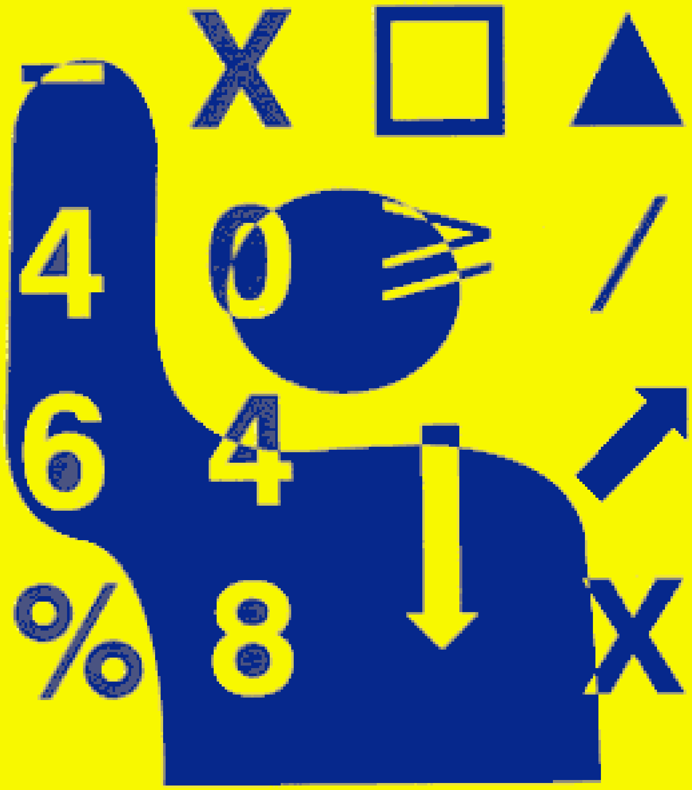
Mathematics Enhancement Programme

Primary Demonstration Project

7A Mensuration

Help Booklet



Support for Primary Teachers in Mathematics

Primary Project
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Mathematics Enhancement Programme

Help Module 7

MENSURATION

Part A

Contents of Part A

Contents of Part B

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PREFACE

This is one of a series of *Help Modules* designed to help you gain confidence in mathematics. It has been developed particularly for primary teachers (or student teachers) but it might also be helpful for non-specialists who teach mathematics in the lower secondary years. It is based on material which is already being used in the *Mathematics Enhancement Programme: Secondary Demonstration Project*.

The complete module list comprises:

1.	ALGEBRA	6.	HANDLING DATA
2.	DECIMALS	7.	MENSURATION
3.	EQUATIONS	8.	NUMBERS IN CONTEXT
4.	FRACTIONS	9.	PERCENTAGES
5.	GEOMETRY	10.	PROBABILITY

Notes for overall guidance:

- Each of the 10 modules listed above is divided into 2 parts. This is simply to help in the downloading and handling of the material.
- Though referred to as 'modules' it may not be necessary to study (or print out) each one in its entirely. As with any self-study material you must be aware of your own needs and assess each section to see whether it is relevant to those needs.
- The difficulty of the material in **Part A** varies quite widely: if you have problems with a particular section do try the one following, and then the next, as the content is not necessarily arranged in order of difficulty. Learning is not a simple linear process, and later studies can often illuminate and make clear something which seemed impenetrable at an earlier attempt.
- In **Part B, Activities** are offered as backup, reinforcement and extension to the work covered in Part A. **Tests** are also provided, and you are strongly urged to take these (at the end of your studies) as a check on your understanding of the topic.
- The marking scheme for the revision test includes B, M and A marks. Note that:

  M *marks* are for method;

  A *marks* are for accuracy (awarded only following a correct M mark);

  B *marks* are independent, stand-alone marks.

We hope that you find this module helpful. Comments should be sent to:

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The full range of Help Modules can be found at

www.ex.ac.uk/cimt/help/menu.htm

1 *Mensuration* Introductory Notes

Historical Background

Once a child – or a civilisation! – formalises the art of *counting*, it is but a short step to using whole numbers to quantify *measures*. The language and structures we use to refer to large positive integers (say up to low thousands) combine fairly naturally with the idea of a given *unit* to create a flexible and powerful way of quantifying amounts. Thus, for smallish **lengths** the ancient civilisations of the near east (Babylonian/Egyptian, c.1700 BC) used *cubits*; one *cubit* referred to the length of the forearm, or *ulna* (so is related to the later English unit – the *ell*). The ancient Greeks and Romans (400 BC to 600 AD) used *palms* – similar to our *hands*, still used for measuring the height of horses; for longer distances they used the *pes* (or foot), the *passus* (equal to 5 *pedes*), and the *stadium* (roughly a furlong). Many ancient cultures measured **volumes** of grain in *basketfuls*.

*T*he most significant mathematical feature of these early *measures* is that although the units themselves may be inexact (What exactly is a *foot*? What is a *basket?*), the number of units is *absolutely exact* (because we are dealing with whole numbers).

The whole idea of using numbers to quantify amounts has two parts:

- The first part is the mathematical **idea** of choosing a fixed *unit* and then replicating that unit to match a given amount, which can then be assigned a certain number of units, or quantity. This **idea** is *abstract* and *exact.*
- The second part is the **practical implementation** of this scheme, by agreeing
  - (a) how to realise the abstract idea of the fixed unit *in practice*; and
  - (b) how to replicate the unit *reliably* and *fairly*.

  This **practical implementation** is inevitably approximate.

It is important to establish these two ideas (one exact, and one approximate) in pupils' minds as separate aspects of measurement.

Introducing partial units (halves and other fractions), raises a new source of approximation: it is tempting to think that *one complete basket* involves no approximation, whereas *two thirds of a basket* clearly involves a degree of estimation. This can add to the confusion as to what is exact and what is approximate – especially if one is unclear about the exact nature of fractions.

Units of length, weight, volume and currency developed locally, so only had to be sufficiently accurate for local needs. Trade between regions encouraged the development of common measures, but without the necessary political interest, change was inevitably slow. Moreover, units of measure could never be more accurate than the available technology allowed. The imposition of *standard units of measure* was at the mercy of political and technological developments. The most striking example is the spread of the *metric* *system.* The planning and introduction of the metric system (in France in the 1790s, and thence into other European countries conquered by Napoleon), was the result of a unique combination of events: namely the rise of a powerful Emperor (Napoleon), who happened to be scientifically educated (being a member of the *Académie des Sciences*, and a keen amateur mathematician) at a time when the necessary technological developments were in place for the first time. (For example, the definition of a *metre* as 'the distance between two marks on a specified platinum bar, stored at a fixed temperature' in a vault in Paris would have been unthinkable – for scientific, technological and political reasons – 100 years earlier.)

The kind of practical developments indicated above tended to obscure the simple mathematical idea which underlies the introduction of standard units – namely:

- Once a unit u is chosen as a basic measure for a quantity (e.g. *length*), we can *in principle* measure any other amount A of the same kind (i.e. another *length*) by a number x using the idea of *proportion*.

the quantity A is measured *exactly* by the number X, whenever the ratio 'amount A : unit u' corresponds to the ratio 'x : 1'.

This principle is as fundamental today as it was when first enunciated by the ancient Greeks. However, our trust in improved technology can lead to confusion between the *exactness in principle* of the idea behind measurement and the *inevitable inaccuracy* of all practical measurements. (We tend to use the misleading expression 'accurate' measurement when we really mean that the inevitable error is small!) The key idea behind all measurement is that

once we choose a particular segment u as our unit of length

(be it a *centimetre*, an *inch*, or just an unspecified *unit*),

any other segment A is measured *exactly in principle* by the number x of times that the segment u 'fits in to A'.

It is this *idea* that is *exact*; in practice measurement introduces its own inexactness. Thus that the act of measuring is best seen as obtaining precise upper and lower estimates for the *exact* measure: when we say that the diagonal of a square with sides of length 1 cm has length '1.41 cm', we are really saying that the true length lies between 1.405 cm and 1.415 cm.

Approximate practical measurement is quite different from the mathematical fact that a square of side length 1 unit has diagonal of length *exactly* $\sqrt{2}$ units. This result is *exact* because the answer is *calculated* rather than *measured*.

Another important mathematical notion related to units of measurement is the fact that once a unit of *length* is chosen, this gives rise to a natural related unit of *area* (namely the 2-dimensional 'magnitude' of a 1 by 1 square), and to a natural unit of *volume* (namely the 3-dimensional 'magnitude' of a 1 by 1 by 1 cube). Moreover, once a unit of *time* is chosen, we can combine this with our unit of length to get related units of *speed* and *acceleration*. Similarly, once a unit of *mass* is fixed, we get a related unit of *density*.

Many civilisations developed rules of thumb to find approximate areas and volumes of familiar everyday shapes (such as the area of special shaped fields, and the volume of special grain containers). However, since they usually felt no need to give precise definitions of those shapes, it is often impossible to tell how good their rules were. More detailed procedures – with some proofs – occur in the mathematics of ancient India, China and Japan, but it is hard to know exactly when their methods were developed. Again it was the ancient Greeks (around 300 BC) who probably first gave precise definitions, and set up a structure within which they could realise their insistence on *proving* that their rules were correct. They gave strict proofs for all the basic results we now know (area of rectangles, parallelograms, triangles, trapezia; volumes of prisms; area and perimeter of circles – including the amazing fact that the same number π appears in both formulas; volumes of cones and pyramids; volumes and surface areas of spheres).

Key Issues

Introduction

This is a long module as it deals with units and measurement, areas and volumes and nets for common 3-D shapes. You will be familiar with many of these topics, so be selective when choosing the sections that you work through.

Language / Notation

This module uses extensive mathematical language which you will need to understand. For example, you need to be familiar with

units : km, cm, mm; mile, yard, feet and inches
litre, gallon, pint;
kg, g, tonne; lb, oz, stone.

shapes : square, rectangle, triangle, cube,
parallelogram, trapezium, kite, rhombus,
cube, cuboid, cylinder, prism, pyramid, cone, sphere.

Key Points

You should be familiar with reading scales and converting units before embarking on the main topics in this module.

$1 \text{ m}^3 = 1000$ litres

1 tonne = 1000 kg

1 gallon = 8 pints

1 kg is about 2.2 lbs

1 gallon is about 4.5 litres

1 litre is about 1.75 pints

5 miles is about 8 km

1 inch is about 2.5 cm

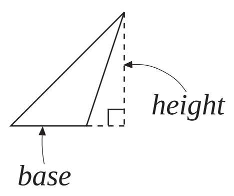
Square:	Area = x^2
Rectangle:	Area = $l\,w$
Triangle:	Area = $\frac{1}{2}b\,h$
Circle:	Area = πr^2 Circumference = $2\pi r$
Parallelogram:	Area = $b\,h$
Trapezium:	Area = $\frac{1}{2}(a+b)h$
Kite:	Area = $\frac{1}{2}a\,b$
Cube:	Volume = a^3
Cuboid:	Volume = $a\,b\,c$
Cylinder:	Volume = $\pi r^2 h$
Prism:	Volume = $A\,l$
Pyramid:	Volume = $\frac{1}{3}A\,h$
Cone:	Volume = $\frac{1}{3}\pi r^2 h$
Sphere:	Volume = $\frac{4}{3}\pi r^3$

Misconceptions

- change of units: remember that $1\text{ cm} = 10\text{ mm}$ and $100\text{ cm} = 1\text{ m}$, so $1\text{ m} = 1000\text{ mm}$.
- square units: $1\text{ m}^2 = 10\,000\text{ cm}^2$ (not 100 cm^2)
- the correct formula for the area of triangle is

  $$\frac{1}{2}\text{ base} \times \text{perpendicular height}$$

  and not just $b \times h$ – you must use the *perpendicular* height.



WORKED EXAMPLES and EXERCISES

7 Mensuration

This module is concerned with measuring, calculating and estimating lengths, areas and volumes, as well as the construction of three-dimensional (3-D) objects.

7.1 Units and Measuring

Different units can be used to measure the same quantities. It is important to use sensible units. Some important units are listed below.

1 km = 1000 m
1 m = 100 cm
1 m = 1000 mm
1 cm = 10 mm

1 tonne = 1000 kg
1 kg = 1000 g

1 litre = 1000 ml
1 m^3 = 1000 litres
1 cm^3 = 1 ml

Worked Example 1

What would be the best units to use when measuring,

(a) the distance between Birmingham and Manchester,

(b) the length of a matchbox,

(c) the mass of a person,

(d) the mass of a letter,

(e) the mass of a lorry,

(f) the volume of medicine in a spoon,

(g) the volume of water in a swimming pool?

Solution

(a) Use km (or miles).

(b) Use mm or cm.

(c) Use kg.

(d) Use grams.

(e) Use tonnes

(f) Use ml.

(g) Use m^3.

7.1

Worked Example 2

(a) How many mm are there in 3.72 m?

(b) How many cm are there in 4.23 m?

(c) How many m are there in 102.5 km?

(d) How many kg are there in 4.32 tonnes?

Solution

(a) 1 m = 1000 mm

So

$$3.72\ \text{m} = 3.72 \times 1000$$
$$= 3720\,\text{mm}$$

(b) 1 m = 100 cm

So

$$4.23\,\text{m} = 4.23 \times 100$$
$$= 423\,\text{cm}$$

(c) 1 km = 1000 m

So

$$102.5\ \text{km} = 102.5 \times 1000$$
$$= 102\,500\,\text{m}$$

(d) 1 tonne = 1000 kg

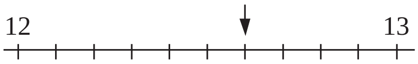
So

$$4.32\ \text{km} = 4.32 \times 1000$$
$$= 4320\,\text{kg}$$

Worked Example 3

What value does each arrow point to?

(a)



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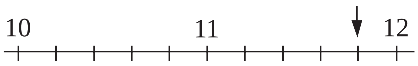

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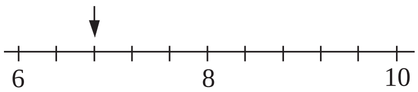


Solution

(a) Here the marks are 0.1 units apart.
So the arrow points to 12.6.

(b) Here the marks are 0.2 units apart.
So the arrow points to 11.8.

(c) Here the marks are 0.4 units apart.
So the arrow points to 6.8.

7.1

Exercises

1. Measure each line below. Give the length to the nearest mm.

   (a)

   (b)

   (c)

   (d)

   (e)

2. Which units do you think would be the most suitable to use when measuring:

   (a) the distance between two towns,

   (b) the length of a sheet of paper,

   (c) the mass of a sheet of paper,

   (d) the mass of a sack of cement,

   (e) the volume of a water in a cup,

   (f) the volume of water in a large tank?

3. (a) How many grams are there in 12.3 kg?

   (b) How many mm are there in 4.7 m?

   (c) How many mm are there in 16.4 cm?

   (d) How many m are there in 3.4 km?

   (e) How many cm are there in 3.7 m?

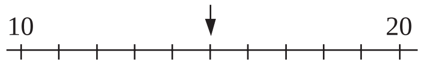
   (f) How many ml are there in 6 litres?

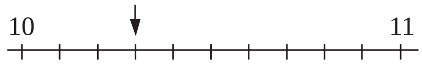
4. Copy and complete the table below.

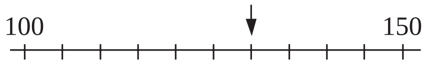
Length in m	*Length in cm*	*Length in mm*
4		
	311	
		1500
	374	
8.62		

7.1

5. Read off the value shown by the arrow on each scale

(a) 

(b) 

(c) 

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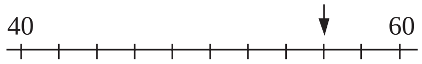
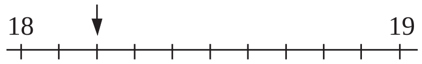
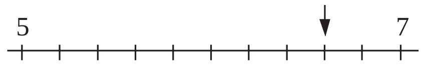
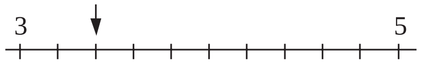
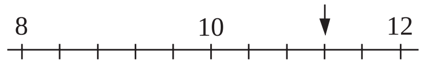
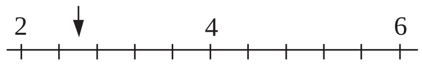
(e) 

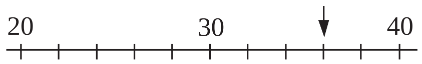
(f) 

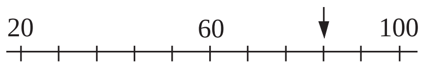
(g) 

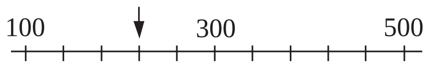
(h) 

(i) 

(j) 

(k) 

(l) 

(m) 

6. A jug contains 1 litre of water.

(a) If 150 ml is poured out, how much water is left?

(b) A glass holds 200 ml of water. How many glasses could be filled from a full jug?

7. State whether the following lengths would be best measured to the nearest m, cm or mm.

(a) Your height.

(b) The length of a ship.

(c) The height of a hill.

(d) The thickness of a book.

(e) The height of a building.

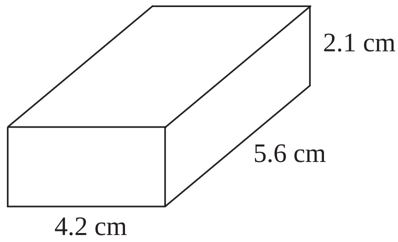
(f) The length of a matchstick.

(g) The width of a matchstick.

7.1

8. A cuboid has sides as shown in the diagram.

Convert the lengths of these sides to mm.



9. Each length below is given in mm. Give each length to the nearest cm.

(a) 42 mm (b) 66 mm (c) 108 mm

(d) 3 mm (e) 7 mm (f) 9.4 mm

10. (a) What metric unit of length would you use to measure the length of a large coach?

(b) Using the unit you gave in part (a) estimate the length of a large coach.

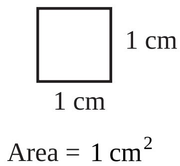
(LON)

Just for Fun

Which is heavier, 1 kg of iron or 1 kg of feathers?

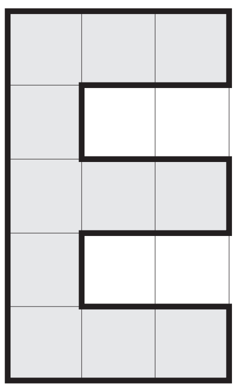
7.2 Estimating Areas

A square with sides of 1 cm has an area of 1 cm^2.



Worked Example 1

Find the area of the shaded shape.



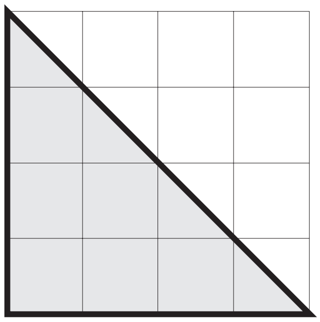
Solution

The shape covers 11 squares, so its area is 11 cm^2.

7.2

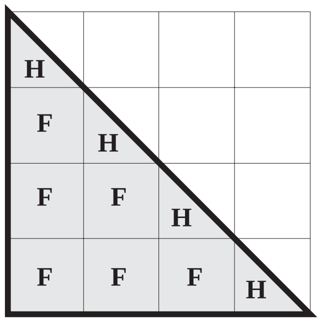
Worked Example 2

Find the area of the shaded triangle.



Solution

The triangle covers 6 full squares marked **F**, and 4 half squares marked **H**.

