

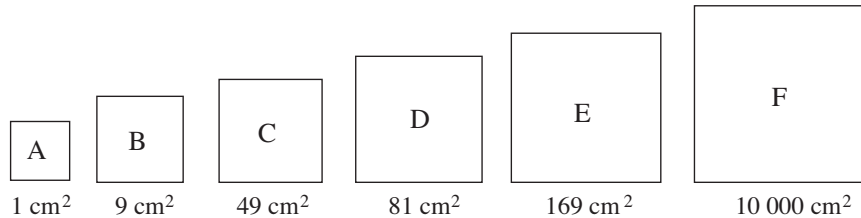
Mathematics Enhancement Programme

TEACHING SUPPORT: Year 6

SOLUTIONS TO EXERCISES

1. Question and Solution

What length is the **perimeter** of each square if its area is:



A: $P =$ 4 cm,

B: $P =$ 12 cm

C: $P =$ 28 cm

D: $P =$ 36 cm

E: $P =$ 52 cm

F: $P =$ 400 cm

Which squares are similar? All of them

(p10, Q4)

Notes

As each shape is a square, we have first to find the side length, say x , such that

$$x \times x = \text{area}$$

For example, for Shape C,

$$\begin{aligned}
 x \times x = 49 \text{ cm}^2 &\Rightarrow x = 7 \text{ cm} \Rightarrow \text{perimeter} = 7 + 7 + 7 + 7 \\
 &= 4 \times 7 \\
 &= 28 \text{ cm}
 \end{aligned}$$

The formula that you are using can be summarised as

$$A = x \times x \quad (\text{we write this as } x^2)$$

$$P = 4 \times x \quad (\text{we write this as } 4x)$$

All the squares are similar (identical in shape but not in size).

2. Question and Solution

Write a plan, estimate, calculate and check your answer in your exercise book.

Write the answer in a sentence here. Underline any data not needed in the calculation.

- a) Christopher bought a painting for £2600. Then he sold it 3 weeks later for £2800. After another 2 weeks, he changed his mind and bought the painting back for £3100. After 1 week, he sold the painting again for £3200.

Did he make a profit or a loss on the painting and how much was it?

Answer: Profit of £300

- b) A box 15 cm deep holds 13 kg of tomatoes and a box 20 cm deep holds 17 kg of tomatoes. What is the total price of all the tomatoes in the 2 boxes if 1 kg of tomatoes costs £2.25.

Answer: £1.75

(p14, Q2)

Notes

The important aspect here is to identify the data needed and that *not* needed. The depths of the boxes is irrelevant. Only the weight and cost of are needed to make the calculation.

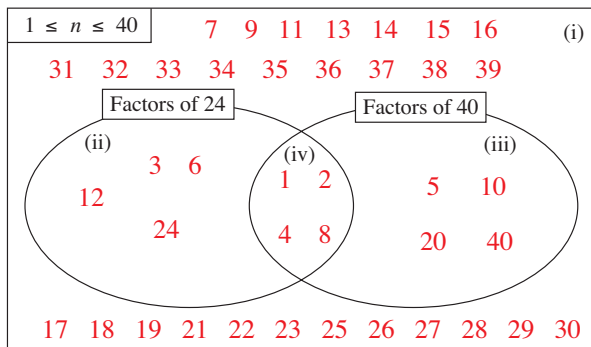
$$\left. \begin{array}{l} \text{a) } 2800 - 2600 = 200 \\ 3200 - 3100 = 100 \end{array} \right\} \text{total profit of } \pounds 300$$

b) Total weight = (13 + 17) kg = 30 kg

$$\begin{aligned} \text{Cost} &= 30 \times \pounds 2.25 \\ &= 3 \times \pounds 22.50 \\ &= \pounds 67.50 \end{aligned}$$

3. Question and Solution

a) Write the natural numbers from 1 to 40 in the Venn diagram.



b) What is the **greatest common factor** of 24 and 40 **8**

c) What is the **lowest common multiple** of 24 and 40? **120**

d) What kind of numbers are in the parts of the diagram labelled

Write a sentence to describe each set.

- i) **Not factors of 24 or of 40**
- ii) **Factors of 24 but not factors of 40**
- iii) **Factors of 40 but not factors of 24**
- iv) **Numbers which are factors of both 24 and 40**

(p20, Q4)

Notes

c) Multiples of 24 are

$$24, 48, 72, 96, \textcircled{120}, 144, 168, \dots,$$

and of 40 are

$$40, 80, \textcircled{120}, 160, 200, \dots,$$

Hence you can see that 120 is the lowest common multiple.

4. Question and Solution

Solve these equations and inequalities. Check your results.

a) $0.332 + a = 10$

$a = 9.668$

b) $5 \times b - 4.07 = 5$

$b = 1.414$

c) $c - 92.7 = 3.8$

$c = 96.5$

d) $d \div 100 = 0.054$

$d = 5.4$

e) $8 \times (e \div 10) = 2.5$

$e = 3.125$

f) $(76.4 - f) + 5 = 80$

$f = -323.6$

g) $0.1 \times 100 < g \leq 1.5 \times 10$

$10 < g \leq 15$

h) $h \div 10 < 2.2 - h$

$h < 2$

(p25, Q5)

Notes

a) $0.332 + a = 10$

Take 0.332 from both sides,

$$0.332 + a - 0.332 = 10 - 0.332$$

i.e.

