Y6

R: Calculations

C: Simple equations and inequalities

E: Simple formulae (in words)

Lesson Plan 141

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $\underline{141} = 3 \times 47$ Factors: 1, 3, 47, 141
- $\underline{316} = 2 \times 2 \times 79 = 2^2 \times 79$
  Factors: 1, 2, 4, 79, 158, 316
- $\underline{491}$ is a prime number Factors: 1, 491
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and $23^2 > 491$)
- $\underline{1141} = 7 \times 163$ Factors: 1, 7, 163, 1141

Individual work, monitored (or whole class activity)

BB: 141, 316, 491, 1141

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

141	3
47	47
1	

316	2
158	2
79	79
1	

1141	7
163	163
1	

8 min

2

Sets

a) Who can tell me a <u>true</u> statement about all the shapes in this base set?

BB: B = {

}

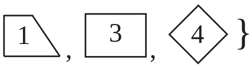
e.g. (They are quadrilaterals.)

For which of the shapes in Set B is this statement is true?

BB: *The quadrilaterals have at least one pair of perpendicular sides.*

T points to each shape in turn and Ps say 'True' or 'False'. Let's call this true subset T. Ps dictate what T should write.

BB: T = {



}

b) Here is a base set of certain integers.

BB: B = {– 5, – 4, – 3, – 2, – 1, 0, 1, 2, 3}

For which of these numbers is this statement true?

BB: $2 - x > 3$

T points to each number in turn and Ps say whether or not it should be in the true subset and why. Class agrees/disagrees.

BB: T = {– 5, – 4, – 3, – 2}

c) This time, the base set is all the <u>even</u> numbers,

BB: B = {Even numbers}

and the true statement is: $6 + x = 9$

Elicit that no <u>even</u> number can make the statement true.

We say that the true set for this statement, based on the given base set of even numbers, is an <u>empty</u> set and write it like this.

d) BB: B = {Positive numbers}

True statement: $2 + y > 0$

Elicit that <u>all</u> the numbers in the base set make the statement true.

The inequality is always true for all the numbers in the base set, so we can write: BB: T = B = {Positive numbers}

Whole class activity

Drawn (stuck) on BB or SB or OHT (or use enlarged copy master)

Agreement, praising

Class shouts out in unison.

Agreement, praising

Ps mark the right angles.

Ps write details on BB. e.g.

$2 - (-5) = 2 + 5 = 7 > 3$ (T)

. . .

$2 - (-1) = 2 + 1 = 3 \not> 3$ (F)

etc.

Ask several Ps what they think. Extra praise if a P notices that

$x = 9 - 6 = \underline{3}$,

but 3 is an <u>odd</u> number so is not in Set B.

BB: T = {∅}

i.e. The true set <u>is</u> the base set.

15 min

Y6

Lesson Plan 141

Activity

Notes

3

PbY6b, page 141

Q.1 Read: *The base set (B) of numbers for this question is:*

B = {– 3, – 2, – 1, 0, 1, 2, 3, 4, 5}

Which of these numbers can be used instead of the letters to make the statements true?

Individual work, monitored, helped

Base set written on BB or SB or OHT

Set a time limit. Ask Ps to write their answers in the form:

T = { . . . }

Differentiation by time limit.

Review with whole class. Ps come to BB to explain reasoning and to write the true set. Class agrees/disagrees. Mistakes discussed and corrected. T points out that the 'true set' is the <u>solution</u> to the equation or inequality.

Reasoning, agreement, self-correction, praising

Solution:

a) $5.4 + \square = 3.4$, $\square = 3.4 - 5.4 = \underline{-2}$ $\underline{T = \{-2\}}$

Extension

If the base set was <u>all</u> the numbers you know, what would the true set be? (It would still be T = {– 2}).

If the base set was only positive numbers, what would the true set be?

Elicit that the true set would be empty: T = {∅}
as no positive number can make the statement true.

b) $-3 + x = 4$, $x = 4 - (-3) = 4 + 3 = \underline{7}$

but 7 is not an element of the base set, so the answer is:
T = {∅}, i.e. it is an empty set.

Extension

If the base set was all numbers, what would the true set be?

T = {7}

c) *y is divisible by 3* T = {– 3, 0, 3} (i.e. multiples of 3)

d) $3 + z < 9$, $z < 9 - 3$, $z < 6$,
so T = B = {– 3, – 2, – 1, 0, 1, 2, 3, 4, 5}

<u>All</u> the numbers in the base set make the statement true.

e) $-2.6 \times t \geq 2$

Ps might try substituting each possible value:

$-2.6 \times (-3) = 7.8$ (T), $-2.6 \times (-2) = 5.2$ (T),
$-2.6 \times (-1) = 2.6$ (T), $-2.6 \times 0 = 0$ (F),
$-2.6 \times 1 = -2.6$ (F), $-2.6 \times 2 = -5.2$ (F),
$-2.6 \times 3 = -7.8$ (F), $-2.6 \times 4 = -10.4$ (F),
$-2.6 \times 5 = -13$ (F)

so the true set is: T = {– 3, – 2, – 1}

or give extra praise for a shorter method of reasoning:

If $t = 0$, the result is 0 and if t is positive, the result is negative, so in Set B only the negative numbers make the statement true.

23 min

Y6

Lesson Plan 141

Activity | ***Notes***

4

PbY6b, page 141

Q.2 Read: *Which numbers in the given **base set** (B) can be used instead of the letters to make the equations true?*

Set a time limit or deal with one or two at a time. Remind Ps to check their answers by substituting each value in the true set for the letter.

Individual work, monitored, helped

Review with whole class. Ps could show numbers on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

Discussion, reasoning, checking, agreement, self-correction, praising

Feedback for T

Solution:

a) $x - 9 = 3$ $B = \{\text{whole numbers}\}$

$x = 3 + 9 = \underline{12}$ *Check:* $\underline{12} - 9 = 3$ ✔

$\underline{T = \{12\}}$

b) $y + 8 = 7$ $B = \{0, 1, 2, 3, 4\}$

$y = 7 - 8 = \underline{-1}$ *Check*: $-1 + 8 = 7$ ✔

but – 1 is <u>not</u> an element of B, so $\underline{T = \{\varnothing\}}$

c) $z - 6 = 6 - z$ $B = \{5\frac{1}{3},\ 5\frac{2}{3},\ 6,\ 6\frac{1}{3},\ 6\frac{2}{3}\}$

$z + z = 6 + 6$

$z = \underline{6}$ *Check*: $\underline{6} - 6 = 6 - \underline{6}$ ✔

$\underline{T = \{6\}}$

d) $3 + t \times 3 = (3 + t) \times 3$ $B = \{-2, -1, 0, 1, 2, 3, 4, 5\}$

$t \times 3 = 9 + t \times 3 - 3$

$t \times 3 = 6 + t \times 3$

By substitution, <u>none</u> of the numbers in the base set makes the equation true, so $\underline{T = \{\varnothing\}}$

e) $3 \times t + 3 = (t + 1) \times 3$ $B = \{-7, -3\frac{1}{5}, -0.21, 0, 0.375, 6\frac{1}{7}\}$

$3 \times (t + 1) = (t + 1) \times 3$

By substitution, <u>all</u> the numbers in set B make the equation true, so $\underline{T = B = \{-7, -3\frac{1}{5}, -0.21, 0, 0.375, 6\frac{1}{7}\}}$

f) $4 \times u - 2 = u + 10$ $B = \{-1, 4, 9, 14\}$

$4 \times u = u + 12$

$3 \times u = 12$

$u = \underline{4}$ *Check*: $4 \times \underline{4} - 2 = 14 = \underline{4} + 10$ ✔

$\underline{T = \{4\}}$

g) $|v + 3| = v + 3$ $B = \{-5, -4, -3, -2, -1, 0, 1, 2, 3\}$

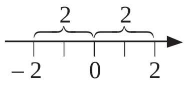
Elicit or remind Ps that $|v + 3|$ is read as 'the <u>absolute value</u> of $v + 3$' and means its distance from zero, i.e. its numerical value disregarding whether it is positive or negative.

Substitute each number in the base set for v to see whether it makes the equation true. (– 5 and – 4 do <u>not</u> make it true.)

$\underline{T = \{-3, -2, -1, 0, 1, 2, 3\}}$

BB: <u>absolute value</u>

e.g. $|-2| = 2$, $|+2| = 2$



e.g. $|-5 + 3| = |-2| = 2$

$\neq -5 + 3 = -2$

Activity

Notes

4

(Continued)

h) $(w + 1) \times (w - 2) = 0 \qquad B = \{-3, -2, -1, 0, 1, 2, 3\}$

By substituting each number in set B for w in the equation:

$(-3 + 1) \times (-3 - 2) = (-2) \times (-5) = 10 \neq 0$

$(-2 + 1) \times (-2 - 2) = (-1) \times (-4) = 4 \neq 0$

$(-1 + 1) \times (-1 - 2) = 0 \times (-3) = 0$ ✔

$(0 + 1) \times (0 - 2) = 1 \times (-2) = -2 \neq 0$

$(1 + 1) \times (1 - 2) = 2 \times (-1) = -2 \neq 0$

$(2 + 1) \times (2- 2) = 3 \times 0 = 0$ ✔

$(3 + 1) \times (3 - 2) = 4 \times 1 = 4 \neq 0$

So $\underline{w = -1}$ or $\underline{w = 2}$

or $\underline{T = \{-1, 2\}}$

Extra praise if a P points out that as the product is zero, one of the factors must equal zero,

i.e. $(w + 1) = 0$, so $\underline{w = -1}$

or $(w - 2) = 0$, so $\underline{w = 2}$

If no P notices this, T draws Ps' attention to it.

T points out that the 'true set' is the solution to the equation.

31 min

5

PbY6b, page 141

Q.3 Read: *Which numbers can be written instead of the letters to make the statements true? Solve the equations and inequalities in your exercise book.*

What is missing from this question? (There is no base set given.) T tells Ps that in such cases, they should take the base set is being all the numbers that they have learned. Elicit what they are: integers (positive and negative whole numbers and 0), positive and negative fractions and decimals.

T: We call all these numbers the set of <u>rational</u> numbers (BB). They are shown mathematically by using a capital 'Q'.

Set a time limit or deal with one at a time. Ps might use trial and error but encourage Ps to work it out logically where possible. Remind Ps to check their results by substitution.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class decides who is correct. Mistakes discussed and corrected.

Solution:

a) $a + b = b + a$ (The set of rational numbers)

e.g. $-\dfrac{1}{3} + 4 = 4 + (-\dfrac{1}{3})$ $\qquad \underline{T = Q}$

b) $x - (-4) = 2.1$

$x = 2.1 + (-4)$ or $x + 4 = 2.1$

$= \underline{-1.9}$ $\qquad x = 2.1 - 4 = \underline{-1.9}$

c) $y \times (-5) = 30$

$y = 30 \div (-5) = -30 \div 5 = \underline{-6}$

d) $2 \times (x + 1) = 2 \times x + 5$

$2x + 2 = 2x + 5$

Impossible, as $2\,x = 2x$ but $2 \neq 5$, so $\underline{T = \{\varnothing\}}$

e) $(u - 2) \times 3 = -6 + 3 \times u$

$u - 2 = -2 + u$ (The set of rational numbers)

or $u + (-2) = -2 + u$ [Similar to part a): $\underline{T = Q}$]

Individual work, monitored, helped

Initial discussion to clarify the base set.

BB: <u>Rational Numbers</u> (Q)
(integers, fractions, decimals)

What other capital letter do you know stands for a set of special numbers?

BB: <u>Natural Numbers</u> (N)
(positive whole numbers)

Agree that <u>any</u> rational number can be substituted for *a* and *b*, as the terms of an addition can be interchanged without affecting the result.

To Ts:

Addition is <u>commutative</u>.

Check: $\underline{-1.9} - (-4)$
$= -1.9 + 4 = 2..1$ ✔

Check: $\underline{-6} \times (-5) = 30$ ✔

(A negative number multiplied by a negative number results in a postive product.)

<u>Any</u> rational number can be substituted for *u*.

Activity

Notes

5

(Continued)

f) $44 \div z < 11$

If $44 \div z = 11$, $z = 44 \div 11 = 4$

As the quotient is <u>less</u> than 11, z must be <u>greater</u> than 4.

i.e. $\underline{z > 4}$.

g) $u \times v = v \times u$ (The set of rational numbers: $\underline{T = Q}$)

(Any rational number can be substituted for u and v and the equation will be true, as the factors in a multiplication can be interchanged and the product will be the same.

h) $t - 3 \geq 2 \times t$

If
$t = 1$, $-2 \ngeq 2$
$t = 2$, $-1 \ngeq 4$
$t = 10$, $7 \ngeq 20$

$t = 0$, $-3 \ngeq 0$
$t = -1$, $-4 \ngeq -2$
$t = -2$, $-5 \ngeq -4$
$t = -3$, $-6 \geq -6$ ✔
$t = -4$, $-7 \geq -8$ ✔
$t = -10$, $-13 \geq -20$ ✔

so $\underline{t \leq -3}$

Check:

$z = 3$: $44 \div 3 = 14\frac{2}{3} \nless 11$

$z = 5$: $44 \div 5 = 8.8 < 11$

Show it on the number line.

To T:

Multiplication is <u>commutative</u>.

In this question, trial and error done in a logical way as opposite might be easier to understand.

Show what is happening on the number line as t becomes greater and smaller.

However, give extra praise if a P suggests taking t away from each sid of the inequality, giving the result: $\underline{-3 \geq t}$

39 min

6

PbY6b, page 141*, *Q.4

Read: *Write an equation about the relationship between the given data. Solve the equation, then check your result in context.*

Deal with one part at a time. T chooses a P to read out the question. Who can write an equation about it? P comes to BB to write and say it. Class agrees/disagrees.

What should we do to solve this equation? Who agrees? Who thinks something else? P writes solution on BB, explaining reasoning. Class checks that the solution is correct by substitution.

Solution:

a) *I think of a number. If I subtract 8 from 3 times my number, the result is 19. What is my number?*

BB: $3 \times n - 8 = 19$ (or $3n - 8 = 19$)

$3 \times n = 19 + 8 = 27$

$n = 27 \div 3 = \underline{9}$ $\underline{n = 9}$

b) *I think of a number. If I divide my number by 5, then subtract 11 from the quotient, the result is 8. What is my number?*

BB: $n \div 5 - 11 = 8$

$n \div 5 = 8 + 11 = 19$

$n = 19 \times 5 = \underline{95}$ $\underline{n = 95}$

c) *I think of a number. I add 28 to 4 times my number, then divide the sum by 4. I subtract my number from the quotient and the difference is 7. What is my number?*

BB: $(n \times 4 + 28) \div 4 - n = 7$

$n + 7 - n = 7$, $n + 7 = 7 + n$ (So n is <u>any</u> rational number: $\underline{T = Q}$)

Whole class activity

(or individual work if Ps wish, reviewed with whole class)

Discussion, reasoning, checking, agreement, praising

Allow Ps to write and explain, with help of class.

T interferes only if Ps are stuck.

Check:

$3 \times \underline{9} - 8 = 27 - 8 = 19$ ✔

Check:

$\underline{95} \div 5 - 11 = 19 - 11 = 8$ ✔

e.g.

$n = \underline{5}$: $(5 \times 4 + 28) \div 4 - 5$
$= 48 \div 4 - 5$
$= 12 - 5 = 7$ ✔

Ps check other numbers too.

45 min

Y6

R: Calculations

C: Relationships in formulae using letters and symbols

E: Generalisations

Lesson Plan
142

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $\underline{142} = 2 \times 71$ Factors: 1, 2, 71, 142
- $\underline{317}$ is a prime number Factors: 1, 317
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and $19^2 > 317$)
- $\underline{492} = 2 \times 2 \times 3 \times 41 = 2^2 \times 3 \times 41$
  Factors: 1, 2, 3, 4, 6, 12, 41, 82, 123, 164, 246, 492
- $\underline{1142} = 2 \times 571$ Factors: 1, 2, 571, 1142
  (571 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23 and $29^2 > 571$)

Individual work, monitored (or whole class activity)

BB: 142, 317, 492, 1142

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

$$\begin{array}{r|l} 142 & 2 \\ 71 & 71 \\ 1 & \end{array} \qquad \begin{array}{r|l} 492 & 2 \\ 246 & 2 \\ 123 & 3 \\ 41 & 41 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 1142 & 2 \\ 571 & 571 \\ 1 & \end{array}$$

8 min

2

Inequalities

Let's show the answer to each question by writing an inequality.

T asks the question and Ps come to BB to write and explain. Class agrees/disagrees. Ask Ps to show some of the inequalities on the number line. Class points out errors.

e.g.

Which whole numbers are:

a) more than – 5 but less than + 3 BB: $-5 < x < 3$

T: To show that x is a whole number, we can also write: $x \in Z$

'Z' stands for the set of whole numbers or integers and $\in$ means 'is an element of'

b) not less than – 6 but less than + 16 BB: $-6 \leq y < 16,\ y \in Z$

c) not greater than – 5 and less than – 2 BB: $z \leq -5,\ z \in Z$

(If z is not greater than – 5, it can be equal to – 5 or less than – 5, and numbers equal to or less than – 5 are also less than – 2)

d) at least – 2 and at most + 1 BB: $-2 \leq u \leq 1,\ u \in Z$

e) positive but less than 4. BB: $0 < v < 4,\ v \in N$

(If v is a positive whole number, it is a natural number.)

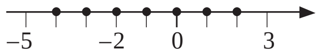
Whole class activity

At a good pace

Agreement, praising

Draw appropriate segments of number line on BB or show on class number line. e.g.

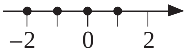
BB:



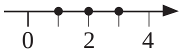
BB: Set of integers (Z)
(whole numbers)

BB:

BB:



BB:



13 min

3

Numbers written as operations

Answer each question by writing an operation. T asks the questions. Ps come to BB or dictate what T should write. Class agrees/disagrees.

T shows the short form of notation where relevant.

Write the number which is:

a) i) 5 more than 3 (3 + 5) ii) x more than 3 $(3 + x)$
ii) 3 more than y $(y + 3)$

b) i) 7 less than x $(x – 7)$ ii) u less than 7 $(7 – u)$
iii) smaller than w by 2 $(w – 2)$

Whole class activity

At a fast pace

Agreement, praising

Elicit or remind Ps that, e.g. $b \times c$ can be written as bc

Praising, encouragement only

Feedback for T

Y6

Lesson Plan 142

Activity ***Notes***

3 (Continued)

c) i) 3 times t ($3 \times t$ or $3t$) ii) s times 5 ($s \times 5$, or $5s$)

iii) b times c ($b \times c$, or bc)

d) i) a quarter of x ($x \div 4$, or $\frac{x}{4}$) ii) a third of y ($y \div 3$, or $\frac{y}{3}$)

ii) 2 sevenths of z ($z \div 7 \times 2$, or $z \times \frac{2}{7}$)

(or $\frac{2 \times z}{7}$ or $\frac{2z}{7}$)

18 min

---

4 ***PbY6b, page 142***

Q.1 Read: *a) Complete this table.*

b) Solve the inequality $2 \times x - 1 < 5$, *if the base set is:*

i) the set of integers (Z)

ii) the set of natural numbers (N)

iii) the set of all the numbers that you know

(Q: rational numbers)

Set a time limit. Ps might write the solutions by listing possible numbers but encourage them to try to write inequalities.

Review with whole class. Ps come to BB to complete table or dictate what T should write, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Then Ps come to BB to write the solution for each part. Who agrees? Who wrote it in a different way? Mistakes discussed and corrected. Ask Ps to show the solutions on the class number line too.

Solution:

a)

x	-3	-2	-1	0	1	2	3	4	5	6
$2 \times x - 1$	-7	-5	-3	-1	1	3	5	7	9	11

b) i) $x = \{-3, -2, -1, 0, 1, 2\}$ or $\underline{x < 3,\ x \in Z}$

ii) $x = 2$ or $x = 1$, or $\underline{x < 3,\ x \in N}$

iii) too many possible numbers to list, so it is better to write the solution as: $\underline{x < 3,\ x \in Q}$

Individual work, monitored, helped

Table (and axes for Extension) drawn on BB or use enlarged copy master or OHT

First elicit that:

- integers are whole numbers
- natural numbers are positive whole numbers
- rational numbers are positive and negative whole numbers, fractions, decimals and zero.

Reasoning, agreement, self-correction, praising

Extension

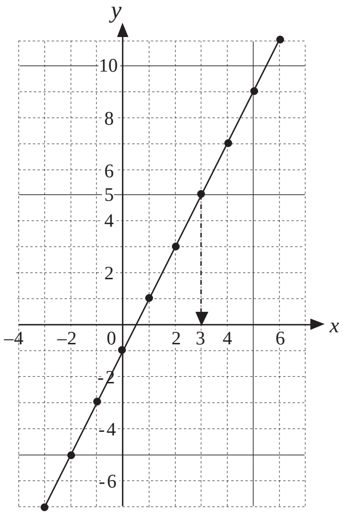
Can anyone think of another way we could show the solutions? T gives a hint if necessary. What do we often draw after completing a table? (a graph)

T has axes already prepared and Ps come to BB to draw a dot for each column in the table. Class points out errors.

Is it correct to join up the dots? (Only to show the rational numbers, as that solution also includes parts of numbers.)

How can we use this graph to solve the inequality?

Ps come to BB to point out the relevant dots or line segement below the horizontal grid line, $y = 5$, and to the left of the vertical grid line, $x = 3$.



23 min

Y6

Lesson Plan 142

Activity ***Notes***

5

PbY6b, page 142

Q.2 Read: *Write a formula about the relationship between the data.*

Set a time limit. Ps write formulae in *Pbs*.

Review with whole class. T chooses Ps to read out the relationship descriptions and Ps show formulae on scrap paper or slates on command. Class decides which formulae are correct. If there is disagreement, ask Ps to explain by drawing a diagram on BB. Mistakes discussed and corrected. [Elicit or show the short algebraic forms where relevant.]

Solution:

a) *the area of a rectangle with sides a and b*

$A = a \times b = ab$

b) *the perimeter of a rectangle with sides e and f*

$P = 2 \times (e + f) = 2 \times e + 2 \times f$ [$= 2e + 2f$]

c) *the area of a square with side c* $A = c \times c = c^2$

d) *the perimeter of a square with side t* $P = 4 \times t = 4t$

e) *the area of a square with diagonal e* $A = \dfrac{e \times e}{2} = \dfrac{e^2}{2}$

f) *the surface area of a cube with edge c*

$A = 6 \times c \times c = 6c^2$

g) *the volume of a cube with edge a* $V = a \times a \times a = a^3$

h) *the volume of a cuboid with edges a, b, c.*

$V = a \times b \times c = abc$

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHT

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Feedback for T

28 min

6

PbY6b, page 142

Q.3 Read: *The difference between two numbers is 19.*

Ps read questions themselves and write the answers as operations involving *x* or *y*.

Review with whole class. T chooses Ps to read out the questions and Ps show answers on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Mistakes discussed and corrected.

Solution:

a) *What is the other number if:*

i) the smaller number is x [$x + 19$]

ii) the greater number is y? [$y - 19$]

b) *Write the sum of the two numbers using only one letter, x.*

$x + x + 19 = 2 \times x + 19$ ($= 2x + 19$)

c) *What are the two numbers if their sum is 40?*

$2x + 19 = 40$, $2x = 40 - 19 = 21$, $x = \underline{10.5}$

The smaller number is <u>10.5</u> and the larger number is 10.5 + 19 = <u>29.5</u>.

Individual work, monitored, (helped)

Differentiation by time limit

Responses shown in unison.

Reasonging, agreement, self-correction, praising

Elicit that: $y - x = 19$

Check: 29.5 – 10.5 = 19 ✔

32 min

Y6		*Lesson Plan 142*
Activity		***Notes***
7	***PbY6b, page 142*** Q.4 Read: *A natural number is 3 times another natural number.* What is a natural number? (A natural number is a positive whole number.) Ps read questions themselves and write the answers as operations involving *y*. Review with whole class. T chooses Ps to read out the questions and Ps show answers on scrap paper or slates on command. Ps with correct answers explain at BB to Ps who were wrong. Mistakes discussed and corrected. *Solution:* a) *If the smaller number is y, what is the greater number?* [The greater number is $3 \times y$ (or $3y$)] b) *Write the sum of the two numbers.* [$y + 3 \times y = 4 \times y$ (or $y + 3y = 4y$)] c) *Calculate the smaller number if the sum of the two numbers is 324.* [$4 \times y = 324$, $y = 324 \div 4 = \underline{81}$] The smaller number is 81. *36 min*	Individual work, monitored, (helped) Differentiation by time limit Responses shown in unison. Reasoning, agreement, self-correction, praising *Check*: $81 + 3 \times 81$ $= 81 + 243 = 324$ ✔
8	***PbY6b, page 142**, **Q.5*** Read: *In a box there are b apples. In a second box there are 7 apples more than b. In a third box there are 5 apples less than b.* T draws 3 'boxes' on BB. a) Read: *How many apples are in each box?* Ps come to BB or dictate what T should write. Class agrees/disagrees. b) Read: *How many apples are in the 3 boxes altogether?* T allows Ps half a minute to think about it and write in *Ex. Bks.* **A**, come and show us what you think. Who agrees? Who thinks it should be something else? Why? BB: Total number of apples: $b + b + 7 + b - 5 = 3 \times b + 2$ c) *How many apples are in the first box if there are 77 apples in all 3 boxes?* I will give you a minute to work it out. Show me . . . now! (25) P answering correctly explains reasoning at BB. Another P checks the answer. Mistakes corrected. BB: $3b + 2 = 77$ $3b = 77 - 2 = 75$ $b = 75 \div 3 = \underline{25}$ *Answer*: There are 25 apples in the first box. *40 min*	Whole class activity (or individual work, reviewed with whole class as usual) BB: $\boxed{b}$ $\boxed{b + 7}$ $\boxed{b - 5}$ Reasoning, agreement, praising (= $3b + 2$) Responses shown in unison. Reasoning, checking, agreement, self-correction, praising *Check*: $25 + (25 + 7) + (25 - 5)$ $= 25 + 32 + 20$ $= 77$ ✔
9	**Secret numbers** a) I am thinking of a natural number. If I add 3 to 4 times my number, I will get 31. What is the number I am thinking of? (7) b) I am thinking of another natural number. If I take it away from 7 times the number, I will get 102. What is my number? (17) *45 min*	Whole class activity Ps show numbers in unison. Reasoning, agreement, praising a) $4x + 3 = 31$, $4x = 28$, $x = \underline{7}$ b) $7y - y = 6y = 102$, $y = \underline{17}$

Y6

R: Calculations

C: Equations and inequalities

E: Relating to the idea of a balance

Lesson Plan 143

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>143</u> = 11 × 13 Factors: 1, 11, 13, 143
- <u>318</u> = 2 × 3 × 53 Factors: 1, 2, 3, 6, 53, 106, 159, 318
- <u>493</u> = 17 × 29 Factors: 1, 17, 29, 493
- <u>1143</u> = 3 × 3 × 127 = 3^2 × 127

  Factors: 1, 3, 9, 127, 381, 1143

1143	3
381	3
127	127
1	

8 min

Individual work, monitored (or whole class activity)

BB: 143, 318, 493, 1143

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

318	2
159	3
53	53
1	

143	11
13	13
1	

493	17
29	29
1	

2

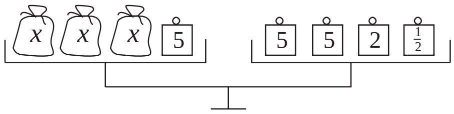
Equations and inequalities

Let's write a mathematical statement about each balance and solve it one step at a time. Unless we are told otherwise, we will assume that the base set of numbers to choose from is all the numbers we have learned.

Ps come to BB to say what they see in the diagram and write an equation or inequality. Elicit that when the two sides of the balance are level, it means that they are <u>equal</u>. If one side of the balance is lower than the other, it means that side is <u>more</u> than the other side. Class discusses how to work out the solution. Stress that what is done to one side of the balance must also be done to the other side to keep the balance in the same position. Ps say what they are doing at each step in a loud voice.

BB:

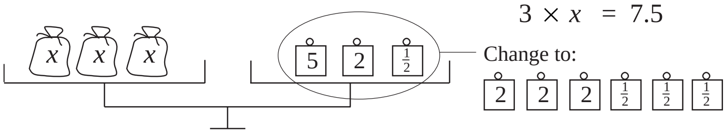
a)



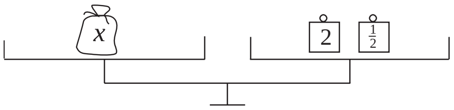
$3 \times x + 5 = 12.5$

Take 5 away from each side:

$3 \times x = 7.5$

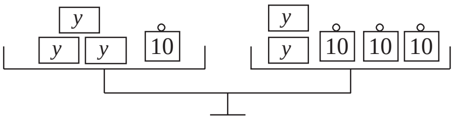


Divide each side by 3:



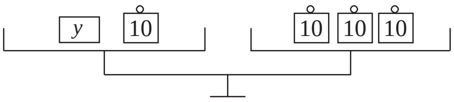
$\underline{x = 2.5}$

b)



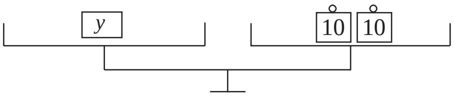
$3y + 10 = 2y + 30$

[Take 2*y* from each side]



$y + 10 = 30$

[Take 10 from each side]



$\underline{y = 20}$

Whole class activity

Drawn (stuck) on BB or use enlarged copy master or OHP

[or use a real balance and equal bags/boxes and weights)

At a good pace

Agreement praising

T helps when necessary.

Ps could think of a context for each diagram.

(e.g. Bags of potatoes + kg weights. How many kg of potatoes are in each bag?)

Elicit that to make 3 equal anmounts on RHS, change the '5' to two '2's + two 'halves' i.e. 2.5 + 2.5 + 2.5)

Check: $3 \times \underline{2.5} + 5$

$= 7.5 + 5 = 12.5$ ✔

(e.g. Boxes of apples and kg weights. How many kg of apples are in each box?)

Check:

LHS: $3 \times 20 + 10 = 70$

RHS: $2 \times 20 + 30 = 70$ ✔

Y6

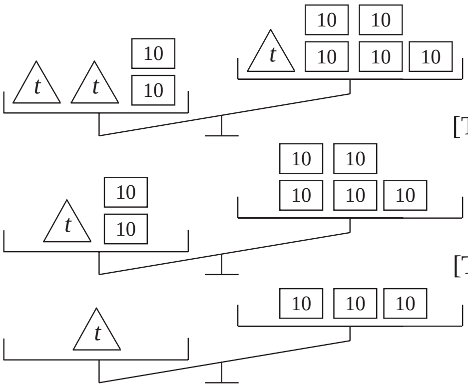
Lesson Plan 143

Activity		*Notes*

2

(Continued)

c) Elicit that LHS is more.



$2t + 20 > t + 50$

[Take t from each side]

$t + 20 > 50$

[Take 20 from each side]

$\underline{t > 30}$

20 min

(e.g. equal packets of tea and 10 g weights. How much tea is in each packet?)

Elicit that we cannot work out exactly how much tea is in each packet but we can say that it is more than 30 g.

3

PbY6b, page 143

Q.1 Read: *Solve the equations and check your results.*

Tell Ps that unless the base set is specified, they should take it as being all the numbers they have learned (the rational numbers).

Set a time limit or deal with one row at a time.

Review with whole class. Ps could show solutions on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Class checks their results mentally by substitution. Mistakes discussed and corrected.

Ask Ps to reason by generalising in words using the names of the components of the operations.

Solution:

a) $x + 2.7 = 11, \; x = 11 - 2.7 = \underline{8.3}$

b) $-6.2 + y = 3, \; y = 3 - (-6.2) = 3 + 6.2 = \underline{9.2}$

To calculate the unkown term in a 2-term addition, subtract the known term from the sum.

c) $z - (-3) = -2, \; z = -2 + (-3) = \underline{-5}$

To calculate the reductant, add the subtrahend to the difference.

d) $\frac{x}{4} \times 3 = \frac{9}{4}, \; \frac{x}{4} = \frac{9}{4} \div 3 = \frac{3}{4}, \; x = \underline{3}$

To calculate the multiplicand, divide the product by the multiplier.

e) $u \div (-3) = 6, \; u = 6 \times (-3) = \underline{-18}$

To calculate the dividend, multiply the quotient by the divisor.

f) $(-42) \div v = 6, \; v = (-42) \div 6 = \underline{-7}$

To calculate the divisor, divide the dividend by the quotient.

25 min

Individual work, monitored, helped

Written on BB or SB or OHT

Differentiation by time limit.

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

Extra praise for reasoning using the balance method. e.g.

a) Subtract 2.7 from each side.

b) Subtract (– 6.2) from, or add 6.2 to, each side.

c) Add (– 3) to each side.

d) Divide each side by 3, then multiply each side by 4.

e) Multiply each side by (– 3).

f) Multiply each side by v:

$-42 = 6 \times v$,

Divide each side by 6:

$\underline{-7} = v$

Y6

Lesson Plan 143

Activity

4

PbY6b, page 143

Q.2 Read: *Solve the inequalities.*

Set a time limit or deal with one at a time. Ps can use any method they wish (including trial and error).

Review with whole class. Ps come to BB to explain reasoning. Who agrees with the answer? Who worked it out a different way? Give extra praise if Ps reason using the 'balance' model.

Ask Ps to show the solution to part a) on the number line.

Solution: e.g.

a) $a + (-5) < -13$

e.g. If $a + (-5) = -13$, $a = -13 - (-5) = -13 + 5 = \underline{-8}$

but $a + (-5) < -13$, so $a < -8$

Check: If $a = -9$: $-9 + (-5) = -14 < -13$ ✔

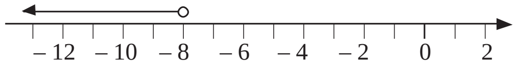
or using the 'balance' model:

$a + (-5) < -13$ [Subtract (– 5) from each side]

$a < -13 - (-5) = -13 + 5$

$\underline{a < -8}$

BB:



b) $b - \left(-\frac{7}{6}\right) \geq 1\frac{5}{9}$

$b + \frac{7}{6} \geq 1\frac{5}{9}$

$b \geq 1\frac{5}{9} - 1\frac{1}{6} = \frac{10-3}{18} = \frac{7}{18}$

$\underline{b \geq \frac{7}{18}}$ [Check for $b = \frac{6}{18}$ too.]

Check:

If $b = \frac{7}{18}$,

$\frac{7}{18} - \left(-\frac{21}{18}\right) = \frac{28}{18}$

$= \frac{14}{9} = 1\frac{5}{9}$ ✔

c) $c - (+63) \leq -17$ [Add 63 to each side.]

$c \leq -17 + 63$

$\underline{c \leq 46}$ *Check*: If $c = 47$, $47 - 63 = -16 \nless -17$ ✗

d) $d + \frac{2}{3} > -2\frac{1}{4}$,

$d + \frac{8}{12} > -2\frac{3}{12}$ [Subtract $\frac{8}{12}$ from each side,]

$\underline{d > -2\frac{11}{12}}$

30 min

Notes

Individual work, monitored, helped

Written on BB or SB or OHT

Differentiation by time limit.

Reasoning, agreement, self-correction, praising

Accept any valid reasoning.

Extension

T helps Ps to check the results. (See below)

If $a = -7$, $-7 + (-5)$

$= -12 \nless -13$ ✗

Elicit/remind Ps about the notation for showing inequalities on the number line:

- *black* (closed) circle above the number if it is included;
- *white* (open) circle above the number if is <u>not</u> included.

If $b = \frac{8}{18}$,

$\frac{8}{18} - \left(-\frac{21}{18}\right) = \frac{29}{18}$

$= 1\frac{11}{18} > 1\frac{10}{18}$ ✔

If $c = 46$, $46 - 63 = -17$ ✔

If $c = 45$, $45 - 63 = -18$

and $-18 < -17$ ✔

Check:

If $d = -2\frac{10}{12}$,

$-2\frac{10}{12} + \frac{8}{12} = -2\frac{2}{12} = -2\frac{1}{6}$

and $-2\frac{1}{6} > -2\frac{1}{4}$ ✔

Y6

Lesson Plan 143

Activity | ***Notes***

5

PbY6b, page 143

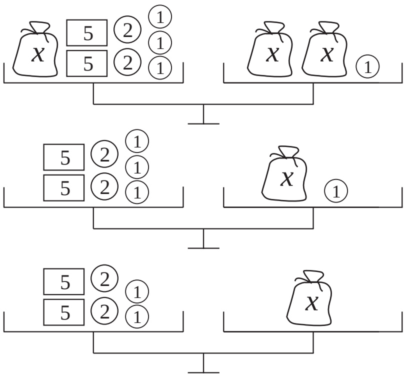
Q.3 Read: *Write an equation about the diagram. Solve the equation by changing the sides equally. Follow the steps.*

What do you notice about this set of balances? (They are level, so the RHS = the LHS) Make sure that the RHS equals the LHS of your equations!

Set a time limit of 2 minutes. Ps write in *Pbs*.

Review with whole class. Ps come to BB to write equations and explain reasoning, saying what they have done to each side. Class agrees/ disagrees. Mistakes discussed and corrected.

Solution:



$x + 17 = 2x + 1$

[Subtract x from each side.]

$17 = x + 1$

[Subtract 1 from each side]

$\underline{16 = x}$

or $\underline{x = 16}$

34 min

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Reasoning, checking:

Check: $\underline{16} + 17 = 33$

$2 \times \underline{16} + 1 = 32 + 1 = 33$ ✔

agreement, self-correction, praising

Elicit or explain that:

$1 \times x = 1x = x$

$2 \times x = 2x$

$2x - x = x$

We could write the solution in a shorter way like this:

BB: $x + 17 = 2x + 1$ [$-x$]

$17 = x + 1$ [-1]

$\underline{16 = x}$

writing what we will do to get the next line in square brackets.

6

PbY6b, page 143

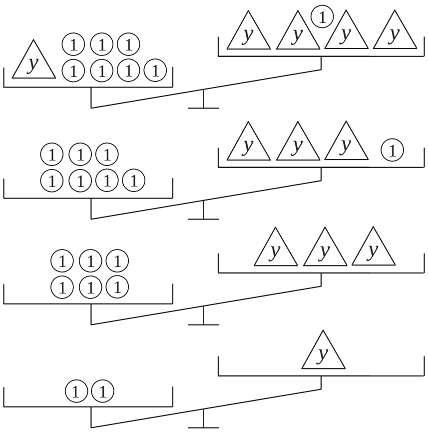
Q.4 Read: *Write an inequality about the diagram. Solve the inequality by changing the sides equally. Follow the steps.*

What do you notice about this set of balances? (LHS is more than RHS) Make sure that the LHS is more than the RHS in your inequalities!

Set a time limit of 4 minutes. Ps write in *Pbs*.

Review with whole class. Ps come to BB to write inequalities and explain reasoning, saying what they have done to each side. Class agrees/ disagrees. Mistakes discussed and corrected.

Solution:



$y + 7 > 4y + 1$ [$-y$]

$7 > 3y + 1$ [-1]

$6 > 3y$ [$\div 3$]

$\underline{2 > y}$ or $\underline{y < 2}$

40 min

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Reasoning, checking, agreement, self-correction, praising

Check:

$y = 1$: $1 + 7 = 8$

$4 \times 1 + 1 = 5$

and $8 > 5$ ✔

$y = 2$: $2 + 7 = 9$

$4 \times 2 + 1 = 9$

and $9 \not> 9$ ✗

Elicit that:

$1 \times y = 1y = y$

$4 \times y - y = 3 \times y = 3y$

$3 \times y \div 3 = 1 \times y = y$

If y is the weight of a sack of of potatoes in kg, what else can we write about y?

BB: $0 < y < 2$

Y6

Lesson Plan 143

Activity

Notes

7

PbY6B, page 143, Q.5

Read: *Draw diagrams to help you solve this equation.*

BB: $3 \times x - 2 = x + 6$

Allow Ps a minute to think about it, discuss with their neighbours and try out ideas. T asks Ps for their suggestions.

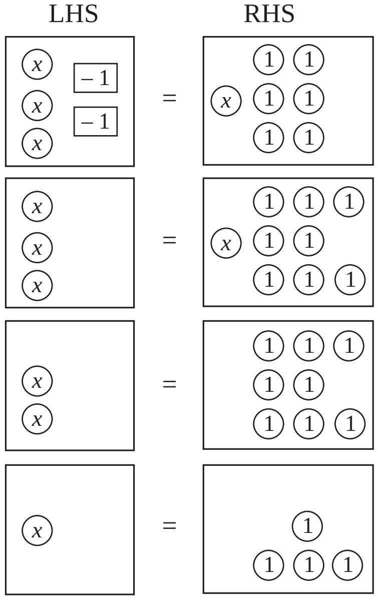
If no P has a good idea (it is difficult because of the – 2) T suggests a cash and debt model. e.g.

① means £1 in cash, i.e. + 1 , $\boxed{-1}$ means £1 in debt, i.e. – 1

Let's show the equation using these symbols. Ps come to BB to draw symbols (or stick pre-prepared cards) on BB. Class points out errors.

Ps suggest what to do to get the following line, and Ps come to BB in pairs to carry it out (redrawing or manipulating the symbols) while a 3rd P writes the matching equation. Check the solution.

BB: e.g.



$3 \times x - 2 = x + 6$ [+ 2]

$3 \times x = x + 8$ [$-x$]

$2 \times x = 8$ [÷ 2]

$\underline{x = 4}$

Whole class activity

(or individual trial first if Ps wish)

T has circles and rectangles already prepared (see diagram).

Discussion, agreement, praising

Extra praise if a P thinks of the cash and debt model.

At a good pace

In good humour.

Reasoning, agreement, praising

Ps could write the equations in *Ex. Bks*. too.

Elicit that:

$-2 + 2 = 0$

$x = 1 \times x$

$3 \times x - x = 2 \times x$

$2 \times x \div 2 = 1 \times x = x$

Check:

LHS: $3 \times \underline{4} - 2 = 10$

RHS: $\underline{4} + 6 = 10$ ✔

Extension

Could we have done it this way? Is it correct? What do you think of it?

T shows prepared solution and asks several Ps what they think.

Elicit that it is correct but adding 7 to both sides does nothing to get closer to the solution. The solution above is better as every step makes progress (although the 1st and 2nd steps could be interchanged).

BB: $3 \times x - 2 = x + 6$ [+ 7]

$3 \times x + 5 = x + 13$ [$-x$]

$2 \times x + 5 = 13$ [÷ 2]

$x + 2.5 = 6.5$ [– 2.5]

$\underline{x = 4}$

45 min

Y6

R: Calculations
C: Equations and inequalities Using the 'balance' method
E: Generalising relationships (Algebraic expressions)

Lesson Plan 144

Activity

Notes

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $\underline{144} = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$ $[= \left(2^2 \times 3\right)^2 = 12^2]$

  Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144

  (Square number. No. of factors: $(4 + 1) \times (2 + 1) = 5 \times 3 = \underline{15}$)
- $\underline{319} = 11 \times 29$ Factors: 1, 11, 29, 319
- $\underline{494} = 2 \times 13 \times 19$ Factors: 1, 2, 13, 19, 26, 38, 247, 494
- $\underline{1144} = 2 \times 2 \times 2 \times 11 \times 13 = 2^3 \times 11 \times 13$

  Factors: 1, 2, 4, 8, 11, 13, 22, 26, ↓

  1144, 572, 286, 143, 104, 88, 52, 44

Individual work, monitored (or whole class activity)

BB: 144, 319, 494, 1144

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

$$\begin{array}{r|l} 144 & 2 \\ 72 & 2 \\ 36 & 2 \\ 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 319 & 11 \\ 29 & 29 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1144 & 2 \\ 572 & 2 \\ 286 & 2 \\ 143 & 11 \\ 13 & 13 \\ 1 & \end{array} \qquad \begin{array}{r|l} 494 & 2 \\ 247 & 13 \\ 19 & 19 \\ 1 & \end{array}$$

8 min

2

Solving equations and inequalities

Let's solve this equation and inequality, keeping in mind the two sides of a balance or set of scales.

What do you think the symbols mean?

($\boxed{-x}$ means – £x or £x in debt, ⓧ means £x in cash

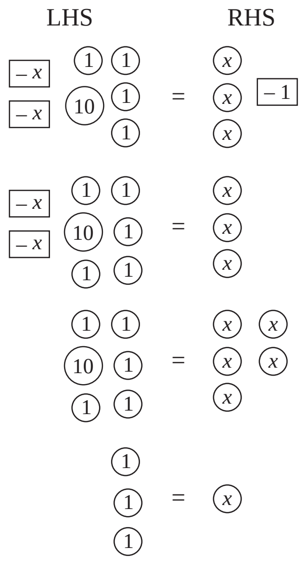
a) Study this diagram. Who can tell me what it means in words?
(£14 in cash and £2x in debt is equal to £3x in cash and £1 in debt.)

Let's write it as a mathematical equation. Ps come to BB or dictate what T should write. Class agrees/disagrees.

Now let's solve it by changing each side equally. T or Ps suggest what to do at each step and Ps come to BB in pairs, one to draw (or stick on) symbols and the other to write the matching equation.

Class points out errors and checks the result.

BB:



e.g.

$\boxed{-2 \times x + 14 = 3 \times x - 1}$ [+ 1]

$-2 \times x + 15 = 3 \times x$ [+ 2 × x]

$15 = 5 \times x$ [÷ 5]

$\underline{3 = x}$ or $\underline{x = 3}$

Whole class activity

Equation and inequality shown as symbols drawn (stuck) on BB. (If stuck on BB, T should have enough symbols prepared to complete each row of the two parts.)

At a good pace

Involve many Ps.

Discussion, reasoning, agreement, praising

Accept any valid method of solution, not necessarily the steps shown here, e.g. Ps might suggest subtracting 14 from each side as the first step)

or

$\boxed{2 \times (-x) + 14 = 3x + (-1)}$

Elicit that:

$-2 \times x = 2 \times (-x) = -2x$

$-2 \times x + 2 \times x = -2x + 2x$

$= \underline{0}$

$3 \times x + 2 \times x = 5 \times x$

(or $3x + 2x = 5x$)

$5 \times x \div 5 = 1 \times x = x$

(or $5x \div 5 = x$)

Check:

LHS: $-2 \times \underline{3} + 14 = 8$

RHS: $3 \times \underline{3} - 1 = = 8$ ✔

Y6 *Lesson Plan 144*

Activity		***Notes***

2 (Continued)

b) Now study this diagram. Who can tell me what it means in words? (£3*x* in debt and £1 in cash is less than £*x* plus £10 in cash.)

Let's write it as an inequality. Ps come to BB or dictate what T should write. Class agrees/disagrees.

Now let's solve it by changing each side equally. T or Ps suggest what to do at each step and Ps come to BB in pairs.

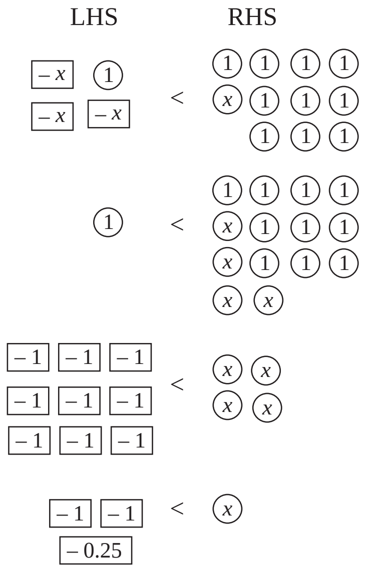
Class points out errors. Check possible values for *x*.

Discussion, reasoning, agreement, praising

At a good pace

Involve several Ps.

BB:



e.g.

$\boxed{-3 \times x + 1 < x + 10}$ [+ 3 × x]

or $\boxed{3 \times (-x) + 1 < x + 10}$

$1 < 4 \times x + 10$ [–10]

Of course, steps 1 and 2 could be interchanged.

$-9 < 4 \times x$ [÷ 4]

$\underline{-\frac{9}{4} < x}$ or $\underline{-2.25 < x}$

or $\underline{x > -\frac{9}{4}}$ or $\underline{x > -2..25}$

Agree that if the context is money, the decimal form of the solution is better.

What does this solution really mean?

(*x* could mean less than £2.25 in debt, or *x* could be £0, or *x* could be a positive amount.)

Discussion, agreement, praising

T directs Ps thinking if necessary.

Let's check that we are correct.

e.g. $x = -3$: $-3 \times (-3) + 1 < -3 + 10$

(> £2.25 in debt) $10 \nless 7$ ✗

$x = -1$: $-3 \times (-1) + 1 < -1 + 10$

(< £2.25 in debt) $4 < 9$ ✔

$x = 0$: $-3 \times 0 + 1 < 0 + 10$

(No debt, no cash) $1 < 10$ ✔

e.g. $x = 2$: $-3 \times 2 + 1 < 2 + 10$

(Cash) $-5 < 12$ ✔

15 min

Y6

Lesson Plan 144

Activity | *Notes*

3

Letters in equations

a) How could we write '17 apples plus 8 apples' in a shorter way?

BB: 17a + 8a = 25a

We have written the first letter as an abbreviation of 'apple'.

What if the letter *a* meant the price of an apple? How would we write the equation then? Ps come to BB or dictate to T.

BB: $17 \times a + 8 \times a = 25 \times a$

but we could also write this too as:

$17a + 8a = 25a$

What else could *a* mean? (e.g. the mass of an apple)

b) How could we write mathematically '84 bananas divided by 12'?

BB: 84b ÷ 12 = 7b (b is an abbreviation for banana)

What if the letter *b* means the mass of a banana? How would we write the division then?

BB: $84 \times b \div 12 = 7 \times b$ or $84b \div 12 = 7b$

What else could *b* mean? (e.g. the price of a banana)

c) What could this mean if c is an abbreviation?

BB: 30c ÷ 5c = 6

e.g. 30 chairs divided into groups of 5 chairs equals 6 (times).

T: We could also think of *c* as being the price of a chair. Then the division could mean the ratio of the chair prices.

BB: $30 \times c \div 5 \times c = 6$

$\rightarrow 30 \times c : 5 \times c = 6 \rightarrow 30c : 5c = 6 \rightarrow 6c : c = 6$

It means that the ratio of the price of 6 chairs to the price of 1 chair is 6, or that 6 chairs cost 6 times as much as 1 chair.

Whole class activity

Ps come to BB or dictate what T should write. Class agrees/ disagrees.

BB: abbreviation
(shortened form of a word)

Discussion, reasoning, agreement, praising

To T:

Note that in printed material (*Pbs*, LPs, etc.) letters which represent unknown amounts (variables) are shown in italic but this of course cannot be done on the BB or in *Ex Bks*.

T leads Ps to realise that in mathematics letters can be used to represent many things but they are always treated in the same way.

($\rightarrow$ 6 : 1 = 6)

20 min

4

PbY6b page 144

Q.1 Read: *Write each operation using abbreviations (e.g. 'a' instead of 'apricots') then do the operation.*

Set a time limit of 3 minutes. Ps work in *Ex. Bks*.

Review quickly with whole class. Ps come to BB or dictate to T. Class agrees/disagrees. Mistakes discussed and corrected.

Solution:

a) 140 apricots + 27 apricots – 52 apricots

[140a + 27a – 52a = 167a – 52a = $\underline{115a}$]

b) 150 apples ÷ 5 [150a ÷ 5 = $\underline{30a}$]

c) 83 boxes × 3 [83b × 3 = $\underline{249b}$]

d) 63 stamps ÷ 9 stamps [63s ÷ 9s = $\underline{7}$] $\underline{\text{times}}$, not stamps!

e) $\frac{4}{7}$ of 84 potatoes [$\frac{4}{7}$ of 84p = $\frac{4}{\cancelto{1}{7}} \times \cancelto{12p}{84p} = \underline{48p}$

or $\frac{4}{7}$ of 84p = 84p ÷ 7 × 4 = 12p × 4 = $\underline{48p}$]

f) 4 apples + 10 apples + 5 bananas – 2 apples – 4 bananas

[$4\underline{a} + 10\underline{a} + 5b - 2\underline{a} - 4b = 12\underline{a} + 1b = \underline{12a + b}$]

Individual trial, monitored, (helped)

Reasoning, agreement, self-correction, praising

63 stamps divided into groups of 9 stamps gives 7 groups

The apples and the bananas are calculated separately!