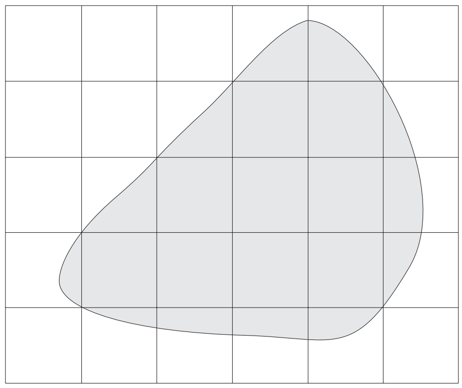


Area $= 6 + 2$

$= 8\,\text{cm}^2$

Worked Example 3

Estimate the area of the shape shaded in the diagram.



Solution

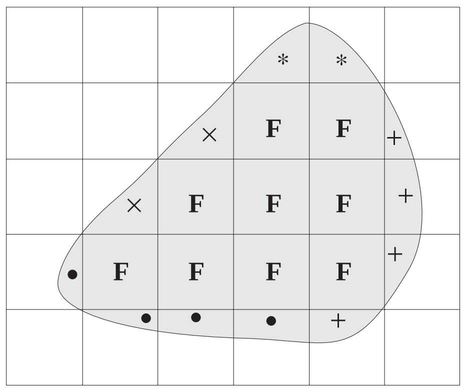
This is a much more complicated problem as there are only 9 full squares marked **F**, but many other part squares. You need to combine part squares that approximately make a whole square. For example,

the squares marked * make about 1 full square;

the squares marked × make about 1 full square;

the squares marked + make about 1 full square;

the squares marked • make about 1 full square.



Thus the total area is approximately

$$9 + 4 = 13\ \text{cm}^2$$

7.2

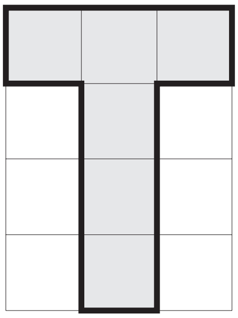
Just for Fun

Use 12 cocktail sticks to form 6 equilateral triangles, all of the same area. Move only 4 cocktail sticks from your figure so as to get 3 equilateral triangles, 2 of which are of the same area.

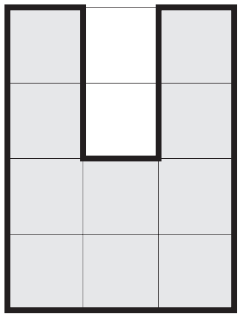
Exercises

1. Find the area of each of the following shapes.

(a)



(b)



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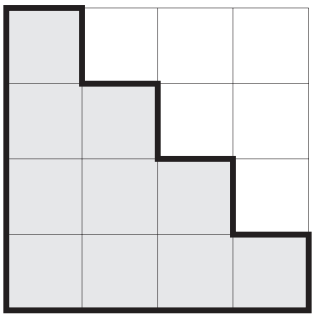

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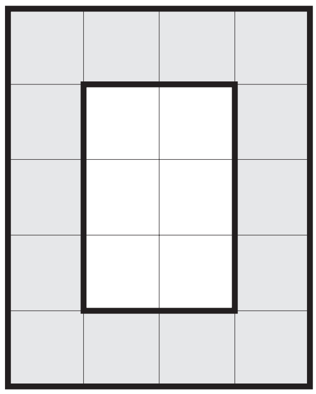
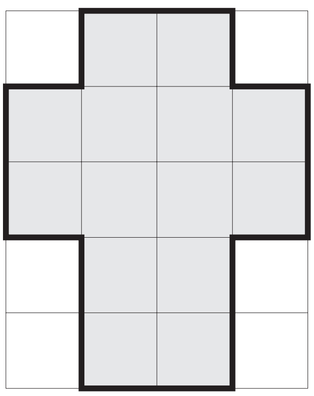
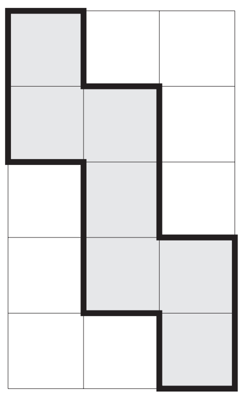


(e)



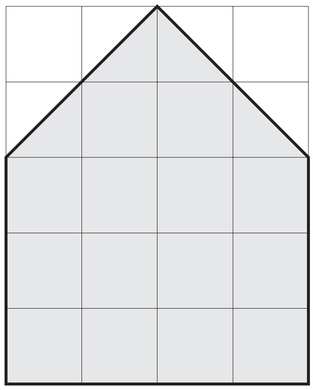
(f)



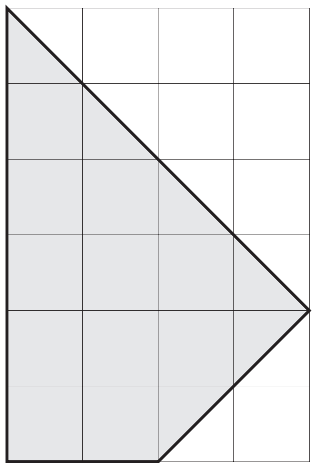
7.2

2. By counting the number of whole squares and half squares, find the area of each of the following shapes.

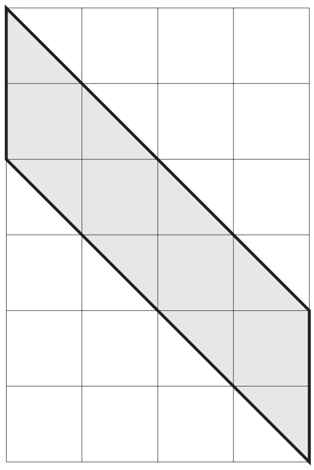
(a)



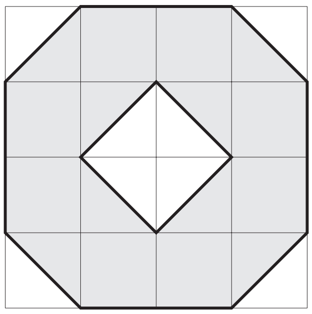
(b)



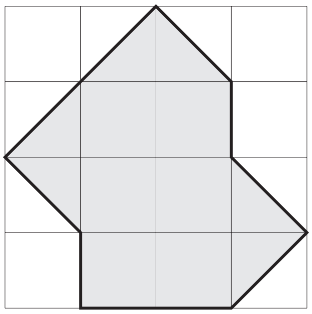
(c)



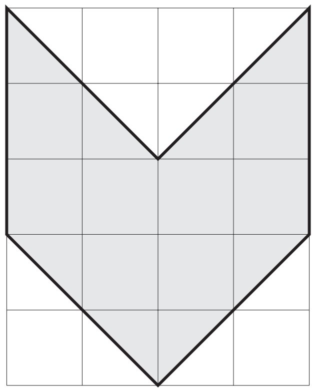
(d)



(e)



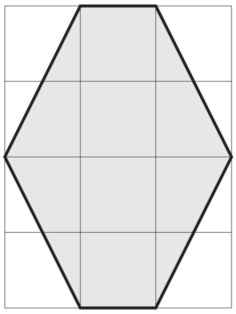
(f)



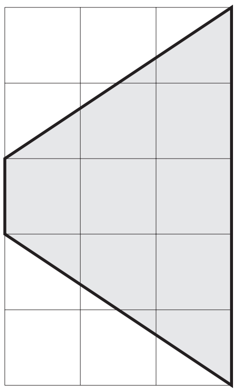
7.2

3. Estimate the area of each of the following shapes.

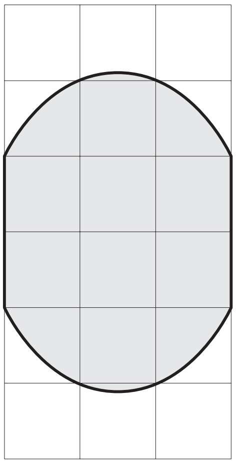
(a)



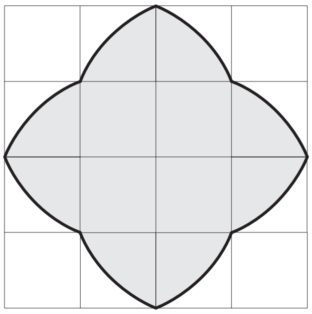
(b)



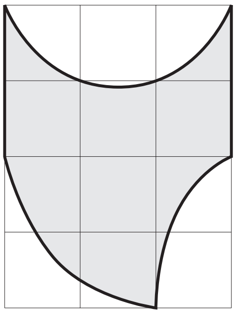
(c)



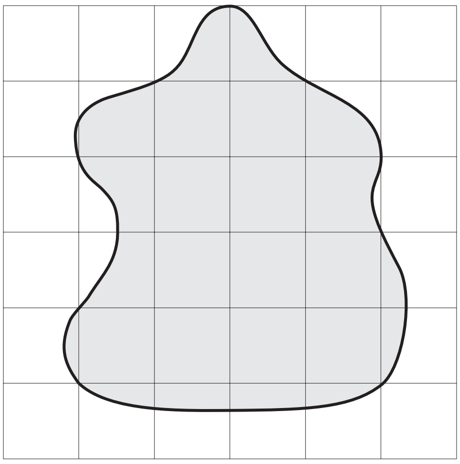
(d)



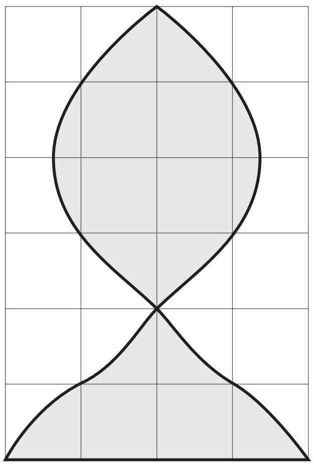
(e)



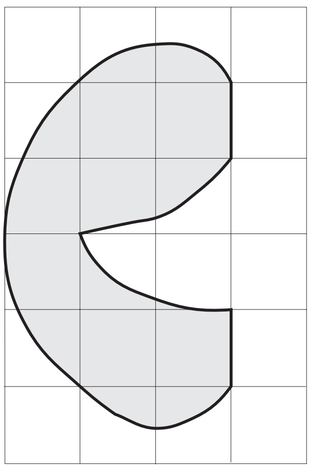
(f)



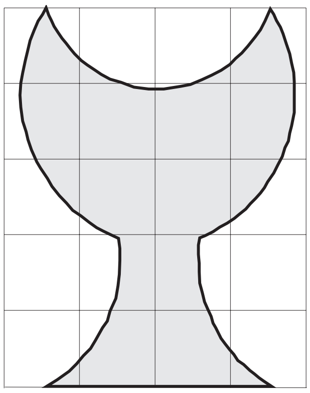
(g)



(h)

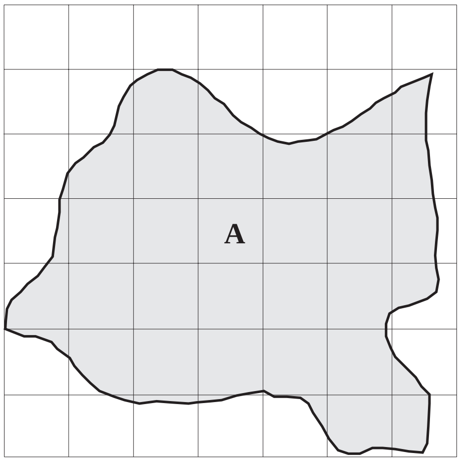


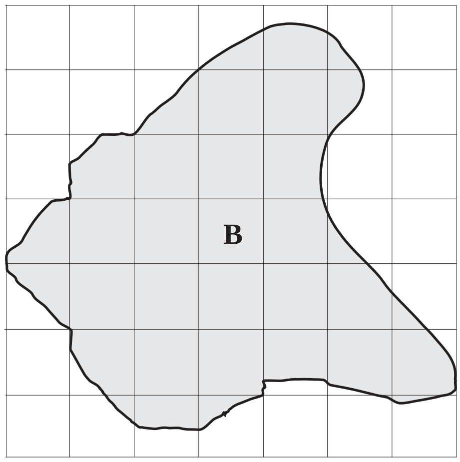
(i)



7.2

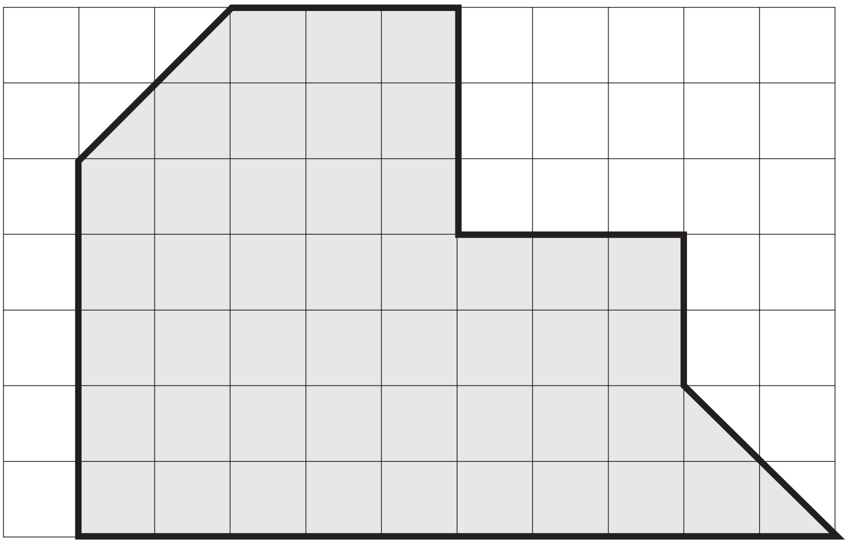
4. The diagrams below shows the outlines of two islands, A and B. The grid squares have sides of length 1 km. Find the approximate area of each island.





5. Each of the squares in this grid has an area of 1 square centimetre.

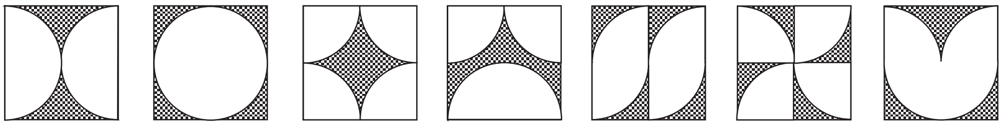
Work out the area of the shaded shape.



(LON)

Investigation

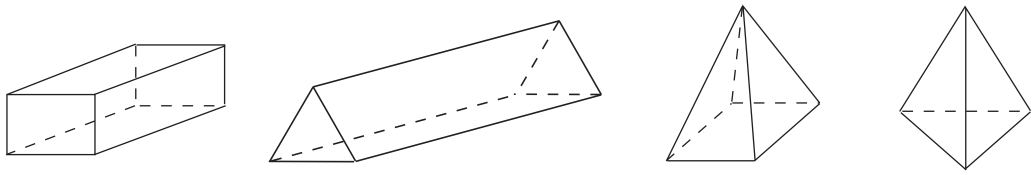
Which of the following shaded figures has the greatest area? The squares are of the same length and the curved lines are all arcs of circles.



7.3 Making Solids Using Nets

A net can be cut out, folded and glued to make a hollow shape.

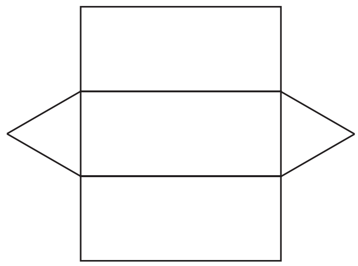
In this Unit, you will be dealing with 3-dimensional shapes such as



cuboid *prism* *pyramid* *tetrahedron*

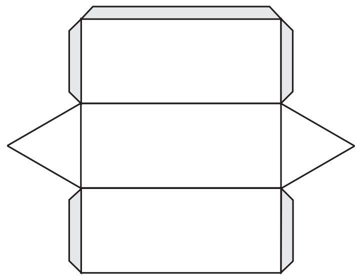
Worked Example 1

What solid is made when the net shown is folded and glued?



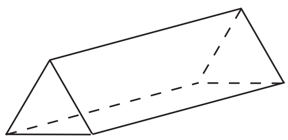
Solution

It is important to add tabs to the net so that it can be glued. You could put tabs on every edge, but this would mean gluing tabs to tabs. The diagram opposite shows one possible position of the tabs.



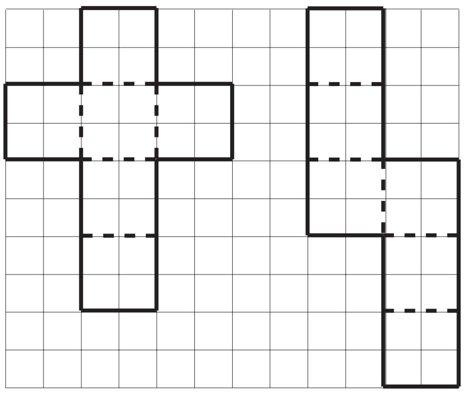
Before gluing, crease all the folds.

The final solid is a triangular prism.



Exercises

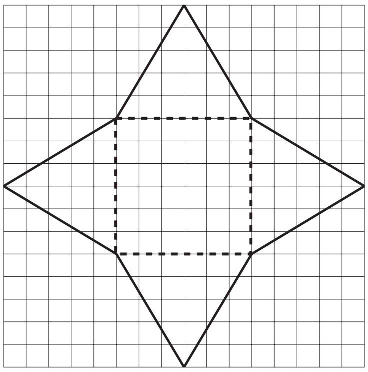
1. Copy and cut out larger versions of the following nets. Fold and glue them to obtain cubes. Do not forget to add tabs to the nets.