$a = 9.668$

b) $5 \times b - 4.07 = 5$

Add 4.07 to both sides,

$$5 \times b - 4.07 + 4.07 = 5 + 4.07$$

$$5 \times b = 9.07$$

Divide both sides by 5 to give,

$$b = 9.07 \div 5$$

$$b = 1.414$$

c) $c - 92.7 = 3.8$

Add 92.7 to both sides to give,

$$92.7 + c - 92.7 = 92.7 + 3.8$$

$$c = 96.5$$

d) $d \div 100 = 0.054$

Multiply both sides by 100 to give,

$$100 \times (d \div 100) = 100 \times 0.054$$

$$\frac{100 \times d}{100} = 100 \times 0.054$$

$$d = 5.4$$

e) $8 \times (e \div 10) = 2.5$

Divide both sides by 8 to give,

$$8 \times (e \div 10) \div 8 = 2.5 \div 8$$

$$(e \div 10) = 0.3125$$

Multiply both sides by 10 to give,

$$10 \times (e \div 10) = 10 \times 0.3125$$

$$e = 3.125$$

$$f) \quad (76.4 - f) \div 5 = 80$$

Multiply both sides by 5,

$$5 \times (76.4 - f) \div 5 = 5 \times 80$$

$$76.4 - f = 400$$

Add f to both sides,

$$f + 76.4 - f = f + 400$$

$$76.4 = f + 400$$

Take 400 from both sides,

$$76.4 - 400 = f + 400 - 400$$

$$-323.6 = f$$

$$\text{i.e.} \quad f = -323.6$$

$$g) \quad 0.1 \times 100 < g \leq 1.5 \times 10$$

$$10 < g \leq 15$$

$$h) \quad h \div 10 < 2.2 - h$$

Multiply both sides by 10,

$$h < 10 \times 2.2 - 10 \times h$$

$$h < 22 - 10h$$

Add $10h$ to both sides,

$$10h + h < 22 - 10h + 10h$$

$$11h < 22$$

Divide both sides by 11,

$$h < \frac{22}{11} = 2$$

$$\text{i.e.} \quad h < 2$$

5. Question and Solution

- a) Convert these fractions to 24ths and write them in increasing order in your exercise book.

$$\frac{1}{2}, \frac{3}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{4}, \frac{4}{3}, \frac{7}{24}, \frac{16}{8}, \frac{5}{6}, 1\frac{5}{12}$$

$$\Rightarrow \frac{7}{24}, \frac{12}{24}, \frac{16}{24}, \frac{18}{24}, \frac{20}{24}, \frac{30}{24}, \frac{32}{24}, \frac{34}{24}, \frac{36}{24}, \frac{48}{24}$$

- b) Convert each fraction to an equivalent fraction with numerator 12. Write them in increasing order in your exercise book.

$$\frac{3}{4}, \frac{2}{11}, \frac{6}{5}, \frac{1}{3}, \frac{6}{7}, \frac{5}{10}, \frac{9}{6}, \frac{4}{5}, \frac{4}{3}, \frac{3}{2}$$

$$\Rightarrow \frac{12}{66}, \frac{12}{36}, \frac{12}{25}, \frac{12}{16}, \frac{12}{15}, \frac{12}{14}, \frac{12}{10}, \frac{12}{9}, \frac{12}{8}, \frac{12}{8}$$

(p31, Q5)

Notes

- a) Rewriting as fractions converted to 24ths gives

$$\frac{12}{24}, \frac{36}{24}, \frac{16}{24}, \frac{18}{24}, \frac{30}{24}, \frac{32}{24}, \frac{7}{24}, \frac{48}{24}, \frac{20}{24}, \frac{34}{24}$$

and in increasing order,

$$\frac{7}{24}, \frac{12}{24}, \frac{16}{24}, \frac{18}{24}, \frac{20}{24}, \frac{30}{24}, \frac{32}{24}, \frac{34}{24}, \frac{36}{24}, \frac{48}{24}$$

$$\text{or } \frac{7}{24}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \frac{5}{4}, \frac{4}{3}, 1\frac{5}{12}, \frac{3}{2}, \frac{16}{8}$$

- b) Rewriting as equivalent fractions with numerator 12 gives,

$$\frac{12}{16}, \frac{12}{66}, \frac{12}{10}, \frac{12}{36}, \frac{12}{14}, \frac{12}{24}, \frac{12}{8}, \frac{12}{15}, \frac{12}{9}, \frac{12}{8}$$

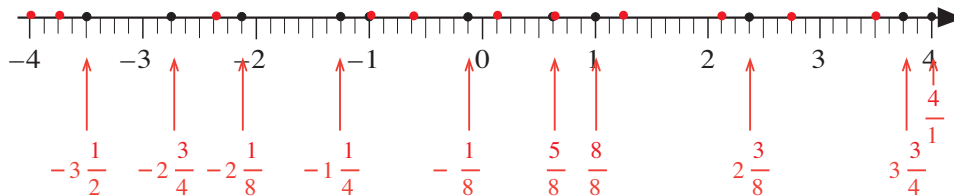
so, in increasing order,

$$\frac{12}{66}, \frac{12}{36}, \frac{12}{24}, \frac{12}{16}, \frac{12}{15}, \frac{12}{14}, \frac{12}{10}, \frac{12}{9}, \frac{12}{8}, \frac{12}{8}$$

6. Question and Solution

A **rational** number is any number which can be written as a fraction, where the numerator and denominator are whole numbers but the denominator is *not* zero.

