A} \\
\text{ } \\
\text{B} \\
\text{ } \\
\text{C} \\
\text{ } \\
\text{D} \\
\text{ } \\
\text{P}_1 \\
\text{ } \\
\text{P}_2 \\
\text{ } \\
\text{P}_4 \\
\end{array}
\]

There are 9 possible points.

Individual trial first

Or whole class activity with \( P_1 \) if Ps are struggling. Once \( P_1 \) has been shown, Ps might be able to find the other points.

\( P_1 \) is the centre point, i.e. the intersection of the 2 diagonals \( \text{ABP}_1 \cong \text{BCP}_1 \cong \text{CDP}_1 \cong \text{DAP}_1 \) (right-angled isosceles triangles)

\( P_2 \) is 2 cm from B and 2 cm from C outside the square.

\( \text{ABP}_2 \cong \text{CDP}_2 \) (obtuse-angled)

\( \text{BCP}_2 \) is equilateral

\( \text{DAP}_2 \) is acute-angled

[Similarly for \( P_3, P_4 \) and \( P_1 \)]

\( P_5 \) is 2 cm from A and from D inside the square.

\( \text{ABP}_5 \cong \text{CDP}_5 \) (acute-angled)

\( \text{BCP}_5 \) is obtuse-angled

\( \text{DAP}_5 \) is equilateral

[Similarly for \( P_7, P_8 \) and \( P_1 \)]
Factorising 170, 345, 520 and 1170. Miscellaneous challenges

**PbY6b, page 170**

**Solutions:**

Q.1  
**a) Plan:** £500 \( \times 0.075 = £37.50 \)  
**Answer:** £37.50 interest would be added to the account.

**b) Plan:** £537.50 \( \times 1.075 = £577.8125 = £577.81 \)  
**Answer:** The account would be worth £577.81 at the end of the 2nd year.

**c) At end of 3rd year:** £577.81 \( \times 1.075 \approx £621.15 \)  
**At end of 4th year:** £621.15 \( \times 1.075 \approx £667.74 \)  
**At end of 5th year:** £667.74 \( \times 1.075 \approx £777.82 \)  
**Answer:** Harvey will have to wait 5 years.

Q.2  
**V = 4 \times 2 \times 5 = 40 cm cubes**  
a) **3 blue faces:** 8 cubes (\( \times \))  
b) **2 blue faces:** 20 cubes (\( \bullet \))  
c) **1 blue face:** 12 cubes (\( \Delta \))  
d) **no blue faces:** none

**Check:** 8 + 20 + 12 = 40 (cm cubes) ✔

Q.3  
\[ \begin{align*}  
&\text{2 colours} \quad \text{3 colours} \quad \text{4 colours} \\
&\text{d) } \quad \text{3 colours} \quad \text{4 colours} \quad \text{4 colours}  
\end{align*} \]

Q.4  
\[ \frac{1}{x} + \frac{1}{y} = \frac{1}{8} \quad (= \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40} = \frac{6}{48} = \frac{7}{56} \ldots) \]

**x = y = 16:**  
\[ \frac{1}{16} + \frac{1}{16} = \frac{2}{16} = \frac{1}{8} \]

**x = 12, y = 24:**  
\[ \frac{1}{12} + \frac{1}{24} = \frac{2}{24} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8} \]

**x = 10, y = 40:**  
\[ \frac{1}{10} + \frac{1}{40} = \frac{4}{40} + \frac{1}{40} = \frac{5}{40} = \frac{1}{8} \]

**x = 9, y = 72:**  
\[ \frac{1}{9} + \frac{1}{72} = \frac{8}{72} + \frac{1}{72} = \frac{9}{72} = \frac{1}{8} \]

and of course the values for \( x \) and \( y \) can be exchanged (e.g. \( x = 24, y = 12 \), so there are 7 possible solutions.

**Notes**

170 \( = 2 \times 5 \times 17 \)  
**Factors:** 1, 2, 5, 10, 17, 34, 85, 170

345 \( = 3 \times 5 \times 23 \)  
**Factors:** 1, 3, 5, 15, 23, 69, 115, 345

520 \( = 2^3 \times 5 \times 13 \)  
**Factors:** 1, 2, 4, 5, 8, 10, 13, 20, 26, 40, 52, 65, 104, 130, 260, 520

1170 \( = 2 \times 3^2 \times 5 \times 13 \)  
**Factors:** 1, 2, 3, 5, 6, 9, 10, 13, 15, 18, 26, 30, 39, 45, 65, 78, 90, 117, 130, 195, 234, 390, 585, 1170

(or set factorising as extra task for homework at the end of Lesson 169 and review at the start of Lesson 170.