25 min

Y6

Lesson Plan 144

Activity ***Notes***

5

PbY56, page 144

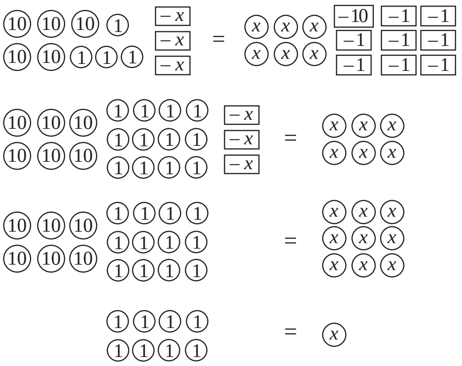
Q.2 Read: *Study the diagram to help you understand the equation. Solve the equation and check the result. (The first step is given.)*

Eicit that the circles mean positive amounts and the rectangles mean negative amounts (e.g. a cash and debt model)

Set a time limit of 3 minutes. Ps copy what is given in *Pbs* in *Ex. Bks* and continue the steps. (If Ps can solve the equation without diagrams, they need not draw them.)

Review with whole class. Ps could show solution on scrap paper or slates on command. Ps with correct answer explain reasoning at BB. (T could have symbols already prepared to make this easier and quicker.) Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution:



$54 - 3 \times x = 6 \times x - 18$ [+ 18]

$72 - 3 \times x = 6 \times x$ [+ 3× x]

$72 = 9 \times x$ [÷ 9]

$\underline{8 = x}$ or $\underline{x = 8}$

Check:

LHS: $54 - 3 \times \underline{8} = 54 - 24 = 30$

RHS: $6 \times \underline{8} - 18 = 48 - 18 = 30$ ✔

Individual work, monitored, helped

Drawn (stuck) on BB or use enlarged copy master or OHP

Accept any valid steps but tell Ps to make sure that, at each stage, both sides of the equation balance (are equal) .

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

Elicit that:

$6x + 3x = 9x$

$$\frac{\overset{1}{\cancel{9}} \times x}{\cancel{9}_1} = 1x = x$$

30 min

6

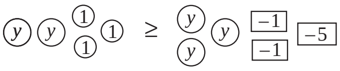
PbY6b, page 144

Q.3 Read: *Write an inequality about the diagram. Solve the inequality and check your result by filling in the table.*

Ask Ps to write the inequality above the diagram in *Pbs* first, then review it and make sure that any mistakes are corrected before Ps solve it in *Ex. Bks* and check it in table in *Pbs*.

Set a time limit. Review with whole class. Ps show solution on scrap paper or slates on command. Ps with correct answer explain reasoning at BB. Who did the same? Who did it a different way? Ps come to BB to complete the table as a check. Class points out errors. Mistakes discussed and corrected.

Solution: $2y + 3 \geq 3y - 7$ [or $2y + 3 \geq 3y + (-7)$]



Check:

y	– 1	0	3	7	9	10	11	16
Left side (L)	1	3	9	17	21	23	25	35
Right side (R)	– 10	– 7	2	14	20	23	26	41
L ≥ R	Yes	Yes	Yes	Yes	Yes	Yes	No	No

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master of OHP

Responses shown in unison.

Accept any correct steps.

Reasoning, agreement, checking, praising

$2y + 3 \geq 3y - 7$ [– 2y]

$3 \geq y - 7$ [+ 7]

$\underline{10 \geq y}$ or $\underline{y \leq 10}$

Ask Ps to show the solution on the class number line.

35 min

Y6

Lesson Plan 144

Activity

Notes

7

PbY6b page 144

Q.4 Read: *Solve the equations and check the results in your exercise book.*

Individual work, monitored, helped

Written on BB or SB or OHT

Set a time limit or deal with one part at a time. Ps may draw diagrams if they wish, but encourage Ps to try without them.

Review with whole class. Ps show solution on scrap paper or slates on command. Ps with correct answers explain reasoning at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected.

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

Solution: e.g.

a) $5 \times x = 2 \times x + 12$ [$- 2 \times x$]

$3 \times x = 12$ [` ÷ 3]

$\underline{x = 4}$

Check:

LHS: $5 \times \underline{4} = 20$

RHS: $2 \times \underline{4} + 12 = 20$ ✔

b) $5 \times y + 3 = 2 \times y + 9$ [$- 3$]

$5 \times y = 2 \times y + 6$ [$- 2 \times y$]

$3 \times y = 6$ [÷ 3]

$\underline{y = 2}$

Check:

LHS: $5 \times \underline{2} + 3 = 13$

RHS: $2 \times \underline{2} + 9 = 13$ ✔

c) $4 \times z - 5 = 10 + (-z)$ [$+ 5$]

$4 \times z = 15 + (-\mathrm{z})$ [$-(-z)$ or $+z$]

$5 \times z = 15$ [÷ 5]

$\underline{z = 3}$

Check:

LHS: $4 \times \underline{3} - 5 = 7$

RHS: $10 + (-\underline{3}) = 7$ ✔

d) $45 - d = 25 - d$ [$+ d$]

$45 = 25$ ✗

[but $45 \neq 25$, so the equation is impossible! i.e. $T = \varnothing$]

Agree that there is no rational number which makes the equation true.

e) $\dfrac{x}{4} + 5 = 8$ [$- 5$]

$\dfrac{x}{4} = 3$ [$\times 4$]

$\underline{x = 12}$

Check:

$\dfrac{12}{4} + 5 = 3 + 5 = 8$ ✔

40 min

Activity ***Notes***

8

PbY6b page 144, Q.5

Read: *We weighed out equal packs in kg. What can you write about the mass, m, of one pack?*

Show the possible values for each pack on an appropriate segment of the number line.

T has inequalities and segment of number line already on BB or SB or OHT. Deal with one at a time. Class reads out the inequality in unison.

Ps come to BB to write each line of the solution, explaining reasoning. Class points out errors. T asks class for possible values of *m* as a check. After agreeement, T chooses a P to show the solution on the number line.

Solution:

a) $7 \times m + 1 \le 22$ [– 1]

$7 \times m \le 21$ [÷ 7]

$m \le 3$

but as the mass of a pack cannot be negative or zero (or there wouldn't be a pack) the correct solution in context is:

$\underline{0 < m \le 3}$

b) $4 \times m + 32 > 12 \times m$ [$- 4 \times m$]

$32 > 8 \times m$ [÷ 8]

$4 > m$

but as the mass of a pack cannot be negative or zero, the solution in context is:

$\underline{4 > m > 0}$

c) $29.5 < 5 \times m + 2 < 32$ [Decrease each part by 2]

$27.5 < 5 \times m < 30$ [Divide each part by 5]

$\underline{5.5 < m < 6}$

Ask Ps to say what each solution means in the context.

a) Each pack weighs more than 0 kg but less than, or equal to, 3 kg.

b) Each pack weighs more than 0 kg but less than 4 kg.

c) Each pack weighs more than 5.5 kg but less than 6 kg.

45 min

Whole class activity

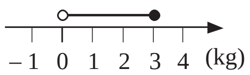
(or individual trial first under a time limit if Ps wish and there is time)

(or if class is able, ask Ps to draw the appropriate number line segment on BB)

Discussion, reasoning, agreement, praising

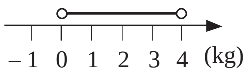
Extra praise for Ps who realise that in the given context, $m \le 3$ is not the whole solution.

BB:

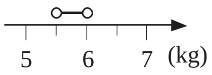


A *white* dot shows that the number is <u>not</u> included in the solution; a *black* dot shows that the number <u>is</u> included.

BB:



BB:



or

Each pack weighs between 5.5 kg and 6 kg.

Y6

Lesson Plan *145*

Activity

Factorising 145, 320, 495 and 1145. Revision and practice.

PbY6b, page 145

Solutions:

Q.1 a) $5 \times n + 4 = 39$ $[-4]$ *Check*: $5 \times 7 + 4 = 39$ ✔

$5 \times n = 35$ $[\div 5]$

$\underline{n = 7}$

Answer: My number is 7.

b) $\frac{n}{2} + 7 = 2 \times n$ $(\times 2)$

$n + 14 = 4 \times n$ $[-n]$

$14 = 3 \times n$ $[\div 3]$

$\underline{4\frac{2}{3} = n}$ or $\underline{n = 4\frac{2}{3}}$

Check:: LHS: $4\frac{2}{3} \div 2 + 7 = 2\frac{1}{3} + 7 = 9\frac{1}{3}$

RHS: $2 \times 4\frac{2}{3} = 8\frac{4}{3} = 9\frac{1}{3}$ ✔

Answer: My number is 4 and 2 thirds.

c) e.g. Let smaller number be x; then greater number is $x + 5$.

$x + x + 5 = 27$ $[-5]$

$2x = 22$ $[\div 2]$

$\underline{x = 11}$ so greater number: $11 + 5 = \underline{16}$

Answer: The two numbers are 11 and 16.

Q.2 a) $A_{\text{triangle}} = \frac{b \times h}{2}$ b) $P_{\text{octagon}} = 8 \times a$ $(= 8a)$

c) $A_{\text{cuboid}} = 2 \times (a \times b + b \times c + a \times c) = \underline{2 \times (ab + bc + ac)}$

d) $V_{\text{cuboid}} = a \times 2a \times 3a = a \times 2 \times a \times 3 \times a$

$= 6 \times a^3 = \underline{6a^3}$

e) $A = 2 \times (a \times a + a \times 2a + a \times 2a)$

$= 2 \times (a^2 + 2a^2 + 2a^2) = 2 \times 5a^2 = \underline{10a^2}$

Q.3 a) $a - (-4) > -2$ $[+(-4)]$

$a > -2 + (-4)$

$\underline{a > -6}$

b) $\frac{4}{5} + (-b) \leq \frac{3}{10}$ $[-\frac{4}{5} = -\frac{8}{10}]$

$-b \leq -\frac{5}{10}$ $[\times -1]$

Show on the number line why the sign needs to change!

$\underline{b \enclose{circle}{\geq} \frac{5}{10}}$ $(= \frac{1}{2})$

Check: $\underline{b = \frac{5}{10}}$:

$\frac{8}{10} + (-\frac{5}{10})$

$= \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$ ✔

Also check that numbers <u>less</u> than a half do <u>not</u> make the inequality true.

Notes

$\underline{145} = 5 \times 29$

Factors: 1, 5, 29, 145

$\underline{320} = 2^6 \times 5$

Factors: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 64, 80, 160, 320

$\underline{495} = 3^2 \times 5 \times 11$

Factors: 1, 3, 5, 9, 11, 15, 33, 45, 55, 99, 165, 495

$\underline{1145} = 5 \times 229$

Factors: 1, 5, 229, 1145

(or set factorising as homework at the end of *Lesson 144* and review at the start of *Lesson 145*.

Check:

$11 + 16 = 27$ ✔

$16 - 11 = 5$ ✔

a) *Check*:

$\underline{a = -7}$: $-7 - (-4) = -3$

and $-3 \not> -2$ ✗

$\underline{a = -6}$: $-6 - (-4) = -2$

and $-2 \not> -2$ ✗

$\underline{a = -5}$: $-5 - (\ 4) = -1$

and $-1 > -2$ ✔

b) *Check*: $\underline{b = \frac{6}{10}}$:

$\frac{8}{10} - \frac{6}{10} = \frac{2}{10} < \frac{3}{10}$ ✔

Y6

Lesson Plan 145

Activity | ***Notes***

Solutions: (continued)

Q.3 c) $c + (+4) \geq +4$ $[-4]$

$\underline{c \geq 0}$

Check: $\underline{c = 0}$: $0 + (+4) = 4$ ✔

$\underline{c = 3}$: $3 + (+4) = 7 > +4$ ✔

$\underline{c = -1}$: $-1 + (+4) = 3 \not\geq 4$ ✗

Q.4 a) $x + 6.2 = 9.3$ $[-6.2]$ *Check*: $\underline{3.1} + 6.2 = 9.3$ ✔

$\underline{x = 3.1}$

b) $-3.7 + y = 5$ $[+3.7]$ *Check*: $-3.7 + \underline{8.7} = 5$ ✔

$\underline{y = 8.7}$

c) $z \times 2 = \frac{1}{4}$ $[\div 2]$ *Check*: $\underline{\frac{1}{8}} \times 2 = \frac{2}{8} = \frac{1}{4}$ ✔

$\underline{z = \frac{1}{8}}$

d) $3 \times a = a + 5$ $[-a]$ *Check*: LHS: $3 \times \underline{2.5} = 7.5$

$2 \times a = 5$ RHS: $\underline{2.5} + 5 = 7.5$ ✔

$\underline{a = 2.5}$

e) $5 \times b + 2 = 3 \times b - 8$ $[-3 \times b]$

$2 \times b + 2 = -8$ $[-2]$

$2 \times b = -10$ $[\div 2]$

$\underline{b = -5}$

Check:

LHS: $5 \times (\underline{-5}) + 2 = -23$

RHS: $3 \times (\underline{-5}) - 8 = -23$ ✔

f) $\frac{c}{3} - 2 = 8$ $[+2]$ *Check*: $\frac{30}{3} - 2 = 10 - 2 = 8$ ✔

$\frac{c}{3} = 10$ $[\times 3]$

$\underline{c = 30}$

Q.5 a)

U	3	4	5	6	7	8	9	10	11	12
E	$35\frac{1}{2}$	$36\frac{3}{4}$	38	$39\frac{1}{4}$	$40\frac{1}{2}$	$41\frac{3}{4}$	43	$44\frac{1}{4}$	$45\frac{1}{2}$	$46\frac{3}{4}$

Details: e.g. $U = 3$:

$$E = \frac{5}{4} \times \underline{3} + \frac{127}{4}$$

$$= \frac{15}{4} + \frac{127}{4} = \frac{142}{4}$$

$$= \frac{71}{2} = \underline{35\frac{1}{2}}$$

After 3 or 4 columns have been completed, Ps might notice that the European sizes form a sequence with rule $+1\frac{1}{4}$. Extra praise for this!

b) i) $E = 40 \rightarrow U = 6\frac{1}{2}$ (between sizes 6 and 7)

ii) $E = 38 \rightarrow U = 5$

iii) $E = 45 \rightarrow U = 10\frac{1}{2}$ (between sizes 10 and 11)

(Ps read data from table – there is no need to do more calculations.)

Y6

R: Calculations

C: Equations and inequalities. Balance model

E: Word problems. Absolute value

Lesson Plan 146

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 146 = 2 × 73 Factors: 1, 2, 73, 146
- 321 = 3 × 107 Factors: 1, 3, 107, 321
- 496 = 2 × 2 × 2 × 2 × 31 = $2^4 \times 31$

  Factors: 1, 2, 4, 8, 16, 31, 62, 124, 248, 496
- 1146 = 2 × 3 × 191

  Factors: 1, 2, 3, 6, 191, 382, 573, 1146

1146	2
573	3
191	191
1	

8 min

Individual work, monitored (or whole class activity)

BB: 146, 321, 496, 1146

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

146	2
73	73
1	

321	3
107	107
1	

496	2
248	2
124	2
62	2
31	31
1	

2

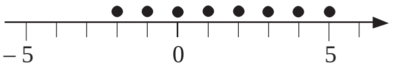
Writing inequalities

Let's write an inequality about the set of numbers marked on these parts of the number line. Why are some shown by dots, some by lines?

Ps come to BB or dictate what T should write. Who agrees? Who can think of another way to write it? T gives hints where necssary. Class checks the inequality with some possible values.

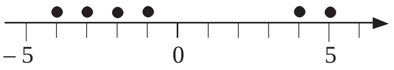
BB: e.g.

a)



$-3 < n < 6,\ n \in Z$

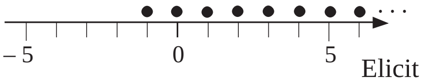
b)



$-4 \leq n \leq -1$ OR

$4 \leq n \leq 5,\ n \in Z$

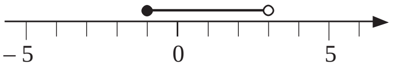
c)



$-1 \leq n,\ n \in Z$

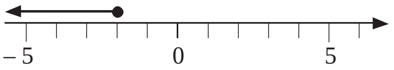
Elicit that '. . .' means 'and so on'.

d)



$-1 \leq n < 3,\ n \in Q$

e)



$n \leq -2,\ n \in Q$

14 min

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Discussion, agreement, praising

or $-2 \leq n \leq 5,\ n \in Z$

or $-2 \leq n < 6,\ n \in Z$

(i.e. *n* is a member of the set of whole numbers or integers)

Two criteria are possible – both must be given.

or $-2 < n,\ n \in Z$

(i.e. *n* is a member of the set of all the numbers we know, or *n* is a <u>rational</u> number)

or $n < -1,\ n \in Q$

3

Solving inequalities

Let's solve these equations and inequalities and show the solution on the number line. The base set is the set of rational numbers. ($n \in Z$)

Ps come to BB to write each row of the solution, explaining reasoning. Class points out errors. When solution has been agreed and checked, Ps come to BB to draw appropriate section of number line (only a rough drawing is needed) and mark the solution.

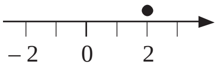
BB:

a) $5 \times x + 3 = x + 11$ [$-x$]

$4 \times x + 3 = 11$ [-3]

$4 \times x = 8$ [$\div 4$]

$\underline{x = 2}$



Whole class activity

Written on BB or SB or OHT

At a good pace

Involve as many Ps as possible.

Reasoning, agreement, checking, praising

Check:

LHS: $5 \times 2 + 3 = 13$

RHS: $2 + 11 = 13$ ✔

Y6

Lesson Plan 146

Activity

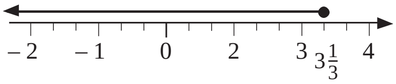
3

(Continued)

b) $5 \times y - 2 \times y \ \leq\ 10$

$3 \times y \ \leq\ 10$ [÷ 3]

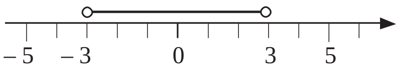
$\underline{y \ \leq\ 3\frac{1}{3}}$



c) $|\,e\,| < 3$

What does this inequality mean?
(The <u>absolute value</u> of *e* is less than 3.)

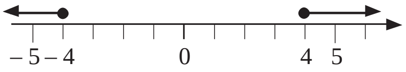
So $\underline{-3 < e < 3}$



d) $|\,f\,| \geq 4$

What does this inequality mean?
(The <u>absolute value</u> of *f* is greater than or equal to 4.)

$\underline{f \leq -4 \text{ OR } f \geq 4}$



Check with various values for *f*. e.g.

$f = -6$: $|-6| = 6 \geq 4$ ✔ $f = -4$: $|-4| = 4$ ✔

$f = -3$: $|-3| = 3 \geq 4$ ✗ $f = 5.5$: $|5.5| = 5.5 \geq 4$ ✔

Notes

Check with some values, e.g.

$y = 3\frac{1}{3}$, $y = 3$, $y = 4$

Ps check in *Ex. Bks* or at side of BB.

Elicit that the absolute value of a number is how far it is from zero.

Check with some values for *e*.

Two sets of numbers are possible (for negative *t* and for positive *t)* and <u>both</u> should be stated in the solution.

21 min

4

PbY6b, page 146

Q.1 Read: *Solve the equations and check your results.*

Set a time limit or deal with one at a time (or do the more difficult ones with the whole class). Ps can use any valid method.

Review with whole class. Ps could show results on scrap paper or slates on command. Ps with correct answers explain reasoning on BB. Who did the same? Who did it a different way? Come and show us. etc. Mistakes discussed and corrected.

Solution: e.g.

a) $2 \times a + a = \frac{21}{40}$

$3 \times a = \frac{21}{40}$ [÷ 3]

$\underline{a = \frac{7}{40}}$

Check:

$2 \times \frac{7}{40} + \frac{7}{40}$

$= \frac{14}{40} + \frac{7}{40} = \frac{21}{40}$ ✔

b) $b \times \frac{4}{9} = b - 1$ [× 9]

$b \times 4 = 9 \times b - 9$ [+ 9]

$4 \times b + 9 = 9 \times b$ [– 4 × *b*]

$9 = 5 \times b$ [÷ 5]

$\underline{\frac{9}{5} = b}$ or $\underline{b = \frac{9}{5}}$

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

Check:

LHS: $\frac{\overset{1}{\cancel{9}}}{5} \times \frac{4}{\underset{1}{\cancel{9}}} = \frac{4}{5}$ ✔

RHS: $\frac{9}{5} - 1 = \frac{9}{5} - \frac{5}{5} = \frac{4}{5}$

Y6

Lesson Plan 146

Activity

4

(Continued)

c) $35 \div c = 14, \quad c = 35 \div 14 = 5 \div 2 = \underline{2\frac{1}{2}}$

d) $7 \times (d - 2) = d - 2$

$7 \times d - 14 = d - 2 \quad [-\,d]$

$6 \times d - 14 = -\,2 \quad [+\,14]$

$6 \times d = 12 \quad [\div\,6]$

$\underline{d = 2}$

or $\quad [-\,(d - 2)]$

$6 \times (d - 2) = 0$

$6 \times d - 12 = 0 \quad [+12]$

$6 \times d = 12 \quad [\div\,6]$

$\underline{d = 2}$

e) $(4 - e) \times 5 = -\,5 \times e + 20$

$20 - 5e = -\,5e + 20 \quad [-\,20]$

$-\,5e = -\,5e$

So e can be <u>any</u> rational number and the solution is: $\underline{e \in Q}$

f) $6 \times f - 3 \times f = \frac{3}{8} + f \times 3$

$3 \times f = \frac{3}{8} + f \times 3 \quad [-\,3 \times f]$

$\cancel{0 = \frac{3}{8}}$

BUT $0 \neq \frac{3}{8}$, so the equation is impossible! $\quad \underline{f = \varnothing}$

27 min

Notes

Check:

$35 \div \frac{5}{2} = \overset{7}{\cancel{35}} \times \frac{2}{\underset{1}{\cancel{5}}} = 14$ ✔

Check:

LHS: $7 \times (\underline{2} - 2) = 7 \times 0 = 0$

RHS: $2 - 2 = 0$ ✔

When the LHS of an equation is <u>identical</u> to the RHS, we say that it is an i<u>dentity</u>:

e.g. $3a = 3a$, $-\,y = -\,y$, etc. and states what is obvious.

Extra praise if Ps did this correctly without T's help.

Elicit the notation for an empty set.

5

PbY5b, page 146, Q.2

Read: *Gerry Giraffe eats the same amount of leaves every day.*

His keeper told the other keepers in the zoo that:

- *three of Gerry's daily portions plus five kilograms are less than 29 kg, but*
- *five of Gerry's daily portions plus 4 kg are more than 34 kg.*

What could be the mass of Gerry's daily portion of leaves?

Let's call the unknown mass of leaves *m*. Who could write an inequality about *m*? Who could write another one? Class agrees/disagrees.

Ps come to BB to solve the inequalities or dictate what T should write.

How could we show the solutions on the number line? Ps suggest the part of the number line needed and come to BB to draw it and mark the solutions of the two inequalities. Which numbers make <u>both</u> solutions true? How could we mark them on the number line? How could we write it as a single inequality? Ps come to BB. Class agrees/disagrees.

Solution:

a) *Write two inequalities about the portion of leaves.*

b) *Solve the inequalities and note the values which make* ***both*** *inequalities true.*

c) *Write your answer in a sentence.*

$3 \times m + 5\text{ kg} < 29\text{ kg} \quad [-\,5\text{ kg}]$, $\qquad 5 \times m + 4\text{ kg} > 34\text{ kg} \quad [-\,4\text{ kg}]$

$3 \times m < 24\text{ kg} \quad [\div\,3] \qquad 5 \times m > 30\text{ kg} \quad [\div\,5]$

$\underline{m < 8\text{ kg}}$ AND $\underline{m > 6\text{ kg}}$

Answer: Gerry eats between 6 kg and 8 kg of leaves each day.

31 min

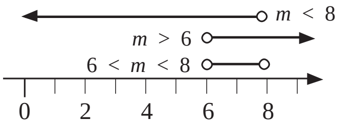
Notes

Whole class activity

(or individual trial first if Ps wish, reviewed as usual)

Discussion, reasoning, agreement, (self-correction), praising

BB:



T asks a P to say the answer in a sentence.

Y6

Lesson Plan 146

Activity

Notes

6

PbY6b, page 146

Q.3 Read: *Show the relationship among the data by writing an equation. Solve the equation and check the result in the given context.*

Deal with one at a time. Ps read question themselves and solve it in *Ex. Bks.* under a short time limit.

Review with whole class. Ps could show answer on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected. T asks a P to say the answer in a sentence.

Individual work, monitored, helped

Encourage Ps to draw digrams to help them understand the problem.

Responses shown in unison

Discussion, reasoning, agreement, self-correction, praising

Accept any valid method.

Solutions: e.g.

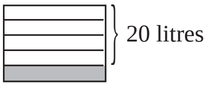
a) *One fifth of a barrel is 20 litres less than the capacity of the whole barrel. What is the capacity of the barrel?*

e.g. Let the capacity in litres be c.

$$c \div 5 = c - 20 \qquad [\times 5]$$
$$c = 5c - 100 \qquad [+ 100]$$
$$c + 100 = 5c \qquad [- c]$$
$$100 = 4c \qquad [\div 4]$$
$$\underline{25 = c} \quad \text{or} \quad \underline{c = 25}$$

Answer: The capacity of the barrel is 25 litres.

BB:



or $\frac{4}{5} \times c = 20$

$$c = 20 \div \frac{4}{5} = \overset{5}{\cancel{20}} \times \frac{5}{\cancel{4}_1}$$
$$= \underline{25} \text{ (litres)}$$

Check: LHS: $25 \div 5 = 5$
RHS: $25 - 20 = 5$ ✔

b) *On Monday, a shop sold x kg of honey. On Tuesday it sold 11 kg more than on Monday, and on Wednesday it sold 5 kg more than on Monday.*

How much honey did the shop sell on each of these days if the total amount of honey sold was 220 kg?

Monday: x, Tuesday: $x + 11$, Wednesday: $x + 5$

Plan: $x + x + 11 + x + 5 = 220$

$$3x + 16 = 220 \qquad [- 16]$$
$$3x = 204 \qquad [\div 3]$$
$$\underline{x = 68}$$

Answer: The shop sold 68 kg of honey on Monday, 79 kg on Tuesday and 73 kg on Wednesday.

Check: $68 + 79 + 73 = 220$ ✔

c) *In one container there is twice as much water as there is in a second container.*

If we took 30 litres of water out of the first container and 12 litres of water out of the second container, both containers would hold the same amount of water.

How much water is in each container?

Amount in 1st container: $2x$, Amount in 2nd container: x

Plan: $2x - 30 = x - 12 \qquad [+ 12]$

$$2x - 18 = x \qquad [+ 18]$$
$$2x = x + 18 \qquad [- x]$$
$$\underline{x = 18} \text{ (litres)}$$

Accept any valid steps towards the solution.

Answer: There are 36 litres of water in the first container and 18 litres of water in the second container.

Check: LHS: $36 - 30 = 6$
RHS: $18 - 12 = 6$ ✔

Y6

Lesson Plan 146

Activity

Notes

6

(Continued)

d) *Let* ① *mean £1,* $\boxed{-1}$ *mean – £1 and* ✉ *mean an £x banknote in an envelope.*

Betty has: ① $\boxed{-1}$ $\boxed{-1}$ $\boxed{-1}$ ✉ ✉

Larry has: ① ① ① ① ① ① ① ① ① $\boxed{-1}$ ✉

If Betty has the same amount of money as Larry, what value of banknote is in each envelope?

Plan: $1 + (-3) + 2 \times x = 9 + (-1) + x$ $[-x]$

$-2 + x = 8$ $[+2]$

$\underline{x = 10}$

Answer: There is a £10 banknote in each envelope.

45 min

Symbols drawn (stuck) on BB.

BB: B = L, ✉ =

Ps could manipulate the symbols on BB to model what is being done to the equation.

Y6

R: Calculations

C: Equations and inequalities. Word problems

E: Advanced problems

Lesson Plan 147

Activity

Notes

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- $\underline{147} = 3 \times 7 \times 7 = 3 \times 7^2$ Factors: 1, 3, 7, 21, 49, 147
- $\underline{322} = 2 \times 7 \times 23$
  Factors: 1, 2, 7, 14, 23, 46, 161, 322
- $\underline{497} = 7 \times 71$ Factors: 1, 7, 71, 497
- $\underline{1147} = 31 \times 37$ Factors: 1, 31, 37, 1147

Individual work, monitored (or whole class activity)

BB: 147, 322, 497, 1147

(Ps use calculators for 1147.)

Reasoning, agreement, self-correction, praising

e.g.

$$\begin{array}{r|l} 322 & 2 \\ 161 & 7 \\ 23 & 23 \\ 1 & \end{array} \qquad \begin{array}{r|l} 497 & 7 \\ 71 & 71 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 147 & 3 \\ 49 & 7 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1147 & 31 \\ 37 & 37 \\ 1 & \end{array}$$

8 min

2

PbY6b, page 147, Q.1

Read: *The b**ase set** is the set of natural numbers. (N)*

Write an equation or an inequality, solve it and check your result.

Which number am I thinking of?

Deal with one part at a time. T chooses a P to read out the description. Allow 1 minute for Ps to solve it in *Ex. Bks*, then Ps show the number on scrap paper or slates on command.

P answering correctly explains reasoning at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected.

Solution:

a) *I add 5 to 3 times my number and the result is 53.*

Plan: $3 \times n + 5 = 53$ [– 5]

$3 \times n = 48$ [÷ 3]

$\underline{n = 16}$

Check: $3 \times \underline{16} + 5 = 48 + 5$

$= 53$ ✔

Answer: The number I am thinking of is 16.

b) *I subtract 18 from 7 times my number and the result is 269.*

Plan: $7 \times n - 18 = 269$ [+ 18]

$7 \times n = 287$ [÷ 7]

$\underline{n = 41}$

Check: $7 \times \underline{41} - 18 = 287 - 18$

$= 269$ ✔

Answer: The number I am thinking of is 41.

c) *I subtract 4 times my number from 7 times my number and the result is 156.*

Plan: $7 \times n - 4 \times n = 156$

$3 \times n = 156$ [÷ 3]

$\underline{n = 52}$

Check: $7 \times \underline{52} - 4 \times \underline{52}$

$= 364 - 208 = 156$ ✔

Answer: The number I am thinking of is 52.

d) *I add 6 to 5 times my number and the result is less than 26.*

Plan: $5 \times n + 6 < 26$ [– 6]

$5 \times n < 20$ [÷ 5]

$\underline{n < 4}$

Answer: The number I am thinking of could be 1, 2 or 3.

Whole class activity but individual calculation

Responses shown in unison.

In good humour.

Reasoning, checking, agreement, self-correction, praising

Feedback for T

Check:

$n = 1$: $5 \times 1 + 6 = 11 < 26$ ✔
$n = 2$: $5 \times 2 + 6 = 16 < 26$ ✔
$n = 3$: $5 \times 3 + 6 = 21 < 26$ ✔
$n = 4$: $5 \times 4 + 6 = 26 \not< 26$ ✗

(as the base set is the set of natural numbers.)

Y6

Lesson Plan 147

Activity

Notes

2 (Continued)

d) *The difference between 7 times and 5 times my number is greater than 50.*

Plan: $7 \times n - 5 \times n > 50$

$2 \times n > 50$ [$\div$ 2]

$\underline{n > 25}$

(i.e. 26, 27, 28, . . .)

Answer: The number I am thinking of could be any natural number greater than 25.

15 min

3 ***PbY6b, page 147***

Q.2 Read: *Solve each problem in two ways, with and without an equation.*

How could you solve the problem if you don't write an equation? (e.g. writing an operation, drawing a digram, explaining in words)

Deal with one part at a time under a short time limit. Ps read problem themselves, solve it in two ways in *Ex. Bks*, check their result and write the answer in a sentence.

Review with whole class. T chooses a P to read out the question. **A**, read us your answer. Who agrees with **A**? Who has another answer? Who agrees with that? Ps come to BB to write the equation and to show other methods of solution. Class checks result and agrees on correct answer. Mistakes discussed and corrected. T asks some Ps which method they prefer and why.

Solution: e.g.

a) *Daffy Duck is twice as old as Donald Duck. If the sum of their ages is 21 months, how old is Daffy and how old is Donald?*

Donald: n $2 \times n + n = 21$

Daffy: $2 \times n$ $3 \times n = 21$ [$\div$ 3]

$\underline{n = 7}$

Check: $2 \times 7 + 7 = 14 + 7 = 21$ ✓

Answer: Daffy is 14 months old and Donald is 7 months old.

b) *A 120 cm long stick is cut into two pieces so that one of the pieces is 30 cm longer than the other piece.*

How long is each piece?

Short piece: x $x + x + 30 = 120$ [– 30]

Long piece: $x + 30$ $2 \times x = 90$ [$\div$ 2]

$\underline{x = 45}$

Check: $45 + (45 + 30) = 45 + 75 = 120$ ✓

Answer: The long piece is 75 cm and the short piece is 45 cm.

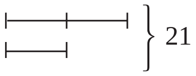
Individual work, monitored, helped

Discuss the meaning of 'equation'here. Elicit that it involves an unknown amount shown by a letter or symbol, whereas an 'operation' involves only numbers and operation signs.

Reasoning, checking, agreement, self-correction, praising

Extra praise if Ps draw diagrams, as shown below.

or



or $21 \div 3 = \underline{7}$

as the sum of their ages is divided into 3 equal parts: Donald's age is 1 part, Daffy's age is 2 parts.

or

If both pieces were the same length as the <u>short</u> piece:

$(120 - 30) \div 2 = 90 \div 2 = \underline{45}$

Long piece: $45 + 30 = \underline{75}$

or

If both pieces were the same length as the <u>long</u> piece:

$(120 + 30) \div 2 = 150 \div 2 = \underline{75}$

Short piece: $75 - 30 = \underline{45}$

Y6

Lesson Plan 147

Activity | ***Notes***

3

(Continued)

c) *Liz has twice as many marbles as Julia. If Liz gave Julia 10 marbles they would both have the same amount. How many marbles do Liz and Julia each have?*

Julia: x $\quad x + 10 = 2 \times x - 10 \quad [+ 10]$

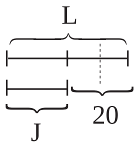
Liz: $2 \times x$ $\quad x + 20 = 2 \times x \quad [- x]$

$\underline{20 = x}$

Check: $20 + 10 = 40 - 10$, and $2 \times 20 = 40$ ✔

Notes:

or, e.g.



Liz must have 2 × 10 = 20 more marbles than Julia.

Answer:
Liz has 40 marbles and Julia has 20 marbles.

25 min

4

PbY6b, Page 147

Q.3 Read: *Solve the problems by writing equations.*

Set a time limit or deal with one at a time. Ps can use any letter or symbol they wish for the unknown amounts.

Ask Ps to think about how to put the explanation in a) i) into words first before they write it in *Ex. Bks*.

Review with whole class. Where possible, Ps show results on slates or scrap paper on command. Ps answering correctly explain reasoning at BB. Class agrees/ disagrees. Mistakes discussed and corrected.

Solution:

a) *Joe's bank balance is £37. Joe's and Charlie's bank balances add up to £25.*

i) *Explain how this is possible.* e.g.

'Charlie's bank balance is negative so the sum of Joe's and Charlie's accounts is less than Joe's balance.

ii) *How much money is in Charlie's account?*

J: £37, J + C: £25, C: x

$37 + x = 25 \quad [- 37]$

$x = \underline{-12}$

Answer: Charlie's balance is – £12 (or Charlie is £12 in debt).

b) *Claire's bank balance is £2.50. Claire's and Mike's balances add up to £31.*

How much money does Mike have in his account?

C: £2.50, C + M: £31, M: x

$2.5 + x = 31 \quad [- 2.5]$

$x = \underline{28.5}$

Answer: Mike has £28.50 in his account.

c) *Colin's balance is £4.50 less than Pete's balance. How much do they each have if the sum of their balances is £2.20?*

C = P – £4.50, P + C = £2.20

$P + P - 4.5 = 2.2 \quad [+ 4.5]$

$2 \times P = 6.7 \quad [\div 2]$

$\underline{P = 3.35}$

C = £3.35 – £4.50 = $\underline{-\text{£}1.15}$

Answer: Peter has £3.35 and Colin has – £1.15.

Notes:

Individual work, monitored, (helped)

Differentiation by time limit.

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

Feedback for T

T asks several Ps to read out their sentences. Ps say what they think of them. If Ps hear a better worded explanation than their own, encourage them to write it in *Ex Bks*. too.

Check:
£37 + (– £12) = £37 – £12
= £25 ✔

Check:
£2.50 + £28.50 = £31 ✔

Check:
3.35 + (– 1.15) = 2.20 ✔

(or Colin is £1.15 in debt.)

35 min

Y6

Lesson Plan 147

Activity

5

PbY6b, page 147*, *Q.4

Read: *Solve the problems by writing equations*.

Deal with one part at a time. Ps decide what to do first and how to continue. T helps and guides where necessary by asking appropriate questions. Class points out any errors or suggests better steps.

[e.g. for a):

What is the unkown amount? What shall we call it? Who can write an amount for Alex (Ben)? Which of them has more? How many times more? Who can write an equation about the relationship between their money? How can we write it in a simpler way? What should we do first to solve the equation? What should we do next? How can we check the result? Who can say the answer in a sentence?]

Ps could write the solution in *Ex. Bks* at the same time.

Solution: e.g.

a) *Alex has £100 in cash, is £300 in debt and has two savings bonds of equal value. Ben is £100 in debt, has £400 in cash and has 3 such saving bonds.*

If Ben has 3 times as much money as Alex, how much is a savings bond worth?

Let a savings bond be worth x.

Then $A = 100 + (-300) + 2 \times x$,

$3 \times A = B = -100 + 400 + 3 \times x$

$$\begin{aligned}
3 \times [100 + (-300) + 2 \times x] &= -100 + 400 + 3 \times x \\
3 \times (-200 + 2 \times x) &= 300 + 3 \times x \\
-600 + 6 \times x &= 300 + 3 \times x \qquad [+ 600] \\
6 \times x &= 900 + 3 \times x \qquad [-3 \times x] \\
3 \times x &= 900 \qquad [\div 3] \\
\underline{x = 300}\ (£)
\end{aligned}$$

Answer: A savings bond is worth £300.

b) *Adam has £100 in cash, is £300 in debt and has a savings bond.*

Matthew has £500 in cash, is £100 in debt and also has a savings bond.

Norah has £100 in cash, is £200 in debt and has two savings bonds.

If the sum of the two boys' money is greater than twice Norah's money, how much can one of their savings bonds be worth?

Let a savings bond be worth x.

BB: $A = 100 + (-300) + x$, $N = 100 + (-200) + 2 \times x$

$M = 500 + (-100) + x$, $A + M > 2 \times N$

$$\begin{aligned}
100 + (-300) + x + 500 + (-100) + x &> 2 \times [100 + (-200) + 2 \times x] \\
600 - 400 + 2 \times x &> 200 - 400 + 4 \times x \\
200 + 2 \times x &> -200 + 4 \times x \qquad [+ 200] \\
400 + 2 \times x &> 4 \times x \qquad [-2 \times x] \\
400 &> 2 \times x \qquad [\div 2] \\
\underline{200 > x} \text{ or } \underline{x < 200}
\end{aligned}$$

Answer: A savings bond is worth less than £200.

Notes

Whole class activity

(or individual trial first if Ps wish and there is time, monitored, <u>helped</u>, corrected)

Involve as many Ps as possible.

At a reasonable pace, in good humour!

Discussion, reasoning, agreement, checking, (self-correction), praising

Feedback for T

Check:

A: $100 + (-300) + 2 \times 300 = -200 + 600 = \underline{400}$

B: $-100 + 400 + 3 \times 300 = 300 + 900 = \underline{1200}$

and $1200 = 3 \times 400$ ✔

Check:

If $x = 200$:

A: $100 + (-300) + 200 = \underline{0}$

M: $500 - 100 + 200 = \underline{600}$

N: $100 - 200 + 400 = \underline{300}$

and $A + M = 600 \not> 2 \times N$ ✗

If $x < 200$, e.g. 100

A: $100 - 300 + 100 = -100$

M: $500 - 100 + 100 = \underline{500}$

N: $100 - 200 + 200 = \underline{100}$

$A + M = 400 > 2 \times N$ ✔

If $x > 200$, e.g. 201

A: $100 - 300 + 201 = 1$

M: $500 - 100 + 201 = \underline{601}$

N: $100 - 200 + 402 = \underline{302}$

$A + M = 602 \not> 2 \times N$ ✗

45 min

Y6

R: Calculations
C: Equations and inequalities
E: Advanced problems

Lesson Plan
148

Activity

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>148</u> $= 2 \times 2 \times 37 = 2^2 \times 37$ Factors: 1, 2, 4, 37, 74, 148
- <u>323</u> $= 17 \times 19$ Factors: 1, 17, 19, 323
- <u>498</u> $= 2 \times 3 \times 83$
  Factors: 1, 2, 3, 6, 83, 166, 249, 498
- <u>1148</u> $= 2 \times 2 \times 7 \times 41 = 2^2 \times 7 \times 41$
  Factors: 1, 2, 4, 7, 14, $\underbrace{28,\ 41}$, 82, 164, 287, 574, 1148

8 min

Notes

Individual work, monitored (or whole class activity)

BB: 148, 323, 498, 1148

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

$$\begin{array}{r|l} 148 & 2 \\ 74 & 2 \\ 37 & 37 \\ 1 & \end{array} \qquad \begin{array}{r|l} 323 & 17 \\ 19 & 19 \\ 1 & \end{array} \qquad \begin{array}{r|l} 498 & 2 \\ 249 & 3 \\ 83 & 83 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1148 & 2 \\ 574 & 2 \\ 287 & 7 \\ 41 & 41 \\ 1 & \end{array}$$

2

Problem

Listen carefully to this problem and note down the important data.

The perimeter of a quadrilateral is 117 cm. Side b is 4 cm shorter than side a. Side c is twice as long as side a and side d is 5 cm less than side c. What length is each side?

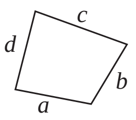
a) Is there any missing or irrelevant data?

(There is just enough data given to solve the problem and no irrelevant data.)

b) What should we do now? (Draw a <u>diagram</u>.)

P draws a quadrilateral on BB and labels its sides. Now what should we do? (Write down what we know.) Other Ps come to BB to write down the <u>relationships</u> (or dictate to T).

BB: e.g.



$P = a + b + c + d = 117$ cm
$b = a - 4$ cm, $c = 2 \times a$
$d = c - 5$ cm $= 2 \times a - 5$ cm

c) What should we do now? (Make a <u>plan</u>.) Let's write the plan as one equation. Ps come to BB or dictate to T. Class points out errors.

BB: *Plan:* $a + (a - 4) + 2 \times a + (2 \times a - 5) = 117$

Is this a good plan? (Yes) Can anyone think of a better one? (No)

d) What should we do before we solve our plan? (<u>Estimate</u> the solution.) How could we estimate the length of side *a*? (e.g. If the sides were equal, then the length of each side would be about 30 cm.)

e) Now let's <u>solve</u> the equation. Ps come to BB to write and explain each step (or dictate steps to T). Class agrees/disagrees.

BB: $a + (a - 4) + 2 \times a + (2 \times a - 5) = 117$

$$\begin{aligned} 6 \times a - 9 &= 117 && [+ 9] \\ 6 \times a &= 126 && [\div 6] \\ \underline{a = 21}\ &\text{(cm)} \end{aligned}$$

Is this the answer to the question? (No, this is only side *a* – we need to work out the lengths of all the sides.) Ps dictate to T. (BB)

f) What should we do now? (Check the result.) How can we do it?

Notes

Whole class activity

T could have problem written on BB or SB or OHT

Revision of the steps needed to solve word problems.

Involve as many Ps as possible.

T leads or directs as necessary.

Discussion, reasoning, agreeement, praising

Ps could write solution in *Ex. Bks.* at the same time.

Elicit that writing *d* as an expression of *a* will help in the solution – it will ensure that only one unknown amount needs to be dealt with.

If no P suggests this equation, T starts and Ps continue.

BB: *E*: $117 \text{ cm} \div 4$
$\approx 120 \text{ cm} \div 4$
$= 30$ cm

So $a = 21$ cm
$b = 17$ cm
$c = 42$ cm
$d = 37$ cm

Check:
$P = 21 + 17 + 42 + 37 = 117$ ✔

Activity

2

(Continued)

h) What is the last thing we should do? (Write the answer in a <u>sentence</u>.) Ps dictate what T should write.

BB: *Answer*: The lengths of the sides of the quadrilateral are 21 cm, 17 cm, 42 cm and 37 cm.

T asks Ps to think about other things too. e.g.

- Is the answer realistic? (Yes, it is)
- Could we have solved the problem in a better way?

  (Elicit/point out that we could have expressed each side in terms of *b* rather than *a* but it would have been more complicated, so we used the best method.)

15 min

Notes

Let's summarise the steps needed to solve a word problem.

1. Note the relevant data.
2. Draw a diagram and/or write down what we know.
3. Look for relationships.
4. Write a plan.
5. Estimate the result.
6. Do the calculation.
7. Check the result in context.
8. Write the answer in a sentence.

3

PbY6b, page 148

Q.1 Read: *Solve the equations and inequalities. Check your results.*

Set a time limit or deal with one at a time. Ps work in *Ex. Bks.*

Review with whole class. Ps could show the solutions on scrap paper or slates on command. Ps responding correctly explain reasoning at BB. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution:

a) $x - \frac{2}{5} = \frac{7}{10}$ $\quad [+ \frac{2}{5} = \frac{4}{10}]$

$x = \frac{11}{10} = \underline{1\frac{1}{10}}$

b) $y - \frac{3}{4} > \frac{3}{4} - y$ $\quad [+ y]$

$2 \times y - \frac{3}{4} > \frac{3}{4}$ $\quad [+ \frac{3}{4}]$

$2 \times y > \frac{6}{4}$ $\quad [\div 2]$

$\underline{y > \frac{3}{4}}$

c) $\frac{4}{5} + u = u + \frac{12}{15}$ $\quad [-u]$

$\frac{4}{5} = \frac{12}{15}$ $\quad$ [Simplify RHS]

$\frac{4}{5} = \frac{4}{5}$

This is an <u>identity</u>, so *u* can be <u>any</u> rational number, i.e. $\underline{u \in Q}$

d) $\frac{2}{3} \times t = \frac{6}{30}$ $\quad \left[\div \frac{2}{3}\right]$

$t = \frac{6}{30} \div \frac{2}{3} = \frac{\overset{3}{\cancel{6}}}{\underset{10}{\cancel{30}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}}}$, $\quad \underline{t = \frac{3}{10}}$

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, checking, agreement, self-correcting, praising

Feedback for T

Checks:

a) $\frac{11}{10} - \frac{4}{10} = \frac{7}{10}$ ✔

b) $y > \frac{3}{4}$: e.g. $y = 1$:

LHS: $1 - \frac{3}{4} = \frac{1}{4}$

RHS $\frac{3}{4} - 1 = -\frac{1}{4}$

so LHS > RHS ✔

If $y = \frac{3}{4}$,

LHS = 0 = RHS ✗

If $y < \frac{3}{4}$, e.g. 0:

LHS: $-\frac{3}{4}$, RHS: $\frac{3}{4}$

so LHS < RHS ✗

Check:

$\frac{2}{3} \times \frac{3}{10} = \frac{6}{30}$ ✔

Y6

Lesson Plan 148

Activity		***Notes***
3	(Continued) e) $0.2 \times v + 0.85 \;\leq\; 1.7 \times v - 0.8$ [+ 0.8] $0.2 \times v + 1.65 \;\leq\; 1.7 \times v$ [– 0.2 × *v*] $1.65 \;\leq\; 1.5 \times v$ [÷ 1.5] $\underline{1.1 \;\leq\; v}$ or $\underline{v \;\geq\; 1.1}$ f) $w + 0.3 = 0$ [– 0.3] $\underline{w = -0.3}$	*C*: $1.65 \div 1.5 = 16.5 \div 15$ $= 3.3 \div 3 = \underline{1.1}$ *Check* for: $v = 1.1$, (✗) $v > 1.1$, e.g. $v = 2$ (✔) $v < 1.1$, e.g. $v = 1$ (✗) *Check*: $-0.3 + 0.3 = 0$ ✔
	22 min	
4	***PbY6b, page 148*** Q.2 Read: *Write an equation for each question. Solve the equation and check your result.* Deal with one at a time or set a time limit. Ps read questions themselves and solve them in *Ex. Bks*. Review with the whole class. Ps could show numbers on scrap paper or slates on command. Ps answering correctly explain reasoning at BB. Who did the same? Who did it a different way? Mistakes discussed and corrected. *Solutions*: e.g. a) *Brian said, "If I add my number to a quarter of my number I get* $12\frac{1}{2}$. *What is my number?"* Let Brian's number be *x*. $x + \frac{x}{4} = 12\frac{1}{2}$ [× 4] $4 \times x + x = 50$ $5 \times x = 50$ [÷ 5] $\underline{x = 10}$ *Answer:* Brian's number is 10. b) *Tom said, "If I add a quarter of my number to half of my number I get the same result as if I had taken 2 away from 4 fifths of my number. What is my number?"* Let Tom's number be *x*. $\frac{x}{2} + \frac{x}{4} = \frac{4}{5} \times x - 2$ [Convert to equivalent fractions with lowest common denominator: 20] $\frac{10 \times x}{20} + \frac{5 \times x}{20} = \frac{16}{20} \times x - 2$ [× 20] $10 \times x + 5 \times x = 16 \times x - 40$ $15 \times x = 16 \times x - 40$ [– 15 × *x*] $0 = x - 40$ [+ 40] $\underline{40 = x}$ *Answer:* Tom's number is 40.	Individual work, monitored, helped Responses shown in unison. Reasoning, checking, agreement, self-correction, praising Accept any valid reasoning. *Check*: $10 + \frac{10}{4} = 10 + 2\frac{1}{2}$ $= 12\frac{1}{2}$ ✔ *Check*: $\frac{40}{2} + \frac{40}{4} = 20 + 10 = 30$ $\frac{4}{5} \times 40 - 2 = 32 - 2 = 30$ ✔

Activity | ***Notes***

4

(Continued)

c) *Two identical bottles contain 2.2 litres of squash altogether.*

One bottle is $\frac{2}{3}$ *full and the other bottle is* $\frac{4}{5}$ *full.*

How much squash is there in a full bottle?

Solution:

Let the capacity of a bottle be *x*.

$$\frac{2}{3} \times x + \frac{4}{5} \times x = 2.2$$ [Convert fractions to lowest common denominator: 15]

$$\frac{10}{15} \times x + \frac{12}{15} \times x = 2.2$$

$$\frac{22}{15} \times x = 2.2 \qquad [\times\ 15]$$

$$22 \times x = 33 \qquad [\div\ 22]$$

$$\underline{x = 1.5} \text{ (litres)}$$

Answer: There are 1.5 litres of squash in a full bottle.

$2.2 \times 15 = 22 + 11 = 33$

$33 \div 22 = 3 \div 2 = 1.5$

Check: $\frac{2}{3}$ of 1.5 + $\frac{4}{5}$ of 1.5 $= 1 + 1.2 = 2.2$ ✔

30 min

5

PbY6b, page 148

Q.3 Read: *Write an equation for each question. Solve the equation and check your result.*

Deal with one at a time. Ps work in *Ex. Bks*.

Review with whole class. T asks a P to read out the question and Ps show answers on scrap paper or slates on command. Ps with correct answer explain reasoning at BB. Class agrees/ disagrees and checks result. Who worked it out another way? Mistakes discussed and corrected.