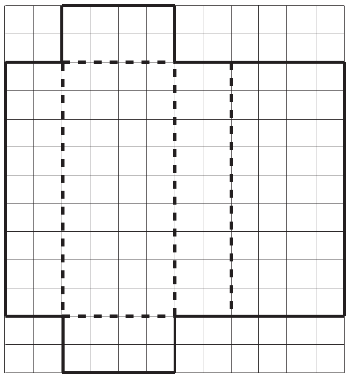
7.3

2. Copy each net shown below make it into a solid. State the name of the solid that you make, if it has one.

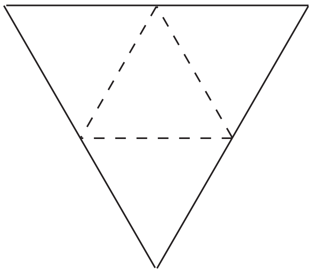
(a)



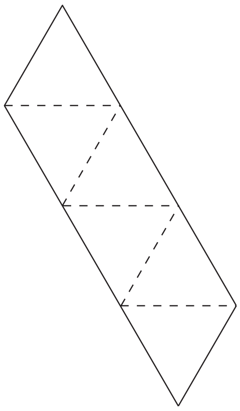
(b)



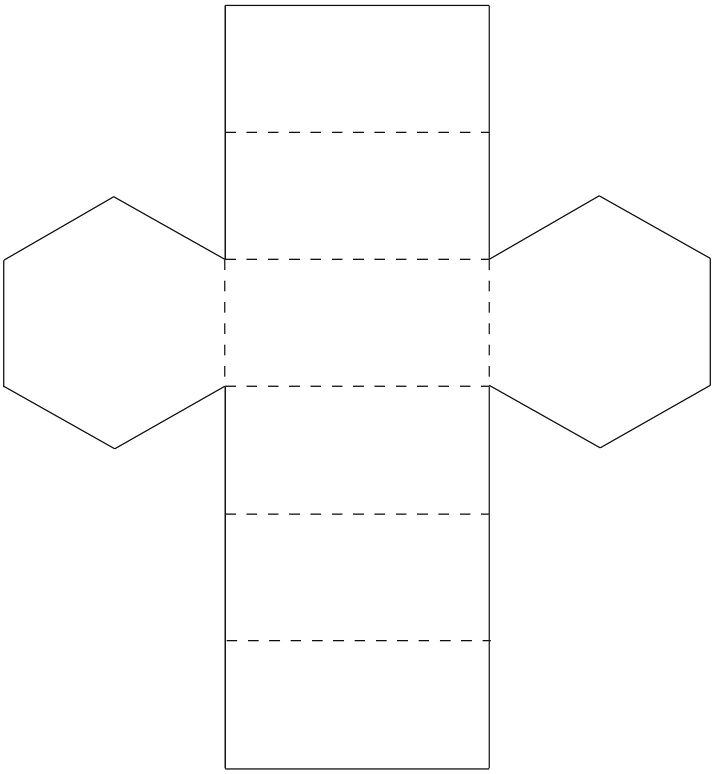
(c)



(d)

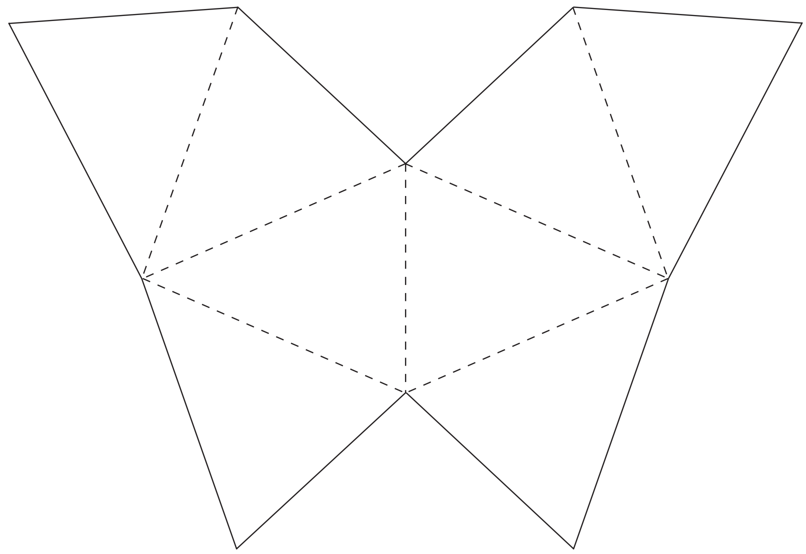


(e)

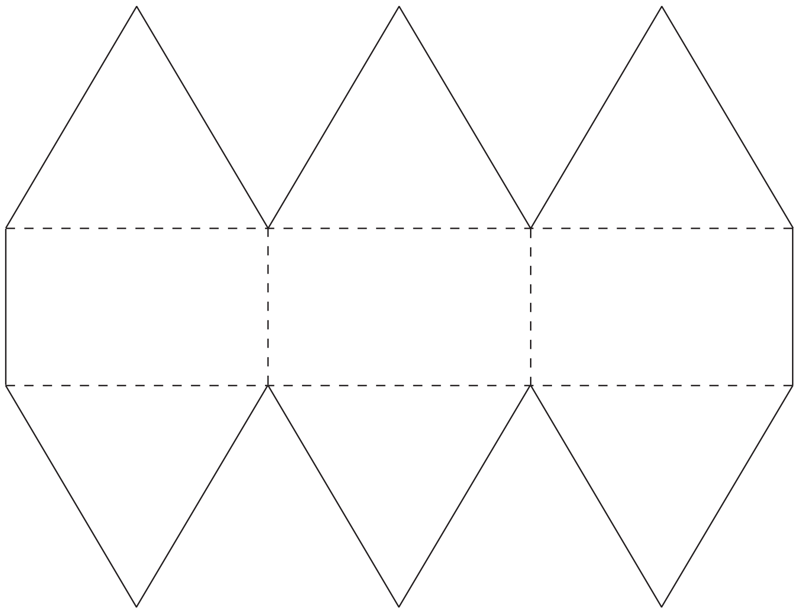


7.3

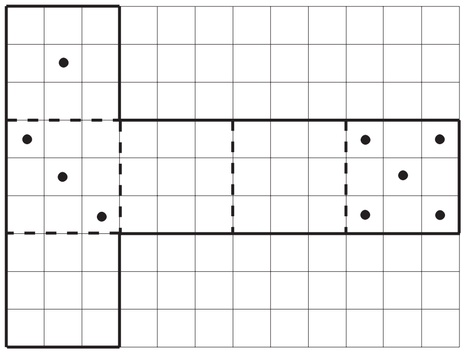
(f)



(g)



3. The diagram shows the net for a dice with some of the spots in place. Fill in the missing spots so that the opposite faces add up to 7. Then make the dice.

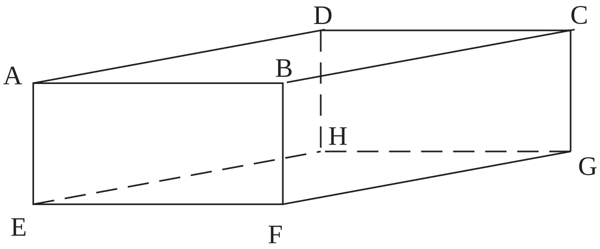


7.4 Constructing Nets

A net for a solid can be visualised by imagining that the shape is cut along its edges until it can be laid flat.

Worked Example 1

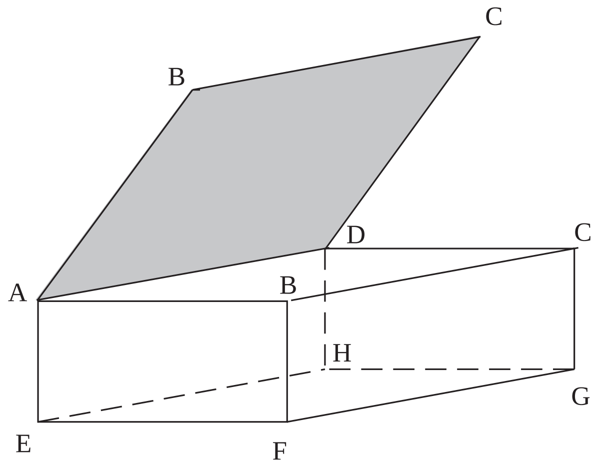
Draw the net for the cuboid shown in the diagram.

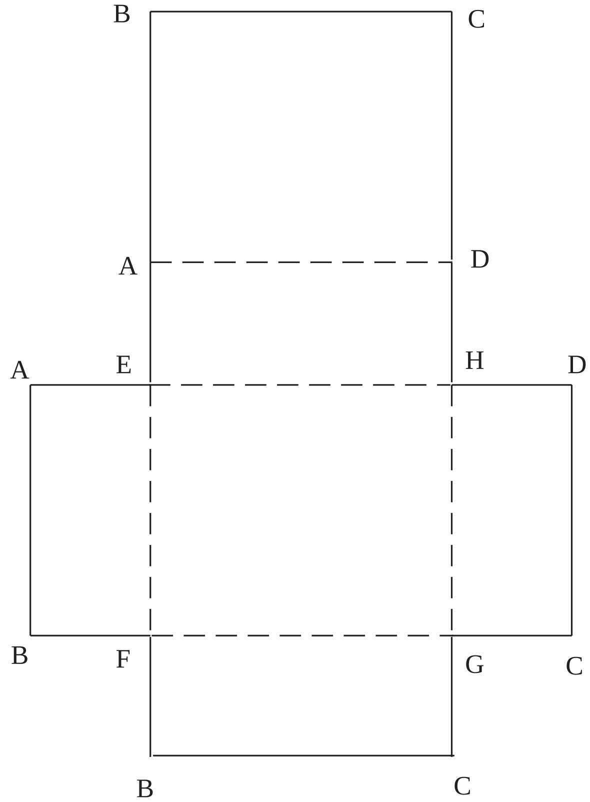


Solution

Imagine making cuts as below:

- cut along the edges AB, BC and CD to open the top like a flap.
- then cut down AE, BF, CG and DH, and press flat to give the net below.

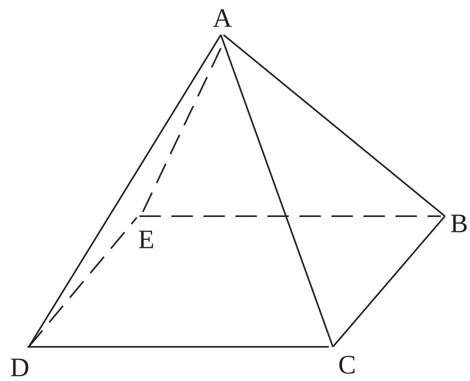




7.4

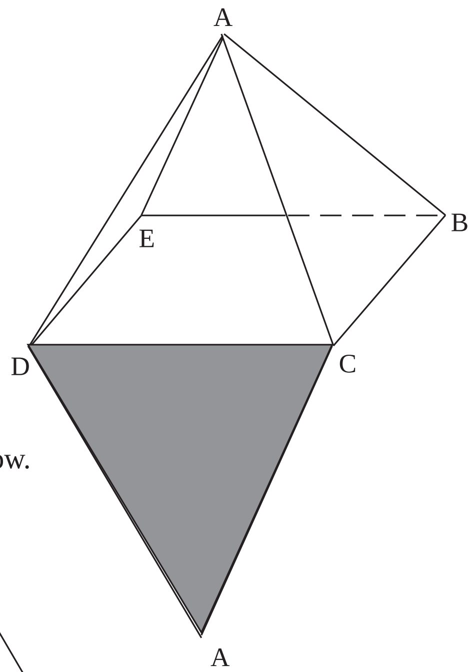
Worked Example 2

Draw the net for this square based pyramid.

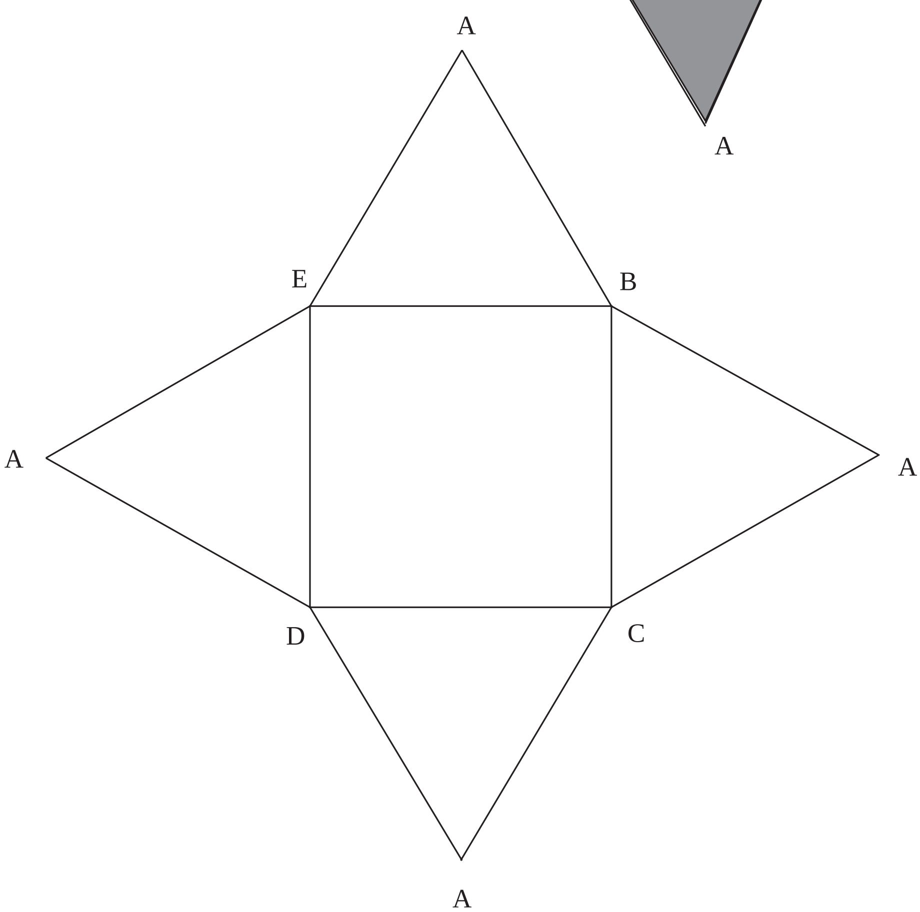


Solution

First imagine cutting down the edges AD and AC and opening out a triangle.



Then cutting down AB and AE gives the net below.

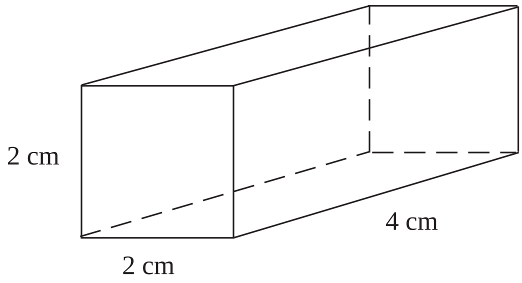


7.4

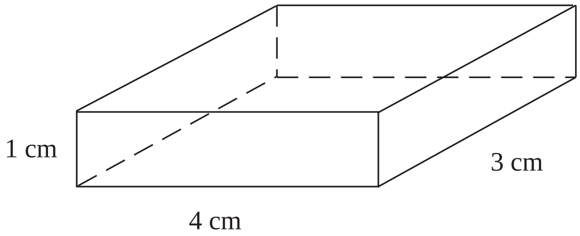
Exercises

1. Draw an accurate net for each cuboid below.

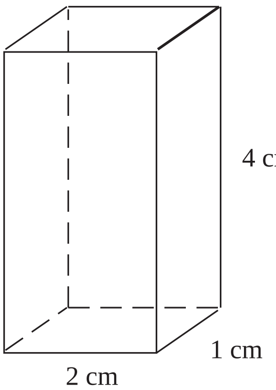
(a)



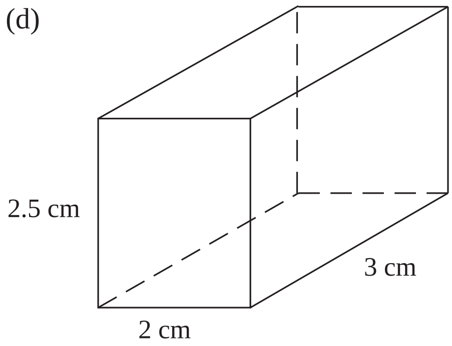
(b)



(c)

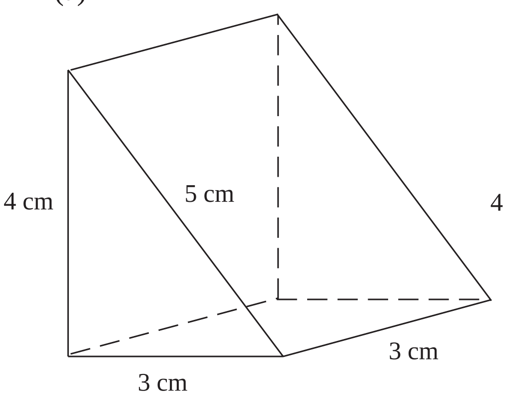


(d)

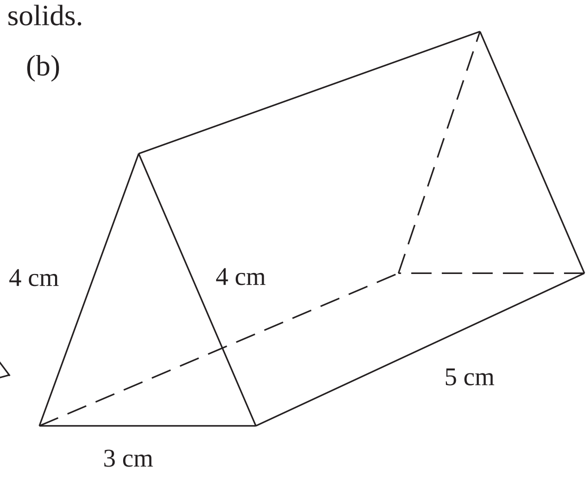


2. Draw a net for each of the following solids.

(a)

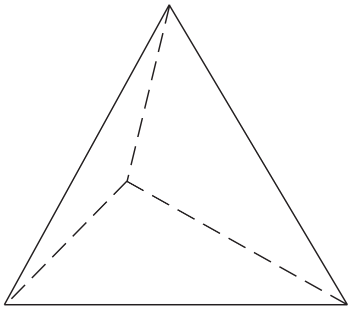


(b)



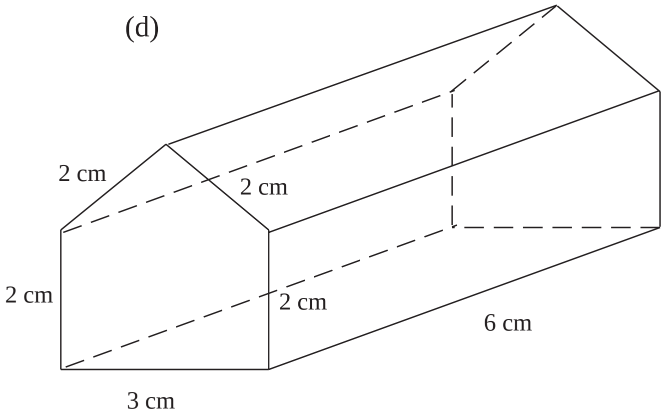
7.4

(c)

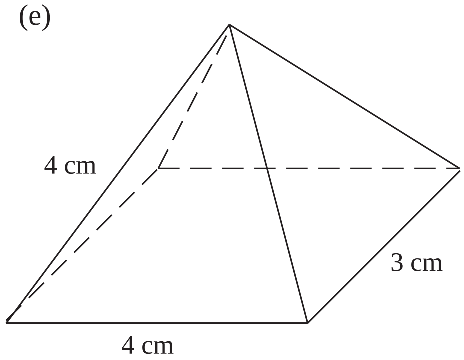


All edges 5 cm

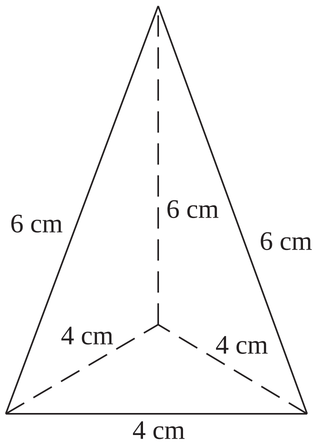
(d)



(e)



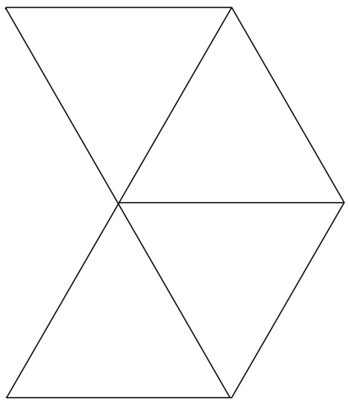
(f)



3. (a) Draw and cut out four equally sized equilateral triangles.

(b) How many different ways can they be arranged with sides joined together?

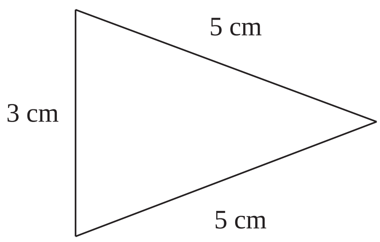
One example is shown.