**Note to T**

Equations such as this which have integer solutions are called  

**Diophantine equations**

after the Greek philosopher and mathematician  

**Diophantos of Alexandria.**

He lived in the 3rd century A.D. and is credited with being the founder of modern algebra. The use of symbols to represent numbers was found in his published material entitled *Arithmetic.*]
## Activity

**Solutions (continued):**

Q.5  

a)  

i) \(5008 473c\)

\[
3 \times (5 + 0 + 4 + 3) + 1 \times (0 + 8 + 7 + c) \\
= 3 \times 12 + 1 \times 15 + c \\
= 36 + 15 + c \\
= 51 + c \\
\]

\[51 + 9 = 60 \text{ (next greater whole 10)} \rightarrow c = 9\]

ii) \(5120 173c\)

\[
3 \times (5 + 2 + 1 + 3) + 1 \times (1 + 0 + 7 + c) \\
= 3 \times 11 + 1 \times 8 + c \\
= 33 + 8 + c \\
= 41 + c \\
\]

\[41 + 9 = 50 \text{ (next greater whole 10)} \rightarrow c = 9\]

iii) \(8300 720c\)

\[
3 \times (8 + 0 + 7 + 0) + 1 \times (3 + 0 + 2 + c) \\
= 3 \times 15 + 1 \times 5 + c \\
= 45 + 5 + c \\
= 50 + c \\
\]

\[50 + 0 = 50 \rightarrow c = 0\]

b) \(5070 4827\)

\[
3 \times (5 + 7 + 4 + 2) + 1 \times (0 + 0 + 8 + 7) \\
= 3 \times 18 + 1 \times 8 + 7 \\
= 54 + 8 + 7 \\
= 69 \neq 70 \\
\]

Therefore one of the numbers in the bracket on the RHS could have been read incorrectly as 1 less than it should be.

As 1 does not look like 0, it is more likely that 9 has been read as 8, so the correct number could have been \(5070 4927\).

However, another possibility is \(5070 4828\), if the check digit had been read incorrectly.
### Y6

#### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- **171** = \(3 \times 3 \times 19\)
  
  Factors: 1, 3, 9, 19, 57, 171
  
- **346** = \(2 \times 173\)
  
  Factors: 1, 2, 173, 346
  
- **521** is a prime number
  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and \(23^2 > 521\))
  
- **1171** is a prime number
  
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and \(37^2 > 1171\))

\[8 \text{ min}\]

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

#### Notes

- **Lesson Plan 171**

Individual work, monitored (or whole class activity)

BB: 171, 346, 521, 1171

Ps can use a calculator.

Reasoning, agreement, self-correction, praising

- **171** | 3 | 346 | 2
- **57** | 3 | 173 | 173
- **19** | 19 | 1

Review the questions interactively with the whole class, whether Ps attempted them or not.

- **PbY6b, page 171**

Q.1 Read: *We want to assign the numbers 1, 2, 3, 4, 5, 6, 7 and 8 to the vertices of a cube so that the sums of the two numbers on each edge are all different.*

*Is this possible? Give a reason for your answer.*

Ask Ps to write down anything they find out. If any Ps think that they have a solution, ask them to show it on diagram on BB, then class checks it and points out the equal sums.

**Solution:**

Points Ps might have noticed or T could ask about:

- 28 different pairs can be formed, giving
- 13 different sums from 3 to 15
- A cube has 8 vertices and 12 edges, so it seems possible to assign 8 different numbers to give 12 different sums.
- The sum of all the 12 sums must be equal to:
  
  \(3 \times (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 3 \times 36 = 108\)
  
  [Each number can be paired with 3 other numbers because 3 edges of the cube meet at each vertex]
  
  or using the names of the vertices:
  
  \[
  (A + B) + (B + C) + (C + D) + (D + A) + (A + E) + (B + F) + (C + G) + (D + H) + (E + F) + (F + G) + (G + H) + (E + H)
  \]
  
  \[= 3A + 3B + 3C + 3D + 3E + 3F + 3G + 3H\]
  
  \[= 3 \times (A + B + C + D + E + F + G + H)\]
  
- The sum of the 13 different possible sums is
  
  \[3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = 117\]

28 possible pairs:

- \((1 + 2), (1 + 3), (1 + 4), (1 + 5)\)
- \((1 + 6), (1 + 7), (1 + 8), (2 + 3)\)
- \((2 + 4), (2 + 5), (2 + 6), (2 + 7)\)
- \((2 + 8), (3 + 4), (3 + 5), (3 + 6)\)
- \((3 + 7), (3 + 8), (4 + 5), (4 + 6)\)
- \((4 + 7), (4 + 8), (5 + 6), (5 + 7)\)
- \((5 + 8), (6 + 7), (6 + 8), (7 + 8)\)

- As 117 is 9 more than 108, the sum of 9 can be left out, so we do not use these pairs:
  
  \((1 + 8), (2 + 7), (3 + 6), (4 + 5)\)
Y6

Activity

2

(Continued)

T then helps Ps to reason the rest of the solution, involving them where possible.