Solution: e.g.

a) *130% of a number is the same as adding 10.8 to 35% of the number. What is the number?*

Let the number be *x*.

$$\frac{130}{100} \times x = \frac{35}{100} \times x + 10.8 \qquad [- \frac{35}{100} \times x\ ]$$

$$\frac{95}{100} \times x = 10.8 \qquad [\times\ 100]$$

$$95 \times x = 1080 \qquad [\div\ 95]$$

$$x = \frac{1080}{95} = \frac{216}{19} = \underline{11\frac{7}{19}}$$

$$\begin{array}{cc|ccc} & & & 1 & 1 \\ \hline 1 & 9 & 2 & 1 & 6 \\ & - & 1 & 9 & \\ & & & 2 & 6 \\ & & - & 1 & 9 \\ & & & & ⑦ \end{array}$$

Check:

130% – 35% = 95%

$$\frac{95}{100} \times 11\frac{7}{19} = \frac{\overset{1}{\cancel{19}}}{\underset{5}{\cancel{20}}} \times \frac{\overset{54}{\cancel{216}}}{\underset{1}{\cancel{19}}} = \frac{54}{5} = 10\frac{4}{5} = 10.8 ✔$$

[N.B. Substituting $11\frac{7}{19}$ for *x* and working out the LHS and RHS is very difficult. This is an easier way to check it.]

Individual work, monitored, <u>helped</u>

(if Ps are struggling, stop individual work and continue as a whole class activity)

T gives hint about how to check the result in a).

Responses shown in unison.

Reasoning, checking, agreement, self-correction, praising

Accept any valid reasoning.

or

$1.3x = 0.35x + 10.8 \quad [- 0.35x]$

$0.95x = 10.8 \quad [\times\ 100]$

$95x = 1080 \quad [\div\ 95]$

$$x = \frac{1080}{95} = 11\frac{7}{19}$$

Elicit that the exact number cannot be given in decimal form, only an approximation.

Extra praise if a P thinks of this or used this concept for the original equation:

95% of *x* = 10.8, etc.

Answer: The number is $11\frac{7}{19}$.

Y6

Lesson Plan 148

Activity

Notes

5 (Continued)

b) *Lilly has £150 in her purse. This amount is £60 less than 1 sixth of all her money. How much money does Lilly have?*

Let the amount of money be *x*.

$$\frac{x}{6} - 60 = 150 \qquad [+\ 60]$$

$$\frac{x}{6} = 210 \qquad [\times\ 6]$$

$$\underline{x = 1260}\ (£)$$

Answer: Lilly has £1260.

c) *I am thinking of a number. When I subtract 5 thirds of my number from the number itself, than add* $-\frac{1}{2}$ *to the difference, the result is* $-\frac{7}{6}$. *What is my number?*

Let my number be *x*.

$$x - \frac{5}{3} \times x + (-\frac{1}{2}) = -\frac{7}{6} \qquad [+\ \frac{1}{2} = \frac{3}{6}]$$

$$\frac{3}{3} \times x - \frac{5}{3} \times x = -\frac{4}{6}$$

$$-\frac{2}{3} \times x = -\frac{2}{3} \qquad [\times\ (-1)]$$

$$\frac{2}{3} \times x = \frac{2}{3} \qquad [\div\ \frac{2}{3},\ \text{i.e.} \times \frac{3}{2}\ ]$$

$$\underline{x = 1}$$

Answer: My number is 1.

38 min

Check:

$$\frac{1260}{6} - 60 = 210 - 60$$

$$= 150$$ ✔

Check:

$$1 - \frac{5}{3} + (-\frac{1}{2}) = -\frac{2}{3} - \frac{1}{2}$$

$$= -\frac{4+3}{6} = -\frac{7}{6}$$ ✔

Elicit that $\frac{3}{2}$ is the reciprocal of $\frac{2}{3}$, so their product is 1.

Activity

6

PbY6b, page 148, Q.4

Read: *Solve each problem with and without an equation.*

Deal with one at a time. T chooses a P to read out the question and allows Ps a minute to think about how to solve it.

Who can solve it by writing an equation? P comes to BB to write equation. Class agrees/disagrees. Ps come to BB to solve it, doing one step each and explaining reasoning. Class points out errors.

T chooses a P to check the result. Class helps where necessary.

Who can think of a way to solve it without using a letter for the unknown amount? P comes to BB to show it. Class decides whether or not is valid. Who thought of another way? etc. Which method do you like best? Why? T chooses a P to say the answer in a sentence.

Solution: e.g.

a) *Tim has covered* $\frac{3}{8}$ *of his planned route plus an additional 2 km.*
He still has 17 km to go. How long is Tim's route?

Let the length of Tim's route be x.

$$\frac{3}{8}\times x + 2 + 17 = x \qquad [-\ \frac{3}{8}\times x]$$

$$19 = \frac{5}{8}\times x \qquad [\div\ \frac{5}{8},\ \text{i.e.}\ \times\ \frac{8}{5}]$$

$$19 \times \frac{8}{5} = x$$

$$\frac{152}{5} = x$$

$\underline{30.4 = x}$, or $\underline{x = 30.4}$ (km)

Check: $\frac{3}{\cancel{8}_1}\times\overset{3.8}{\cancel{30.4}}$ km = 11.4 km

and 11.4 + 2 + 17 = 30.4 (km) ✔

Answer: Tim's route is 30.4 km.

b) *Belinda spent half of her money plus another £40. Then she spent half of what was left plus £40. Her money has just run out.*
How much money did Belinda have at first?

Let Belinda's money be x.

$$\text{e.g.}\quad \left(\frac{x}{2} - 40\right) \div 2 - 40 = 0 \qquad [+40]$$

$$\left(\frac{x}{2} - 40\right) \div 2 = 40 \qquad [\times 2]$$

$$\frac{x}{2} - 40 = 80 \qquad [+40]$$

$$\frac{x}{2} = 120 \qquad [\times 2]$$

$\underline{x = 240}$ (£)

Check: £240 ÷ 2 = £120, £120 – £40 = £80, £80 ÷ 2 = £40, £40 – £40 = £0 ✔

Answer: Belinda had £240 at first.

45 min

Notes

Whole class activity

(or individual trial first if Ps wish and there is time)

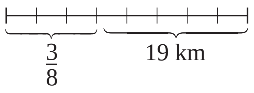
Involve as many Ps as possible.

At a good pace, in good humour.

Discussion, reasoning, checking, agreement, (self-correction), praising

T gives hints or directs Ps thinking if Ps have no ideas.

Accept any valid equation or other method of solution

or 

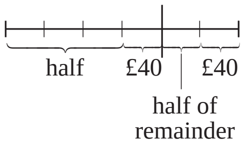
$1 - \frac{3}{8} = \frac{5}{8} \rightarrow$ 19 km

$\frac{1}{8} \rightarrow$ 3.8 km

$\frac{8}{8} \rightarrow$ $\underline{30.4\text{ km}}$

or 19 km ÷ $\frac{5}{8}$ = 19 km × $\frac{8}{5}$

= $\underline{30.4\text{ km}}$

or 

or

If half of the remainder + £40 is the remainder, then the remainder must be £80.

So half of the money is £80 + £40 = £120, and the whole amount is £240.

or (£40 × 2 + £40) × 2

= (£80 + £40) × 2

= £120 × 2

= $\underline{£240}$

Y6

R: Calculations

C: Equations and inequalities. Word problems

E: Advanced problems

Lesson Plan 149

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>149</u> is a prime number Factors: 1, 149

  (as not exactly divisible by 2, 3, 5, 7, 11 and $13^2 > 149$)
- <u>324</u> $= 2 \times 2 \times 3 \times 3 \times 3 \times 3 = 2^2 \times 3^4$ [$= \left(2 \times 3^2\right)^2 = 18^2$]

  Factors: 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 81, 108, 162, 324
- <u>499</u> is a prime number Factors: 1, 499

  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and $23^2 > 499$)
- <u>1149</u> $= 3 \times 383$ Factors: 1, 3, 383, 1149

  (383 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, $23^2 > 383$)

Individual work, monitored (or whole class activity)

BB: 149, 324, 499, 1149

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

Elicit that: 324 is a <u>square</u> number; it has $3 \times 5 = 15$ factors; its <u>square root</u> is 18.

BB: $\sqrt{324} = 18$, because $18 \times 18 = 324$

1149	3
383	2
1	

324	2
162	2
81	3
27	3
9	3
3	3
1	

8 min

2

Inequality signs 1

Let's write a suitable relationship sign between the two numbers in each pair to make a true statement. What are the relationship signs? Ps dictate and T writes on BB. (=, <, >)

Ps come to BB to write the signs, say the statements and check them on the class number line. Class agrees/disagrees.

BB:

a) $2 \boxed{<} 5$ but $-2 \boxed{>} -5$ b) $5.1 \boxed{>} -3$ but $-5.1 \boxed{<} 3$

c) $-2 \boxed{<} 4$ but $2 \boxed{>} -4$ d) $-2 \boxed{<} -1$ but $2 \boxed{>} 1$

e) $0 \boxed{<} 2$ but $0 \boxed{>} -2$ f) $0 \boxed{>} -2.5$ but $0 \boxed{<} 2.5$

Whole class activity

Written on BB or use enlarged copy master or OHP

At a good pace. Involve different Ps at each step.

Reasoning, agreement, praising

Feedback for T

(If there is time, Ps suggest other pairs of numbers and choose Ps to write the signs.)

12 min

3

Inequality signs 2

a) T write an inequality on BB. e.g. $2.6 < 3.4$

Follow my instructions for writing the next inequality. Ps come to BB or dictate what T should write. Class agrees/disagrees.

BB: $2.6 < 3.4$ [Take the <u>opposite</u> value of each side.]

$-2.6 > -3.4$ [Multiply each side by (– 2)]

$5.2 < 6.8$ [Divide each side by (– 4)]

$-1.3 > -1.7$ [Divide each side by (– 1)]

$1.3 < 1.7$

What do you notice about the signs? Elicit that when the numbers are divided or multiplied by a negative number, not only do the numbers change to their opposite value but the relationship sign changes to the opposite sign.

Whole class activity

At a good pace

Reasoning, agreement, praising

Elicit/remind Ps that the <u>opposite</u> value of a positive number has the same digit(s) but its sign is negative, and the opposite value of a negative number has the same digits but its sign is positive.

Opposite numbers are an equal distance from zero but on opposite sides of zero.

Y6

Lesson Plan 149

Activity ***Notes***

3

(Continued)

b) Let's solve this inequality. We can do it in two ways.

T (Ps) suggest what to do at each step and Ps come to BB to write the instruction in square brackets and then to write and say the next line. Class points out errors.

i) Avoiding multiplying or dividing by a negative number.

BB: $(-7) \times x > \frac{1}{3}$ $[+ 7 \times x]$

$0 > 7 \times x + \frac{1}{3}$ $[-\frac{1}{3}]$

$-\frac{1}{3} > 7 \times x$ $[\div 7]$

$\underline{-\frac{1}{21} > x}$ or $\underline{x < -\frac{1}{21}}$

ii) Dividing by a negative number.

BB: $(-7) \times x > \frac{1}{3}$ $[\div (-7)]$

$\underline{x < -\frac{1}{21}}$

(The signs change to their opposites: '–' to '+', '+' to '–', '>' to '<')

c) Let's think of different ways to solve this inequality.

Ps suggest what to do at each step, then come to BB or dictate what T should write. Class agrees/disagrees.

BB: $3 \times (-x) < 9$ $[\div 3]$ or $3 \times (-x) < 9$ $[\div -3]$

$(-x) < 3$ $[\times (-1)]$ $\underline{x > -3}$

$\underline{x > -3}$

(Multiplying or dividing by a negative number changes the signs.]

Check: $x = -3$: $3 \times [-(-3)] = 3 \times 3 = 9$ ✗

e.g. $x = -2$: $3 \times [-(-2)] = 3 \times 2 = 6$ ✔

$x = -4$: $3 \times [-(-4)] = 3 \times 4 = 12$ ✗

Elicit that to divide a fraction by an integer, either divide the numerator where possible or multiply the denominator.

Check by substituting values of x less than, equal to, and greater than $-\frac{1}{21}$. (Only values of x less than $-\frac{1}{21}$ make the inequality true.)

or $3 \times (-x) < 9$ $[+ 3 \times x]$

$0 < 9 + 3 \times x$ $[-9]$

$-9 < 3 \times x$ $[\div 3]$

$\underline{-3 < x}$ or $\underline{x > -3}$

20 min

4

PbY6b, page 149

Q.1 Read: *Solve the equations and inequalities in your exercise book.*

Deal with a), b) and c) one at a time under a time limit.

Review with whole class. T chooses Ps to BB to explain their solutions at BB. Who got the same solution? Who did it a different way? If disagreement, Ps check by substitution on BB. Mistakes discussed and corrected.

Solution: e.g.

a) i) $3.7 \times x - 2.4 < 4.9 \times x + 1.2$ $[-3.7 \times x]$

$-2.4 < 1.2 \times x + 1.2$ $[-1.2]$

$-3.6 < 1.2 \times x$ $[\div 1.2]$

$\underline{-3 < x}$ or $\underline{x > -3}$

Individual work, monitored <u>helped</u>

Written on BB or SB or OHT

Differentiation by time limit.

Discussion, reasoning, agreement, self-correction, praising

Extra praise for Ps who realised that they need only solve the first inequality in a). The other two can be deduced from the first solution!

Y6

Lesson Plan 149

Activity ***Notes***

4

(Continued)

a) ii) $3.7 \times x - 2.4 = 4.9 \times x + 1.2, \quad \underline{x = -3}$

iii) $3.7 \times x - 2.4 > 4.9 \times x + 1.2, \quad \underline{x < -3}$

b) i) $\frac{3}{14} + x < -2 \qquad [-\frac{3}{14}]$

$$\underline{x < -2\frac{3}{14}}$$

Check with some values for *x* if there is disagreement.

ii) $\frac{3}{14} + x = -2, \quad \underline{x = -2\frac{3}{14}}$

iii) $\frac{3}{14} + x > -2, \quad \underline{x > -2\frac{3}{14}}$

c) i) $3 \times x - (-10) < 20 \qquad [+ (-10)]$

$3 \times x < 10 \qquad [\div 3]$

$$\underline{x < 3\frac{1}{3}}$$

ii) $3 \times x - (-10) = 20: \quad \underline{x = 3\frac{1}{3}}$

iii) $3 \times x - (-10) > 20: \quad \underline{x > 3\frac{1}{3}}$

26 min

5

PbY6b, page 149,

Q.2 Read: *Solve the problem with or without an equation.*

Set a time limit of 3 minutes. Ps read problem themselves and use any method they wish. Ps can work in pairs if they wish.

Review with whole class. Who managed to solve it? **X**, come and show us what you did. Who agrees? Who did it a different way? Mistakes disussed and corrected. Ps who did not solve it, copy solution in *Ex Bks*. If no P managed to solve it correctly, T draws diagram on BB and leads Ps through the solution, involving them where possible

Individual trial, monitored helped

If Ps are struggling, stop individual work and continue as a whole class activity.

Discussion, reasoning, agreement, self-correction, praising

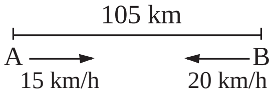
Solution:

Two cities, A and B, are 105 km apart.

A cyclist starts from A and cycles to B at a steady speed of 15 km per hour. At exactly the same time, another cyclist starts from B and cycles to A at a steady speed of 20 km per hour.

a) *When will they meet each other?*

BB:



Let *t* be the number of hours after they started that they met.

Plan: $15 \times t + 20 \times t = 105$

$35 \times t = 105$

$t = 105 \div 35 = 21 \div 7 = \underline{3}$ (hours)

Answer: They will meet each other 3 hours after they started.

Accept and praise this trial and error method too. T might show it if no P thought of it.

Time (h)	1	2	3
A (km)	15	30	45
B (km)	20	40	60
A + B	35	70	105

Y6

Lesson Plan 149

Activity

Notes

5

(Continued)

b) *Where will the cyclists meet?*

From A: 3×15 km = $\underline{45\text{ km}}$, or

From B: 3×20 km = $\underline{60\text{ km}}$ or 105 km – 45 km = $\underline{60\text{ km}}$

Answer: The cyclists will meeet at the point which is 45 km from A (and 60 km from B).

c) *When will they arrive at their destinations?*

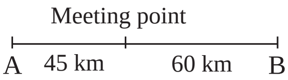
A to B: $105 \div 15 = 21 \div 3 = \underline{7}$ (hours)

B to A: $105 \div 20 = 21 \div 4 = \underline{5\frac{1}{4}}$ (hours)

Answer:

The cyclist going from A to B will arive at B 7 hours after he or she started. The cyclist going from B to A will arrive at A 5 and a quarter hours after he or she started.

BB:



We do not know what time they started so we cannot give exact times as the answer.

30 min

6

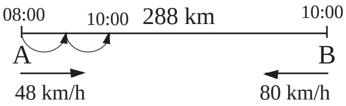
PBY6b, page 149**, **Q.3

T chooses a P to read out the information given in the question.

Read: *Town A is 288 km from town B. Cindy leaves A at 08:00 and drives at a steady speed of 48 km per hour to B. Dan leaves B at 10:00 and drives at a steady speed of 80 km per hour to A.*

What can we do with this information to help us understand it better? (Draw a diagram.) P comes to BB to draw and label diagram, with prompts from class where necessary. Rest of Ps work in in *Ex. Bks*.

BB: e.g.



a) Read: *When will they meet each other?*

A, how would you work it out? Who would do the same as **A**? Who can think of another way to do it? T gives hints if Ps have no ideas.

Solution: e.g.

Let t be the number of hours from the time Cindy starts until they meet, so the number of hours from Dan's start time until they meet is $t - 2$.

Then $48 \times t + 80 \times (t - 2) = 288$

$48 \times t + 80 \times t - 160 = 288$ [+ 160]

$128 \times t = 448$ [÷ 128]

$$t = \frac{448}{128} = \frac{112}{32} = \frac{14}{4} = \frac{7}{2} = \underline{3\frac{1}{2}}\text{ (h)}$$

C will meet D at: 8 h + 3.5 h = $\underline{11.5\text{ h}}$

or Let t be the number of hours from the time Dan starts until they meet, so the number of hours from Cindy's start time until they meet is $t + 2$.

Then $48 \times (t + 2) + 80 \times t = 288$

$48 \times t + 96 + 80 \times t = 288$ [– 96]

$128 \times t = 192$ [÷ 128]

$$t = \frac{192}{128} = \frac{48}{32} = \frac{12}{8} = \frac{3}{2} = \underline{1\frac{1}{2}}\text{ (h)}$$

D will meet C at: 10 h + 1.5 h = $\underline{11.5\text{ h}}$

Whole class activity

(or individual trial first if Ps wish and there is time.)

Agreement, praising

Allow Ps time to think about the method of solution.

Discussion, reasoning, agreement, praising

Involve several Ps.

or C travels 96 km before D starts, so they approach each other over 288 – 96 = 192 (km) for t hours.

$48 \times t + 80 \times t = 192$

$128 \times t = 192$

$t = 192 \div 128 = 24 \div 16$

$= 3 \div 2 = \underline{1.5\text{ (h)}}$

So D will travel for 1.5 hours and C will travel for 3.5 hours before they meet.

Answer:

Cindy and Dan will meet at 11:30.

Activity ***Notes***

6 (Continued)

b) *Where will they meet each other?*

From A: 48 km × 3.5 = 144 km + 24 km = <u>168 km</u> , or

From B: 80 km × 1.5 = 80 km + 40 km = <u>120 km</u>

Answer: Cindy and Dan will meeet each other at the point which is 168 km from A (and120 km from B).

Accept either way.

Only one calculation is required for the answer, although both could be shown.

c) Read: *When will they reach their destinations?*

Ps come to BB or suggest what T should write. Class agrees/disagrees. Class (T) helps/corrects. e.g.

Cindy: 288 km ÷ 48 km = 24 km ÷ 4 km = <u>6</u> (hours)

Dan: 288 km ÷ 80 km = 28.8 km ÷ 8 km = <u>3.6</u> (hours)

Answer: Cindy will arrive at B at 14:00 (or 2.00 pm) and Dan will arrive at A at 13:36 (or 1.36 pm).

Reasoning, agreement, praising

Elicit that reducing the dividend and the divisor by the same number of times does not change the result.

Ps say the answer in a sentence in different ways.

36 min

7 ***PbY6b, Page 149***

Q.4 Read: *Solve the problems by writing equations.*

Deal with one at a time. T chooses a P to read out the question. Allow Ps a minute to work it out in *Ex. Bks*, then Ps show results on scrap paper or slates on command. Ps with different answers explain reasoning at BB. Class points out erors and decides on the correct answer. Mistakes discussed/corrected.

T chooses a P to say the answer in a sentence.

Solutions : e.g.

Individual work but class kept together on the questions.

Encourage Ps to draw diagrams where relevant.

Responses shown in unison.

Reasoning, agreeement, self-correction, praising

T helps when necessary.

a) *Piggy runs off at a speed of 5 metres per second. Two seconds later, Doggy chases Piggy at a speed of 7 metres per second.*

When and where will Doggy catch up with Piggy?

i) Let t be the number of seconds that Piggy runs for.

Then the number of seconds that Doggy runs for is $t - 2$. But the distance they each run is the same, so

Plan:

$$\begin{aligned} 5 \times t &= 7 \times (t-2) \\ 5 \times t &= 7 \times t - 14 && [-(5 \times t)] \\ 0 &= 2 \times t - 14 && [+14] \\ 14 &= 2 \times t && [\div 2] \\ \underline{7 = t}, &\text{ or } \underline{t = 7} \text{ (seconds)} \end{aligned}$$

[If there is no time to cover all the questions during the lesson, set the rest for homework and review before the start of *Lesson 150*.]

ii) Distance *Piggy* runs before she is caught : 7 × 5 m = <u>35 m</u>

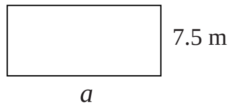
Answer: *Doggy* will catch up with *Piggy* 7 seconds after she started, and 35 metres from their starting point.

b) *The area of a rectangular garden is 150 m*$^{2.}$ *If its length is 7.5 m, what is its width?*

Plan: $a \times 7.5 = 150$ [÷ 7.5]

$$a = \frac{150}{7.5} = \frac{\overset{2}{\cancel{15}} \times 10}{\cancel{7.5}_{\,1}} = \underline{20} \text{ (m)}$$

Answer: The width of the garden is 20 m.



$A = a \times b = 150 \text{ m}^2$

Y6

Lesson Plan 149

Activity

Notes

7 (Continued)

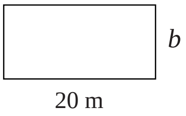
c) *The perimeter of a rectangular garden is 55 m and its length is 20 m. What is its width?*

Plan: $2 \times (20 + b) = 55$ $[\div 2]$

$20 + b = 27.5$ $[-20]$

$b = \underline{7.5}$ (m)

Answer: The width of the garden is 7.5 m.



$P = 2 \times (a + b) = 55$ m

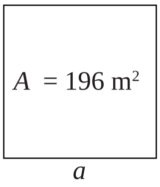
d) *The area of a square garden is 196 m^2. How long is each side?*

$A = a \times a = 196$ m^2

Factorise 196: $\underline{196} = 2 \times 2 \times 7 \times 7 = (2 \times 7)^2 = 14^2$

$a = \sqrt{196}$ m^2 $= \underline{14 \text{ m}}$ (as $14 \times 14 = 196$)

Answer: Each side of the square is 14 metres long.



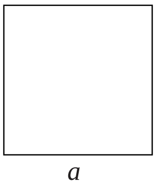
e) *The perimeter of a square garden is* $60\frac{4}{5}$ *m .*

How long is each side?

$P = 4 \times a = 60\frac{4}{5}$ m, $a = 60\frac{4}{5} \div 4 = \underline{15\frac{1}{5}}$ (m)

Answer: Each side is $15\frac{1}{5}$ metres long.

$P = 60\frac{4}{5}$ m



f) *The surface area of a cuboid is 58 cm^2. Its base edges are 4 cm and 2 cm. What is the height of the cuboid?*

$A = 2 \times (a \times b + a \times c + b \times c) = 58$ cm^2

$2 \times (4 \times 2 + 4 \times c + 2 \times c) = 58$

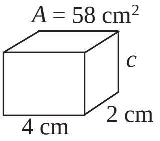
$2 \times (8 + 6 \times c) = 58$ $[\div 2]$

$8 + 6 \times c = 29$ $[-8]$

$6 \times c = 21$ $[\div 6]$

$\underline{c = 3.5}$ (cm)

Answer: The height of the cuboid is 3.5 centimetres.



g) *The volume of a cuboid is 28 cm^3. Its base edges are 4 cm and 2 cm. What is its height?*

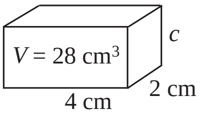
$V = a \times b \times c = 28$ cm^3

$4 \times 2 \times c = 28$

$8 \times c = 28$ $[\div 8]$

$\underline{c = 3.5}$ (cm)

Answer: The height of the cuboid is 3.5 centimetres.



45 min

Y6

Lesson Plan *150*

Activity

Factorising 150, 325, 500 and 1150. Revision and practice.

PbY6b, page 150

Solutions:

Q.1 a) $\frac{n}{2} + \frac{n}{4} = 10.5$ [× 4]

$2 \times n + n = 42$

$3 \times n = 42$ [÷ 3]

$\underline{n = 14}$

Check: $\frac{14}{2} + \frac{14}{4} = 7 + 3.5$

$= 10.5$ ✔

Answer: Russell's number is 14.

b) $\frac{2}{3} \times n + 5 = n - 10$ $[(- \frac{2}{3} \times n\ ]$

$5 = \frac{n}{3} - 10$ [+ 10]

$15 = \frac{n}{3}$ [× 3]

$\underline{45 = n}$ or $\underline{n = 45}$

Check:: LHS: $\frac{2}{\cancel{3}_1} \times \overset{15}{\cancel{45}} + 5 = 30 + 5 = 35$

RHS: $45 - 10 = 35$ ✔

Answer: Margaret's number is 45.

c) $\frac{n}{10} + 10 = \frac{n}{5} + 5$ [× 10]

$n + 100 = 2 \times n + 50$ [– n]

$100 = n + 50$ [– 50]

$\underline{50 = n}$ or $\underline{n = 50}$

Answer: Liz's number is 50.

Q.2 a) Let t be the number of hours from when the train starts from City A to the passing point.

e.g. $125 \times t + 100 \times (t - 1) = 350$

$125 \times t + 100 \times t - 100 = 350$ [+ 100]

$225 \times t = 450$ [÷ 225]

$\underline{t = 2}$ (hours)

Answer: The trains will pass each other at 11.00.

b) From A: 125 km × 2 = 250 km

Answer: The trains will pass each other at the point which is 250 km from A and 100 km from B.

c) Arrival at B: 350 km ÷ 125 km = $\frac{350}{125} = \frac{14}{5} = \underline{2.8}$ (h)

Arrival at A: 350 km ÷ 100 km = $\underline{3.5}$ (hours)

Answer: The first train will arrive at City B at 11:48 and the second train will arrive at City A at 13:30.

Notes

$\underline{150} = 2 \times 3 \times 5^2$

Factors: 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150

$\underline{325} = 5^2 \times 13$

Factors: 1, 5, 13, 25, 65, 325

$\underline{500} = 2^2 \times 5^3$

Factors: 1, 2, 4, 5, 10, 20, 25, 50, 100, 125, 250, 500

$\underline{1150} = 2 \times 5^2 \times 23$

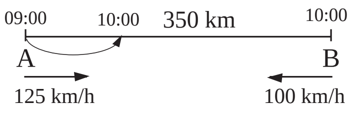
Factors: 1, 2, 5, 10, 23, 25, 46, 50, 115, 230, 575, 1150

(or set factorising as homework at the end of *Lesson 149* and review at the start of *Lesson 150.*

Check:

LHS: $\frac{50}{10} + 10 = 5 + 10 = 15$

RHS: $\frac{50}{5} + 5 = 10 + 5 = 15$ ✔



9 h + 2 h = $\underline{11\text{ h}}$

or from B: 100 km × 1

(4 × 25 = 100)

(60 min × 0.8 = 48 min)

Activity

Notes

Solutions (continued)

Q.3 a) Let x be the length of the 3rd piece, so
length of long piece: $x + 20$; length of short piece: $x - 20$

$$x + x + 20 + x - 20 = 240$$
$$3 \times x = 240 \qquad [\div 3]$$
$$\underline{x = 80} \text{ (cm)}$$

Answer: The length of the 3rd piece of wire was 80 cm.

Check:
$80 + (80 + 20) + (80 - 20)$
$= 80 + 100 + 60 = 240$ ✔

b) Let x be the number of sweets that Louis has.

$$3 \times x - 8 = x + 8 \qquad [-x]$$
$$2 \times x - 8 = 8 \qquad [+8]$$
$$2 \times x = 16 \qquad [\div 2]$$
$$\underline{x = 8}$$

Answer: Louis has 8 sweets and Sarah has 24 sweets.

Check:
LHS: $3 \times 8 - 8 = 16$
RHS: $8 + 8 = 16$ ✔

c) Let the money that George had at first be x.

$$x - \frac{x}{2} - \frac{x}{4} - \frac{x}{8} = 2 \qquad [\times 8]$$
$$8x - 4x - 2x - x = 16$$
$$\underline{x = 16} \text{ (£)}$$

Answer: George had £16 at first.

Check:
$$16 - \frac{16}{2} - \frac{16}{4} - \frac{16}{8}$$
$$= 16 - 8 - 4 - 2 = 2 \text{ ✔}$$

Q.4 a)
$$3a + 2a = 12$$
$$5a = 12 \qquad [\div 5]$$
$$\underline{a = 2.4}$$

Check: $3 \times 2.4 + 2 \times 2.4$
$= 7.2 + 4.8 = 12$ ✔

b)
$$42 \div b = 3 \qquad [\times b]$$
$$42 = 3 \times b \qquad [\div 3]$$
$$\underline{14 = b}$$

Check: $42 \div 14$
$= 21 \div 7 = 3$ ✔

c)
$$2 \times (c + 2) = 3$$
$$2 \times c + 4 = 3 \qquad [-4]$$
$$2 \times c = -1 \qquad [\div 2]$$
$$\underline{c = -0.5}$$

Check::
$2 \times (-0.5 + 2)$
$= 2 \times 1.5 = 3$ ✔

d)
$$2d + 5d = 3d + \frac{1}{2}$$
$$7d = 3d + \frac{1}{2} \qquad [-3d]$$
$$4d = \frac{1}{2} \qquad [\div 4]$$
$$\underline{d = \frac{1}{8}}$$

Check:
LHS: $2 \times \frac{1}{8} + 5 \times \frac{1}{8} = \frac{7}{8}$
RHS: $3 \times \frac{1}{8} + \frac{1}{2}$
$= \frac{3}{8} + \frac{4}{8} = \frac{7}{8}$ ✔

Y6

Lesson Plan 150

Activity

Notes

Solutions (continued)

e) $\frac{e}{9} - 2 \div 9 = e \div 15$ [× 9]

$e - 2 = \frac{9 \times e}{15} = \frac{3 \times e}{5}$ [× 5]

$5e - 10 = 3e$ [– 3*e*]

$2e - 10 = 0$ [+ 10]

$2e = 10$ [÷ 2]

$\underline{e = 5}$

Check

LHS: $\frac{5}{9} - 2 \div 9 = \frac{3}{9} = \frac{1}{3}$

RHS: $5 \div 15 = 1 \div 3 = \frac{1}{3}$

✔

f) $f^2 = f \times 111$ [÷ *f*]

$\underline{f = 111}$

Check:

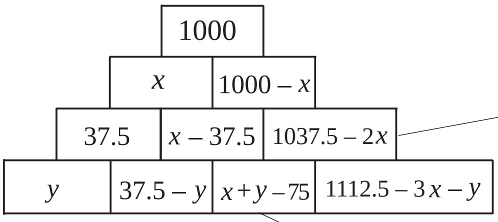
LHS: 111 × 111

RHS: 111 × 111 ✔

Q.5 Accept any valid rule with any numbers which make the rule correct but challenge the more able Ps to generalise, as below.

Rule: e.g. The sum of every two adjacent numbers is the number directly above them.

Let *x* be one of the terms of the addition which sums to 1000, then the other term is 1000 – *x*, etc. Note that in the bottom row a second variable needs to be used.



C:

$1000 - x - (x - 37.5)$

$= 1000 - x - x + 37.5$

$= 1037.5 - 2x$

$1037.5 - 2x - (x + y - 75)$

$= 1037.5 - 2x - x - y + 75$

$= 1112.5 - 3x - y$

$x - 37.5 - (37.5 - y)$

$= x - 37.5 - 37.5 + y$

$= x + y - 75$

Y6

R: Definitions and properties

C: Revision: 3-D and 2-D shapes. Polygons. Triangles and quadrilaterals

E: Problems and challenges

Lesson Plan 151

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>151</u> is a prime number Factors: 1, 151
  (as not exactly divisible by 2, 3, 5, 7, 11 and $13^2 > 151$)
- <u>326</u> = 2 × 163 Factors: 1, 2, 163, 326
- <u>501</u> = 3 × 167 Factors: 1, 3, 167, 501
- <u>1151</u> is a prime number Factors: 1, 1151
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and $37^2 > 1151$)

Individual work, monitored (or whole class activity)

BB: 151, 326, 501, 1151

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 326 & 2 \\ 163 & 163 \\ 1 & \end{array} \qquad \begin{array}{r|l} 501 & 3 \\ 167 & 167 \\ 1 & \end{array}$$

8 min

2

Properties of 3-D shapes

T has various solids on table at front of class. (e.g. sphere, cylinder, cone, prisms with various polygons as their base, cuboid, cube, other polyhedra) What do these shapes all have in common? (3-D shapes or solids) T holds up each in turn and Ps say its name if they know it.

Let's put them into sets. T specifies a set and Ps come to front of class to choose the appropriate solids and to say why they chose them. Class agrees/disagrees. Ps can choose the criteria for a set too.

e.g.

a) It has only plane faces (i.e. it is a polyhedron); it has no plane faces; it has some plane faces.

b) It is a convex shape; it is a concave shape.

For the polyhedra only:

c) Group according to the number of edges (faces, vertices)
   T elicits or reminds Ps of the relationships in Euler's theorem.
   (No. of faces + no. of vertices = no. of edges + 2)

d) It has parallel faces; it has no parallel faces

e) It has perpendicular faces; it has no perpendicular faces

f) It has only regular faces; it has no regular faces.
   (Ps name and analyse the faces of the polyhedra: square, triangle, rectangle, parallelogram, trapezium, etc.)

Whole class activity

Shapes already prepared.

Involve majority of Ps.

At a good pace.

Reasoning, agreement, praising

BB: <u>3–D: Polyhedron</u>
(a solid with only plane faces)

<u>2–D: Polygon</u>
(a plane shape with only straight sides)

BB: $f + v = e + 2$
(Euler's theorem)

Extra praise for unexpected (clever) criteria

16 min

Y6

Lesson Plan 151

Activity | **_Notes_**

3

PbY6b, page 151

Q.1 Read: *If the statement is true, write T in the box and if it is false, write F.*

I wll give you 3 minutes to do it. Start . . . now! . . . Stop!

Review with whole class. T chooses a P to read out the statement. Ps show T or F on scrap paper or slates (or use pre-agreed actions) on command. Ps with different answers explain reasoning by giving an example or counter-example. Class agrees on the correct response. Mistakes corrected.

Solution:

a) A **cuboid** has 8 vertices, 6 faces and 10 edges. [F]

b) Every **cube** has 6 faces, 8 vertices and 12 edges. [T]

c) A **circle** is a 2-dimensional shape. [T]

d) A **line segment** is a 2-D shape. [F]

e) Every cuboid is a **prism**. [T]

(Elicit that a prism is a polyhedron with at least one pair of parallel, congruent faces.)

f) Any prism is a cuboid. [F]

(If its base is neither a rectangle nor a square, the prism is <u>not</u> a cuboid.)

g) If the diagonals of a quadrilateral are equal and **bisect** each other, the quadrilateral is a rectangle. [T]

h) If a quadrilateral has 2 lines of symmetry it is a **rhombus**. [F]

(Elicit that a rhombus is a quadrilateral with equal sides.)

Individual work, monitored

Written on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, praising

Use models where necessary.

Feedback for T

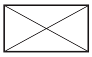
(A cuboid has 12 edges.)

(It has 1 dimension: length)

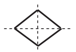
[It is a rectangular-based prism]

e.g. Show a a triangular prism.

BB: e.g.

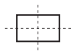


e.g.



is a rhombus

but



is not.

21 min

4

PbY6b, page 151

Q.2 Read: *Construct an **isosceles** triangle which has a base side of 5 cm:*

a) and its other two sides are 3 cm long

b) and its height is 2.5 cm

c) and the angles at its base are 75°

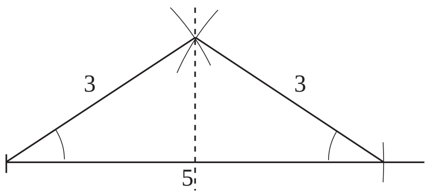
d) and it is a regular triangle.

What is an isosceles triangle? (A triangle which has at least 2 equal sides) What instruments should you use to construct thetriangles? (Ruler and compasses)

Deal with one at a time under a short time limit. Ps finished first construct the triangle on BB using BB instruments, explaining what they are doing at each step. Class points out errors. Ps' own mistakes are discussed and corrected. What else do you notice about this isosceles triangle? (See below)

Solution:

a)



obtuse-angled isosceles triangle

Individual work, monitored, helped

Less able Ps could use a protractor for c).

Demonstration, explanation, agreement, self-correction, praising

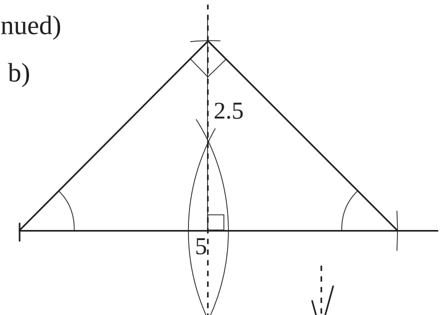
Feedback for T

Elicit that it has one line of symmetry (the bisector of the obtuse angle and perpendicular bisector of the base)

Activity

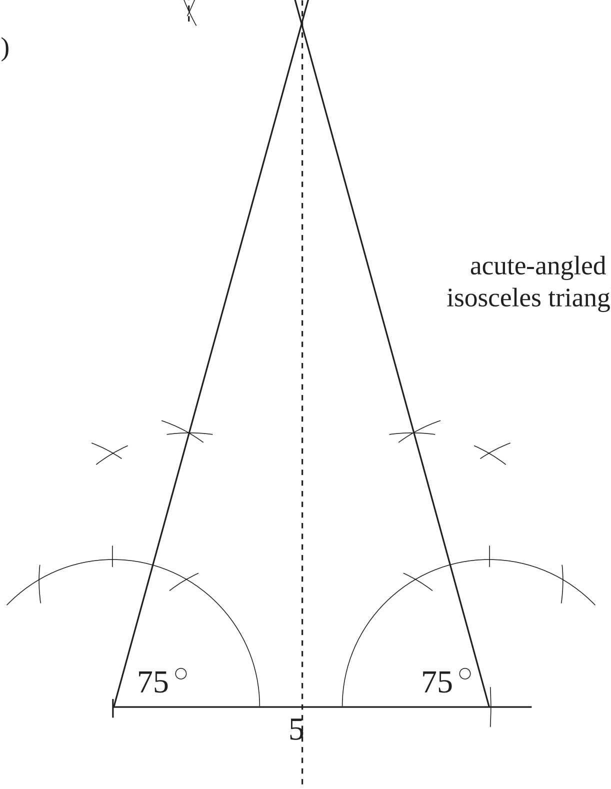
4 (Continued)

b)



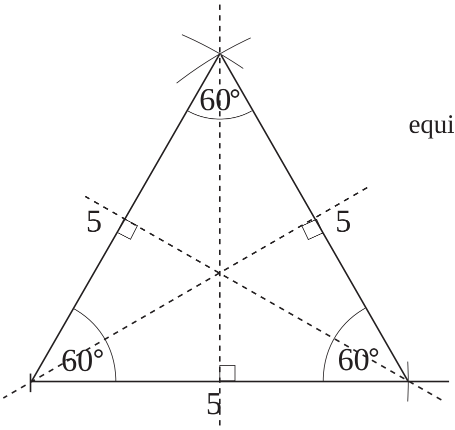
right-angled
isosceles triangle

c)



acute-angled
isosceles triangle

d)



equilateral triangle

28 min

Notes

Elicit that the height of the triangle is the perpendicular distance from its base to the opposite vertex.

Its line of symmetry bisects its base.

To construct a 75° angle, construct two 60° angles, bisect one of them to form two 30° angles, then bisect one of these to form a 15° angle: 15° + 60° = 75°.

Extend both arms until they intersect.

Elicit that its line of symmetry is the perpendicular bisector of its base.

Elicit that a regular triangle has equal sides and equal angles, so each angle is:

180° ÷ 3 = 60°

It has 3 lines of symmetry.

Y6

Lesson Plan 151

Activity

5

PbY6b, page 151, Q.3

Read: *These triangles are made up of* ***congruent*** *triangles. The triangles in b), d) and e) are* ***isosceles*** *triangles.*

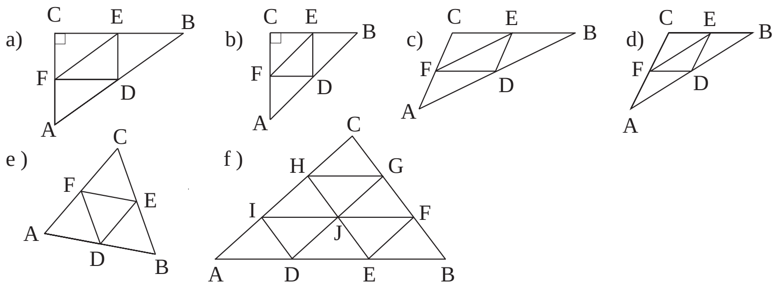
What are congruent triangles? (Triangles which are exactly the same size and shape.) What is an isosceles triangle? (It is a triangle which has at least two equal sides.) Is an equilateral Δ an isosceles Δ? (Yes)

Read: *Find relationships for each shape and write mathematical statements about them.*

What could we do to make it easier to write statements about the triangles? (Label the vertices.) T suggests it if Ps do not.

Deal with one triangle at a time. Ps come to BB or dictate what T should write. Class agrees/disagrees. T prompts if Ps run out of ideas or writes a statement and asks Ps if it is correct.

Solution: e.g.



e.g.

a) $\Delta ABC \sim \Delta FEC$; Ratio of areas of $\Delta ABC : \Delta FEC = 4:1$;

$AF = FC = DE$; $AB = 2 \times AD$; FDEC is a rectangle;
ADEF is a parallelogram; $\angle A + \angle B = 90°$; $EF \parallel AB$;

$DE \perp CB$; $\dfrac{EF}{AB} = \dfrac{1}{2}$; $\Delta ADF \cong \Delta DEF$, etc.

f) $\Delta\ CHG \sim \Delta CIF \sim \Delta CAB$ in the ratio 1 : 2 : 3;

Ratio of areas of $\Delta\ CHG : \Delta CIF : \Delta CAB = 1:4:9$;

Extra praise if Ps notice that the ratio of the areas is the ratio of the sides squared. ($1^2 = 1$, $2^2 = 4$, $3^2 = 9$)

$HG \parallel IF \parallel AB$; $GH = \dfrac{IF}{2} = \dfrac{AB}{4}$; etc.

37 min

Notes

Whole class activity

Drawn (stuck) on BB or use enlarged copy master or OHP

Elicit the sign which means 'congruent to'

BB: $\cong$

Ps label triangles in *Pbs* too.

Involve as many Ps as possible.

At a good pace.

Agreement, praising

Extra praise for unexpected statements

Ps write a different type of statement below each triangle in *Pbs*.

Triangles b) to e) can also have similar statements made about them, except that the shape of FDEC is a square in b), a parallelogram in c) and a rhombus in d) and e);

In b): $\angle A = \angle B = 45°$, etc.

In f), as an extension, T could suggest marking the midpoint of AB (e.g. K) and elicit the ratios:

CJ : JK = 2 : 1,

CJ : CK = 2 : 3,

KJ : KC = 1 : 3

Y6

Lesson Plan 151

Activity

Notes

6

PbY6b, page 151

Q.4 Read: *Mark the midpoints of the sides of each quadrilateral and join them up in order. Write the names of the polygons you have made.*

Set a time limit. Ps use rulers or compasses to mark the midpoints.

Review with whole class. T could have solution already prepared and uncover each shape as it is dealt with. Ps say the name of the original shape and the name of the shape they made by joining the midpoints of the sides. Mistakes corrected.

Which of the original shapes are symmetrical? Ps come to BB to draw the lines of symmetry. Class agrees/disagrees. Are these lines of symmetry the same for the new shapes? (Only in e) do they not match up: a rectangle has 2 lines of symmetry but the deltoid has only one)

Individual work, monitored, helped

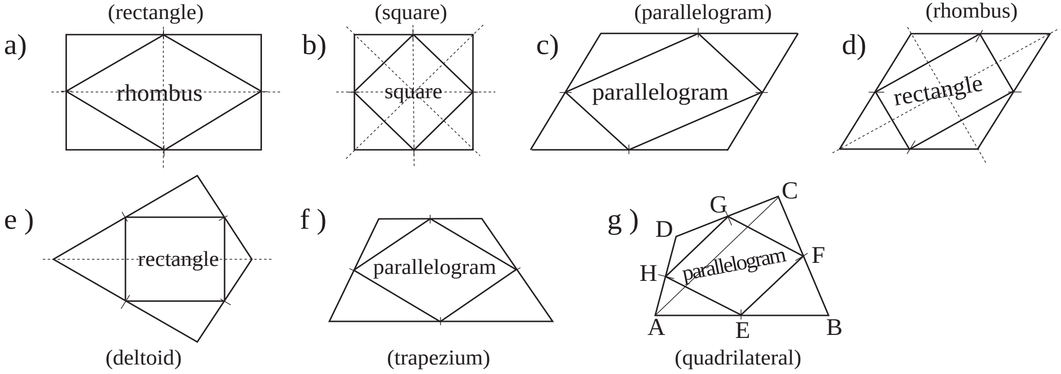
Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, self-correction, praising

Whole class activity

Agreement, praising

Solution:



Extension

How can we prove that, in g), EFGH is a parallelogram?

If Ps have no ideas, lead Ps through the proof for c) and g), involving Ps when possible but have no expectations of them understanding it.

e.g. in g): Join AC.

In triangle ACD, HG is parallel to AC and is half its length.
(as HG joins the midpoints of AD and DC)

In triangle ABC, EF is parallel to AC and is half its length.

So HG is parallel and equal to EF.

Therefore EFGH is a parallelogram.

or

Triangle DHG is similar to triangle DAC in the ratio of 1 : 2,
so HG is parallel to AC and HG : AC is 1 : 2, i.e. HG is half of AC.

Triangle BFE is similar to triangle BCA in the ratio of 1 : 2,
so EF is parallel to AC and EF : AC is 1 : 2, i.e. EF is half of AC.

So EF is equal and parallel to HG.

Therefore EFGH is a parallelogram.

Whole class activity

Review the properties of a parallelogram first. (Quadrilateral with opposite sides equal and parallel)

Elicit that if one pair of opposite sides are equal and parallel, then the other pair of opposite sides must also be equal and parallel.

T starts to explain and Ps try to follow the reasoning.

Allow Ps who understood the first part to explain the 2nd part of the reasoning in their own words, with T's help.

Similar reasoning can be applied to c).

45 min

Y6

R: Calculations

C: Review: Reflection in an axis and symmetry

E: Problems and challenges

Lesson Plan
152

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>152</u> $= 2 \times 2 \times 2 \times 19 = 2^3 \times 19$

  Factors: 1, 2, 4, 8, 19, 38, 76, 152
- <u>327</u> $= 3 \times 109$ Factors: 1, 3, 109, 327
- <u>502</u> $= 2 \times 251$ Factors: 1, 2, 251, 502
- <u>1152</u> $= 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^7 \times 3^2$

  Factors: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 32, ↓

  1152, 576, 384, 288, 192, 144, 128, 96, 72, 64, 48, 36

  [No. of factors: $(7 + 1) \times (2 + 1) = 8 \times 3 =$ <u>24</u>]

Individual work, monitored (or whole class activity)

BB: 152, 327, 502, 1152

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

e.g.

152	2
76	2
38	2
19	19
1	

327	3
109	109
1	

502	2
251	251
1	

1152	2
576	2
288	2
144	2
72	2
36	2
18	2
9	3
3	3
1	

8 min

2

Reflection in an axis

A, come an mark a point on the BB and label it A. **B**, come and draw a mirror line and label it *e*. How can we reflect point A in line *e*?

P comes to BB to draw the reflection, explaining what he/she is doing in a loud voice. Class agrees/disagrees. Who can think of another way to do it? Come and show us.

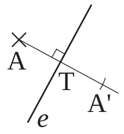
T chooses a P to summarise the steps needed (see below).

What are the main properties of the reflection? Ps come to BB or dictate what T should write. Class agrees/disagrees.

Repeat in a similar way for reflecting a line segment and a circle.

a) <u>Reflection of a point in an axis</u>

e.g.



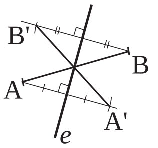
$AT = TA'$

$AA' \perp e$

1) Draw (using set square and ruler or compasses) a perpendicular line from A through *e* . Label the point of intersection, e.g. T.
2) Using compasses (or ruler), measure the distance from A to T and then mark the same distance on the opposite side of T. Label the marked point A'. A' is the mirror image of A.

b) <u>Reflection of a line segment in an axis</u>

e.g.



$AB = A'B'$, $AA' \perp e$, $BB' \perp e$,

$AA' \parallel BB'$, $AB' = A'B$,

AB and A'B' intersect on line *e*.

1) Reflect the points A and B in *e* and label them A' and B'.
2) Join A' to B'. A'B' is the mirror image of AB.

Whole class activity

T should have BB instruments available for Ps to use.

Involve several Ps.

Discussion, reasoning, agreement, praising

Ps could do drawings in *Ex. Bks* too.

Feedback for T

Elicit that a line segment has a start and end point whereas a line is never-ending (infinite) in both directions.

Y6

Lesson Plan 152

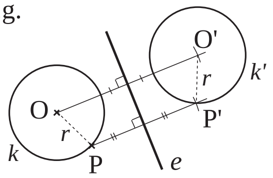
Activity | ***Notes***

2

(Continued)

c) Reflection of a circle in an axis

e.g.



$OO' \perp e$, $PP' \perp e$, $OO' \parallel PP'$,

$OP = O'P' =$ radius (r)

$k = k'$

1) Reflect the centre point, O, and a point, P, on the circumference, *k*. Label the mirror images O' and P'.

2) With compasses set to length OP, draw a circle around O' passing through P'. Label the circumference k'.

Help Ps to use BB compasses to draw the circles.

15 min

3

PbY6b, page 152

Q.1 Read: ***Reflect*** *each shape in the given mirror line. Use a ruler and a pair of compasses.*

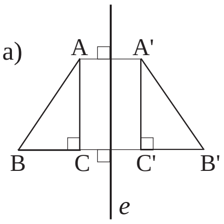
Set a time limit or deal with one at a time. Ps finished early can be asked to write mathematical statements about their reflections.

Review with whole class. T could have solution already prepared and ask Ps just to explain the steps to save time. Class agrees/disagrees. Mistakes discussed and corrected.

Ask Ps to make true statements about the reflections.

Solution:

a)



$\Delta ABC \cong \Delta A'B'C'$

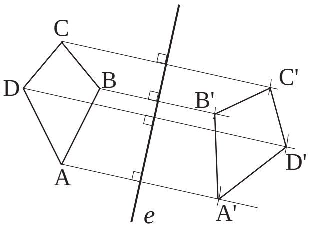
$AC = A'C'$, $AC \parallel A'C' \parallel e$

Line $BC \equiv$ line $C'B' \perp e$

$AA' \parallel BB'$, $BA = B'A'$,

$\angle A = \angle A'$, $\angle B = \angle B'$, etc.

b)



$ABCD \cong A'B'C'D'$ (deltoids)

$AB = A'B'$, etc.