- a) Label in fraction form the
- rational**
- numbers marked on the number line.



- b) Mark the
- opposite**
- value of each number in
- red*
- .

- c) Write each of the marked numbers as a decimal.

$-4, -3.75, -3.5, -2.75, -2.375, -2.125, -1.25, -1, -0.125, 0.125,$
 $1, 1.25, 2.125, 2.375, 2.75, 3.5, 3.75, 4$

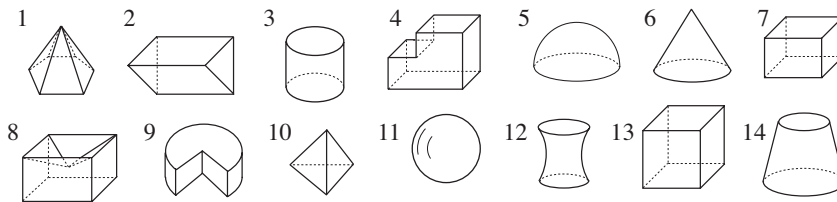
- d) What is their sum?
- 0**

*(p40, Q1)**Notes*

This is a good exercise in finding fractions on a number line and then converting them to decimals. In part d), the sum has to be zero as each number has its opposite number.

7. Question and Solution

Count the faces, edges and vertices of these solids which have **only** plane faces. Complete the table.



Solid	1	2	4	7	8	10	13	
Number of faces	7	5	8	6	9	4	6	
Number of edges	12	9	18	12	16	6	12	
Number of vertices	7	6	12	8	9	4	8	

(p41, Q2)

Notes

This is a straightforward question included to ensure that students are clear about the identification of

faces (*F*), edges (*E*) and vertices (*V*)

(The extra column in the table is included to make students think -a typical Hungarian-type ploy!)

There is an extension that you could set for able students:

'What is the connection between *F*, *E* and *V*?'

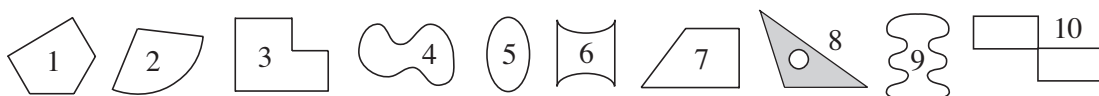
This relationship, known as Euler's formula, is

$$F + V = E + 2$$

If your students can find this, you could then ask them to find other solids with plane faces and check whether or not the formula still holds.

8. Question and Solution

Study these plane shapes. What do some of them have in common?



What do the shapes listed below have in common? Write a sentence about each set.

- a) 1, 3, 7 **All are polygons.**
- b) 4, 5, 9 **Each shape is enclosed by single curved line.**
- c) 2, 4, 5, 6, 8, 9 **Each shape has at least one curved side.**
- d) 1, 3, 6, 7, 10 **All have at least one pair of parallel sides.**
- e) 1, 3, 7, 10 **All have at least one pair of perpendicular sides.**
- f) 3, 4, 6, 8, 9, 10 **All are concave shapes.**
- g) 1, 2, 5, 6, 9 **All have at least one line of symmetry.**
- h) Make up a set which is different from those given. Write a sentence about it.

E.g. 6, 7, 8

The shapes each have 4 sides.

(p45, Q1)

Notes

This question is designed to get students to identify characteristics about these solids. We have given the obvious answers but there will be others and you should encourage your students to be creative. This is particularly relevant for part h).

9. Question and Solution

Precise measurements are important in design, technology, engineering, chemistry, medicine, etc. In everyday life, it is enough to use rough estimates and conversions.

Complete the missing items. Write as fractions and as decimals.

- a) If 1 inch \approx 2.5 cm, then 1 cm \approx $\frac{2}{5}$ in \approx 0.4 inches
- b) If 1 foot \approx 0.3 m, then 1 m \approx $\frac{10}{3}$ ft \approx 3.3 feet
- c) If 1 metre \approx 1.1 yards, then 1 yard \approx $\frac{10}{11}$ m \approx 0.9 metres
- d) If 1 mile \approx 1.6 km, then 1 km \approx $\frac{5}{8}$ miles \approx 0.675 miles
- e) If 1 ounce \approx 28 g, then 1 g \approx $\frac{1}{28}$ oz \approx 0.036 ounce
- f) If 1 kg \approx 2.2 lb, then 1 lb \approx $\frac{5}{11}$ kg \approx 0.45 kilograms
- g) If 1 pint \approx 0.57 litres, then 1 litre \approx $\frac{100}{57}$ pt \approx 1.75 pints
- h) If 1 gallon \approx 4.5 litres, then 1 litre \approx $\frac{2}{9}$ gal \approx 0.22 gallons

(p47, Q1)

Notes

Unfortunately we have two systems of units in common use, the 'imperial' system of

length (inches, feet, yards, miles)

mass (ounces, pounds, stones)

and the 'metric' system (known as SI units, abbreviated from French: *Système international d'unités*) of

length (millimetres, mm; centimetres, cm; metres, m; kilometres, km)

mass (grams, g; kilograms, kg; tonnes, t).