Since the 12 remaining sums must each be used once only, we could use, e.g.

\[ \begin{align*}
3 &= 1 + 2, & 4 &= 1 + 3, & 5 &= 1 + 4 \\
[5] &= 2 + 3 \text{ is impossible, as 2 and 3 are not on the same edge.} \\
\end{align*} \]

Then we could use \[ 6 = 1 + 5 \]

\[ 6 = 2 + 4 \text{ is impossible, as 2 and 4 are not on the same edge.} \]

but \( 1 + 5 \) is impossible too, as there are only 3 edges meeting at vertex 1, and we have used them all already, so we cannot use 6 as a sum, and the task is impossible.

Answer: e.g.

It is impossible to assign the numbers 1 to 8 to the vertices of a cube so that the sums of the numbers on each edge are all different because at least one of the 12 required sums cannot occur.

3

HMC: Hungarian Mathematics Competition
1987
Age 11

PbY6b, page 171

Q.2 Read: The numbers 1, 2, 3, . . . , 10 and 11 were each written on a small piece of paper. The pieces of paper were mixed up and put into two boxes. Adam added the numbers in one box and Becky added the numbers in the other box. Becky said, "Isn't it interesting? The sum of my numbers is exactly six times the sum of Adam's numbers."

Adam said, "I think there must be a mistake in our calculations."

Is Adam correct? Give a reason for your answer.

Solution:

The total sum of the 11 numbers is: \( \frac{1 + 11}{2} \times 11 = 66 \)

(or \( 5 \times 12 + 6 = 66 \), as: 1 2 3 4 5 6 7 8 9 10 11

and \( 1 + 11 = 2 + 10 = 3 + 9 = 4 + 8 = 5 + 7 = 12 \))

Let Adam's numbers sum to \( a \), and Becky's numbers sum to \( b \).

If \( a = 6 \times b \), then \( 6 \times b + b = 66 \)

\( 7 \times b = 66 \)

but 66 is not exactly divisible by 7, so there must be a mistake in their calculations.

Answer: Adam is correct, there is a mistake, because 6 times his sum added to Becky's sum does not result in the sum of all the eleven numbers.

Individual trials first

If no P has solved it in a given time (e.g. 5 minutes) continue as a whole class activity

Praise any Ps who managed a positive step (e.g. calculating the total sum of the numbers or calling the unknown sums by letters)

If necessary, T gives hints or directs Ps' thinking, involving them whenever possible.
**Activity**

<table>
<thead>
<tr>
<th>Y6</th>
<th>Lesson Plan 171</th>
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<tr>
<td>4</td>
<td></td>
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</tbody>
</table>

**PbY6b, page 171**

**Q.3** Read: **Write the numbers from 1 to 12 in the two concentric circles so that:**

- each inner number is even
- the sum of the outer numbers is twice the sum of the inner numbers.

**Solution:**

The sum of all 12 numbers is: \( \frac{1 + 12}{2} \times 12 = 78 \) (or \( 13 \times 6 \))

Ratio of sums: Inner : Outer = 1 : 2

so Inner sum is \( \frac{1}{3} \) of 78 = 26 and Outer sum is \( 26 \times 2 = 52 \)

Possible arrangements (but the numbers in each ring can be in any order)

<table>
<thead>
<tr>
<th>BB:</th>
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<tbody>
<tr>
<td>9</td>
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<tr>
<td>3</td>
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<tr>
<td>12</td>
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<tr>
<td>2</td>
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<tr>
<td>6</td>
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<tr>
<td>8</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>12</td>
</tr>
</tbody>
</table>

**Notes**

Individual trial, monitored

Drawn on BB or SB or OHT

Elicit that concentric circles share a common centre point.

Accept trial and error but give extra praise for logical reasoning

If necessary, T gives hint about calculating the sum of the 12 numbers first.

**Inner ring:** 2, 4, 8, 12

or 2, 6, 8, 10

**Outer ring:** remaining numbers

---

**Q.4** Read: **The members of a club rented a room for their meeting.**

Ten members attended the meeting and they each paid the same amount towards the hire of the room. If another five members of the club had attended the meeting, everyone would each have paid £10 less.

How much did it cost to hire the meeting room?

**Solution:** e.g.

Let the amount that each of the 10 members paid be \( x \) (in £s) then the total amount paid would be \( 10 \times x \).

If there were 15 members, they would each pay \( x - 10 \) and the total amount paid would be \( 15 \times (x - 10) = 15x - 150 \).

As the room hire cost would be the same in both cases, then:

\[
10x = 15x - 150 \quad [+ 150] \\
10x + 150 = 15x \quad [- 10x] \\
150 = 5x \quad [\div 5] \\
30 = x \\
\]

and \( 10 \times 30 = £300 \)

**Answer:** The cost of the room was £300.

**Check:**

\[
15 \times (£30 - £10) = £300 \checkmark
\]

---

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### Activity

**6**  

**HMC:** Hungarian Mathematics Competition 1998  
**Age 11**

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#### Y6 Lesson Plan 171

**Notes**

- Individual trial
- (left open as optional homework if Ps do not have time to solve it during the lesson)
- Accept any valid method of solution.