$\angle A = \angle A'$, etc.

$AA' \parallel BB' \parallel CC' \parallel DD'$

$AA' \perp e$, etc.

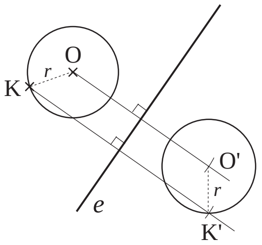
Individual work, monitored, helped, corrected

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

Feedback for T

c)



The 2 circles are congruent.

$OK = O'K' =$ radius (r)

$OO' \parallel KK'$,

$OO' \perp e$, $KK' \perp e$,

KK'O'O is a trapezium.

23 min

Activity ***Notes***

4

PbY6b, page 152

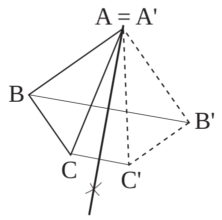
Q.2 Read**:** *Draw the mirror line in the correct place for each shape and its mirror image***.**

Set a time limit. Ps use rulers and compasses.

Review with whole class. Ps come to BB to draw the mirror lines, explaining what they are doing. Who did the same? Who did it a different way? etc. Class points out errors. Mistakes discussed and corrected.

Solution:

a)

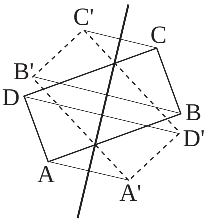


The point A ≡ A' is on the mirror line.

The midpoints of BB' and CC' are on the mirror line.

The perpendicular bisector of BB' (and of CC') is the mirror line.

b)



The points of intersection of C'D' and CD, and of AB and A'B' lie on the mirror line.

Draw a straight line through these two intersections. This is the mirror line.

Elicit that in each part the shape and its mirror image are congruent. The shapes in a) are right-angled triangles, the shapes in b) are rectangles, the shapes in c) are hexagons.

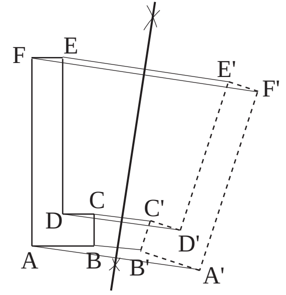
Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

Reasoning, agreement, self-correction, praising

(If majority of Ps are struggling, stop individual work and continue as a whole class activity, with Ps working in *Ex. Bks*. while a P works on BB.

c)



The perpendicular bisector of EE' and of AA' (and of BB', CC', DD') is the mirror line.

(Only 2 points are needed to draw a straight line.)

30 min

5

PbY6b, page 152

Q.3 Read: *Which shapes are symmetrical? Draw the lines of symmetry where appropriate. Write the number of lines of symmetry below each shape.*

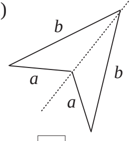
Set a time limit of 3 minutes. Ps use rulers and set squares.

Review with whole class. Ps come to BB to draw mirror lines, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected. Ask Ps to name the shapes if they can.

Who had all 9 correct? Let's give them a clap!

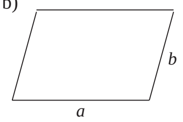
Solution:

a) deltoid



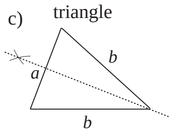
1

b) parallelogram



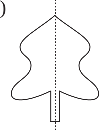
0

c) isosceles triangle



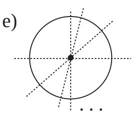
1

d) plane shape



1

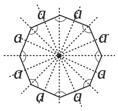
e)



Infinite ∞

circle

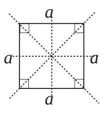
f)



8

octagon

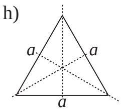
g)



4

square

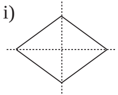
h)



3

equilateral triangle

i)



2

rhombus

Individual work, monitored

Drawn on BB or use enlarged copy master or OHP

Differentiation by time limit.

Reasoning, agreement, self-correction, praising

Elicit or remind Ps of the symbol which means an infinite number of times (i.e. never-ending)'.

BB: ∞ means an infinite number (infinity)

Feedback for T

35 min

Y6

Lesson Plan 152

Activity

6

PbY6b, page 152*, *Q.4

Read: *Draw the path of the billiard ball afer it has rebounded off the edge of the billiard table.*

Who has played billiards? What are the rules? (T tells them if no P knows.) Who can explain what the diagrams mean? (The white circle is the ball, the shaded thick lines are parts of the surround of the billiard table, the arrow shows the path of the billiard ball after it has been hit by the billiard cue.)

a) How can we tell at which point the ball will hit the edge of the table? (Extend the arrow line until it meets the shaded area.) P comes to BB to draw it. Rest of Ps draw it in *Pbs.*

Where do you think the ball will go after that? Ask several Ps what they think. T gives hint about reflection if Ps have no ideas. (The ball will rebound off the edge at the same angle as it hits it.)

How can we we draw these equal angles? (Draw a line perpendicular to the edge of the table at the point where the ball hits it, then either measure the angle made by the arrow line and the perpendicular and measure the same angle on the opposite side, or draw a mirror image of the arrow line.) Elicit that this time the arrrowhead will point away from the edge of the table. T (P) works on BB and Ps work in *Pbs.*

b) Once Ps have been given the idea, they might be able to do part b) more easily (but elicit that the procedure has to be done twice).

T (or Ps) work on BB and Ps work in *Pbs.*

What do you notice about the path of the ball after it has rebounded the second time? (Its path is parallel to the path of the first hit but is moving in the opposite direction.)

[If T has cue and ball and there is an expert snooker or billiard player in the class, the P could demonstrate the hit.]

Notes

Whole class activity

(or individual trial first)

Drawn on BB or use enlarged copy master or OHT

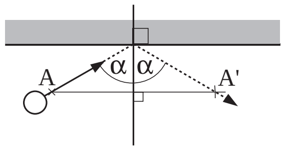
[If possible, T could have a real cue and balls to show.]

Discussion involving many Ps. T gives hints if necessary.

Reasoning, agreement, praising

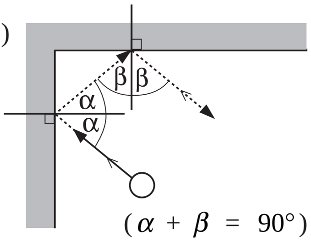
Solution:

a)



- Measure $\angle\,\alpha$ and draw an equal angle on opposite side of perpendicular, or
- mark a point on original arrow, reflect it and join it to the rebound point.

b)



$(\alpha + \beta = 90°)$

40 min

7

PbY6b, page 152

Q.5 Read: *We want the black billiard ball to hit the white ball after rebounding off the edge of the billiard table.*

Draw the path it should take. Explain why you drew it.

Set a time limit of 4 minutes. Ps can work in pairs if they wish and discuss with their neighbours. It is likely that most Ps will use trial and error and gradually get closer to the correct paths.

If no P is on the right track, T could give a hint about using the edge of the table as a mirror line for one of the balls (but which ball?)

If no P has solved it, either lead Ps through the solution or leave the problem open as homework.

Solution:

1. Reflect the *white* ball in the given table edge (by reflecting the centre point and drawing around it a circle of equal radius).
2. Join its centre to the centre of the black ball.
3. Join the point where it hits the edge of the table to the centre of the white ball and draw the appropriate arrowheads.

Ps could draw a perpendicular axis at the rebound point and check that the second path is a reflection of the first path.

Individual (paired) trial first, monitored

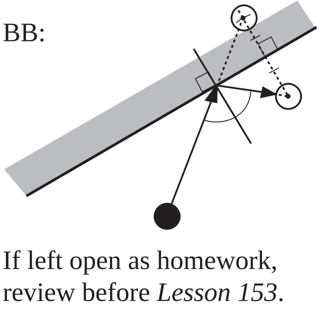
(or whole class activity if time is short or Ps are not very able)

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

If a P thinks of the idea below without hints from T, class gives them '3 cheers'!

BB:



If left open as homework, review before *Lesson 153*.

45 min

Y6

R: Calculations

C: Congruence. Reflection in a point, translation, rotation

E: Problems

Lesson Plan
153

Activity

Notes

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>153</u> $= 3 \times 3 \times 17 = 3^2 \times 17$
  Factors: 1, 3, 9, 17, 51, 153
- <u>328</u> $= 2 \times 2 \times 2 \times 41 = 2^3 \times 41$
  Factors: 1, 2, 4, 8, 41, 82, 164, 328
- <u>503</u> is a prime number Factors: 1, 153
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and $23^2 > 503$)
- <u>1153</u> is a prime number Factors: 1, 1153
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, and $37^2 > 1153$)

Individual work, monitored (or whole class activity)

BB: 153, 328, 503, 1153

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 153 & 3 \\ 51 & 3 \\ 17 & 17 \\ 1 & \end{array} \qquad \begin{array}{r|l} 328 & 2 \\ 164 & 2 \\ 82 & 2 \\ 41 & 41 \\ 1 & \end{array}$$

8 min

2

PbY6b, page 153

Q.1 Read: *a) Measure the length of segment AC and mark A' on the* ***ray*** *so that it is the reflection of A in C.*

b) Complete the statements.

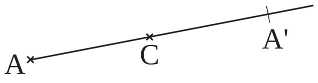
What is a ray? (A straight line starting from a point and extending in only one direction.)

Set a time limit of 3 minutes. Advise Ps to use compasses rather than a ruler. (Set width of compasses to AC, then with point of compasses on C, draw an arc to cut the ray on the opposite side of C.)

Review with whole class. Ps come to BB to show and explain the construction (using BB compasses) and to fill in the missing items. Who agrees? Who wrote something else? Mistakes discussed and correcte.d

Solution:

a)



b) i) AC $\boxed{=}$ CA' ii) C is the midpoint of $\boxed{AA'}$.

T: We say that A' is the reflection of A in point C, or
when A is reflected in point C, its mirror image is A'.

Individual work, monitored, helped, construction corrected

Drawn/written on BB or SB or OHT

Agreement, praising

Using compasses is quicker and more accurate than measuring with a ruler.

Reasoning, agreeement, self-correction, praising

Feedback for T

12 min

Activity ***Notes***

3

Properties of reflection

Follow my instructions. T calls Ps to work on BB while rest of Ps work in *Ex. Bks* or on plain sheets of paper.

Mark 3 points, A, B and C, in your *Ex. Bks* . Arrange them like this. (BB) Let's reflect point A in point C. How can we do it? Ps dictate the steps. (Join A to C and extend the line on the other side of C. Set width of compasses to AC then with point of compasses on C, draw an arc on the opposite side of C. Label the point A'.)

Now let's reflect point B in point C. (Ps again dictate the steps to get B'.)

Let's join A to B. Where is the mirror image of AB reflected in C? (A'B')

Who can tell me true statements about the diagram? Ps come to BB to write mathematical statements or to explain in words. Class agrees/ disagrees. T prompts if Ps miss important properties. e.g.

BB: AB = A'B', AC = CA', BC = CB', AB' = BA',

$\Delta ABC \cong A'B'C$, $\angle A = \angle A'$, $\angle B = \angle B'$, $B\hat{C}A = B'\hat{C}A'$

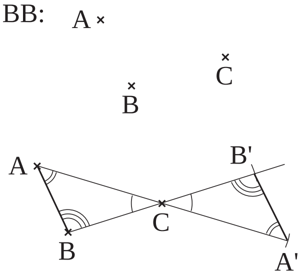
AB || A'B', AB' || BA', ABA'B' is a parallelogram,

Line segement AB can be rotated onto A'B' by 180° around C.

T might suggest that the direction of A'B' is the opposite of AB (draw appropriate arrowheads on B and B') and ask class if it is true. Elicit that the same could be said about AB' and A'B if we joined them up.

Whole class activity but individual drawing, monitored

BB:



Involve many Ps.

Have no expectations!

Reasoning, agreement, praising only

or e.g. AA' : AC = 2 : 1

[Preparation for the concept of vectors.]

17 min

4

PbY6b, page 153

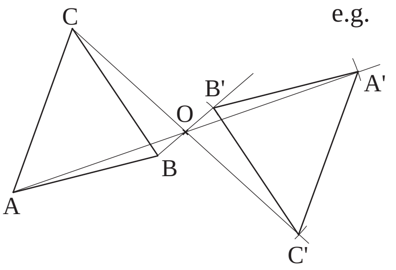
Q.2 Read: ***Reflect*** *triangle ABC in point O. Use a ruler and a pair of compasses.*

Who can tell me how we will use the ruler and compasses? (Use a ruler to draw a ray from each vertex through point O and extend the ray on the other side of O. Use compasses to measure and mark the mirror images of the vertex. Use a ruler to join up the mirror images of the vertices to form triangle A'B'C'.)

Set a time limit of 3 minutes. Review with whole class. Ps finished first show and explain construction on BB (or T has construction already prepared and Ps just explain what has been done to save time). Mistakes discussed and corrected

Let's think of true statements about the diagram. Ps tell class in words and/or come to BB to write it mathematically. Class agrees/disagrees. T suggests some too and asks if they are correct.

Solution:



e.g.

AB = A'B', etc.

AO = OA', etc.

$\Delta ABC \cong A'B'C$

$\Delta AOC \cong A'OC'$

$\angle A = \angle A'$, etc.

AC || A'C', AB || A'B'

AC'A'C is a parallelogram,

ΔABC can be rotated onto $\Delta A'B'C'$ by 180° around O.

The mirror image of the mirror image of triangle ABC is itself.

Individual construction, monitored, helped, corrected

Drawn on BB or use enlarged copy master or OHP

T repeats Ps' steps in a clearer way if necessary.

Reasoning, agreement, self-correction, praising

Wholc class activity

Involve several Ps.

At a fast pace

Have no expectations.

Accept and praise any valid statement.

Extra praise for clever statements.

Feedback for T

etc.

23 min

Y6

Lesson Plan 153

Activity

Notes

5

PbY6b, page 153*, *Q.3

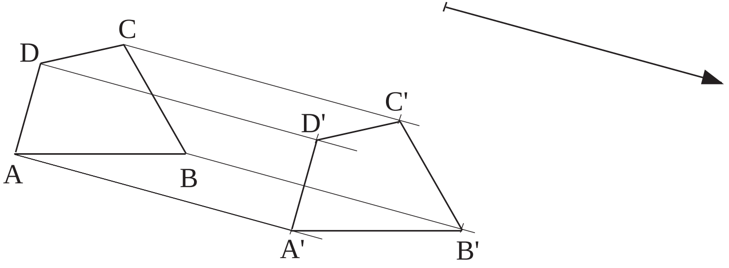
Read: ***Translate*** *quadrilateral ABCD in the direction, and by the distance, shown by the arrow. Use a ruler and a pair of compasses.*

What does translate mean? (Move in the plane.) How can we translate the shape? (Translate each vertex, then join up the new points.)

How does the arrow help us to translate point A? Ps who have ideas come to BB to demonstrate. Who agrees? Who thinks we should do it another way? T gives hints if necessary. Elicit that the length of the arrow tells us how far the point should move and the direction in which the arrow is pointing shows us the angle it should be moved. Elicit that the path of point A will be equal and parallel to the arrow.

Revise how to draw parallel lines (using either ruler and set square or 2 rulers). T demonstrates if Ps have forgotten. Ps come to BB to draw parallel lines (with T's help), measure and mark the images of the points with compasses, label them, then join up the points to form A'B'C'D'. Rest of Ps follow in *Pbs*.

Solution:



Let's think of true statements about the diagram. Ps come to BB or dictate to T. Class agrees/disagrees. (See opposite.)

Extension

We could say that AD and A'D' are pointing in the same direction. T or P draws arrowheads at D and D'.

T: A line segment which has direction is called a vector. We write vectors with an arrow above them like this.

BB: $\overrightarrow{AD} = \overrightarrow{A'D'}$ and read it as, 'Vector AD equals vector A'D'.'

Who can see another pair of vectors in the diagram? (Ps come to BB.)

Notes:

Whole class activity

Drawn on BB or use enlarged copy master or OHP, for demonstration only

Discuss the steps required for the translation.

Involve several Ps.

T could have a template of the quadrilateral cut out and use it to demonstrate the translation.

Lay ruler (or set square) with one of its edges along the arrow. Place ruler beneath 1st ruler (or set square) and slide it along until it rests on A. Draw a ray from A along base of ruler.

Praising, encouragement only

Statements: e.g.

BB: ABCD $\cong$ A'B'C'D'

AD = A'D', AD || A'D', etc.

AA' || BB' || CC' || DD'

$\angle$A = $\angle$A', etc.

The labelling in both has the same orientation (anticlock-wise)

Have no expectations!

Extra praise for unexpected criteria.

BB: Vector

a line segment with direction

29 min

6

PbY6b, page 153

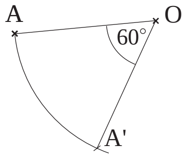
Q.4 Read: ***Rotate*** *point A around centre O by* 60° *anticlockwise.*

First demonsrate the rotation with strips of card or straws. Discuss the steps needed to do draw it. (Join AO. Set compasses to width AO and with compass point on A, then on O, draw 2 arcs. Join O to the point of intersection, A'.)

This construction uses the concept of an equilateral triangle but Ps might also suggest using a protractor to measure the angle. Accept either method.

A P works on BB with T's help while rest of Ps work in *Pbs*.

Solution:



Notes:

Individual work after initial discussion on method of construction, monitored, helped, corrected

Points drawn on BB or SB or OHT

Discussion, reasoning, agreement, praising

Elicit true statments about the diagram. e.g.

AO = A'O = AA'

AO and OA' are radii of a circle with centre O

$\angle$A = $\angle$A' = $\angle$O = 60°

Y6

Lesson Plan 153

Activity ***Notes***

6

(Continued

T markes another point, B, below A. Ps mark it in *Pbs*. too. Let's rotate B around O by 60°. P works on BB and rest of Ps work in *Pbs.* Let's label the image B'.

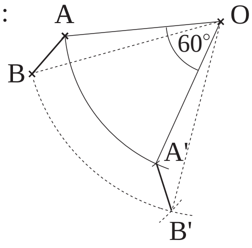
If we join up AB, where could we find its rotational image? (A'B')

What true statement could we write about the two line segments?

BB: AB = A'B' (the only one possible, as they are not parallel and are not facing in the same direction)

Who can tell us how to rotate any line segement or shape around a point? (Rotate each vertex, then join up the rotational images.)

BB:



Remind Ps or elicit that rotation by a positive angle means rotation anti-clockwise and by a negative angle means rotation clockwise

35 min

7

PbY6b, page 153

Q.5 Read: ***Rotate*** *the shape around centre O by* 90° *clockwise* .
Use a ruler and a pair of compasses (and a ptotractor if you wish).

T has template already prepared. What kind of shape is it? (concave hexagon) Who can come and show us the rotation? Class decides whether it is correct. Elicit that a rotation clockwise means a rotation by a <u>negative</u> angle, i.e. by – 90° .

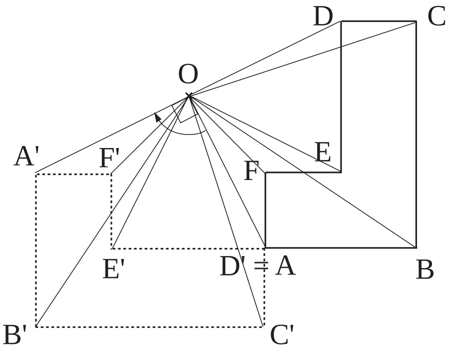
Do rotation of point A on BB with help of a P while rest of Ps follow in *Pbs*. (First join O to A, then using the right angle on a set square or a corner of a ruler, or by construction of two 60° angles and bisecting one of them, or by using a protractor, draw a right angle, then set compasses to width OA and with point of compasses on O, mark point A' on the other arm of the angle.)

Set a time limit of 5 minutes for Ps to rotate the other vertices and join up the images.

Review with whole class. T could have construction already prepared (or Ps finished early complete the rotation on BB). Mistakes discussed and corrected.

Collect properties of the rotation (or if times runs out, set this task for homework and review before the start of *Lesson 154*).

Solution:



Individual work, monitored, helped, corrected after whole class discussion of the steps

Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

(Ps finished early could write some true statements in *Pbs*.)

Agreement, (self-correction), praising

At a good pace. Involve many Ps. Praising only

e.g.

AB = A'B', AB $\perp$ A'B', etc.

ABCDEF $\cong$ A'B'C'D'E'F'

$\angle A = \angle A' = \angle B = \angle B' = \angle C = \angle C' = \angle D = \angle D' = \angle F = \angle F' = 90°$

$\angle E = \angle E' = 270°$

etc.

45 min

Y6

R: Calculations

C: Circle: names of its components, circumference

E: Introducing π

Lesson Plan 154

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 154 = 2 × 7 × 11 Factors: 1, 2, 7, 11, 14, 22, 77, 154
- 329 = 7 × 47 Factors: 1, 7, 47, 329
- 504 = $2 \times 2 \times 2 \times 3 \times 3 \times 7 = 2^3 \times 3^2 \times 7$

  Factors: 1, 2, 3, 4, 6, 7, 8, 9, 12, 14, 18, 21 ↓
  504, 252, 168, 126, 84, 72, 63, 56, 42, 36, 28, 24
- 1154 = 2 × 577 Factors: 1, 577, 1154

  (577 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, and $29^2 > 577$)

Individual work, monitored (or whole class activity)

BB: 154, 329, 504, 1154

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

154	2
77	7
11	11
1	

329	7
47	47
1	

504	2
252	2
126	2
63	3
21	3
7	7
1	

1154	2
577	577
1	

6 min

2

Components of a circle

Follow my instructions on drawing a circle. Ps draw in *Ex. Bks*. or on plain sheets of paper which they can stick in *Ex Bks*. T chooses different Ps to work on BB at each step.

a) Set your compasses to width 3 cm. Mark a point O and draw a circle around O. What do we call the line around the edge of a circle? (circumference)

b) Mark a point A on the circumference and draw OA. What part of the circle is OA? (radius) Write along the radius: r = 3 cm.

c) Draw a straight line through centre O and mark the points where it crosses the circumference as B and C. Label the line *e*.

Line *e* is a line of symmetry of the circle because it divides it in half. How many lines of symmetry does a circle have? (an infinite number)

What part of the circle is line segment BC? (diameter)

What is the relationship between the length of the radius, *r*, and the length of the diameter? BB: $d = 2 \times r$

c) Draw a line, *f*, which crosses the circumference at 2 points, D and E, but does not pass through O. What do we call line *f* ?
(We say that line *f* is an intersector of the circle.)

What part of the circle is the line segment DE? (chord)

What name do we give to a part of the circumference, e.g. the part betwen A and B? (arc) What other arcs can you see?

Colour *red* the part of the plane inside the circle which is enclosed by the the radii OA and OB and by the smaller arc AB. (It is like a slice of pizza.) What name to we call this part of the circle? (sector)

We say that ∠BOA in this sector is a central angle of the circle.

Colour *blue* the part of the plane inside the circle which is enclosed by the chord DE and by the smaller arc DE.
What name to we give to this part of the circle? (segment)

Note that ECBD is also a segment of the circle.

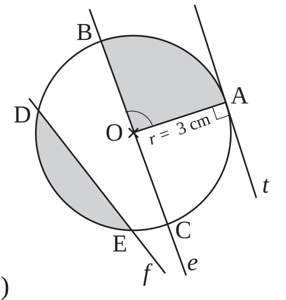
Whole class activity but individual drawing, monitored closely, helped, corrected

T chooses a diifferent P for each step shown on BB.

T could have flash cards of the underlined names prepared and stuck to side of BB. Ps choose the appropriate card at each step.

Allow Ps the opportunity to say the name if they know it.

BB: e.g.



e)

Draw a line, *t*, which touches the circle at just one point, A. What do we call such lines? (We say that line *t* is a tangent to the circle at point A.)

A tangent is perpendicular to the radius which meets it at a common point.

BB: OA $\perp$ *t*

18 min

Y6

Lesson Plan 154

Activity | *Notes*

3

PbY6b, page 154

Q.1 Read: *Complete the statements about the diagram.*

T could leave the flash cards on show to help Ps to spell the missing words. Set a time limit of 4 minutes.

Review with whole class. T chooses a P to read out the sentence, saying 'something' instead of the box and another to identify the componenet on the diagram.

Show me the missing word . . . now! Ps with mistakes correct them and repeat the sentence again correctly.

Solution:

a) OT is a radius of the circle.

b) O is the centre of the circle.

c) AB is a diameter of the circle.

d) **Line segment** CD is a chord of the circle.

e) The smaller shape EOF is a sector of the circle.

f) The curve EF is an arc on the **circumference** of the circle.

g) $\angle$EOF (= α) is the central angle of the smaller sector EOF.

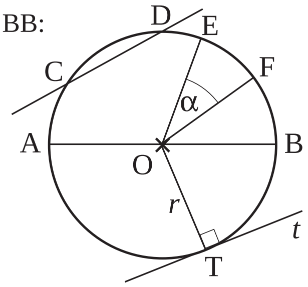
h) Line CD is an intersector of the circle.

i) *t* is a tangent to the circle.

Individual work, monitored, helped

Drawn on BB or use enlarged copy master or OHP

BB:



Responses shown in unison.

Agreement, self-correction, praising

Feedback for T

Extension

What other statements can you make about the diagram?
(e.g. OA = OE = OF = OB = OT = r, OT $\perp$ t,
Line t touches the circle at point T, or T is the common point of OT and t, etc.)

Whole class activity

Praising only

24 min

4

Ratio of the circumference and diameter of a circle

Ps have cylinders (e.g. empty cans or wooden solids) and string or thick thread on desks. (Use at least 3 different sizes.)

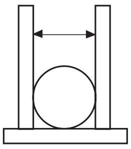
a) What shape is the base of your cylinder? (a circle) Let's measure its circumference and then its diamater and compare them.

How can we measure its circumference? (T tells Ps what to do if no P has a suggestion.)

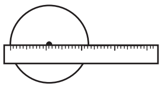
1. Coil the string tightly around the outside of the can close to the base, keeping it an equal distance from the base, and mark where the string meets itself.
2. Lay the the string tightly along a ruler and note the marked length.

How can we measure the diameter of its base? Ps make suggestions.

e.g. If its centre is not marked, use 3 rulers as in 1st diagram; if its centre is marked, only one ruler is needed (2nd diagram)



or



Note the length of the diameter.

b) Let's calculate the ratio of the lengths of the circumference and diameter. How should we do it? (Divide the length of the circumference by the length of the diameter.) Ps do calculations in *Ex. Bks* or on scrap paper and note the result.

Whole class activity but paired work in measuring and recording, monitored, helped, corrected

T gives instructions and Ps follow them.

T could have a large model for demonstration.

Some Ps could measure in inches and some in cm.

Tell Ps to round to 2 decimal places if their result is not exact.

Activity ***Notes***

4

c) Let's collect the different data and results in this table. A P from each pair dictates their results and the other P writes them in the table.

BB: e.g.

c	18.8 cm	6.3 inches	25 cm
d	6 cm	2 inches	8 cm
$\frac{c}{d}$	$3.1\dot{3}$	3.15	3.125

etc.

At a fast pace.

Table drawn on BB or SB or OHT (one column for each pair of Ps)

d) What do you notice? (The ratio in the bottom row is between 3.1 and 3.2 each time, whatever the length of line or unit of measure.)

Agreement, praising

T: The ratio of the circumference of a circle and its diameter is a constant value (i.e. it does not change) and is about 3.14.

The exact value extends to an infinite number of decimal places. We call this value 'pi' and write it using a Greek letter.

T explains and Ps listen.

BB: Ratio of the circumference of a circle to its diameter:

$$\frac{\text{circumference}}{\text{diameter}} = \pi \text{ (pi)} \approx 3.14$$

Ps write the ratio on the blank page at the back of their *Pbs.*

30 min

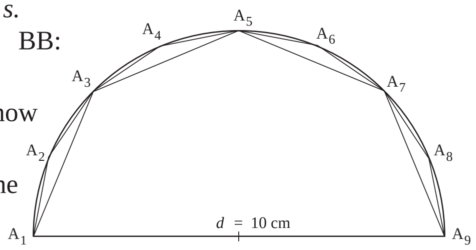
5

PbY6b, page 154

Q.2 Read: *This* **semicircle** *has a radius of 5 cm. The length of its curved line is s.*

Individual work in measuring and calculating but class kept together on the tasks.

What is a semicircle? BB: (Half of a circle)



Drawn on BB or use enlarged copy master or OHP

Who can come and show us s on the diagram?

What is s? (Half of the circumference of the whole circle)

What is d on the diagram? (The diameter of the whole circle.)

What are A_1, A_2, etc? (Points on the circumference) Let's mark the points more clearly (with 'ticks'). Ps do it in *Pbs* too.

This will help Ps to measure more accurately.

a) Read: *Measure the length of the two broken lines.*

Set a short time limit. Ps use compasses and rulers to measure each part, find their sum and write lengths in *Pbs*.

Ps might notice that the distances betwen each pair of points is about equal, measure one part and multiply by the number of parts.

Review with whole class. Ps could show lengths on scrap paper or slates on command. Accept slight inaccuracies but Ps who are very inaccurate should measure again (with the help of a more able P).

Responses shown in unison.

Solution:

Length of $A_1 A_3 A_5 A_7 A_9 \approx$ <u>15.2 cm</u>

Length of $A_1 A_2 A_3 A_4 A_5 A_6 A_7 A_8 A_9 \approx$ <u>15.6 cm</u>

Agreement, self-correction, praising

b) Read: *Write the lengths of the curved line, s, and the two broken lines in increasing order.*

Ps write it as an inequality inside semicircle in *Pbs*.

T quickly checks each P.

T chooses a P to write it on BB and class agrees/disagrees. Mistakes discussed and corrected.

Reasoning, agreement, self-correction, praising

Solution: 15.2 cm < 15.6 cm < s

Agree that the length of s will be nearer 15.6 cm than 15.2 cm.

Y6

Lesson Plan 154

Activity

Notes

5

(Continued)

c) Read: *If the ratio of the* ***circumference*** *to the* ***diameter*** *of any circle is about 3.14, what is the length of the curved line of the semi-circle?*

Allow Ps a minute to think about it and discuss with their their neighbours if they wish. Who thinks they know what to do? Come and explain to us. Who agrees? Who would do it another way? etc. If no P is on the right track, T directs Ps' thinking.

Solution:

s = half the length of the circumference of the whole circle

$\frac{c}{d} \approx 3.14$, so $c \approx 3.14 \times d = 3.14 \times 10$ cm = 31.4 cm

and $s \approx 31.4$ cm $\div 2 =$ 15.7 cm

Answer: The length of the curved line of the semi-circle is about 15.7 cm.

d) Read: *Compare the lengths of the 3 lines and write their ratio.*

What can you tell me about the accuracy of the 3 lengths?

(The shorter broken line is a rough estimate of *s*; the longer broken line is a better estimate of *s* and the calculated value is very close to the actual length.) Who can write the ratio of the 3 lengths?

BB: Shorter Broken Line : Longer BL : Calculated Value
= 15.2 : 15.6 : 15.7 = 152 : 156 : 157

Discussion, reasoning, agreement, praising

Ps copy solution in *Ex. Bks.*

T chooses a P to say the answer in asentence.

Discussion, agreement, praising

Elicit that the more points there are on the curved line, the closer the length will get to the actual value of *s*.

37 min

6

PbY6b, page 154

Q.3 Read: *If the circumference of a circle with diameter 1 unit is about 3.14 units, calculate the circumference of a circle which has these lengths.*

What is the relationship between circumference and diameter?

BB: $c \approx 3.14 \times d$

Set a time limit of 4 minutes. Ps write answers in *Ex. Bks.*

Review orally with whole class. T chooses Ps to give their answers and explain their reasoning. Class agrees/disagrees. Mistakes discussed and corrected. If there is disagreement, ask Ps to show details of the calculations on BB.

Solution:

a) a 1 cm diameter: $c \approx 3.14 \times 1$ cm = 3.14 cm
b) a 7 cm diameter: $c \approx 3.14 \times 7$ cm = 21.98 cm
c) a 1 m diameter: $c \approx 3.14 \times 1$ m = 3.14 m
d) a 5 m diameter: $c \approx 3.14 \times 5$ m = 15.7 m
e) a 1 cm radius: $c \approx 3.14 \times 2$ cm = 6.28 cm
f) a 3 cm radius: $c \approx 3.14 \times 6$ cm = 18.84 cm
g) a 1 m radius: $c \approx 3.14 \times 2$ m = 6.28 m
h) a 2 m radius: $c \approx 3.14 \times 4$ m = 12.56 m

Individual work, monitored, (helped)

Differentiation by time limit

Reasoning, agreement, self-correction, praising

Elicit that $d = 2 \times r$

Feedback for T

42 min

Y6		*Lesson Plan 154*
Activity		***Notes***
7	***PbY6b, page 154, Q.4*** T reads out the question, Ps calculate mentally or in *Ex. Bks*. and stand up when they have an answer. T goes to them and Ps whisper answer in T's ear. T tells them whether they are correct or not. If wrong they must sit down and calculate again. When a few Ps are standing, T asks P who stood up first to explain how he/she got the answer so quickly. Ps correct their mistakes. *Solution:* *If* $\pi \approx 3.14$*, calculate the circumference of a circle which has a:* *a) 10 cm diameter* [$c \approx 3.14 \times 10 \text{ cm} = \underline{31.4 \text{ cm}}$] *b) 8 m radius* [$c \approx 3.14 \times 2 \times 8 \text{ m} = 6.28 \times 8 \text{ m} = \underline{50.\ 24 \text{ m}}$] *c) 4 m radius* [$c \approx 3.14 \times 2 \times 4 \text{ m} = 3.14 \times 8 \text{ m} = \underline{25.\ 12 \text{ m}}$] *d) radius r* [$c \approx 3.14 \times 2 \times r = 6.28 \times r$ $(= 6.28r)$] *45 min*	Whole class activity but individual calculation In good humour! Praising, encouragement only Class applauds quickest P(s). (or $50.24 \text{ m} \div 2 = 25.12 \text{ m}$)
Homework	Set Question 5 for homework and review before the start of *Lesson 155*. ***PbY6b, page 154, Q.5*** a) $c = \pi \times 22 \text{ cm}$ $(= 22\pi \text{ cm})$ b) $c = \pi \times 2.5 \text{ m}$ $(= 2.5\pi \text{ m})$ c) $c = \pi \times d$ $(= \pi d)$ d) $c = 2 \times \pi \times \text{r}$ $(= 2\pi\text{r})$	Optional Praise Ps who tried it and give extra praise to Ps who were correct. T shows the short forms.

Y6

Lesson Plan
155

Activity

Factorising 155, 330, 505 and 1155. Revision and practice.

PbY6b, page 155

Solutions:

Q.1 a) *Every* ***isosceles triangle*** *has angles of* 60°. [F]

(An isosceles triangle can be acute-angled, right-angled or obtuse-angled.)

b) *No* ***rectangle*** *has adjacent equal sides.* [F}

(A square has adjacent equal sides and is a rectangle.)

c) *The* ***diameter*** *of a circle is twice the length of its radius.* [T]

d) The **circumference** of a circle is its radius multiplied by π. [F]

(The circumference of a circle is its <u>diameter</u> multipled by π.)

f) *There is a* ***prism*** *which has congruent faces.* [T]

(e.g. A cube has congruent faces and is a prism.

g) *If the diagonals of a quadrilateral* ***bisect*** *each other at right angles, the quadrilateral is a* ***rhombus***. [T]

(A square fits this description but it too is a rhombus.)

h) *A* ***tangent*** *to a circle can touch the circle at more than 1 point.* [F]

(A tangent touches the circle at only one point.)

Notes

<u>155</u> = 5 × 31

Factors: 1, 5, 31, 155

<u>330</u> = 2 × 3 × 5 × 11

Factors: 1, 2, 3, 5, 6, 10, 11, 15, 22, 30, 33, 55, 66, 110, 165, 330

<u>505</u> = 5 ×101

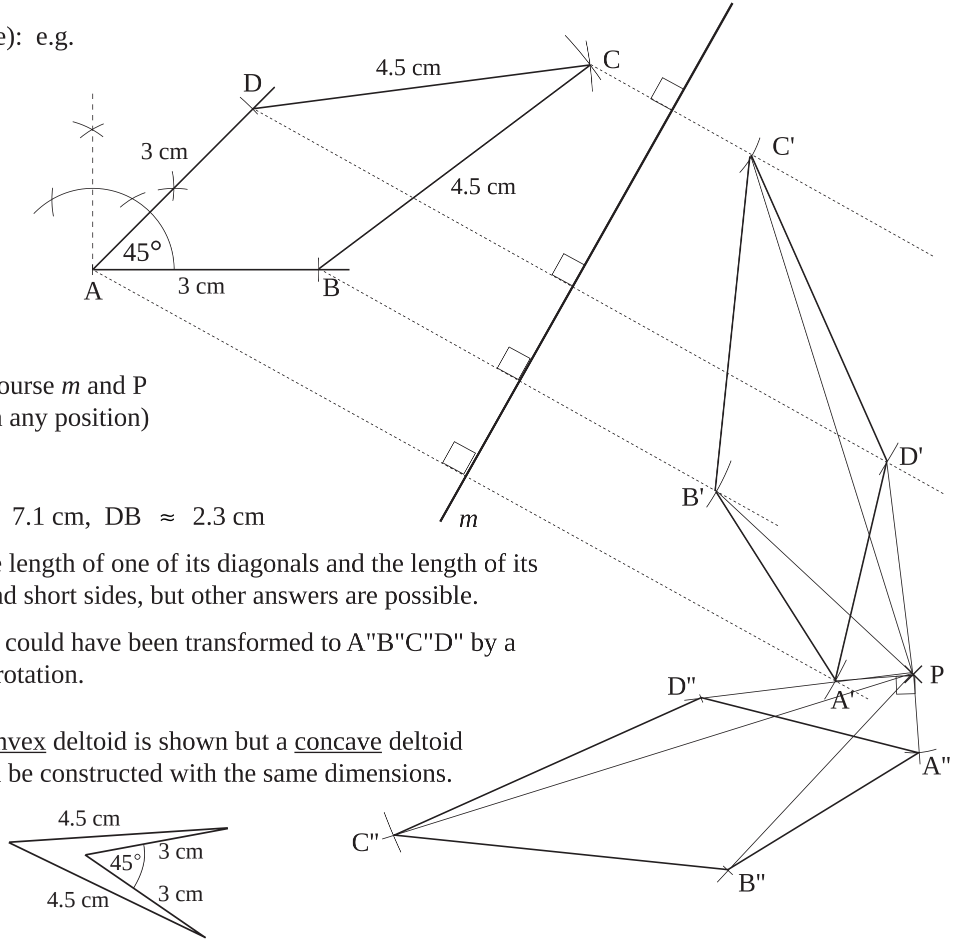
Factors: 1, 5, 101, 505

<u>1155</u> = 3 × 5 × 7 × 11

Factors: 1, 3, 5, 7, 11, 15, 21, 33, 35, 55, 77, 105, 165, 231, 385, 1155

(or set factorising as extra task for homework at the end of *Lesson 154* and review at the start of *Lesson 155.*

Q.2 a), d) and e): e.g.



(but of course *m* and P can be in any position)

b) AC ≈ 7.1 cm, DB ≈ 2.3 cm

c) e.g. the length of one of its diagonals and the length of its long and short sides, but other answers are possible.

f) ABCD could have been transformed to A"B"C"D" by a single rotation.

N.B. A <u>convex</u> deltoid is shown but a <u>concave</u> deltoid could be constructed with the same dimensions.

Activity ***Notes***

Solutions (continued)

Q.3 a) i) $c \approx 1.5 \text{ cm} \times 3.14 = \underline{4\ .71 \text{ cm}}$

ii) $c \approx 3.5 \text{ cm} \times 3.14 = \underline{10.99 \text{ cm}}$

iii) $c = 2 \times 3 \times \pi \times \pi = 6 \times \pi^2 = \overline{6\pi^2 \text{ units}}$

or $\approx 2 \times 3 \times \pi \times 3.14 = \overline{18.84\pi \text{ uints}}$

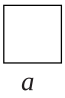
or $\approx 2 \times 3 \times 3.14 \times 3.14 \approx \underline{59.16 \text{ units}}$ (to 2 d.p.)

Accept any of these answers.

b) i) $c = \overline{8.9\pi \text{ cm}}$ ii) $c = \overline{3.24\pi \text{ cm}}$ iii) $\overline{c = \pi \text{ cm}}$

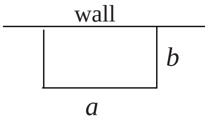
Q.4 a) The largest area is enclosed by the most regular shape, so the pen should be a square and the length of each side will be:

$P = 4a = 200 \text{ m}$, so $a = 200 \text{ m} \div 4 = \underline{50 \text{ m}}$



b) This is probably best done by trial and error using a table to show different possible values for *a* and *b*.

Elicit that: $a + 2b = 200$, so $b = (200 - a) \div 2$



e.g.

a	10	20	40	60	80	100	110	104	101
b	95	90	80	70	60	50	45	48	49.5
A	950	1800	3200	4200	4800	5000	4950	4992	4999.5

The greatest area is enclosed when the longer side is 100 m and the two shorter sides are 50 m.

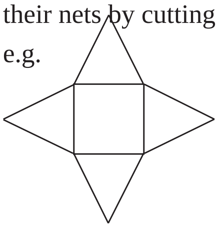
c) $A_1 = 50 \times 50 = 2500 \text{ (m}^2\text{)}$, $A_2 = 50 \times 100 = 5000 \text{ (m}^2\text{)}$

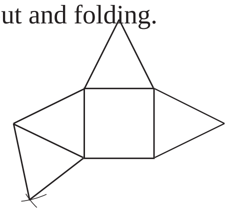
Ratio: $\dfrac{A_2}{A_1} = \dfrac{5000}{2500} = \dfrac{2}{1} \rightarrow 200\%$

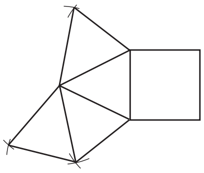
or $\dfrac{A_1}{A_2} = \dfrac{2500}{5000} = \dfrac{1}{2} \rightarrow 50\%$

Q.5 Accept any valid net. Ps use rulers and compasses. Ps check their nets by cutting out and folding.

e.g.







Y6

R: Calculations

C: Revision: Perimeter, area, volume

E: Problems and challenges

Lesson Plan 156

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>156</u> $= 2 \times 2 \times 39 = 2^2 \times 39$ Factors: 1, 2, 4, 39, 78, 156
- <u>331</u> is a prime number Factors: 1, 331
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and $19^2 > 331$)
- <u>506</u> $= 2 \times 11 \times 23$
  Factors: 1, 2, 11, 22, 23, 46, 253, 506
- <u>1156</u> $= 2 \times 2 \times 17 \times 17 = 2^2 \times 17^2$ [$= (2 \times 17)^2 = 34^2$]
  Factors: 1, 2, 4, 17, 34, 68, 289, 578, 1156 (square number)

8 min

Individual work, monitored (or whole class activity)

BB: 156, 331, 506, 1156

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

156	2
78	2
39	39
1	

506	2
253	11
23	23
1	

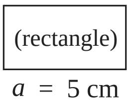
1156	2
578	2
289	17
17	17
1	

2

Perimeter and area

a) Let's calculate the perimeter and area of these polygons.

i)



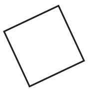
$b = 2$ cm

$a = 5$ cm

$P = 2 \times (5 + 2)$ cm $= \underline{14 \text{ cm}}$

$A = (5 \times 2)$ cm$^2 = \underline{10 \text{ cm}^2}$

ii)



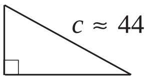
(square)

$a = 1.1$ m

$P = 4 \times 1.1$ m $= \underline{4.4 \text{ m}}$

$A = (1.1 \times 1.1)$ m$^2 = \underline{1.21 \text{ m}^2}$

iii)



$a = 2$ km

$c \approx 4472$ m

$b = 4$ km

(right-angled triangle)

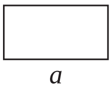
$P \approx (2 + 4 + 4.472)$ km

$= \underline{10.472 \text{ km}}$

$A = \dfrac{4 \times \cancel{2}^{1}}{\cancel{2}_{1}}$ km$^2 = \underline{4 \text{ km}^2}$

b) Let's see if you are clever enough to do it using letters instead of numbers! Elicit that each letter could stand for any value so the resulting equations are <u>general formulae</u> for perimeter and area.

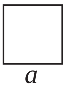
i)



$P = 2 \times (a + b)$

$A = a \times b = ab$

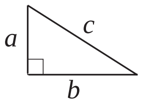
ii)



$P = 4 \times a = 4a$

$A = a \times a = a^2$

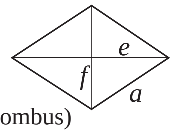
iii)



$P = a + b + c$

$A = \dfrac{a \times b}{2}$

iv)

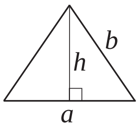


(rhombus)

$P = 4 \times a = 4a$

$A = \dfrac{e \times f}{2}$

v)



(isosceles triangle)

$P = a + b + b = a + 2b$

$A = \dfrac{a \times h}{2}$

Whole class activity

Shapes drawn on BB or use enlarged copy master or OHP

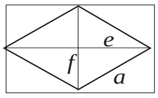
Ps first name the shape, then come to BB or dictate what T should write. Class points out errors.

At a good pace.

Reasoning, agreement, praising

Feedback for T

If necessary, remind Ps how to calculate the area of a rhombus by drawing the surrounding rectangle. e.g.



Elicit that the diagonals of a rhombus bisect each other at right angles.

Activity ***Notes***

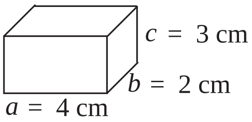
2

(Continued)

c) Let's write the surface area and volume of each of these polyhedra.

What is a polyhedron? (3-D shape with many plane faces)

i)



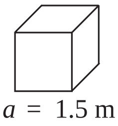
(cuboid)

$A = 2 \times (4 \times 2 + 4 \times 3 + 2 \times 3\ )\ \text{cm}^2 = 2 \times (8 + 12 + 6)\ \text{cm}^2$

$= 2 \times 26\ \text{cm}^2 = \underline{52\ \text{cm}^2}$

$V = (4 \times 2 \times 3)\ \text{cm}^3 = \underline{24\ \text{cm}^3}$

ii)

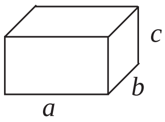


(cube)

$A = 6 \times (1.5 \times 1.5)\ \text{m}^2 = 6 \times 2.25\ \text{m}^2 = \underline{13.5\ \text{m}^2}$

$V = (1.5 \times 1.5 \times 1.5)\ \text{m}^3 = 1.5 \times 2.25\ \text{m}^3 = \underline{3.375\ \text{m}^3}$

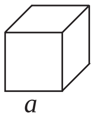
iii)



$A = 2 \times (a \times b + a \times c + b \times c\ )$

$V = a \times b \times c$ [= abc]

iv)



$A = 6 \times (a \times a\ ) = 6 \times a^2$ [= $6a^2$]

$V = a \times a \times a = a^3$

Notes:

(2.25 + 1.125 = 3.375)

[= $2\,(ab + ac + bc)$]

Elicit the short forms also.

20 min

3

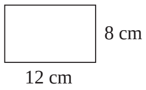
PbY6b, page 156

Q.1 Read: *Write below each* ***polygon*** *its perimeter and area.*

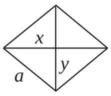
What is a polygon? (a plane shape with many straight sides)
Set a time limit of 3 minutes. Ps calculate mentally (or in *Ex. Bks*) and write results in *Pbs*.

Review with whole class. First elicit the name of the shape then Ps show results on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

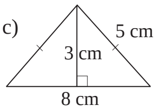
Solution:

a)


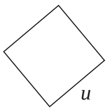
(rectangle)

b)


(rhombus)

c)


(isosceles triangle)

d)


(square)

a) $P = 2 \times (12 + 8)\ \text{cm} = 2 \times 20\ \text{cm} = \underline{40\ \text{cm}}$

$A = (12 \times 8)\ \text{cm}^2 = \underline{96\ \text{cm}^2}$

b) $P = 4 \times a$ (= $4a$); $A = \dfrac{x \times y}{2}$ (= $\dfrac{x\,y}{2}$)

c) $P = 8\ \text{cm} + 2 \times 5\ \text{cm} = \underline{18\ \text{cm}}$; $A = \dfrac{3 \times \cancel{8}^{\,4}}{\cancel{2}_{\,1}}\ \text{cm}^2 = \underline{12\ \text{cm}^2}$

d) $P = 4 \times u$ (= $4u$); $A = u \times u = u^2$

Notes:

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, self-correcting, praising

Extra praise if Ps give the short forms too.

Feedback for T

24 min

Y6

Lesson Plan 156

Activity

Notes

4

PbY6b, page 156

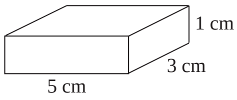
Q.2 Read: *Write below each* **polyhedron** *its surface area and volume.*

What is a polyhedron? [a solid (or 3-D shape) which has many plane faces (or only faces which are polygons)]

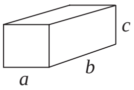
Set a time limit of 4 minutes. Ps do necessary calculations in *Ex. Bks* and write results in *Pbs*.

Review with whole class. First elicit the name of the shape then Ps show results on scrap paper or slates on command. Ps responding correctly explain reasoning to Ps who were wrong. Class agrees/disagrees. Mistakes discussed and corrected.

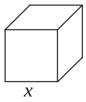
Solution:

a) 

(cuboid)

b) 

(cuboid)

c) 

(cube)

a) $A = 2 \times (5 \times 3 + 5 \times 1 + 3 \times 1)$ cm^2

$= 2 \times 23$ cm^2 $=$ $\underline{46 \text{ cm}^2}$

$V = (5 \times 3 \times 1)$ cm^3 $=$ $\underline{15 \text{ cm}^2}$

b) $A = 2 \times (a \times b + a \times c + b \times c)$ [$= 2\,(ab + ac + bc)$]

$V = a \times b \times c$ [$= abc$]

d) $A = 6 \times x \times x = 6 \times x^2$ [$= 6x^2$];

$V = \text{x} \times x \times x = x^3$

30 min

Individual work, monitored, (helped)

Drawn on BB or use enlarged copy master or OHP (or show actual models)

Responses shown in unison.

Reasoning, agreement, self-correcting, praising

Extra praise if Ps give the short forms too.

Feedback for T

Extension

What are the general formulae for the surface area and volume of a square-based cuboid?

$A = 4 \times a \times b + 2 \times a \times a$

$= 4 \times a \times b + 2 \times a^2$

[$= 4ab + 2a^2$]

$V = a \times b \times a = a^2 \times b$

[$= a^2 b$]

5

HMC:

Hungarian Mathematics Competition

1982

Age 11

PbY6b, page 156

Q.3 Read: *In the diagram, the points on the two sides are midpoints. What part of the square has been shaded?*

What shape has been shaded? (a concave deltoid)

Allow Ps a couple of minutes to think and try to solve it.
(If no P is on the right track, T gives hint about labelling the vertices and calculating the area of each unshaded part.)

If you have an answer, show me it . . . now! $\left(\dfrac{1}{4}\right)$

P with correct answer explains reasoning at BB to class. Class agrees/disagrees.

If no P has the correct answer, T leads Ps through the solution, involving them once they understand what to do.

Ps write solution in *Ex. Bks* too.

Solution: e.g.

Let each side of the square be 1 unit. Then:

Area of AEGF $= \dfrac{1}{2} \times \dfrac{1}{2} = \dfrac{1}{4}$ (square unit)

Area of Δ EBC = Area of Δ CDF $= \{\dfrac{1}{2} \times 1) \div 2$

$= \dfrac{1}{2} \div 2 = \dfrac{1}{4}$ (sq. unit)

34 min

Individual trial first, monitored, helped

(Revert to a whole class activity if no P has an idea)

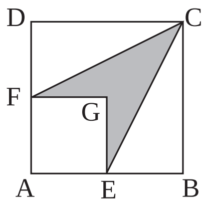
Drawn on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, self-correction, praising

BB:

e.g.