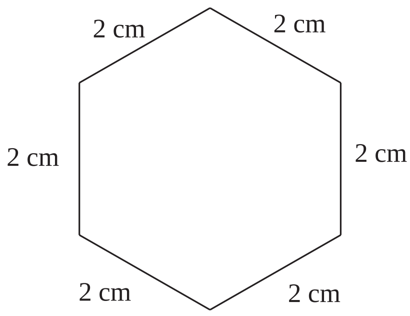
(c) Which of your arrangements of triangles form a net for a tetrahedron?

4. The diagrams below show the ends of two of prisms that each have length of 8 cm. Draw a net for each prism.

(a)

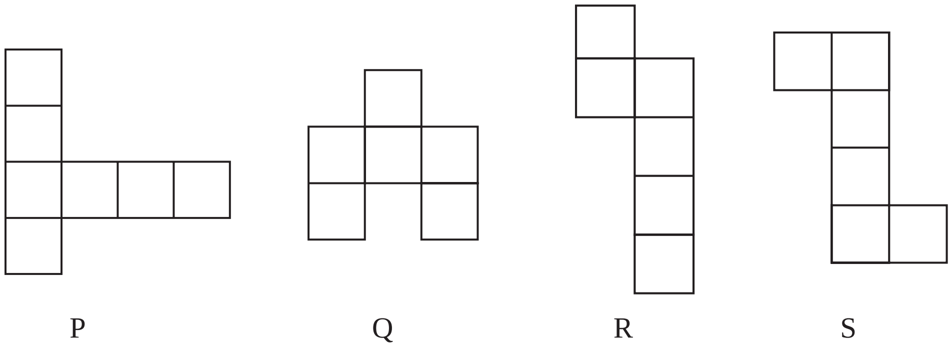


(b)



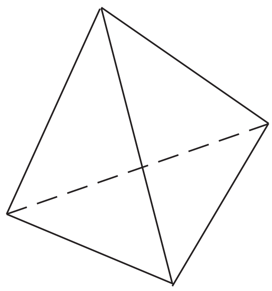
7.4

5. Which one of these nets can be folded to make a cube?



(SEG)

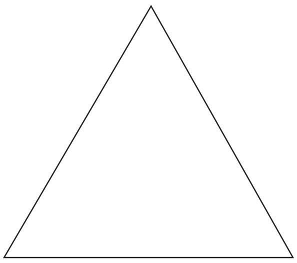
6.



The diagram above shows a pyramid with four equal triangular faces.

Each edge is 4 cm long.

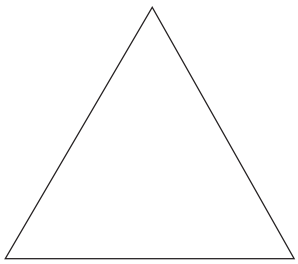
Below is one of the faces.



(a) What is the special name given to this kind of triangle?

(b) What is the size of each angle of this triangle?

(c) Construct an accurate net for the pyramid.
One face has been drawn for you.



(NEAB)

7.5 Conversion of Units

It is useful to be aware of both metric and imperial units and to be able to convert between them.

Imperial Units

1 foot	=	12 inches
1 yard	=	3 feet
1 pound (lb)	=	16 ounces
1 stone	=	14 pounds
1 gallon	=	8 pints

Conversion Facts

1 kg is about 2.2 lbs.

1 gallon is about 4.5 litres.
1 litre is about 1.75 pints.

5 miles is about 8 km.
1 inch is about 2.5 cm.
1 foot is about 30 cm.

Worked Example 1

John is measured. His height is 5 feet and 8 inches.

Find his height in:

(a) inches, (b) centimetres (c) metres.

Solution

(a) There are 12 inches in one foot, so

$$\begin{aligned} \text{John's height} &= 5 \times 12 + 8 \\ &= 60 + 8 \\ &= 68 \text{ inches} \end{aligned}$$

7.5

(b) 1 inch is about 2.5 cm, so

$$\text{John's height} = 68 \times 2.5$$
$$= 170 \text{ cm}$$

(c) 1 metre = 100 cm, so

$$\text{John's height} = 1.7 \text{ m}$$

Worked Example 2

A family travels 365 miles on holiday. Convert this distance to km.

Solution

As 5 miles is approximately equal to 8 km, first divide by 5 and then multiply by 8.

$$365 \div 5 = 73$$
$$73 \times 8 = 584$$

So 365 miles is approximately the same as 584 km.

Worked Example 3

Jared weighs 8 stone and 5 pounds. Find Jared's weight in:

(a) pounds,

(b) kg.

Solution

(a) There are 14 pounds in 1 stone, so

$$\text{Jared's weight} = 8 \times 14 + 5$$
$$= 112 + 5$$
$$= 117 \text{ lbs}$$

(b) As 1 pound is about 0.45 kg,

$$\text{Jared's weight} = 117 \times 0.45$$
$$= 53 \text{ kg (to the nearest kg)}$$

Worked Example 4

A line is 80 cm long. Convert this length to inches.

Solution

1 inch = 2.5 cm

$\frac{80}{2.5} = 32$, so the line is about 32 inches long.

7.5

Exercises

1. Convert each quantity to the units given.

(a) 3 inches to cm
(b) 18 stone to pounds
(c) 6 lbs to ounces
(d) 6 feet 3 inches to inches
(e) 15 kg to lbs
(f) 3 yards to inches
(g) 3 feet to cm
(h) 5 gallons to litres
(i) 120 inches to cm
(j) 45 kg to lbs
(k) 9 litres to pints
(l) 45 gallons to litres
(m) 8 litres to pints
(n) 6 gallons to pints

2. Convert each quantity to the units given. Give answers to 1 decimal place.

(a) 8 lbs to kg
(b) 3 lbs to kg
(c) 16 pints to litres
(d) 10 cm to inches
(e) 400 cm to feet
(f) 80 ounces to pounds
(g) 182 lbs to stones
(h) 50 litres to gallons
(i) 84 inches to feet
(j) 52 cm to inches
(k) 16 litres to gallons
(l) 3 pints to litres
(m) 6 lbs to kg
(n) 212 cm to feet

3. The table gives the distances between some towns in miles. Convert the distances to km, giving your answer to the nearest km.

	Norwich	Great Yarmouth	Lowestoft
Great Yarmouth	19		
Lowestoft	27	11	
Beccles	18	20	9

4. A car travels on average 10 km for every litre of petrol. The car is driven from Leicester to Peterborough, a distance of 41 miles.

(a) How far does the car travel in km?
(b) How many litres of petrol are used?
(c) How many gallons of petrol are used?

7.5

5. A recipe for a large cake includes the following ingredients.

$\frac{3}{4}$ pint	orange juice
3 lbs	flour
$\frac{1}{2}$ lb	butter
2 lbs	mixed fruit

Convert these units to litres or kg, giving your answers to 2 decimal places.

6. The Krishnan family is going on holiday with their caravan. The length of their car is 12 feet 10 inches and the length of their caravan is 16 feet 8 inches.

   Find the total length of the car and caravan in

   (a) inches, (b) cm, (c) metres.

7. James is 6 feet 2 inches tall and weighs 11 stone 5 pounds.

   Michael is 180 cm tall and weighs 68 kg.

   Who is the taller and who is the heavier?

8. Jane and Christopher go strawberry picking. Jane picks 8 kg and Christopher picks 15 lbs. Who has picked the greater weight of strawberries?

9. A customer asks for a sheet of glass 15 inches by 24 inches, What would be the area of the glass in cm^2?

10. Rohan is going to buy a new car. He tries out two different ones.

    The first car he tries out travels 50 miles on 2 gallons of petrol.

    The second car travels 100 km on 12 litres of petrol.

    Find the petrol consumption in litres per km for both cars.

    Which is the more economical?

11. Here is a rule to change miles into kilometres.

Multiply the number of miles by 8 Divide by 5

(a) Use this rule to change 30 miles into kilometres.

(b) Write down an equation connecting kilometres (K) and miles (M).

(c) Use your equation to find the value of M when $K = 100$.

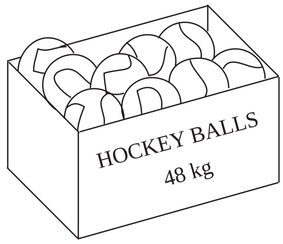
(NEAB)

7.5

12. (a) Convert 48 kg to grams.

A box contains 280 hockey balls.

The hockey balls weigh 48 kg.



(b) Calculate the weight of one hockey ball to the nearest gram.

One kilogram is approximately 2.2 pounds.

(c) Estimate the weight of the box of hockey balls in pounds.

(SEG)

13. The same quantity can sometimes be measured in different units.

(a) Write out the statement below, filling in the missing unit.

Choose the unit from this list:

millimetres, centimetres, metres, kilometres

1 inch = 2.54 . . . .

(b) Write out the statement below, filling in the missing unit.

Choose the unit from this list:

millimetres, litres, gallons, cubic metres

4 pints = 2.27 . . . .

(MEG)

14. (a) Megan is 5 feet 3 inches tall.

1 cm = 0.394 inches

12 inches = 1 foot

Calculate Megan's height in centimetres. Give your answer to an appropriate degree of accuracy.

(b) An electronic weighing scale gives Megan's weight as 63.4792 kg.

Give her weight correct to an appropriate degree of accuracy.

(SEG)

15. A ball bearing has mass 0.44 pounds.

1 kg = 2.2 pounds.

(a) Calculate the mass of the ball bearing in kilograms.

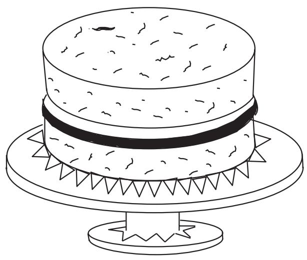
(b) $\text{Density} = \dfrac{\text{Mass}}{\text{Volume}}$

When the mass is measured in kg and the volume is measured in cm^3, what are the units of the density?

(SEG)

7.5

16. A recipe for a cake for four people uses



4 eggs

8 ounces sugar

4 ounces butter

14 ounces flour

$\frac{1}{4}$ pint milk

16 ounces = 1 pound

James finds a 500 g bag of flour in the cupboard.

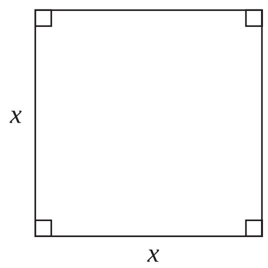
Will he have enough flour for this recipe?

Clearly explain your reasoning.

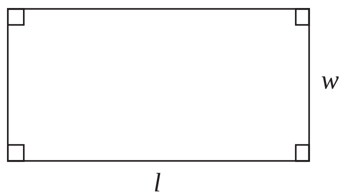
(NEAB)

7.6 Squares, Rectangles and Triangles

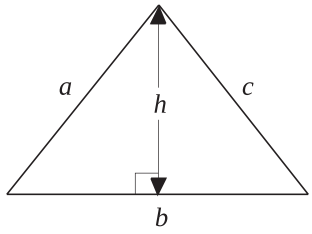
For a *square*, the area is given by $x \times x = x^2$ and the perimeter by $4x$, where x is the length of a side.



For a *rectangle*, the area is given by lw and the perimeter by $2(l + w)$, where l is the length and w the width.



For a *triangle*, the area is given by $\frac{1}{2}bh$ and the perimeter by $a + b + c$, where b is the length of the base, h the height and a and c are the lengths of the other two sides.

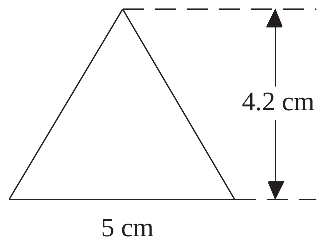


7.6

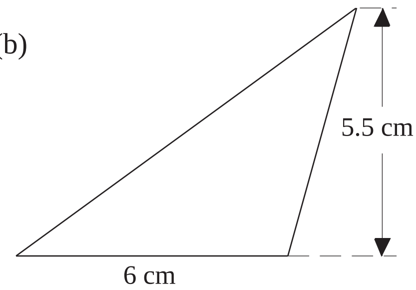
Worked Example 1

Find the area of each triangle below.

(a)



(b)



Solution

Use Area $= \frac{1}{2}bh$ or $\frac{1}{2} \times \text{base} \times \text{height}$.

(a) Area $= \frac{1}{2} \times 5 \times 4.2$

$= 10.5 \text{ cm}^2$

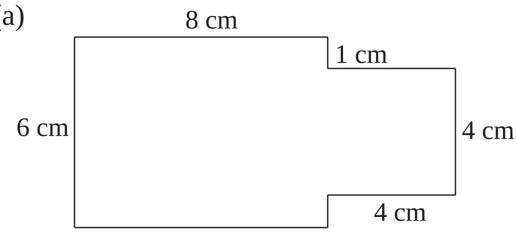
(b) Area $= \frac{1}{2} \times 6 \times 5.5$

$= 16.5 \text{ cm}^2$

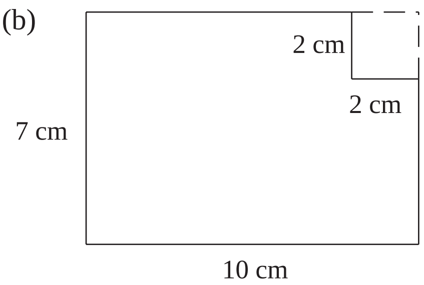
Worked Example 2

Find the perimeter and area of each shape below.

(a)



(b)



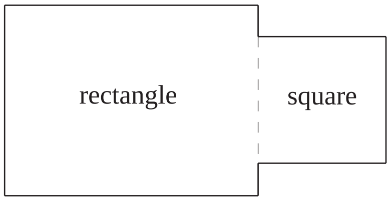
Solution

(a) The perimeter is found by adding the lengths of all the sides.

$P = 6 + 8 + 1 + 4 + 4 + 4 + 1 + 8$

$= 36 \text{ cm}$

To find the area, consider the shape split into a rectangle and a square.



Area = Area of rectangle + Area of square

$= 6 \times 8 + 4^2$

$= 48 + 16$

$= 64 \text{ cm}^2$

7.6

(b) Adding the lengths of the sides gives

$$P = 10 + 7 + 8 + 2 + 2 + 5$$

$$= 34 \text{ cm}$$

The area can be found by considering the shape to be a rectangle with a square removed from it.

$$\text{Area of shape} = \text{Area of rectangle} - \text{Area of square}$$

$$= 7 \times 10 - 2^2$$

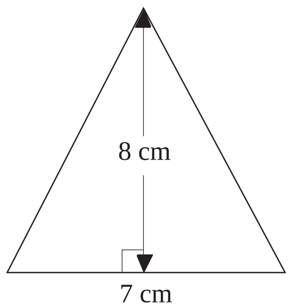
$$= 70 - 4$$

$$= 66 \text{ cm}^2$$

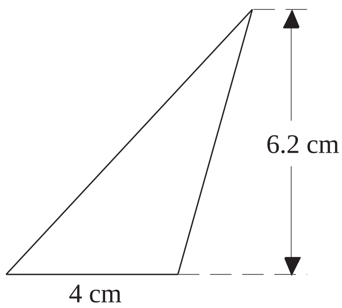
Exercises

1. Find the area of each triangle.

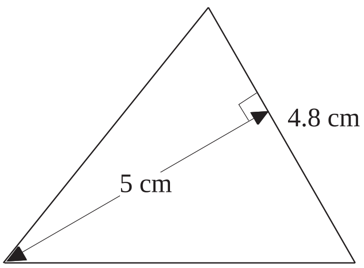
(a)



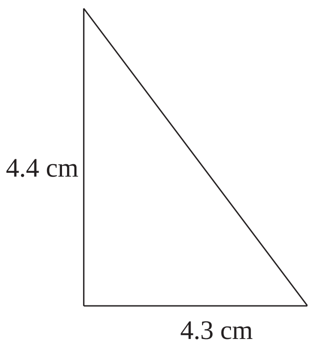
(b)



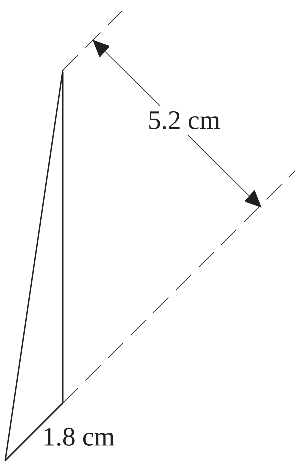
(c)



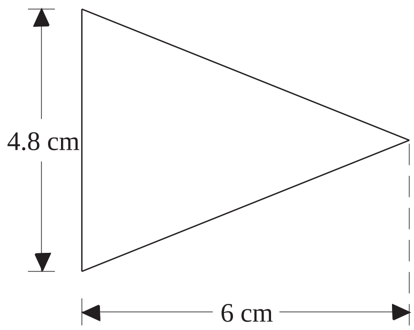
(d)



(e)



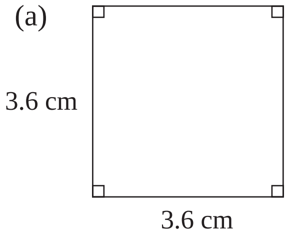
(f)



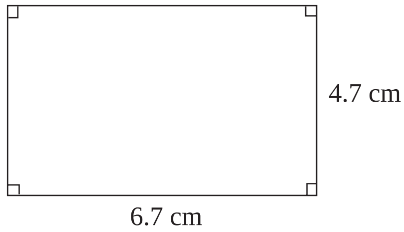
7.6

2. Find the perimeter and area of each shape below.

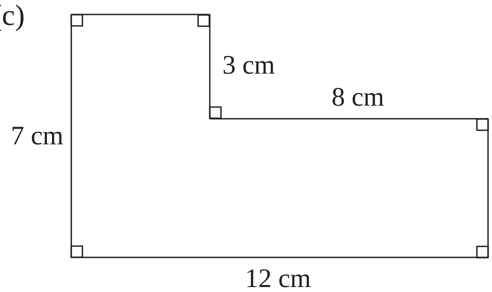
(a)



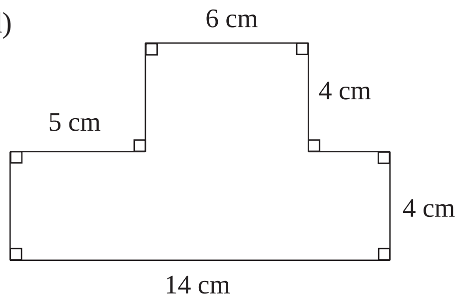
(b)



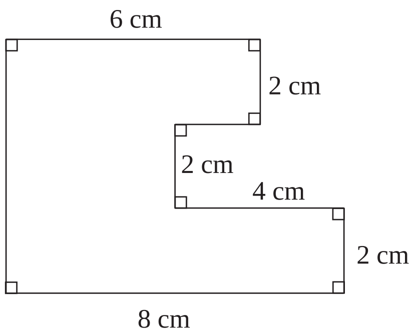
(c)



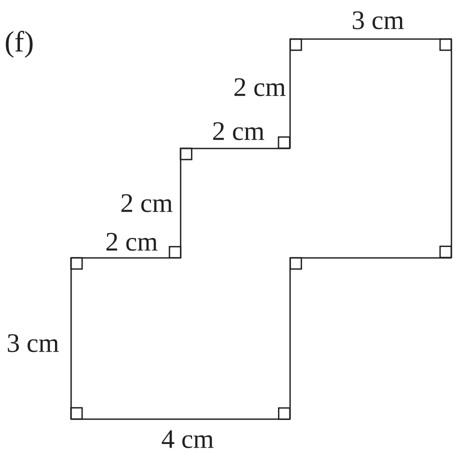
(d)



(e)