---

#### Q5 Read:

**Ben picked some apples from his apple tree and put them in a box in his garage.**

*That day, Ben made an apple pie with one third of the apples in the box.*

*The next day he ate one third of the remaining apples and on the following day he gave one third of what was left to his neighbour.*

*If 8 apples were left in the box, how many apples did Ben pick from the tree?*

**Solution:** e.g.

Let the number of apples Ben picked be $n$.

- Apples left after 1st day: $\frac{2}{3} \times n$
- Apples left after 2nd day: $\frac{2}{3} \times (\frac{2}{3} \times n)$
- Apples left after 3rd day: $\frac{2}{3} \times (\frac{2}{3} \times \frac{2}{3} \times n) = \frac{8}{27} \times n = 8$

or starting on the 3rd day with the 8 apples left:

- Apples at beginning of 3rd day: $8 \div 2 \times 3 = 12$
- Apples at beginning of 2nd day: $12 \div 2 \times 3 = 18$
- Apples picked on 1st day: $18 \div 2 \times 3 = 27$

**Answer:** Ben picked 27 apples from the tree.
**Lesson Plan**

**Y6**

**R:** Calculations  
**C:** Problems  
**E:** Puzzles and challenges

### Activity 1

**Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Elicit that:

- \(172 = 2 \times 2 \times 43 = 2^2 \times 43\)  
  Factors: 1, 2, 4, 43, 86, 172

- 347 is a prime number  
  Factors: 1, 347
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and \(19^2 > 347\))

- \(522 = 2 \times 3 \times 3 \times 29 = 2 \times 3^2 \times 29\)  
  Factors: 1, 2, 3, 6, 9, 18, 29, 58, 87, 174, 261, 522

- \(1172 = 2 \times 2 \times 293 = 2^2 \times 293\)  
  Factors: 1, 2, 4, 293, 586, 1172

---

8 min

**Notes**

Individual work, monitored (or whole class activity)

BB: 172, 347, 522, 1172

T decides whether Ps can use a calculator.

Reasoning, agreement, self-correction, praising

Review the questions interactively with the whole class, whether Ps attempted them or not.

### Individual trial

**PbY6b, page 172**

Q.1 Read: The sides of a square are each divided into 4 equal parts. Some of the points are joined up as shown in the diagram. What part of the area of the whole square is the area of the shaded part?

T could suggest that Ps trace or redraw the square, cut it out and rearrange the pieces if no P has thought of doing it.

**Solution:**

If the square is cut into 3 pieces, as shown, the pieces can be rearranged to give the shape on the RHS.

BB:

The shape on the RHS is made from 17 unit squares, so this must also be the area of the first square.

In the RHS diagram, it can be seen that the shaded area makes 2 unit squares, so is \(\frac{2}{17}\) of the area of the shape.

As the shaded area is the same in both diagrams, it must also be \(\frac{2}{17}\) of the original square.

**Answer:** The shaded part is 2 seventeenths of the area of the square.
**Activity**

**PbY6b, page 172**

**Q.2** Read: *What is the sum of the shaded angles? Explain how you worked out the solution.*

If Ps are struggling, T could suggest labelling the vertices and points of intersection and hint about the sum of the angles in a triangle.

**Solution:**

Label the diagram as shown.

The sum of the angles in a triangle is 180°.

In ΔACQ, \( \triangle AQC = 180° -(\angle A + \angle C) \),

so \( \triangle DQP = 180° - [180° -(\angle A + \angle C)] \) (as AD is a straight line)

In ΔBPE, \( \triangle BPE = 180° -(\angle B + \angle E) \),

so \( \triangle QPD = 180° - [180° -(\angle B + \angle E)] \) (as BD is a straight line)

In ΔQPD, \( \triangle Q + \triangle P + \triangle D = 180° \)

i.e. \( (\angle A + \angle C) + (\angle B + \angle E) + \angle D = 180° \)

Or T gives instructions and Ps follow these practical steps.

1. Lay a pencil along AD, pointing from A to D.
2. Rotate the pencil through \( \angle A \) so that it lies along AC.
3. Rotate the pencil through \( \angle C \) so that it lies along EC.
4. Rotate the pencil through \( \angle E \) so that it lies along EB.
5. Rotate the pencil through \( \angle B \) so that it lies along DB.
6. Rotate the pencil through \( \angle D \) so that it lies along DA.

The pencil is now back in its original position but facing in the opposite direction, so it has turned through an angle of 180°.

i.e. \( \angle A + \angle B + \angle C + \angle D + \angle E = 180° \)

**Notes**

Individual or paired trial

Drawn on BB or use enlarged copy master or OHP

Remind Ps of the different notation for identifying certain angles.

If no P can solve it, T directs Ps’ thinking, involving Ps when they understand.

Less able Ps might find this practical method easier.