So Area of ECFG =

$1 - \dfrac{1}{4} - \dfrac{1}{4} - \dfrac{1}{4} = \dfrac{1}{4}$ (sq. unit)

Answer: One quarter of the square has been shaded.

Y6

Lesson Plan 156

Activity

6

HMC: Hungarian Mathematics Competition 1995 Age 11

PbY6b, page 156

Q.4 Read: *In the diagram, the sides of the large square are 3 units long. The sides of the large square have been divided into 3 equal parts and some of the dividing points have been joined up.*

What is the area of the shaded square?

Use the same idea to solve this question as we used in Q.3. Set a time limit of 3 minutes.

Review with whole class. Ps with answers show results on scrap paper or slates oncommand. P with corrrect answer explains reasoning at BB. Who agrees? Who did it another way? etc. (If no P has the correct answer, T directs Ps' thinking.)

Ps correct their mistakes or if they could not solve it, write the solution in *Ex. Bks*.

Solution:

Area of Δ AEH = Area of Δ EBF = Area of Δ FCG

= Area of Δ GDH = $\frac{1 \times \cancel{2}^{1}}{\cancel{2}_{1}} = \underline{1}$ (square unit)

Area of ABCD = $3 \times 3 = \underline{9}$ (square units)

Area of EFGH = $9 - 4 \times 1 = 9 - 4 = \underline{5}$ (square units)

Answer: The area of the shaded square is 5 square units.

37 min

Notes

Individual work, monitored, helped

(Revert to a whole class activity if no P has an idea)

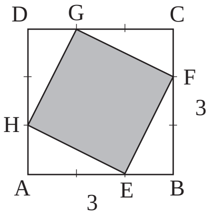
Drawn on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, self-correction, praising

BB:

e.g.



Class applauds Ps who solved the problem without help.

7

HMC: Hungarian Mathematics Competition 1993 Age 12

PbY6b, page 156, Q.5

Read: *A wooden cube was cut by some planes. The cuts were parallel to two opposite faces. The sum of the surface area of the pieces formed is 3 times the surface area of the cube. How many planes cut the cube?*

Allow 2 minutes for Ps to think and discuss with their neighbours if they wish. Who thinks they know what to do? Come and explain it to us. Who agrees? Who thinks something else? If no P has an idea, T directs Ps' thinking and class solves it together.

Solution: e.g.

Let the length of an edge of the cube be a, then

area of each face = a^2

surface area of the cube = $6 \times a^2$

When we make one cut along a suitable plane, the 2 pieces formed have 2 extra faces, i.e. their total surface area increases by $2 \times a^2$.

If we let the number of cuts be n, then

BB: $6 \times a^2 + n \times 2 \times a^2 = 18 \times a^2$ $[-(6 \times a^2)]$

$n \times 2 \times a^2 = 12 \times a^2$ $[\div (2 \times a^2)]$

$\underline{n = 6}$

Answer: Six planes cut the cube.

41 min

Whole class activity

(or individual trial first if Ps wish)

Discussion, reasoning, agreement, (self-correction) praising

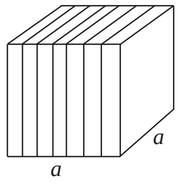
Extra praise if a P has a good idea of what to do.

T involves different Ps where possible.

Ps could copy the solution in *Ex. Bks*. too

Show the cuts in a diagram (or demonstrate on a model made from inter-locking plastic cubes)

BB:



Y6

Lesson Plan 156

Activity

8

HMC:
Hungarian
Mathematics
Competition
1996
Age 11

PbY6b, page 156, Q.6

Read: *Imagine a cube built from 27 small 1 cm cubes.*

The middle cube in each face is removed and so is the small cube at the centre of the large cube.

What is:

a) the surface area of the remaining solid

b) the volume of the reminaing solid?

Allow Ps a minute to think about it and discuss with their neighbours.

Who thinks they know what to do? Come and explain it to us. Who agrees? Who thinks something else? If no P has an idea, T directs Ps' thinking and class solves it together.

Solution: e.g.

a) Original volume is 27 cm^3, and 27 = 3 × 3 × 3, so the length of each edge is 3 cm.

Area of each face of the cube = (3 × 3) cm^2 = 9 cm^2

Surface area of the original cube = 6 × 9 cm^2 = 54 cm^2

After removing the 7 unit cubes:

Area lost = 6 × 1 cm^2 = 6 cm^2

Area gained = 6 × 4 cm^2 = 24 cm^2

So surface area of solid = 54 – 6 + 24 = $\underline{72}$ (cm^2)

b) After removing 7 unit cubes, the volume of the remaining solid is

27 cm^3 – 7 cm^3 = $\underline{20\text{ cm}^3}$

45 min

Notes

Whole class activity

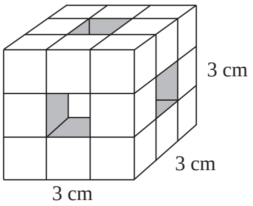
(or individual or paired trial if Ps wish)

If the class is not very able, T (or Ps) could build the model first but if Ps are able, let them imagine it first, using enlarged copy master or OHP, then confirm with a model.

Discussion, reasoning, agreement, praising

Involve several Ps.

BB:



Extra praise for Ps who work out the solution without help.

N.B. If done as an individual challenge, leave the problem open as homework if no P can solve it in the time remaining.

Y6

R: Calculations

C: Revision: Perimeter, area, volume, angles

E: Problems and challenges

Lesson Plan 157

Activity

Notes

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 4 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>157</u> is a prime number Factors: 1, 157
  (as not exactly divisible by 2, 3, 5, 7, 11, and $13^2 > 157$)
- <u>332</u> = $2 \times 2 \times 83 = 2^2 \times 83$ Factors: 1, 2, 4, 83, 166, 332
- <u>507</u> = $3 \times 13 \times 13 = 3 \times 13^2$ Factors: 1, 3, 13, 39, 169, 507
- <u>1157</u> = 13×89 Factors: 1, 13, 89, 1157

6 min

Individual work, monitored (or whole class activity)

BB: 157, 332, 507, 1157

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

332	2
166	2
83	83
1	

507	3
169	13
13	13
1	

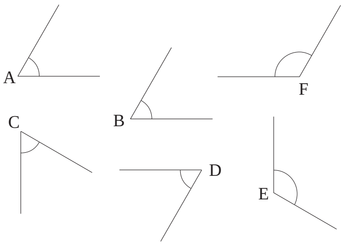
1157	13
89	89
1	

2

Angles

a) Which of these angles do you think are equal? Ps study them carefully then come to BB or dictate to T. Class agrees/disagrees. How did you decide whether they were equal or not? (Ps will probably mention imagining them being moved together or turned around in their heads, or noticing parallel or perpendicular arms, etc.)

BB:



$\angle A = \angle B = \angle D = \angle C$; $\angle E = \angle F$

b) Which pairs of angles do you think make an angle of 180°?

Ps come to BB or dictate to T. Class agrees/disagrees. How did you decide? (By imagining, e.g. $\angle A$ and $\angle F$ being moved together so that their vertices are at one point, then of course, each of the 2 angles can be replaced by an angle which is equal to it.)

BB: $\angle A + \angle F = 180°$, $\angle A + \angle E = 180°$, $\angle B + \angle F = 180°$,
$\angle B + \angle E = 180°$, $\angle C + \angle F = 180°$, $\angle C + \angle E = 180°$,
$\angle D + \angle F = 180°$, $\angle D + \angle E = 180°$

12 min

Whole class acti vity

Drawn on BB or use enlarged copy master or OHP

Involve as many Ps as possible.

Discussion, reasoning, agreement, checking, praising

[Practice in recognising equal angles and supplementary angles by comparing the position of their arms]

T: Angles which together form an angle of 180° are called supplementary angles.

BB: <u>Supplementary angles</u>

(The 3 angles in a triangle are supplementary angles.)

3

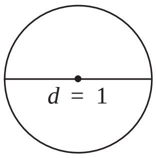
Circumference

What length is the circumference of each of these circles?

Ps come to BB or dictate to T. Class agrees/disagrees. T helps Ps to express the exact length using π.

BB:

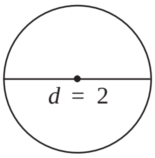
a)



$P \approx 3.14$ units

$P = \pi$ units

b)



$P \approx 6.28$ units

$P = 2 \times \pi$ units

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Reasoning agreement, praising

Elicit/remind Ps that the ratio of the circumference of a circle to its diameter is π (pi) and $\pi \approx 3.14$.

Y6

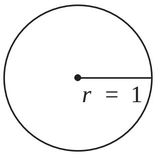
Lesson Plan 157

Activity ***Notes***

3

(Continued)

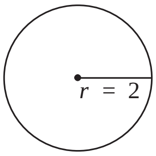
c)



$P \approx 6.28$ units

$P = 2 \times \pi$ units

d)



$P \approx 4 \times 3.14 = 12.56$ (units)

$P = 4 \times \pi$ units

Stress that when we use π, the value is exact but when we use 3.14, the value is only approximate.

17 min

4

PbY6b, page 157, Q.1

Read: *The circumference of a circle with radius 1 unit is* *$2 \times \pi \approx 6.28$ units.*
Colour an arc of the given length on the circumference of the circle.

a) If the whole circumference (T draws finger around it) is $2 \times \pi$, what part of the cirucmference is π? (half) **A**, come and show us the arc with your finger. Who agrees? Who can show us it another way? Elicit that the arc is a semicircle and there are lots of ways to show it but agree that it is easiest to start from the point on the circumference where the radius meets it on the digram (although the semicircle could be shown above or below the radius)

T colours the agreed arc on BB and Ps colour it in *Pbs* too.

Continue in this way, dealing with one part at a time but if T thinks that most Ps understand what to do, change to individual work under a time limit and review as usual with the whole class.

Whole class activity

Drawn on BB or use enlarged copy master or OHP

At a good pace

Involve several Ps.

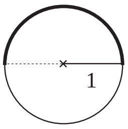
Discussion, reasoning, agreement, praising

Extension

What size of angle is formed by the two radii at the endpoints of each arc? Ps come to BB or dictate what T should write. Elicit that the whole central angle of a circle is $360°$, so, e.g. in a) the angle formed is half of $360°$, which is $180°$. etc.

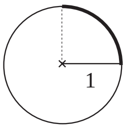
Solution:

a) π



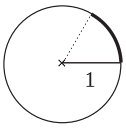
$180°$

b) $\frac{\pi}{2}$



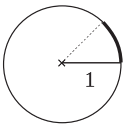
$90°$

c) $\frac{\pi}{3}$



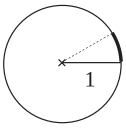
$60°$

d) $\frac{\pi}{4}$



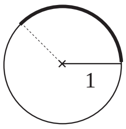
$45°$

e) $\frac{\pi}{6}$



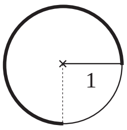
$30°$

f) $\frac{3}{4}\pi$



$135°$

g) $\frac{3}{2}\pi$



$270°$

Elicit that for circles with radius 1 unit:

$\pi = \frac{1}{2}$ of the circumference,

$\frac{\pi}{2} = \frac{1}{4}$ of the circumference,

$\frac{\pi}{3} = \frac{1}{6}$ of the circumference,

$\frac{\pi}{4} = \frac{1}{8}$ of the circumference

$\frac{\pi}{6} = \frac{1}{12}$ of the circumference

$\frac{3}{4}\pi = \frac{3}{8}$ of the circumference

$\frac{3}{2}\pi = \frac{3}{4}$ of the circumference

T: Mathematicians used to give the size of an angle using π in exactly this way before the unit we use now for measuring angles (degrees) had been thought of.

23 min

Y6

Lesson Plan 157

Activity

5

HMC:
Hungarian
Mathematics
Competition
1992
Age 11

PbY6b, page 157

Q.2 a) Read: *Is there a square in which the numerical value of its perimeter length (in cm) is equal to the numerical value of its area (in cm^2)?*

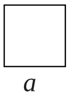
Allow Ps 2 minutes to think about it and discuss it with their neighbours if they wish.

Stand up if you think there is. Show me the length of one of its sides . . . .now! (4 cm)

P with correct answer explains reasoning to class. Who agrees? Who thought in another way? etc. If no P was correct, T leads Ps through the reasoning below. Ps who were wrong write the correct solution in *Ex. Bks*.

Solution:

Let the length of a side of the square be *a*. BB:



BB: $P = 4 \times a$ and $A = a \times a$

$$4 \times a = a \times a \quad [\div a]$$

$$\underline{4 = a}$$

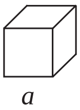
Answer: Yes, there is such a square and it has sides of 4 cm.

b) Read: *Is there a cube in which the numerical value of its surface area (in cm^2) is equal to the numerical value of its volume (in cm^3)?*

Again, set a time limit of 2 minutes. Continue in a similar way to a) but more Ps might be able to solve it this time.

Solution:

Let the length of an edge of the cube be *a*. BB:



BB: $A = 6 \times a \times a$ and $V = a \times a \times a$

$$6 \times a \times a = a \times a \times a \quad [\div (a \times a)]$$

$$\underline{6 = a}$$

Answer: Yes, there is such a cube and it has edges of 6 cm.

Notes

Individual trial first, monitored, helped

Deal with one part at a time.

Ps work in *Ex. Bks.*

Responses shown in unison.

Discussion, reasoning, checking, agreement, self-correction, praising

Class applauds Ps who solved both questions without help.

Check:

$P = 4 \times 4 = \underline{16}$ (cm)

$A = 4 \times 4 = \underline{16}$ (cm^2) ✔

Check:

$A = 6 \times 6 \times 6$

$= 36 \times 6 = \underline{216}$ (cm^2)

$V = 6 \times 6 \times 6 = \underline{216}$ (cm^3) ✔

28 min

6

HMC:
Hungarian
Mathematics
Competition
1993
Age 11

PbY6b, page 157

Q.3 Read: *You have 60* ***congruent*** *small cubes and you want to build different kinds of cuboids.*

You must use all the unit cubes to build each cuboid.

How many different sized cuboids could be built?

Set a time limit of 3 minutes. Ps work in *Ex. Bks*. (If Ps are struggling, T gives hints about factorising and drawing a table.)

Review with whole class. Ps who have an answer show result on scrap paper or slates on command. P anwering correctly explains at BB to Ps who were wrong. Who agrees? Who did it a different way? Mistakes discussed and corrected.

Solution: e.g. Let the edges of the cuboid be *a*, *b* and *c*.

$V = 60$	*a*	1	1	1	1	1	1	2	2	2	3
	b	1	2	3	4	5	6	2	3	5	4
	c	60	30	20	15	12	10	15	10	6	5

Answer: Ten different sized cuboids could be built.

Individual work, monitored, helped

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Ps who could not do it, copy solution in *Ex. Bks*.

Factors of 60:

1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

60	2
30	2
15	3
5	5
1	

32 min

Y6

Lesson Plan 157

Activity

Notes

7

HMC:
Hungarian Mathematics Competition
1983
Age 12

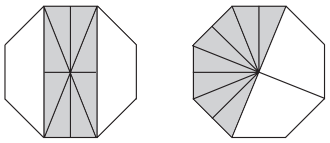
PbY6b, page 157, Q.4

Read: *The two regular octagons are congruent.*
Show that the two shaded areas are equal.

What is a regular octagon? (Polygon with 8 equal sides and angles)

Allow Ps a minute to think about the problem and discuss with their neighbours if they wish. Who has an idea? Who agrees? Who thinks something else? etc. If no P is on the right track T hints about dividing up the two shaded areas into equal parts. How could we do it? Ps come to BB to show it. Class agrees/disagrees. (Elicit that the shaded areas can be divided into congruent right-angled triangles: each triangle has base length half the side of the octagon, height from the centre of the octagon perpendicular to a side, and <u>hypotenuse</u> from the centre of the octagon to a vertex.] Ps do the same in *Pbs* too.

Solution:



To find the centre of each octagon:

LHS: draw diagonals of rectangle

RHS: join up another pair of opposite vertices

Answer: Each of the two shaded areas contains 8 congruent right-angled triangles which form half of the octagon, so the shaded areas are equal.

Whole class activity

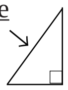
Drawn on BB or use enlarged copy master or OHP

Discussion, reasoning, agreement, praising

Extra praise for Ps who think of this without help.

Remind Ps of the name of the side opposite the right angle in a right-angled triangle

BB: <u>hypotenuse</u>



Class agrees on a form of words for the answer and Ps write it in *Pbs*.

36 min

8

HMC:
Hungarian Mathematics Competition
1991
Age 12

PbY6b, page 157

Q.5 Read: *The shorter side of a rectangle is 2 units and each of its diagonals is 4 units.*

a) What size are the angles formed by the diagonals?

b) What size are the angles formed by the diagonals and the sides?

What should we do first? (Draw a diagram and label it.)

What do we know about the diagonals of a rectangle? (They are equal and bisect one other.)

Set a time limit of 3 minutes. Ps work in *Ex. Bks*.

Review with whole class. Ps show angles on scrap paper or slates on command. Ps answering correctly explain reasoning at BB, with T's help if necessary. Who did the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution: e.g.

a) AD = BC = 2, AC = BD = 4

In Δ AOD:

$$AD = 2, \text{ and } AO = DO = \frac{4}{2} = 2$$

Δ AOD is equilateral so each of its angles is: $\dfrac{180°}{3} = \underline{60°}$.

$A\hat{O}D = C\hat{O}B = 60°$, (opposite angles)

so $A\hat{O}B = C\hat{O}D = 180° - 60° = \underline{120°}$

(as BD is a straight line)

Answer: The smaller angles formed by the diagonals are 60° and the larger angles are 120°.

Indvidual trial first, monitored, helped

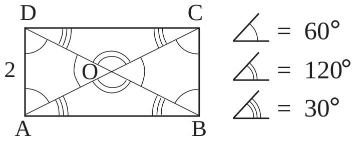
[If Ps are struggling, stop individual work and continue as a whole class activity, with T working on BB with help of class and Ps working in *Ex. Bks*.]

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Extra praise for Ps who realised that two different sized angles are formed by the diagonals.

BB:



b) $D\hat{A}B = 90°$, so

$C\hat{A}B = 90° - 60° = \underline{30°}$

Answer: The angles formed by the diagonals with the vertical sides are 60° and by the diagonals with the horizontal sides are 30°.

40 min

Y6

Lesson Plan 157

Activity

9

HMC:
Hungarian Mathematics Competition
1994
Age 12

PbY6b, page 157, Q.6

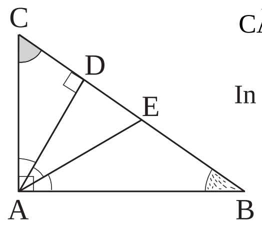
Read: *In this right-angled triangle, the lines DA and EA divide the right angle CAB into 3 equal parts.*

DA is perpendicular to the ***hypotenuse*** *BC. E is the midpoint of BC.*

What size are the acute angles of triangle ABC?

Allow Ps a minute to think about it and discuss with their neighbours if they wish. Who thinks they know what to do? Come and show us. Who agrees? Who would do it another way? If no P has an idea, T gives hint about the sum of the angles in a triangle. If still no P can do it, T leads Ps through the solution below, involving Ps once they understand. Ps write the solution in *Ex Bks*.

BB: e.g.



$$C\hat{A}D = D\hat{A}E = E\hat{A}B = \frac{90^\circ}{3} = 30^\circ$$

$$\text{In } \Delta\,ADC,\quad D\hat{C}A = 180^\circ - 90^\circ - 30^\circ$$

$$= 90^\circ - 30^\circ = \underline{60^\circ}$$

$$\text{In } \Delta\,ABC,\quad \angle A + \angle B + \angle C = 180^\circ$$

$$\text{so}\quad A\hat{B}C = 180^\circ - 90^\circ - 60^\circ = \underline{30^\circ}$$

Answer: In triangle ABC, angle C is 60° and angle B is 30°.

45 min

Notes

Whole class activity

Drawn on BB or use enlarged copy master or OHP

Ps decide what to do first and how to continue.

Discussion, reasoning, agreement, praising

Feedback for T

[Other methods are possible, e.g. Δ ABE is an isosceles Δ, with base AB and EA = EB, so $E\hat{A}B = E\hat{B}A = \underline{30^\circ}$
or
obtain angle B from Δ ABD, etc.]

Y6

R: Calculations

C: Revision: Number systems with bases other than 10

E: Problems and challenges

Lesson Plan
158

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>158</u> = 2 × 79 Factors: 1, 2, 79, 158
- <u>333</u> = 3 × 3 × 37 = 3^2 × 37 Factors: 1, 3, 9, 37, 111, 333
- <u>508</u> = 2 × 2 × 127 = 2^2 × 127
  Factors: 1, 2, 4, 127, 254, 508
- <u>1158</u> = 2 × 3 × 193 =
  Factors: 1, 2, 3, 6, 193, 386, 579, 1158

Individual work, monitored (or whole class activity)

BB: 158, 333, 508, 1158

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$\begin{array}{r|l} 158 & 2 \\ 79 & 79 \\ 1 & \end{array}$ $\begin{array}{r|l} 333 & 3 \\ 111 & 3 \\ 37 & 37 \\ 1 & \end{array}$

$\begin{array}{r|l} 508 & 2 \\ 254 & 2 \\ 127 & 127 \\ 1 & \end{array}$ $\begin{array}{r|l} 1158 & 2 \\ 579 & 3 \\ 193 & 193 \\ 1 & \end{array}$

8 min

2

Different number systems

a) Do you remember in Year 3 learning about number systems which were not based on 10? We called them Numberlands. We imagined creatures called Dizzy Dombles who lived in Threeland. They used the number 3 as their base number instead of 10, and this is one of their numbers. Who can read it out? What does it mean? Ps come to BB or dictate to T. Class agrees/disagrees.

Whole class activity

Written on BB.

Discussion, reasoning, agreement, praising

Involve as many Ps as possible.

BB: $2102_{\textcircled{3}}$ (read as 'two one zero two, in base 3')

means in <u>base 10</u>: 2 × (3 × 3 × 3) + 1 × (3 × 3) + 0 × 3 + 2 × 1

$= 2 \times 3^3 + 1 \times 3^2 + 0 \times 3 + 2 \times 1$

$= 2 \times 27 + 1 \times 9 + 0 \times 3 + 2 \times 1$

= 54 + 9 + 0 + 2 = <u>65</u>

('sixty five' in the base 10 number system: 6 × 10 + 5 × 1)

[Tell Ps that when a base is <u>not</u> specified, we assume that the number is in base 10]

Who can read out this number? (one zero two two, in base 3)
What does it mean? Ps come to BB or dictate to T. Class agrees/disagrees.

BB: $1022_{\textcircled{3}} = 1 \times 3^3 + 0 \times 3^2 + 2 \times 3 + 2 \times 1$

$= 1 \times 27 + 0 \times 9 + 2 \times 3 + 2 \times 1$

= 27 + 6 + 2 = <u>35</u> (base 10)

i.e. 'thirty-five'
3 × 10 + 5 × 1

Let's show the numbers from 1 to 10 in base 3 in a table.

Ps come to BB or dictate what T should write. Class agrees/disagrees.

BB:

In base 10	1	2	3	4	5	6	7	8	9	10	. . .
In base 3	1	2	10	11	12	20	21	22	100	101	. . .

Table already prepared and Ps fill in the numbers.

Let's read out the base 3 numbers in increasing order. (One, two, one zero, one one, one two, two zero, two one, two two, one zero zero, one zero one, etc.)

In unison. Praising

Activity ***Notes***

2

(Continued)

b) Who can say these numbers? What do they mean? Ps come to BB or dictate to T. Class agrees/disagrees.

BB: i) $413_{\textcircled{7}} = 4 \times 7^2 + 1 \times 7 + 3 \times 1$

$= 4 \times 49 + 1 \times 7 + 3 \times 1 = 196 + 7 + 3$

$= \underline{206}$ (base 10)

ii) $101101_{\textcircled{2}} = 1 \times 2^5 + 0 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^2 + 1$

$= 1 \times 32 + 1 \times 8 + 1 \times 4 + 1 = \underline{45}$ (base 10)

c) Let's code the number 74 in the base 5 number system.

What would the value of the place-value columns be? Ps come to BB or dictate to T.

BB: . . . 5^3 5^2 5 1

i.e. (. . . 125 25 5 1)

How can we do it? (Divide 74 by 25, then divide the remainder by 5, then the remainder after that is the number of 'ones'.) Ps come to BB or dictate what T should write.

BB: $74 \div 25 = \underline{2}$, r 24

$24 \div 5 = \underline{4}$, r $\underline{4}$ $\rightarrow$ $74 = 244_{\textcircled{5}}$

Notes:

'four one three, in base 7'

'one zero one one zero one, in base 2'

Ps say in unison:

'Seventy-four is two four four in base 5.'

20 min

3

HMC:

Hungarian Mathematics Competition

1980

Age 11

PbY6b, page 158**, **Q.1

Read: *This number is in a number system which is based on a number less than 10. We know that:*

- *when the number is divided by 2, there is a remainder of 1,*
- *when the number is divided by 3, there is a remainder of 0,*
- *when the number is divided by 4, there is a remainder of 3.*

What is the base number?

What can we tell about the base number? (It cannot be 1, 2 or 3 because the base must be greater than the digits in the number. It must be even, because if the base was odd, the sum of the digits would be even and it would not fulfil the first condition above.)

Which numbers are left? (4, 6, 8) Let's try them out. T leads Ps through the solution below, letting Ps take over when they understand.

Solution: e.g. Let's call the base number *b*.

BB: $123_{\textcircled{b}} = 1 \times b^2 + 2 \times b + 3 \times 1 = b \times b + 2 \times b + 3$

If $b = 4$: $123_{\textcircled{4}} = 16 + 8 + 3 = \underline{27}$

($27 \div 2 = 13$, r 1 ✔, $27 \div 3 = 9$ ✔, $27 \div 4 = 6$, r 3 ✔)

If $b = 6$: $123_{\textcircled{6}} = 36 + 12 + 3 = \underline{51}$

($51 \div 2 = 25$, r 1 ✔, $51 \div 3 = 17$ ✔, $51 \div 4 = 12$, r 3 ✔)

If $b = 8$: $123_{\textcircled{8}} = 64 + 16 + 3 = \underline{83}$

($83 \div 2 = 41$, r 1 ✔, $83 \div 3 = 27$, r 2 ✗, so $b \neq 8$

Notes:

Whole class activity

(or individual trial if Ps wish)

Written on BB

BB: 1 2 3☐

Discussion, reasoning, agreement, praising

(e.g. If ☐ = 3, number is

$1 \times 9 + 2 \times 3 + 3 = 1\underline{8}$)

Extra praise if Ps think of this, otherwise T gives hints or suggests it and asks Ps if the reasoning is correct

Reasoning, agreement, praising

Ps write solution in *Ex. Bks*.

[$b = 4$ is a solution]

[$b = 6$ is also a solution]

Answer:

The base number is 4 or 6.

25 min

Y6

Lesson Plan 158

Activity

4

HMC:
Hungarian
Mathematics
Competition
1986
Age 11

PbY6b, page 158

Q.2 a) Read: *Number the days of 2004 in order. This is their* ***ordinal value*** *.*

Example: 1st of January is ***1****;*
*5th of February is 31 + 5 = **36**, etc.*

What is special about the year 2004? [It was a leap year (divisible by 4) so there were 29 days in February.]

Quickly revise the number of days in the other months.

Set a short time limit. Ask Ps to list just the first day of each month and its ordinal value (it would take too long to include every day) in *Ex. Bks.* (Less able Ps could have a 2004 calendar on desks.)

Review with whole class. T has days already listed and Ps dictate the ordinal values. Ps check their own list and correct any mistakes.

If we let any date in a month be n, who can think of another way to write the ordinal values? If necessary, T does the first one, then Ps dictate the others.

b) Read *Multiply each date in every month by the ordinal value of the month. This is their* ***product value****.*

*Example: 11 April: 11 × 4 = **44**;*
*31 October: 31 × 10 = **310**, etc.*

Do we need to write out the product values of every day in every month? Elicit that, again, we could use n for any date in a month. Ps make another column in *Ex. Bks* and list the product values for the months (as opposite).

Review quickly with whole class. Ps dictate to T and check and correct their own lists.

c) Read: *How many days were there in the year 2004 when the* ***ordinal value*** *and the* ***product value*** *were equal?*

How do you think we can do this without having to compare the two values for all 366 days? (If the two expressions involving n are equal, n should work out as a whole day.)
T gives hint if Ps cannot think of it. Do a random example on BB, with Ps dictating what T should write, e.g.

BB: Jul: $182 + n = 7n$ $[-n]$
$182 = 6n$ $[\div 6]$
$30\frac{2}{6} = n$ (impossible, so <u>none</u> in July)

Ps do the rest in *Ex. Bks.* under a time limit.

How many such days did you find? Show me . . .now! (32)

P answering correctly explains reasoning. Who agrees? Who found another day? Come and show us. Class points out errors. Mistakes corrected. T chooses a P to say the answer in a sentence.

Answer: In 2004, there were 32 days when the ordinal value and product value were equal.

[N.B. Ps could, of course, list ordinal and product values for all 366 days and then count those which are equal – a lot of work!]

Notes

Individual work, monitored, (helped), one part at a time

BB:

<u>2004 Ordinal Values</u>

1 Jan → 1 or n
1 Feb → 32 or $31 + n$
1 Mar → 61 or $60 + n$
1 Apr → 92 or $91 + n$
1 May → 122 or $121 + n$
1 Jun → 153 or $152 + n$
1 Jul → 183 or $182 + n$
1 Aug → 214 or $213 + n$
1 Sep → 245 or $244 + n$
1 Oct → 275 or $274 + n$
1 Nov → 306 or $305 + n$
1 Dec → 336 or $335 + n$

<u>2004 Product Values</u>

1 Jan → $1 \times n = n$
1 Feb → $2 \times n = 2n$
1 Mar → $3 \times n = 3n$
1 Apr → $4 \times n = 4n$
1 May → $5 \times n = 5n$
1 Jun → $6 \times n = 6n$
1 Jul → $7 \times n = 7n$
1 Aug → $8 \times n = 8n$
1 Sep → $9 \times n = 9n$
1 Oct → $10 \times n = 10n$
1 Nov → $11 \times n = 11n$
1 Dec → $12 \times n = 12n$

Solution:

Jan: $n = n$ (Identity – <u>any</u> day)
so <u>31</u> such days in January

Feb: $31 + n = 2n$ $[-n]$
$\underline{31 = n}$
but there is no 31st of February!

Mar: $60 + n = 3n$ $[-n]$
$60 = 2n$ $[\div 2]$
$\underline{30 = n}$
so <u>1</u> day in March (30th)

. . .

Dec: $335 + n = 12n$ $[-n]$
$335 = 11n$ $[\div 11]$
$30\frac{5}{11} = n$ (impossible)
so no such days in December.

35 min

Y6

Lesson Plan 158

Activity ***Notes***

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done in class time as individual challenges or as whole class activities, with any remaining questions set as optional homework and reviewed interactively before the start of *Lesson 159*. Or T could divide the class into teams of roughly equal ability and set the remaining questions as a 'maths challenge' competition.

Make sure that the questions are reviewed by the whole class, whether Ps attempted them or not.

T could have a prize for the team which solves most questions correctly.

5

HMC:
Hungarian Mathematics Competition
1988
Age 11

PbY6b, page 158

Q.3 Read: *I cut a rectangle into two parts by drawing a straight line. Then I cut one of the two parts into two polygons by drawing another straight line. Then I cut one of the two polygons by drawing another straight line, and so on.*

After I had drawn 100 dividing lines, I counted the vertices of all the polygons I had formed. I counted 300 vertices.

Is this possible? Give a reason for your answer.

Solution: e.g.

After 100 cuts, we would have 101 polygons. Even if they were all triangles (the polygon with the least number of vertices), the number of vertices would be

$= 101 \times 3 = \underline{303}$ and $303 > 300$

Answer: No, it is not possible, as the number of vertices must be at least 303.

Individual trial first

Ps could have paper and scissors on desks

(or whole class activity, with Ps demonstrating the cutting in front of class)

There is no need to do all 100 cuts, just enough for Ps to undertand what is happening.

6

HMC:
Hungarian Mathematics Competition
1993
Age 11

PbY6b, page 158

Q.4 Read: ***Prove*** *that if all the natural numbers from 1 up to and including a number which has units digit 5 (in a base 10 number system), the sum will be divisible by 5.*

Solution: e.g.

$1 + 2 + 3 + 4 + 5 = 15$, and 15 is divisible by 5

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15$

$= 16 \times 7 + 8 = 112 + 8 = 120$, and 120 is divisible by 5, etc.

or ' the sum of any 5 consecutive natural numbers is a multiple of 5 because the remainders will be 1, 2, 3, 4 and 0, which sum to a multiple of 5, so any number of such groups will also sum to a multiple of 5'.

Whole class activity

(Note that: $1 + 15 = 16$,
$2 + 14 = 16$.
. . .
$7 + 9 = 16$)

T gives hint about using the remainders if no P thinks of it.

7

HMC:
Hungarian Mathematics Competition
1984
Age 12

PbY6b, page 158

Q.5 Read: *Liz started to write the whole numbers from 1 and now she is writing the 2893rd digit. Which whole number is she now writing?*

Solution:

There are 9 1-digit, 90 2-digit and 900 3-digit numbers.

The number of digits is: $1 \times 9 + 2 \times 90 + 3 \times 900$

$= 9 + 180 + 2700 = \underline{2889}$

Liz is writing the 2893rd digit: $2893 - 2889 = 4$

So Liz is writing the 4th digit after 999, i.e. the first 4-digit number, 1000, and she has reached the last '0' of that number.

Individual trial

1-digit: 1 to 9
2-digits: 10 to 99
3-digits: 100 to 999

Activity

8

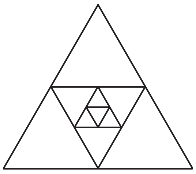
HMC:
Hungarian
Mathematics
Competition
1984
Age 12

PbY6b, page 158

Q.6 Read: *Four equilateral triangles have been drawn, one inside the other. The area of the innermost, smallest triangle is 1 square unit.*

What is the sum of the areas of the 4 triangles?

Solution: e.g.



A of ▽ = 1

A of △ = 4

A of ▽ = 16

A of △ = 64

Sum of the 4 areas:
1 + 4 + 16 + 64
= <u>85</u> (square units)

Notes

Individul trial

Y6

R: Calculations
C: Sequences, factors, divisibility
E: Complex problems, challenges

Lesson Plan 159

Activity | *Notes*

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>159</u> = 3 × 53 Factors: 1, 3, 53, 159
- <u>334</u> = 2 × 167 Factors: 1, 2, 167, 334
- <u>509</u> is a prime number Factors: 1, 509
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and $23^2 > 509$
- <u>1159</u> = 19 × 61 Factors: 1, 19, 61, 1159

8 min

Individual work, monitored (or whole class activity)

BB: 159, 334, 509, 1159

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 159 & 3 \\ 53 & 53 \\ 1 & \end{array} \qquad \begin{array}{r|l} 334 & 2 \\ 167 & 167 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1159 & 19 \\ 61 & 61 \\ 1 & \end{array}$$

2

PbY6b, pahe 159

Q.1 Read: *The first 10 positive integers are multiplied together.*
How mny zeros are at the right-hand side of the product?

Set a time limit of 4 minutes. Ps work in *Ex. Bks.*

Review with whole class. Ps could show number of zeros on scrap paper or slates on command. P answering correctly explains reasoning. Who thought the same? Who did it a different way? etc. Mistakes discussed and corrected.

Solution:

$1 \times 2 \times 3 \times 4 \times 5 \times 6 \times 7 \times 8 \times 9 \times 10$

$= 1 \times 2 \times 3 \times 2^2 \times 5 \times (2 \times 3) \times 7 \times 2^3 \times 3^2 \times (2 \times 5)$

This product has two prime factors which are 5 and more than two prime factors which are 2, so there are <u>two</u> factors which are (2 × 5 = 10).

So the product is divisible by 10 ×10 = 100,

but as there are no more factors of 10, it is <u>not</u> divisible by 1000.

Answer: There are 2 zeros at the right-hand side of the product.

14 min

Individual work, monitored, helped

Responses shown in unison .

Reasoning, agreement, self-correction, praising

Check with a calculator.

Check:

1 × 2 × 3 × 4 × 5 × 6 × 7 × 8 × 9 × 10 = 3 628 8<u>00</u>

3

HMC:
Hungarian Mathematics Competition
1986
Age 12

PbY6b, page 159, Q.2

Read: *The first 100 positive whole numbers are multiplied together.*
In the product, which digit is in the place value 24th from the right?

How can we show the multiplication without writing all the factors?

BB: 1 × 2 × 3 × 4 × 5 × . . . × 98 × 99 × 100

Which digits do we know will be 1st and 2nd from the right? (00, as one of the factors is 100). What other factor will produce a zero in the product? (10) Let's see if we can make 100s or 10s from the factors.

How many factors of 5 × 5 = 25 are in the multiplication? (4)

Elicit that there are more than 4 factors of 4 in the multiplication, so there are <u>4</u> factors involving 25 × 4 = 100.

How many factors of 5 are in the multiplication other than those we have used for the 25s? (16) Elicit that there are more than 16 factors containing 2, so there are <u>16</u> factors of 5 × 2 = 10 in the multiplication.

This means that the 4 factors of 100 and 16 factors of 10 will give at least 4 × 2 + 16 = 8 + 16 = <u>24</u> zeros on the RHS of the product.

20 min

Whole class activity

Ask Ps if anyone has an idea what to do (without having to multiply all the numbers) but if Ps cannot think of anything, T leads Ps through the solution.

Elicit that:

100 = 25 × 4 and 10 = 5 × 2

Factors of 25: 25, 50, 75, 100

Factors of 5 <u>not</u> involving 25:

5, 10, 15, 20, 30, 35, 40, 45, 55, 60, 65, 70, 80, 85, 90, 95

Answer: The last 24 digits of the product are zeros, so the 24th digit from the right is 0.

Y6

Lesson Plan 159

Activity

Notes

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done in class time as individual challenges or as whole class activities, with any remaining questions set as optional homework and reviewed interactively before the start of*Lesson 160*. Or T could divide the class into teams of roughly equal ability and set the remaining questions as a 'maths challenge' competition.

Make sure that the questions are reviewed by the whole class, whether Ps attempted them or not.

T could have a prize for the team which solves most questions correctly.

4

PbY6b, page 159

Q.3 Read: *Imagine that this fraction is simplified as far as possible.*

$$\frac{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10}{2^{10}}$$

[2^{10} *means the product of 10 factors and each factor is 2.]*

Which number will be the denominator of the simplified fraction?

Set a time limit. Ps work in *Ex. Bks* and discuss with neighbours if they wish.

Review with whole class. Ps show number on scrap paper or slates on command. P answering correctly explains reasoning to class. Who agrees? Who thought in a different way? etc. Mistakes discussed and corrected.

Solution: e.g

$$\frac{1\times \cancel{2}\times 3\times \overset{\cancel{2}}{\cancel{4}}\times 5\times \overset{3}{\cancel{6}}\times 7\times \overset{\overset{\cancel{2}}{\cancel{4}}}{\cancel{8}}\times 9\times \overset{5}{\cancel{10}}}{\cancel{2}\times \cancel{2}\times \cancel{2}\times \cancel{2}\times \cancel{2}\times \cancel{2}\times \cancel{2}\times \cancel{2}\times 2\times 2}$$

$$= \frac{1\times 3\times 5\times 3\times 7\times 9\times 5}{2\times 2} = \frac{3\times 5\times 3\times 7\times 9\times 5}{4}$$

and it cannot be simplified further.

<u>or</u>

$$\frac{1\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10}{2^{10}}$$

$$= \frac{1\times 2\times 3\times \left(2^2\right)\times 5\times (2\times 3)\times 7\times \left(2^3\right)\times 9\times (2\times 5)}{2^{10}}$$

$$= \frac{2^8\times\ (3\times\ 5\times 3\times 7\times 9\times 5)}{2^{10}}$$

$$= \frac{3\times 5\times 3\times 7\times 9\times 5}{2^2} = \frac{3\times 5\times 3\times 7\times 9\times 5}{4}$$

Answer: The denominator of the simplifed fraction will be 4.

Individual trial first

Written on SB or OHT

Also accept an explanation in words: e.g.

There are 5 even factors in the numerator, so 5 '2's can be cancelled out in the numerator and denominator.

There are 2 factors which are divisible by 4, so another two '2's can be cancelled out in the numerator and denominator.

There is one factor which is divisible by 8, so another '2' in the numerator and denominator can be cancelled out.

So 8 '2's can be cancelled out in both the numerator and the denominator, leaving 2 '2's in the denominator, so the denominator is 4.

Activity ***Notes***

5

HMC:
Hungarian
Mathematics
Competition
1987
Age 12

PbY6b, page 159

Q.4 Read: *Imagine that this fraction is simplified as far as possible.*

$$\frac{1 \times 2 \times 3 \times 4 \times 5 \times \ldots \times 98 \times 99 \times 100}{2^{100}}$$

[2^{100} *means the product of 100 factors and each factor is 2.]*

Which number will be the denominator of the simplified fraction?

Solution: e.g

$100 \div 2 = \underline{50}$ (no. of even factors)

$100 \div 4 = \underline{25}$ (no. of factors which are divisible by 2^2)

$100 \div 8 = \underline{12}$, r . . . (no. of factors which are divisible by 2^3)

$100 \div 16 = \underline{6}$, r . . . (no. of factors which are divisible by 2^4)

$100 \div 32 = \underline{3}$, r . . . (no. of factors which are divisible by 2^5)

$100 \div 64 = \underline{1}$, r. . . (no. of factors which are divisible by 2^6)

So no. of '2's in numerator which can cancel '2's in denominator:

BB: $50 + 25 + 12 + 6 + 3 + 1 = 97$

$$\frac{2^{100}}{2^{97}} = 2^3 = 2 \times 2 \times 2 = \underline{8}$$

Answer: The denominator of the simplified fraction will be 8.

Whole class activity

Written on BB or SB or OHT

Elicit that the ellipsis stands for the numbers not shown.

Agree that it would take too long to write out every number in the numerator, so ask Ps to think of another way of solving the problem.

T directs Ps thinking if necessary.

[We are assuming that the base set of numbers is the set of natural numbers!]

6

HMC:
Hungarian
Mathematics
Competition
1990
Age 12

PbY6b, page 159, Q.5

Read: *Some* ***consecutive*** *whole numbers, from 1 to a positive whole number which is greater than 1, are addded together.*

Which digit can be in the units place value in the sum?

(Give a reason for your answer.)

What does consecutive mean? (one following the other in order)

e.g. 1 + 2 + 3 + . . . Ps dictate the sums for 2, 3, 4, etc. numbers.

BB: $1 + 2 = \underline{3}$, $1 + 2 + 3 = \underline{5}$, $1 + 2 + 3 + 4 = 1\underline{0}$,

$1 + 2 + 3 + 4 + 5 = 1\underline{5}$, $1 + 2 + 3 + 4 + 5 + 6 = 2\underline{1}$,

$1 + 2 + 3 + 4 + 5 + 6 + 7 = 2\underline{8}$, $1 + 2 + \ldots + 8 = 3\underline{6}$,

$1 + 2 + 3 + \ldots + 9 = 5\underline{5}$, $1 + 2 + 3 + \ldots + 10 = 6\underline{5}$,

So it seems as if the units digit in the sum can be 0, 1, 3, 5, 6 or 8.

[Or T suggests calling the number of terms (i.e. the greatest term) n.

BB: $1 + 2 + 3 + \ldots + (n - 3) + (n - 2) + (n - 1) + n = \dfrac{1 + n}{2} \times n$

Subsitituting numbers for n: $\dfrac{1 \times 2}{2} = \underline{1}$, $\dfrac{2 \times 3}{2} = \underline{3}$, $\dfrac{3 \times 4}{2} = \underline{6}$,

$\dfrac{4 \times 5}{2} \rightarrow \underline{0}$, $\dfrac{5 \times 6}{2} \rightarrow \underline{5}$, $\dfrac{6 \times 7}{2} \rightarrow \underline{1}$, $\dfrac{7 \times 8}{2} \rightarrow \underline{8}$,

$\dfrac{8 \times 9}{2} \rightarrow \underline{6}$, $\dfrac{9 \times 10}{2} \rightarrow \underline{5}$, $\dfrac{10 \times 11}{2} \rightarrow \underline{5}$, etc.]

Whole class activity
(or individual trial if Ps wish)

$1 + 2 + 3 + \ldots + 11 = 7\underline{6}$

$1 + 2 + 3 + \ldots + 12 = 8\underline{8}$

etc.

or Sum $= \dfrac{n\,(n + 1)}{2}$

Answer:
Any of the digits 0, 1, 3, 5, 6 or 8 can be in the units place-value column in the sum.

Y6

Lesson Plan 159

Activity ***Notes***

7

HMC:
Hungarian Mathematics Competition
1991
Age 12

PbY6b, page 159

Q.6 Read: *A new volume in a series of books is published every 7 years. When the 7th volume was published, the sum of all the year numbers in which a book was published was 13 727.*

In which year was the first volume in the series published?

Solution: e.g.

Let the 1st year of publication be *n*. Then

$n + (n + 7) + (n + 14) + (n + 21) + (n + 28) + (n + 35) + (n + 42) = 13\,727$

$7 \times n + 147 = 13\,727$ [– 147]

$7 \times n = 13\,580$ [÷ 7]

$\underline{n = 1940}$

Answer: The first volume was published in 1940.

Individual work

Check:

$$\begin{array}{r} 1940 \\ 1947 \\ 1954 \\ 1961 \\ 1968 \\ 1975 \\ +\ 1982 \\ \hline 13727 \\ \hline \end{array} \checkmark$$

8

HMC:
Hungarian Mathematics Competition
1999
Age 12

PbY6b, page 159, Q.7

Read: *The whole numbers from 1 to 1999 are added together.*

Is the sum the square of a natural number?

Give a reason for your answer.

Ps decide what to do first and how to continue. T gives hint or directs Ps' thinking only if necessary. Class agrees/disagrees. Ps write solution in *Ex. Bks.*

Solution: e.g.

$$1 + 2 + 3 + \ldots + 1997 + 1998 + 1999 = \frac{1 + 1999}{2} \times 1999$$

$$= 1000 \times 1999 = 1999\,000$$

1999 000 cannot be a square number, as it is divisible by 10^2 but not by $\left(10^2\right)^2 = 10^4$.

Check:

$$\begin{array}{r|l} 1999\,000 & 2 \\ 999\,500 & 2 \\ 499\,750 & 2 \\ 249\,875 & 5 \\ 49\,975 & 5 \\ 9\,995 & 5 \\ 1\,999 & 1999 \\ 1 & \end{array}$$

$1999\,000 = 2^3 \times 5^3 \times 1999$

which is not a square number.

Elicit or point out that the prime factors of a square number have even powers.

e.g. $144 = 2^4 \times 3^2 = \left(2^2 \times 3\right)^2$

Whole class activity

and 1999 is a prime number (as it is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and $47^2 > 1999$)

Y6

Lesson Plan
160

Activity

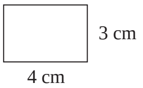
Factorising 160, 335, 510 and 1160. Revision and practice.

PbY6b, page 160

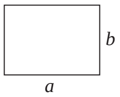
Solutions:

Q.1

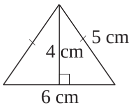
a)



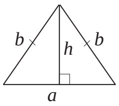
b)



c)



d)



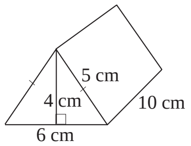
a) $P = 2 \times (4 + 3)$ cm $=$ $\underline{14\text{ cm}}$; $A = 4 \times 3 = \underline{12}$ (cm^2)

b) $P = 2 \times (a + b) = \underline{2\,(a + b)}$; $A = a \times b = \underline{ab}$

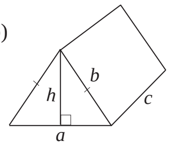
c) $P = 6$ cm $+ (2 \times 5)$ cm $=$ $\underline{16\text{ cm}}$; $A = \dfrac{6 \times \cancel{4}^{\,2}}{\cancel{2}_{1}}$ cm $=$ $\underline{12\text{ cm}}$

d) $P = a + 2 \times b = \underline{a + 2b}$; $A = \dfrac{a \times h}{2} = \underline{\dfrac{ah}{2}}$

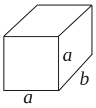
Q.2 a)



b)



c)



a) $A = 2 \times \dfrac{6 \times \cancel{4}^{\,2}}{\cancel{2}_{1}}$ cm^2 $+ 2 \times (5 \times 10)$ cm^2 $+ (6 \times 10)$ cm^2

$= 2 \times 12$ cm^2 $+ 2 \times 50$ cm^2 $+ 60$ cm^2

$= 24$ cm^2 $+ 100$ cm^2 $+ 60$ cm^2

$=$ $\underline{184\text{ cm}^2}$

$V = \dfrac{6 \times \cancel{4}^{\,2}}{\cancel{2}_{1}} \times 10 = 12 \times 10 = \underline{120}$ (cm^3)

b) $A = \overset{1}{\cancel{2}} \times \dfrac{a \times h}{\cancel{2}_{1}} + 2 \times (b \times c) + a \times c = \underline{a\,h + 2\,b\,c + ac}$

$V = \dfrac{a \times h}{2} \times c = \dfrac{a\,h\,c}{2}$

c) $A = 2 \times a \times a + 4 \times (a \times b) = 2a^2 + 4ab$

$V = a \times a \times b = a^2b$

Q.3 First factorise 40, then show the possibilities in a table.

$$\begin{array}{r|l} 40 & 2 \\ 20 & 2 \\ 10 & 2 \\ 5 & 5 \\ 1 & \end{array}$$

Factors: 1, 2, 4, 5, 8, 10, 20, 40

$V = 40$	a	1	1	1	1	2	2
	b	1	2	4	5	2	4
	c	40	20	10	8	10	5

Answer: Six different cuboids could be built.

Notes

$\underline{160} = 2^5 \times 5$

Factors: 1, 2, 4, 5, 8, 10, 16, 20, 32, 40, 80, 160

$\underline{335} = 5 \times 67$

Factors: 1, 5, 67, 335

$\underline{510} = 2 \times 3 \times 5 \times 17$

Factors: 1, 2, 3, 5, 6, 10, 15, 17, 30, 34, 51, 85, 102, 170, 255, 510

$\underline{1160} = 2^3 \times 5 \times 29$

Factors: 1, 2, 4, 5, 8, 10, 20, 29, 40, 58, 116, 145, 232, 290, 580, 1160

(or set factorising as extra task for homework at the end of *Lesson 159* and review at the start of *Lesson 160.*

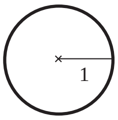
Activity

Notes

Solutions (continued)

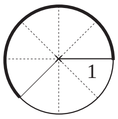
Q.4

a) 2π



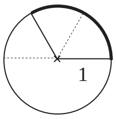
360°

b) $\frac{5\pi}{4}$



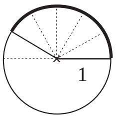
225°

c) $\frac{2\pi}{3}$



120°

d) $\frac{5\pi}{6}$



150°

Elicit that for circles with radius 1 unit:

$\pi = \frac{1}{2}$ of the circumference,

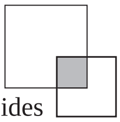
$\frac{\pi}{4} = \frac{1}{8}$ of the circumference, so $\frac{5\pi}{4} = \frac{5}{8}$ of the circumference

$\frac{\pi}{3} = \frac{1}{6}$ of the circumference, so $\frac{2\pi}{3} = \frac{1}{3}$ of the circumference

$\frac{\pi}{6} = \frac{1}{12}$ of the circumference, so $\frac{5\pi}{6} = \frac{5}{12}$ of the circumference

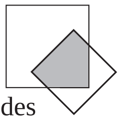
Q.5

a)



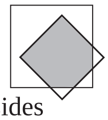
4 sides

b)



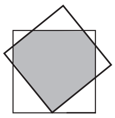
5 sides

c)



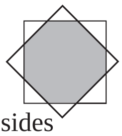
6 sides

d)



7 sides

e)



8 sides

f) 10 sides

Impossible!