The imperial system is still used in the USA and, in part, in the UK but most of the world now uses the SI system.

To complete each part of the question we use the 'unity' method. For example, in part a),

$$1 \text{ inch} \approx 2.5 \text{ cm}$$

$$\Rightarrow \frac{1}{2.5} \text{ inch} \approx 1 \text{ cm}$$

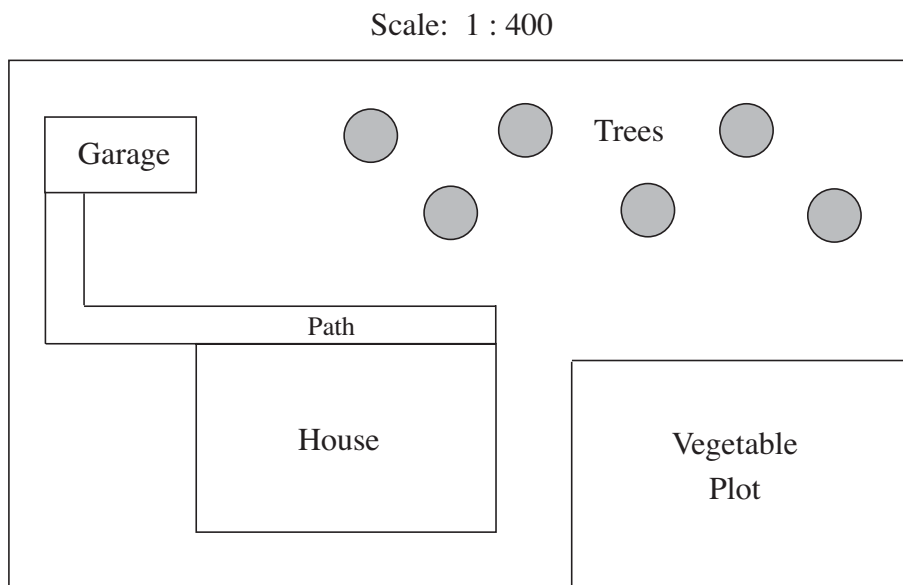
But $\frac{1}{2.5} = \frac{10}{25} = \frac{2}{5} \Rightarrow 1 \text{ cm} \approx \frac{2}{5} \text{ inch}$

and $\frac{2}{5} = \frac{4}{10} = 0.4 \Rightarrow 1 \text{ cm} \approx 0.4 \text{ inch}$

Note that the symbol ' \approx ' means 'approximately equal to'.

10. Question and Solution

This is the plan of a house and its garden.



- a) Measure on the plan, then calculate the real lengths and widths of:

	<i>Length</i>	<i>Width</i>
i) the house	16 m	10 m
ii) the garage	8 m	4 m
iii) the vegetable plot	18 m	12 m
iv) the whole garden.	48 m	28 m

- b) Calculate: i) the perimeter of the vegetable plot **60 m**
 ii) the area of the garden **1344 m²**

(p49, Q1)

Notes

This example illustrates how scale drawings are used and how to convert from the real context to the scale drawing and vice versa.

For example, for the House, on the scale drawing it measures 4 cm by 2.5 cm.

As the scale is 1 : 400, the actual dimensions are

$$4 \times 400 \text{ cm by } 2.5 \times 400 \text{ cm}$$

i.e. 1600 cm by 1000 cm \Rightarrow 16 m by 10 m

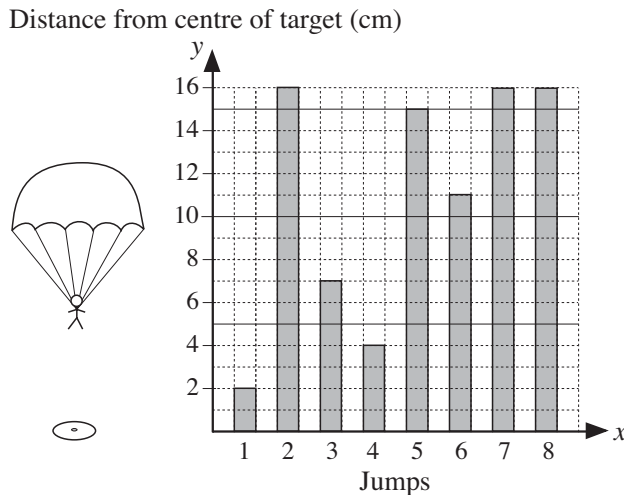
11. Question and Solution

In a parachute target jumping competition, each competitor makes 8 jumps.