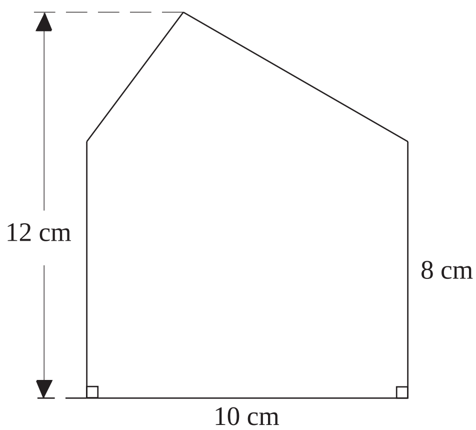


(f)

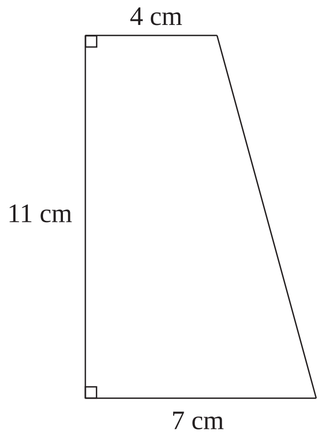


3. Find the area of each shape.

(a)

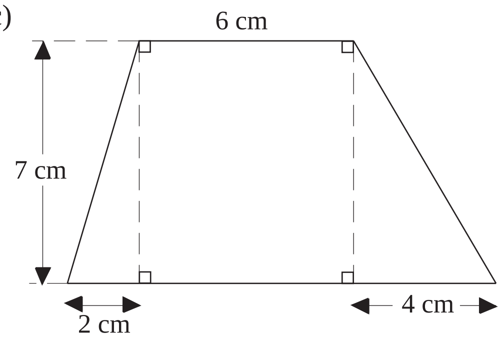


(b)

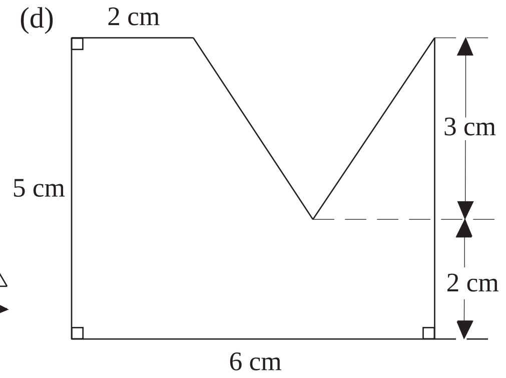


7.6

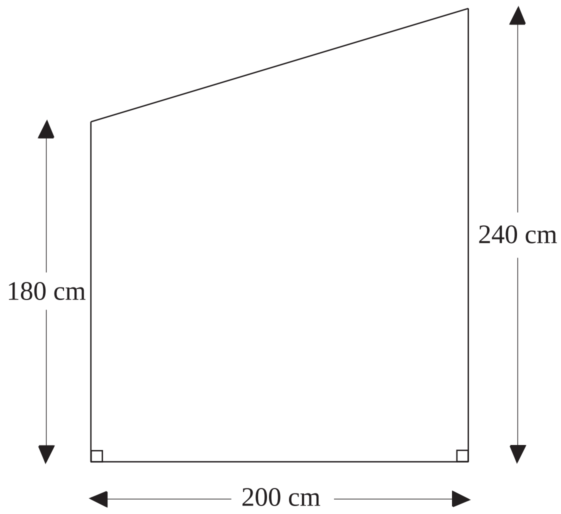
(c)



(d)



4. The diagram shows the end wall of a shed built out of concrete bricks.

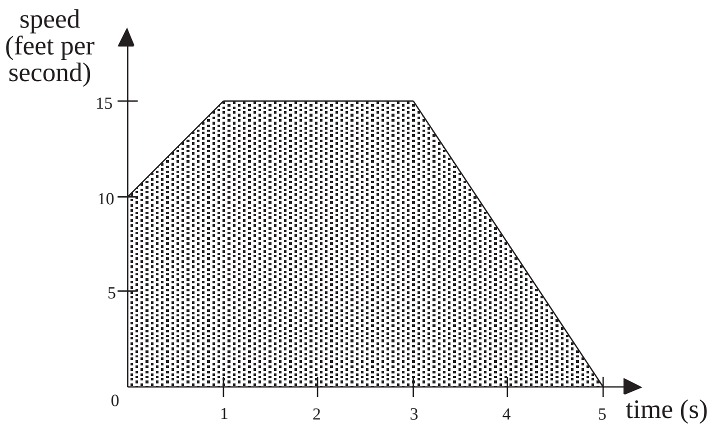


(a) Find the area of the wall.

(b) The blocks are 45 cm by 23 cm in size.

How many blocks would be needed to build the wall? (The blocks can be cut.)

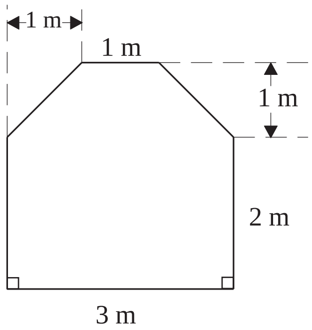
5. The shaded area on the speed time graph represents the distance travelled by a car. Find the distance.



7.6

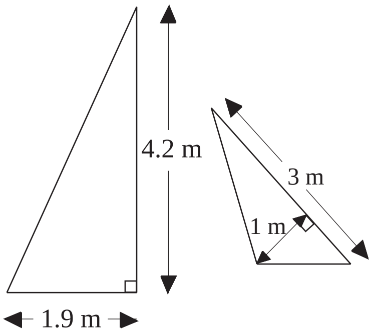
6. The plan shows the base of a conservatory.

   Find the area of the base.



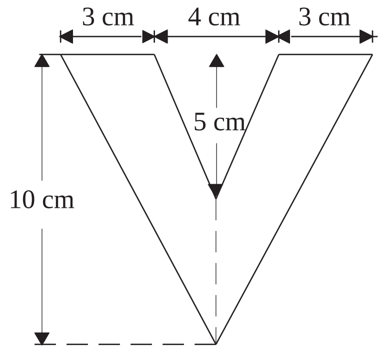
7. The diagram shows the two sails from a dinghy.

   Find their combined area.

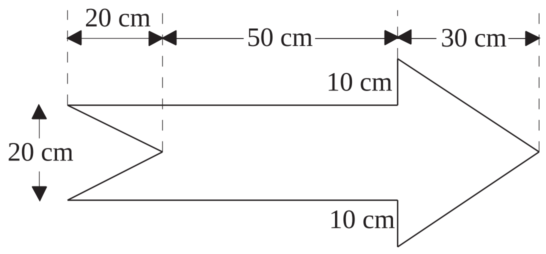


8. The diagram shows the letter V.

   Find the area of this letter.



9. Find the area of the arrow shown in the diagram.

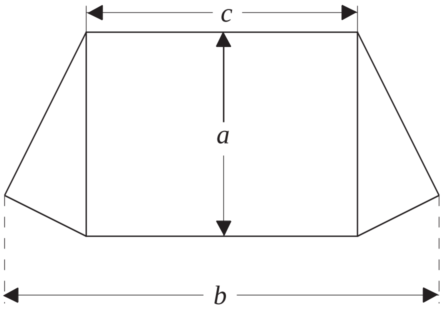


10. The diagram shows how the material required for one side of a tent is cut out.

    (a) Find the area of the material shown if: $b = 3.2$ m, $c = 2$ m and

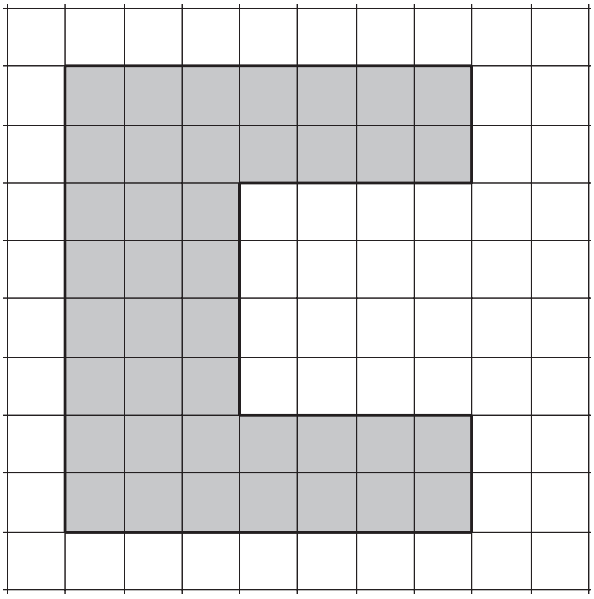
    (i) $a = 1.5$ m (ii) $a = 2$ m

    (b) Find the area if $a = 1.6$ m, $b = 3.4$ m, and $c = 2$ m.



7.6

11.



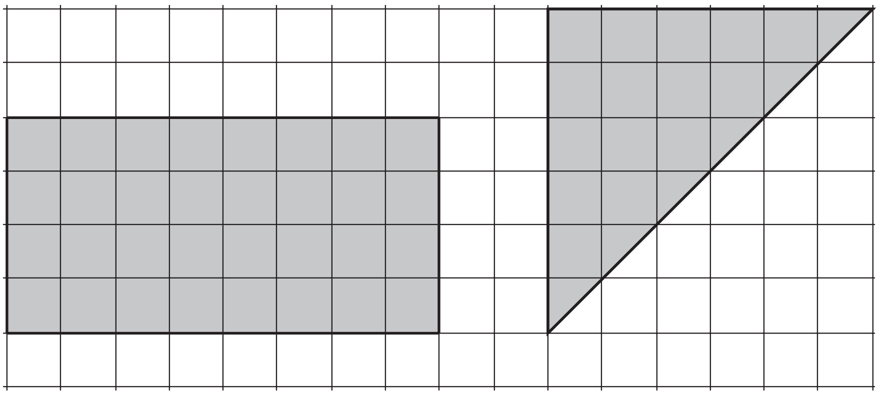
The  shape above is shaded on centimetre squared paper.

(a) Find the perimeter of this shape.

(b) Find the area of this shape.

(MEG)

12.



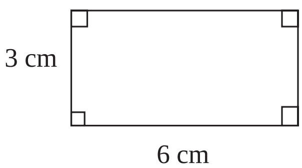
(a) What is the perimeter of the rectangle?

(b) What is the area of the triangle?

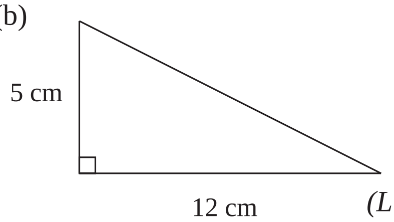
(SEG)

13. Work out the areas of these shapes.

(a)



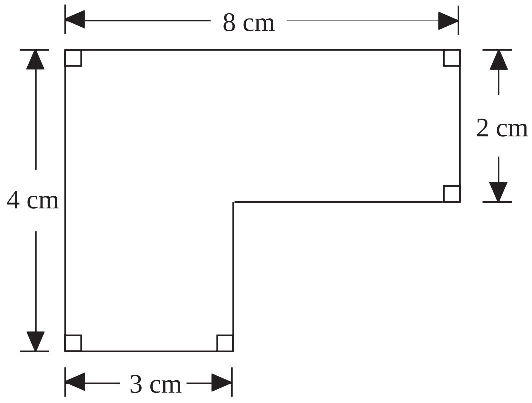
(b)



(LON)

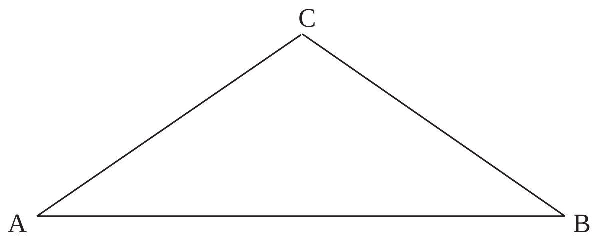
7.6

14. Calculate the area of this shape.



(LON)

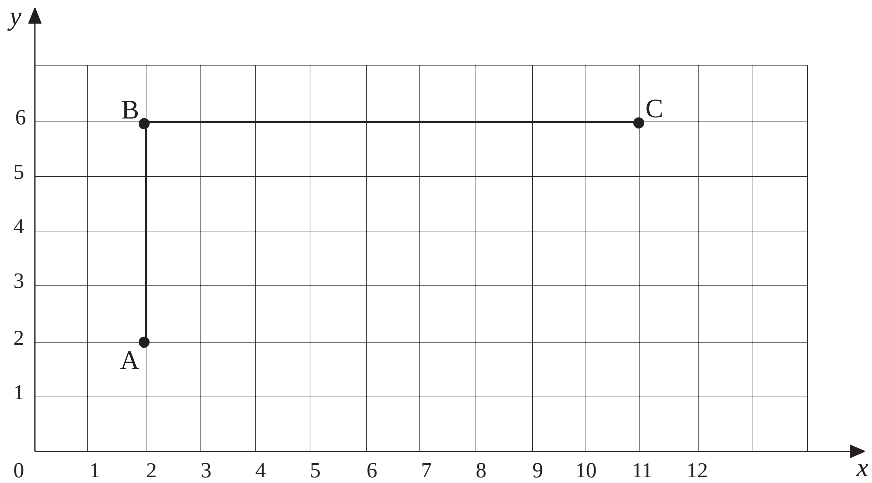
15.



By making and using appropriate measurements, calculate the area of triangle ABC in square centimetres. State the measurements that you have made and show your working clearly.

(MEG)

16.



(a) Write down the coordinates of the mid-point of AC.

(b) Copy the diagram and mark and label a point D so that ABCD is a rectangle.

(c) (i) Find the perimeter of the rectangle ABCD.

(ii) Find the area of the rectangle ABCD.

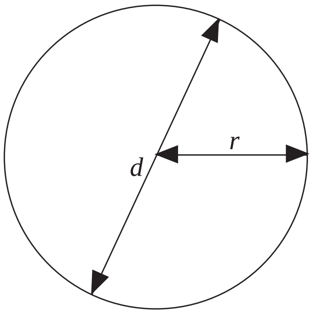
(d) The rectangle has reflective (line) symmetry.

Describe another type of symmetry that it has.

(MEG)

7.7 Area and Circumference of Circles

The *circumference* of a circle can be calculated using



$$C = 2\pi r \quad \text{or} \quad C = \pi d$$

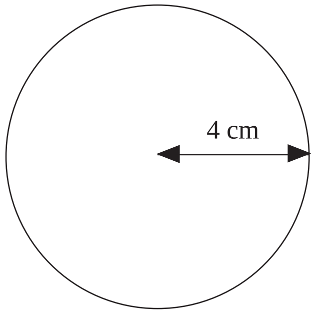
where r is the radius and d the diameter of the circle.

The *area* of a circle is found using

$$A = \pi r^2 \quad \text{or} \quad A = \frac{\pi d^2}{4}$$

Worked Example 1

Find the circumference and area of this circle.



Solution

The circumference is found using $C = 2\pi r$, which in this case gives

$$C = 2\pi \times 4$$

$$= 25.1 \text{ cm} \qquad \text{(to one decimal place)}$$

The area is found using $A = \pi r^2$, which gives

$$A = \pi \times 4^2$$

$$= 50.3 \text{ cm}^2 \qquad \text{(to one decimal place)}$$

Worked Example 2

Find the radius of a circle if:

(a) its circumference is 32 cm,

(b) its area is 14.3 cm^2.

Solution

(a) Using $C = 2\pi r$ gives

$$32 = 2\pi r$$

and dividing by 2π gives

$$\frac{32}{2\pi} = r$$

so that

$$r = 5.10 \text{ cm} \quad \text{(to 2 decimal places)}$$

(b) Using $A = \pi r^2$ gives

$$14.3 = \pi r^2$$

7.7

Dividing by π gives

$$\frac{14.3}{\pi} = r^2$$

Then taking the square root of both sides gives

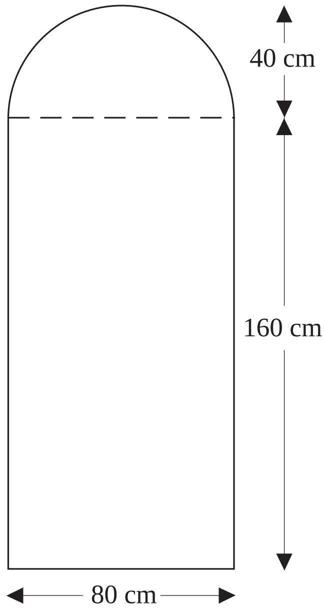
$$\sqrt{\frac{14.3}{\pi}} = r$$

so that

$$r = 2.13 \text{ cm} \quad \text{(to 2 decimal places)}$$

Worked Example 3

Find the area of the door shown in the diagram.
The top part of the door is a semicircle.



Solution

First find the area of the rectangle.

$$\begin{aligned}\text{Area} &= 80 \times 160 \\ &= 12\,800 \text{ cm}^2\end{aligned}$$

Then find the area of the semicircle.

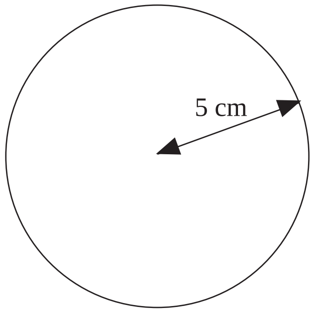
$$\begin{aligned}\text{Area} &= \frac{1}{2} \times \pi \times 40^2 \\ &= 2513 \text{ cm}^2\end{aligned}$$

$$\begin{aligned}\text{Total area} &= 12\,800 + 2513 \\ &= 15\,313 \text{ cm}^2 \quad (\text{to the nearest cm}^2)\end{aligned}$$

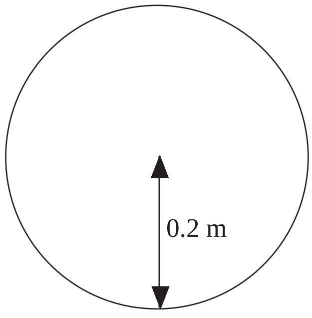
Exercises

1. Find the circumference and area of each circle shown below.

(a)

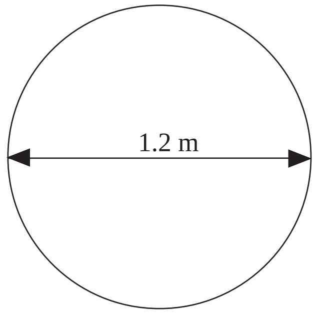


(b)

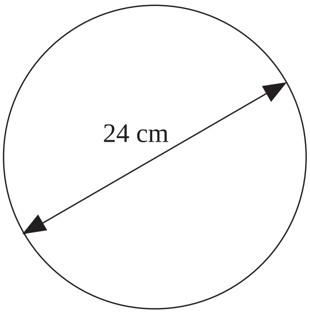


7.7

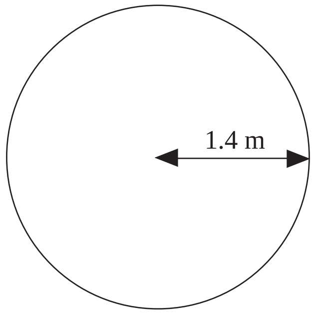
(c)



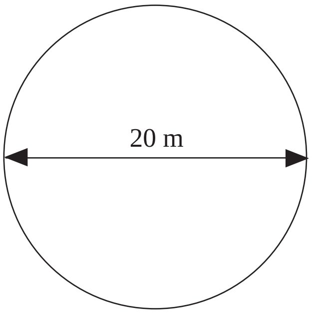
(d)



(e)



(f)



2. Find the radius of the circle which has:

   (a) a circumference of 42 cm,

   (b) a circumference of 18 cm,

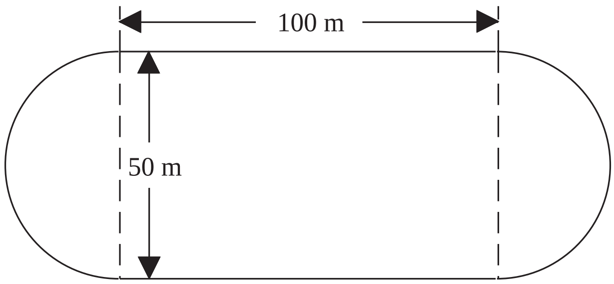
   (c) an area of 69.4 cm^2,

   (d) an area of 91.6 cm^2.