Q.6 First calculate the RH square in top row:

(1) $42 - (10 + 26) = 42 - 36 = \underline{6}$

(2) Let the middle unknown square be a, then the other unknown squares can be expressed in terms of a. e.g.

(3) $42 - 6 - a = \underline{36 - a}$

(4) $42 - 26 - a = \underline{16 - a}$

(5) $42 - 10 - a = \underline{32 - a}$

(6) $42 - 6 - (32 - a) = \underline{4 + a}$

(7) $42 - a - (4 + a) = \underline{38 - 2a}$

10	**26**	(1) 6
(7) $38 - 2a$	(2) a	(6) $4 + a$
(3) $36 - a$	(4) $16 - a$	(5) $32 - a$

→

As any column, row or diagonal sums to 42, then, e.g. taking the LH column:

$10 + (38 - 2a) + (36 - a) = 42$

$84 - 3a = 42$ [$+ 3a$]

$84 = 3a + 42$ [$- 42$]

$42 = 3a$ [$\div 3$]

$\underline{14 = a}$

Now the values of all the unknown squares can be calculated, as opposite.

Check that each row, column and diagonal sums to 42.

Bold numbers are given.

10	**26**	6
10	14	18
22	2	18

(3) $36 - 14 = \underline{22}$

(4) $16 - 14 = \underline{2}$

(5) $32 - 14 = \underline{18}$

(6) $4 + 14 = 18$

(7) $38 - 2 \times 14 = 38 - 28$
$= \underline{10}$

Y6

R: Calculations

C: Word problems. Logic. Sets

E: Problems

Lesson Plan 161

Activity | *Notes*

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>161</u> = 7 × 23 Factors: 1, 7, 23, 161
- <u>336</u> = $2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$
  Factors: 1, 2, 3, 4, 6, 7, 8, 12, 14, 16, ↓
  336, 168, 112, 84, 56, 48, 42, 28, 24, 21
- <u>511</u> = 7 × 73 Factors: 1, 7, 73, 511
- <u>1161</u> = $3 \times 3 \times 3 \times 43 = 3^3 \times 43$
  Factors: 1, 3, 9, $\underbrace{27,\ 43}$, 129, 387, 1161

Individual work, monitored (or whole class activity)

BB: 161, 336, 511, 1161

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 161 & 7 \\ 23 & 23 \\ 1 & \end{array} \qquad \begin{array}{r|l} 336 & 2 \\ 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 511 & 7 \\ 73 & 73 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1161 & 3 \\ 387 & 3 \\ 129 & 3 \\ 43 & 43 \\ 1 & \end{array}$$

8 min

2

HMP

Hungarian Mathematics Problem Book

2001
Y5–6

PbY6b, page 161

Q.1 Read: *Decide whether each statement is true or false and write T or F in the box.*

Set a time limit of 4 minutes. Remind Ps to think of examples or counter examples to support their answers.

Review with whole class. T chooses a P to read out each statement and Ps show 'T' or 'F' on slates or scrap paper (or by pre-agreed actions) on command. Ps with different responses explain their reasoning, drawing diagrams where ncessary. Class decides who is correct. Mistakes discussed and corrected.

Solution:

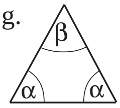
a) *The **product** of two numbers can be less than each of the two numbers.* [T]

b) *The **arithmetic mean** of two negative numbers can be positive.* [F]

(The sum of two negative numbers is always negative, so half that sum is also negative.)

c) *There is an **isosceles** triangle which has two right angles.* [F]

(e.g.



$\alpha + \alpha + \beta = 180°$

If $\alpha = 90°$, then $\beta = 0°$, which is impossible.)

d) *There is a positive fraction less than 1 which is equal to its **reciprocal.*** [F]

(Impossible – if the reciprocal of a is b, then $a \times b = 1$.)

When $0 > a < 1$, then $b = \dfrac{1}{a} > 1$)

e) *If a **product** is zero, at least one of its factors is zero.* [T]

(If $a \neq 0$ and $b \neq 0$, then $a \times b \neq 0$)

f) *If the areas of two triangles are equal, the triangles are **congruent**.* [F]

(A right-angled triangle and an isosceles triangle can have the same area but they are <u>not</u> congruent.)

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

Responses shown in unison.

Reasoning, agreement, self-correction praising

Feedback for T

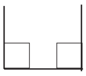
e.g. $\dfrac{1}{2} \times \dfrac{1}{3} = \dfrac{1}{6}$,

$\dfrac{1}{6} < \dfrac{1}{2}$ and $\dfrac{1}{6} < \dfrac{1}{3}$

e.g. $[(-4) + (-6)] \div 2$

$= -10 \div 2 = -5$

or e.g.



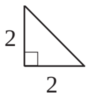
A triangle with 2 right angles at its base is impossible!

e.g. If $a = \dfrac{1}{3}$, $b = 3$

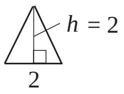
If $a = \dfrac{1}{10}$, $b = 10$

e.g. $5 \times 0 = 0$, $0 \times 0 = 0$
$3 \times 4 \neq 0$

e.g.



and



$A = \dfrac{2 \times \cancel{2}^{1}}{\cancel{2}_{1}} = 2$ (sq. units)

Y6

Lesson Plan 161

Activity

Notes

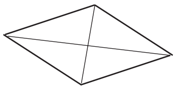
2

(Continued)

g) *There is a quadrilateral which is both a* ***deltoid*** *and a* ***parallelogram*** *but is not a square.* [T]

(A <u>rhombus</u> which does not have right angles is a deltoid and also a parallelogram.)

e.g.



- 2 pairs of adjacent equal sides
- Opposite sides equal and parallel

14 min

N.B. We do not wish to prescribe timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done in class time as individual challenges or as whole class activities, with any remaining questions set as optional homework and reviewed interactively before the start of *Lesson 162*, or T could divide the class into teams of roughly equal ability and set the remaining questions as a 'maths challenge' competition.

Make sure that the questions are reviewed by the whole class, whether Ps attempted them or not.

(T could have a prize for the team which solves most questions correctly.)

3

HMP

Hungarian Mathematics Problem Book

2001

Y5–6

PbY6b, page 161

Q.2 Read: *Frank, Charlie, Johnny and George went to visit their friend. The surnames of the 4 boys, in no particular order, are Little, Grant, Tailor and Miller.*

Miller arrived first, then Johnny, then Little. George arrived last.

Each boy gave his friend a present. Miller gave a magic cube, Frank gave a pen, George gave a bar of chocolate and Tailor gave a book.

What is the full name of each boy?

Individual trial or whole class activity

(or whole class activity to start with, then individual completion)

Allow Ps time to think about it and try to solve it. If Ps are struggling, stop individual work and suggest showing the information in a table. Ps dictate what T should write.

Deal with the information in the 2nd paragraph with the whole class, then either Ps complete the table as individual work if they wish, or the class completes it together, with T's help.

If a P manages to solve it without help, ask him or her to explain their thinking to the class.

If they did not use a table, T could suggest this method of solution and ask Ps what they think of it.

Solution:

From 2nd paragraph:

	Little	Grant	Tailor	Miller
Frank				
Charlie				
Johnny	×			×
George	×			×

Miller is not Johnny.
Miller is not George.
Little is not Johnny.
Little is not George.

From 3rd paragraph:

	Little	Grant	Tailor	Miller
Frank	✔	×	×	×
Charlie	×	×	×	✔
Johnny	×	×	✔	×
George	×	✔	×	×

Miller is not Frank, so Miller must be Charlie.

Therefore Charlie is not Little, Grant or Tailor.

Frank must be Little, so Frank is not Grant or Tailor.

Tailor is not George, so Tailor must be Johnny, so Grant is George.

Answer: The four boys are called Frank Little, Charlie Miller, Johnny Tailor and George Little.

Y6

Lesson Plan 161

Activity

Notes

4

HMC:
Hungarian
Mathematics
Competition
1997
Age 12

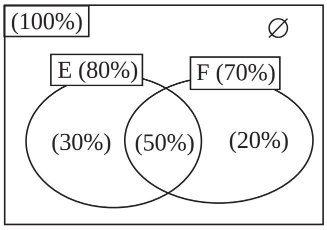
PbY6b, page 161

Q.3 Read: *In a Canadian city, 80% of the population speaks English and 70% speaks French. Every inhabitant can speak either French or English.*

What percentage of the population can speak both languages?

If Ps are struggling, suggest that they show the information in a Venn diagram, as below.

Solution:



Reasoning: e.g.

80% + 70% = 150%,

which is 50% more than 100% so 50% speak both languages.

Check: 30% + 50% + 20%
= 100% ✔

Answer: 50% of the population can speak English and French.

Individual trial

5

HMC:
Hungarian
Mathematics
Competition
2000
Age 11

PbY6b, page 161

Q.4 Read: *Ten pupils took part in a mathematics competition in which 5 problems were set. Thirty-five answers were handed in.*

We know that there was a pupil who handed in only 1 answer, a pupil who handed in 2 answers and a pupil who handed in 3 answers.

Show that there must be a pupil who answered all five problems.

Solution: e.g.

Let's suppose that exactly 1 pupil solved only 1 problem, exactly 1 pupil solved 2 problems and exactly 1 pupil solved 3 problems.

Then 7 pupils would have handed in 29 answers, but 29 ÷ 7 = 4, $\underline{\text{r } 1}$, so at least one of the pupils must have answered all 5 problems.

or

Suppose that the other 7 pupils handed in 4 answers each, then in total there would be 1 + 2 + 3 + 7 × 4 = 6 + 28 = $\underline{34}$ answers but as there are 35 answers, at least one pupil must have done all five problems.

Individual trial
(or whole class activity)

[If the number of pupils who solved 1 (2, 3) problems was more than 1, the problem would be easier to solve.]

Y6

Lesson Plan 161

Activity ***Notes***

6

HMC:
Hungarian
Mathematics
Competition
1985
Age 12

PbY6b, page 161*, *Q.5

Read: *A year 6 class of 42 pupils took part in a special Physical Education lesson. The pupils could choose from basketball, swimming and gymnastics.*

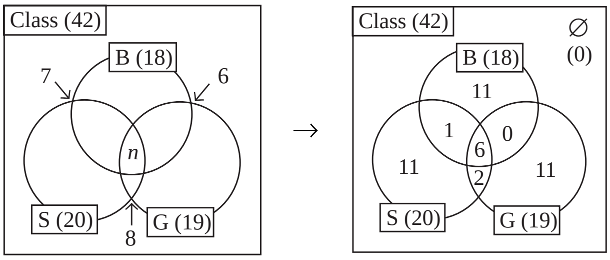
We know that 20 of them did swimming, 19 did gymnastics and 18 played basketball. We also know that 7 pupils swam and played basketball, 8 pupils swam and did gymnastics and 6 pupils did gymnastics and played basketball.

How many pupils took part in all 3 sports?

Allow Ps time to think about it for a few minutes and discuss with their neighbours. If any Ps have good ideas, T helps them to develop the solution, involving other Ps where possible. If no P has an idea, T suggests drawing a Venn diagram and calling the number of pupils who took part in all 3 sports n. Then Ps might be able to proceed from there, otherwise T directs Ps thinking and class solves the problem together.

Solution: e.g.

BB:



Write an equation involving n:, solve it then calculate the other values as a check.

BB:
$$20 + 19 + 18 - (8 + 7 + 6) + n = 42$$
$$57 - 21 + n = 42$$
$$36 + n = 42 \qquad [-36]$$
$$\underline{n = 6}$$

Answer: Six pupils took part in all three sports.

Whole class activity

(or individual trial first if Ps wish)

(or alternative equation to the one given in the solution:

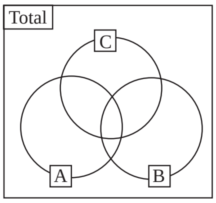
$$20 + (18 - 7) + (19 - 6 - 8 + n) = 42$$
$$20 + 11 + 5 + n = 42$$
$$36 + n = 42$$
$$\underline{n = 6}$$

Check: ✔

$11 + 1 + 11 + 11 + 6 + 2 = 42$

∅ means 'empty set'

[**To T**: ∩ means 'intersection', ∪ means 'union'



$A \cup B \cup C = A + B + C - (A \cap B + A \cap C + B \cap C) + A \cap B \cap C$]

7

HMC:
Hungarian
Mathematics
Competition
2000
Age 11

PbY6b, page 161, Q.6

Read: *A shooting practice target is shaped like an equilateral triangle and each of its sides is 1 metre long. If 10 shots hit the target, show that two of the shots must be less than 34 cm apart.*

Allow Ps time to think about it and discuss with their neighbours. Ps who have ideas develop them with help of T and class. If no P is on the right track, T gives hints or directs Ps' thinking, involving Ps where possible.

Solution: e.g.

First draw an equilateral triangle. As 34 cm $\approx \frac{1}{3}$ m, divide each side of the triangle into thirds to form 9 smaller, congruent, equilateral triangles. Any point in one of these small triangles is <u>at most</u> 33 and 1 third cm away from another point on that triangle.

The worst possible scenario is that the first 9 shots hit different small triangles.

However, the 10th shot <u>must</u> hit one of these 9 triangles. As the distance between any 2 points on a triangle is at most 33 and 1 third cm apart, then at least two of the shots must be less than 34 cm apart.

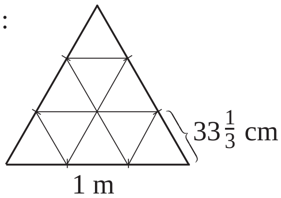
Whole class activity

(or individual challenge if Ps wish)

If done as an individual challenge and no P has solved it in class, it could be left open as homework.

(Make sure that the scale of the diagram is such that the sides are easily divided into 3.)

BB:



Y6

R: Calculations

C: Revision: arithmetic, algebra

E: Problems and challenges

Lesson Plan *162*

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 162 = $2 \times 3 \times 3 \times 3 \times 3 = 2 \times 3^4$

  Factors: 1, 2, 3, 6, 9, 18, 27, 54, 81, 162
- 337 is a prime number Factors: 1, 337

  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and $19^2 > 337$)
- 512 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^9$ $[= \left(2^3\right)^3]$

  Factors: 1, 2, 4, 8, 16, 32, 64, 128, 256, 512 (cubic number)
- 1162 = $2 \times 7 \times 83$ Factors: 1, 2, 7, 14, 83, 166, 581, 1162

Individual work, monitored (or whole class activity)

BB: 162, 337, 512, 1162

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 162 & 2 \\ 81 & 3 \\ 27 & 3 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1162 & 2 \\ 581 & 7 \\ 83 & 83 \\ 1 & \end{array} \qquad \begin{array}{r|l} 512 & 2 \\ 256 & 2 \\ 128 & 2 \\ 64 & 2 \\ 32 & 2 \\ 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array}$$

8 min

2

Arithmetic laws

Let's complete these equations using the letters, then check that they are true using numbers.

Ps come to BB or dictate what T should write. Class agrees/disagrees.

After each equation has been checked with numbers, ask Ps to explain the general rule or law in words using the components of the 4 operations.

BB: e.g.

a) $(a + b) \times c = [a \times c + b \times c]$

$$\text{LHS}: (2 + \frac{3}{4}) \times 5 = 2\frac{3}{4} \times 5 = \frac{11}{4} \times 5 = \frac{55}{4} = 13\frac{3}{4}$$

$$\text{RHS}: 2 \times 5 + \frac{3}{4} \times 5 = 10 + \frac{15}{4} = 10 + 3\frac{3}{4} = 13\frac{3}{4} \checkmark$$

[Multiplying a sum by a number has the same result as multiplying each term of the sum by the number and adding the products.]

b) $a + (-b) = [a - b]$ e.g. $5 + (-1.3) = 5 - 1.3 = 3.7$ ✔

[Adding a negative number has the same result as subtracting the opposite positive number.]

c) $a - (-b) = [a + b]$ e.g. $\frac{4}{9} - \left(-\frac{1}{3}\right) = \frac{4}{9} + \frac{1}{3} = \frac{7}{9}$ ✔

[Subtracting a negative number has the same result as adding the opposite positive number.]

d) $\frac{a - b}{c} = \left[\frac{a}{c} - \frac{b}{c}\right]$ e.g. LHS: $\frac{15 - 6}{3} = \frac{9}{3} = 3$

RHS: $\frac{15}{3} - \frac{6}{3} = 5 - 2 = 3$ ✔

[Dividing a difference by a number has the same result as dividing the reductant and the subtrahend by that number and subtracting the two quotients.]

Whole class activity

Written on BB or SB or OHT

At a good pace

Reasoning, checking, agreement, praising only

T chooses Ps to decide on the number each letter represents, then those Ps choose other Ps to do the calculaations.

Feedback for T

Activity ***Notes***

2

(Continued)

e) $\dfrac{a}{b} \times c = [\dfrac{a \times c}{b}]$ e.g. $\dfrac{5}{8} \times 3 = \dfrac{5 \times 3}{8} = \dfrac{15}{8}$ $[= 1\dfrac{7}{8}]$

[To multiply a fraction by a whole number, multiply the numerator by that number.]

f) $\dfrac{a}{b} + \dfrac{c}{b} = \left[\dfrac{a+c}{b}\right]$ e.g. LHS: $\dfrac{4}{7} + \dfrac{2}{7} = \dfrac{6}{7}$, RHS: $\dfrac{4+2}{7} = \dfrac{6}{7}$

[To add two fractions which have the same denominator, add the numerators and keep the same denominator.]

g) $\dfrac{a}{b} + \dfrac{c}{d} = [\dfrac{a \times d}{b \times d} + \dfrac{c \times b}{b \times d} = \dfrac{a \times d + c \times d}{b \times d}]$

e.g. $\dfrac{2}{3} + \dfrac{5}{8} = \dfrac{2 \times 8}{3 \times 8} + \dfrac{5 \times 3}{3 \times 8} = \dfrac{2 \times 8 + 5 \times 3}{3 \times 8} = \dfrac{16+15}{24}$

$= \dfrac{31}{24}$ $[= 1\dfrac{7}{34}]$

[To add two fractions with different denominators, multiply each numerator by the denominator of the other fraction, add the two products, then divide by the product of the two denominators.

T could show this one if Ps cannot do it and Ps check that it is correct by substituting numbers for the letters.

16 min

3

PbY6b, page 162

Q.1 Read: *Complete the* ***arithmetic laws****. Try them with numbers if necessary.*

Set a time limit of 5 minutes. Ps check mentally or in *Ex. Bks.* (The more difficult equations can be done with the whole class.

Review with whole class. Ps come to BB to complete the equations, explaining reasoning. Who agrees? Who wrote something else? etc. Mistakes discussed and corrected.

Ask Ps to explain the laws in words where appropriate.

Solution:

a) $a + (-b) - (+c) - (-d) = [a - b - c + d]$

b) $(a - b) \times c = [a \times c - b \times c]$ $(= ac - bc)$

c) $x \times y + x \times z = [x \times (y + z)]$ $(= x\,(y + z))$

d) $(a - b) \div c = [a \div c - b \div c]$

e) $u \div w + v \div w = [(u + v) \div w]$

f) $2 \times f + 3 \times f - 4 \times f = [(2 + 3 - 4) \times f = 1 \times f = f]$

g) $6t - 4t - 9t = [-7t]$

h) $\dfrac{a \times c}{b \times c} = \left[\dfrac{a}{b}\right]$ i) $\dfrac{a+b}{c} = \left[\dfrac{a}{c} + \dfrac{b}{c}\right]$

j) $\dfrac{a}{b} - \dfrac{c}{d} = \left[\dfrac{a \times d - b \times c}{b \times d} = \dfrac{ad - bc}{bd}\right]$ k) $\dfrac{a \times n}{n} = [a]$

l) $\dfrac{a}{b} \times b = [a]$ m) $\dfrac{a}{b} \div c = \left[\dfrac{a \div c}{b} = \dfrac{a}{b \times c} = \dfrac{a}{bc}\right]$

n) $\dfrac{a}{b} \times \dfrac{c}{d} = \left[\dfrac{a \times c}{b \times d} = \dfrac{ac}{bd}\right]$

o) $\dfrac{a}{b} \div \dfrac{c}{d} =$

$[\dfrac{a}{b} \times \dfrac{d}{c} = \dfrac{a \times d}{b \times c} = \dfrac{ad}{bc}]$

Individual work, monitored, helped

Written on BB or use enlarged copy master or OHP

(If class is not very able, deal with one or two at a time.)

Reasoning, checking with actual values, agreement, self-correction, praising

Feedback for T

Elicit that, e.g. $6t = 6 \times t$

25 min

Y6

Lesson Plan 162

Activity

Notes

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions with the whole class, whether Ps attempted them or not.

4

HMC:

Hungarian Mathematics Competition

1985

Age 11

PbY6b, page 162

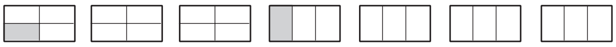
Q.2 Read: *How is it possible to share 7 equal sized loaves of bread among 12 hungry people* **without** *cutting each loaf into 12 pieces?*

Try to solve it using as few cuts as possible.

Solution: e.g.

$$7 = 3 + 4 = 12 \times \frac{1}{4} + 12 \times \frac{1}{3},$$

so the loaves could be cut like this, using 14 cuts:

BB: 

Each person would get $\frac{1}{4} + \frac{1}{3} = \frac{3+4}{12} = \frac{7}{12}$ of the bread.

Individual trial

or

Each person would get $\frac{7}{12}$ of the bread, but

$$\frac{7}{12} = \frac{3}{12} + \frac{4}{12} = \frac{1}{4} + \frac{1}{3},$$

so the loaves should be cut so that there are 12 quarters and 12 thirds.

5

HMC:

Hungarian Mathematics Competition

1991

Age 11

PbY6b, page 162

Q.3 Read: *What is the smallest positive whole number which has units digit 6 and if we move this digit from the right-hand side to the left-hand side but leave the other digits unchanged, we get 4 times the original number?*

Solution: e.g.

Let's write the multiplication using the digits we know about and using an <u>ellipsis</u> (dots) for those we don't.

BB: $\begin{array}{r} \ldots 6 \times 4 \\ \hline 6\ldots \end{array}$ (6 × 4 = 2$\underline{4}$, so 4 must be the 10s digit, and 2 is carried over to the next greater place-value column)

$= \begin{array}{r} \ldots 46 \times 4 \\ \hline 6\ldots 4 \end{array}$ (4 × 4 = 16, 16 + 2 = 1$\underline{8}$, so 8 must be the 100s digit, and 1 is carried over to the next greater place-value column)

$= \begin{array}{r} \ldots 846 \times 4 \\ \hline 6\ldots 84 \end{array}$ (8 × 4 = 32, 32 + 1 = 3$\underline{3}$, so 3 must be the 1000s digit, and 3 is carried over to the next greater place-value column)

$= \begin{array}{r} ..3846 \times 4 \\ \hline 6..384 \end{array}$ (3 × 4 = 12, 12 + 3 = 1$\underline{5}$, so 5 must be the 10 000s digit, and 1 is carried over to the next greater place-value column)

$= \begin{array}{r} ..53846 \times 4 \\ \hline 6..5384 \end{array}$ (5 × 4 = 20, 20 + 1 = 2$\underline{1}$, so 1 must be the 100 000s digit, and 2 is carried over to the next greater place-value column)

$= \begin{array}{r} .153846 \times 4 \\ \hline 615384 \end{array}$ (1 × 4 = 4, 4 + 2 = $\underline{6}$, there is nothing to be carrried over and we have reached digit 6 in the product)

Answer: The number is 153 846.

Whole class activity

(or individual trial if Ps wish)

Allow Ps a minute or two to think and discuss with their neighbours. Ps who have ideas develop them with the whole class.

If no P has an idea, T starts solution and involves Ps when they understand what is happening.

Check:

1	5	3	8	4	6
				×	4
6	1	5	3	8	4
	1	3	1	2	

✔

Y6

Lesson Plan 162

Activity ***Notes***

6

HMC:
Hungarian
Mathematics
Competition
1980
Age 12

PbY6b, page 162

Q.4 Read: *We multiply the digits of a 3-digit whole number, then multiply the digits of the product. We can represent the number and the two products in this way:*

The same shape means the same digit.

What was the original number? Explain your reasoning.

Solution: e.g.

We can write the products as multiplications:

BB: $\triangle \times \bigcirc \times \bigcirc = \triangle\square$; $\triangle \times \square = \square$

If $\triangle = 1$, then $\bigcirc \neq 0, 1, 2$ or 3 (as product is 1-digit)

if $\bigcirc = 4 \rightarrow 144$, 16, and $\square = 6$

so $\underline{144}$ is o.k. ✔

also, $\bigcirc \neq 5, 6, 7, 8$ or 9

(as tens digit in product is not 1)

If $\triangle = 2$, then $\bigcirc \neq 0, 1, 2$ (as product is 1-digit)

$\bigcirc \neq 3, 4, 5, 6, 7, 8\ 9$

(as digits in products do not match the shapes)

Also $\triangle \neq 3, 4, 5, 6, 7, 8$ or 9,

(as digits in product do not match the shapes)

The only possible answer is: $\triangle = 1$, $\bigcirc = 4$, $\square = 6$

Individual trial first

Drawn on BB or SB or OHT

BB:

△○○ ; △□ ; □

The easiest method is to use trial and error, done in a logical way.

Check:

$\underline{1}44$: $1 \times 4 \times 4 = \underline{1}6$,
$1 \times \underline{6} = \underline{6}$

e.g. $\underline{1}55$: $1 \times 5 \times 5 = \underline{2}5$

e.g. $\underline{2}33$: $2 \times 3 \times 3 = \underline{1}8$

e.g. $\underline{5}44$: $5 \times 4 \times 4 = \underline{8}0$

7

HMC:
Hungarian
Mathematics
Competition
1992
Age 12

PbY6b, page 162, Q.5

Read: *We put £255 into 8 envelopes, seal the envelopes and write on each how much money it contains. There is a different amount in each envelope.*

Without opening any of the envelopes we can pay any whole amount from £1 to £255. How much money is in each envelope?

Ps make suggestions and class tries them out. If necessary, T suggests starting at £1 and seeing what notes are needed.

BB: £1 → $\underline{£1}$, £2 → $\underline{£2}$, £3 → £2 + £1, £4 → $\underline{£4}$,
£5 → £4 + £1, £6 → £4 + £2, £7 → £4 + £2 + £1,
£8 → $\underline{£8}$, £9 → £8 + £1, £10 → £8 + £2, etc.

After a while, Ps might realise that what are needed are the powers of 2 (2^0 to 2^7), i.e. the place values in the base 2 number system.

Solution:

The 8 envelopes contain, in increasing order:

BB: £1, £2, £4, £8, £16, £32, £64, £128

(2^0), (2^1), (2^2), (2^3), (2^4), (2^5), (2^6), (2^7)

[N.B. $2^8 = 2 \times 128 = 256 > 255$, so is not needed.]

Whole class trials and solution

(or individual challenge if Ps wish, left open as homework if no P can solve it during the lesson)

There is no need to check every number to 255.

Once Ps have realised what is needed, ask Ps to suggest some larger numbers to check.

e.g. $\underline{213} = 128 + 64 + 16 + 4 + 1$

[$213 \div 2 = 106$, r $\underline{1}$ (1's)
$106 \div 2 = 53$, r $\underline{0}$
$53 \div 2 = 26$, r $\underline{1}$
$26 \div 2 = 13$, r $\underline{0}$
$13 \div 2 = 6$, r $\underline{1}$
$6 \div 2 = 3$, r $\underline{0}$
$3 \div 2 = 1$, r $\underline{1}$
$1 \div 2 = 0$, r $\underline{1}$ (128's)

so $213 = 11010101_{②}$]

Y6

R: Calculations
C: Algebra, equations
E: Word problems

Lesson Plan *163*

Activity | *Notes*

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 163 is a prime number Factors: 1, 163
  (as not exactly divisible by 2, 3, 5, 7, 11, and $13^2 > 163$)
- $\underline{338} = 2 \times 13 \times 13 = 2 \times 13^2$ Factors: 1, 2, 13, 26, 169, 338
- $\underline{513} = 3 \times 3 \times 3 \times 19 = 3^3 \times 19$
  Factors: 1, 3, 9, 19, 27, 57, 171, 513
- 1163 is a prime number Factors: 1, 1163
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 and 31, and $37^2 > 1163$)

Notes:

Individual work, monitored (or whole class activity)

BB: 163, 338, 513, 1163

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 338 & 2 \\ 169 & 13 \\ 13 & 13 \\ 1 & \end{array} \qquad \begin{array}{r|l} 513 & 3 \\ 171 & 3 \\ 57 & 3 \\ 19 & 19 \\ 1 & \end{array}$$

8 min

2

PbY6b, page 163

Q.1 Read: *Solve the equations and inequalities. Check your results. (The **base set** is in brackets.)*

What kind of numbers are in the given base sets?

[Z: the set of integers (whole numbers); Q: the set of rational number, i.e. all the numbers we have learned about (positive and negative integers, fractions and decimals); N: the set of natural numbers (positive whole numbers)]

Deal with one part at a short time under a time limit.

Review with whole class. Ps show solutions on scrap paper or slates on command. Ps answering correctly explain reasoning to Ps who were wrong. Mistakes discussed and corrected.

Solution:

a) $x - 5 > -5 \quad [+ 5]$

$\underline{x > 0}$, $(x \in \mathrm{Z})$ [$\in$ means 'is a member of the set']

b) $(-3) \times (4 \times y) + 8 \leq 2 \times (5 \times y + 6)$

$$\begin{aligned} -12 \times y + 8 &\leq 10 \times y + 12 && [-8] \\ -12 \times y &\leq 10 \times y + 4 && [+12 \times y\,] \\ 0 &\leq 22 \times y + 4 && [-4] \\ -4 &\leq 22 \times y && [\div 22] \end{aligned}$$

$$\underline{-\frac{4}{22} \leq y}, \quad \text{or } \underline{y \geq -\frac{2}{11}} \quad (y \in \mathrm{Q})$$

Check: e.g.

$\underline{y = -\frac{4}{11}}$: LHS : $\frac{48}{11} + 8 = 4\frac{4}{11} + 8 = 12\frac{4}{11}$;

RHS: $-\frac{40}{11} + 12 = -3\frac{7}{11} + 12 = 8\frac{4}{11}$ ✗

$\underline{y = -\frac{2}{11}}$: LHS : $\frac{24}{11} + 8 = 2\frac{2}{11} + 8 = 10\frac{2}{11}$;

RHS: $-\frac{20}{11} + 12 = -1\frac{9}{11} + 12 = 10\frac{2}{11}$ ✓

Notes:

Individual work, monitored, helped

Written on BB or SB or OHT

Agreement, praising

T reminds Ps if they have forgotten.

Responses shown in u nison

Reasoning, checking, agreement, self-correction, praising

Revise the 'balance method' for solving equations if necessary.

Check with easy values less than, equal to and greater than the solution.

Check: e.g.

a) $\underline{x = -1}$: $-1 - 5 = -6$ ✗

$\underline{x = 0}$: $0 - 5 = -5$ ✗

$\underline{x = 2}$: $2 - 5 = -3$ ✓

$\underline{y = 1}$:

LHS: $-12 + 8 = -4$

RHS: $2 \times 11 = 22$ ✓

Y6

Lesson Plan 163

Activity

2

(Continued)

c) $\frac{3\times t}{8} - 2 + \frac{2\times t}{3} = -\frac{5}{6} + \frac{5\times t}{12}$ [Convert to 24ths]

$\frac{9\times t}{24} - \frac{48}{24} + \frac{16\times t}{24} = -\frac{20}{24} + \frac{10\times t}{24}$ [× 24]

$9\times t - 48 + 16\times t = -20 + 10\times t$ [+ 48]

$25\times t = 28 + 10\times t$ [– 10 × *t*]

$15\times t = 28$ [÷ 15]

$t = \frac{28}{15} = \underline{1\frac{13}{15}}$

Check: RHS:

$-\frac{5}{6} + \frac{\overset{1}{\cancel{5}}\times\overset{7}{\cancel{28}}}{\underset{3}{\cancel{12}}\times\underset{3}{\cancel{15}}} = -\frac{5}{6} + \frac{7}{9} = -\frac{15}{18} + \frac{14}{18} = -\frac{1}{18}$ ✔

d) $\frac{3\times v + 5}{-2} = -4$ [× (– 2)]

$3\times v + 5 = 8$ [– 5]

$3\times v = 3$ [÷ 3]

$\underline{v = 1}$

Notes

Check:

LHS:

$\frac{\overset{1}{\cancel{3}}\times\overset{7}{\cancel{28}}}{\underset{2}{\cancel{8}}\times\underset{5}{\cancel{15}}} - 2 + \frac{2\times 28}{3\times 15}$

$= \frac{7}{10} - 2 + \frac{56}{45}$

$= -1\frac{3}{10} + 1\frac{11}{45}$

$= -\frac{27}{90} + \frac{22}{90}$

$= -\frac{5}{90} = -\frac{1}{18}$

Check:

$\frac{3\times 1 + 5}{-2} = \frac{8}{-2} = -4$ ✔

15 min

3

HMC:
Hungarian
Mathematics
Competition
1985
Age 11

PbY6b, page 163, Q.2

Read: *I have 18 coins (2 p and 5 p pieces) in my pocket.*

If I had as many 5 p coins as I have 2 p coins and as many 2 p coins as I have 5 p coins, I would have twice as much money as I have now.

How much money do I have?

Allow Ps a couple of minutes to think about how to solve it. Ps who have ideas explain them to the class. Class decides whether they are valid. If no P has a good idea, T gives hint about using a letter for the number of one type of coin and helps class to form an equation. Then Ps come to BB or dictate what T should write to solve it, check the solution and agrree on a form of words for the answer.

Solution: e.g.

Let x be the number of 2 p coins, then the number of 5 p coins is $18 - x$ and the amount of money in my pocket is

BB: $2\times x + 5\times(18 - x)$

If I did what is suggested, then the amount in my pocket would be:

$5\times x + 2\times(18 - x)$

and it would be twice as much as I have now. So now we can write:

BB: $[2\times x + 5\times(18 - x)]\times 2 = 5\times x + 2\times(18 - x)$

$(-3\times x + 90)\times 2 = 3\times x + 36$

$-6\times x + 180 = 3\times x + 36$ [+ 6 × *x*]

$180 = 9\times x + 36$ [– 36]

$144 = 9\times x$ [÷ 9]

$\underline{16 = x}$

Answer: I have 16 2 p coins and two 5 p coins, so I have 42 p altogether.

Whole class activity

(or individual trial first if Ps wish)

Discussion, reasoning, agreement, checking, (self-correction), praising

Involve many Ps.

Ps could write solution in *Ex. Bks.* too.

Check:

5 p × 16 + 2 p × (18 – 16)

= 80 p + 2 p × 2

= 80 p + 4 p

= 84 p

= 2 × 42 p ✔

21 min

Y6		*Lesson Plan 163*
Activity		***Notes***
	N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize) Any questions not done in class could be set as voluntary homework.	Review the questions with the whole class, whether Ps attempted them or not.)
4 *HMC*: Hungarian Mathematics Competition 1981 Age 11	***PbY6b, page 163*** Q.3 Read: *Steve and a dog can be balanced on a seesaw by 5 equal-sized boxes.* *The dog and 2 cats can be balanced on the seesaw by 3 of the boxes and the dog can be balanced by 4 cats.* *How many cats are needed to balance Steve?* *Solution:* e.g. Let Steve's weight be S, a dog's weight be D, a cat's weight be C and a box's weight be B. Then $S + D = 5 \times B$, $D + 2 \times C = 3 \times B$, $D = 4 \times C$ Writing $4 \times C$ for D in the first two equations: $S + 4 \times C = 5 \times B$ . . . . (1) and $4 \times C + 2 \times C = 3 \times B$ $6 \times C = 3 \times B$ [÷ 3] $2 \times C = B$ Writing $2 \times C$ for B in the first substituted equation (1), $S + 4 \times C = 5 \times (2 \times C)$ $S + 4 \times C = 10 \times C$ [– 4 × C] $\underline{S = 6 \times C}$ *Answer*: Six cats are needed to balance Steve.	Individual work, monitored helped (Revert to a whole class activity if Ps are struggling.) Stress that although there are many unknown amounts in this problem, we must try to end up with an equation involving Steve's weight and a cat's weight. Extra praise for Ps who manage to solve it without help from the T.
5 *HMC*: Hungarian Mathematics Competition 1991 Age 11	***PbY6b, page 163*** Q.4 Read: *The average age of the 11 members of a football team is 22 years. When one member of the team was sent off because of a bad tackle, the average age of the rest of the team was 21 years.* *How old is the player who was sent off?* *Solution:* e.g. Total age of the 11 players: 11 × 22 years = 242 years Total age of 10 players: 10 × 21 years = 210 years So age o fthe 11th player: 242 – 210 = <u>32</u> (years) *Answer*: The player who was sent off was 32 years old.	Individual trial What does average age mean? (As if the older players cancelled out the younger players and they were all the same age. or If all their ages were added together and then divided by the number of players the result would be their average age.)

Y6

Lesson Plan 163

Activity

Notes

6

HMC:
Hungarian Mathematics Competition
1992
Age 11

PbY6b, page 163

Q.5 Read: *If I had four times as much money as I have now, my money would be as much over £1000 as the amount I have now is less than £1000.*

How much money do I have?

Solution: e.g.

Let the amount of money I have now be £x , then:

$$\begin{aligned} 4 \times x - 1000 &= 1000 - x && [+ 1000] \\ 4 \times x &= 2000 - x && [+ x] \\ 5 \times x &= 2000 && [\div 5] \\ \underline{x} &\underline{= 400} \end{aligned}$$

Answer: I have £400 now.

Individual work, monitored

Check:

LHS: 4 × £400 – £1000
= £1600 – £1000 = £600

RHS: £1000 – £400 = £600

✔

7

HMC:
Hungarian Mathematics Competition
1995
Age 11

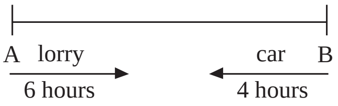
PbY6b, page 163, Q.6

Read: *A lorry and a car started from two cities at the same time and travelled towards each other at steady speeds. The lorry took 6 hours to cover the distance between the two cities and the car took 4 hours.*

After what amount of time did they pass each other?

What should we do first? (Draw a diagram) Ps come to BB to draw and explain. Class agrees/disagrees.

BB:



What part of the distance did they each cover every hour?

(Lorry: $\frac{1}{6}$ of the distance; Car: $\frac{1}{4}$ of the distance)

So every hour they approach each other by $(\frac{1}{6} + \frac{1}{4})$ of the distance.

If they meet in x hours and we think of the whole distance as 1 unit, then

BB:
$$\begin{aligned} \frac{1}{6} + \frac{1}{4} &= \frac{1}{x} && [\times\ 12x] \\ 2x + 3x &= 12 \\ 5x &= 12 && [\div 5] \\ \underline{x} &\underline{= 2.4} \text{ (hours)} \end{aligned}$$

Answer: They passed each other after 2 hours 24 minutes.

Whole class activity

(or individual or paired trial if Ps wish)

Y6

Lesson Plan 163

Activity

8

HMC:
Hungarian Mathematics Competition
1984
Age 12

PbY6b, page 163

Q.7 Read: *A matchbox contains some matches. If we double the number of matches then take away 8, then double the number of matches left and take away 8 again, then do the same for a third time, the box will be empty.*

How many matches are in the matchbox?

Solution: e.g.

Let the number of matches in the matchbox be *n*.

Then number in box after:

Action 1: $n \times 2 - 8$

Action 2: $(n \times 2 - 8) \times 2 - 8$

Action 3: $[\,(n \times 2 - 8) \times 2 - 8] \times 2 - 8$

$$\begin{aligned}
\text{But}\quad [\,(n \times 2 - 8) \times 2 - 8] \times 2 - 8 &= 0 && [+\,8]\\
\text{so}\quad [\,(n \times 2 - 8) \times 2 - 8] \times 2 &= 8 && [\div\,2]\\
(n \times 2 - 8) \times 2 - 8 &= 4 && [+\,8]\\
(n \times 2 - 8) \times 2 &= 12 && [\div\,2]\\
n \times 2 - 8 &= 6 && [+\,8]\\
n \times 2 &= 14 && [\div\,2]\\
\underline{n} &\underline{= 7}
\end{aligned}$$

$$\begin{aligned}
\text{or}\quad [\,(n \times 2 - 8) \times 2 - 8] \times 2 - 8 &= 0\\
(n \times 4 - 16 - 8) \times 2 - 8 &= 0\\
(n \times 4 - 24) \times 2 - 8 &= 0\\
n \times 8 - 48 - 8 &= 0\\
n \times 8 - 56 &= 0 && [+\,56]\\
n \times 8 &= 56 && [\div\,8]\\
\underline{n} &\underline{= 7}
\end{aligned}$$

Answer: There are 7 matches in the matchbox.

Notes

Individual challenge, left open as homework if not solved during the lesson.

(If no P can solve it, T leads Ps through the solution, involving them whenever possible.)

Check:

Action 1: $\underline{7} \times 2 - 8 = 6$

Action 2: $6 \times 2 - 8 = 4$

Action 3: $4 \times 2 - 8 = 0$ ✔

<u>Another method of solution</u>:

Start at 0 and do the opposite operations in reverse order:

$$\begin{aligned}
&\{[(0 + 8) \div 2 + 8] \div 2 + 8\} \div 2\\
&= (12 \div 2 + 8) \div 2\\
&= 14 \div 2\\
&= \underline{7}
\end{aligned}$$

Y6

R: Calculations
C: Revision: projections
E: Problms, challenges

Lesson Plan 164

Activity | *Notes*

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>164</u> = $2 \times 2 \times 41 = 2^2 \times 41$
  Factors: 1, 2, 4, 41, 82, 164
- <u>339</u> = 3×113 Factors: 1, 3, 113, 339
- <u>514</u> = 2×257 Factors: 1, 2, 257, 514
  (257 is not exactly divisible by 2, 3, 5, 7, 11, 13 and $17^2 > 257$)
- <u>1164</u> = $2 \times 2 \times 3 \times 97 = 2^2 \times 3 \times 97$
  Factors: 1, 2, 3, 4, 6, 12, 97, 194, 291, 388, 582, 1164

Individual work, monitored (or whole class activity)

BB: 164, 339, 514, 1164

T decides whether Ps can use calculators.

Reasoning, agreement, self-correction, praising

164	2
82	2
41	41
1	

514	2
257	257
1	

339	3
113	113
1	

1164	2
582	2
291	3
97	97
1	

8 min

2

PbY6b, page 164

Q.1 Read: *Find a relationship between the corresponding values and complete the table.*

Show the data in a graph in your exercise book.

Deal with one table at a time. Class agrees on one form of the rule then Ps complete the table and write the rule in different ways.

Why must the rule in the form $\dfrac{y}{x}$ have the extra condition that x cannot be equal to zero? (Because it is nonsense to divide by 0.)

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agres/disagrees. Mistakes discussed/corrected.

Ps write diferent forms of the rule and choose other Ps to check them using values from the table.

Draw the graph with the whole class. Ps work on BB and rest of class work in *Ex. Bks.* First agree on the range of values needed for the two axes, then Ps draw the axes and label them,

Ps come to BB one after the other to choose a column in the table and plot that point. Class points out errors. Ps do the same in *Ex Bks*. Is it correct to join up the points with a straight line? [In a) it is, but in b) 2 curved lines fit the points better,as shown.]

In b) plot extra points (e.g. the white dots shown) to confirm where the curve should lie.

Solution:

a)

x	0	-1	$\frac{1}{2}$	2	$\frac{2}{3}$	$-\frac{1}{6}$	2.5	-4	$\mathbf{\frac{1}{6}}$	**0.2**	$\mathbf{-\frac{1}{2}}$
y	0	-3	$\frac{3}{2}$	6	**2**	$\mathbf{-\frac{1}{2}}$	**7.5**	**– 12**	$\frac{1}{2}$	0.6	$-\frac{3}{2}$

Rule: $y = 3 \times x,\ x = y \div 3 = \dfrac{1}{3} \times y,\ \dfrac{y}{x} = 3\ (x \neq 0)$

Who can think of another form of the rule? $[\dfrac{x}{y} = \dfrac{1}{3}\ (y \neq 0)]$

What does the graph show us about the relationship between x and y? (x and y are in <u>direct</u> proportion to one another.)

Individual work in completing the tables and writing the rules.

Whole clas activity in drawing the graphs.

Drawn on BB or use enlarged copy master or OHT

The agreed form of the rule could be in words to start with.

At a good pace

Reasoning, agreement, self-correction, praising

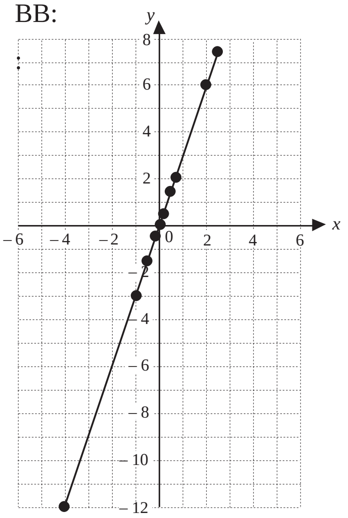
Less able Ps could use the axes on the copy masters.

At a good pace.

Agreement, praising

Discuss the graph line.

BB:



Y6

Lesson Plan 164

Activity

Notes

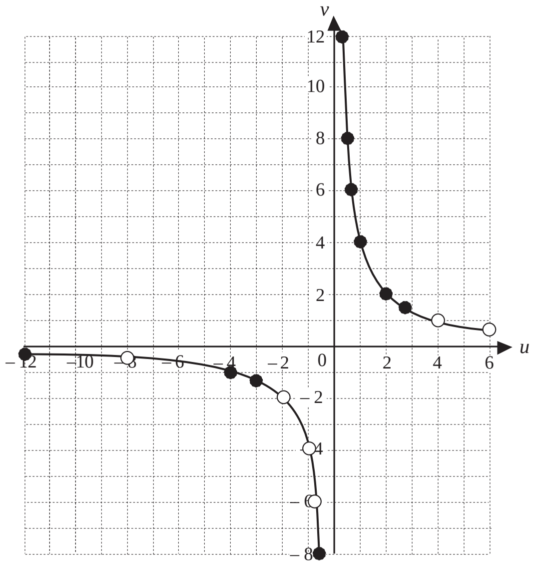
2 (Continued)

Solution:

b)

u	1	$-\frac{1}{2}$	2	$\frac{1}{3}$	-4	$\frac{1}{2}$	$\boldsymbol{\frac{8}{3}}$	$\boldsymbol{\frac{2}{3}}$	-3	-12	—	$\boldsymbol{-4}$
v	4	-8	2	12	$\boldsymbol{-1}$	$\boldsymbol{8}$	$\frac{3}{2}$	6	$\boldsymbol{-\frac{4}{3}}$	$\boldsymbol{-\frac{1}{3}}$	0	-1

Rule: $v = \dfrac{4}{u}$ $(u \neq 0)$, $u = \dfrac{4}{v}$ $(v \neq 0)$, $u \times v = 4$



Note that neither u nor v can be zero.

What does the graph show us about the relationship between u and v? (u and v are in <u>inverse</u> proportion to one another.)

(As one value increases by a certain amount, the other value decreases by that amount, and vice versa.)

20 min

3 ***PbY6b, page 164***

Q.2 Read: *Solve the problems. Think about the ratio between the quantities.*

Deal with one at a time. Ps read the problem themselves and solve it in *Ex. Bks.* under a time limit.

Review with whole class. Ps show results on scrap paper or slates on command. P answering correctly explains reasoning at BB. Who did the same? Who solved it in a different way? etc. Mistakes discussed and corrected.

Elicit whether the quantities are in direct or inverse proportion.

Solution:

a) *If $\frac{4}{5}$ kg of apples cost £2.40, what is the price of $\frac{2}{3}$ kg of apples?*

Plan: $\pounds 2.40 \div \dfrac{4}{5} \times \dfrac{2}{3} = \overset{\overset{0.20}{\cancel{0.60}}}{\cancel{\pounds 2.40}} \times \dfrac{5}{\cancel{4}_1} \times \dfrac{2}{\cancel{3}_1} = \underline{\underline{\pounds 2}}$

or $\pounds 2.40 \div 4 \times 5 \div 3 \times 2 = \pounds 3 \div 3 \times 2 = \underline{\underline{\pounds 2}}$

Answer: The price of $\frac{2}{3}$ kg of apples is £2.

[Price and quantity are in <u>direct</u> proportion to one another.]

Individual work, monitored, helped

Differentiation by time limit

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Feedback for T

or

$\frac{4}{5}$ kg → £2.40

$\frac{1}{5}$ kg → £2.40 ÷ 4 = 60 p

1 kg → 60 p × 5 = £3

$\frac{1}{3}$ kg → £3 ÷ 3 = £1

$\frac{2}{3}$ kg → £1 × 2 = $\underline{\underline{\pounds 2}}$

Activity

3 (Continued)

b) *$\frac{1}{5}$ kg of strawberries costs £1.50, including the cost of the punnet. If all punnets cost 10 p, what would you pay for 400 g of strawberries in a punnet?*

Plan: $\frac{1}{5}$ kg = 200 g $\rightarrow$ £1.40 + £0.10 = £1.50

400 g $\rightarrow$ £1.40 × 2 + £0.10 = <u>£2.90</u>

Answer: You would pay £2.90 for 400 g of strawberries in a punnet.

c) *When our car used $5\frac{1}{3}$ litres of petrol every 100 km, a full tank lasted for 864 km. If our car had used $7\frac{1}{5}$ litres of petrol every 100 km, how far could we have driven with a full tank?*

Plan: 100 km $\rightarrow$ $5\frac{1}{3}$ litres

864 km $\rightarrow$ $5\frac{1}{3}$ litres × 8.64 = 43.20 + 2.88 (litres)

= 46.08 litres

(This is the capacity of the tank.)

46.08 litres ÷ $7\frac{1}{5}$ litres × 100 = 4608 ÷ 7.2 = 46080 ÷ 72

= 5120 ÷ 8 = <u>640</u> (km)

or <u>Consumption</u> <u>Distance covered</u>

$$\begin{tikzcd} 5\frac{1}{3} \text{ litres (per 100 km)} \arrow[r] \arrow[d, bend right, "\times \frac{7\frac{1}{5}}{5\frac{1}{3}}"'] & 864 \text{ km} \arrow[d, bend left, "\times \frac{5\frac{1}{3}}{7\frac{1}{5}}"] \\ 7\frac{1}{5} \text{ litres (per 100 km)} \arrow[r] & ? \text{ km} \end{tikzcd}$$

C: $864 \times \dfrac{\frac{16}{3}}{7.2} = 864 \times \dfrac{16}{3 \times 7.2}$

$= \overset{40}{\cancel{864}} \times \dfrac{16}{\underset{1}{\cancel{21.6}}} = 40 \times 16 = \underline{640}$ (km)

Answer: W could have driven 640 km with a full tank.

30 min

Notes

The price and quantity are <u>not</u> in direct proportion and <u>not</u> in inverse proportion – they are not in proportion at all.

(As the mass increases by a certain number of times, the price does not increase or decrease by that same number of times.)

Amount of petrol in tank and the distance covered (for the same consumption) are in <u>direct</u> proportion. (The more petrol there is in the tank, the further the distance which can be covered.)

Consumption and distance covered (with the same amount of petrol in the tank at the start) are in <u>inverse</u> proportion.

(The higher the consumption of the engine, the shorter the distance covered.)

T could show this method of multiplying by the <u>reciprocal</u> value if no P used it and ask Ps if it is correct.

(864 ÷ 21.6 = 108 ÷ 2.7
= 12 ÷ 0.3 = 120 ÷ 3 = 40)

Y6

Lesson Plan 164

Activity

Notes

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions with the whole class, whether Ps attempted them or not.

4

HMC:
Hungarian Mathematics Competition
1990
Age 11

PbY6b, page 164**, **Q.3

Read: *The big hand and the little hand of a clock coincide at 12 o'clock. When will the two hands of the clock next be in a straight line?*

Allow Ps a minute or two to think and discuss with their neighbours. Ps who have ideas come to BB to explain and develop them, with help of T and class. If no Ps have ideas, T gives hint about the angle covered by each hand in 1 minute. If Ps still cannot suggest what to do, T directs Ps thinking and helps class to solve it together, involving Ps where possible.