The target is a circle with radius 16 cm. The scores range from 0 cm to 16 cm, depending on how far away from the centre of the target circle the parachutist lands.

If the parachutist misses the target completely, the lowest score they can get is 16 cm.

The bar chart shows the scores of one competitor.



- a) Write the scores in decreasing order. **16 cm, 16 cm, 16 cm, 15 cm, 11 cm, 7 cm, 4 cm, 2 cm**
- b) What is the range of the data? **Range:** cm
- c) What is the mode of the data? **Mode:** cm
- d) Calculate the mean value. **Mean:** cm
- e) Calculate the mean of the two middle scores. **Median:** cm
(p52, Q1)

Notes

You might find it helpful to discuss this with students to ensure that they understand the context. Each number along the x -axis represents the number of a jump; the y -axis gives the distance the parachutist landed away from the target.

- a) The competitor had the following scores, in decreasing order, on their 8 jumps,
16 cm, 16 cm, 16 cm, 15 cm, 11 cm, 7 cm, 4 cm, 2 cm
These are the distances they landed away from the target, so the lowest numbers are the best scores (they represent jumps landing nearest to the target).
- b) The range is the (largest value – smallest value),
i.e. $16 \text{ cm} - 2 \text{ cm} = 14 \text{ cm}$
- c) The mode is the number with the highest frequency, so the mode is 16 cm.
- d) To find the mean, we have to add up all the values and divide this by the total number of scores,
i.e. $(3 \times 16 + 15 + 11 + 7 + 4 + 2) \div 8 = (48 + 39) \div 8$
 $= 87 \div 8 = 10.875$

So the mean value of these scores is 10.875 cm

- e) The median is the mean of the two middle scores.

$$\frac{15 + 11}{2} = \frac{26}{2} = 13$$

So the median of the scores is 13 cm.

12. Question and Solution

A bag contains 2 red, 3 yellow and 5 green marbles.



If you took out a marble with your eyes closed, what chance would you give to each of these outcomes?

Join each outcome to the appropriate level of chance.

- a) The marble taken out is *green*.
 b) The marble taken out is *red*.
 c) The marble taken out is either *red or yellow*.
 d) The marble taken out is **not** *yellow*.
 e) The marble taken out is *black*.
 f) The marble taken out is **not** *black*.
- Certain
Likely
Equally likely as unlikely
Unlikely
Impossible

(p60, Q1)

Notes

- a) The probability of getting a *green* (G)

$$= \frac{5}{10} \left(= \frac{1}{2} \right)$$

as there are 5 green marbles and $2 + 3 + 5 = 10$ marbles in total,

so $p(G) = \frac{1}{2} \Rightarrow$ equally likely as unlikely.

- b) $p(R) = \frac{2}{10} = \frac{1}{5} \Rightarrow$ unlikely

- c) $p(R \text{ or } Y) = \frac{(2 + 3)}{10} = \frac{5}{10} = \frac{1}{2} \Rightarrow$ equally likely or unlikely

- d) $p(\text{not } Y) = p(R \text{ or } G) = \frac{(2 + 5)}{10} = \frac{7}{10} \Rightarrow$ likely

- e) $p(B) = 0$ as there are no *black* marbles

- f) $p(\text{not } B) = 1$ as this is certain to happen

13. *Question and Solution*

Do the multiplications in your exercise book. **Simplify** the fractions first if possible.

$$\text{a) i) } \frac{3}{4} \times \frac{2}{9} = \frac{1}{6} \quad \text{ii) } \frac{3}{4} \times \frac{9}{2} = \frac{27}{8} \quad \text{iii) } \frac{4}{3} \times \frac{2}{9} = \frac{8}{27} \quad \text{iv) } \frac{4}{3} \times \frac{9}{2} = 6$$

$$\text{b) i) } \frac{4}{15} \times \frac{12}{5} = \frac{16}{25} \quad \text{ii) } \frac{15}{4} \times \frac{12}{5} = 9 \quad \text{iii) } \frac{4}{15} \times \frac{5}{12} = \frac{1}{9} \quad \text{iv) } \frac{15}{4} \times \frac{5}{12} = \frac{25}{16}$$

$$\text{c) i) } \frac{1}{3} \times \frac{3}{5} \times \frac{5}{7} \times \frac{7}{9} = \frac{1}{9} \quad \text{ii) } \frac{1}{2} \times \frac{4}{8} \times \frac{8}{16} \times \frac{32}{64} \times \frac{128}{256} = \frac{1}{32}$$

(p70, Q5)

Notes

These questions give practice in multiplying fractions. There are many ways of completing these. For example, in c) i), we could multiply out and then simplify to give,

$$\begin{aligned} \frac{1}{3} \times \frac{3}{5} \times \frac{5}{7} \times \frac{7}{9} &= \frac{1 \times 3 \times 5 \times 7}{3 \times 5 \times 7 \times 9} = \frac{105}{945} \\ &= \frac{21}{189} \quad (\div \text{ numerator and denominator by } 5) \\ &= \frac{7}{63} \quad (\div \text{ numerator and denominator by } 3) \\ &= \frac{1}{9} \quad (\div \text{ numerator and denominator by } 7) \end{aligned}$$