**PbY6b, page 172**

**Q.3** Read: *We say that two circles touch each other if they have exactly one common point.

How many circles which touch each of the 3 circles in the diagram can you imagine in the plane?*

Ps could draw the circles roughly and lightly in pencil.

**Solution:**

There are 8 such circles.

[It is clearer to show the circles on different diagrams.]
**Activity**

5

HMC:
Hungarian Mathematics Competition
1986
Age 12

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**Notes**

Individual or paired trial

If nobody can solve it, T gives hints or directs Ps’ thinking to the solution, involving Ps where possible.

(e.g. If only 2 dots were on the same line, how many lines would be drawn? If we have drawn fewer lines, what does that mean? etc.)

---

**Lesson Plan 172**

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**Activity**

6

HMC:
Hungarian Mathematics Competition
1990
Age 12

---

**Notes**

Individual or paired trials first

If Ps are struggling, change to a whole class activity, with T giving hints or directing Ps’ thinking and involving Ps where possible.

**Check:** (Ps dictate)

£195

= £5 × 1 + £2 × 5 × 19

= £5 × 3 + £2 × 5 × 18

= £5 × 5 + £2 × 5 × 17

= £5 × 7 + £2 × 5 × 16

= £5 × 9 + £2 × 5 × 15

= £5 × 11 + £2 × 5 × 14

= £5 × 13 + £2 × 5 × 13

= £5 × 15 + £2 × 5 × 12

= £5 × 17 + £2 × 5 × 11

= £5 × 19 + £2 × 5 × 10

= £5 × 21 + £2 × 5 × 9

= £5 × 23 + £2 × 5 × 8

= £5 × 25 + £2 × 5 × 7

= £5 × 27 + £2 × 5 × 6

= £5 × 29 + £2 × 5 × 5

= £5 × 31 + £2 × 5 × 4

= £5 × 33 + £2 × 5 × 3

= £5 × 35 + £2 × 5 × 2

= £5 × 37 + £2 × 5

= £5 × 39         (20 ways)
### Activity

<table>
<thead>
<tr>
<th>Y6</th>
<th>Lesson Plan 172</th>
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</thead>
<tbody>
<tr>
<td><strong>PbY6b, page 172, Q.6</strong></td>
<td><strong>Notes</strong></td>
</tr>
</tbody>
</table>
| Read: *What is the smallest, positive, whole number which gives:* | **Whole class activity**
|   • a remainder of 1 when it is divided by 3 | (or individual trial if Ps wish) |
|   • a remainder of 2 when it is divided by 4 | Allow Ps time to think and to |
|   • a remainder of 3 when it is divided by 5 | discuss the method of solution. |
|   • a remainder of 4 when it is divided by 6? | If no P has a good idea, T |
| **Solution:** e.g. | gives hints or leads Ps through |
|   In each case, the remainder is 2 less than the divisor, so the | the reasoning and asks Ps |
|   dividend must be 2 less than the lowest common multiple of | to check it. |
|   3, 4, 5 and 6. | **Check:** 58 \(\div\) 3 = 19, r \(\underline{1}\) ✓
| As 4 = 2 \(\times\) 2 and 6 = 2 \(\times\) 3, the lowest common multiple | 58 \(\div\) 4 = 14, r \(\underline{2}\) ✓
|   of 3, 4, 5 and 6 is: | 58 \(\div\) 5 = 11, r \(\underline{3}\) ✓
|   \(2 \times 2 \times 3 \times 5 = 60.\) | 58 \(\div\) 6 = 9, r \(\underline{4}\) ✓
| **Answer:** The smallest positive whole number which fulfils the | [or set this question as an |
|   given conditions is 60 – 2 = 58. | optional homework challenge] |
**Activity**

1. **Factorisation**

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:
- **173** is a prime number
  - Factors: 1, 173
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, and $17^2 > 173$)
- **348** = $2 \times 2 \times 3 \times 29 = 2^2 \times 3 \times 29$
  - Factors: 1, 2, 3, 4, 6, 12, 29, 58, 87, 116, 174, 348
- **523** is a prime number
  - Factors: 1, 523
  - (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and $23^2 > 523$)
- **1173** = $3 \times 17 \times 23$
  - Factors: 1, 3, 17, 23, 51, 69, 391, 1173

8 min

2. **PbY6b, page 173**

Q.1 Read: A father gave £400 to his son. Another father gave £200 to his son. The two sons count their money and notice that they have £400 altogether. How is that possible?

I will give you 3 minutes to think about it and to write an explanation in your Ex. Bks. Start . . . now! . . . Stop!

Stand up if you think it is possible. T chooses Ps sitting and standing to explain their reasoning. Class decides who is correct.

Solution: e.g. BB: Grandad $\rightarrow$ Dad $\rightarrow$ Son

Grandad gave £400 to Dad who gave £200 from the £400 to his son, so the Dad and the Son had £200 each, or £400 altogether.

12 min

3. **PbY6b, page 173**

Q.2 Read: A joiner worked on his own to mend the 4 legs of a large, heavy table. The table was lying top down on the floor. When he had mended the table, the joiner was not strong enough to lift it onto its legs.