Solution: e.g.

When the hands are next in a straight line, the angle between the two hands will have increased from 0° to 180°.

<u>Angle speed</u> of the hands:

Hour hand (small): 60 minutes → 30°
1 minute → 30° ÷ 60 = 0.5°

Minute hand (big): 60 minutes → 360°
1 minute → 360° ÷ 60 = 6°

So every minute, the angle between the 2 hands increases by

$$6° - 0.5° = 5.5°$$

but the angle has to increase by 180°, so the time needed is:

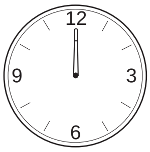
$$180° \div 5.5° = \frac{180}{5.5} = \frac{360}{11} = \underline{32\frac{8}{11}} \text{ (minutes)}$$

Answer: The two hands will next form a straight line at 12 h 32 $\frac{8}{11}$ min.

Whole class activity
(or individual or paired trial first if Ps wish)

Use real or model clock or draw diagram on BB or use enlarged copy master or OHP.

BB:

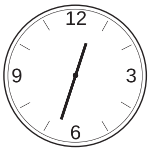


Discussion, reasoning, agreement, praising

Ps could write agreed solution in *Ex. Bks*.

Check solution by showing the position of the hands on a real clock or on the diagram.

BB:



5

HMC:
Hungarian Mathematics Competition
2000
Age 11

PbY6b, page 164

Q.4 Read: *Dad could dig the garden in 2 hours. His elder son, Benny, could dig the garden in 3 hours. His younger son, Charlie, could dig the garden in 6 hours.*

If they all worked together, how long would it take the three of them to dig the garden?

Ps can use the given digram as the being the whole garden and shade the appropriate parts, or amend it to form a table. Both methods are shown.

Solution: e.g.

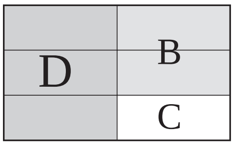
Every hour:

Dad digs half the garden,

Benny digs 1 third of the garden,

Charlie digs 1 sixth of the garden and it is all dug over.

BB:



Answer:

If they all worked together, they would dig the garden in 1 hour.

Individual work, monitored

Diagram drawn on BB or SB or OHT

Alternative method:

Let the time taken together be x, then

	Time needed	Part dug per hour
Dad	2 h	$\frac{1}{2} = \frac{3}{6}$
Benny	3 h	$\frac{1}{3} = \frac{2}{6}$
Charlie	6 h	$\frac{1}{6}$
Together	x	$\frac{1}{x}$

$$\frac{1}{x} = \frac{3}{6} + \frac{2}{6} + \frac{1}{6} = \frac{6}{6} = \underline{1}$$

$\underline{x = 1}$ (hour)

Y6

Lesson Plan 164

Activity

Notes

6

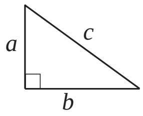
PbY6b, page 164

Q.5 Read: *The ratio of the lengths of the sides of a right-angled triangle is 3 : 4 : 5.*

If the area of the triangle is 24 cm^2*, what is the length of each of its sides?*

Advise Ps to draw a diagram first.

Solution: e.g.



$a : b : c = 3 : 4 : 5$

$$A = \frac{a \times b}{2} = 24 \text{ cm}^2 \qquad [\times 2]$$

$$a \times b = 48 \text{ cm}^2$$

If $a : b : c = 3$ cm : 4 cm : 5 cm, $a \times b = 12 \text{ cm}^2$ ✗

If $a : b : c = 6$ cm : 8 cm : 10 cm, $a \times b = 48 \text{ cm}^2$ ✓

Answer: The length of the sides of the triangle are 6 cm, 8 cm and 10 cm.

Individual work, monitored

or

Let x be the scale factor required, then

$$A: \frac{3x \times 4x}{2} = 24 \quad [\times 2]$$

$$3x \times 4x = 48$$

$$12 \times x^2 = 48 \quad [\div 12]$$

$$x^2 = 4$$

$$\underline{x = 2}$$

So the lengths of the sides are 6 cm, 8 cm and 10 cm.

Y6

Lesson Plan
165

Activity

Factorising 165, 340, 515 and 1165. Revision and practice.

PbY6b, page 165

Notes

<u>165</u> = 3 × 5 × 11

Factors: 1, 3, 5, 11, 15, 33, 55, 165

<u>340</u> = 2^2 × 5 × 17

Factors: 1, 2, 4, 5, 10, 17, 20, 34, 68, 85, 170, 340

<u>515</u> = 5 × 103

Factors: 1, 5, 103, 515

<u>1165</u> = 5 × 233

Factors: 1, 5, 233, 1165

(or set factorising as extra task for homework at the end of *Lesson 164* and review at the start of *Lesson 165.*

Solutions:

Q.1 a) *If the areas of two rectangles are equal, the rectangles are* ***congruent.*** [F]

(e.g. A = 12 cm^2: sides could be 3 cm and 4 cm, or 2 cm and 6 cm, or 1 cm and 12 cm)

b) *All equilateral triangles are* ***similar.*** [T]

(All equilateral triangles have angles of 60°, so they are the same shape, but not necessarily the same size.)

c) *The* ***arithmetic mean*** *of two numbers is always positive.* [F]
(e.g. the mean of – 4 and – 8 is – 6)

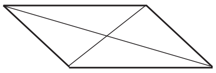
d) *There is an i****sosceles*** *triangle which has three equal angles.* [T]

(An isosceles triangle has <u>at least 2</u> equal sides and angles, so an equilateral triangle is also an isosceles triangle.)

e) *The diagonals of a* ***parallelogram*** *intersect at right angles.* [F]

(Only if the parallelogram is a rhombus, i.e. it has equal sides)

e.g.



f) *If the areas of two squares are equal, the squares are* ***congruent****.* [T]
(All squares are similar, so if two squares have the same area, they must also be exactly the same size.)

Q.2 a) $(a + b) \times c = a \times c + b \times c = ac + bc$

b) $(a + b) \div c = a \div c + b \div c = \dfrac{a}{c} + \dfrac{b}{c} = \dfrac{a + b}{c}$

c) $\dfrac{a}{b} + \dfrac{c}{d} = \dfrac{a \times d + c \times b}{b \times d} = \dfrac{ad + cb}{bd}$

d) $\dfrac{a}{b} \times \dfrac{b}{a} = 1$ (reciprocal values)

e) $\dfrac{a}{b} \div a = \dfrac{1}{b}$

f) $\dfrac{a - b}{c} = \dfrac{a}{c} - \dfrac{b}{c}$

Q.3 a) $\dfrac{1}{x} + \dfrac{1}{y} = \dfrac{1}{8} + \dfrac{1}{24} = \dfrac{3 + 1}{24} = \dfrac{4}{24} = \dfrac{1}{6}$

b) $x = 7, y = 42$; $x = 9, y = 18$; $x = 10, y = 15$;

and $x = 12, y = 12$

Q.4 a) V = 24 unit cubes

b) $A = 2 \times 4 + 4 \times 12 = 8 + 48 =$ <u>56</u> (unit squares)

c) i) Cuboid with **smallest** possible surface area has dimensions 2 units × 3 units × 4 units

(i.e. the most <u>symmetrical</u> shape)

$A = 2 \times (6 + 8 + 12)$
$= 2 \times 26 =$ <u>52</u> (sq. units)

Y6		*Lesson Plan 165*
Activity		***Notes***

(*Solutions*: Q.4 continued)

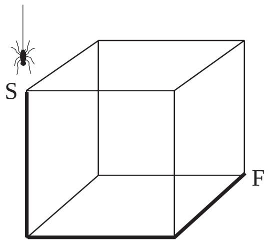
c) ii) Cuboid with the greatest possible surface area has dimensions 24 units × 1 unit × 1 unit

(i.e. the most <u>asymmetrical</u> shape)

$A = 2 \times 1 + 4 \times 24$

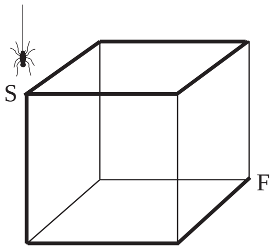
$= 2 + 96 = \underline{98}$ (sq. units)

Q.5 a) e.g.



Shortest path from S to F is <u>3 units</u>.

b) e.g.



Longest path from S to F is <u>7 units</u>.

c) 2 + 4 + 4 + 4 = <u>14</u> differentt paths are possible.

Y6

R: Calculations
C: Revision : Ratio, percentage
E: Word problems

Lesson Plan
166

Activity | *Notes*

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>166</u> = 2 × 83 Factors: 1, 2, 83, 166
- <u>341</u> = 11 × 31 Factors: 1, 11, 31, 341
- <u>516</u> = 2 × 2 × 3 × 43 = 2^2 × 3 × 43
  Factors: 1, 2, 3, 4, 6, 12, 43, 86, 129, 172, 258, 516
- <u>1166</u> = 2 × 11 × 53
  Factors: 1, 2, 11, 22, 53, 106, 583, 1166

Notes:

Individual work, monitored (or whole class activity)

BB: 166, 341, 516, 1166

Ps try it <u>without</u> calculators.

Reasoning, agreement, self-correction, praising

166	2
83	83
1	

516	2
258	2
129	3
43	43
1	

341	11
31	31
1	

1166	2
583	11
53	53
1	

8 min

2

PbY6b, page 166

Q.1 Read: *Which is more, A or B? Circle the appropriate letter.*

Deal with one at a time. or set a time limit. Ps calculate mentally or write calculations in *Ex. Bks.*

Review with whole class. Ps could show their answer with an equation or inequality on scrap paper or slates on command. Ps with different answers explain reasoning. Class points out errors and agrees on correct answer. Who worked it out in the same way? Who thought in a different way? etc. Mistakes discussed and corrected.

Solution: e.g.

a) *0.15 times A is 3300 kg. 0.25 times B is 4000 kg.* [A > B]

A = 3300 kg ÷ 0.15 = 330 000 kg ÷ 15 = <u>22 000 kg</u>

B = 4000 kg ÷ 0.25 = 400 000 kg ÷ 25 = 80 000 kg ÷ 5
= <u>16 000 kg</u>

b) $\frac{47}{100}$ *of A is 564 litres.* $\frac{55}{100}$ *of B is 605 litres.* [A > B]

A = 564 litres ÷ 0.47 = 56 400 litres ÷ 47 = <u>1200 litres</u>

B = 605 litres ÷ 0.55 = 60 500 litres ÷ 55 = <u>1100 litres</u>

c) *A is 75% of 900 m. B is 120% of 562.5 m.* [A = B]

A = $\overset{9}{\cancel{900}}$ m × $\frac{75}{\cancel{100}_1}$ = <u>675 m</u>

B = 562.5 m × 1.2 = <u>675 m</u>

d) *A is 30% more than £5000. 80% of B is £5000.* [A > B]

A = 130% of £5000 = £5000 × 1.3 = <u>£6500</u>

B = £5000 ÷ 0.8 = £50 000 ÷ 8 = <u>£6250</u>

Notes:

Individual work, monitored, helped

Written on BB or SB or OHT

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept any valid form of calculation.

Feedback for T

			1	2	0	0
4	7	5	6	4	0	0
	–	4	7			
			9	4		
		–	9	4		
				0	0	0

22 min

Y6

Lesson Plan 166

Activity

3

PbY6b, page 166

Q.2 Read: *Solve the problems and check your results in context.*

Deal with one at a time under a short time limit. Ps read problem themselves, write a plan, estimate the result, do the calculation, check that the result is feasible, then write the answer in a sentence.

Review with whole class. T chooses a P to read out the question and Ps show results on scrap paper or slates on command. P with correct answer explains reasoning at BB. Class agrees/disagrees. Who did the same? Who worked it out in another way? etc. Mistakes discussed and corrected.

T chooses a P to say the answer in a sentence.

Solutions: e.g.

a) *£10 000 was put into a bank account with a yearly interest rate of 6.5%.*

How much would be in the account after one year if the money was not touched?

Plan: £10 000 × 1.065 = <u>£10 650</u>

Answer: After one year, there would be £10 650 in the account.

b) *How much money should we put into a bank account with a yearly interest of 6.5% if we want to have £21 300 in the account after one year?*

Let the amount of money we put into the account be *A*.

Plan: $A \times 1.065 = £21\,300$

$A = £21\,300 \div 1.065$

$= \underline{£20\,000}$

Answer: We should put £20 000 into the bank account.

c) *If we put 1200 g of fresh meat on a barbecue, we would only get 780 g of cooked meat to eat. What percentage of the meat is lost through cooking?*

Plan: Meat lost: 1200 g – 780 g = 420 g

Percentage lost: $\dfrac{\overset{35}{\cancel{420}}}{\underset{\underset{1}{\cancel{12}}}{\cancel{1200}}} \times \overset{1}{\cancel{100}}\% = \underline{35\%}$

Answer: 35% of the meat is lost through cooking.

d) *One of the sides of a rectangle was reduced by 40%. By what percentage should the adjacent side be increased so that the area of the rectangle stays the same?*

Sides of original rectangle: *a* and *b*, $A = a \times b$

Sides of new rectangle: $a \times 0.6$ and b', $A' = a \times 0.6 \times b'$

$a \times 0.6 \times b' = a \times b \quad [\div a]$

$0.6 \times b' = b \quad [\div 0.6]$

$b' = b \div 0.6 = b \times \dfrac{10}{6} = b \times 1.\dot{6}$

$\rightarrow \quad 166.\dot{6}\,\%$ of b

Answer: The adjacent side should be increased by $66.\dot{6}$ %.

Notes

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept and praise any valid method of solution.

Feedback for T

E: a little more than £10 000

or *Plan*:

$£10\,000 + \dfrac{65}{\cancel{1000}} \times £10\,\cancel{000}$

$= £10\,000 + £650 = \underline{£10\,650}$

E: a little less than £21 300

C: $21\,300 \div 1.065$

$= 21\,300\,000 \div 1065$

$= 4260\,000 \div 213$

$= \underline{20\,000}$

E: 1200 – 800 = 400

$\dfrac{400}{1200} = \dfrac{1}{3} \approx 33\%$

or $\dfrac{\overset{65}{\cancel{780}}}{\underset{1}{\cancel{1200}}} \times \cancel{100}\% = 65\%$

Lost: 100% – 65% = <u>35%</u>

Elicit that if a side is reduced by 40%, it is 60% or 6 tenths or 0.6 of the original length.

If no P used this method of solution, T could lead Ps through it and ask what they think of it.

[i.e. $b' = 1\dfrac{2}{3}$ of *b*]

32 min

Y6

Lesson Plan 166

Activity

4

Erratum

In *Pbs*:
in d), area should be 8470 cm^2
<u>not</u>
9430 cm^2

PbY6b, page 166

Q.2 Read: *Solve the problems and check your results in context.*

Deal with one at a time under a short time limit. Ps read problem themselves, solve it and check the result, then write the answer in a sentence.

Review with whole class. T chooses a P to read out the question and Ps show results on scrap paper or slates on command. P with correct answer explains reasoning at BB. Class agrees/disagrees. Who did the same? Who worked it out in another way? etc. Mistakes discussed and corrected.

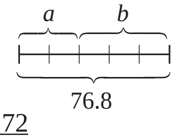
T chooses a P to say the answer in a sentence.

Solutions: e.g.

a) *The sum of two numbers is 76.8 and their ratio is 2 : 3. What are the two numbers?*

Let the two numbers be *a* and *b*.

Then $a + b = 76.8$, $a : b = 2 : 3$



$$a = \frac{2}{5} \text{ of } 76.8 = 0.4 \times 76.8 = \underline{30.72}$$

$$b = \frac{3}{5} \text{ of } 76.8 = 0.6 \times 76.8 = \underline{46.08}$$

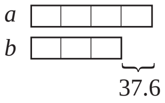
or $b = 76.8 - 30.72 = \underline{46.08}$

Answer: The two numbers are 30.72 and 46.08.

b) *The difference between two positive numbers is 37.6 and their ratio is 4 : 3. What are the two numbers?*

Let the two numbers be *a* and *b*.

Then $a - b = 37.6$, $a : b = 4 : 3$



$a = 4 \times 37.6 = \underline{150.4}$

$b = 3 \times 37.6 = \underline{112.8}$

or $b = 150.4 - 37.6 = \underline{112.8}$

Answer: The two numbers are 150.4 and 112.8.

c) *The ratio of two angles in a quadrilateral is 2 : 7. The third angle is 40° less, and the 4th angle is 60° less, than the largest angle. What sizes are the angles in the quadrilateral?*

The sum of the angles in any quadrilateral is 360°, so

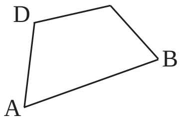
$\angle A + \angle B + \angle C + \angle D = 360°$

$\angle A : \angle B = 2 : 7$,

so if we let the scale factor be α,

$\angle A = 2\alpha$, $\angle B = 7\alpha$,

$\angle C = 7\alpha - 40°$, $\angle D = 7\alpha - 60°$



Adding the angles:

$$2\alpha + 7\alpha + 7\alpha - 40° + 7\alpha - 60° = 360°$$

$$23\alpha - 100° = 360° \quad [+ 100°]$$

$$23\alpha = 460° \quad [\div 23]$$

$$\underline{\alpha = 20°}$$

Notes

Individual work, monitored, helped

Responses shown in unison.

Reasoning, agreement, self-correction, praising

Accept and praise any valid method of solution.

Feedback for T

or

$76.8 \div (2 + 3) = 76.8 \div 5$
$= 15.36$

$a = 2 \times 15.36 = \underline{30.72}$

$b = 3 \times 15.36 = \underline{46.08}$

Check:

$30.72 + 46.08 = 76.8$ ✔

or

$$\frac{4}{7} - \frac{3}{7} = \frac{1}{7} \rightarrow 37.6$$

$$a = \frac{4}{7} \rightarrow 37.6 \times 4 = \underline{150.4}$$

$$b = \frac{3}{7} \rightarrow 37.6 \times 3 = \underline{112.8}$$

Check: $150.4 - 112.8 = 37.6$ ✔

$\angle A = 40°$
$\angle B = 140°$
$\angle C = 100°$
$\angle D = 80°$
$\overline{\underline{360°}}$ ✔

Answer: The angles in the quadrilateral are 40°, 80°, 100° and 140°.

Y6

Lesson Plan 166

Activity

Notes

4

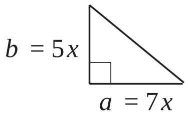
(Continued)

d) *The ratio of the two shorter sides of a right-angled triangle is 7 : 5 and its area is 8470 cm*^{2}*.*

How long are these two sides?

Let the two side lengths be a and b.

Then $a : b = 7 : 5$, $A = \dfrac{a \times b}{2}$



If we let the scale factor be x, then

$a = 7x$, $b = 5x$, $A = \dfrac{7x \times 5x}{2} = 8470 \text{ cm}^2$

Solve: $\dfrac{7x \times 5x}{2} = 8470$ [$\times$ 2]

$35 \times x^2 = 16\,940$ [$\div$ 35]

$x^2 = 484$

$x \times x = 4 \times 121 = (2 \times 11) \times (2 \times 11)$

$x = 2 \times 11 = \underline{22}$

Now we can work out the values of a and b.

$a = 7 \times 22$ (cm) $=$ <u>154 cm</u>, $b = 5 \times 22$ (cm) $=$ <u>110 cm</u>

Answer: The lengths of the two shorter sides of the triangle are 154 cm and 110 cm.

$16\,940 \div 35 = 3388 \div 7$
$= 484$

or factorise 484:

484	2
242	2
121	11
11	11
1	

Check:

$(154 \times 110) \div 2$

$= 16\,940 \div 2 = 8470$ ✔

e) *The ratio of the lengths of 3 edges meeting at a vertex of a cuboid is 2 : 4 : 5. The volume of the cuboid is 320 cm*^{3}*. What lengths are the edges of the cuboid?*

Let the 3 edge lengths be a, b and c.

Then $a : b : c = 2 : 4 : 5$, $V = a \times b \times c$

If we let the scale factor be x, then

$a = 2x$, $b = 4x$, $c = 5x$, $V = 2x \times 4x \times 5x$

Solve: $2x \times 4x \times 5x = 320$

$40 \times x^3 = 320$ [$\div$ 40]

$x^3 = 8$

$\underline{x = 2}$

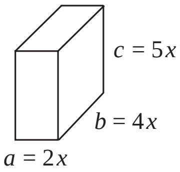
Now we can work out the values of a, b and c.

$a = 2 \times 2$ (cm) $=$ <u>4 cm</u>, $b = 4 \times 2$ (cm) $=$ <u>8 cm</u>,

$c = 5 \times 2$ (cm) $=$ <u>10 cm</u>

Answer: The lengths of the 3 edges of the cuboid are 4 cm, 8 cm and 10 cm.

e.g.



i.e. $x \times x \times x = 2 \times 2 \times 2$

Check:

$4 \times 8 \times 10 = 320$ ✔

40 min

Y6

Lesson Plan 166

Activity

Notes

5

PbY6b, page 166

Q.4 Read: *The perimeter of an irregular* ***pentagon*** *is 54 cm.*
The length of its sides are in the ratio
1 : 2 : 5 : 6 : 7 : 9.
Calculate the length of each side.

Set a time limit of 3 minutes. Ps work individually or in pairs.

Review with whole class. T asks a P for the lengths of the sides. Who agrees with **A**? Who has different lengths? Who agrees with **B**? Ps with different lengths explain reasoning at BB. Class agrees on the correct answer. Who had the correct answer but worked it out in another way? etc. Mistakes discussed and corrected.

Solution: e.g.

Let the scale factor be x, then

BB: $x + 2x + 5x + 6x + 7x + 9x = 54$

$30x = 54$ [÷ 30]

$\underline{x = 1.8}$

Check: 1.8 + 3.6 + 9 + 10.8 + 12.6 + 16.2 = 54 ✔

Answer: The lengths of the sides are 1.8 cm, 3.6 cm, 9 cm, 10.8 cm, 12.6 cm and 16.2 cm.

45 min

Individual (paired) work, monitored, helped

Differentiation by time limit.

Reasoning, agreement, checking, self-correction, praising

[If no P had the correct answer, the question could be left open for voluntary homework and reviewed before the start of *Lesson 167*.]

or $1 + 2 + 5 + 6 + 7 + 9 = 30$

$\frac{1}{30}$ of 54 cm = 1.8 cm,

$\frac{2}{30}$: 3.6 cm, $\frac{5}{30}$: 9 cm,

$\frac{6}{30}$: 10.8 cm, $\frac{7}{30}$: 12.6 cm

$\frac{9}{30}$: 16.2 cm

Y6

R: Calculations

C: Revision: Ratio, proportion

E: Problems and challenges

Lesson Plan
167

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>167</u> is a prime number Factors: 1, 167
  (as not exactly divisible by 2, 3, 5, 7, 11 and $13^2 > 167$)
- <u>342</u> = $2 \times 3 \times 3 \times 19 = 2 \times 3^2 \times 19$
  Factors: 1, 2, 3, 6, 9, 18, 19, 38, 57, 114, 171, 342
- <u>517</u> = 11×47 Factors: 1, 11, 47, 517
- <u>1167</u> = 3×389 Factors: 1, 3, 389, 1167
  (389 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17 , 19 and $23^2 > 389$)

Individual work, monitored (or whole class activity)

BB: 167, 342, 517, 1167

Ps can use a calculator for 1167.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 342 & 2 \\ 171 & 3 \\ 57 & 3 \\ 19 & 19 \\ 1 & \end{array} \qquad \begin{array}{r|l} 517 & 11 \\ 47 & 47 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1167 & 3 \\ 389 & 389 \\ 1 & \end{array}$$

8 min

2

PbY6b, page 167, Q.1

a) Read: *Write five numbers using* ***non-zero*** *digits so that their ratio is 1 : 2 : 3 : 4 : 5.*

Use each digit only once.

What is the obvious solution? (1, 2, 3, 4, 5) Who can think of another solution? Is it correct? What scale factor has been used? Is there another solution?

Elicit that using scale factor:

2: 2, 4, 6, 8, 10 (not valid, as zero used)

3: 3, 6, 9, 12, 15 (not valid, as two '1's used)

4, 5, 6, 7, 8: (not valid either)

9: 9, 18, 27, 36, 45 ✔

What do you notice about the solution using scale factor 9? (Every non-zero digit has been used once.)

If we wanted only this answer, how should we have worded the question? (Write five numbers using <u>all the</u> non-zero digits . . .)

b) Read: *Use* ***all*** *possible digits once each to make five numbers in the ratio 1 : 2 : 3 : 4 : 5.*

What is different about this question? (We must use zero.)

Give Ps a minute or two to try out some numbers, then Ps come to BB or dictate them to teacher. Class agrees/disagrees.

BB: 18, 36, 54, 72, 90 (scale factor 18, i.e. 2×9)

Who found a different solution? (No other solution is possible.)

T: When there is only one solution to a problem, we say that the solution is <u>unique</u>.

Whole class activity

Involve many Ps.

(If Ps cannot come up with another solution, T leads class through the digits, trying out each one.)

Discussion, reasoning, agreement, praising

Extra praise for Ps who managed to solve it without help

14 min

Y6

Lesson Plan 167

Activity ***Notes***

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions interactively with the whole class, whether Ps attempted them or not.

3

PbY6b, page 167

Q.2 Read: *The perimeter of an* ***isosceles*** *triangle is 36.8 cm. The length of the base side is 2 thirds of the length of the adjacent side.*

a) What length are the sides of the triangle?

b) What is the ratio of the 3 sides?

Solution: e.g.

a) Let the base length be *a* and the length of the sides adjacent to the base be *b*.

$$P = a + 2 \times b = 36.8 \text{ cm}, \quad a = \frac{2}{3} \times b$$

Substituting $\frac{2}{3} \times b$ for *a* in the perimeter equation:

$$\frac{2}{3} \times b + 2 \times b = 36.8 \text{ cm}$$

$$2\frac{2}{3} \times b = 36.8 \text{ cm}$$

$$\frac{8}{3} \times b = 36.8 \text{ cm} \qquad [\times 3, \div 8]$$

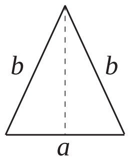
$$\underline{b = 13.8 \text{ cm}}$$

Answer: The lengths of the sides are 13.8 cm and 9.2 cm.

b) Ratio of the sides: $9.2 : 13.8 : 13.8 = \frac{2}{3} : 1 : 1$

Individual trial

BB:



so $a = \frac{2}{\cancel{3}_1} \times \overset{4.6}{\cancel{13.8}}$ cm

$= \underline{9.2 \text{ cm}}$

$[\frac{9.2}{13.8} = \frac{92}{138} = \frac{46}{69} = \frac{2}{3}]$

4

PbY6b, page 167

Q.3 Read: *The ratio of two natural numbers is 3 : 2 and they are both multiples of 6. If we divide them by 6, the first quotient is 4 greater than the second qotient.*

What are the two numbers?

Solution: e.g.

Let the two numbers be *a* and *b*, then $a : b = 3 : 2$.

Let the scale factor be *x*, then $a = 3x$ and $b = 2x$

$$3x \div 6 = 2x \div 6 + 4 \qquad [\times 6]$$

$$3x = 2x + 24 \qquad [-2x]$$

$$\underline{x = 24}$$

So $a = 3 \times 24 = \underline{72}$ and $b = 2 \times 24 = \underline{48}$

Answer: The two numbers are 48 and 72.

Individual trial

Accept any valid method of solution.

Check:

$72 : 48 = 36 : 24 = 3 : 2$ ✔

$72 \div 6 = 12$, $48 \div 6 = 8$

and $12 = 8 + \underline{4}$ ✔

Y6

Lesson Plan 167

Activity

Notes

5

HMC:
Hungarian Mathematics Competition
1987
Age 11

PbY6b, page 167

Q.4 Read: *In a mathematics competition, 9 pupils got through to the final round. In the final round, 6 tenths of the girls solved at least two problems correctly.*

How many boys and how many girls reached the final of the competition?

Solution: e.g.

Let the number of girls be g and the number of boys be b.

Then $b + g = 9$

$$\frac{6}{10} = \frac{3}{5}, \text{ so } \frac{3}{5} \text{ of } g \text{ must be a natural number, and } 0 < g \leq 9$$

$$\frac{3}{5} \text{ of } 5 \ (= 3) \text{ gives the only whole number among } \frac{3}{5} \text{ of '1 to 9'}$$

So $g = \underline{5}$ and $b = 9 - 5 = \underline{4}$

Answer: Four boys and five girls reached the final.

Individual or paired trial

(Revert to a whole class activity if Ps are struggling)

We cannot have part of a girl!

6

HMC:
Hungarian Mathematics Competition
1990
Age 11

Erratum

In *Pbs* the question should be:

'How much money do I have in each pocket?'

PbY6b, page 167

Q.5 Read: *I have £2 in my two pockets altogether.*

If I transfer a quarter of the money that I have in one pocket plus an additional 20 p from the same pocket to the other pocket, I would have an equal amount of money in each pocket. How much money do I have in each pocket?

Solution: e.g.

Let the amount of money in one pocket be a (in pence), and in the other pocket be b (in pence).

Then $a + b = 200$, $b = 200 - a$

$$a - \frac{a}{4} - 20 = 200 - a + \frac{a}{4} + 20 \qquad [\times 4]$$

$$4a - a - 80 = 800 - 4a + a + 80$$

$$3a - 80 = 880 - 3a \qquad [+ 3a]$$

$$6a - 80 = 880 \qquad [+ 80]$$

$$6a = 960 \qquad [\div 6]$$

$$\underline{a = 160} \text{ (p)}$$

So $b = 200 - 160 = \underline{40}$ (p)

Answer: I have £1.60 in one pocket and 40 p in the other pocket.

Individual trial

Stress that <u>every</u> term on each side of the equation must be multiplied by 4.

Check:

LHS: 160 – (40 + 20) = 100

RHS: 40 + (40 + 20) = 100 ✔

Y6

Lesson Plan 167

Activity | ***Notes***

7

HMC:
Hungarian Mathematics Competition
1997
Age 11

PbY6b, page 167, Q.6

Read: *Sally owns a hotel. She has seen some material which matches the colour scheme in her public rooms exactly. She needs 51 m*^{2} *of material to make cushions and drapes.*

However, Sally has been told that when the material is washed, it shrinks by $\frac{1}{16}$ *of its length and by* $\frac{1}{18}$ *of its width, so she intends to wash the material before she uses it.*

How many square metres of unshrunk material should she buy?

Solution: e.g.

Let the amount of material to be bought be x (in square metres).

If the material shrinks by $\frac{1}{16}$ of its length and by $\frac{1}{18}$ of its width then the shrunken material will be $\frac{15}{16}$ of the original length and $\frac{17}{18}$ of the original width.

BB: $x \times \frac{15}{16} \times \frac{17}{18} = 51\text{ m}^2$ [$\times$ 16 $\times$ 18]

$x \times 15 \times 17 = 51 \times 16 \times 18\ (\text{m}^2)$ [$\div$ (15 $\times$ 17)]

$$x = \frac{{}^{3}\cancel{51} \times 16 \times \cancel{18}^{6}}{{}_{5}\cancel{15} \times \cancel{17}_{1}}\ \text{m}^2$$

$$= \frac{3 \times 96}{5}\ \text{m}^2 = \frac{288}{5}\ \text{m}^2 = \underline{57.6\ \text{m}^2}$$

Answer: Sally should buy 57.6 m^2 of unshrunk material.

Whole class activity

(or individual trial if Ps wish)

Allow Ps a minute or two to think about it and discuss with their neighbours.

Ps develop their ideas with the help of T and class.

If no P is on the right track, T gives hints or directs Ps thinking, involving them as much as possible.

[Note that it is easier here to leave the divisor as a 2-term multiplication, rather than their product, 255, especially if Ps are not using calculators.]

8

HMC:
Hungarian Mathematics Competition
1983
Age 11

PbY6b page 167, Q.7

Read: *Which is more:* $\frac{3}{4}$ *or* $\frac{3000001}{4000001}$*?*

Solution: e.g.

Let's think of 3 quarters as its equivalent fraction $\frac{3\,000\,000}{4\,000\,000}$.

BB: If $\frac{3\,000\,000}{4\,000\,000} > \frac{3000001}{4000001}$ [$\times$ 4 000 000 $\times$ 4 000 001]

$3\,000\,000 \times 4\,000\,001 > 3\,000\,001 \times 4\,000\,000$

$3\,000\,000 \times 4\,000\,000 + 3\,000\,000 > 3\,000\,000 \times 4\,000\,000 + 4\,000\,000$ [– 3 000 000 $\times$ 4 000 000]

$3\,000\,000 > 4\,000\,000$ ✗ *Not true!*

or If $\frac{3\,000\,000}{4\,000\,000} < \frac{3000001}{4000001}$ [$\times$ 4 000 000 $\times$ 4 000 001]

$3\,000\,000 \times 4\,000\,001 < 3\,000\,001 \times 4\,000\,000$

$3\,000\,000 \times 4\,000\,000 + 3\,000\,000 < 3\,000\,000 \times 4\,000\,000 + 4\,000\,000$ [– 3 000 000 $\times$ 4 000 000]

$3\,000\,000 < 4\,000\,000$ ✔ *True!*

Answer: $\frac{3000001}{4000001}$ is more than $\frac{3}{4}$.

Whole class activity

(or individual challenge, left open as voluntary homework if not solved in class time)

Y6

R: Calculations

C: Miscellaneous problems

E: Problems and challenges

Lesson Plan 168

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>168</u> = $2 \times 2 \times 2 \times 3 \times 7 = 2^3 \times 3 \times 7$

  Factors: 1, 2, 3, 4, 6, 7, 8, 12
  168, 84, 56, 42, 28, 24, 21, 14 ↓

- <u>343</u> = $7 \times 7 \times 7 = 7^3$ (cubic number)

  Factors: 1, 7, 49, 343

- <u>518</u> = $2 \times 7 \times 37$ Factors: 1, 2, 7, 14, 37, 74, 259, 518

- <u>1168</u> = $2 \times 2 \times 2 \times 2 \times 73 = 2^4 \times 73$

  Factors: 1, 2, 4, 8, 16, 73, 146, 292, 584, 1168

Individual work, monitored (or whole class activity)

BB: 168, 343, 518, 1168

Ps could try it without a calculator.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 168 & 2 \\ 84 & 2 \\ 42 & 2 \\ 21 & 3 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 518 & 2 \\ 259 & 7 \\ 37 & 37 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 343 & 7 \\ 49 & 7 \\ 7 & 7 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1168 & 2 \\ 584 & 2 \\ 292 & 2 \\ 146 & 2 \\ 73 & 73 \\ 1 & \end{array}$$

8 min

2

PbY6b, page 168, Q.1

Read: *Freddie Fox decided that in future he would tell lies on Mondays, Wednesdays and Fridays but he would always tell the truth on the other days of the week,*

One day, Freddie said, "Tomorrow I will tell the truth."

On what day of the week could he have said it?

Allow Ps a minute or two to think about it and disucss with their neighbours if they wish. Who thinks that they know the answer? Why do you think so? Who agrees? Who thinks something else? etc. If no P has the correct explanation, try each day in turn.

Could he have said it on a Monday (Tuesday, etc.) Why not?

Solution:

Freddie could not have said it on:

- Monday, Wednesday or Friday, as he tells lies on these days and as on the days following them he tells the truth, he would not be telling a lie.
- Tuesday, Thursday or Sunday, as he tells the truth on these days and as on the days following them he tells lies, he would not be telling the truth.

The only possible day is a <u>Saturday</u>, as he tells the truth on Saturdays and also tells the truth on Sundays, so his statement is true.

Whole class activity

[A similar problem was used in Year 5, *Lesson 174* (Q.2) so some Ps might remember the logic needed.]

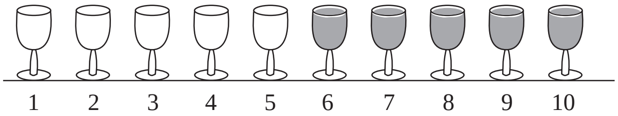
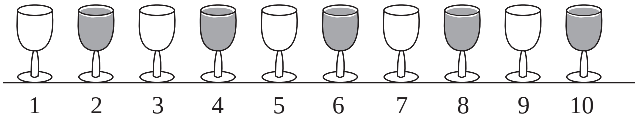
Discussion involving many Ps.

Reasoning, agreement, praising

13 min

Y6

Lesson Plan 168

Activity		***Notes***
	N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.	Review the questions interactively with the whole class, whether Ps attempted them or not.
3	***PbY6b, page 168*** Q.2 Read: *Is it possible for four whole numbers to have an odd number as their sum and an odd number as their product? If so, write the numbers. If not, say why not.* *Solution:* To get an odd sum, there must be an odd number of odd terms. e.g. 4 + 8 + 10 + 17 is <u>odd</u> (1 odd number) but 4 × 8 × 10 × 17 is <u>even</u> (units digit 0) 20 + 3 + 41 + 15 is <u>odd</u> (3 odd numbers) but 20 × 3 × 41 × 15 is <u>even</u> (units digit zero) It is <u>impossible</u> for the sum and product of 4 whole numbers to be odd. To get an odd sum there must be an odd number of odd terms and to get an odd product, there must be an odd number of factors which are <u>all</u> odd numbers.	Individual trial first Ps try out different numbers in *Ex. Bks*. Then whole class discussion and agreement on the correct form of words for the answer.
4	***PbY6b, page 168*** Q.3 Read: *Five empty glasses and five glasses full of grape juice are standing in a row.* BB:  *How can you make the empty and full glasses alternate by touching only 2 glasses?* *Solution* Pour the contents of glasses 7 and 9 into empty glasses 2 and 4. BB: 	Individual trial first Drawn (stuck) on BB or use enlarged copy master or OHT (or have transparent plastic beakers and coloured water for demonstration) They cannot exchange places as that would involve touching 4 glasses!

Y6

Lesson Plan 168

Activity

Notes

5

PbY6b, page 168

Q.4 Read: *On an old smudged sheet of paper we can see this writing:*

BB: 72 barrels: £● 37.8●

but the digits marked ● are illegible. What could the price of a barrel have been?

Solution: e.g.

Get rid of the decimal by changing the amount to pence.

BB: 72 barrels: ☐ 378 ☐ p

If ☐ 378 ☐ is exactly divisible by 72, it must also be exactly divisible by 8 and by 9, as 8 × 9 = 72.

☐ 378 ☐ is divisible by 8 only if 78 ☐ is divisible by 8.

78 ☐ is divisible by 8 only if ☐ = <u>4</u> ,

so ☐ 378 ☐ = ☐ 378<u>4</u>

If ☐ 3784 is exactly divisible by 9, its digits must add up to a multiple of 9.

BB: 4 + 8 + 7 + 3 = 22, 22 + <u>5</u> = 27 (the only digit possible)

so ☐ 3784 = <u>53784</u> (p)

BB: 72 barrels: £537.84

1 barrel: £537.84 ÷ 8 ÷ 9 = £67.23 ÷ 9 = <u>£7.47</u>

Answer: The price of a barrel was £7.47.

Notes:

Individual trial first

Change to a whole class activity if Ps are struggling.

T directs' Ps thinking.

(as whole thousands are exactly divisible by 8)

Extra praise for Ps who worked out the answer without help.

6

Pb Y6b, page 168*, *Q.5

Read: *Ben had to make a 4-digit number, choosing from the digits 1, 2, 3, 4, 5 and 6. He was allowed to use a digit more than once.*

Ben wrote his number on a piece of paper and put it in his pocket. The rest of the class had to guess Ben's number.

The first suggestion was 4215. Ben said that two digits were correct but only one of them was in the correct place-value column.

The second suggestion was 2365. Ben said that again two digits were correct but only one of them was in the correct place-value column.

The third suggestion was 5525. This time Ben said that no digits were correct.

What to you think Ben's number could be?

Solution:

3rd clue: 5525 (No digits correct, so number does <u>not</u> contain 2 or 5)

2nd clue: 2365 (3 and 6 correct, so number has 3H or 6T)

1st clue: 4215 (4 and 1 correct, so number has 4Th or 1T)

If 4Th is correct, then 1T is <u>not</u> correct, so 6T must be correct.

If 6T is correct, then 3H is <u>not</u> correct, so number is <u>4163</u>.

If 4Th is <u>not</u> correct, then 1T is correct, so 6T is <u>not</u> correct.

If 6T is <u>not</u> correct, then 3H is correct, so number is <u>6314</u>.

Answer: Ben's number could be 4163 or 6314.

Notes:

Whole class activity

This seems complicated but if we take one clue at a time, it is quite straightforward.

Allow Ps to say what to do first and how to continue.
T intervenes only if necessary.

	Th	H	T	U
BB:	4	☐	6	☐
	4	☐	6	3
	4	1	6	3
or	☐	☐	1	☐
	☐	3	1	☐
	6	3	1	4

Y6

Lesson Plan 168

Activity

Notes

7

HMC:
Hungarian
Mathematics
Competition
1981
Age 11

PbY6b, page 168

Q.6 Read: *We know this information about a certain square and a certain rectangle.*

- *Their areas are equal.*
- *The perimeter of the square is 4 fifths of the perimeter of the rectangle.*
- *The long side of the rectangle is 4 times the length of its short side.*
- *The perimeters, areas and sides of the 2 shapes are whole numbers less than 100.*

What could be the lengths of the sides of the square and the rectangle?

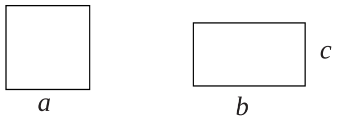
Individual or paired trial first (left open as voluntary homework if Ps have not time to solve it during the lesson)

If all Ps are struggling, stop individual work and continue as a whole class activity, with T directing Ps' thinking and involving them whenever possible.

Ps could write agreed solution in *Ex. Bks*.

Solution: e.g.

BB:



Clue 1: $a \times a = b \times c$ (1)

Clue 2: $4 \times a = \frac{4}{5} \times 2 \times (b + c)$ (2)

Clue 3: $\underline{b = 4 \times c}$ (3)

Clue 4: $a, b, c, 4a, \ 2 \times (b + c), \ a^2, \ b \times c < 100$ and whole numbers

Equations can be numbered so that they may be referred to more easily in the solution.

Substitute $4 \times c$ for b in equation (2):

$$4 \times a = \frac{4}{5} \times 2 \times (4 \times c + c)$$

$$4 \times a = \frac{4}{5} \times 2 \times 5 \times c \qquad [\div 4]$$

$$a = \frac{1}{5} \times 10 \times c$$

$$\underline{a = 2 \times c} \qquad (4)$$

Substitute $2 \times c$ for a in equation (1):

$$2 \times c \times 2 \times \text{c} = b \times c$$

$$4 \times c \times c = b \times c$$

but $b \times c < 100$ and is a whole number,

so $4 \times c \times c < 100$, and is a whole number [÷ 4]

$\underline{c \times c < 25}$, and is a whole number

If $\underline{c = 1}$, then from (3) $\underline{b = 4}$, and from (4) $\underline{a = 2}$

If $\underline{c = 2}$, $\underline{b = 8}$, $\underline{a = 4}$

If $\underline{c = 3}$, $\underline{b = 12}$, $\underline{a = 6}$

If $\underline{c = 4}$, $\underline{b = 16}$, $\underline{a = 8}$

Check:

$A_{\text{square}} = 4 = A_{\text{rectangle}}$ ✔

$A_{\text{square}} = 16 = A_{\text{rectangle}}$ ✔

$A_{\text{square}} = 36 = A_{\text{rectangle}}$ ✔

$A_{\text{square}} = 64 = A_{\text{rectangle}}$ ✔

Answer: The sides of the square could be 2, 4, 6 or 8 units.
The sides of the rectangle could be 1 unit by 4 units, or 2 units by 8 units, or 3 units by 12 units, or 4 units by 16 units.

(Corresponding to the relevant side *a*)

Y6

R: Calculations
C: Miscellaneous problems
E: Challenges

Lesson Plan 169

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>169</u> = 13 × 13 = 13^2 Factors: 1, 13, 169 (square number)
- <u>344</u> = 2 × 2 × 2 × 43 = $2^3 \times 43$
  Factors: 1, 2, 4, 8, 43, 86, 172, 344
- <u>519</u> = 3 × 173 Factors: 1, 3, 173, 519
- <u>1169</u> = 7 × 167 Factors: 1, 7, 167, 1169

$$\begin{array}{r|l} 344 & 2 \\ 172 & 2 \\ 86 & 2 \\ 43 & 43 \\ 1 & \end{array}$$

8 min

Individual work, monitored (or whole class activity)

BB: 169, 344 519, 1169

T decides whether Ps can use a calculator.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 169 & 13 \\ 13 & 13 \\ 1 & \end{array} \qquad \begin{array}{r|l} 519 & 3 \\ 173 & 173 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1169 & 3 \\ 167 & 167 \\ 1 & \end{array}$$

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions interactively with the whole class, whether Ps attempted them or not.

2

HMC:
Hungarian Mathematics Competition
1981
Age 11

PbY6b, page 169

Q.1 Read: *Can you pay a bill of £500 using exactly 12 notes which are £5, £20, or £50 notes? Give a reason for your answer.*

Solution: e.g.

Let the number of £5 notes be *a*, and *a* is a whole number and the number of £20 notes be *b*, and *b* is a whole number, then the number of £50 notes is 12 – *a* – b.

$$\begin{aligned} \text{BB: } 5 \times a + 20 \times b + 50 \times (12 - a - b) &= 500 \\ 5a + 20b + 600 - 50a - 50b &= 500 \quad [-500] \\ 100 - 45a - 30b &= 0 \quad [+45a + 30b] \\ 100 &= 45a + 30b \quad [\div 5] \\ \underline{20 = 9a + 6b} &\quad \ldots\ldots (1) \end{aligned}$$

Substitute possible values for *a* in equation (1).

If $a = 0$, $6b = 20$, so $b = \dfrac{20}{6} = 3\dfrac{1}{3}$ Impossible!

If $a = 1$, $6b = 20 - 9 = 11$, so $b = \dfrac{11}{6} = 1\dfrac{5}{6}$ Impossible!

If $a = 2$, $6b = 20 - 18 = 2$, so $b = \dfrac{2}{6} = \dfrac{1}{3}$ Impossible!

a cannot be greater than 2, as 9 × 3 = 27, which is more than 20, so there is <u>no</u> solution to the equation.

Answer: It is impossible to pay a bill of £500 using exactly 12 notes which are £5, £20 or £50 notes, because there is no combination of 12 such notes which can make £500.

Individual trial first

Revert to a whole class activity if Ps are struggling.

We cannot have part of a £20 note – it would not be valid currency!

Y6

Lesson Plan 169

Activity

Notes

3

HMC: Hungarian Mathematics Competition
1980
Age 11

PbY6b, page 169

Q.2 Read: *The square base of a solid wooden cuboid has 4 cm edges. The height of the cuboid is 3 cm. The outside of the cuboid is painted red.*

If the cuboid is cut into 1 cm cubes, how many of the unit cubes will have:

a) 3 red faces *b) 2 red faces*
c) 1 red face *d) no red faces?*

Solution:

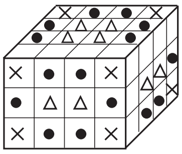
The cuboid is made from $4 \times 4 \times 3 = \underline{48 \text{ cm}}$ cubes

a)	3 *red* faces:	8 cubes	1 at each vertex (✕)
b)	2 *red* faces:	20 cubes	remaining cubes on edges (●)
c)	1 *red* face:	16 cubes	middle cubes on each face (Δ)
d)	no *red* faces:	4 cubes	at centre of cuboid (unseen)
	Check:	48 cubes ✔	

Individual or paired trial

Less able Ps could have unit cubes on desk to build the solid

and/or T has model prepared from unit cubes to confirm Ps' solution



$(8 \times 2 + 4 \times 1)$
$(4 \times 2 + 2 \times 4)$

4

HMC: Hungarian Mathematics Competition
1983
Age 11

PbY6b, page 169

Q.3 Read: *We marked the midpoints of the edges of a cube. Then we joined up each point to the next with straight lines and cut the corners off the cube along these lines.*

The surface of the remaining solid is made up of triangular and square faces.

a) How many triangles and how many squares make up its surface?

b) How many vertices and edges does this solid have?

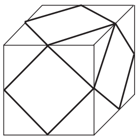
c) Draw this solid.

Solution:

a) <u>8 triangles</u> and <u>6 squares</u> make up its surface.

b) It has <u>12 vertices</u> and <u>24 edges</u>.

c)



Individual trial

T could show a model at the start if Ps need some help in imagining what the solid would look like.

(or Ps could make their own rough models from plasticine).

What is this solid called?

(polyhedron: a 3-D shape with many plane faces)

Extension

Who remembers the relationship between the number of faces, vertices and edges of a polyhedron? T reminds Ps if necessary and class checks that it is true for this polyhedron.

BB: $f + v = e + 2$ [Euler's theorem]

LHS: $(8 + 6) + 12 = 26$
RHS: $24 + 2 = 26$ ✔

Extra praise if a P remembers.

Ps could write the theorem in the blank page at back of *Pbs*. to help them remember it.

Activity

Notes

5

HMC:
Hungarian Mathematics Competition
1984
Age 11

PbY6b, page 169

Q.4 Read: *At Primary school, Peter was asked for a clue about his age. This is what he said.*

'The current age of my father can be written with two digits and his age when I was born could be written with the same two digits.'

How old is Peter?

Solution:

Let the 2 digits in Peter's father's age be a and b.

$ab - ba < 12$ (as Peter is in Primary school)

so $10 \times a + b - (10 \times b + a) < 12$

$9 \times a - 9 \times b < 12$

$9 \times (a - b) < 12$

so $a - b = 1$ (they must be whole numbers)

and $9 \times (a - b) = \underline{9}$

Answer: Peter is 9 years old.

Individual trial first

(T gives hint about naming the 2 digits or reverts to a whole class activity if Ps are struggling.)

e.g. Peter's father could have been 23 years old when Peter was born and be 32 years old now (or 34 and 43)

6

HMC:
Hungarian Mathematics Competition
2001
Age 11

PbY6b, page 169

Q.5 Read: *There were 25 cars in a car park. There were 3 times as many Renaults as Hondas and twice as many Peugeots as Fords. The Hondas were not the same colour.*

How many of each type of car was in the car park?

Solution:

Data: 25 cars, $R = 3 \times H$, $P = 2 \times F$

$H \neq 1$ (as there was more than one colour)

If $H = 2$, then $R = 6$, and $F + 2 \times F = 25 - (6 + 2)$

$3 \times F = 17$

So $H \neq 2$ $F = 5\frac{2}{3}$ Impossible!

If $H = 3$, then $R = 9$, and $F + 2 \times F = 25 - (9 + 3)$

$3 \times F = 13$

So $H \neq 3$ $F = 4\frac{1}{3}$ Impossible!

If $H = 4$, then $R = 12$, and $F + 2 \times F = 25 - (12 + 4)$

$3 \times F = 9$

So $\underline{H = 4}$, $\underline{R = 12}$, $\underline{P = 6}$ $\underline{F = 3}$ ✔

If $H = 5$, then $R = 15$, and $F + 2 \times F = 25 - (15 + 5)$

$3 \times F = 5$

So $H \neq 5$ $F = 1\frac{2}{3}$ Impossible!

If $H = 6$, then $R = 18$, and $F + 2 \times F = 25 - (18 + 6)$

So $H \neq 6$ $3 \times F = 1$ Impossible!

$H < 7$, as if $R = 21$, $21 + 7 = 28$ and $28 > 25$

Answer: There were 4 Hondas, 12 Renaults, 6 Peugeots and 3 Fords in the car park.

Individual or paired trial first

H, R, P and F must be whole numbers, as there could not be part of a car.

If Ps are struggling, T leads them through the reasoning for $H = 2$, then Ps try the other possible values by themselves.

Check: $4 + 12 + 6 + 3 = 25$ ✔
All the conditions are fulfilled.

So $\underline{H = 4}$ is the only possible solution.