It is, though, more efficient to simplify first; that is

$$\begin{aligned} \frac{1}{\cancel{3}} \times \frac{\cancel{3}^1}{\cancel{5}_1} \times \frac{\cancel{5}^1}{\cancel{7}_1} \times \frac{\cancel{7}^1}{9} &= \frac{1 \times 1 \times 1 \times 1}{1 \times 1 \times 1 \times 9} \\ &= \frac{1}{9} \end{aligned}$$

Certainly in part c) ii), simplification should come first as each fraction is $\frac{1}{2}$, so that

$$\begin{aligned} \frac{1}{2} \times \frac{4}{8} \times \frac{8}{16} \times \frac{32}{64} \times \frac{128}{256} &= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \\ &= \frac{1 \times 1 \times 1 \times 1 \times 1}{2 \times 2 \times 2 \times 2 \times 2} \\ &= \frac{1}{32} \end{aligned}$$

Note that the rule we are using is

$$\boxed{\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}}$$

when a, b, c, d are integers and $b, d \neq 0$.

14. Question and Solution

- a) What is $\frac{5}{6}$ of 45.6 kg? **3.8 kg** b) What is 70% of 45.6 kg? **31.92 kg**
- c) $\frac{5}{8}$ of a quantity is 450 m. What is the whole quantity? **720 m**
- d) 62.5% of a quantity is 450 m. What is the whole quantity? **720 m**

(p80, Q3)

Notes

This is an exercise on using fractions in context.

Parts a) and b) are straightforward, that is

$$\begin{aligned}\frac{5}{6} \times 45.6 \text{ kg} &= 5 \times \left(\frac{45.6}{6}\right) \text{ kg} \\ &= 5 \times 7.6 \text{ kg} \\ &= 38.0 \text{ kg} \quad \text{i.e. } 38 \text{ kg}\end{aligned}$$

and

$$\begin{aligned}70\% \text{ of } 45.6 \text{ kg} &= \frac{70}{100} \times 45.6 \text{ kg} \\ &= \frac{7}{10} \times 45.6 \text{ kg} \\ &= \frac{319.2}{10} \text{ kg} \\ &= 31.92 \text{ kg}\end{aligned}$$

Parts c) and d) are more of a challenge!

In c), we have

$$\frac{5}{8} \text{ of a quantity} = 450 \text{ m}$$

so $\frac{1}{8}$ of the quantity = $\frac{450}{5}$ m (dividing both sides by 5)

and quantity = $8 \times \frac{450}{5}$ m (multiplying both sides by 8)

$$\begin{aligned}&= 8 \times 90 \text{ m} \\ &= 720 \text{ m}\end{aligned}$$

Similarly in d),

$$62.5\% \text{ of a quantity} = 450$$

$$1\% \text{ of the quantity} = \frac{450}{62.5}$$

$$\frac{1}{100} \text{ of the quantity} = \frac{450}{62.5}$$

$$\text{quantity} = \frac{100 \times 450}{62.5} \quad (\text{multiply both sides by } 100)$$

$$= \frac{45000}{62.5}$$

$$= \frac{450000}{625} \quad (\text{multiply numerator and denominator by } 10)$$

$$= \frac{90000}{125} * \quad (\text{divide numerator and denominator by } 5)$$

$$= \frac{18000}{25} \quad (\text{divide numerator and denominator by } 5)$$

$$= \frac{3600}{5} \quad (\text{divide numerator and denominator by } 5)$$

$$= 720$$

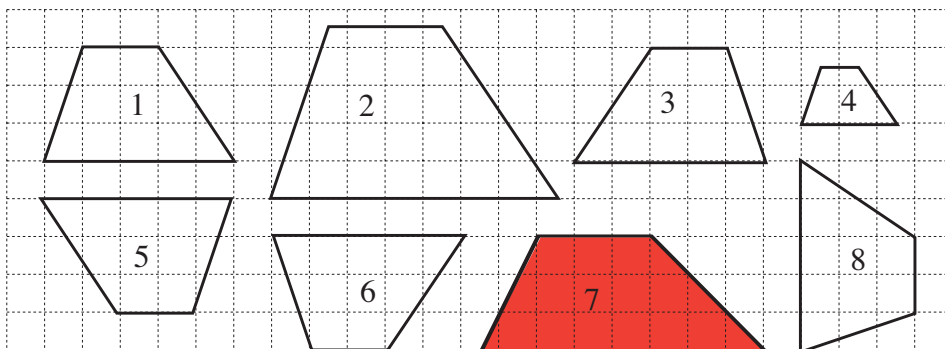
(* Another way of proceeding from here is to notice that

$$125 \times 8 = 1000$$

$$\begin{aligned} \text{hence } \frac{90000}{125} \times \frac{8}{8} &= \frac{710000}{1000} \\ &= 720) \end{aligned}$$

15. Question and Solution

What has been done to Shape 1 to form the other shapes? Describe each **transformation** in your exercise book. Colour the shape which is **not** similar.