He thought of a way of checking whether the table would be stable when it was the right way up by using two pieces of string.

a) How could he have done it?

b) If the table had 3 legs, would it need to be checked in the same way?

Solution: e.g.

a) Pin each piece of string to the tops of two opposite legs. If the 2 strings touch each other, the tops of the legs are exactly the same length, so the table is stable. (See diagram)

b) A table with 3 legs is always stable (but of course its top will not necessarily be horizontal). [T demonstrates if possible.]

16 min

**Notes**

Individual work, monitored (or whole class activity)

BB: 173, 348 523, 1173

T decides whether Ps can use a calculator.

Reasoning, agreement, self-correction, praising

<table>
<thead>
<tr>
<th>348</th>
<th>2</th>
<th>174</th>
<th>2</th>
<th>1173</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>87</td>
<td>3</td>
<td>391</td>
<td>17</td>
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<td>29</td>
<td>29</td>
<td>23</td>
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<td>1</td>
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</table>

Individual work, monitored

T notes Ps who have explained their reasoning clearly.

In unison

Reasoning, agreement, self-correction, praising

Individual or paired trial for 2 or 3 minutes.

Elicit that 'stable' means the table does not move or rock.

Ps who have an answer explain reasoning to class. T helps them to explain. e.g.

BB:

If the 2 joining lines touch, the 4 points are in the same plane.

Any 3 different points which are not on the same line can always be on the same plane.
### Activity

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

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<tbody>
<tr>
<td><strong>PbY6b, page 173, Q.3</strong></td>
</tr>
<tr>
<td><strong>Read:</strong> We divided two numbers, 313 and 390, by the same 2-digit number. In each case the remainder was the same. Which number could we have divided by?</td>
</tr>
<tr>
<td><strong>Solution:</strong> e.g.</td>
</tr>
<tr>
<td>Let the divisor be ( d ) and the remainder be ( m ).</td>
</tr>
<tr>
<td>Then both ((313 - m)) and ((390 - m)) are exactly divisible by ( d ).</td>
</tr>
<tr>
<td>If ((313 - m)) and ((390 - m)) are exactly divisible by ( d ), their difference is also exactly divisible by ( d ).</td>
</tr>
<tr>
<td>BB: ((390 - m) - (313 - m) = 390 - m - 313 + m = 390 - 313 = 77)</td>
</tr>
<tr>
<td>If 77 is exactly divisible by ( d ), then ( d ) must be a factor of 77, i.e. 1, 7, 11 or 77, but the divisor is a 2-digit number, so only 11 and 77 are possible. Let's check both of them.</td>
</tr>
<tr>
<td>If ( d = 11): (313 \div 11 = 28, \text{ r } 5), (390 \div 11 = 35, \text{ r } 5) ✅</td>
</tr>
<tr>
<td>If ( d = 77): (313 \div 77 = 4, \text{ r } 5), (390 \div 77 = 5, \text{ r } 5) ✅</td>
</tr>
<tr>
<td><strong>Answer:</strong> The number that we divided by could have been 11 or 77.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5</th>
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<tbody>
<tr>
<td><strong>PbY6b, page 173</strong></td>
</tr>
<tr>
<td>Q.4 <strong>Read:</strong> Once a time, a king asked a farmer to work for him for a year and promised to pay him 12 gold coins and a horse. The farmer did not like the work he had to do in the palace and longed to be back in his farm. After 7 months he decided to leave his job and asked the king for his wages. The king gave the farmer a horse and 2 gold coins, which the farmer agreed was fair. How many gold coins was the horse worth?</td>
</tr>
<tr>
<td><strong>Solution:</strong> e.g.</td>
</tr>
<tr>
<td>12 months (\rightarrow) 12 gold coins + 1 horse</td>
</tr>
<tr>
<td>7 months (\rightarrow) 2 gold coins + 1 horse</td>
</tr>
<tr>
<td>5 months (\rightarrow) 10 gold coins</td>
</tr>
<tr>
<td>1 month (\rightarrow) 2 gold coins</td>
</tr>
<tr>
<td>So after 12 months he should have received 24 gold coins but he was promised only 12 gold coins + 1 horse, so the horse must have been worth 12 gold coins.</td>
</tr>
<tr>
<td><strong>Check:</strong></td>
</tr>
<tr>
<td>7 months: 2 coins + 1 horse = 2 coins + 12 coins = 14 coins</td>
</tr>
<tr>
<td>2 coins (\times) 7 = 14 coins ✅</td>
</tr>
</tbody>
</table>
**PbY6b, page 173, Q.5** Read: How can this rectangle be cut into two pieces so that the two pieces will form a square?

**Solution:**

The square has sides 12 cm long and its area is 144 cm².

**PbY6b, page 173, Q.6**

Read: The sides of an equilateral triangle were divided into 3 equal parts. Some points were joined up to form another equilateral triangle, as shown in the diagram.