3. The diagram shows a running track.

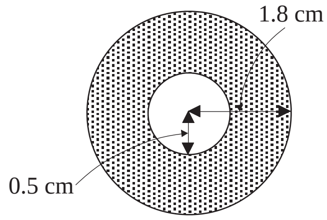
   (a) Find the length of one complete circuit of the track.

   (b) Find the area enclosed by the track.



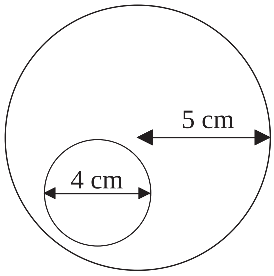
4. A washer has an outer radius of 1.8 cm and an inner radius of 0.5 cm.

   Find the area that has been shaded in the diagram, to the nearest cm^2.



7.7

5. An egg, fried perfectly, can be thought of as a circle (the yolk) within a larger circle (the white).



(a) Find the area of the smaller circle that represents the surface of the yolk.

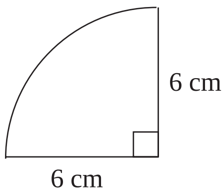
(b) Find the area of the surface of the whole egg.

(c) Find the area of the surface of the white of the egg, to the nearest cm^2 .

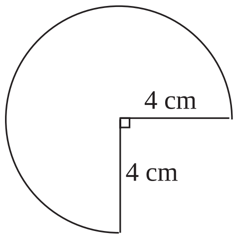
6. The shapes shown below were cut out of card, ready to make cones.

Find the area of each shape.

(a)



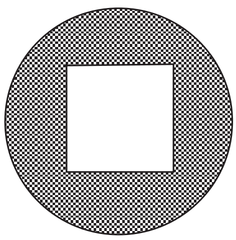
(b)



7. A circular hole with diameter 5 cm is cut out of a rectangular metal plate of length 10 cm and width 7 cm. Find the area of the plate when the hole has been cut out.

8. Find the area of the wasted material if two circles of radius 4 cm are cut out of a rectangular sheet of material that is 16 cm long and 8 cm wide.

9. A square hole is cut in a circular piece of card to create the shape shown.

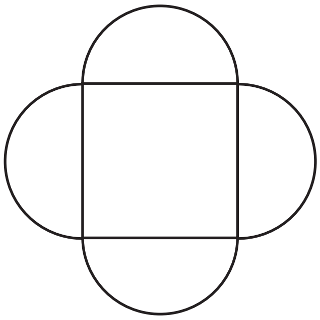


(a) Find the shaded area of the card if the radius of the circle is 5.2 cm and the sides of the square are 4.8 cm.

(b) Find the radius of the circle if the shaded area is 50 cm^2 and the square has sides of length 4.2 cm.

7.7

10. Four semicircles are fixed to the sides of a square as shown in the diagram, to form a design for a table top.



(a) Find the area of the table top if the square has sides of length 1.5 m.

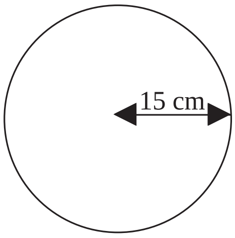
(b) Find the length of the sides of the square and the total area of the table top if the area of each semicircle is 1 m^2.

11. The radius of a circle is 8 cm.

Work out the area of the circle.
(Use $\pi = 3.14$ or the π button on your calculator.)

(LON)

12. A circle has a radius of 15 cm.



Calculate the area of the circle.

Take π to be 3.14 or use the π key on your calculator.

(SEG)

13. Louise does a sponsored bicycle ride.

Each wheel of her bicycle is of radius 25 cm.

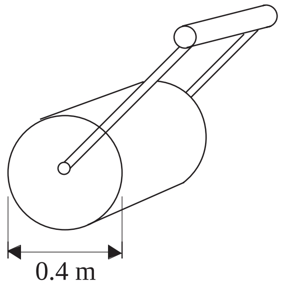
(a) Calculate the circumference of one of the wheels

(b) She cycles 50 km. How many revolutions does a wheel make during the sponsored ride?

(NEAB)

7.7

14. The diameter of a garden roller is 0.4 m.



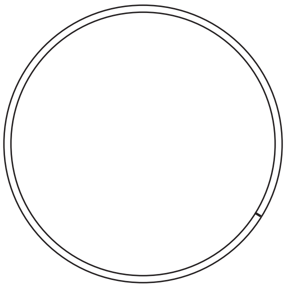
The roller is used on a path of length 20 m.

Calculate how many times the roller rotates when rolling the length of the path once.

Take π to be 3.14 or use the π key on your calculator.

(SEG)

15.



A piece of rope is 12 metres long. It is laid on the ground in a circle, as shown in the diagram.

(a) Using 3.14 as the value of π, calculate the diameter of the circle.

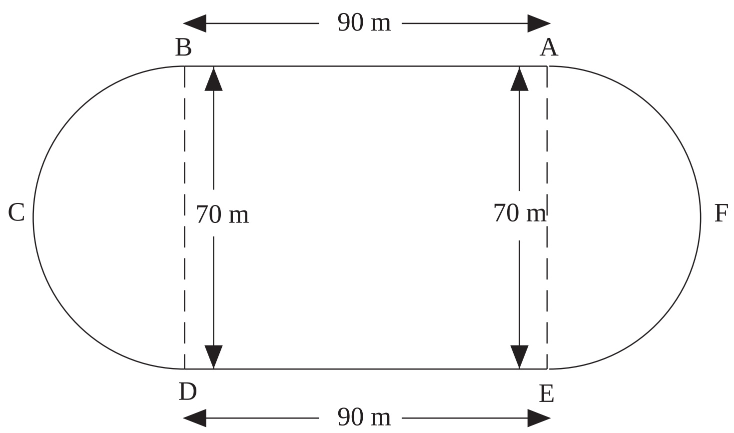
(b) Explain briefly how you would check the answer to part (a) mentally.

The cross-section of the rope is a circle of radius 1.2 cm.

(c) Calculate the area of the cross-section.

(MEG)

16.



The diagram shows a running track.

BA and DE are parallel and straight. They are each of length 90 metres.

BCD and EFA are semicircular. They each have a diameter of length 70 metres.

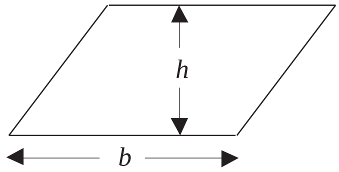
(a) Calculate the perimeter of the track.

(b) Calculate the total area inside the track.

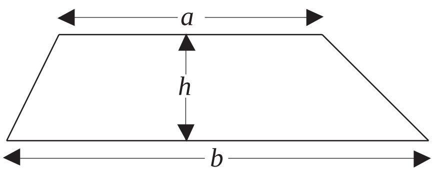
7.8 Areas of Parallelograms, Trapeziums, Kites and Rhombuses

The formulae for calculating the areas of these shapes are:

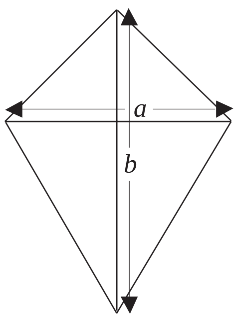
Parallelogram $A = bh$



Trapezium $A = \frac{1}{2}(a+b)h$



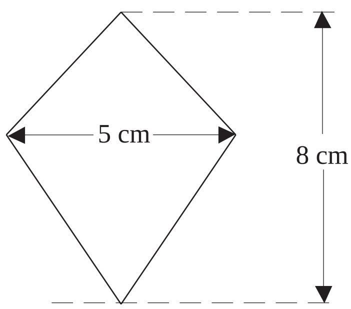
Kite $A = \frac{1}{2}ab$



The area of a *rhombus* can be found using either the formula for a kite or the formula for a parallelogram.

Worked Example 1

Find the area of this kite.



Solution

Using the formula $A = \frac{1}{2}ab$

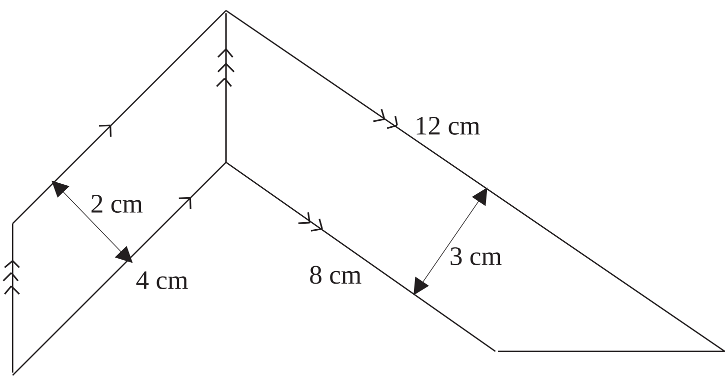
with $a = 5$ and $b = 8$ gives

$$A = \frac{1}{2} \times 5 \times 8$$

$$= 20 \text{ cm}^2$$

Worked Example 2

Find the area of this shape.



7.8

Solution

The shape is made up of a parallelogram and a trapezium.

$$\text{Area of parallelogram} = 2 \times 4 = 8 \text{ cm}^2$$

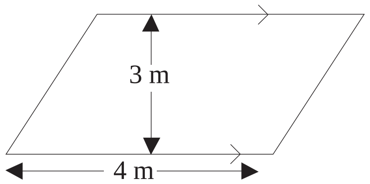
$$\text{Area of trapezium} = \frac{1}{2}(8 + 12) \times 3 = 30 \text{ cm}^2$$

$$\text{Total area} = 8 + 30 = 38 \text{ cm}^2$$

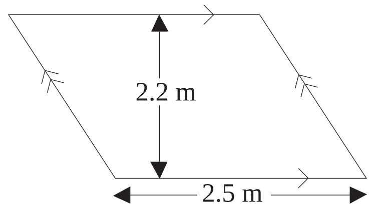
Exercises

Find the area of each of the following shapes.

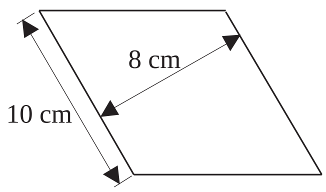
1. (a)



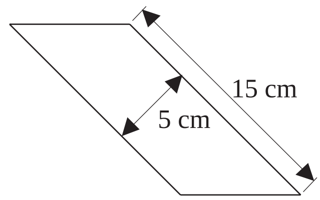
(b)



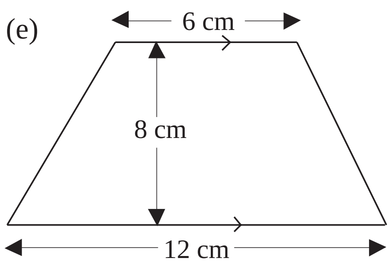
(c)



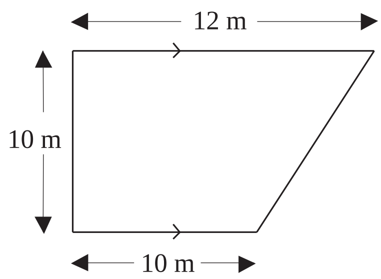
(d)



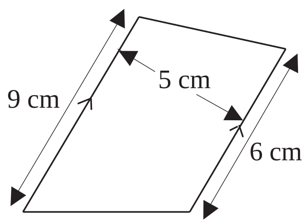
(e)



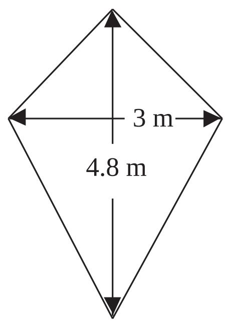
(f)



(g)

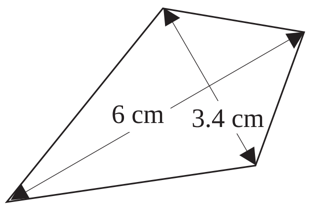


(h)



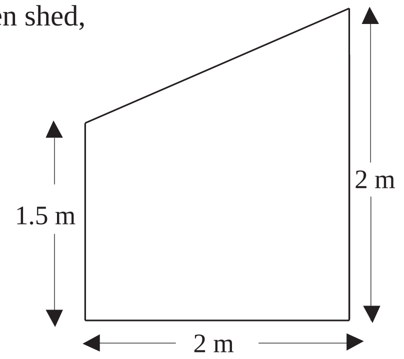
7.8

(i)



2. The diagram shown the end wall of a wooden garden shed,

(a) Find the area of this end of the shed.

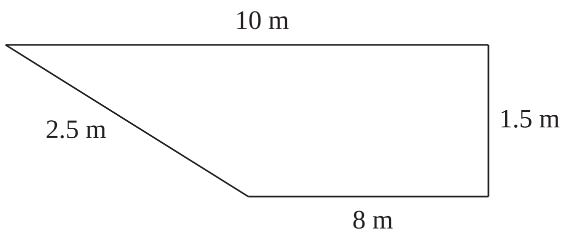


The other end of the shed is identical. The sides are made up of two rectangles of length 30 m.

(b) Find the area of each side of the shed.

(c) Find the total area of the walls of the shed.

3. The diagram shows the vertical side of a swimming pool.



(a) Find the area of the side of the pool.

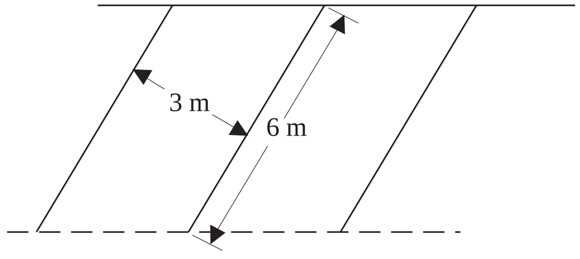
The width of the swimming pool is 4 m.

(b) Find the area of the rectangular end of the swimming pool.

(c) Find the area of the horizontal base of the pool.

(d) Find the total area of the sides and horizontal base of the pool.

4. In a car park, spaces are marked out in parallelograms.

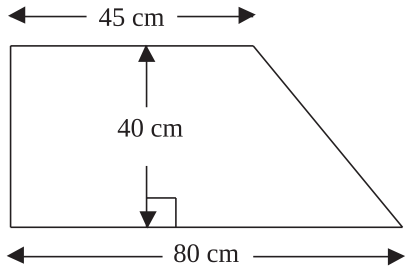


Find the area of each parking space.

7.8

5. The diagram shows a window of a car.

   Find the area of the window.

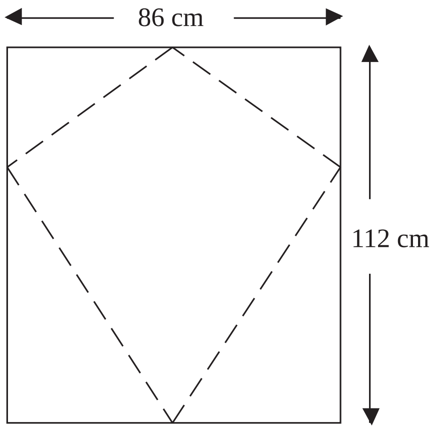


6. A kite is cut out of a sheet of plastic as shown.

   (a) Find the area of the kite.

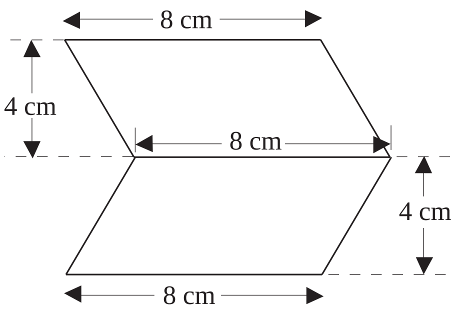
   (b) Find the area of the plastic that would be wasted.

   (c) Would you obtain similar results if you cut a kite out of a rectangle of plastic with dimensions 140 cm by 80 cm?

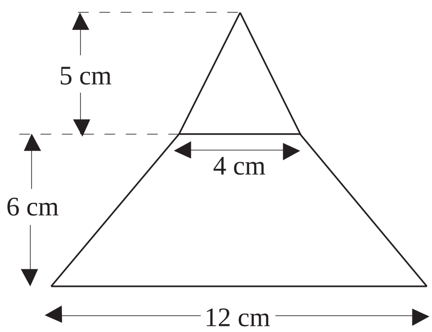


7. Find the area of each of the following shapes.

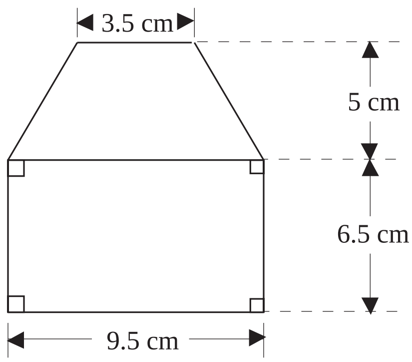
(a)



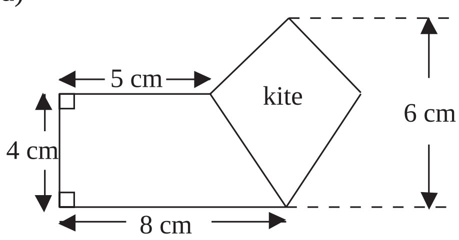
(b)



(c)

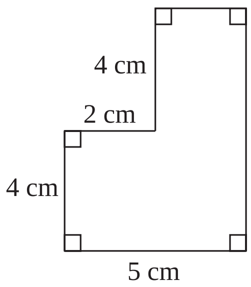


(d)

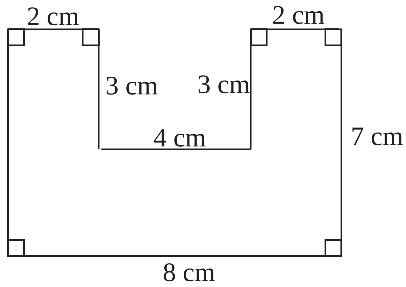


7.8

(e)

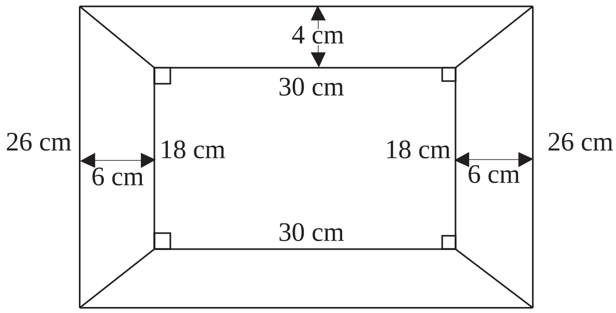


(f)

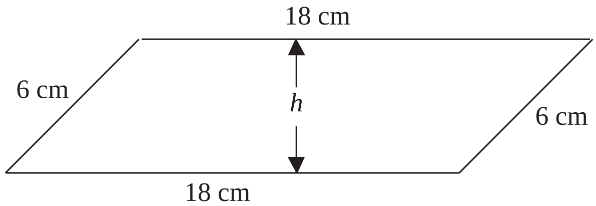


8. A simple picture frame is made by joining four trapezium shaped strips of wood.

Find the area of each trapezium and the total area of the frame.