Activity ***Notes***

7

HMC:
Hungarian Mathematics Competition
1981
Age 12

PbY6b, page 169**, **Q.6

Read: *Six bars of plain milk chocolate cost the same as 4 bars of fruit and nut chocolate or 5 bars of dark chocolate.*

If we buy two bars of each type of chocolate, we are given change from £1.

What is the price of each type of chocolate bar?

Ps decide what to do first and how to continue. T directs Ps' thinking if Ps have no ideas.

Solution: e.g.

Let the price of a bar of: plain milk chocolate be m (pence)
fruit and nut chocolate be f (pence)
dark chocolate be d (pence)

$6 \times m = 4 \times f, \quad 6 \times m = 5 \times d$

so $f = \frac{6}{4} \times m = \frac{3}{2} \times m$ and $d = \frac{6}{5} \times m$

Ratio of $m : f : d = 1 : \frac{3}{2} : \frac{6}{5} = 10 : 15 : 12$

But $2 \times m + 2 \times f + 2 \times d < 100$ $[\div 2]$

$m + f + d < 50$

So the only possible solution is: $\underline{m = 10}$, $\underline{f = 15}$, $\underline{d = 12}$

Answer: The price of a plain milk chocolate bar is 10 p, the price of a fruit and nut chocolate bar is 15 p and the price of a dark chocolate bar is 12 p.

Notes:

Whole class activity

(or individual or paired trial first if Ps wish)

Check:

6×10 p = 60 p

4×15 p = 60 p

5×12 p = 60 p

2×10 p + 2×15 p + 2×12 p

= 20 p + 30 p + 24 p

= 74 p < £1 ✔

(i.e. the scale factor can be only 1, otherwise the sum would be more than 50)

8

HMC:
Hungarian Mathematics Competition
2001
Age 11

PbY6b, page 169

Q.7 Read: *Draw a square, ABCD, with 2 cm sides.*

Draw a point P in the plane of the square so that these isosceles triangles are formed.

ABP, BCP, CDP, DAP

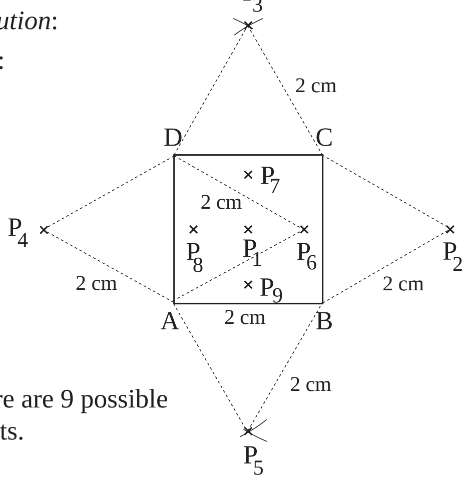
Find more than one solution!

Suggest that Ps label the possible points P_1, P_2, P_3, etc and hint that Ps should use compasses to find them.

When a P has found a point, ask him or her to show it on a digram on BB or SB or OHT and elicit the types of triangles formed.

Solution:

BB:



There are 9 possible points.

Notes:

Individual trial first

Or whole class activity with P_2 if Ps are struggling. Once P_2 has been shown, Ps might be able to find the other points.

P_1 is the centre point, i.e. the intersection of the 2 diagonals

$ABP_1 \cong BCP_1 \cong CDP_1 \cong DAP_1$

(right-angled isosceles triangles)

P_2 is 2 cm from B and 2 cm from C <u>outside</u> the square.

$ABP_2 \cong CDP_2$ (obtuse-angled)

BCP_2 is equilateral

DAP_2 is acute-angled

[Similarly for P_3, P_4 and P_5]

P_6 is 2 cm from A and from D <u>inside</u> the square.

$ABP_6 \cong CDP_6$ (acute-angled)

BCP_6 is obtuse-angled

DAP_6 is equilateral

[Similarly for P_7, P_8 and P_9]

Y6

Lesson Plan *170*

Activity

Factorising 170, 345, 520 and 1170. Miscellaneous challenges

PbY6b, page 170

Solutions:

Q.1 a) *Plan*: £500 × 0.075 = £37.50

Answer: £37.50 interest would be added to the account.

b) *Plan*: £537.50 × 1.075 = £577.8125 ≈ £577.81

Answer: The account would be worth £577.81 at the end of the 2nd year.

c) At end of 3rd year: £577.81 × 1.075 ≈ £621.15

At end of 4th year: £621.15 × 1.075 ≈ £667.74

At end of 5th year: £667.74 × 1.075 ≈ £777.82 ✔

Answer: Harvey will have to wait 5 years.

Q.2 $V = 4 \times 2 \times 5 = 40$ cm cubes

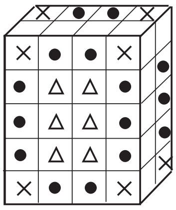
a) 3 *blue* faces: 8 cubes (×)

b) 2 *blue* faces: 20 cubes (●)

c) 1 *blue* face: 12 cubes (Δ)

d) no *blue* faces: none

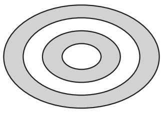
Check: 8 + 20 + 12 = 40 (cm cubes) ✔



Q.3

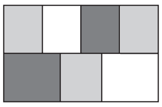
2 colours

a)



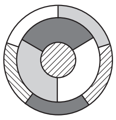
3 colours

b)

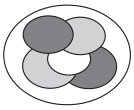


4 colours

c)

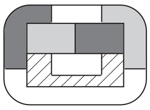


d)



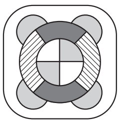
3 colours

e)



4 colours

f)



4 colours

Q.4 $\frac{1}{x} + \frac{1}{y} = \frac{1}{8}$ $\left(= \frac{2}{16} = \frac{3}{24} = \frac{4}{32} = \frac{5}{40} = \frac{6}{48} = \frac{7}{56} \ldots\right)$

$x = y = 16$: $\frac{1}{16} + \frac{1}{16} = \frac{2}{16} = \frac{1}{8}$

$x = 12,\ y = 24$: $\frac{1}{12} + \frac{1}{24} = \frac{2}{24} + \frac{1}{24} = \frac{3}{24} = \frac{1}{8}$

$x = 10,\ y = 40$: $\frac{1}{10} + \frac{1}{40} = \frac{4}{40} + \frac{1}{40} = \frac{5}{40} = \frac{1}{8}$

$x = 9,\ y = 72$: $\frac{1}{9} + \frac{1}{72} = \frac{8}{72} + \frac{1}{72} = \frac{9}{72} = \frac{1}{8}$

and of course the values for x and y can be exchanged (e.g. $x = 24,\ y = 12$, so there are 7 possible solutions.

Notes

$170 = 2 \times 5 \times 17$

Factors: 1, 2, 5, 10, 17, 34, 85, 170

$345 = 3 \times 5 \times 23$

Factors: 1, 3, 5, 15, 23, 69, 115, 345

$520 = 2^3 \times 5 \times 13$

Factors: 1, 2, 4, 5, 8, 10, 13, 20, 26, 40, 52, 65, 104, 130, 260, 520

$1170 = 2 \times 3^2 \times 5 \times 13$

Factors: 1, 2, 3, 5, 6, 9, 10, 13, 15, 18, 26, 30, 39, 45, 65, 78, 90, 117, 130, 195, 234, 390, 585, 1170

(or set factorising as extra task for homework at the end of *Lesson 169* and review at the start of *Lesson 170.*

[**Note to T**

Equations such as this which have integer solutions are called

Diophantine equations

after the Greek philosopher and mathematician

Diophantos of Alexandria.

He lived in the 3rd century A.D. and is credited with being the founder of modern algebra. The use of symbols to represent numbers was found in his published material entitled *Arithmetic*.]

Y6

Lesson Plan 170

Activity

Notes

Solutions (continued):

Q.5 a) i) $5008\ 473c$

$3 \times (5 + 0 + 4 + 3) + 1 \times (0 + 8 + 7 + c)$

$= 3 \times 12 + 1 \times 15 + c$

$= 36 + 15 + c$

$= 51 + c$

$51 + \underline{9} = 60$ (next greater whole 10) $\rightarrow$ $\underline{c = 9}$

ii) $5120\ 173c$

$3 \times (5 + 2 + 1 + 3) + 1 \times (1 + 0 + 7 + c)$

$= 3 \times 11 + 1 \times 8 + c$

$= 33 + 8 + c$

$= 41 + c$

$41 + \underline{9} = 50$ (next greater whole 10) $\rightarrow$ $\underline{c = 9}$

iii) $8300\ 720c$

$3 \times (8 + 0 + 7 + 0) + 1 \times (3 + 0 + 2 + c)$

$= 3 \times 15 + 1 \times 5 + c$

$= 45 + 5 + c$

$= 50 + c$

$50 + \underline{0} = 50$ $\rightarrow$ $\underline{c = 0}$

b) $5070\ 482\mathit{7}$

$3 \times (5 + 7 + 4 + 2) + 1 \times (0 + 0 + 8 + \mathit{7})$

$= 3 \times 18 + 1 \times 8 + \mathit{7}$

$= 54 + 8 + \mathit{7}$

$= 62 + \mathit{7} = 69 \neq 70$

Therefore one of the numbers in the bracket on the RHS could have been read incorrectly as 1 less than it should be.

As 1 does not look like 0, it is more likely that 9 has been read as 8, so the correct number could have been $5070\ 4\underline{9}27$.

However, another possibility is $5070\ 482\underline{8}$, if the check digit had been read incorrectly.

Y6

R: Calculations
C: Problems
E: Puzzles and challenges

Lesson Plan 171

Activity ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- <u>171</u> = 3 × 3 × 19 = $3^2 \times 19$
  Factors: 1, 3, 9, 19, 57, 171
- <u>346</u> = 2 × 173 Factors: 1, 2, 173, 346
- <u>521</u> is a prime number Factors: 1, 521
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, and $23^2 > 521$)
- <u>1171</u> is a prime number Factors: 1, 1171
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23, 29 31, and $37^2 > 1171$)

Individual work, monitored (or whole class activity)

BB: 171, 346 521, 1171

Ps can use a calculator.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 171 & 3 \\ 57 & 3 \\ 19 & 19 \\ 1 & \end{array} \qquad \begin{array}{r|l} 346 & 2 \\ 173 & 173 \\ 1 & \end{array}$$

8 min

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions interactively with the whole class, whether Ps attempted them or not.

2

HMC:
Hungarian Mathematics Competition
1986
Age 11

PbY6b, page 171

Q.1 Read: *We want to assign the numbers 1, 2, 3, 4, 5, 6, 7 and 8 to the vertices of a cube so that the sums of the two numbers on each edge are all different.*

Is this possible? Give a reason for your answer.

Ask Ps to write down anything they find out. If any Ps think that they have a solution, ask them to show it on diagram on BB, then class checks it and points out the equal sums.

Solution:

Points Ps might have noticed or T could ask about:

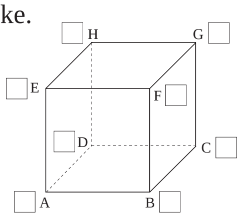
- 28 different pairs can be formed, giving
- 13 different sums from 3 to 15
- A cube has 8 vertices and 12 edges, so it seems possible to assign 8 different numbers to give 12 different sums.
- The sum of all the 12 sums must be equal to:
  3 × (1 + 2 + 3 + 4 + 5 + 6 + 7 + 8) = 3 × 36 = <u>108</u>
  [Each number can be paired with 3 other numbers because 3 edges of the cube meet at each vertex]
  or using the names of the vertices:
  (A + B) + (B + C) + (C + D) + (D + A) + (A + E) + (B + F) + (C + G) + (D + H) + (E + F) + (F + G) + (G + H) + (E + H)
  = 3A + 3B + 3C + 3D + 3E + 3F + 3G + 3H
  = 3 × (A + B + C + D + E + F + G + H)]
- The sum of the 13 different possible sums is
  $3 + 4 + 5 + 6 + 7 + 8 + \boxed{9} + 10 + 11 + 12 + 13 + 14 + 15 = 117$

Individual or paired trials first (e.g. for 5 minutes), then whole class discussion and agreement on solution

Drawn on BB or use enlarged copy master or OHP

Praise any clever points that Ps make.

BB:



28 possible pairs:

(1 + 2), (1 + 3), (1 + 4), (1 + 5)
(1 + 6), (1 + 7), (1 + 8), (2 + 3)
(2 + 4), (2 + 5), (2 + 6), (2 + 7)
(2 + 8), (3 + 4), (3 + 5), (3 + 6)
(3 + 7), (3 + 8), (4 + 5), (4 + 6)
(4 + 7), (4 + 8), (5 + 6), (5 + 7)
(5 + 8), (6 + 7), (6 + 8), (7 + 8)

- As 117 is 9 more than 108, the sum of 9 can be left out, so we do not use these pairs:
  (1 + 8), (2 + 7), (3 + 6), (4 + 5)

Y6

Lesson Plan 171

Activity

2

(Continued)

T then helps Ps to reason the rest of the solution, involving them where possible.

Since the 12 remaining sums must each be used once only, we could use, e.g.

3 = 1 + 2, 4 = 1 + 3, 5 = 1 + 4

[5 = 2 + 3 is impossible, as 2 and 3 are not on the same edge.]

Then we could use 6 = 1 + 5

[6 = 2 + 4 is impossible, as 2 and 4 are not on the same edge.]

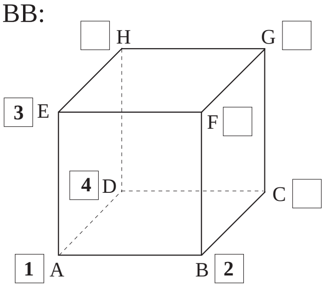
but 1 + 5 is impossible too, as there are only 3 edges meeting at vertex 1, and we have used them all already, so we cannot use 6 as a sum, and the task is impossible.

Answer: e.g.

It is impossible to assign the numbers 1 to 8 to the vertices of a cube so that the sums of the numbers on each edge are all different because at least one of the 12 required sums cannot occur.

Notes

BB:



N.B. 12 edges, so 12 different sums are required but there are only 12 different sums to choose from, so if one of them is shown to be impossible, there is no need to try any of the others!

3

HMC:

Hungarian Mathematics Competition

1987

Age 11

PbY6b, page 171

Q.2 Read: *The numbers 1, 2, 3, . . ., 10 and 11 were each written on a small piece of paper. The pieces of paper were mixed up and put into two boxes. Adam added the numbers in one box and Becky added the numbers in the other box.*

Becky said, "Isn't it interesting? The sum of my numbers is exactly six times the sum of Adam's numbers."

Adam said, "I think there must be a mistake in our calculations."

Is Adam correct? Give a reason for your answer.

Solution:

The total sum of the 11 numbers is: $\frac{1 + 11}{2} \times 11 = 66$

(or $5 \times 12 + 6 = 66$, as : 1 2 3 4 5 ⑥ 7 8 9 10 11

and $1 + 11 = 2 + 10 = 3 + 9 = 4 + 8 = 5 + 7 = 12$)

Let Adam's numbers sum to *a*, and Becky's numbers sum to *b*.

If $a = 6 \times b$, then $6 \times b + b = 66$

$7 \times b = 66$

but 66 is not exactly divisible by 7, so there must be a mistake in their calculations.

Answer: Adam is correct, there is a mistake, because 6 times his sum added to Becky's sum does not result in the sum of all the eleven numbers.

Individual trials first

If no P has solved it in a given time (e.g. 5 minutes) continue as a whole class activity

Praise any Ps who managed a positive step (e.g. calculating the total sum of the numbers or calling the unknown sums by letters)

If necessary, T gives hints or directs Ps' thinking, involving them whenever possible.

Y6

Lesson Plan 171

Activity

Notes

4

HMC:
Hungarian
Mathematics
Competition
1989
Age 11

PbY6b, page 171

Q.3 Read: *Write the numbers from 1 to 12 in the two* ***concentric*** *circles so that:*

- *each inner number is even*
- *the sum of the outer numbers is twice the sum of the inner numbers.*

Solution:

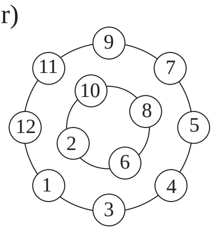
The sum of all 12 numbers is: $\dfrac{1+12}{2} \times 12 = 78$ (or = 13×6)

Ratio of sums: Inner : Outer = 1 : 2

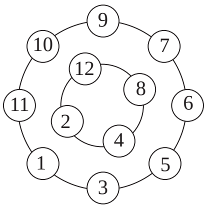
so Inner sum is $\dfrac{1}{3}$ of 78 = <u>26</u> and Outer sum is 26×2 = <u>52</u>

Possible arrangements (but the numbers in each ring can be in any order)

BB:



or



Individual trial, monitored

Drawn on BB or SB or OHT

Elicit that *concentric* circles share a common centre point.

Accept trial and error but give extra praise for logical reasoning

If necessary, T gives hint about calculating the sum of the 12 numbers first.

<u>Inner ring</u>: 2, 4, 8, 12
or 2, 6, 8, 10

<u>Outer ring</u>: remaining numbers

5

HMC:
Hungarian
Mathematics
Competition
1997
Age 11

PbY6b, page 171

Q.4 Read: *The members of a club rented a room for their meeting.*

Ten members attended the meeting and they each paid the same amount towards the hire of the room. If another five members of the club had attended the meeting, everyone would each have paid £10 less.

How much did it cost to hire the meeting room?

Solution: e.g.

Let the amount that each of the 10 members paid be x (in £s) then the total amount paid would be $10 \times x$.

If there were 15 members, they would each pay $x - 10$ and the total amount paid would be $15 \times (x - 10) = 15x - 150$.

As the room hire cost would be the same in both cases, then:

$$10x = 15x - 150 \qquad [+\,150]$$
$$10x + 150 = 15x \qquad [-\,10x]$$
$$150 = 5x \qquad [\div\, 5]$$
$$\underline{30 = x}$$

and $10 \times £30$ = <u>£300</u>

Answer: The cost of the room was £300.

Individual trial

Check:
$15 \times (£30 - £10) = £300$ ✔

Y6

Lesson Plan 171

Activity

6

HMC:
Hungarian Mathematics Competition
1998
Age 11

PbY6b, page 171

Q.5 Read: *Ben picked some apples from his apple tree and put them in a box in his garage.*

That day, Ben made an apple pie with one third of the apples in the box.

The next day he ate one third of the remaining apples and on the following day he gave one third of what was left to his neighbour.

If 8 apples were left in the box, how many apples did Ben pick from the tree?

Solution: e.g.

Let the number of apples Ben picked be n.

Apples left after 1st day: $\frac{2}{3} \times n$

Apples left after 2nd day: $\frac{2}{3} \times (\frac{2}{3} \times n)$

Apples left after 3rd day: $\frac{2}{3} \times (\frac{2}{3} \times \frac{2}{3} \times n) = 8$

$$\frac{8}{27} \times n = 8$$

$$\frac{1}{27} \times n = 1$$

$$\underline{n = 27}$$

or starting on the 3rd day with the 8 apples left:

Apples at beginning of 3rd day: $8 \div 2 \times 3 = 12$

Apples at beginning of 2nd day: $12 \div 2 \times 3 = 18$

Apples picked on 1st day: $18 \div 2 \times 3 = \underline{27}$

Answer: Ben picked 27 apples from the tree.

Notes

Individual trial

(left open as optional homework if Ps do not have time to solve it during the lesson)

Accept any valid method of solution.

If Ben used 1 third of the apples, there would be 2 thirds left.

Check:

1st day: Used: $27 \div 3 = 9$
Left: $27 - 9 = 18$

2nd day: Used: $18 \div 3 = 6$
Left: $18 - 6 = 12$

3rd day: Used: $12 \div 3 = 4$
Left: $12 - 4 = \underline{8}$ ✔

Y6

R: Calculations
C: Problems
E: Puzzles and challenges

Lesson Plan
172

Activity | ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 172 $= 2 \times 2 \times 43 = 2^2 \times 43$ Factors: 1, 2, 4, 43, 86, 172
- 347 is a prime number Factors: 1, 347
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and $19^2 > 347$)
- 522 $= 2 \times 3 \times 3 \times 29 = 2 \times 3^2 \times 29$
  Factors: 1, 2, 3, 6, 9, 18, 29, 58, 87, 174, 261, 522
- 1172 $= 2 \times 2 \times 293 = 2^2 \times 293$
  Factors: 1, 2, 4, 293, 586, 1172

Individual work, monitored (or whole class activity)

BB: 172, 347 522, 1172

T decides whether Ps can use a calculator.

Reasoning, agreement, self-correction, praising

172	2
86	2
43	43
1	

522	2
261	3
87	3
29	29
1	

1172	2
586	2
293	293
1	

8 min

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions interactively with the whole class, whether Ps attempted them or not.

2

HMC:
Hungarian Mathematics Competition
1980
Age 12

PbY6b, page 172

Q.1 Read: *The sides of a square are each divided into 4 equal parts. Some of the points are joined up as shown in the diagram.*

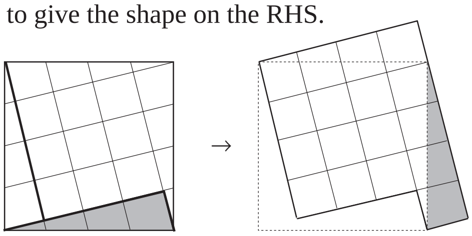
What part of the area of the whole square is the area of the shaded part?

T could suggest that Ps trace or redraw the square, cut it out and rearrange the pieces if no P has thought of doing it.

Solution:

If the square is cut into 3 pieces, as shown, the pieces can be rearrranged to give the shape on the RHS.

BB:



The shape on the RHS is made from 17 unit squares, so this must also be the area of the first square.

In the RHS diagram, it can be seen that the shaded area makes 2 unit squares, so is $\frac{2}{17}$ of the area of the shape.

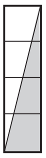
As the shaded area is the same in both diagrams, it must also be $\frac{2}{17}$ of the original square.

Answer: The shaded part is 2 seventeenths of the area of the square.

Individual trial

Drawn (stuck) on BB or use enlarged copy master or OHP

T could have large models prepared so that T or Ps can demonstrate the cutting and rearranging .



Shaded area is half of 4 unit squares.

Y6

Lesson Plan 172

Activity

Notes

3

HMC:
Hungarian
Mathematics
Competition
1984
Age 12

PbY6b, page 172

Q.2 Read: *What is the sum of the shaded angles? Explain how you worked out the solution.*

If Ps are struggling, T could suggest labelling the vertices and points of intersection and hint about the sum of the angles in a triangle.

Solution:

Label the diagram as shown.

The sum of the angles in a triangle is $180°$.

In $\Delta\,ACQ$, $A\hat{Q}C = 180° - (\angle A + \angle C)$,

so $\underline{D\hat{Q}P} = 180° - [180° - (\angle A + \angle C)]$ (as AD is a straight line)

$= \underline{\angle A + \angle C}$

In $\Delta\,BPE$, $B\hat{P}E = 180° - (\angle B + \angle E)$,

so $\underline{Q\hat{P}D} = 180° - [180° - (\angle B + \angle E)]$ (as BD is a straight line)

$= \underline{\angle B + \angle E}$

In $\Delta\,QPD$, $\angle Q + \angle P + \angle D = 180°$

i.e. $(\angle A + \angle C) + (\angle B + \angle E) + \angle D = \underline{180°}$

Or T gives instructions and Ps follow these practical steps.

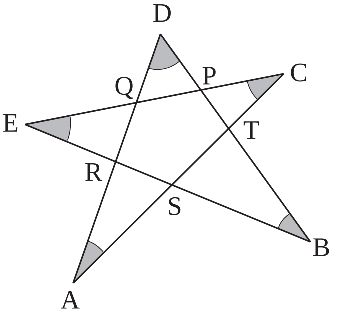
1. Lay a pencil along AD, pointing from A to D.
2. Rotate the pencil through $\angle A$ so that it lies along AC.
3. Rotate the pencil through $\angle C$ so that it lies along EC.
4. Rotate the pencil through $\angle E$ so that it lies along EB.
5. Rotate the pencil through $\angle B$ so that it lies along DB.
6. Rotate the pencil through $\angle D$ so that it lies along DA.

The pencil is now back in its original position but facing in the opposite direction, so it has turned through an angle of $180°$.

i.e. $\angle A + \angle B + \angle C + \angle D + \angle E = 180°$

Individual or paired trial

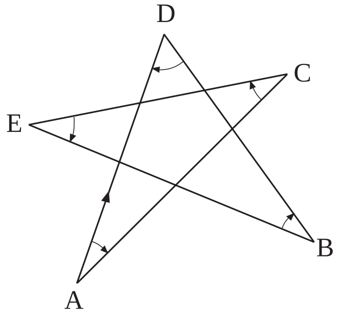
Drawn on BB or use enlarged copy master or OHP



Remind Ps of the different notation for identifying certain angles.

If no P can solve it, T directs Ps' thinking, involving Ps when they understand.

Less able Ps might find this practical method easier.



4

HMC:
Hungarian
Mathematics
Competition
1985
Age 12

PbY6b, page 172

Q.3 Read: *We say that two circles touch each other if they have exactly* ***one*** *common point.*

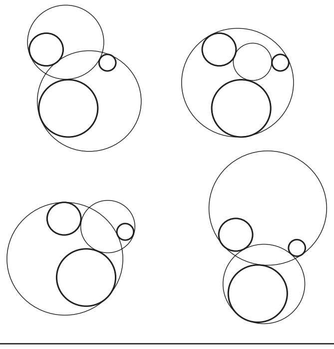
How many circles which touch each of the 3 circles in the diagram can you imagine in the plane?

Ps could draw the circles roughly and lightly in pencil.

Solution:

There are 8 such circles.

[It is clearer to show the circles on different diagrams.]



Individual trial

Drawn on BB or use enlarged copy master or OHP

Ps could show how many circles they drew on scrap paper or slates on command. Ps with most circles show them on diagram on BB.

Who found this one too? Who found another circle that we have not shown yet? etc.

What is the name of a straight line which touches a circle at just one point? (tangent)

Y6

Lesson Plan 172

Activity

Notes

5

HMC:
Hungarian Mathematics Competition
1986
Age 12

PbY6b, page 172

Q.4 Read: *We marked 7 points on a plane and joined them up so that any two different points are on a straight line.*

When we had finished, we had drawn 14 different straight lines.

Show how the 7 points could have been drawn.

Ps try it out in *Ex. Bks* or on scrap paper first, then T helps them to explain their reasoning for the solution.

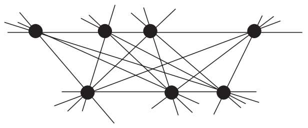
Solution:

If no 3 points (i.e. only 2 points) from the 7 points were on the same straight line, the number of lines needed would be

$$\frac{7 \times 6}{2} = \underline{21}\,,$$

but we drew only 14 different lines, so some of the lines must have 3 or more of the 7 points on them.

BB: e.g.



$$4 \times 3 + 1 + 1 = \underline{14}$$

Individual or paired trial

If nobody can solve it, T gives hints or directs Ps' thinking to the solution, involving Ps where possible.

(e.g. If only 2 dots were on the same line, how many lines would be drawn?

If we have drawn fewer lines, what does that mean? etc.)

[Each of the 4 dots on one line is joined to 3 dots on another line, i.e. (4 × 3) lines + the line joining the 4 dots + the line joining the 3 dots]

6

HMC:
Hungarian Mathematics Competition
1990
Age 12

PbY6b, page 172

Q.5 Read: *David asked his friend to guess how much money he had. He gave him this clue.*

My money could be made up in 20 different ways using just £5 notes and £2 coins but it could not be made up with only £2 coins.

How much money did David have?

Solution: e.g.

As it could not be made up with only £2 coins, it must be an odd amount. As the only odd amount we have is £5, it must be a multiple of 5. It cannot be just £5, otherwise the £2 would not be mentioned. Try the next greater odd multiples in turn. e.g.

$\pounds 15 = \pounds 5 \times 1 + \pounds 2 \times 5\,(\times\,\underline{1}) = \pounds 5 \times \underline{3}$ [2 ways]

$\pounds 25 = \pounds 5 \times 1 + \pounds 2 \times 5 \times \underline{2} = \pounds 5 \times 3 + \pounds 2 \times 5 = \pounds 5 \times 5$ (3 ways)

$\pounds 35 = \pounds 5 \times 1 + \pounds 2 \times 5 \times \underline{3} = \pounds 5 \times 3 + \pounds 2 \times 5 \times 2$
$= \pounds 5 \times 5 + \pounds 2 \times 5 = \pounds 5 \times 7$ (4 ways)

$\pounds 45 = \pounds 5 \times 1 + \pounds 2 \times 5 \times \underline{4} = \pounds 5 \times 3 + \pounds 2 \times 5 \times 3$
$= \pounds 5 \times 5 + \pounds 2 \times 5 \times 2 = \pounds 5 \times 7 + \pounds 2 \times 5$
$= \pounds 5 \times 9$ (5 ways)

. . .

Ps might now see a pattern emerging (the underlined numbers are 1 less than the number of ways).

So for 20 ways, the amount is: $\pounds 5 \times 1 + \pounds 2 \times 5 \times \underline{19}$
$= \pounds 5 + \pounds 190 = \underline{\pounds 195}$

Answer: David had £195.

Individual or paired trials first

If Ps are struggling, change to a whole class activity, with T giving hints or directing Ps' thinking and involving Ps where possible.

Check: (Ps dictate)

$\underline{\pounds 195}$
$= \pounds 5 \times 1 + \pounds 2 \times 5 \times \underline{19}$
$= \pounds 5 \times 3 + \pounds 2 \times 5 \times 18$
$= \pounds 5 \times 5 + \pounds 2 \times 5 \times 17$
$= \pounds 5 \times 7 + \pounds 2 \times 5 \times 16$
$= \pounds 5 \times 9 + \pounds 2 \times 5 \times 15$
$= \pounds 5 \times 11 + \pounds 2 \times 5 \times 14$
$= \pounds 5 \times 13 + \pounds 2 \times 5 \times 13$
$= \pounds 5 \times 15 + \pounds 2 \times 5 \times 12$
$= \pounds 5 \times 17 + \pounds 2 \times 5 \times 11$
$= \pounds 5 \times 19 + \pounds 2 \times 5 \times 10$
$= \pounds 5 \times 21 + \pounds 2 \times 5 \times 9$
$= \pounds 5 \times 23 + \pounds 2 \times 5 \times 8$
$= \pounds 5 \times 25 + \pounds 2 \times 5 \times 7$
$= \pounds 5 \times 27 + \pounds 2 \times 5 \times 6$
$= \pounds 5 \times 29 + \pounds 2 \times 5 \times 5$
$= \pounds 5 \times 31 + \pounds 2 \times 5 \times 4$
$= \pounds 5 \times 33 + \pounds 2 \times 5 \times 3$
$= \pounds 5 \times 35 + \pounds 2 \times 5 \times 2$
$= \pounds 5 \times 37 + \pounds 2 \times 5$
$= \pounds 5 \times 39$ (20 ways)

Y6

Lesson Plan 172

Activity		***Notes***
7 *HMC*: Hungarian Mathematics Competition 1995 Age 12	***PbY6b, page 172, Q.6*** Read: *What is the smallest, positive, whole number which gives:* • *a remainder of 1 when it is divided by 3* • *a remainder of 2 when it is divided by 4* • *a remainder of 3 when it is divided by 5* • *a remainder of 4 when it is divided by 6?* *Solution*: e.g. In each case, the remainder is 2 less than the divisor, so the dividend must be 2 less than the lowest common multiple of 3, 4, 5 and 6. As $4 = 2 \times 2$ and $6 = 2 \times 3$, the lowest common multiple of 3, 4, 5 and 6 is: $2 \times 2 \times 3 \times 5 = 60$. *Answer*: The smallest positive whole number which fulfils the given conditions is $60 – 2 = \underline{58}$.	Whole class activity (or individual trial if Ps wish) Allow Ps time to think and to discuss the method of solution. If no P has a good idea, T gives hints or leads Ps through the reasoning and asks Ps to check it. *Check*: $58 \div 3 = 19$, r $\underline{1}$ ✔ $58 \div 4 = 14$, r 2 ✔ $58 \div 5 = 11$, r 3 ✔ $58 \div 6 = 9$, r 4 ✔ [or set this question as an optional homework challenge]

Y6

R: Calculations
C: Problems
E: Puzzles and challenges

Lesson Plan 173

Activity — ***Notes***

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 173 is a prime number Factors: 1, 173
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, and $17^2 > 173$)
- 348 = $2 \times 2 \times 3 \times 29 = 2^2 \times 3 \times 29$
  Factors: 1, 2, 3, 4, 6, 12, 29, 58, 87, 116, 174, 348
- 523 is a prime number Factors: 1, 523
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19 and $23^2 > 523$)
- 1173 = $3 \times 17 \times 23$
  Factors: 1, 3, 17, 23, 51, 69, 391, 1173

Individual work, monitored (or whole class activity)

BB: 173, 348 523, 1173

T decides whether Ps can use a calculator.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 348 & 2 \\ 174 & 2 \\ 87 & 3 \\ 29 & 29 \\ 1 & \end{array} \qquad \begin{array}{r|l} 1173 & 3 \\ 391 & 17 \\ 23 & 23 \\ 1 & \end{array}$$

8 min

2

PbY6b, page 173

Q.1 Read: *A father gave £400 to his son. Another father gave £200 to* ***his*** *son. The two sons count their money and notice that they have £400 altogether.*

How is that possible?

I will give you 3 minutes to think about it and to write an explanation in your *Ex. Bks*. Start . . . now! . . . Stop!

Stand up if you think it **is** possible. T chooses Ps sitting and standing to explain their reasoning. Class decides who is correct.

Solution: e.g. BB: $\text{Grandad} \xrightarrow{£400} \text{Dad} \xrightarrow{£200} \text{Son}$

Grandad gave £400 to Dad who gave £200 from the £400 to his son, so the Dad and the Son had £200 each, or £400 altogether.

Individual work, monitored

T notes Ps who have explained their reasoning clearly.

In unison

Reasoning, agreement, self-correction, praising

12 min

3

PbY6b, page 173

Q.2 Read: *A joiner worked on his own to mend the 4 legs of a large, heavy table. The table was lying top down on the floor.*

When he had mended the table, the joiner was not strong enough to lift it onto its legs.

He thought of a way of checking whether the table would be stable when it was the right way up by using two pieces of string.

a) How could he have done it?

b) If the table had 3 legs, would it need to be checked in the same say?

Solution: e.g.

a) Pin each piece of string to the tops of two opposite legs. If the 2 strings touch each other, the tops of the legs are exactly the same length, so the table is stable. (See diagram)

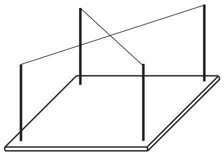
b) A table with 3 legs is always stable (but of course its top will not necessarily be horizontal). [T demonstrates if possible.]

Individual or paired trial for 2 or 3 minutes.

Elicit that 'stable' means the table does not move or rock.

Ps who have an answer explain reasoning to class. T helps them to explain. e.g.

BB:



If the 2 joining lines touch, the 4 points are in the same plane.

Any 3 different points which are not on the same line can always be on the same plane.

16 min

Activity ***Notes***

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions interactively with the whole class, whether Ps attempted them or not.

4

PbY6b, page 173, Q.3

Read: *We divided two numbers, 313 and 390, by the same 2-digit number. In each case the remainder was the same.*

Which number could we hawe divided by?

Solution: e.g.

Let the divisor be d and the remainder be m.

Then both $(313 - m)$ and $(390 - m)$ are exactly divisible by d.

If $(313 - m)$ and $(390 - m)$ are exactly divisible by d, their difference is also exactly divisible by d.

BB: $$(390 - m) - (313 - m) = 390 - m - 313 + m = 390 - 313 = \underline{77}$$

If 77 is exactly divisible by d, then d must be a factor of 77, i.e. 1, 7, 11 or 77, but the divisor is a 2-digit number, so only 11 and 77 are possible. Let's check both of them.

If $d = 11$: $313 \div 11 = 28$, r $\underline{5}$, $390 \div 11 = 35$, r $\underline{5}$ ✔

If $d = 77$: $313 \div 77 = 4$, r $\underline{5}$, $390 \div 77 = 5$, r $\underline{5}$ ✔

Answer: The number that we divided by could have been 11 or 77.

Whole class activity
(or individual trial if Ps wish)

If Ps have no good ideas, T gives hints or directs Ps' thinking, involving Ps where possible or allowing Ps to take over when they realise where it is leading.

Both numbers are possible.

5

PbY6b, page 173

Q.4 Read: *Once a time, a king asked a farmer to work for him for a year and promised to pay him 12 gold coins and a horse.*

The farmer did not like the work he had to do in the palace and longed to be back in his farm. After 7 months he decided to leave his job and asked the king for his wages.

The king gave the farmer a horse and 2 gold coins, which the farmer agreed was fair.

How many gold coins was the horse worth?

Solution: e.g.

12 months → 12 gold coins + 1 horse
7 months → 2 gold coins + 1 horse

5 months → 10 gold coins
1 month → 2 gold coins

So after 12 months he should have received 24 gold coins but he was promised only 12 gold coins + 1 horse, so the horse must have been worth $\underline{12}$ gold coins.

Individual or paired trial

Check:
7 months: 2 coins + 1 horse
= 2 coins + 12 coins
= $\underline{14}$ coins

2 coins × 7 = $\underline{14}$ coins ✔

Y6

Lesson Plan 173

Activity

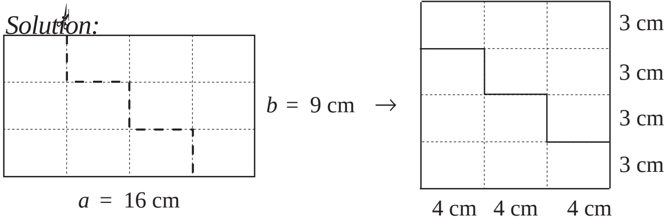
Notes

6

PbY6b, page 173

Q.5 Read: *How can this rectangle be cut into two pieces so that the two pieces will form a square?*

Solution:



The square has sides 12 cm long and its area is 144 cm^2.

Individual trial

[If Ps have difficulty in visualising the solution in their heads, allow them to draw copies of the rectangle and cut them up.]

T could have large model prepared to check the solution with the class.

Check: Area of rectangle

= (9 × 16) cm^2

= $\underline{144 \text{ cm}^2}$ ✔

7

PbY6b, page 173, Q.6

Read: *The sides of an equilateral triangle were divided into 3 equal parts. Some points were joined up to form another equilateral triangle, as shown in the diagram.*

What **part** *of the area of the original triangle is the area of the smaller equilateral triangle?*

Solution:

Let the sides of the larger equilateral triangle be 1 unit.

Then the shaded right-angled triangle has height *h*, base $\frac{1}{3}$ unit and hypotenuse $\frac{2}{3}$ of a unit, but so have the other 2 right-angled triangles, so all 3 right-angled triangles are <u>congruent</u>.

The area of each right-angled triangle is

$$\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$$

of the area of the large equilateral triangle

(as its base is $\frac{1}{3}$ of the base of the large triangle and its height, *h*, is $\frac{2}{3}$ of the height of the large triangle – see diagram)

So the area of the small equilateral triangle is:

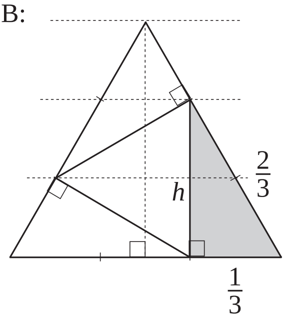
$$1 - \frac{2}{\cancel{9}_3} \times \cancel{3}^1 = 1 - \frac{2}{3} = \underline{\frac{1}{3}}$$

Answer: The area of the smaller equilateral triangle is one third of the area of the original triangle.

Whole class activity

Drawn on BB or use enlarged copy master or OHP

BB:



Ps say what they notice about the diagram and compare the sides and heights of the triangles. (T could draw horizontal lines joining the marked points to help Ps to compare the perpendicular heights.)

If no P can explain the solution, T leads Ps through the reasoning opposite, involving Ps where possible.

Y6

R: Calculations

C: Problems

E: Puzzles and challenges

Lesson Plan
174

Activity

Notes

1

Factorisation

Factorise these numbers in your exercise book and list their positive factors. T sets a time limit of 6 minutes.

Review with whole class. Ps come to BB or dictate to T, explaining reasoning. Class agrees/disagrees. Mistakes discussed and corrected.

Elicit that:

- 174 = 2 × 3 × 29 Factors: 1, 2, 3, 6, 29, 58, 87, 174
- 349 is a prime number Factors: 1, 349
  (as not exactly divisible by 2, 3, 5, 7, 11, 13, 17, and $19^2 > 349$)
- 524 = 2 × 2 × 131 = 2^2 × 131 Factors: 1, 2, 4, 131, 262, 524
- 1174 = 2 × 587 Factors: 1, 2, 587, 1174
  (587 is not exactly divisible by 2, 3, 5, 7, 11, 13, 17, 19, 23 and $29^2 > 587$)

Individual work, monitored (or whole class activity)

BB: 174, 349 524, 1174

T decides whether Ps can use a calculator.

Reasoning, agreement, self-correction, praising

$$\begin{array}{r|l} 174 & 2 \\ 87 & 3 \\ 29 & 29 \\ 1 & \end{array} \qquad \begin{array}{r|l} 524 & 2 \\ 262 & 2 \\ 131 & 131 \\ 1 & \end{array}$$

$$\begin{array}{r|l} 1174 & 2 \\ 587 & 587 \\ 1 & \end{array}$$

8 min

N.B. We have not prescribed timings for the following questions as it is important that Ps have time to think and try out their ideas. The questions can be done as individual challenges or as whole class activities or as a team or individual competition (with a prize). Any questions not done in class could be set as voluntary homework.

Review the questions interactively with the whole class, whether Ps attempted them or not.

2

PbY6b, page 174

Q.1 Read: *A small group of soldiers need to cross a river but the bridge has been destroyed.*

The river is very deep and its current is so swift that it is too dangerous for the soldiers to swim across.

Two children are playing in a boat on the river bank. This boat is so small that only the two children or a single soldier can fit inside it.

Is it possible for the group of soldiers to cross the river using the boat?

Give a reason for your answer.

Solution:

Yes, it is possible if these steps are used.

The 2 children row across the river. One child stays on the opposite bank and the other child rows the boat back.

A solder rows to the opposite bank and the child who was left there brings the boat back.

The 2 children row across the river. One child stays on the . . .

Continue in this way until all the soldiers are across the river.

Individual or paired trial

Allow time for Ps to think and discuss with their neighbours.

Ps who think it is possible stand up n command. T asks several Ps to explain their reasoning to class.

[If there is time, demonstrate the solution with a group of Ps moving from one side of the classroom to the other.]

Activity | *Notes*

3

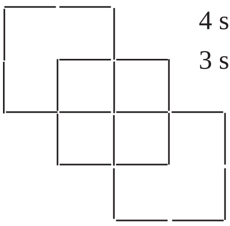
PbY6b, page 174

Q.2 Read: *a) Using 24 matchsticks of equal length, form 4 squares with 1 unit sides and 3 squares with 2 unit sides.*

b) Form 6 equilateral triangles from 12 matchsticks of equal length.

Solution:

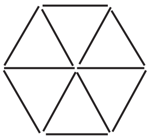
a)



4 squares: 1 × 1

3 squares: 2 × 2

b)



regular hexagon

Individual trials

Ps have used matchsticks or cocktail sticks or Cuisenaire rods on desks.

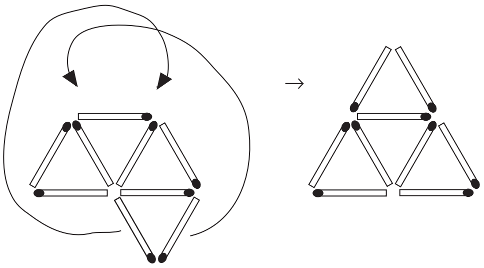
If possible, T has large models to stick on BB (or Ps could lay matchsticks on an OHP) to demonstrate the solutions to the class.

4

PbY6b, page 174

Q.3 Read: *Change the position of only 2 matchsticks so that there are 5 triangles.*

Solution:



Individual trials

Ps make the shape on desks then try out changes.

T has large models to stick on BB, or uses an OHP.

5

PbY6b, page 174

Q.4 Read: *Complete the diagram so that the sum of every two adjacent numbers is the number directly above them.*

Solution:

				848				
			358		490			
		222		136		**354**		
	154		68		68		286	
134		20		**48**		20		**266**

Reasoning: e.g.

In bottom row, let the number between 134 and 48 be x.

$$\begin{aligned} \text{Then}\quad 134 + x + 48 + x &= 222 \\ 182 + 2x &= 222 \qquad [-\,182] \\ 2x &= 40 \qquad [\div\ 2] \\ \underline{x} &\underline{= 20} \end{aligned}$$

In bottom row, let the number between 48 and 266 be y.

$$\begin{aligned} \text{Then}\quad 48 + y + 266 + y &= 354 \\ 314 + 2y &= 354 \qquad [-\,314] \\ 2y &= 40 \qquad [\div\ 2] \\ \underline{y} &\underline{= 20} \end{aligned}$$

Individual trial

Drawn on BB or use enlarged copy master or OHP

Bold numbers are given.

T helps Ps to explain their reasoning in a mathematical way.

Check:

222 + 136 + 354 + 136

= 358 + 490

= 848 ✔

Y6

Lesson Plan 174

Activity

Notes

6

PbY6b, page 174

Q.5 Read: *A farm goose saw a flock of wild geese land on his pond. The farm goose said, "There must be a hundred geese in your flock!"*

One of the wild geese overheard him and said, "There aren't one hundred of us but if there were twice as many of us, then another half of us, then another quarter of us and if you joined our flock, **then** *there would be a hundred geese in our flock."*

How many wild geese landed on the pond?

Solution:

Let the number of wild geese be *g*.

Then

$$\begin{aligned} 2 \times g + \frac{g}{2} + \frac{g}{4} + 1 &= 100 & [\times 4] \\ 8 \times g + 2 \times g + g + 4 &= 400 & [-4] \\ 11 \times g &= 396 & [\div 11] \\ \underline{g} &\underline{= 36} \end{aligned}$$

Answer: Thirty-six geese landed on the pond.

Individual trial

Ps show results on scrap paper or slates on command.

Ps with different answers explain reasoning. Class points out errors and decides who is correct.

[If Ps are struggling, stop individual work and continue as a whole class activity, with Ps coming to BB.]

Check:

$2 \times 36 + 18 + 9 + 1$

$= 72 + 28 = 100$ ✔

7

PbY6b, page 174

Q.6 Read: *We have 30 silver coins. Although they all look the same, we know that one of the coins is fake and is lighter than the others.*

If we tried to find out which coin is fake using a 2-pan balance, what is the **least** *number of weighings we would need to do?*

Solution:

1) Divide the 30 coins into 3 groups of 10.

Weigh Group 1 against Group 2. If they balance, the fake coin must be in Group 3. If they do not balance, the fake coin is in the lighter group.

2) Divide the 10 coins in the lightest group into 3 groups (3, 4, 4).

Weigh the two groups of 4. If they balance, the fake coin must be in the group of 3. If they do not balance, the fake coin must be in the lighter group of 4.

3) If the fake coin is in the group of 3, weigh one coin against another coin. If they balance, the 3rd coin is fake. If they do not balance, the lighter coin is the fake.

or If the fake coin is in a group of 4, weigh 2 coins agains the other 2 coins. The fake coin is in the lighter pair.

4) Then weigh each coin in the lighter pair against the other. The fake coin is the lighter of the two.

Answer: To find the fake coin we would need to do at least 3 weighings and at most 4 weighings.

Individual or paired trial

(or whole class activity, with Ps suggesting what to do and demonstrating the weighings with coins or marbles prepared by T)

Extra praise if Ps think of these strategies without help from T.

Y6

Lesson Plan 174

Activity

8

PbY6b, page 174

Q.6 Read: *The product of the length of a ship in metres, the age of its captain and the number of children he has is 32 118. Each of the three numbers is a whole number.*

How old is the captain of the ship?

Solution:

First factorise 32 118, then write the possible values for length, age and number of children in a table. e.g.

BB: $L \times A \times N = 32\,118$ (= $2 \times 3 \times 53 \times 101$)

32118	2
16059	3
5353	53
101	101
1	

		Most likely		
Length (m)	53	101	202	
Age of the captain	101	53	53	
Number of children	6	6	3	
	Unlikely!		*Unlikely!*	

The lengh of the ship is unlikely to be 1 m or 2 m or 3 m (and the age of the captain cannot be 1, 2, or 3 years) so write the other possible lengths and ages in a table.

The captain cannot be 101 years old (the Navy would have retired him by then) and the length of the ship is unlikely to be 202 m (although it is not impossible!), so the circled column is the most likely answer.

Answer: The captain of the ship is 53 years old.

Notes

Individual trial first

(If Ps are struggling, T hints about factorising and writing possible values in a table.)

T leads the discussion about which values are possible and which are not.

In good humour!

(T could have ready a local example of a distance of 202 m, or Ps could search on the internet for the range of lengths of 'normal' ships.)

[Whether the ship is 101 m or 202 m, the age of the captain is the same.]

Y6

Lesson Plan
175

Activity

Factorising 175, 350, 525 and 1175. Miscellaneous challenges

PbY6b, page 175

Solutions:

Q.1 a) $50 < n < 60$

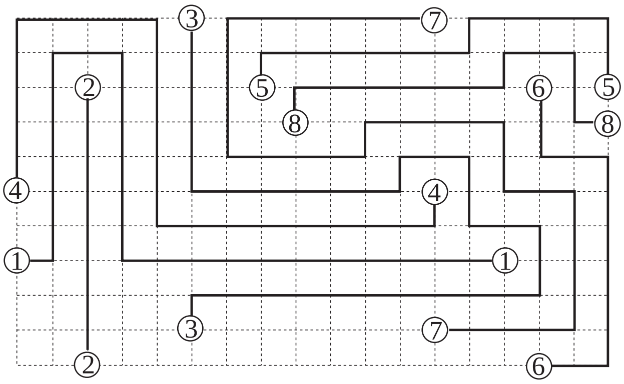
n: <u>51</u> (treble17), <u>54</u> (treble 18), <u>57</u> (treble 19)

b) i) highest possible score is <u>180</u> (3 × treble 20)

ii) lowest possible score is <u>3</u> (3 × 1)

c) 163

Q.2



Q.3 a) i) 2 rectangles (1×6, 3×2)

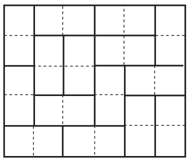
ii) 3 rectangles (1×12, 2×6, 3×4)

iii) 2 rectangles (1×22, 2×11)

iv) the number of pairs of factors of $2 \times n$ (as the area of each domino is 2 unit squares)

b) 5×6 (unit squares) e.g.

This has no fault line.



Q.4 a) 2 dots → 4 different codes (2×2)

b) 3 dots → 8 different codes ($2 \times 2 \times 2$)

c) 4 dots → 16 different codes ($2 \times 2 \times 2$)

d) 6 dots → 64 different codes ($2 \times 2 \times 2 \times 2 \times 2 \times 2$)

Point out that in real life when using the 6-dot code of Braille, the arrangement of 6 flat dots cannot be felt, so is not used. There are only 63 different codes in Braille.

Notes

<u>175</u> = $5^2 \times 7$

Factors: 1, 5, 7, 25, 35, 175

<u>350</u> = $2 \times 5^2 \times 7$

Factors: 1, 2, 5, 7, 10, 14, 25, 35, 50, 70, 175, 350

<u>525</u> = $3 \times 5^2 \times 7$

Factors: 1, 3, 5, 7, 15, 21, 25, 35, 75, 105, 175, 525

<u>1175</u> = $5^2 \times 47$

Factors: 1, 5, 25, 47, 235, 1175

(or set factorising as extra task for homework at the end of *Lesson 174* and review at the start of *Lesson 175)*

If possible, T has a real dartboard and the game could be explained (and played)by 'expert' pupils.

Ideally, Ps have sets of dominoes on desks.

T could supply, or Ps could find out, the Braille alphabet and write messages in code.

If possible, T could have a page of Braille for Ps to see and feel.