What part of the area of the original triangle is the area of the smaller equilateral triangle?

**Solution:**

Let the sides of the larger equilateral triangle be 1 unit.

Then the shaded right-angled triangle has height \( h \), base \( \frac{1}{3} \) unit and hypotenuse \( \frac{2}{3} \) of a unit, but so have the other 2 right-angled triangles, so all 3 right-angled triangles are congruent.

The area of each right-angled triangle is

\[
\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}
\]

of the area of the large equilateral triangle

(as its base is \( \frac{1}{3} \) of the base of the large triangle and its height, \( h \), is \( \frac{2}{3} \) of the height of the large triangle – see diagram)

So the area of the small equilateral triangle is:

\[
1 - \frac{2}{9} \times \frac{1}{3} = 1 - \frac{2}{3} = \frac{1}{3}
\]

**Answer:** The area of the smaller equilateral triangle is one third of the area of the original triangle.
R: Calculations
C: Problems
E: Puzzles and challenges

Lesson Plan

174

Notes

Individual work, monitored (or whole class activity)
BB: 174, 349, 524, 1174
T decides whether Ps can use a calculator.
Reasoning, agreement, self-correction, praising

Individual or paired trial
Allow time for Ps to think and discuss with their neighbours.

Ps who think it is possible stand up and command. T asks several Ps to explain their reasoning to class.

[If there is time, demonstrate the solution with a group of Ps moving from one side of the classroom to the other.]

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- \(174 = 2 \times 3 \times 29\) Factors: 1, 2, 3, 6, 29, 58, 87, 174
- \(349\) is a prime number Factors: 1, 349 (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and \(19^2 > 349\))
- \(524 = 2 \times 2 \times 131 = 2^2 \times 131\) Factors: 1, 2, 4, 131, 262, 524
- \(1174 = 2 \times 587\) Factors: 1, 2, 587, 1174 (587 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23 and \(29^2 > 587\))

Review the questions interactively with the whole class, whether Ps attempted them or not.

2

PbY6b, page 174

Q.1 Read:

A small group of soldiers need to cross a river but the bridge has been destroyed.

The river is very deep and its current is so swift that it is too dangerous for the soldiers to swim across.

Two children are playing in a boat on the river bank.

This boat is so small that only the two children or a single soldier can fit inside it.

Is it possible for the group of soldiers to cross the river using the boat?

Give a reason for your answer.

Solution:

Yes, it is possible if these steps are used.

The 2 children row across the river. One child stays on the opposite bank and the other child rows the boat back.

A soldier rows to the opposite bank and the child who was left there brings the boat back.

The 2 children row across the river. One child stays on the ...

Continue in this way until all the soldiers are across the river.
Activity

3  PbY6b, page 174
Q.2  Read:  
   a) Using 24 matchsticks of equal length, form 4 squares with 1 unit sides and 3 squares with 2 unit sides.
   b) Form 6 equilateral triangles from 12 matchsticks of equal length.

Solution:
   a)  
      4 squares: 1 x 1
      3 squares: 2 x 2
   b)  regular hexagon

4  PbY6b, page 174
Q.3  Read:  Change the position of only 2 matchsticks so that there are 5 triangles.

Solution:
   \[ \rightarrow \]

5  PbY6b, page 174
Q.4  Read:  Complete the diagram so that the sum of every two adjacent numbers is the number directly above them.

Solution:

\[
\begin{array}{cccc}
848 & 358 & 490 \\
222 & 136 & 354 \\
154 & 68 & 68 & 286 \\
134 & 20 & 48 & 20 & 266 \\
\end{array}
\]

Reasoning:  e.g.
In bottom row, let the number between 134 and 48 be \( x \).
Then  \( 134 + x + 48 + x = 222 \)
   \[ 182 + 2x = 222 \]  \([−182]\]
   \[ 2x = 40 \]  \( [÷ 2] \]
   \[ x = 20 \]
In bottom row, let the number between 48 and 266 be \( y \).
Then  \( 48 + y + 266 + y = 354 \)
   \[ 314 + 2y = 354 \]  \([−314]\]
   \[ 2y = 40 \]  \( [÷ 2] \]
   \[ y = 20 \]

Notes

Individual trials
Ps have used matchsticks or cocktail sticks or Cuisennaire rods on desks.
If possible, T has large models to stick on BB (or Ps could lay matchsticks on an OHP) to demonstrate the solutions to the class.

Individual trials
Ps make the shape on desks then try out changes.

T has large models to stick on BB, or uses an OHP.

Individual trial
Drawn on BB or use enlarged copy master or OHP

Bold numbers are given.
T helps Ps to explain their reasoning in a mathematical way.