9. Four rods are joined together to form a parallelogram.



(a) Find the area of the parallelogram if:

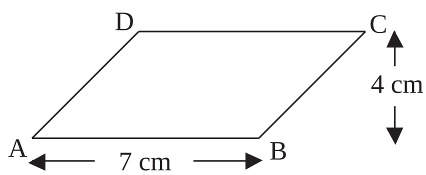
(i) $h = 2$ cm (ii) $h = 4$ cm (iii) $h = 5$ cm

(b) Can h be higher than 6 cm?

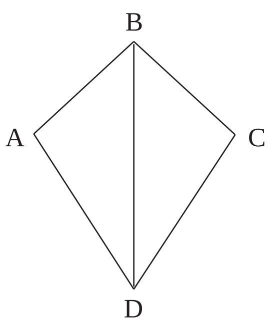
(c) What is the maximum possible area of the parallelogram?

10. (a) Find the area of parallelogram ABCD.

(b) Find the area of the triangle ABC.



11.



Not to scale

Why is the area of the kite ABCD equal to twice the area of the triangle ABD?

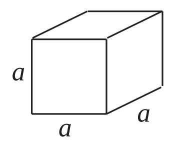
(MEG)

7.9 Volumes of Cubes, Cuboids, Cylinders and Prisms

The volume of a *cube* is given by

$$V = a^3$$

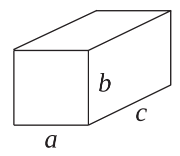
where a is the length of each side of the cube.



For a *cuboid* the volume is given by

$$V = abc$$

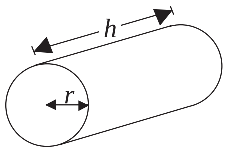
where a, b and c are the lengths shown in the diagram.



The volume of a *cylinder* is given by

$$V = \pi r^2 h$$

where r is the radius of the cylinder and h is its height.



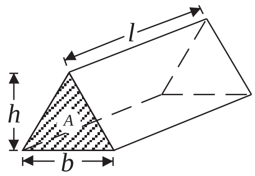
The volume of a *triangular prism* can be expressed in two ways, as

$$V = Al$$

where A is the area of the end and l the length of the prism, or as

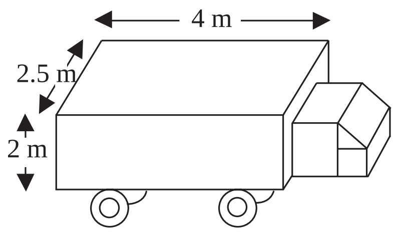
$$V = \frac{1}{2}bhl$$

where b is the base of the triangle and h is the height of the triangle.



Worked Example 1

The diagram shows a lorry.



Find the volume of the load-carrying part of the lorry.

7.9

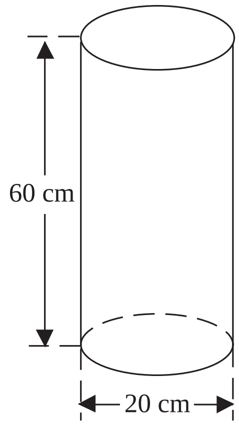
Solution

The load-carrying part of the lorry is represented by a cuboid, so its volume is given by

$$V = 2 \times 2.5 \times 4$$
$$= 20 \text{ m}^3$$

Worked Example 2

The cylindrical body of a fire extinguisher has the dimensions shown in the diagram. Find the maximum volume of water the extinguisher could hold.



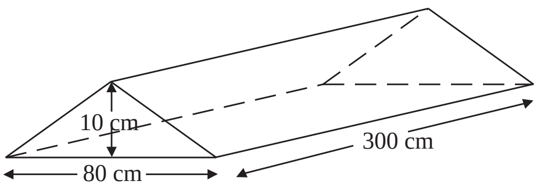
Solution

The body of the extinguisher is a cylinder with radius 10 cm and height 60 cm, so its volume is given by

$$V = \pi \times 10^2 \times 60$$
$$= 18850 \text{ cm}^3 \quad \text{(to the nearest cm}^3\text{)}$$

Worked Example 3

A 'sleeping policeman' (traffic calming road hump) is made of concrete and has the dimensions shown in the diagram. Find the volume of concrete needed to make one 'sleeping policeman'.



Solution

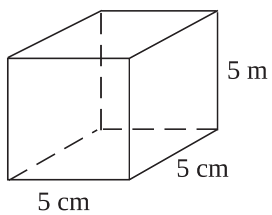
The shape is a triangular prism with $b = 80$, $h = 10$ and $l = 300$ cm. So its volume is given by

$$V = \frac{1}{2} \times 80 \times 10 \times 300$$
$$= 120\,000 \text{ cm}^3.$$

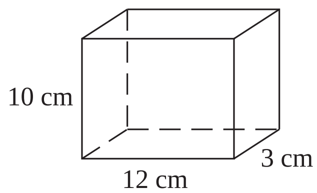
Exercises

1. Find the volume of each solid shown below.

(a)

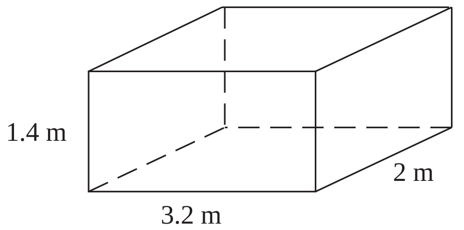


(b)

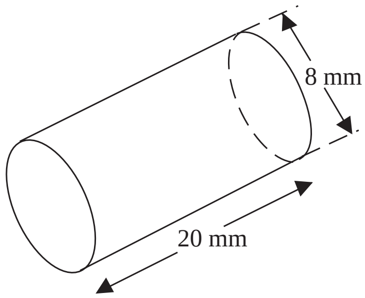


7.9

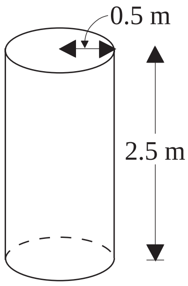
(c)



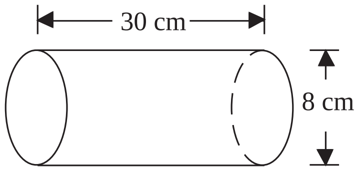
(d)



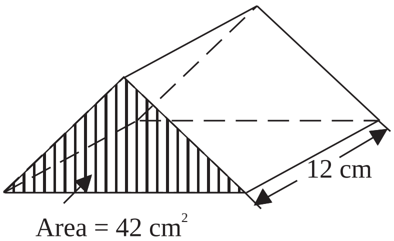
(e)



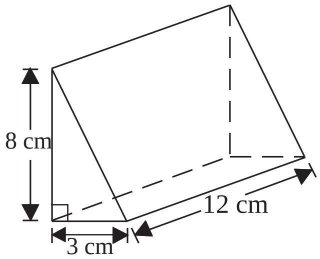
(f)



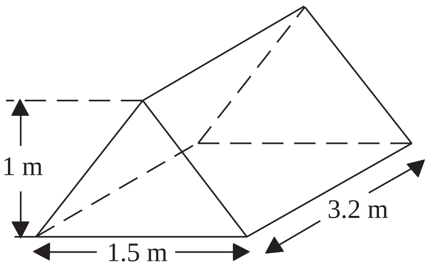
(g)



(h)

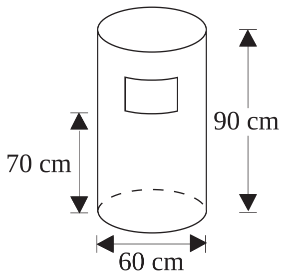


(i)



2. (a) Find the volume of the litter bin shown in the diagram, in m^3 to 2 decimal places.

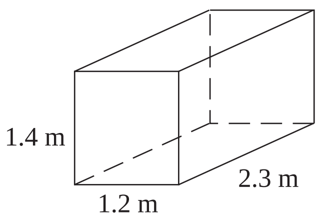
   (b) Find the volume of rubbish that can be put in the bin, if it must all be below the level of the hole in the side, in m^3 to 2 decimal places.



3. A water tank has the dimensions shown in the diagram.

   (a) Find the volume of the tank.

   (b) If the depth of water is 1.2 m, find the volume of the water.



7.9

4. A concrete pillar is a cylinder with a radius of 20 cm and a height of 2 m.

   (a) Find the volume of the pillar.

   The pillar is made of concrete, but contains 10 steel rods of length 1.8 m and diameter 1.2 cm.

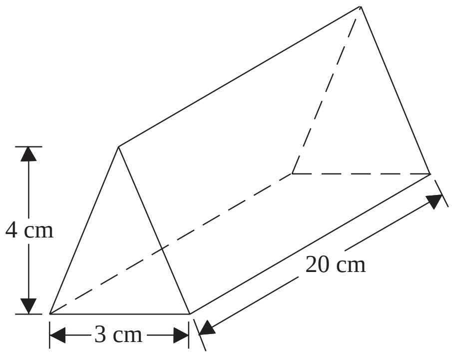
   (b) Find the volume of one of the rods and the volume of steel in the pillar.

   (c) Find the volume of concrete contained in the pillar.

5. The box shown in the diagram contains chocolate.

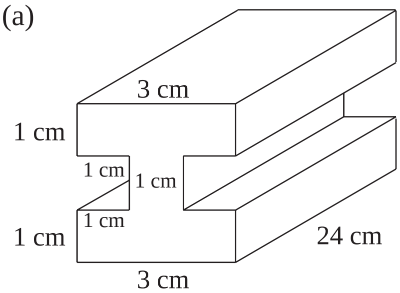
   (a) Find the volume of the box.

   (b) If the box contains 15 cm^3 of air, find the volume of the chocolate.

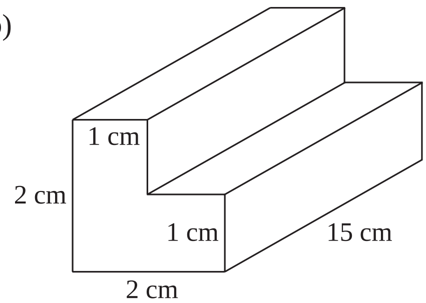


6. Find the volume of each prism below.

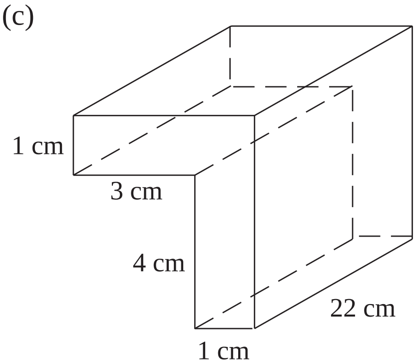
(a)



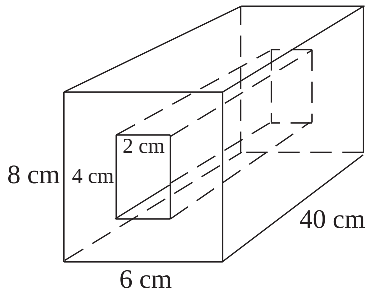
(b)



(c)

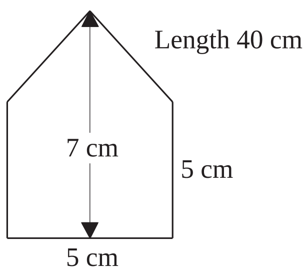


(d)

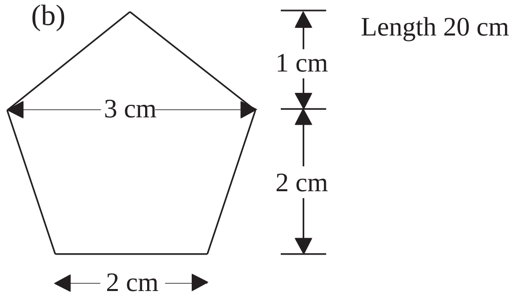


7. Each diagram below shows the cross section of a prism. Find the volume of the prism, given the length specified.

(a)

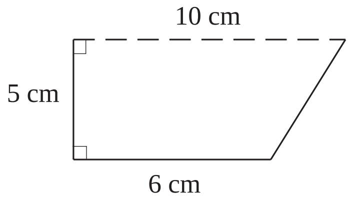


(b)

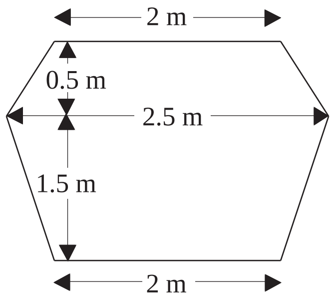


7.9

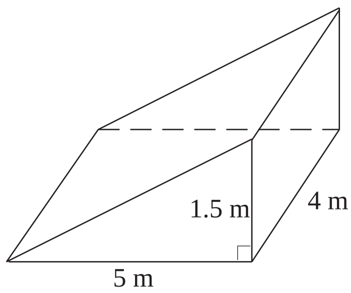
8. The diagram shows the cross section of a length of guttering. Find the maximum volume of water that a 5 m length of guttering could hold.



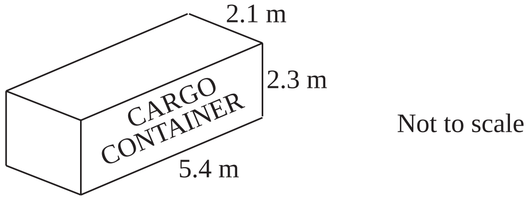
9. The diagram shows the cross section of a skip that is 15 m in length and is used to deliver sand to building sites. Find the volume of sand in the skip when it is filled level to the top.



10. A ramp is constructed out of concrete. Find the volume of concrete contained in the ramp.



11. The diagram shows a cargo container.



Calculate the volume of the container.

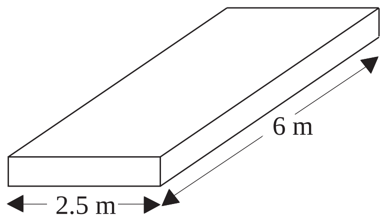
(SEG)

Just for Fun

A man wishes to take 4 litres of water out of a big tank of water, but he has only one 5-litre and one 3-litre jar. How can he do it?

7.9

12.



Not to scale

A garage has a rectangular concrete base 6 m long and 2.5 m wide.

The base is shown in the diagram.

(a) Calculate the area of the garage floor.

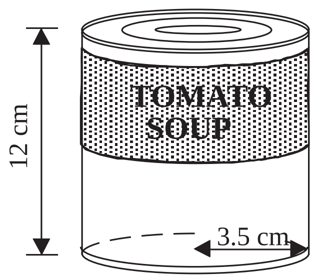
(b) The concrete is 0.2 m thick.
Calculate the volume of the concrete base.

(c) Write 0.2 m in millimetres.

(MEG)

13. Tomato soup is sold in cylindrical tins.

Each tin has a base radius of 3.5 cm and a height of 12 cm.

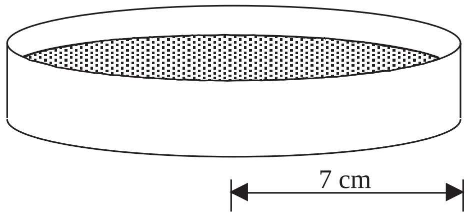


Not to scale

(a) Calculate the volume of soup in a full tin.

Take π to be 3.14 or use the π key on your calculator.

(b) Mark has a full tin of tomato soup for dinner. He pours the soup into a cylindrical bowl of radius 7 cm.



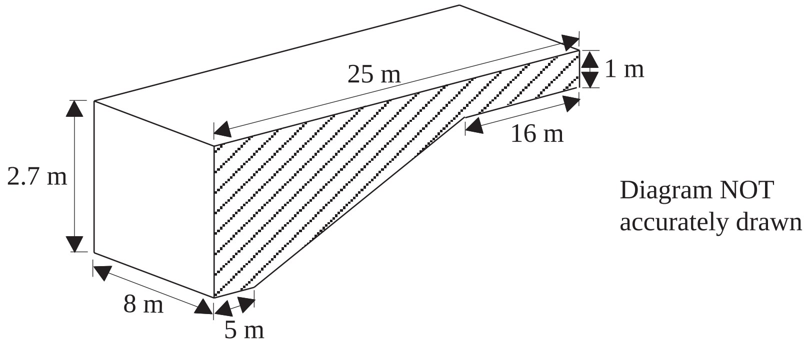
Not to scale

What is the depth of the soup in the bowl?

(SEG)

7.9

14.



The diagram represents a swimming pool.
The pool has vertical sides.
The pool is 8 m wide.

(a) Calculate the area of the shaded cross section.

The swimming pool is completely filled with water.

(b) Calculate the volume of water in the pool.

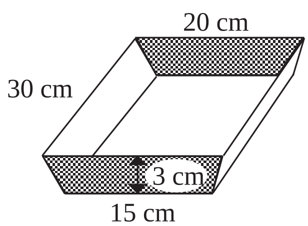
64 m^3 leaks out of the pool.

(c) Calculate the distance by which the water level falls.

(LON)

15. The diagram shows a paint trough in the shape of a prism.

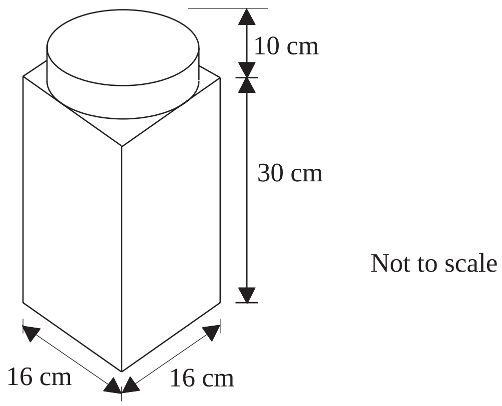
Each shaded end of the trough is a vertical trapezium.



Calculate the *volume* of paint which the trough can hold when it is full.

(SEG)

16. The diagram shows a lamp.



7.9

(a) The base of the lamp is a cuboid.

Calculate the volume of the base.

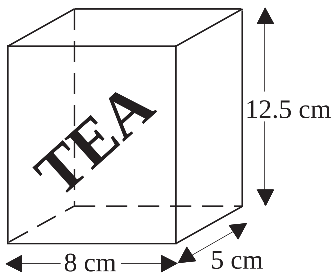
(b) The top of the lamp is a cylinder.

(i) Calculate the circumference of the cylinder.
Take π to be 3.14 or use the π key on your calculator.

(ii) Calculate the volume of the cylinder.

(SEG)

17. The diagram represents a tea packet in the shape of a cuboid.



(a) Calculate the volume of the packet.

There are 125 grams of tea in a full packet.

Jason has to design a new packet that will contain 100 grams of tea when it is full.

(b) (i) Work out the volume of the new packet.

(ii) Express the weight of the new tea packet as a percentage of the weight of the packet shown.

The new packet of tea is in the shape of a cuboid.
The base of the new packet measures 7 cm by 6 cm.

(c) (i) Work out the area of the base of the new packet.

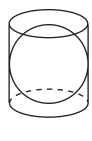
(ii) Calculate the height of the new packet.

(LON)

Information

Archimedes (287BC-212BC), a Greek Mathematician, was once entrusted with the task of finding out whether the King's crown was made of pure gold. While taking his bath, he came up with a solution and was so excited that he dashed out into the street naked shouting "Eureka" (I have found it). The container that you use in the Science laboratory to measure the volume of an irregular object is known as an Eureka can (named after this incident). Archimedes was so engrossed in his work that when his country was conquered by the Romans, he was still working hard at his mathematics. When a Roman soldier ordered him to leave his desk, Archimedes replied, "Don't disturb my circles." He was killed by that soldier for disobeying orders.