1 → 2 enlargement, scale factor $1\frac{1}{2}$

1 → 3 reflection

1 → 4 reduction, scale factor $\frac{1}{2}$

1 → 5 rotation

1 → 6 reflection and translation

1 → 7 stretched in horizontal direction

1 → 8 rotation

Shape 7 is not similar to Shape 1

(p85, Q1)

Notes

This is a straightforward application of transformation, with more than one answer to some of the questions. For example, for 1 → 5, you could rotate 180° in either the clockwise or anticlockwise direction.

16. Question and Solution

Fill in the missing words.

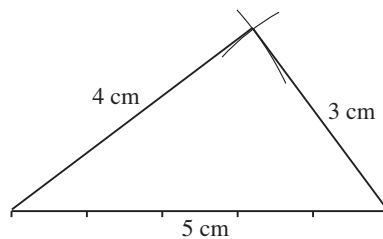
- a) An **equilateral** triangle has angles of ° and has three sides.
- b) An **isosceles** triangle has at least equal (angles).
- c) An **equilateral** triangle is also an triangle.
- d) A triangle which has sides in the ratio of 3 : 4 : 5 is a triangle.
- e) A triangle with 3 different sides is called a triangle.
- f) There is no triangle which has a angle.
- g) The sum of the angles of any triangle is °.

(p90, Q4)

Notes

The responses here reinforce knowledge of the definitions related to triangles.

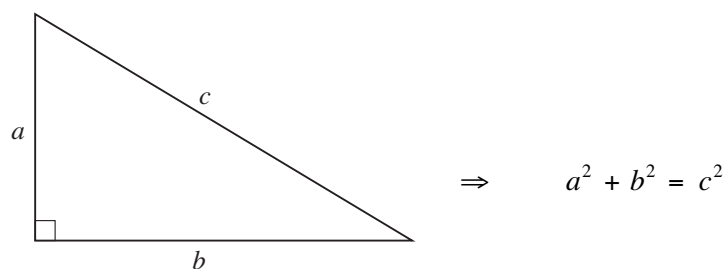
Part d) gives a special example of a right angled triangle. If you construct a 3 cm by 4 cm by 5 cm triangle by drawing a horizontal line of 5 cm and then constructing lines of 4 cm and 3 cm using a compass and the point of intersection, then measure the enclosed angle, which will be 90°.



This is, in fact, an example of a Pythagorean Triple, as

$$3^2 + 4^2 = 9 + 16 = 25 = 5^2$$

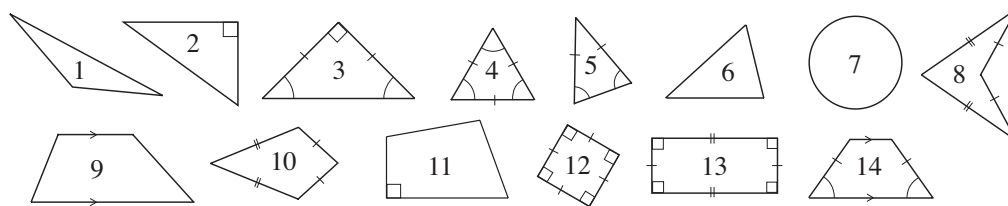
which, using Pythagoras' Theorem, shows that this is a right angle. The full result is shown below.



There are many other Pythagorean Triples, for example, 6, 8, 10 (which is a multiple of 3, 4, 5) and 12, 5, 13 but they are not that easy to find.

17. Question and Solution

List the numbers of the shapes which match the descriptions.



- a) It has line symmetry. 3, 4, 5, 7, 8, 10, 12, 13, 14
- b) It has rotational symmetry. 4, 7, 12, 13
- c) It is a regular shape. 4, 7, 12
- d) It has an obtuse angle. 1, 9, 10, 11, 14
- e) It has **only** acute angles. 4, 5, 6
- f) It is a trapezium. 9, 12, 13, 14
- g) It is a deltoid. 8, 10, 12
- h) It is a rhombus. 12
- i) It is **not** a polygon. 7

(p95, Q1)

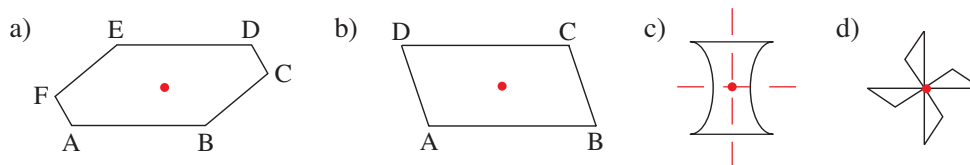
Notes

For f), remember that the definition of a *trapezium* is 'a quadrilateral with at least one pair of parallel sides'. Hence not only are shapes 9 and 14 trapeziums, but so are shapes 12 and 13.

Also note that the definition of a *deltoid* is 'a quadrilateral with two adjacent pairs of sides equal'. This definition applies to shape 8 (concave) and shape 10 (convex) and also to shape 12 (as all sides are equal).

18. Question and Solution

Draw any lines of symmetry and mark the centres of rotation.