Check:
\[
222 + 136 + 354 + 136 = 538 + 490 = 848 \checkmark
\]
<table>
<thead>
<tr>
<th>Activity</th>
<th>Lesson Plan 174</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y6</td>
<td>Notes</td>
</tr>
</tbody>
</table>
| **6**    | Individual trial
|
| **Q.5** Read: A farm goose saw a flock of wild geese land on his pond. The farm goose said, "There must be a hundred geese in your flock!"
|
| One of the wild geese overheard him and said, "There aren't one hundred of us but if there were twice as many of us, then another half of us, then another quarter of us and if you joined our flock, then there would be a hundred geese in our flock."
|
| How many wild geese landed on the pond? |
| **Solution:** |
| Let the number of wild geese be \( g \).
|
| Then \( 2 \times g + \frac{g}{2} + \frac{g}{4} + 1 = 100 \) \ [\times 4]  
| \( 8 \times g + 2 \times g + g + 4 = 400 \) \ [-4]  
| \( 11 \times g = 396 \) \ [+11]  
| \( g = 36 \)  
| **Answer:** Thirty-six geese landed on the pond. |

| **7**    | Individual or paired trial  
| **Q.6** Read: We have 30 silver coins. Although they all look the same, we know that one of the coins is fake and is lighter than the others.  
| If we tried to find out which coin is fake using a 2-pan balance, what is the least number of weighings we would need to do? |
| **Solution:** |
| 1) Divide the 30 coins into 3 groups of 10.  
| Weigh Group 1 against Group 2. If they balance, the fake coin must be in Group 3. If they do not balance, the fake coin is in the lighter group.  
| 2) Divide the 10 coins in the lightest group into 3 groups (3, 4, 4).  
| Weigh the two groups of 4. If they balance, the fake coin must be in the group of 3. If they do not balance, the fake coin must be in the lighter group of 4.  
| 3) If the fake coin is in the group of 3, weigh one coin against another coin. If they balance, the 3rd coin is fake. If they do not balance, the lighter coin is the fake.  
| or If the fake coin is in a group of 4, weigh 2 coins against the other 2 coins. The fake coin is in the lighter pair.  
| 4) Then weigh each coin in the lighter pair against the other.  
| The fake coin is the lighter of the two.  
| **Answer:** To find the fake coin we would need to do at least 3 weighings and at most 4 weighings. |

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Solution:
First factorise 32 118, then write the possible values for length, age and number of children in a table. e.g.

\[
BB: L \times A \times N = 32118 \quad (= 2 \times 3 \times 53 \times 101)
\]

<table>
<thead>
<tr>
<th></th>
<th>Length (m)</th>
<th>Age of the captain</th>
<th>Number of children</th>
</tr>
</thead>
<tbody>
<tr>
<td>32118</td>
<td>2</td>
<td>53</td>
<td>6</td>
</tr>
<tr>
<td>16059</td>
<td>3</td>
<td>101</td>
<td>6</td>
</tr>
<tr>
<td>5353</td>
<td>53</td>
<td>53</td>
<td>3</td>
</tr>
<tr>
<td>101</td>
<td>101</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The length of the ship is unlikely to be 1 m or 2 m or 3 m (and the age of the captain cannot be 1, 2, or 3 years) so write the other possible lengths and ages in a table.

The captain cannot be 101 years old (the Navy would have retired him by then) and the length of the ship is unlikely to be 202 m (although it is not impossible!), so the circled column is the most likely answer.

Answer: The captain of the ship is 53 years old.
Factorising 175, 350, 525 and 1175. Miscellaneous challenges  
*PbY6b, page 175*

**Solutions:**

Q.1  
**a)** \(50 < n < 60\)  
\(n\): \(51\) (treble 17), \(54\) (treble 18), \(57\) (treble 19)  
**b)**  
\(i\) highest possible score is \(180\) (\(3 \times\) treble 20)  
\(ii\) lowest possible score is \(3\) (\(3 \times 1\))  
\(c\) 163  

Q.2  
![Diagram of a dartboard with numbers 1 to 7]  

Q.3  
**a)**  
\(i\) 2 rectangles \((1 \times 6, 3 \times 2)\)  
\(ii\) 3 rectangles \((1 \times 12, 2 \times 6, 3 \times 4)\)  
\(iii\) 2 rectangles \((1 \times 22, 2 \times 11)\)  
\(iv\) the number of pairs of factors of \(2 \times n\)  
(as the area of each domino is 2 unit squares)  

**b)** \(5 \times 6\) (unit squares) e.g.  
This has no fault line.  

Q.4  
**a)** 2 dots \(\rightarrow\) 4 different codes \((2 \times 2)\)  
**b)** 3 dots \(\rightarrow\) 8 different codes \((2 \times 2 \times 2)\)  
**c)** 4 dots \(\rightarrow\) 16 different codes \((2 \times 2 \times 2)\)  
**d)** 6 dots \(\rightarrow\) 64 different codes \((2 \times 2 \times 2 \times 2 \times 2)\)  
Point out that in real life when using the 6-dot code of Braille, the arrangement of 6 flat dots cannot be felt, so is not used.  
There are only 63 different codes in Braille.