Archimedes' greatest contribution was the discovery that the volume of a sphere is $\frac{2}{3}$ *that of a cylinder whose diameter is the same as the diameter of the sphere. At his request, the sphere in the cylinder diagram was engraved on his tombstone.*



Answers to Exercises

7.1 Units and Measuring

1. All ± mm (a) 125 mm (b) 24 mm (c) 70 mm (d) 107 mm
   (e) 7 mm

2. (a) km or miles (b) cm (c) mg or grams (d) kg (e) ml
   (f) m^3 or litres

3. (a) 12 300 g (b) 4 700 mm (c) 164 mm (d) 3 400 m
   (e) 370 cm (f) 6 000 ml

4.

Length in m	*Length in cm*	*Length in mm*
4	400	4 000
3.11	311	3 110
1.5	150	1 500
3.74	374	3 740
8.62	862	8 620

5. (a) 15 (b) 10.3 (c) 130 (d) 45 (e) 56 (f) 18.2
   (g) 6.6 (h) 3.4 (i) 11.2 (j) 2.6 (k) 36 (l) 84
   (m) 220

6. (a) 850 ml (b) 5

7. (a) cm (b) m (c) m (d) cm/mm
   (e) m (f) cm/mm (g) mm

8. 42 mm , 56 mm , 21 mm

9. (a) 4 cm (b) 7 cm (c) 11 cm (d) 0 cm (e) 1 cm (f) 1 cm

10. (a) m (b) 10 m

7.2 Estimating Areas

1. (a) 6 cm^2 (b) 10 cm^2 (c) 10 cm^2 (d) 14 cm^2
   (e) 14 cm^2 (f) 7 cm^2

2. (a) 16 cm^2 (b) 14 cm^2 (c) 8 cm^2 (d) 12 cm^2
   (e) 9 cm^2 (f) 12 cm^2

3. (a) 8 cm^2 (b) 9 cm^2 (c) between 10 cm^2 and 12 cm^2 (11 cm^2)
   (d) between 8 cm^2 and 10 cm^2 (9 cm^2)
   (e) between 6 cm^2 and 8 cm^2 (7 cm^2)
   (f) betweeen 16 cm^2 and 18 cm^2 (17 cm^2)
   (g) between 10 cm^2 and 12 cm^2 (11 cm^2)
   (h) between 9 cm^2 and 10 cm^2
   (i) between 9 cm^2 and 11 cm^2 (10 cm^2)

Answers

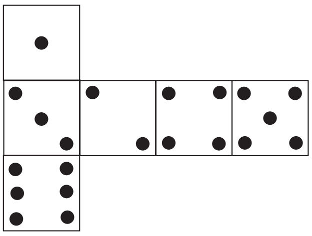
7.2

4. The area of each island is between 25 km^2 and 27 km^2
   (Remark: The left island is slightly bigger than the right one)

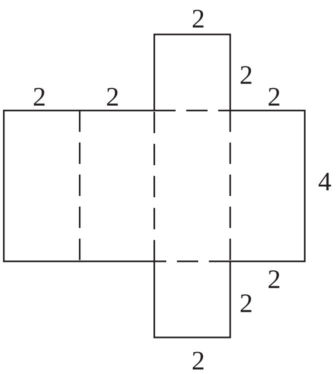
5. 47 cm^2

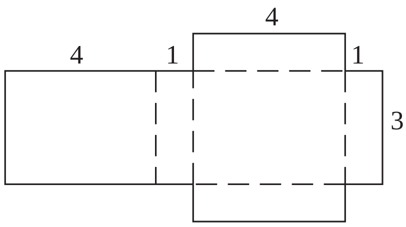
7.3 Making Solids Using Nets

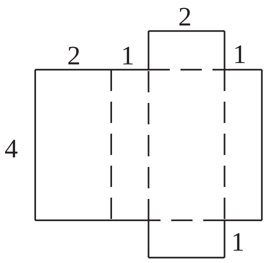
2. (a) square-based pyramid (b) cuboid (c) tetrahedron (d) hexahedron
   (e) hexagonal prism (f) octahedron

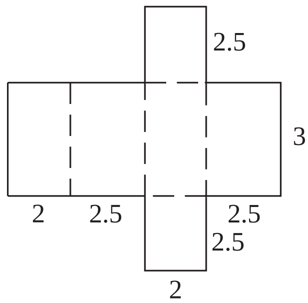
3. 

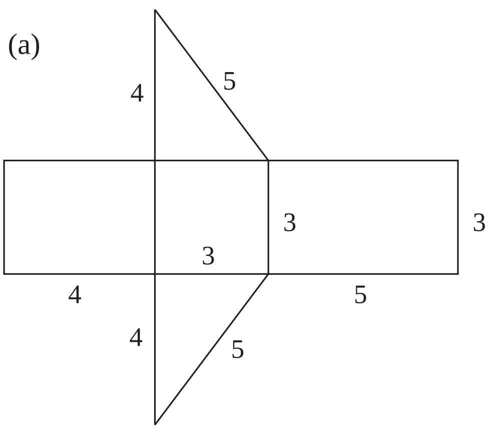
7.4 Constructing Nets

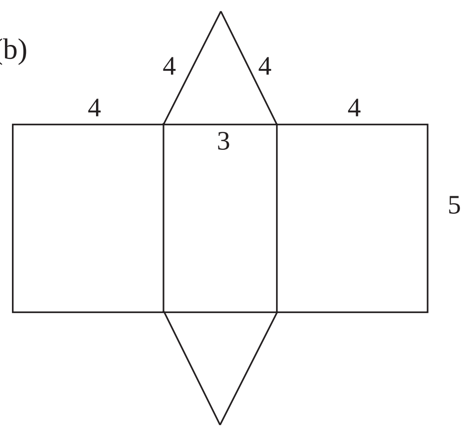
1. (a) 

(b) 

(c) 

(d) 

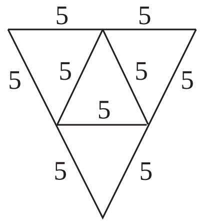
2. (a) 

(b) 

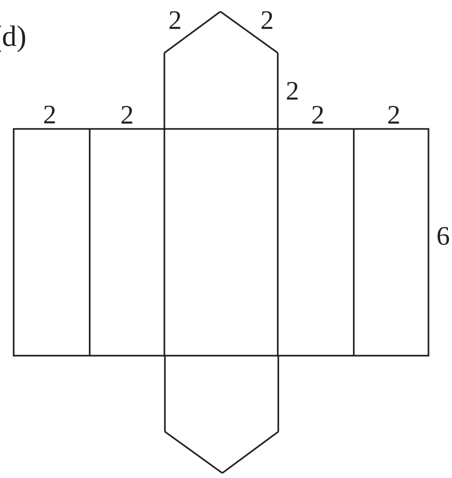
Answers

7.4

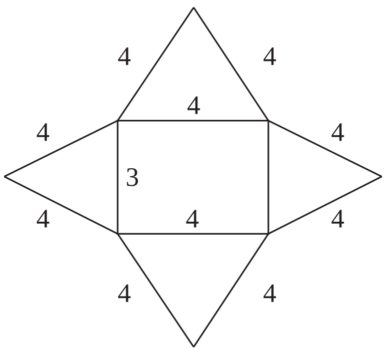
(c)



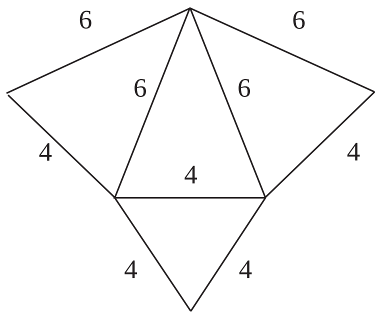
(d)

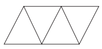
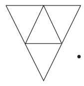


(e)

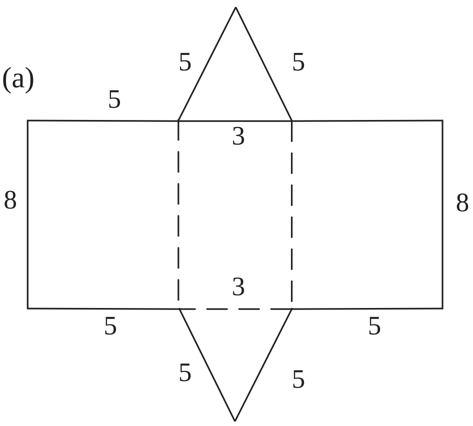


(f)

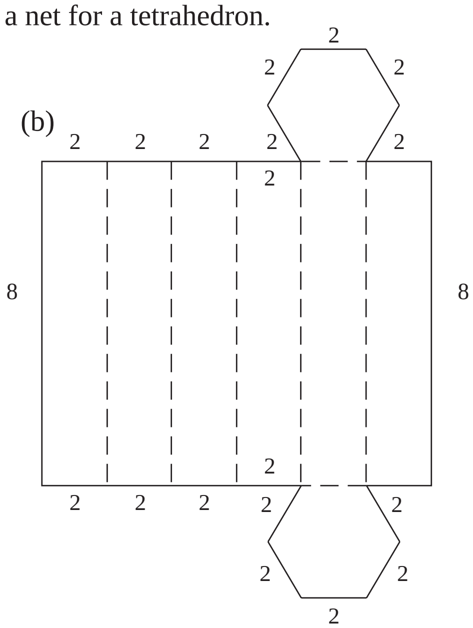


3. (b) Three ways; as in question and  and  .
Only the latter two configurations form a net for a tetrahedron.

4. (a)

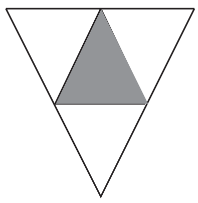


(b)



5. S

6. (a) Equilateral triangle (b) 60° (c)



Answers

7.5 Conversion of Units

1. (a) 7.5 cm (b) 252 lbs (c) 96 ounces (d) 75 inches (e) 33 lbs
   (f) 108 inches (g) 90 cm (h) 22.5 litres (i) 300 cm (j) 99 lbs
   (k) 15.75 pints (l) 202.5 litres (m) 14 pints (n) 48 pints

2. (a) 3.6 kg (b) 1.4 kg (c) 9.1 litres (d) 4.0 inches (e) 13.3 feet
   (f) 5.0 lbs (g) 13.0 stones (h) 11.1 gallons (i) 7.0 feet (j) 20.8 inches
   (k) 3.6 gallons (l) 1.7 litres (m) 2.7 kg (n) 7.1 feet

3.

	Norwich	Great Yarmouth	Lowestoft
Great Yarmouth	30		
Lowestoft	43	18	
Beccles	29	32	14

4. (a) 65.6 km (b) 6.56 litres (c) 1.46 gallons

5. 0.43 litres of orange juice 1.36 kg of flour
   0.23 kg of butter 0.91 kg of mixed fruit

6. Total length = 354 inches , 885 cm, 8.85 m

7. James is both taller and heavier than Michael (James is 185 cm tall and weighs over 70 kg).

8. Jane picked the greater weight; she picked about 17.6 lbs.

9. The area of the sheet of glass is 2250 cm^2. Its dimensions are 37.5 cm by 60 cm.

10. The first car consumes 0.1125 litres per km, and the second car consumes 0.12 litres per km. Hence, the first car is the more economical.

11. (a) 48 km (b) $K = \dfrac{M \times 8}{5}$ (c) = 62.5

12. (a) 48 000 g (b) 171 g (to the nearest gram) (c) 105.6 lbs

13. (a) cm (b) litres

14. (a) 160 cm (to the nearest cm) (b) 63.5 kg

15. (a) 0.2 kg (b) kg/cm^3

16. Yes he will. He needs less than 1 lb of flour for this recipe, and he has more than 1 lb of flour (500 g = $\dfrac{1}{2}$ kg $\cong$ 1.1 lbs).

Answers

7.6 Squares, Rectangles and Triangles

1. (a) 28 cm^2 (b) 12.4 cm^2 (c) 12 cm^2 (d) 9.46 cm^2
   (e) 4.68 cm^2 (f) 14.4 cm^2

2. (a) 14.4 cm, 12.96 cm^2 (b) 22.8 cm, 31.49 cm^2
   (c) 38 cm, 60 cm^2 (d) 44 cm, 80 cm^2 (e) 32 cm, 36 cm^2
   (f) 28 cm, 28 cm^2

3. (a) 100 cm^2 (b) 60.5 cm^2 (c) 63 cm^2 (d) 24 cm^2

4. (a) 42 000 cm^2 (b) 41 blocks

5. $57\frac{1}{2}$ feet

6. 8 m^2

7. 3.99 m^2 + 1.5 m = 5.49 m^2

8. 40 cm^2

9. 1800 cm^2

10. (a) (i) 3.9 m^2 (ii) 5.2 m^2 (b) 4.32 m^2

11. (a) 38 cm (b) 40 cm^2

12. (a) 24 cm (b) 18 cm^2

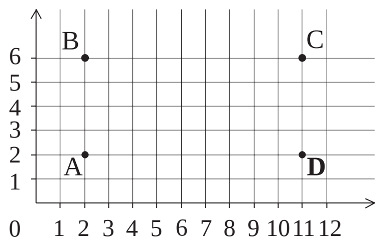
13. (a) 18 cm^2 (b) 30 cm^2

14. 22 cm^2

15. AB = 7 cm. Perpendicular height from C to AB = 2.4 cm ; area $\approx$ 8.4 cm^2.

16. (a) (6.5, 4)

    (b)



D is (11, 2)

    (c) (i) 26 cm (ii) 36 cm^2 (d) Rotational symmetry

Answers

7.7 Area and Circumference of Circles

1. (a) C = 31.4 cm (to 1 d.p.)
   A = 78.5 cm^2 (to 1 d.p.)

   (b) C = 1.26 m (to 2 d.p.)
   A = 0.13 m^2 (to 2 d.p.)

   (c) C = 3.77 m (to 2 d.p.)
   A = 1.13 m^2 (to 2 d.p.)

   (d) C = 75.40 cm (to 1 d.p.)
   A = 452.39 cm^2

   (e) C = 8.80 m (to 2 d.p.)
   A = 6.16 m^2 (to 2 d.p.)

   (f) C = 62.83 m (to 2 d.p.)
   A = 314.16 m^2 (to 2 d.p.)

2. (a) $r = 6.7$ cm (to 1 d.p.) (b) $r = 2.9$ cm (to 1 d.p.)
   (c) $r = 4.7$ cm (to 1 d.p.) (d) $r = 5.4$ cm (to 1 d.p.)

3. (a) 357 m (to the nearest m)
   (b) $\left(\frac{50}{2}\right)^2 \pi + 50 \times 100 = 6963 \text{ m}^2$ (to the nearest m^2)

4. $(1.8)^2 \pi - (0.5)^2 \pi \approx 9.39 \text{ cm}^2$

5. (a) 12.57 cm^2 (to 2 d.p.) (b) 78.54 cm^2 (to 2 d.p.)
   (c) 66 cm^2 (to the nearest cm^2)

6. (a) 28.27 cm^2 (b) 37.70 cm^2

7. $10 \times 7 - \left(\frac{5}{2}\right)^2 \pi \approx 50.37$ cm

8. $16 \times 8 - 2 \times 4^2 \times \pi \approx 27.47 \text{ cm}^2$

9. (a) 61.91 cm^2 (b) $r^2\pi - (4.2)^2 = 50 \text{ cm}^2 \Rightarrow r \approx 4.64$ cm

10. (a) 5.78 m^2 (b) 1.6 m, 6.55 m^2

11. 201 cm^2 (to the nearest cm^2)

12. 707 cm^2 (to the nearest cm^2)

13. (a) C = 50π = 157 cm (to the nearest cm)
    (b) 31848 revs (C = 157 cm) or 31831 revs (C = 50π)

14. 16 times

15. (a) 3.82 m (b) 3.82 m is slightly less than 4 m and $\pi = 3.14$ is slightly more than 3 m, hence their product (which gives the length of the rope) is about 12 m.
    (c) 4.52 cm^2

16. (a) 400 m (to the nearest m) (b) 10148.5 m^2 (to 1 d.p.)

Answers

7.8 Areas of Parallelograms, Trapeziums, Kites and Rhombuses

1. (a) $12\ m^2$ (b) $5.5\ m^2$ (c) $80\ cm^2$ (d) $75\ cm^2$
   (e) $72\ cm^2$ (f) $110\ m^2$ (g) $37.5\ cm^2$ (h) $7.2\ m^2$
   (i) $10.2\ cm^2$

2. (a) $3.5\ m^2$ (b) $45\ m^2$ (c) $112\ m^2$

3. (a) $13.5\ m^2$ (b) $6\ m^2$ (c) $32\ m^2$ (d) $75\ m^2$

4. $18\ m^2$

5. $2500\ cm^2$

6. (a) $4816\ cm^2$ (b) $4816\ cm^2$
   (c) The area of the wasted plastic would be equal to the area of the kite (each of them would be $5600\ cm^2$, which is half the area of the rectangular sheet used to make the kite).

7. (a) $64\ cm^2$ (b) $58\ cm^2$ (c) $94.25\ cm^2$ (d) $44\ cm^2$
   (e) $32\ cm^2$ (f) $44\ cm^2$

8. $132\ cm^2$, $144\ cm^2$. Total area: $552\ cm^2$

9. (a) (i) $36\ cm^2$ (ii) $72\ cm^2$ (iii) $90\ cm^2$
   (b) No, since h is a side of a right-angled triangle whose hypotenuse is 6 cm long .
   (c) The maximum area is reached when $h = 6$ cm, and is equal to $108\ cm^2$.
   (The maximum area is reached when the parallelogram is actually a rectangle.)

10. (a) $28\ cm^2$ (b) $14\ cm^2$ (half the area of the parallelogram)

11. The kite ABCD is made of the two congruent triangles ABD and CBD. Hence, its area covers twice the area of each of these triangles.

7.9 Volumes of Cubes, Cuboids, Cylinders and Prisms

1. (a) $125\ cm^3$ (b) $360\ cm^3$ (c) $8.96\ m^3$
   (d) $1005\ mm^3$ (to the nearest mm^3) (e) $2\ m^3$ (to the nearest m^3)
   (f) $1508\ cm^3$ (to the nearest cm^3) (g) $504\ cm^3$
   (h) $144\ cm^3$ (i) $2.4\ m^3$

2. (a) $0.25\ m^3$ (b) $0.20\ m^3$ (to the nearest m^3)

3. (a) $3.864\ m^3$ (b) $3.312\ m^3$

4. (a) $251327\ cm^3$ (b) $203.6\ cm^3$, $2036\ cm^3$
   (c) $249291\ cm^3$ (to the nearest cm^3)

Answers

7.9

5. (a) 120 cm^3 (b) 105 cm^3

6. (a) 168 cm^3 (b) 45 cm^3 (c) 176 cm^3 (d) 1600 cm^3

7. (a) 1200 cm^3 (b) 130 cm^3

8. 20 000 cm^3

9. 67.5 m^3

10. 15 m^3

11. 26 m^3 (to the nearest m^3)

12. (a) 15 m^2 (b) 3 m^3 (c) 200 mm

13. (a) 462 cm^3 (to the nearest cm^3) (b) 3 cm

14. (a) 36 .9 m^2 (b) 295.2 m^3 (c) 0.32 m

15. 1575 cm^3

16. (a) 7680 cm^3
    (b) (i) 50.3 cm (ii) 2011 cm^3 (to the nearest cm^3)

17. (a) 500 cm^3 (b) (i) 400 cm^3 (ii) 80%
    (c) (i) 42 cm^2 (ii) 9.5 cm (to the nearest mm)