(p96, Q4)

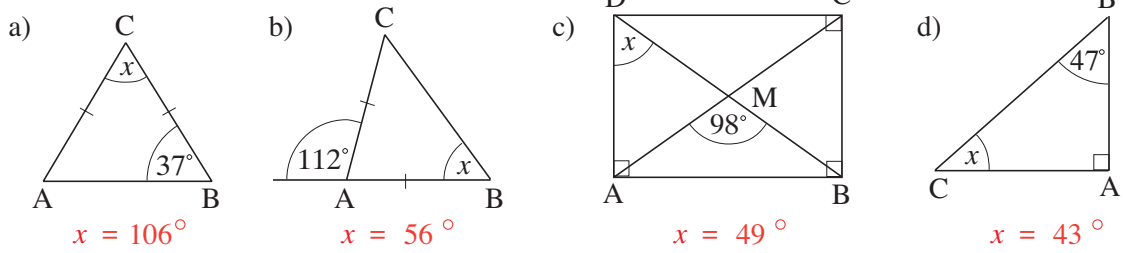
Notes

This question illustrates *two* concepts:

- lines of symmetry (that is, mirror lines where the reflections must be identical)
- rotational symmetry (where the shape is repeated when it is rotated).

19. Question and Solution

Calculate the size of angle x . The diagrams are not drawn to scale.

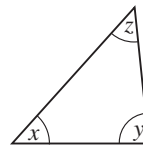


(p100, Q4)

Notes

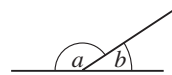
The angle facts needed for these questions are

- angles in a triangle sum to 180°



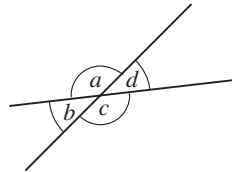
$$x + y + z = 180^\circ$$

- angles on a straight line sum to 180°



$$a + b = 180^\circ$$

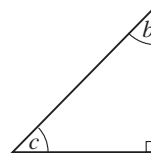
- opposite angles are equal.



$$a = c \text{ and } b = d$$

- in a right angled triangle, the other two angles sum to 90°

(this follows from $x + y + z = 180^\circ$, with, for example, $z = 90^\circ$)



$$a + b = 90^\circ$$

20. Question and Solution

Use a calculator to work out the missing values.

- a) If $\text{£}1 \approx 1.43$ Euros,
 1 Euro \approx £
- b) If 1 Euro ≈ 7.47 Danish Kroner,
 1 DK \approx EUR
- c) If 1 USD ≈ 0.62 GBP,
 1 GBP \approx USD
- d) If $\text{£}1 \approx 183.2$ JPY,
 1 JPY \approx £

(p105, Q1)

Notes

This is an application of the *unitary* method (i.e., first calculating for 1 unit), so that, in part a)

$$\begin{aligned}
 1.43 \text{ Euros} &\approx \text{£}1 \\
 1 \text{ Euro} &\approx \text{£} \frac{1}{1.43} \\
 &\approx 70 \text{ p} \quad (\text{from calculator})
 \end{aligned}$$

21. Question and Solution

- a) The surface area of each face of an ice cube is 49 cm^2 . Calculate:
- the volume of the ice cube **343 cm^3**
 - its mass, if 1 cm^3 of ice weighs 0.91 g **312 g**
- b) The surface area of a square-based prism is 64 cm^2 and its base edge is 2 cm .
What is the volume of the prism? **28 cm^3**

(p107, Q6)

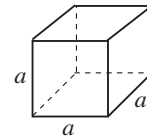
Notes

In part a), you need to note that it is a cube so that, if one side length is a , then $a \times a = 49$,

giving $a = 7 \text{ cm}$

The volume is given by

$$\begin{aligned} a \times a \times a &= 7 \times 7 \times 7 \\ &= 49 \times 7 \\ &= 343 \text{ cubic cm or } 343 \text{ cm}^3 \end{aligned}$$

**22. Question and Solutions**

Solve these problems in your exercise book.

- a) Tom had blackcurrants, gooseberries, loganberries, raspberries and strawberries in his garden. One summer he gathered all the fruit and put it in his freezer.
This is the ratio of the fruit in Tom's freezer. $B : G : L : R : S = 6 : 7 : 5 : 4 : 2$
If there are 36 lbs of fruit in the freezer, how many lbs of each type of fruit did Tom pick?
 $B \ 9 \text{ lbs}; G \ 10\frac{1}{2} \text{ lbs}; L \ 7\frac{1}{2} \text{ lbs}; R \ 6 \text{ lbs}; S \ 3 \text{ lbs}$
- b) In a school with 1350 pupils, the ratio of boys to girls is $11 : 14$ and the ratio of teachers to pupils is $2 : 45$.
- How many girls and how many boys are in the school? **$756 \text{ girls}, 594 \text{ boys}$**
 - How many teachers are in the school? **60 teachers**
- c) The ratio of *red* to *blue* to *green* beads in a jar is $7 : 13 : 17$. If there are 126 *red* beads, how many *blue* and how many *green* beads are in the jar?
 $234 \text{ blue}, 306 \text{ green}$

(p115, Q2)

Notes

- a) The key here is to convert the ratios to fractions.
To do this, we add

$$6 + 7 + 5 + 4 + 2 = 24$$